TÉCNICO LISBOA

MEFT
Boosting the Future

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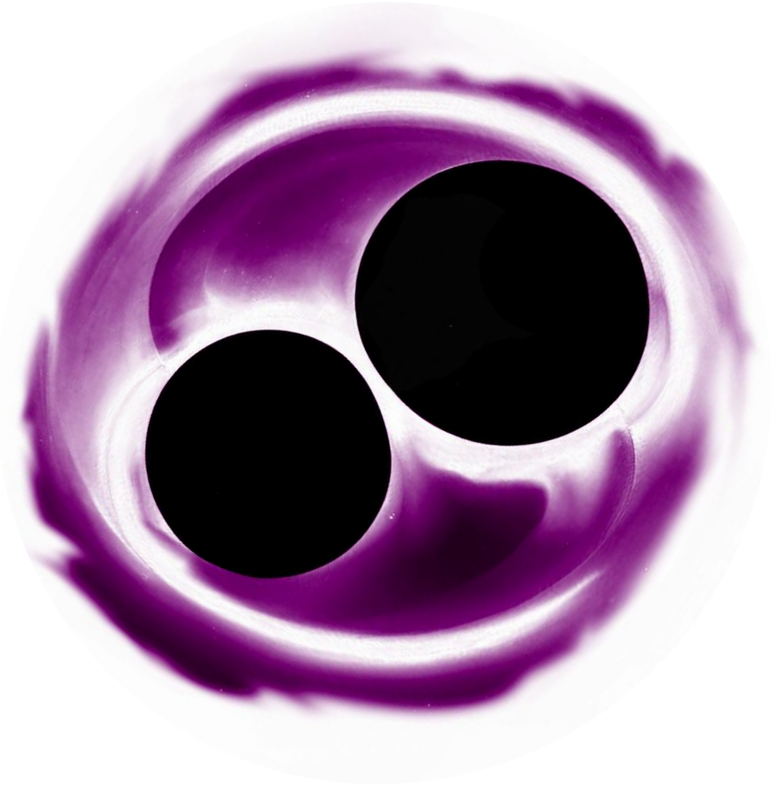

Improved Binary Black Hole Initial Data

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David Hilditch

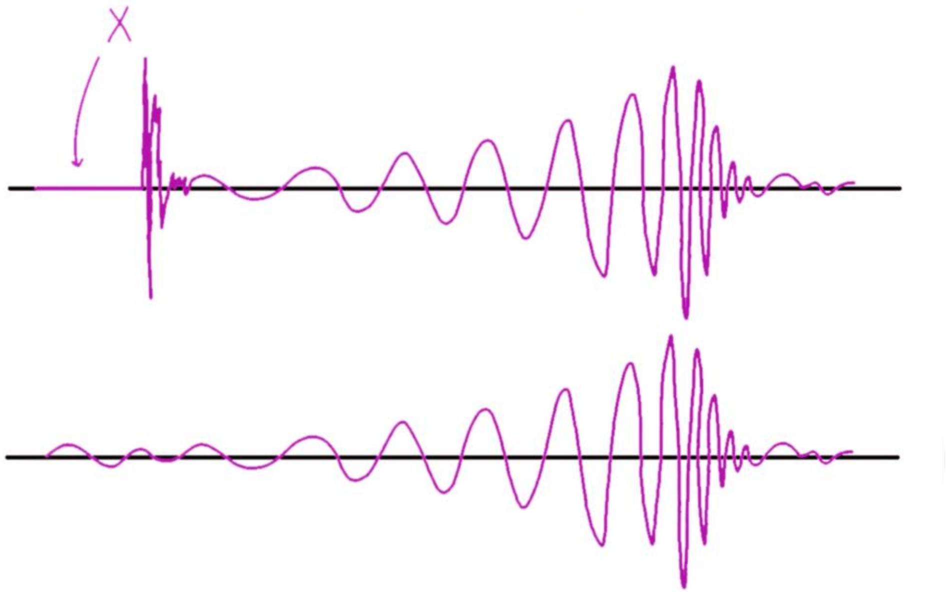
Spanish and Portuguese Relativity Meeting (EREP 2024)

Motivation: Gravitational Wave Detection

- November 2021: GWTC-3, catalogue of GW candidates up to 90
- Identification techniques: Matched Filtering Method
- Construction of template banks with numerical simulations

- Problem: Unrealistic initial data

- Solution: Radiative content in initial data



Numerical Relativity

- Einstein Field Equations $$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$
- 3 + 1 Decomposition
- 12 evolution + 4 constraint Equations

- Extended Conformal Thin-Sandwich (XCTS)

Input: $\overline{\gamma}_{ij} \quad \overline{u}_{ij} \quad K \quad \partial_t K$

Solved for: $\beta^i \quad \psi \quad \alpha\psi$

Output: $\gamma_{ij} = \psi^4 \overline{\gamma}_{ij} \qquad K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij} K$

Post-Newtonian Formalism for BBH

$$\epsilon = 1/c$$

- Post-Newtonian Expansion of the spatial metric

$$\gamma_{ij}^{PN} = \psi_{PN}^4 \delta_{ij} + h_{ij}^{TT} \qquad \psi_{PN} = 1 + \sum_{a=1}^{2} \frac{E_a}{2r_a} + O(\epsilon^6) \qquad E_a = (\epsilon^2) m_a + (\epsilon^4)\left(\frac{p_a^2}{2m_a} - \frac{m_1 m_2}{2r_{12}}\right)$$

$$h_{ij}^{TT} = h_{ij}^{TT\ (NZ)} + h_{ij}^{TT\ (remainder)} + O(\epsilon^5) \qquad H_{ij}^{TT\ a}[\vec{u}] = H_{ij}^{TT\ a}[\vec{u};t] + H_{ij}^{TT\ a}[\vec{u};t_a^r] + H_{ij}^{TT\ a}[\vec{u};t_a^r \to t]$$

$$(\epsilon^4) h_{(4)ij}^{TT} + (\epsilon^5) h_{(5)ij}^{TT} \qquad H_{ij}^{TT\ 1}\left[\frac{\vec{p_1}}{\sqrt{m_1}}\right] + H_{ij}^{TT\ 2}\left[\frac{\vec{p_2}}{\sqrt{m_2}}\right] + H_{ij}^{TT\ 1}\left[\sqrt{\frac{m_1 m_2}{2r_{12}}}\hat{n_{12}}\right] + H_{ij}^{TT\ 2}\left[\sqrt{\frac{m_1 m_2}{2r_{12}}}\hat{n_{12}}\right]$$

- Post-Newtonian Expansion of the extrinsic curvature

PhysRevD.57.7274(1998) Class.QuantumGrav.28.134003(2011) PhysRevD.67.064008(2003)

Post-Newtonian-Hamiltonian for Past Evolution

- Radiative term in the metric $t - t_a^r - r_a(t_a^r) = 0$

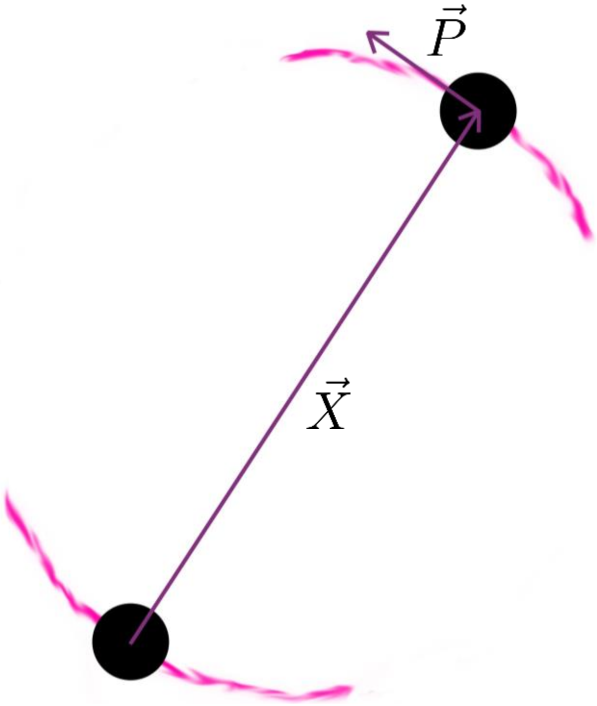
$$H_{ij}^{TT\ a}[\vec{u}] = H_{ij}^{TT\ a}[\vec{u};t] + H_{ij}^{TT\ a}[\vec{u};t_a^r] + H_{ij}^{TT\ a}[\vec{u};t_a^r \to t]$$

- Hamiltonian Evolution in Post-Newtonian expansion

$$\frac{dX^i}{dt} = \frac{\partial H}{\partial P_i} \qquad \frac{dP_i}{dt} = -\frac{\partial H}{\partial X^i} + F_i \qquad F_i = \frac{1}{\omega|\vec{L}|}\frac{dE}{dt}P_i$$

$$H(\vec{X},\vec{P}) = \mu\left[\hat{H}_{Newt}(\vec{q},\vec{p}) + \hat{H}_{1PN}(\vec{q},\vec{p}) + \hat{H}_{2PN}(\vec{q},\vec{p}) + \hat{H}_{3PN}(\vec{q},\vec{p})\right]$$

$$\vec{q} \equiv \vec{X}/M \quad \vec{p} \equiv \vec{P}_1/\mu \quad \mu = m_1 m_2/M = m_1 m_2/(m_1+m_2)$$



PhysRevD.77.044037(2008) PhysRevD.74.104005(2006)

