Can relativistic effects explain galactic dynamics without dark matter?

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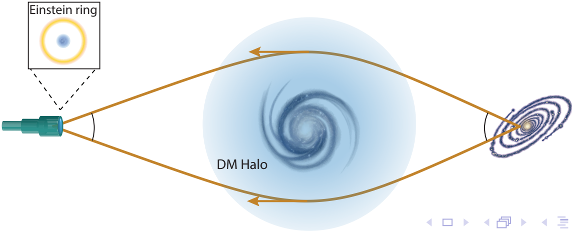
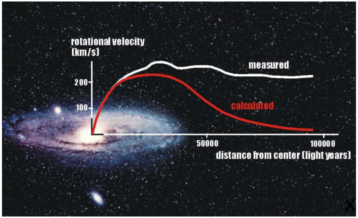
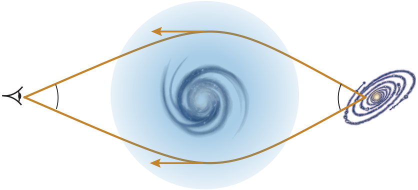


Missing mass problem

Galactic flat rotation curves



Gravitational lensing



- Weak field analysis:
  - these effects cannot be accounted for based only on the visible baryonic matter
  - for stars in galaxy, $v \lesssim 10^{-3} \quad \Rightarrow \quad \dfrac{d^2\vec{x}}{dt^2} = \vec{G}_{\rm N} + O(|\vec{G}_{\rm N}|10^{-6})$

    $\Rightarrow$ relativistic corrections *one million* times smaller than needed to impact rotation curves
- Can full general relativity explain them, without dark matter?

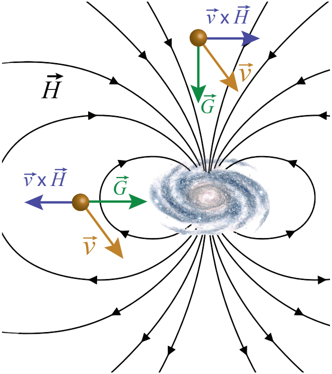
Quasi-Maxwell formalism

Stationary spacetime: $ds^2 = -e^{2\Phi}(dt - \mathcal{A}_i dx^i)^2 + h_{ij} dx^i dx^j$

Space part of time-like geodesic equation:

$$\frac{\tilde{D}\vec{U}}{d\tau} = \gamma \left[\gamma \vec{G} + \vec{U} \times \vec{H}\right]$$

($\tilde{D}/d\tau \equiv$ covariant derivative wrt to h_{ij})



Space part of null geodesic equation:

$$\frac{\tilde{D}\vec{k}}{d\lambda} = \nu \left[\nu \vec{G} + \vec{U} \times \vec{k}\right]$$

- analogous to Lorentz force $D\vec{U}/d\tau = (q/m)[\gamma \vec{E} + \vec{U} \times \vec{B}]$
- $G_i = -\Phi_{,i} \equiv$ "gravitoelectric" field
- $H^i = e^{\Phi} \epsilon^{ijk} \mathcal{A}_{k,j} \equiv$ "gravitomagnetic" field
- $h_{ij} \equiv$ space (or radar) metric

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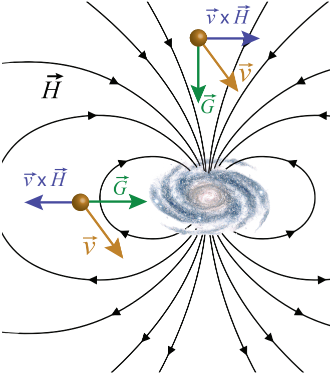

