Field interactions from broken diffeomorphisms

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Motivation

- Observational data → **accelerated expansion** of the Universe → **dark energy** domination
- Dark energy models: cosmological constant (**ΛCDM**), *quintessence*, *k-essence*, *phantom dark energy*…
- Matter sector → mostly in the form of **dark matter.**

Unknown nature of the **dark sector** → study **modifications of gravity at cosmological scales.**

Motivation

- **Unimodular gravity (UG)**

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break invariance under diffeomorphisms

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- **Transverse diffeomorphisms (TDiff)**
- Weyl rescalings
- Solution to the vacuum-energy problem

A. Einstein, Siz. Preuss. Acad. Scis. (1919).
W. G. Unruh, Phys. Rev. D 40 (1989) 1048.

- Study of **TDiff** invariant theories
- Modify the **geometry sector** or the **matter sector**.

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Interesting phenomenology in **cosmological contexts**

A. L. Maroto (2023), JCAP 04, 037, arXiv:2301.05713 [gr-qc].

Objectives and methodology

- Study shift-symmetric **TDiff invariant theories** in the matter sector, consisting of multiple kinetically driven scalar fields, applications to **cosmology.**

- Application to the study of the **dark sector** and comparison with **observational data** from type Ia supernovae.

Single-field TDiff theories

- Variation of the action under **coordinate transformations**

$$x^\mu \to x^\mu + \xi^\mu(x) \quad ; \quad S = \int d^4x f(g)\mathcal{L}$$

$$\delta_\xi S = \int d^4x \partial_\mu \xi^\mu \big(f(g) - 2gf'(g)\big)\mathcal{L}$$

TDiff: $\partial_\mu \xi^\mu = 0$

- Single-field **TDiff theories**

$$S = \int d^4x \left(\frac{1}{2} f_K(g)\partial_\mu\phi\partial^\mu\phi - f_V(g)V(\phi)\right)$$

- Flat **Robertson-Walker** spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = b(\tau)^2 d\tau^2 - a(\tau)^2 \mathbf{dx^2}$$

Single-field TDiff theories

- **Energy-momentum tensor:** perfect fluid approach $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$ (when $\partial_\mu \phi$ is a timelike vector) (*)

- Homogeneous case $\phi = \phi(\tau)$: energy density and pressure

$$\rho = \frac{f_K(g)}{b^2\sqrt{g}}\left(1 - F_K(g)\right)(\phi')^2 + \frac{2 f_V(g)}{\sqrt{g}} F_V(g) V(\phi)$$

$$p = \frac{f_K(g)}{b^2\sqrt{g}} F_K(g)(\phi')^2 - \frac{2 f_V(g)}{\sqrt{g}} F_V(g) V(\phi)$$

D. Jaramillo-Garrido, A. L. Maroto, and P. Martín-Moruno, JHEP 03, 084, arXiv:2307.14861 [gr-qc].

Single-field TDiff theories

- **Einstein Equations** ($G_{\mu\nu} = 8\pi G T_{\mu\nu}$)

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho b^2$$

- **Field equations**

$$\phi'(\tau) = C_\phi \frac{b^2(\tau)}{f_{K(g)}}$$

- **Conservation of** the energy-momentum tensor $\nabla_\nu T^{\mu\nu} = 0$ (preserved through Bianchi identities)

$$\rho' + 3\frac{a'}{a}(\rho + p) = 0$$

Gives rise to geometrical constrains on g which allow to obtain b

- **Model:** kinetically driven fields and power-law couplings

Kinetic regime: constrain $g^{1-\alpha}|2\alpha - 1| = C_g a^6$

Equation of state parameter: $w = \frac{p}{\rho} = \frac{\alpha}{1-\alpha}$

Multi-field TDiff theories. Presentation of the model

- **Particular model:** two kinetically driven fields with power-law couplings

$$S = \sum_i \int d^4x f_i(g)\left(\frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi^i\right) \quad ; f_{K_1}(g) = k g^{\alpha} \;\; ; f_{K_2}(g) = \lambda g^{\beta}$$