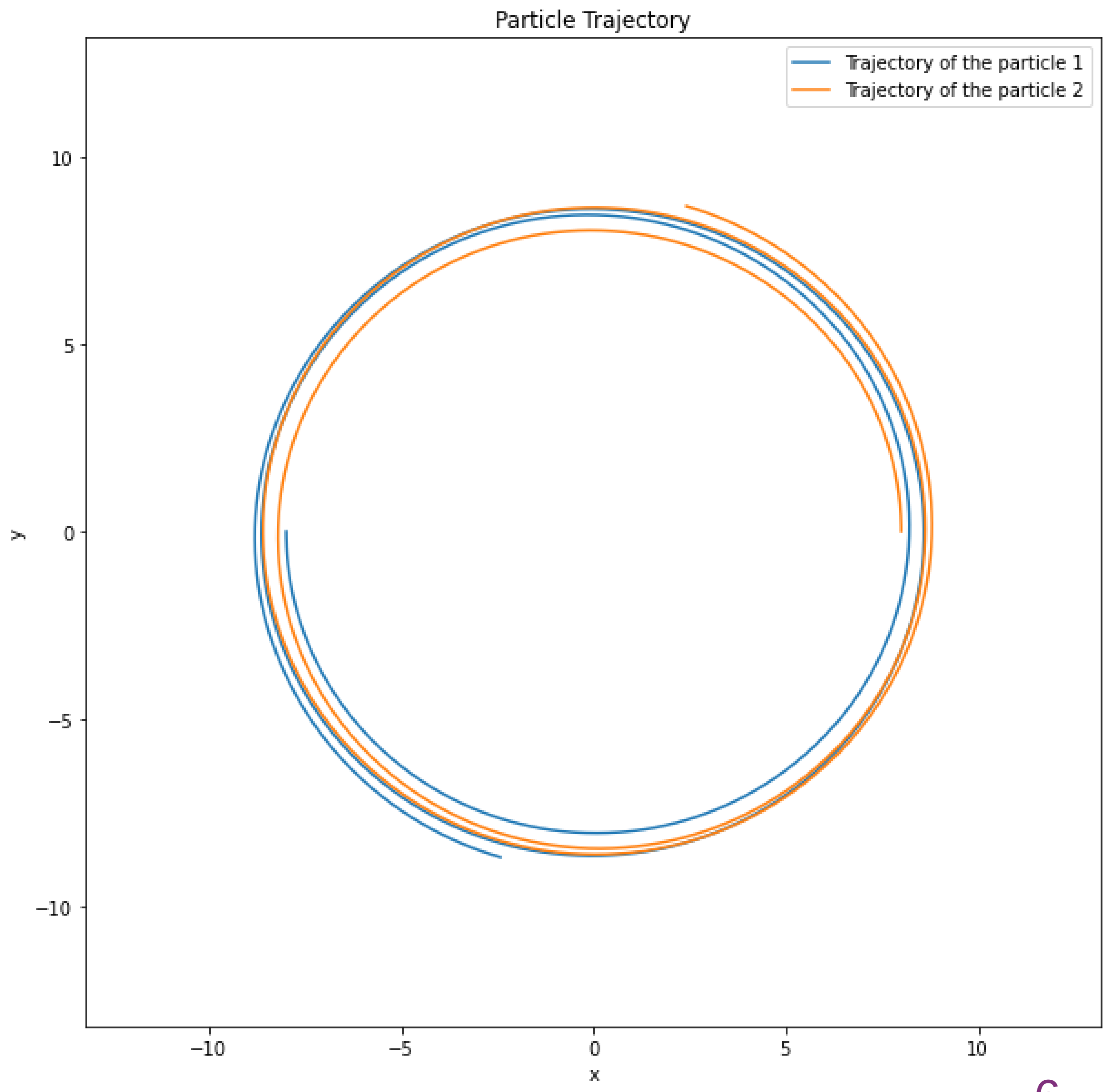
Numerical Implementation

- Toy Model Code to check calculations
- C++ based, straight forward computations of the expressions
- Graphic representations

- Implementation with SpECTRE
- Already implemented XCTS Solver
- Template Metaprogramming, parallelization and optimization techniques

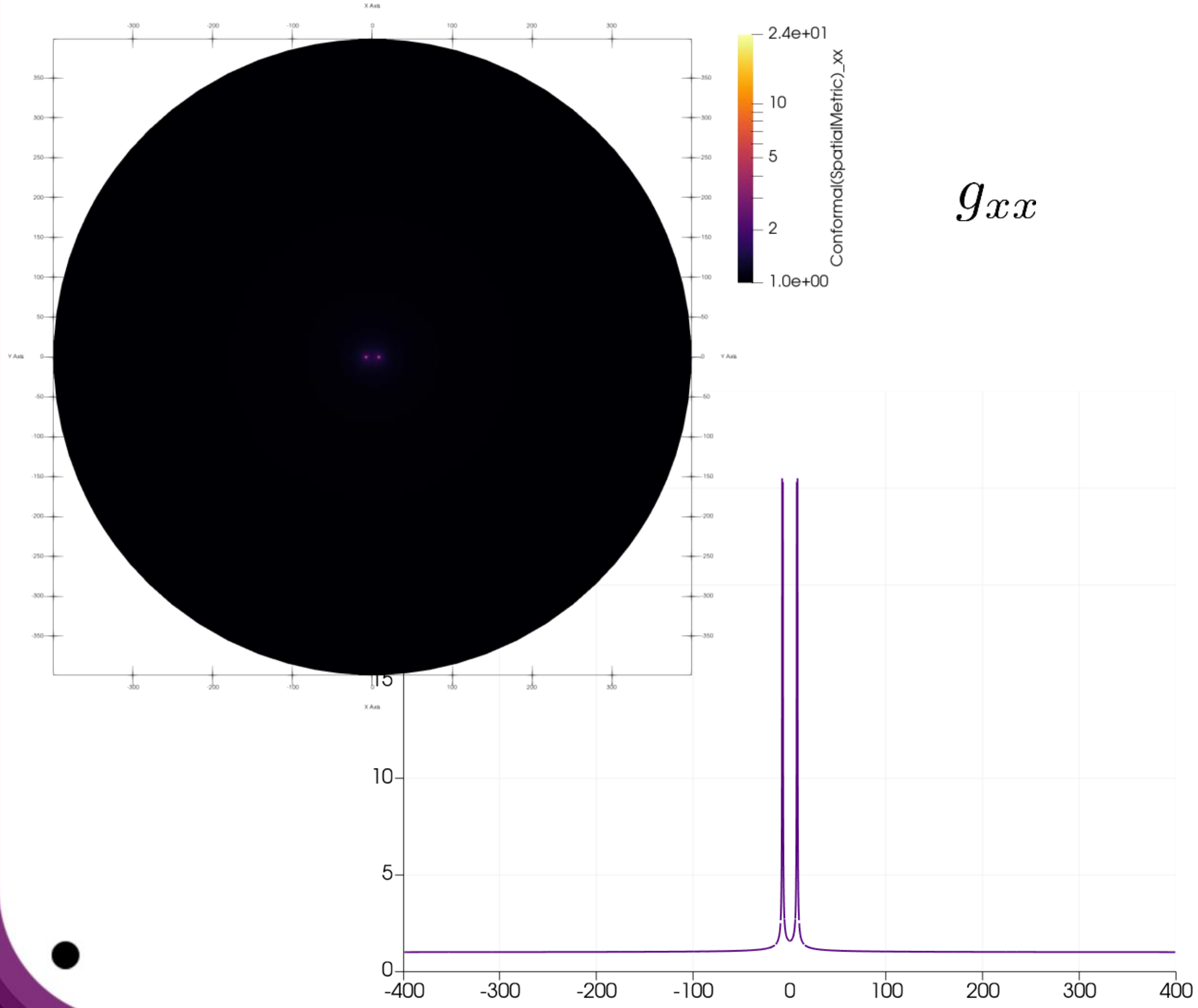
Changes to the metric

- System:
  - Mass 1: 1.0
  - Mass 2: 1.0
  - Separation: 8.0 M
- Momentum:
  - Calculated with PN expression for circular orbits