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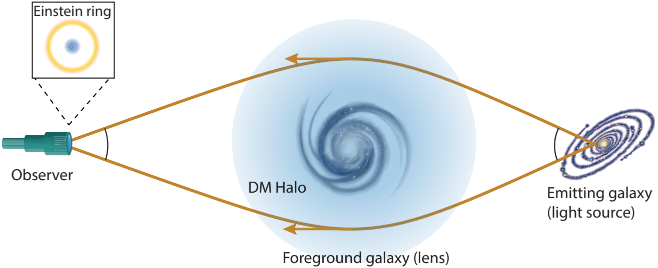
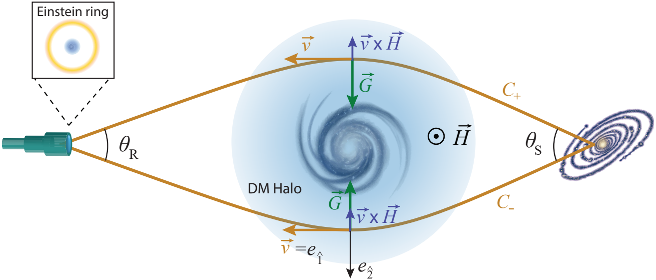
- $h_{ij} \equiv$ space (or radar) metric
    - $\vec{G} = \vec{G}_{\rm N} +$ non-linear terms
    - if GR was to explain the missing mass problem, would have to be either through $\vec{H}$, or the non-linear terms in $\vec{G}$

Gravitational lensing



- Nearly spherical lens: when the light source, lens, and observer are aligned, an Einstein ring forms in the observer's sky.
- Nearly perfect Einstein rings have been detected (e.g. "Cosmic Horseshoe", B1938+666);
- impossible to explain based only on the visible baryonic matter.
- Consistent with dark matter halos roughly spherical or moderately deformed

Gravitational lensing — $\vec{H}$ cannot be the culprit



- The gravitomagnetic field $\vec{H}$ cannot mimic dark matter
  - in the equatorial plane, GM "force" $\vec{v} \times \vec{H}$ deflects rays on both sides of the body in the same direction;
  - creates no convergence of rays along axis connecting source and lens

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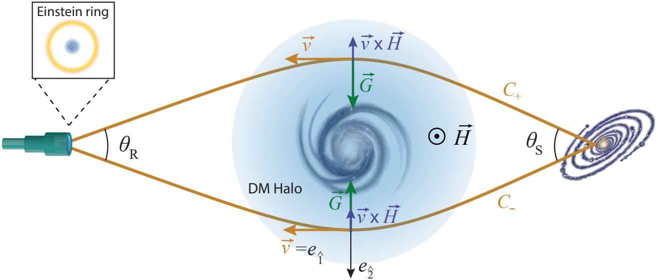

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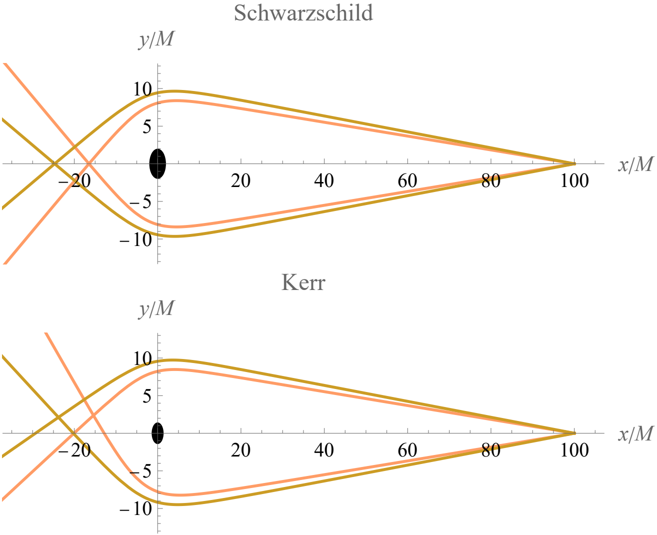
Gauss-Bonnet theorem applied to 2-surface $\mathcal{S}$ on the space manifold (of metric h_{ij}), bounded by C_+ and C_-:

$$\theta_{\rm R} = \iint_{\mathcal{S}} K d\mathcal{S} + \int_{C_+} \kappa_{\rm g} d\lambda - \int_{C_-} \kappa_{\rm g} d\lambda - \theta_{\rm S}$$