- **Geometrical constrain:**

$$C_1 g^{1-\alpha}|2\alpha - 1| + C_2 g^{1-\beta}|2\beta - 1| = C_g a^6$$

→ **effective interactions** between the fields:

$$\rho_1' + 3\frac{a'}{a}(\rho_1 + p_1) = Q = -\rho_2' - 3\frac{a'}{a}(\rho_2 + p_2)$$

- **Energy densities:**

$$\rho_i = C_i(1 - \alpha_i)\frac{b^{1-2\alpha_i}}{a^{6\alpha_i+3}}$$

- No interacting terms in the action, the equation of state parameters are **constant** (w_i)

$$w_1 = \frac{\alpha}{1-\alpha} \quad ; \quad w_2 = \frac{\beta}{1-\beta}$$

Multi-field TDiff theories. Domination regimes

- **Domination regimes:** assuming one field (ϕ_1) dominates, it will approximately satisfy its individual conservation law, and $b \propto a^{3w_1}$:

$$\rho_1(a) \propto a^{-3(1+w_1)} \quad ; \quad \rho_2(a) \propto a^{-3\left(1+w_{eff}\right)}$$

- The subdominant component sees its **decay behavior** altered, now parameterized by

$$\boxed{w_{eff} = \frac{2w_2 - w_1 + w_1 w_2}{1 + w_2}}$$

Multi-field TDiff theories. Domination regimes

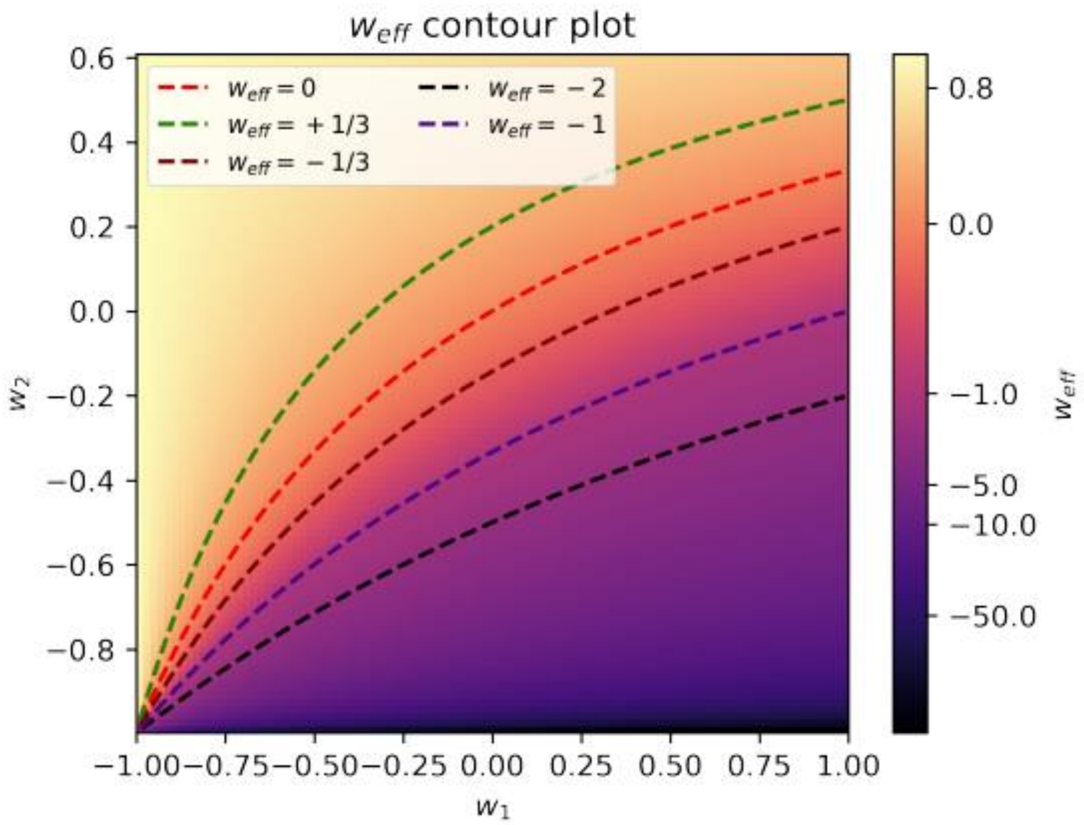

Fig.1. Effective equation of state parameter for the subdominant field in terms of w_1, w_2.