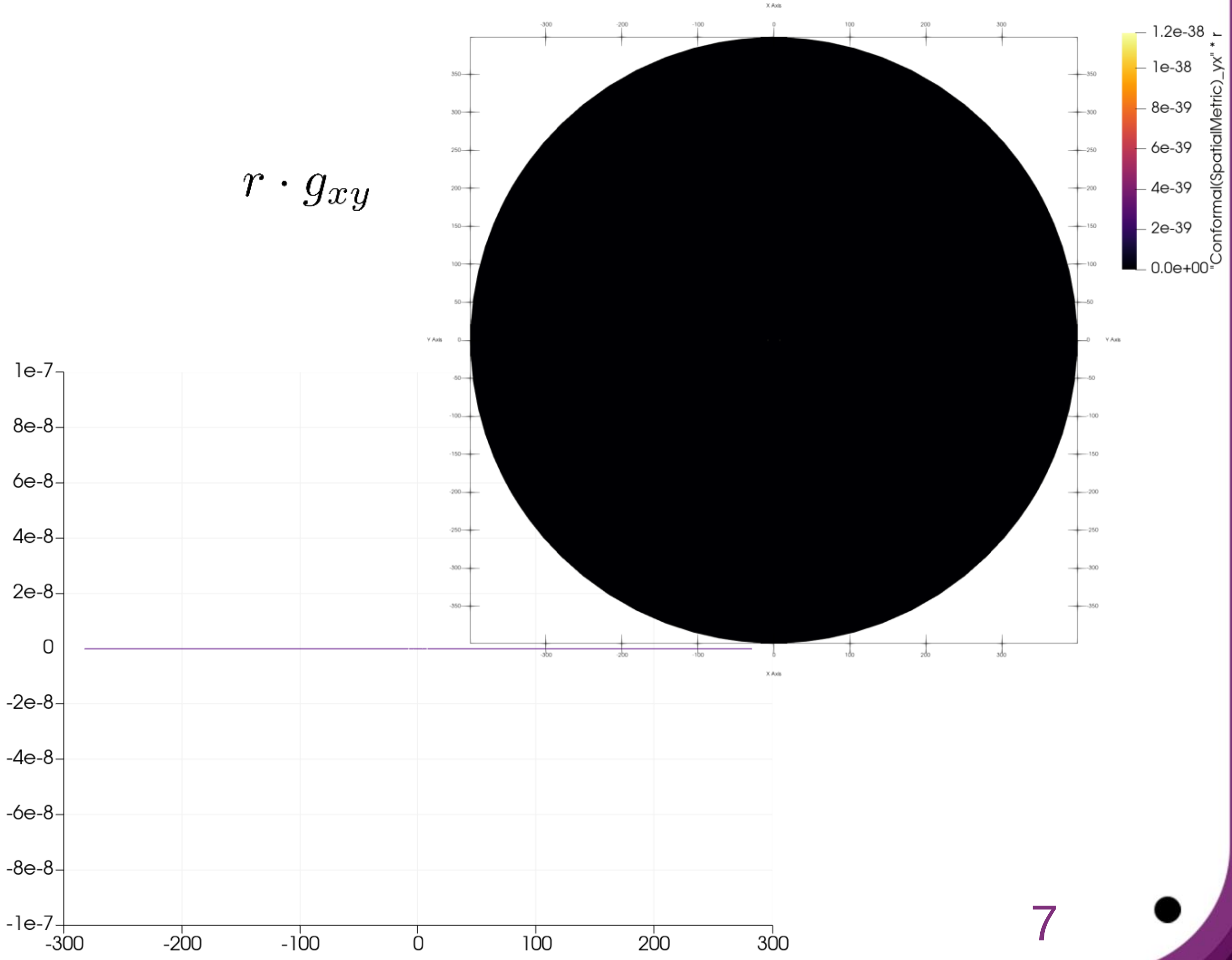
Changes to the metric

- Conformal term



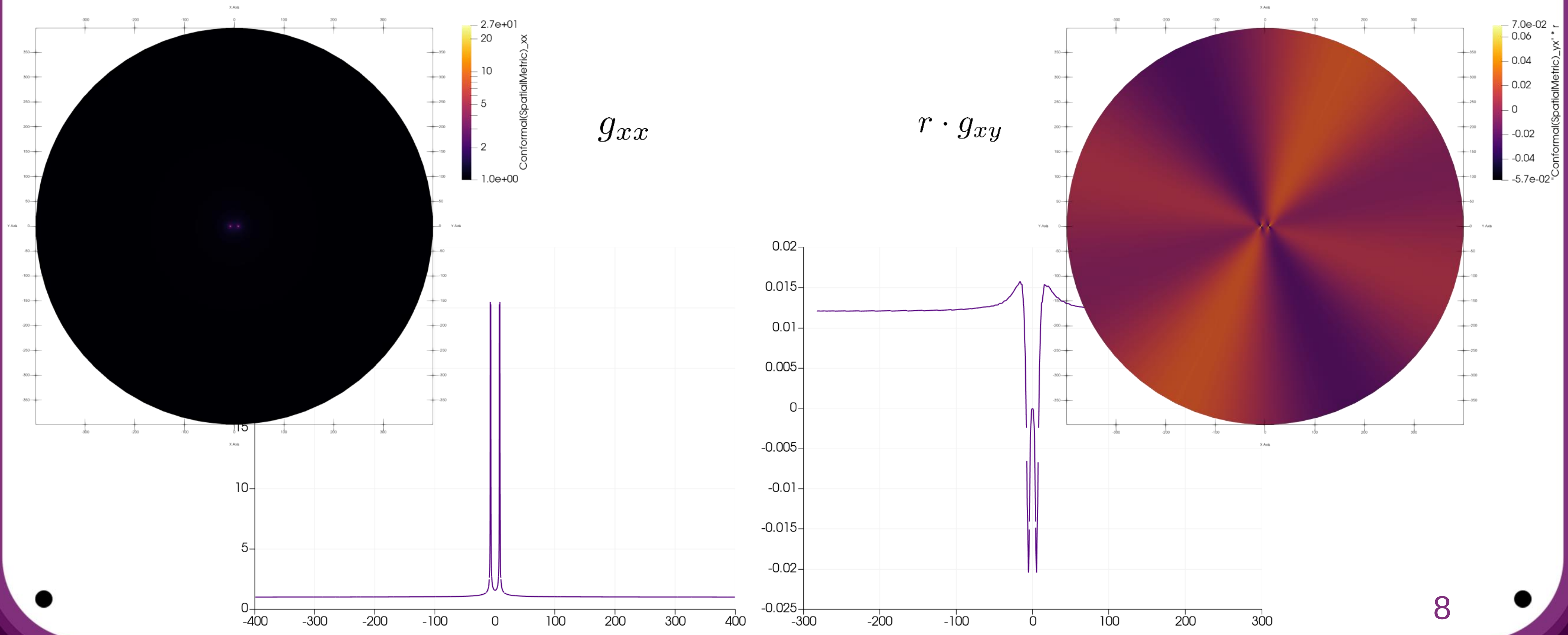
g_{xx}

$r \cdot g_{xy}$



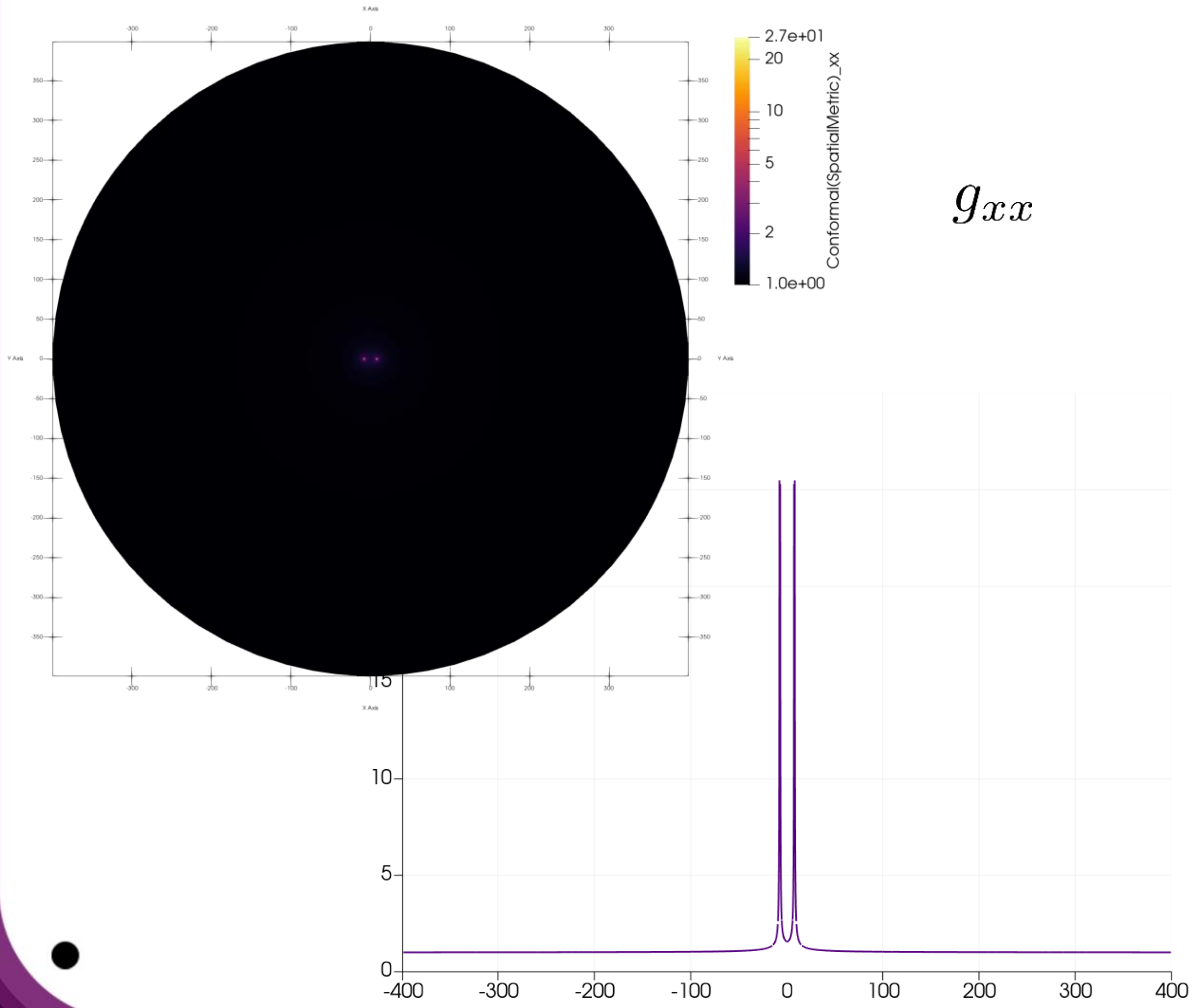