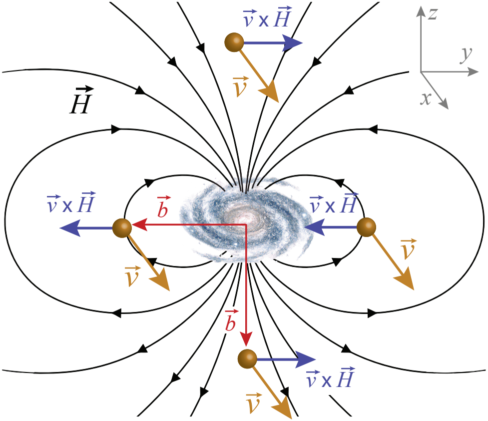
- $\kappa_{\rm g} = G^{\hat{2}} + (\vec{v} \times \vec{H})^{\hat{2}} \Rightarrow$ gravitomagnetic contributions to $\theta_{\rm R}$ cancel out

Gravitational lensing — $\vec{H}$ cannot be the culprit



- Kerr: rays starting at equal (in magnitude) angles will not cross along the lens-source axis (x-axis)
- those that do cross along the axis, arrive at different angles.

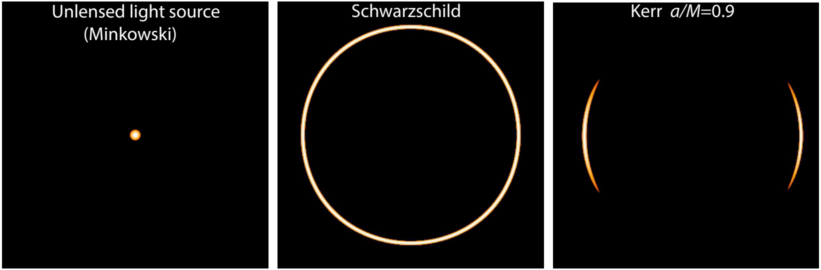
Gravitational lensing — $\vec{H}$ cannot be the culprit



Dipole-like $\vec{H}$:

- rays with impact parameter $\vec{b}$ orthogonal to the equatorial plane are deflected orthogonally to $\vec{b}$
  - creates no convergence
  - deflection direction the same for $\pm\vec{b}$; but opposite to equatorial plane

Gravitational lensing — $\vec{H}$ cannot be the culprit

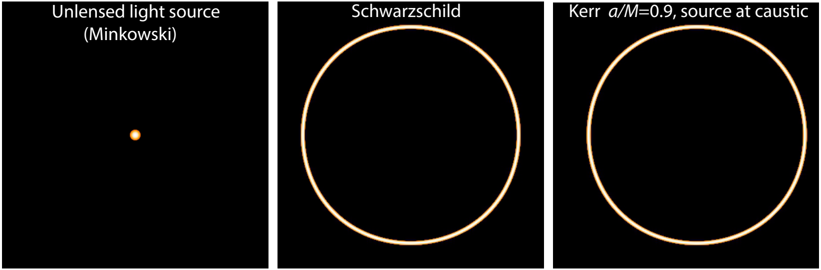


(Images generated with the GYOTO ray tracing code)

Kerr:

- for aligned setting, ring is weakened at the poles or splits into pair of arcs.
- Image shifted orthogonally to $\vec{S}$.

Gravitational lensing — $\vec{H}$ cannot be the culprit



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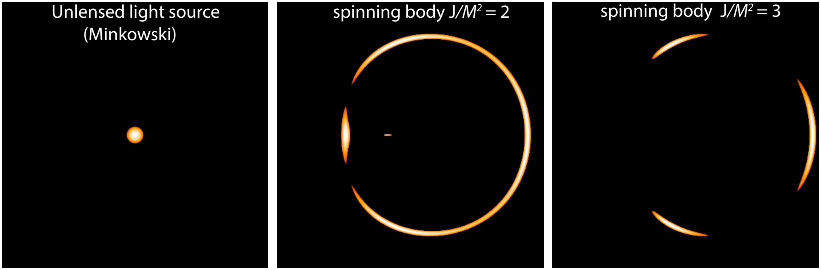
Kerr:

