Kerr-de Sitter and algebraically special metrics with prescribed asymptotics in all dimensions

Carlos Peón Nieto
Joint work with Marc Mars

VNiVERSiDAD D SALAMANCA
CAMPUS DE EXCELENCIA INTERNACIONAL

POLITÉCNICA | UNIVERSIDAD POLITÉCNICA DE MADRID

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- Kerr-de Sitter (and related metrics) also characterized by alignment [Mars, Senovilla '14].

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$$\mathrm{Cot}(\gamma) = \frac{K_1}{|\xi|^5}(\boldsymbol{\xi} \otimes \boldsymbol{\xi})^{tf}, \qquad D = \frac{K_2}{|\xi|^5}(\boldsymbol{\xi} \otimes \boldsymbol{\xi})^{tf}, \quad K_1, K_2 \in \mathbb{R}.$$

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Aim of today's talk: new characterization in all dim

$\{$ Kerr-de Sitter-like $\} = \{$ Algebraically special + (locally) conformally flat $\mathscr{I}$ $\}$

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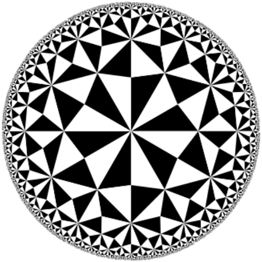

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$(M, g; \Omega)$ conformal extension of $(\widetilde{M}, \widetilde{g})$

- $g = \Omega^2 \widetilde{g}$
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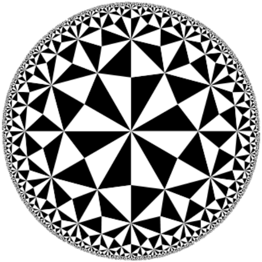

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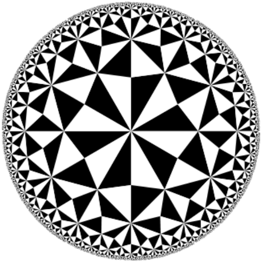

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  - $\mathscr{I}$ smooth conformal structure $[\gamma]$: each $\gamma \in [\gamma]$ is a boundary metric.

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$(CEE) \Longrightarrow$ Fefferman-Graham (FG) asymptotic expansion for g_Ω

Fefferman-Graham expansion: n odd case

$$g_\Omega \sim \sum_{r=0}^{(n-1)/2} g_{(2r)}\Omega^{2r} + \sum_{r=n}^{\infty} g_{(r)}\Omega^r \quad \left\{ \begin{array}{l} \bullet\ \gamma \text{ prescribes } g_{(0)} \text{ and generates } g_{(2r<n)}. \\ \bullet\ g_{(n)} \text{ is independent of } \gamma \text{ except for} \\ \qquad \mathrm{Tr}_\gamma g_{(n)} = 0, \qquad \mathrm{div}_\gamma g_{(n)} = 0 \qquad (\star) \end{array} \right.$$

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$(g_{(0)}, g_{(n)})$ seed data FG expansion $\rightarrow$ Asymptotic data:
- $(\Sigma, \gamma) \equiv$ Riemannian *n*-manifold
- $g_n \equiv$ 2-tensor satisfying $(\star)/(\star\star)$

Asymptotic initial value problem in $n+1$ dimensions

- General theorem from multiple contributions: [Friedrich '86, Anderson '05, Anderson & Chruściel '05, Kamiński '21, Rodnianksi & Shlapentokh-Rothman '18, Hintz '23]

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Asymptotic initial value problem at spacelike $\mathscr{I}$

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Geometric identification of asymptotic data [Mars & P.-N. '21, Hollands *et al.* '05]

Let $(\mathscr{I}, \gamma)$ be locally conformally flat. Then and only then

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Conformal properties of $\xi \rightarrow$ generate all Kerr-de Sitter-like metrics as limits or analytic extensions of Kerr-de Sitter.

Theorem. *The Kerr-de Sitter-like class in all dimensions*

$$\widetilde{g} = \widetilde{g}_{dS} + \widetilde{\mathcal{H}}\widetilde{k} \otimes \widetilde{k}, \qquad \text{must have} \quad \widetilde{\mathcal{H}} = \frac{2M\rho^{n-2}}{\Xi \prod_{i=1}^{q}(1+\rho^2 a_i^2)}, \quad M \in \mathbb{R},$$

and

a) Kerr-de Sitter family,

$$\widetilde{g}_{dS} = -W\frac{(\rho^2-\lambda)}{\rho^2}\mathrm{d}t^2 + \frac{\Xi}{\rho^2-\lambda}\frac{\mathrm{d}\rho^2}{\rho^2} + \delta_{p,q}\frac{\mathrm{d}\alpha_{p+1}^2}{\rho^2} + \sum_{i=1}^{q}\frac{1+\rho^2 a_i^2}{\rho^2}\left(\mathrm{d}\alpha_i^2 + \alpha_i^2\mathrm{d}\phi_i^2\right) + \frac{(\rho^2-\lambda)}{\lambda W\rho^2}\frac{\mathrm{d}W^2}{4}.$$