Changes to the metric

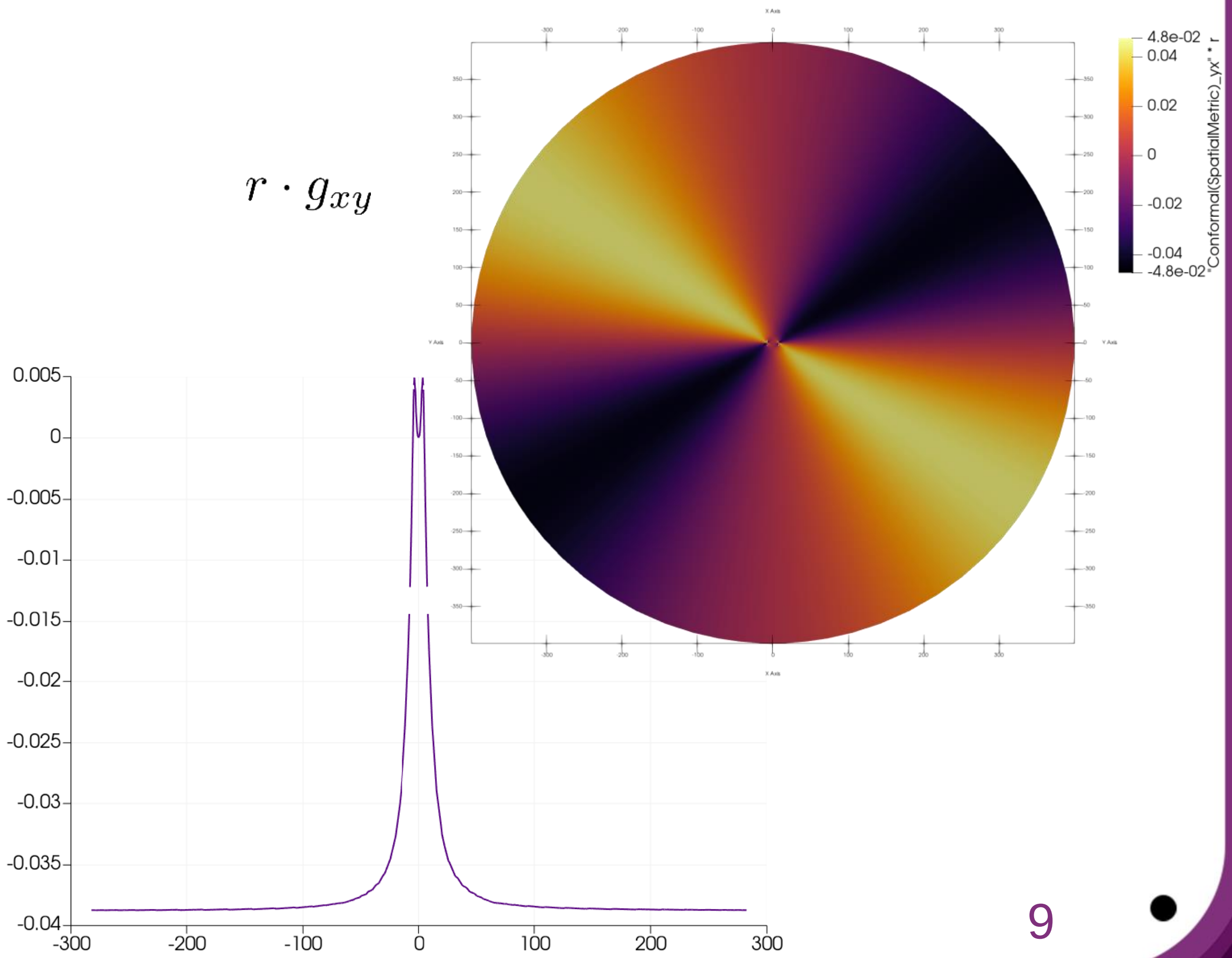
- Conformal term + Near zone term



Changes to the metric

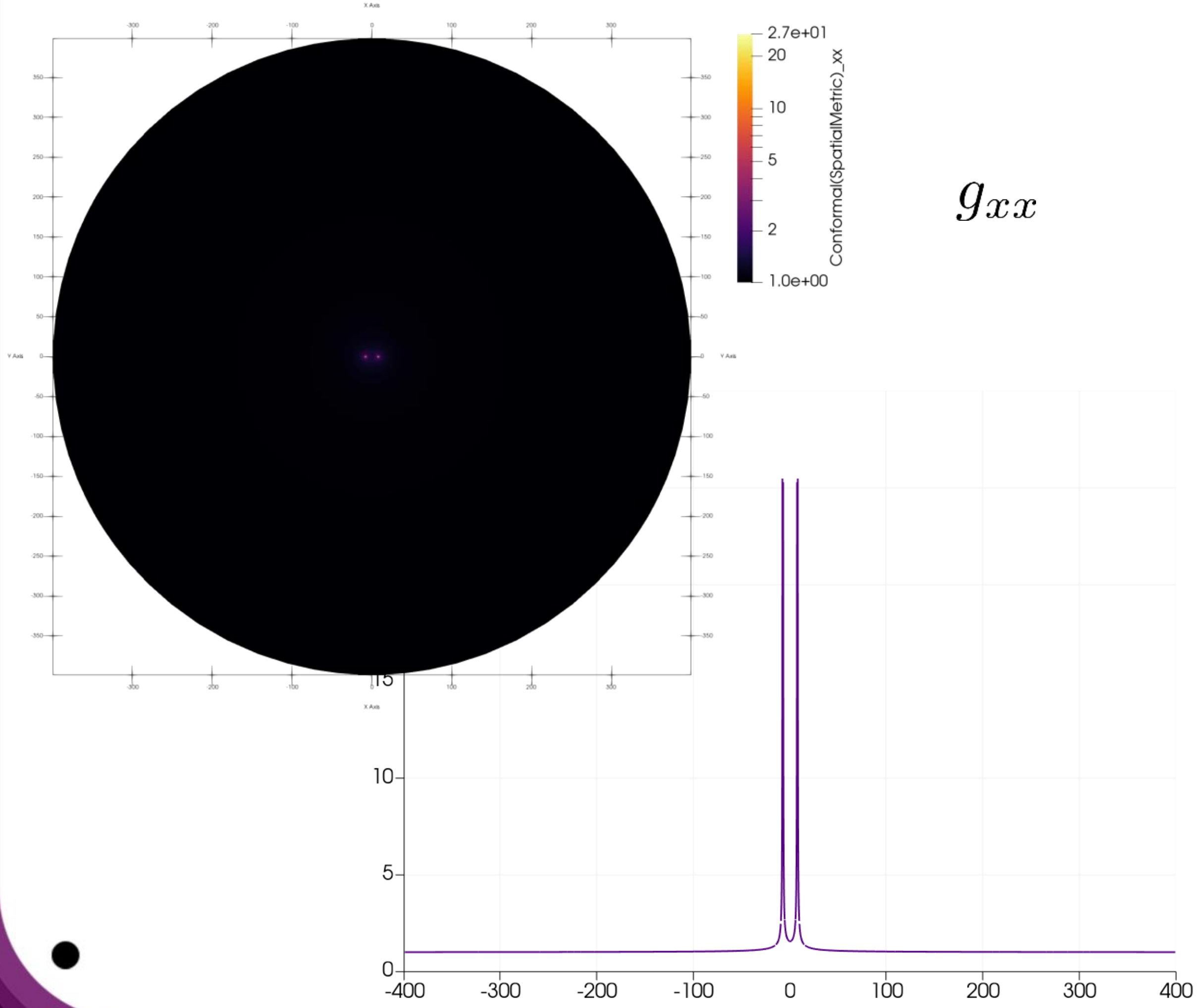
- Conformal term + Near zone term + Present term