- for aligned setting, ring is weakened at the poles or splits into pair of arcs.
- Image shifted orthogonally to $\vec{S}$.
- for source at the primary caustic (off the optical axis), covering the whole caustic section: nearly perfect, shifted ring forms
- similar (*for source wider than caustic*) to non-aligned Schwarzschild lens
  - same angular diameter $\Rightarrow$ $\vec{H}$ does not contribute to lens power
- smaller sources: rings to do not form anywhere

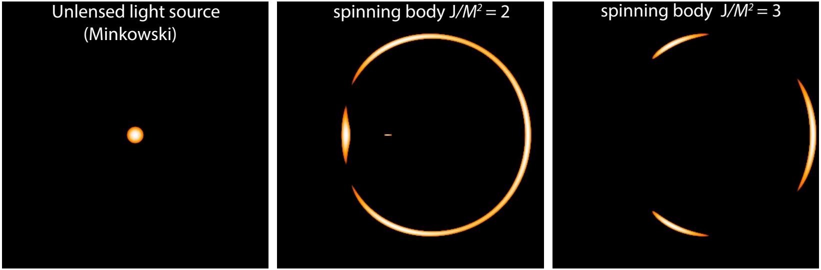
Gravitational lensing — $\vec{H}$ cannot be the culprit



(Images generated with the GYOTO ray tracing code)

- For $S/M^2 > 1$ (possible only for extended bodies, like stars) the ring's deformation is unavoidable
  - in general, the ring does not even form
- still $H < G$ (typically $H \ll G$) along the ray trajectory: $H/G \sim v_{\rm rot} R/r < 1$.

Gravitational lensing — $\vec{H}$ cannot be the culprit



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- For $S/M^2 > 1$ (possible only for extended bodies, like stars) the ring's deformation is unavoidable
  - in general, the ring does not even form
- still $H < G$ (typically $H \ll G$) along the ray trajectory: $H/G \sim v_{\rm rot} R/r < 1$.
- But a much larger $\vec{H}$ would be needed in order to have an impact on galactic rotation.

Gravitational lensing — $\vec{H}$ cannot be the culprit

Space part of time-like geodesic equation:

$$\frac{\tilde{D}\vec{U}}{d\tau} = \gamma^2 \left[\vec{G} + \vec{v} \times \vec{H}\right] \qquad v \lesssim 10^{-3} \text{ for stars in galaxy}$$

Space part of null geodesic equation:

$$\frac{\tilde{D}\vec{k}}{d\lambda} = \nu^2 \left[\vec{G} + \vec{v} \times \vec{H}\right] \qquad v = 1 \text{ for light}$$

- In order for gravitomagnetic force $\vec{v} \times \vec{H}$ to have impact on rotation curves, one needs $|\vec{H}| \sim 10^3 |\vec{G}|$
  - impossible for rotating body $H/G \sim v_{\mathrm{rot}} R/r < 1$
  - would imply, for photons, $|\vec{v} \times \vec{H}| \sim 10^3 |\vec{G}|$ (GM force 3 orders of magnitude larger than Newtonian force!)
  - $$\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{in}} \approx 2 \int_{-\infty}^{\infty} \vec{G} dt + \int_{-\infty}^{\infty} \vec{v} \times \vec{H} dt$$
    $\Rightarrow$ bending angles *orders of magnitude larger* than observed!
- $\vec{H}$ cannot be the driver of galactic dynamics

Non-linear GR effects work *against* attraction

Geodesic equation for a star in a galaxy, constrained by observed lensing to be:

$$\frac{\tilde{D}\vec{U}}{d\tau} \approx \vec{G} \qquad v_{\rm circ} = \sqrt{rG_r} + O(10^{-6})$$

- Remains only to clarify whether non-linear effects can amplify $\vec{G}$ in order to sustain the rotation curves without dark matter