b) $\{a_i \to \infty\}$-limit-Kerr-de Sitter,

$$\widetilde{g}_{dS} = \frac{\lambda\alpha_{p+1}^2}{\rho^2}\mathrm{d}t^2 - \frac{\Xi}{\lambda}\frac{\mathrm{d}\rho^2}{\rho^2} + \delta_{p+1,q}\frac{\alpha_{p+1}^2\mathrm{d}\phi_q^2}{\rho^2} + \sum_{i=1}^{p}\frac{1+\rho^2 a_i^2}{\rho^2}\left(\mathrm{d}\alpha_i^2 + \alpha_i^2\mathrm{d}\phi_i^2\right) + \left(\frac{1}{\lambda} + \frac{\sum_{i=1}^{p}\alpha_i^2}{\rho^2\hat{\alpha}_{p+1}^2}\right)\mathrm{d}\alpha_{p+1}^2 - \frac{2\mathrm{d}\alpha_{p+1}}{\rho^2\alpha_{p+1}}\left(\sum_{i=1}^{p}\alpha_i\mathrm{d}\alpha_i\right).$$

c.1) Wick-rotated-Kerr-de Sitter for n even,

$$\widetilde{g}_{dS} = \frac{\lambda W}{\rho^2}\mathrm{d}t^2 - \frac{\Xi}{\lambda}\frac{\mathrm{d}\rho^2}{\rho^2} + \sum_{i=1}^{q}\frac{1+\rho^2 a_i^2}{\rho^2}\left(\mathrm{d}\alpha_i^2 + \alpha_i^2\mathrm{d}\phi_i^2\right) - \frac{1}{W\rho^2}\frac{\mathrm{d}W^2}{4}.$$

c.2) Wick-rotated-Kerr-de Sitter for n odd,

$$\widetilde{g}_{dS} = W\frac{(\rho^2+\lambda)}{\rho^2}\mathrm{d}t^2 - \frac{\Xi}{\rho^2+\lambda}\frac{\mathrm{d}\rho^2}{\rho^2} - \frac{\mathrm{d}\alpha_{p+1}^2}{\rho^2} + \sum_{i=1}^{p}\frac{1+\rho^2 a_i^2}{\rho^2}\left(\mathrm{d}\alpha_i^2 + \alpha_i^2\mathrm{d}\phi_i^2\right) + \frac{(\rho^2+\lambda)}{\lambda W\rho^2}\frac{\mathrm{d}W^2}{4}.$$

and

Case	Constraint on $\{\alpha_i\}$	W	Ξ	$\widetilde{k}$
a)	$\sum_{i=1}^{p+1}(1+\lambda a_i^2)\alpha_i^2 = 1$	$\sum_{i=1}^{p+1}\alpha_i^2$	$\sum\limits_{i=1}^{p+1}\frac{1+\lambda a_i^2}{1+\rho^2 a_i^2}\alpha_i^2$	$W\mathrm{d}t - \frac{\Xi}{\rho^2-\lambda}\mathrm{d}\rho - \sum\limits_{i=1}^{q} a_i\alpha_i^2\mathrm{d}\phi_i$
b)	$\alpha_{p+1}^2 + \sum_{i=1}^{p}\lambda a_i^2\alpha_i^2 = 1$	α_{p+1}^2	$\alpha_{p+1}^2 + \sum\limits_{i=1}^{p}\frac{\lambda a_i^2}{1+\rho^2 a_i^2}\alpha_i^2$	$W\mathrm{d}t + \frac{\Xi}{\lambda}\mathrm{d}\rho - \sum\limits_{i=1}^{p} a_i\alpha_i^2\mathrm{d}\phi_i$
c.1)	$\sum_{i=1}^{p+1}\lambda a_i^2\alpha_i^2 = 1$	$\sum_{i=1}^{p+1}\alpha_i^2$	$\sum\limits_{i=1}^{p+1}\frac{\lambda a_i^2}{1+\rho^2 a_i^2}\alpha_i^2$	$\frac{\Xi}{\lambda}\mathrm{d}\rho - \sum\limits_{i=1}^{q} b_i\alpha_i^2\mathrm{d}\phi_i$
c.2)	$\alpha_{p+1}^2 - \sum_{i=1}^{p}(1-\lambda a_i^2)\alpha_i^2 = 1$	$\alpha_{p+1}^2 - \sum_{i=1}^{p}\alpha_i^2$	$\alpha_{p+1}^2 - \sum\limits_{i=1}^{p}\frac{1-\lambda a_i^2}{1+\rho^2 a_i^2}\alpha_i^2$	$W\mathrm{d}t + \frac{\Xi}{\rho^2+\lambda}\mathrm{d}\rho - \sum\limits_{i=1}^{q} a_i\alpha_i^2\mathrm{d}\phi_i$

Table 1: *Functions defining the Kerr-Schild-de Sitter families.*

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Thank you for your attention