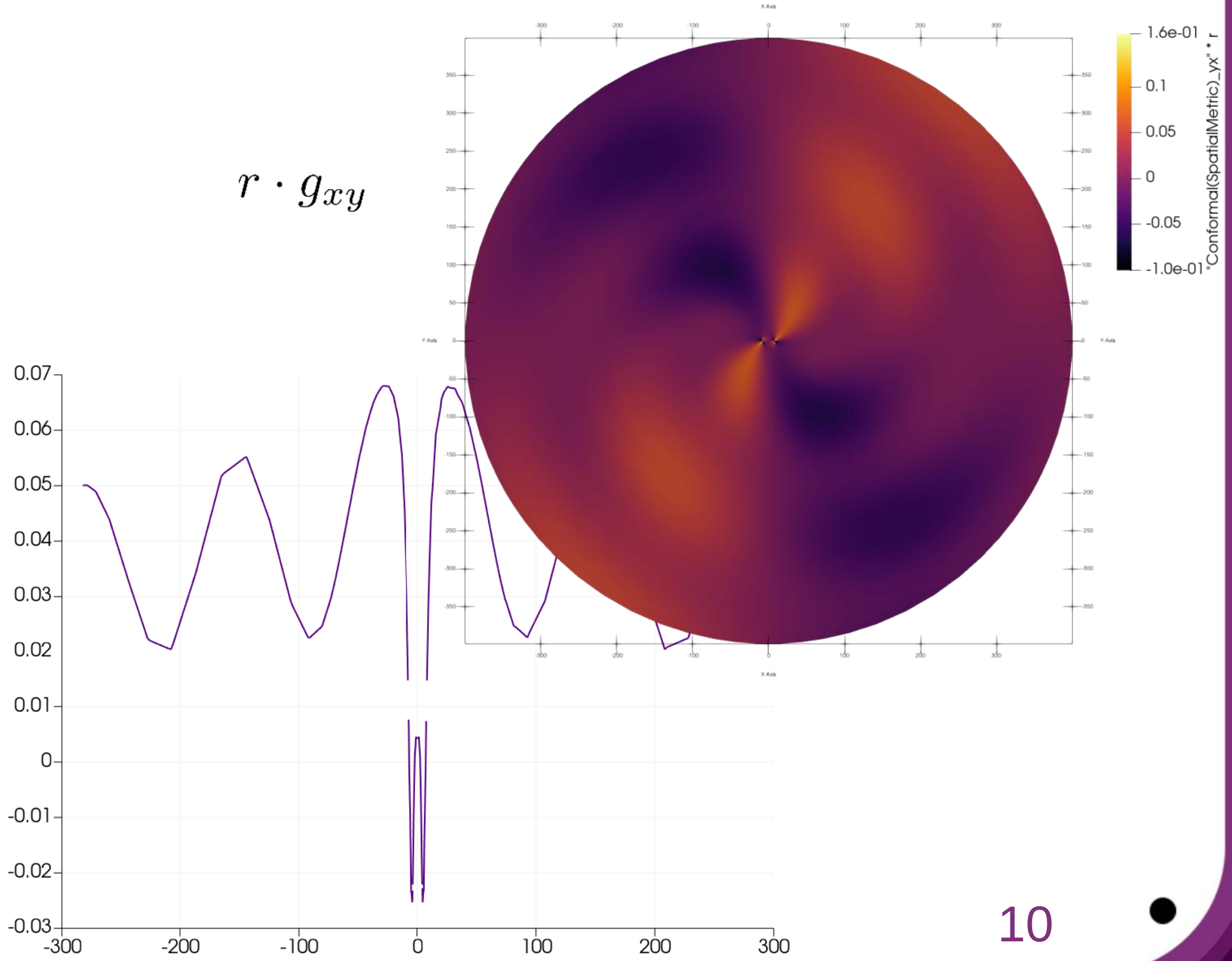




Changes to the metric

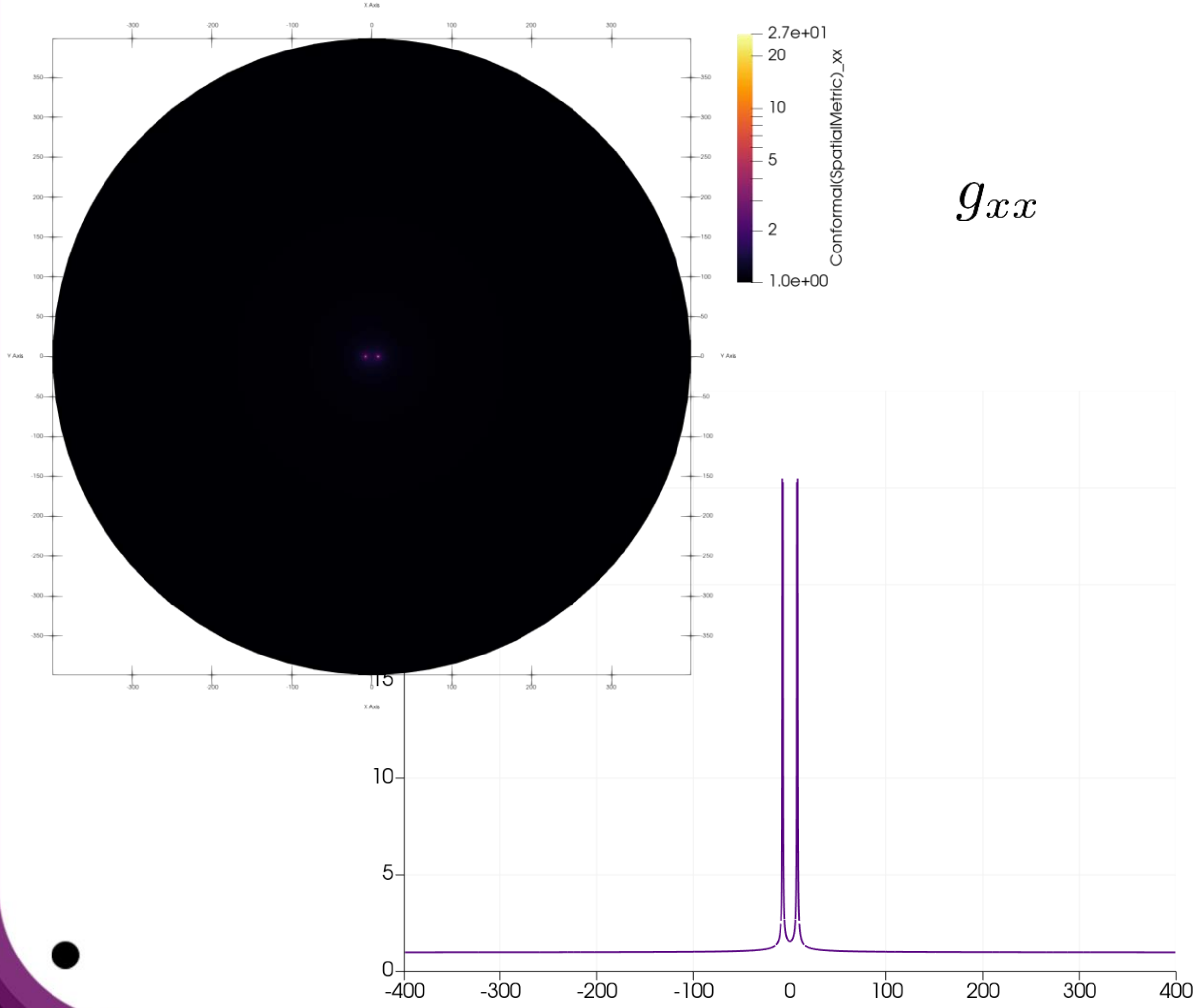
- Conformal term + Near zone term + Present term + Past term

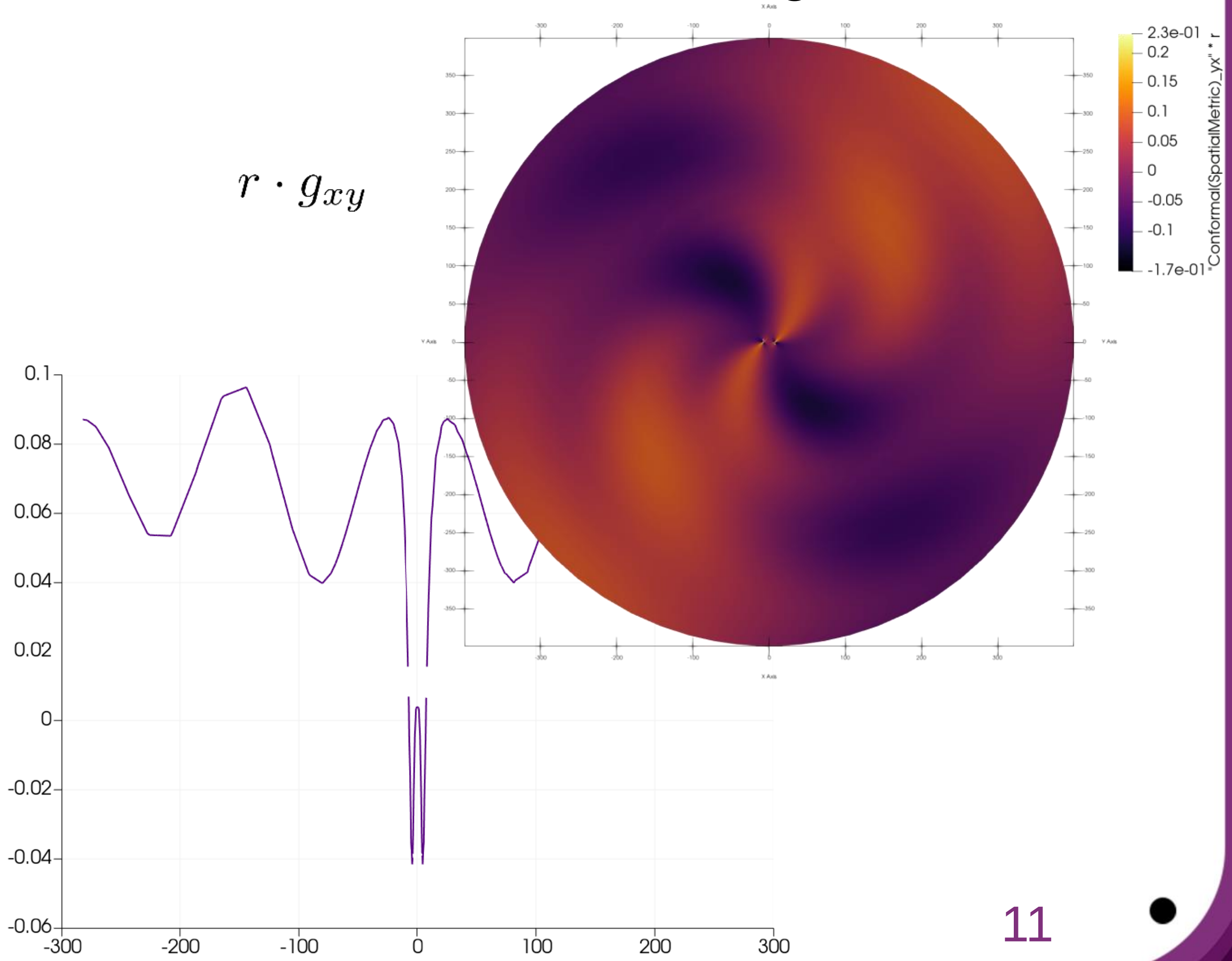




Changes to the metric

- Conformal term + Near zone term + Present term + Past term + Integral term

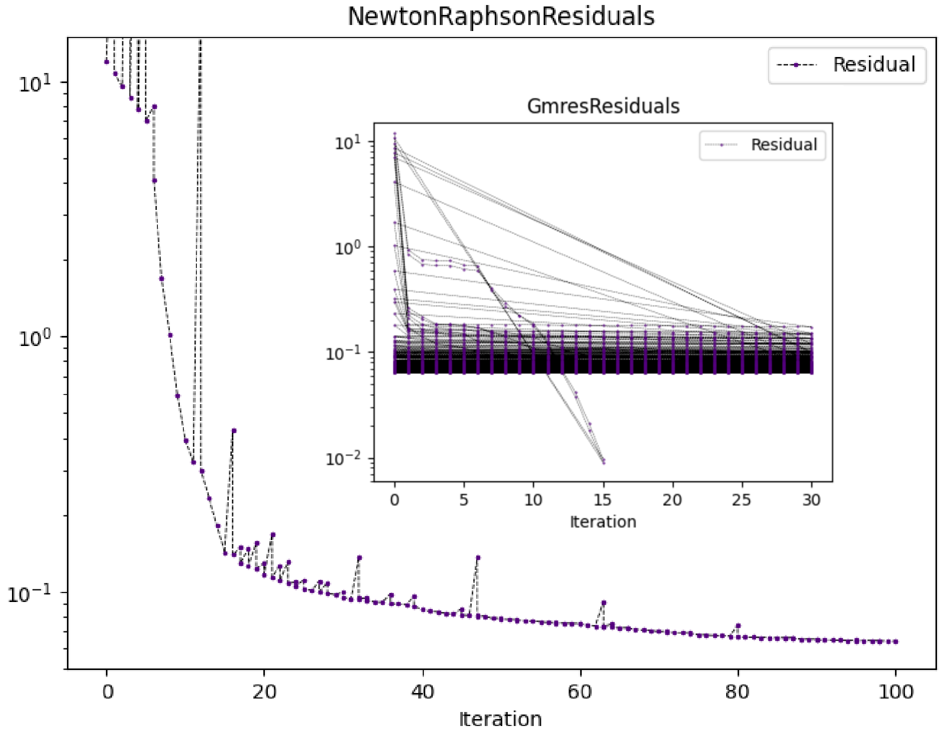




Problems with PN initial data

- Solving the XCTS equations with SpECTRE
- Choose inner boundary conditions
  - Dirichlet
  - Neumann
  - Quasiequilibrium apparent horizon
- Schwarzschild Isotropic
  - Not horizon penetrating (r < 0.5 m)
  - Coordinate change near BHs
  - Maximal Horizon Penetrating Isotropic