- **Wide phenomenology.**
- Possible to have a **cosmological constant** in subdominant regimes.
- Possibility of ***phantom-crossing***.

Multi-field TDiff theories. Analytical case

- **Analytical case:** $\alpha = 0,\ \beta = -\frac{1}{2}\ (w_2 = -\frac{1}{3})$. Constrain with analytical solution:

$$C_1 g + 3C_2 g^2 = C_g a^6$$

- Allows us to obtain $b(a)$ and the energy densities analytically:

$a \gg 1$ limit:

$$\rho_1(a) \propto \left(\sqrt{A}a^{-\frac{9}{2}} - \frac{1}{\sqrt{2A}} a^{-\frac{15}{2}}\right)$$

$$\rho_2(a) \propto \left(-3\sqrt{A}a^{-\frac{9}{2}} + 2a^{-\frac{3}{2}}A^{\frac{3}{2}}\right)$$

$$A \equiv \sqrt{12C_2C_g/C_1^2}$$

$a \ll 1$ limit:

$$\rho_1(a) \propto \left(a^{-3} - \frac{3C_2C_g}{2C_1^2}a^3\right)$$

$$\rho_2(a) \propto a^3$$

- Dark matter dominates at **shorter times** and dark energy at **later times**.

Multi-field TDiff theories. Analytical model

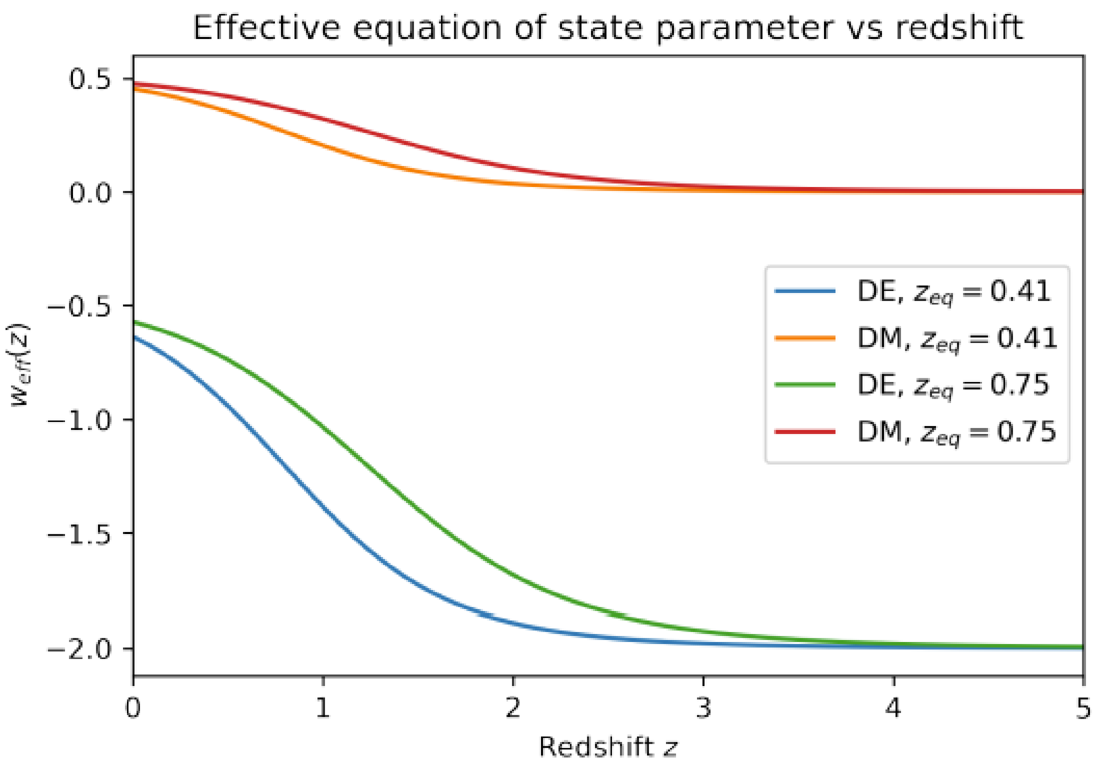

Fig.2. Effective equation of state parameter for both components in terms of the redshift.