Field equations for $\vec{G}$ and $\vec{H}$

$$\tilde{\nabla} \cdot \vec{G} = -4\pi(2\rho + T^{\alpha}_{\ \alpha}) + \vec{G}^2 + \tfrac{1}{2}\vec{H}^2$$
$$\tilde{\nabla} \times \vec{H} = -16\pi\vec{J} + 2\vec{G} \times \vec{H}$$

- time-time and time-space projections of $R_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\alpha}_{\ \alpha}\right)$

$$\tilde{\nabla} \times \vec{G} = 0$$
$$\tilde{\nabla} \cdot \vec{H} = -\vec{G} \cdot \vec{H}$$

- Identities

Non-linear terms $\vec{G}^2$ and $\vec{H}^2/2$ act as effective negative "energy" sources for $\vec{G}$

- *counter* the attractive effect of $2\rho + T^{\alpha}_{\ \alpha}$
- aggravate the missing mass problem

Non-linear GR effects work *against* attraction — Post-Newtonian approximation

- static point mass

$$\vec{G} = -\frac{M}{r^3}\left(1 - \frac{2M}{r}\right)\vec{r} \;<\; -\frac{M}{r^3}\vec{r} \;\equiv\; \vec{G}_{\mathrm{N}}$$

angular velocity of circular orbit:

$$\Omega_{\mathrm{circ}} = \left[\sqrt{\frac{M}{r^3}} - \frac{3}{2}\sqrt{\frac{M^3}{r^5}}\right] \;<\; \sqrt{\frac{M}{r^3}} \;\equiv\; \Omega_{\mathrm{N}}$$

$\Rightarrow$ non-linear term *slows down* rotation

- self gravitating disks (Mach-Malec, 2015)

$$\Omega_{\mathrm{circ}} = \Omega_{\mathrm{N}}\left[1 - \frac{2}{1-\delta}\Omega_{\mathrm{N}}^2 r^2 - \frac{4h_{\mathrm{N}}}{1-\delta}\right] - \frac{\mathcal{A}_\phi}{r^2(1-\delta)} \;;$$

$\delta \in [-\infty, 0] \setminus \{-1\}$

$\Rightarrow$ non-linear term *slows down* rotation

Balasin-Grumiller "galactic" model

$$ds^2 = -(dt - \mathcal{A}_\phi(r,z)d\phi)^2 + h_{ij}(r,z)dx^i dx^j$$

$$\mathcal{A}_\phi(r,z) = V_0(R - r_0) + \frac{V_0}{2}\left[d_{r_0} + d_{-r_0} - d_R - d_{-R}\right]$$

$$h_{ij}dx^i dx^j = r^2 d\phi^2 + e^{\nu(r,z)}(dr^2 + dz^2)$$

$$d_R \equiv \sqrt{r^2 + (z-R)^2}$$

$$d_{-R} \equiv \sqrt{r^2 + (z+R)^2}$$

$$d_{r_0} \equiv \sqrt{r^2 + (z-r_0)^2}$$

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$r_0 \equiv$ radius of galactic bulge; $R \equiv$ radius of galactic disk

$V_0 = \text{const.} \equiv$ dust velocity, wrt ZAMOS, in the "flat regime"

Claimed to describe, in *comoving coordinates*, a rotating dust with a flat velocity profile matching the Milky Way's. But:

- $g_{\alpha\beta}$ time-independent $\Rightarrow$ dust at rest in rigid frame
  $\Rightarrow$ incompatible with flat rotation curve (demands non-constant $\vec{\Omega}$)
- $\vec{G} = 0$, $\lim_{r\to\infty} \vec{H} = 0$ $\Rightarrow$ *asymptotically inertial rigid* frame
  - dust **static** with respect the asymptotic inertial frame (Costa *et al*, 2023)
    $\Rightarrow$ non-rotating with respect to the distant quasars
- Cannot describe any galaxy.