$\beta^i \quad \psi \quad \alpha\psi$

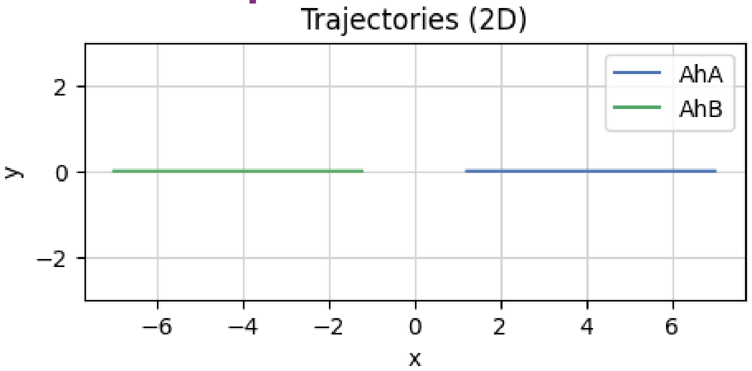


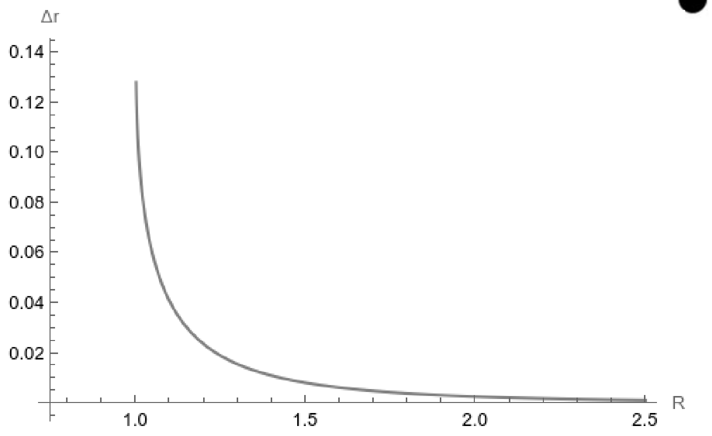
Problems with HP+PN initial data

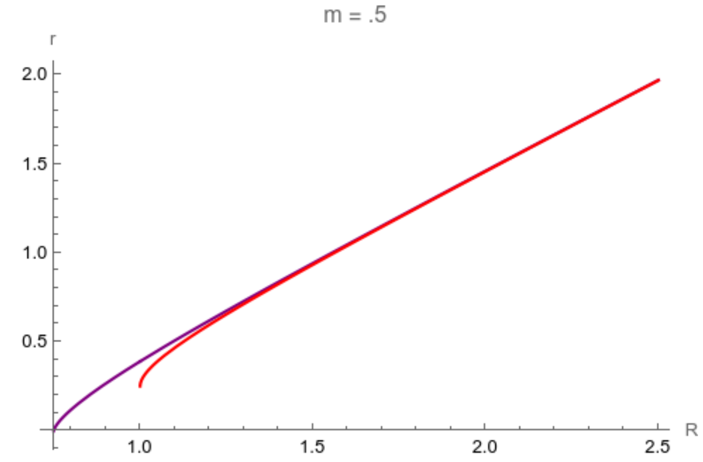
- Time coordinate
  - Past Evolution might not be compatible
  - $\overline{u}_{ij}$
- Set up:

$$\psi = \psi_{HP} \qquad \beta^i = \beta^i_{HP} \qquad \alpha\psi = \alpha_{HP}\psi_{HP}$$

$$\gamma_{ij} = \delta_{ij} + \frac{h^{TT}_{ij}}{\psi^4_{HP}} \qquad K = K_{PN} \approx 0$$







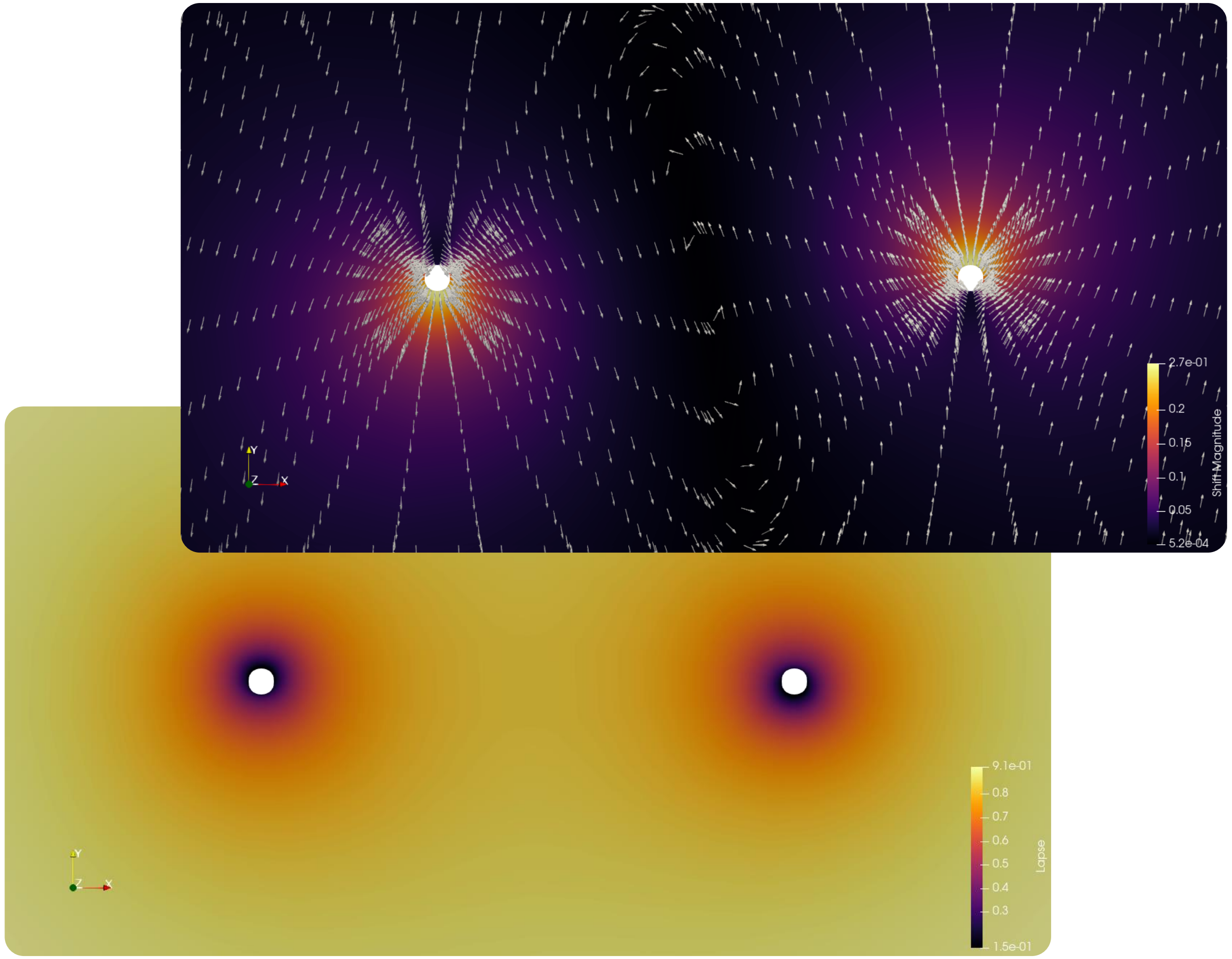
$$\psi = 1 \qquad \beta^i = \beta^i_{HP} \qquad \alpha\psi = \alpha_{HP}$$

$$\gamma_{ij} = \psi^4_{HP}\delta_{ij} + f\dot{h}^{TT}_{ij} \qquad K = K_{evol}$$

Phys.Rev.D.75.067502(2007)

Working on: Boosted HP + PN

- Problem:
  - Missing momentum in HP+PN
  - Compatibility with $\overline{u}_{ij}$
- Solution?
  - Apply boost to the single HP solutions
  - Velocities = PN past evolution
  - Superpose the two BHs
  - Add PN radiative term



Thank you!
Questions?

Auxiliary Slides - NR

- ADM equations

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho \qquad D_j(K^{ij} - \gamma^{ij}K) = 8\pi j^i$$

$$\rho = T_{\mu\nu}n^\mu n^\nu$$

$$j^i = -T_{\mu\nu}n^\mu\gamma^{\nu i}$$

$$S^{ij} = T_{\mu\nu}\gamma^{\mu i}\gamma^{\nu j}$$

$$\partial_t\gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = \alpha(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}) - D_iD_j\alpha - 8\pi\alpha(S_{ij} - \tfrac{1}{2}\gamma_{ij}(S-\rho)) + \beta^k\partial_k K_{ij} + K_{ik}\partial_j\beta^k + K_{kj}\partial_i\beta^k$$

- XCTS Equations

$$(\overline{\Delta}_L\beta)^i - (\overline{L}\beta)^{ij}\overline{D}_j ln(\overline{\alpha}) = \overline{\alpha}\overline{D}_j(\overline{\alpha}^{-1}\overline{u}^{ij}) + \tfrac{4}{3}\overline{\alpha}\psi^6\overline{D}^i K + 16\pi\overline{\alpha}\psi^{10}S^i$$

$$\overline{D}^2\psi - \tfrac{1}{8}\psi\overline{R} - \tfrac{1}{12}\psi^5K^2 + \tfrac{1}{8}\psi^{-7}\overline{A}_{ij}\overline{A}^{ij} = -2\pi\psi^5\rho$$

$$\overline{A}^{ij} = \tfrac{1}{2\overline{\alpha}}((\overline{L}\beta)^{ij} - \overline{u}^{ij})$$

$$\overline{D}^2(\alpha\psi) = \alpha\psi(\tfrac{7}{8}\psi^{-8}\overline{A}_{ij}\overline{A}^{ij} + \tfrac{5}{12}\psi^4K^2 + \tfrac{1}{8}\overline{R} + 2\pi\psi^4(\rho + 2S)) - \psi^5\partial_t K + \psi^5\beta^i\overline{D}_i K$$