- **Parameter**: redshift at the moment of equality.
- We can define an **effective equation of state parameter** in terms of the redshift,

$$\rho_i' + 3\frac{a'}{a}\left(1 + w_{eff,i}(z)\right)\rho_i = 0$$

$$w_{eff,i}(z) = -\frac{1}{3}\left(-\frac{1+z}{b(z)}(1-2\alpha_i)\frac{db}{dz} - 6\alpha_i - 3\right) - 1$$

A TDiff model for dark sector interactions

- We study **general dark sector models** with two interacting fields: dark matter ($\alpha = 0$) and dark energy ($\beta < -1/2$). Two free parameters (fixing the *gauge condition* $a(t_0) = b(t_0) = 1$):

- **Coupling exponent** of dark energy (β).
- **Ratio** between the abundances today (λ)

$$\lambda = \frac{\Omega_{DMT}}{\Omega_{DET}}$$

$$\frac{C_1}{C_2} g^{1-\alpha}|2\alpha - 1| + g^{1-\beta}|2\beta - 1| = \frac{C_g}{C_2} a^6$$

- Start from **Friedmann equation** with baryons and TDiff dark sector:

$$H^2(z) = H_0^2 \left[\Omega_B(1+z)^3 + (1-\Omega_B)\left(1+\frac{1}{\lambda}\right)^{-1} b(z)(1+z)^3 + \frac{1}{\lambda}(1-\Omega_B)\left(1+\frac{1}{\lambda}\right)^{-1} b(z)^{1-2\beta}(1+z)^{6\beta+3}\right]$$

A TDiff model for dark sector interactions

- Dark matter dominates at shorter times ➡ we can define an **effective matter density parameter** at high redshift, which is constant:

$$\Omega_M^{eff} \equiv \Omega_B + (1-\Omega_B)\left(1+\frac{1}{\lambda}\right)^{-1} b_{early}$$

- b_{early} is the value of b at high redshift, which is a **constant** under matter domination.

$$b_{early} = \sqrt{\frac{\lambda(1-\beta)+|2\beta-1|}{\lambda(1-\beta)}}$$

A TDiff model for dark sector interactions

- We can fit both parameters (w_2, Ω_M^{eff}) to observational data. Using the **Union2** data from type Ia supernovae and do the analysis using the **distance moduli.**

$$\mu^{th}(z) = 5\log_{10}\big(H_0 d_L(z)\big) + M \equiv \hat{\mu}(z) + M$$

- The **best fit** will be that minimizing the value of the χ^2 estimator:

$$\chi^2 = \sum_i \frac{\left(\mu^{obs}(z_i) - \mu^{th}(z_i)\right)^2}{E_i^2}$$

- The absolute magnitude M will be marginalized:

$$M = \sum_i \left(\frac{1}{\sigma}\frac{\left(\hat{\mu}(z_i) - \mu^{obs}(z_i)\right)}{E_i^2}\right) \; ; \sigma = \sum_i E_i^{-2}$$

A TDiff model for dark sector interactions

Results and comparison with wCDM

- **Two parameter** fit (w_2, Ω_M^{eff}):

	Best fit	$\chi^2_{\rm min}$
TDiff	$w_2 = -0.813^{+0.102}_{-0.060}$, $\Omega_M^{\rm eff} = 0.387^{+0.056}_{-0.078}$	542.16
wCDM	$w = -1.063^{+0.225}_{-0.183}$, $\Omega_M = 0.294^{+0.077}_{-0.069}$	542.64