Balasin-Grumiller "galactic" model — non-linearity

$$ds^2 = -(dt - \mathcal{A}_\phi(r,z)d\phi)^2 + h_{ij}(r,z)dx^i dx^j$$

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- $\vec{G} = 0$ and $\vec{J} = 0$ (comoving coordinates):

$$\tilde{\nabla} \cdot \vec{G} = -4\pi\rho + \frac{1}{2}\vec{H}^2 = 0$$

- Linearizing yields empty space equation $\rho = 0$
  - purely non-linear solution
    (no linear, or Newtonian limit)

- extreme repulsive action of $\vec{H}^2/2$ *cancels out* exactly the attractive effect of the dust's energy density ρ ("freezes" the dust!)

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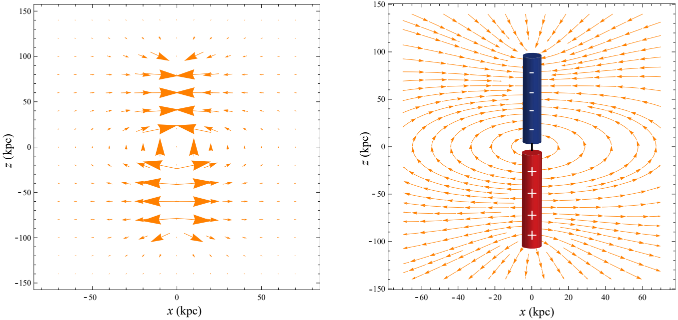
    - purely non-linear solution (no linear, or Newtonian limit)
  - extreme repulsive action of $\vec{H}^2/2$ *cancels out* exactly the attractive effect of the dust's energy density ρ ("freezes" the dust!)
  - $\vec{H}$ generated by singularities along the axis, not by motion of matter.

BG "galactic" model — gravitomagnetic field $\vec{H}$



- This is the gravitomagnetic field of a pair of oppositely charged NUT rods along z-axis, of gravitomagnetic charges

$$Q_{\mathrm{NUT}} = \frac{1}{4\pi}\int_{\mathcal{S}} \mathrm{d}\boldsymbol{\mathcal{A}} = \frac{1}{4\pi}\int_{\mathcal{S}} \vec{H}\cdot \vec{d\mathcal{S}} = \mp V_0(R-r_0)/2$$

- matches the magnetic field $\vec{B}_{\mathrm{rods}}$ of a pair of magnetically charged rods, identifying $V_0/2$ with charge density λ_{M}: $(B_{\mathrm{rods}})_i \overset{\lambda_{\mathrm{M}}\to V_0/2}{=} H_i$. (length of the rods approximately equal to galactic diameter...)

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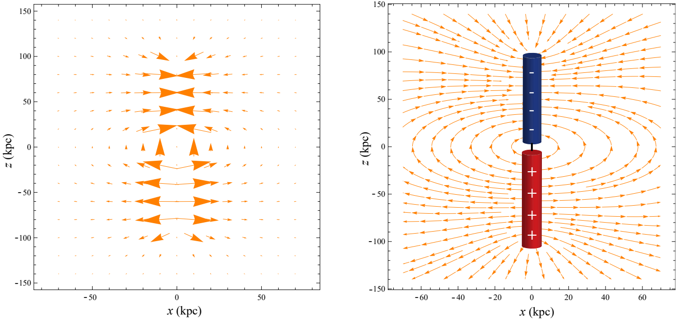