Auxiliary Slides – PN BBH

$$h_{(4)}^{TT\ ij} = \frac{1}{4}\sum_a \frac{1}{m_a r_a}\Big\{[p_a^2 - 5(\hat{n}_a\cdot\vec{p}_a)^2]\delta^{ij} + 2p_a^i p_a^j + [3(\hat{n}_a\cdot\vec{p}_a)^2 - 5p_a^2]n_a^i n_a^j + 12(\hat{n}_a\cdot\vec{p}_a)n_a^{(i}p_a^{j)}\Big\} + \frac{1}{8}\sum_a\sum_{b\neq a} m_a m_b\Big\{-\frac{32}{s_{ab}}(\frac{1}{r_{ab}}+\frac{1}{s_{ab}})n_{ab}^i n_{ab}^j + 2(\frac{r_a+r_b}{r_{ab}^3}+\frac{12}{s_{ab}^2})n_a^i n_b^j + 32(\frac{2}{s_{ab}^2}-\frac{1}{r_{ab}^2})n_a^{(i}n_{ab}^{j)} + [\frac{5}{r_{ab}r_a}-\frac{1}{r_{ab}^3}(\frac{r_b^2}{r_a}+3r_a)-\frac{8}{s_{ab}}(\frac{1}{r_a}+\frac{1}{s_{ab}})]n_a^i n_a^j + [5\frac{r_a}{r_{ab}^3}(\frac{r_a}{r_b}-1)-\frac{17}{r_{ab}r_a}+\frac{4}{r_a r_b}+\frac{8}{s_{ab}}(\frac{1}{r_a}+\frac{4}{r_{ab}})]\delta^{ij}\Big\}$$

$$K_{PN}^{ij} = -\psi_{PN}^{-10}\left[(\epsilon^3)\tilde{\pi}_{(3)}^{ij} + (\epsilon^5)\frac{1}{2}\dot{h}_{(4)ij}^{TT} + (\epsilon^5)(\phi_{(2)}\tilde{\pi}_{(3)}^{ij})^{TT}\right] + O(\epsilon^6)$$

$$\tilde{\pi}_{(3)}^{ij} = \frac{1}{16\pi}\sum_a p_a^k\{-\delta_{ij}(\frac{1}{r_a})_{,k} + 2[\delta_{ik}(\frac{1}{r_a})_{,j} + \delta_{jk}(\frac{1}{r_a})_{,i}] - \frac{1}{2}r_{a,ijk}\}$$

Auxiliary Slides – PN BBH

$$H_{ij}^{TT\ a}[\vec{u};t] = -\frac{1}{4r_a(t)}\Big\{[u^2 - 5(\vec{u}\cdot\hat{n}_a)^2]\delta_{ij} + 2u^iu^j + 3(\vec{u}\cdot\hat{n}_a)^2 - 5u^2]n_a^in_a^j + 12(\vec{u}\cdot\hat{n}_a)u^{(i}n_a^{j)}\Big\}_t$$

$$H_{ij}^{TT\ a}[\vec{u};t_a^r] = -\frac{1}{r_a(t_a^r)}\Big\{[-2u^2+2(\vec{u}\cdot\hat{n}_a)^2]\delta^{ij}+4u^iu^j+[2u^2+2(\vec{u}\cdot\hat{n}_a)^2]n_a^in_a^j - 8(\vec{u}\cdot\hat{n}_a)u^{(i}n_a^{j)}\Big\}_{t_a^r}$$

$$H_{ij}^{TT\ a}[\vec{u};t_a^r\to t] = -\int_{t_a^r}^{t} d\tau\frac{(t-\tau)}{r_a(\tau)^3}\Big\{[-5u^2 + 9(\vec{u}\cdot\hat{n}_a)^2]\delta^{ij} + 6u^iu^j - 12(\vec{u}\cdot\hat{n}_a)u^{(i}n_a^{j)} + [9u^2 - 15(\vec{u}\cdot\hat{n}_a)^2]n_a^in_a^j\Big\} - \int_{t_a^r}^{t} d\tau\frac{(t-\tau)^3}{r_a(\tau)^5}\Big\{[u^2 - 5(\vec{u}\cdot\hat{n}_a)^2]\delta^{ij} + 2u^iu^j - 20(\vec{u}\cdot\hat{n}_a)u^{(i}n_a^{j)} + [-5u^2 + 35(\vec{u}\cdot\hat{n}_a)^2]n_a^in_a^j\Big\}$$

Auxiliary Slides – PN Hamiltonian

$$\hat{H}_{Newt}(\vec{q},\vec{p}) = \frac{p^2}{2} - \frac{1}{q}$$

$$\hat{H}_{1PN}(\vec{q},\vec{p}) = \frac{1}{8}(3\eta - 1)(p^2)^2 - \frac{1}{2q}\Big[(3+\eta)p^2 + \eta(\vec{n}\cdot\vec{p})^2\Big] + \frac{1}{2q^2}$$

$$\hat{H}_{2PN}(\vec{q},\vec{p}) = \frac{1}{16}(1-5\eta+5\eta^2)(p^2)^3 + \frac{1}{8q}\Big[(5-20\eta-3\eta^2)(p^2)^2 - 2\eta^2(\vec{n}\cdot\vec{p})^2p^2 - 3\eta^2(\vec{n}\cdot\vec{p})^4\Big] + \frac{1}{2q^2}\Big[(5+8\eta)p^2 + 3\eta(\vec{n}\cdot\vec{p})^2\Big] - \frac{1}{4q^3}(1+3\eta)$$

$$\hat{H}_{3PN}(\vec{q},\vec{p}) = \frac{1}{128}(-5+35\eta-70\eta^2+35\eta^3)(p^2)^4 + \frac{1}{16q}\Big[(-7+42\eta-53\eta^2 - 5\eta^3)(p^2)^3 + (2-3\eta)\eta^2(\vec{n}\cdot\vec{p})^2(p^2)^2 + 3(1-\eta)\eta^2(\vec{n}\cdot\vec{p})^4p^2 - 5\eta^3(\vec{n}\cdot\vec{p})^6\Big] + \frac{1}{16q^2}\Big[(-27+136\eta+109\eta^2)(p^2)^2 + (17+30\eta)\eta(\vec{n}\cdot\vec{p})^2p^2 + \frac{4}{3}(5+43\eta)\eta(\vec{n}\cdot\vec{p})^4\Big] + \frac{1}{q^3}\Big[\Big(-\frac{25}{8} + \big(\frac{\pi^2}{64} - \frac{335}{48}\big)\eta - \frac{23}{8}\eta^2\Big)p^2 + \Big(-\frac{85}{16} - \frac{3\pi^2}{64} - \frac{7}{4}\eta\Big)\eta(\vec{n}\cdot\vec{p})^2\Big] + \frac{1}{q^4}\Big[\frac{1}{8} + \Big(\frac{109}{12} - \frac{21\pi^2}{32}\Big)\eta\Big]$$

Auxiliary Slides – PN Hamiltonian

$$\frac{dE}{dt} = -\frac{32}{5}\eta^2 v_\omega^{10} \times$$
$$\times \{1 + f_2(\eta)v_\omega^2 + f_3(\eta)v_\omega^3 + f_4(\eta)v_\omega^4 + f_5(\eta)v_\omega^5 + f_6(\eta)v_\omega^6 + f_{l6}(\eta)v_\omega^6 ln(4v_\omega) + f_7(\eta)v_\omega^7\}$$

$$f_2(\eta) = -\frac{1247}{336} - \frac{35}{12}\eta$$

$$f_3(\eta) = 4\pi$$

$$f_4(\eta) = -\frac{44711}{9072} + \frac{9271}{504} + \frac{65}{18}\eta^2$$

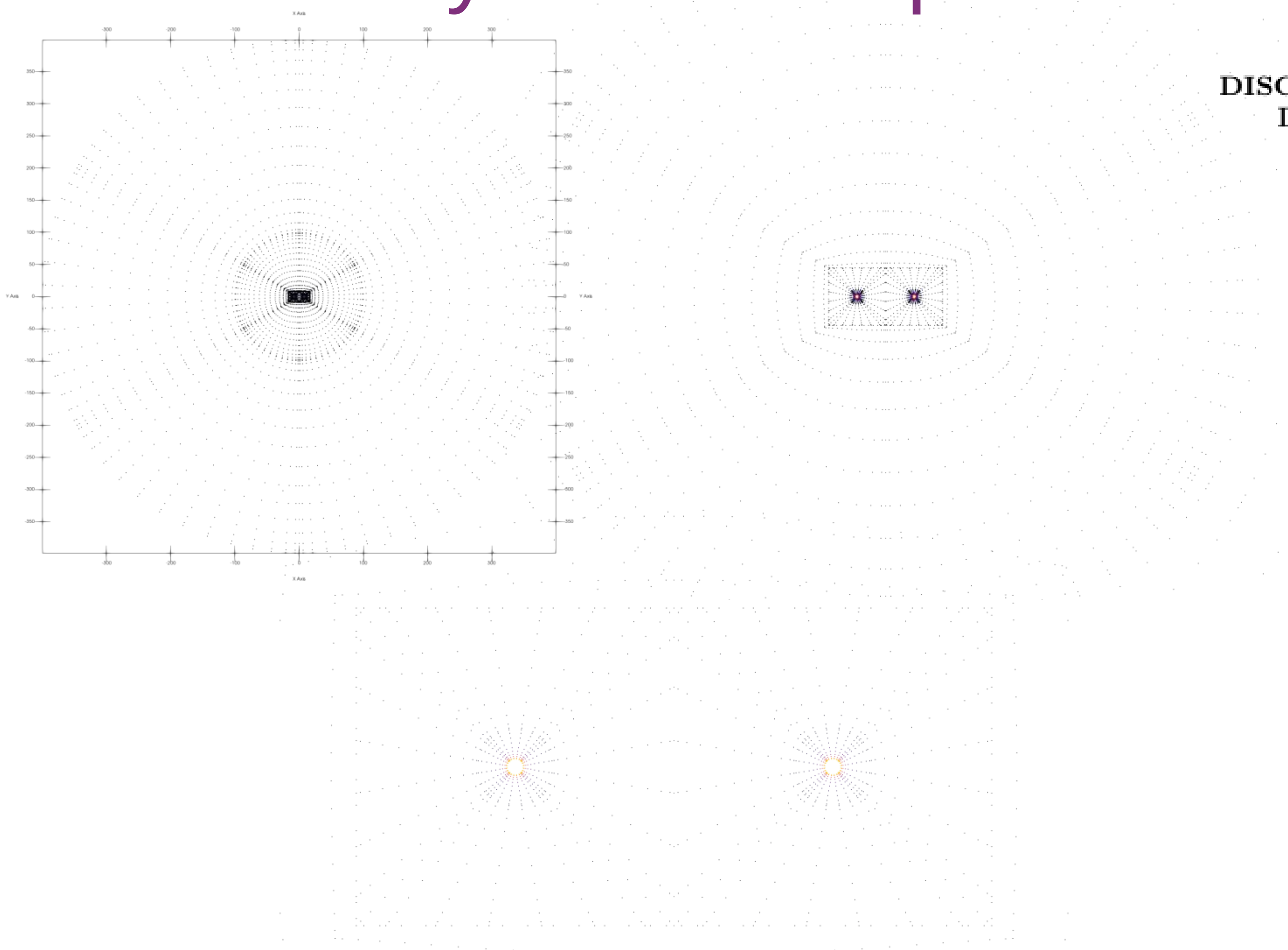
$$f_5(\eta) = -\left(\frac{8191}{672} + \frac{583}{24}\eta\right)\pi$$