Best fit: two-parameter case

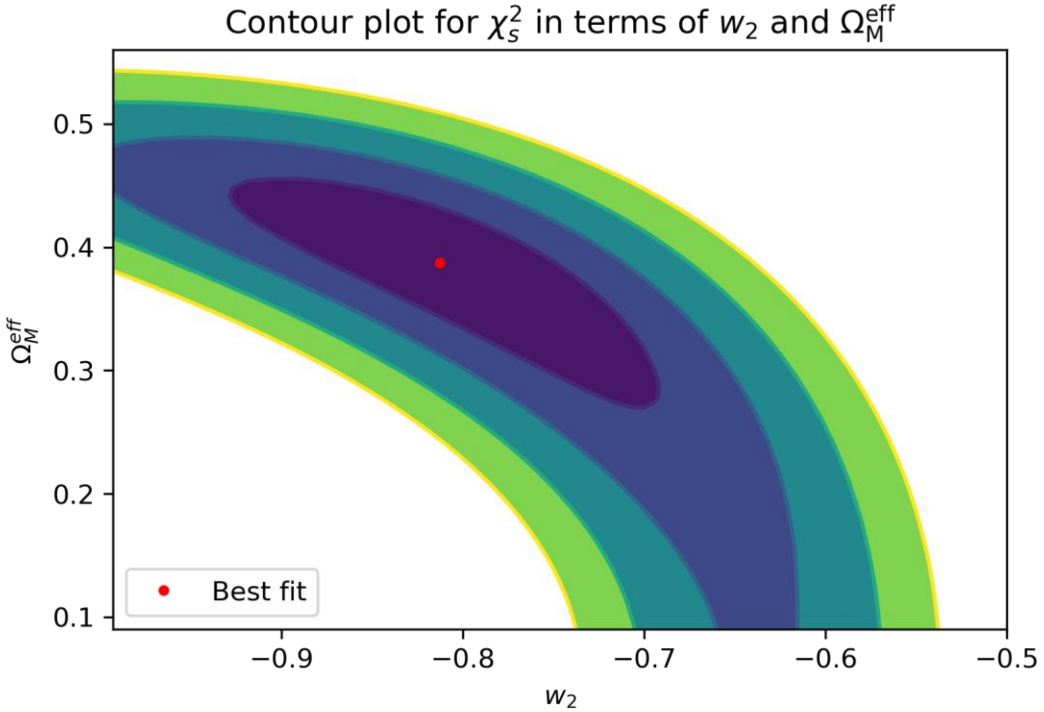

Fig.4. Contour regions up to the four sigma region for both parameters.

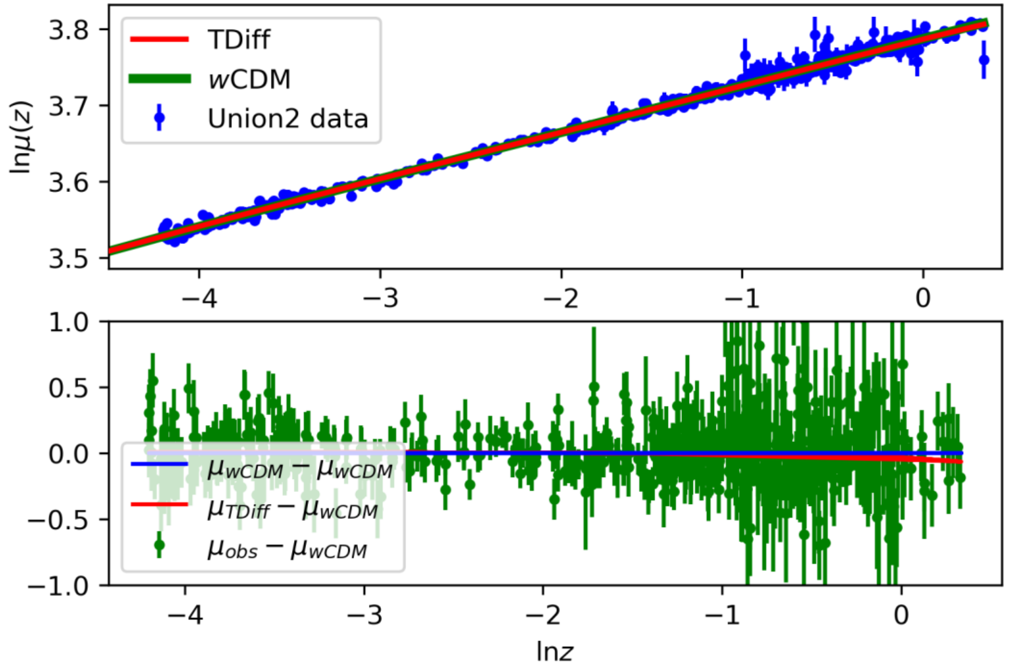

Fig.5. Comparison between the best fit, observations and wCDM.

A TDiff model for dark sector interactions

Best fit: two parameter case

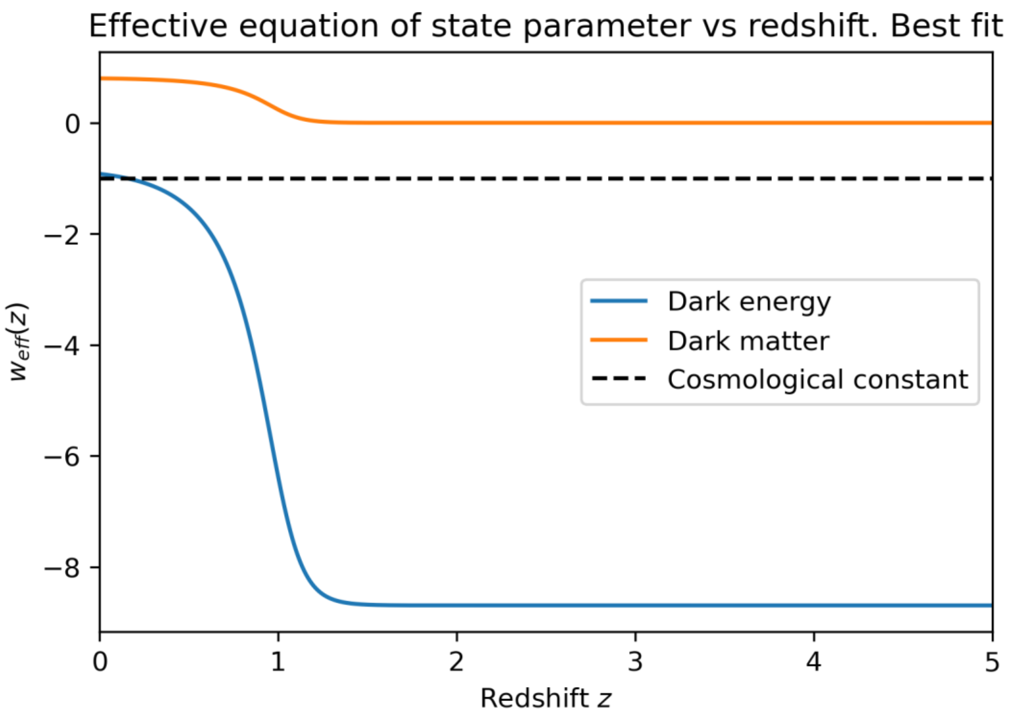

Fig.6. Effective equation of state parameters for each component in terms of the redshift. Best fit.

- Results are **compatible** in the one sigma region with the CMB measurement for Ω_M.
- TDiff and wCDM are in accordance, they start to deviate at higher redshift.
- Dark energy exhibits a **dynamical behavior:** phantom in the past, *quintessence* in the future and approximately constant at recent times. There is **phantom-crossing**.
- Dark matter starts to decay **faster** tan expected from its equation of state as dark energy starts to dominate.

Conclusions and future work

- **Breaking** the invariance under diffeomorphisms in a theory with multiple scalar fields leads to **interactions** of geometrical nature between both components.

- In cosmological contexts, matter fields exhibit **dynamical decay behaviors** that vary with the expansion of the universe, even if the individual equation of state parameters are constant.

- It is possible to have dark energy interpolating from ***phantom*** (in the subdominant regime) and ***quintessence*** (in the future), without introducing non canonical terms or ghosts in the action.

- The model shows **good accordance** with observations and it is **compatible** with the CMB measurement in the one sigma region.

Conclusions and future work

- **Future work**:
  - Stability of the theory under **perturbations** and **structure formation**.
  - Break the symmetry in the **geometric sector** too.
  - Study **more general couplings** and non-homogeneous fields.
  - Consider models in the **potential or mixed regimes**.
  - Extend to more **data sets** (Pantheon, CMB, BAO...)