$$f_{l6}(\eta) = -\frac{1712}{105}$$

$$f_6(\eta) = \frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\eta - \frac{94403}{3024}\eta^2 - \frac{775}{324}\eta^3$$

$$f_7(\eta) = \left(-\frac{16285}{504} + \frac{214745}{1728}\eta + \frac{193385}{3024}\eta^2\right)\pi$$

Auxiliary Slides – SpECTRE SolveXcts



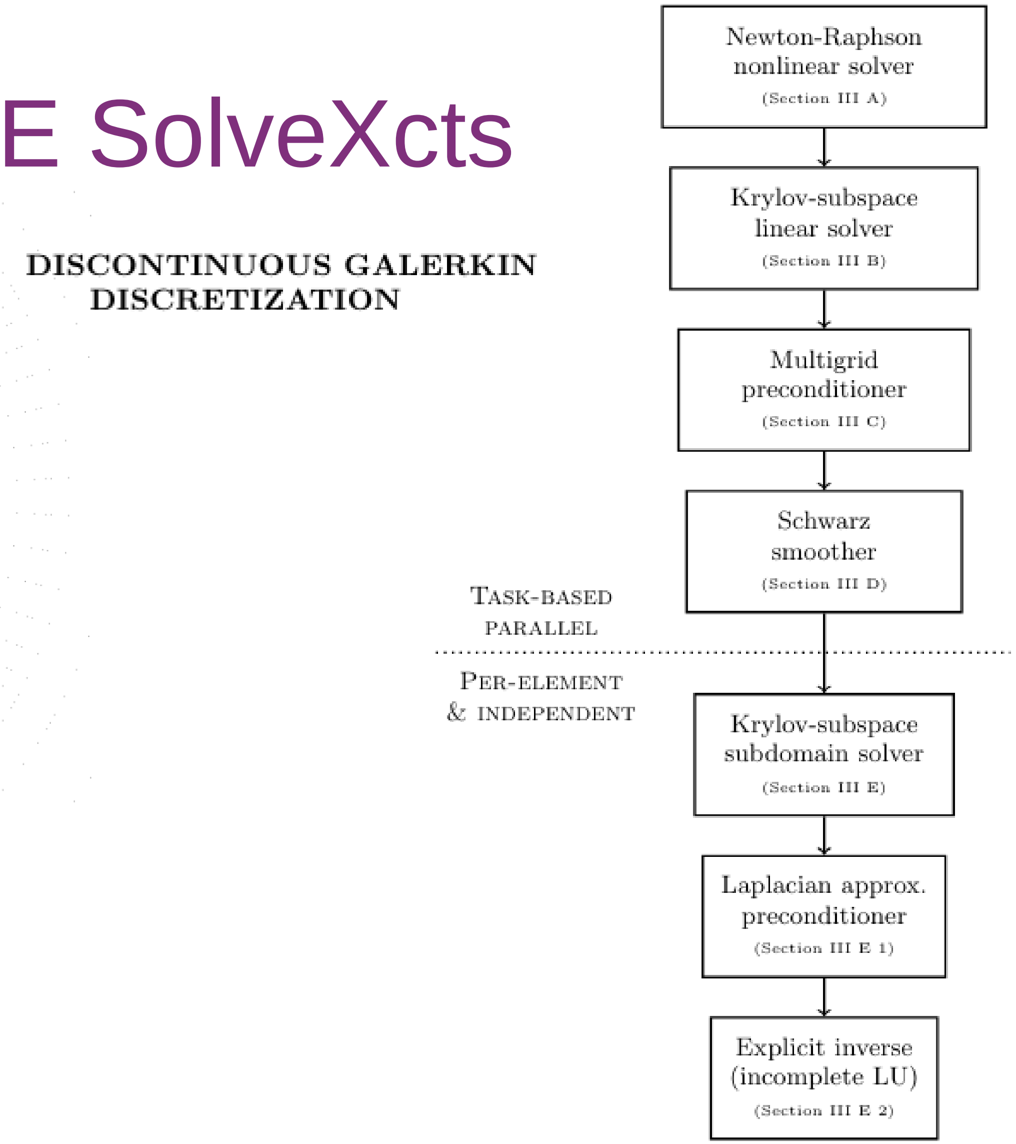


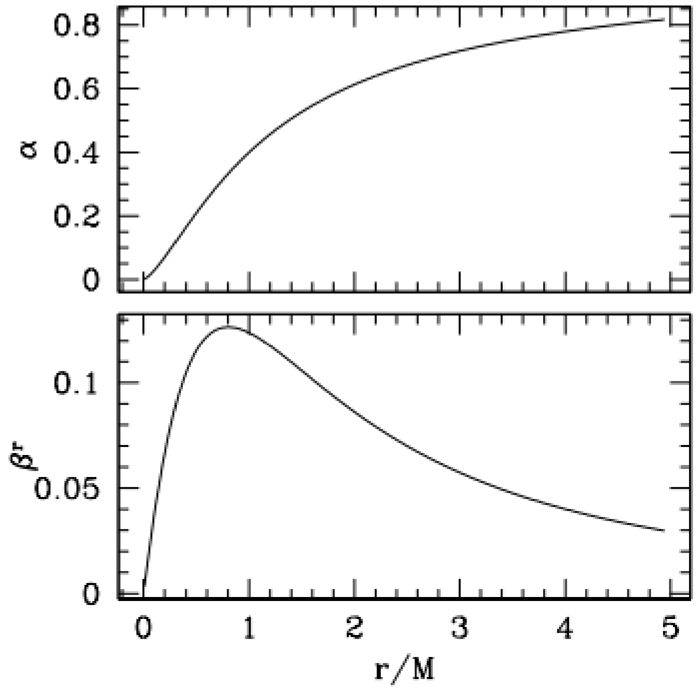
FIG. 3. Overview of the technology stack we employ to solve the discretized elliptic problem (1). All algorithms above the dotted line follow SpECTRE's task-based parallelism paradigm. The algorithms below the dotted line run within a task, and on all elements independently.

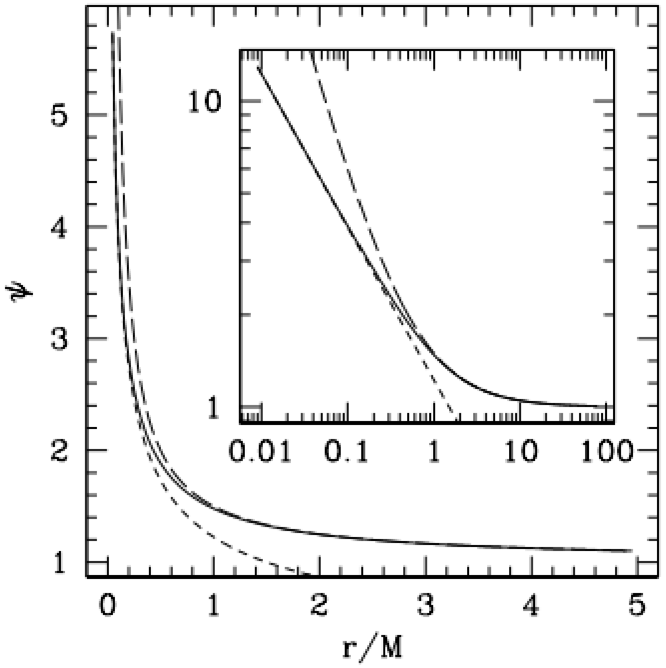
Nils L. Vu and others. A scalable elliptic solver with task-based parallelism for the SpECTRE numerical relativity code. Phys. Rev. D, 105(8):084027, apr 2022. arXiv:2111.06767

Auxiliary Slides – HP BH

$$r = \left[\frac{2R+m+(4R^2+4mR+3m^2)^{1/2}}{4}\right]\left[\frac{(4+3\sqrt{2})(2R-3m)}{8R+6m+3(8R^2+8mR+6m^2)^{1/2}}\right]^{1/\sqrt{2}}$$

$$\psi = \left[\frac{4R}{2R+m+(4R^2+4mR+3m^2)^{1/2}}\right]^{1/2} \quad \alpha = \left(1 - \frac{2m}{R} + \frac{27m^4}{16R^4}\right)^{1/2} \quad \beta^r = \frac{3\sqrt{3}m^2}{4}\frac{r}{R^3}$$





Horizon: r ~ 0.8 m

Phys.Rev.D.75.067502(2007)