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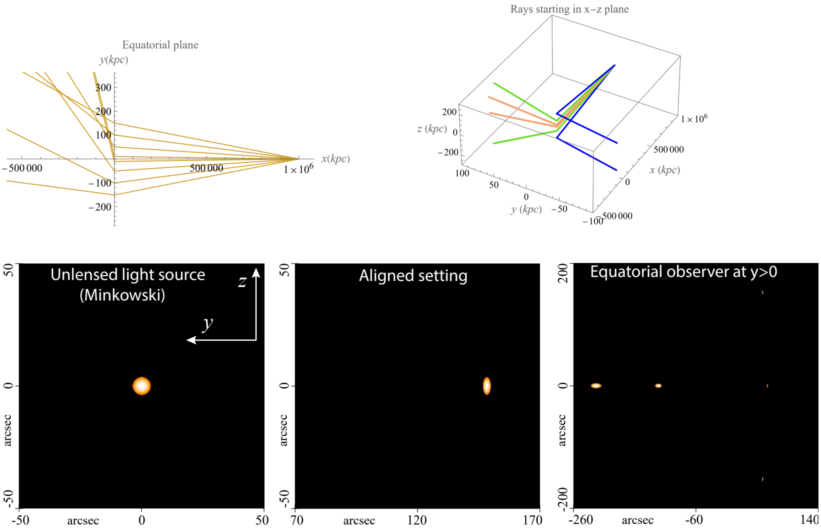
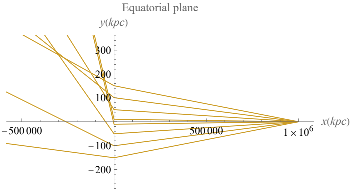
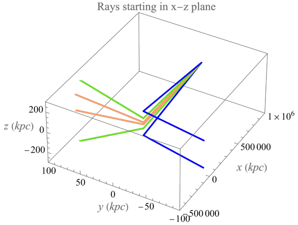
- matches the magnetic field $\vec{B}_{\mathrm{rods}}$ of a pair of magnetically charged rods, identifying $V_0/2$ with charge density λ_{M}: $(B_{\mathrm{rods}})_i \overset{\lambda_{\mathrm{M}}\to V_0/2}{=} H_i$.
(length of the rods approximately equal to galactic diameter...)
- Plus a curl-free term in $\boldsymbol{\mathcal{A}} \Rightarrow$ potential of an infinite spinning cosmic string, of angular momentum per unit mass $j = -V_0(R-r_0)/4$.

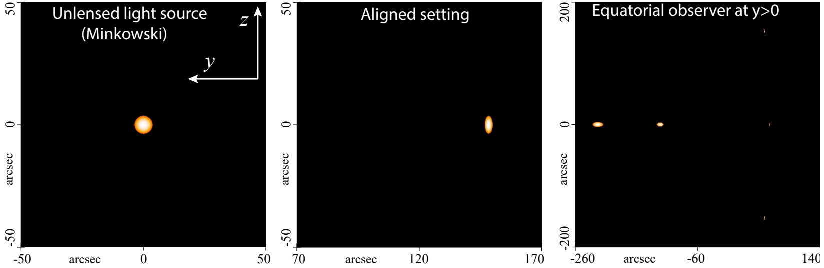


(Images generated with the GYOTO ray tracing code)

- Rays do not cross along optical axis for aligned setting
- Multiple images at equator for $y > 0$, where light rays cross
- No Einstein rings







(Images generated with the GYOTO ray tracing code)

- Deflection angles much larger than observed (spherical lens with Milky Way's mass $M = 10^{12} M_\odot$, yields Einstein ring $18\mathrm{arcsec}$ wide).

BG "galactic" model — origin of claimed rotation curves

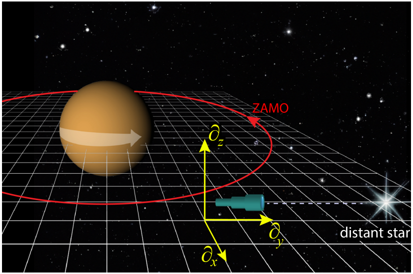
- BG velocity is measured wrt zero angular momentum observers (ZAMOs)

(azimuthal angular momentum: u_ϕ)

- ZAMOs: $(u_{\mathrm{Z}})_\phi = 0$
- have angular velocity

  $$\Omega_{\mathrm{Z}} \equiv \frac{u_{\mathrm{Z}}^\phi}{u_{\mathrm{Z}}^0} = -\frac{g_{0i}}{g_{00}} = \frac{e^{2\Phi}\mathcal{A}_\phi}{g_{\phi\phi}}$$

  relative to asympt. inertial frame
- are *dragged* by $\boldsymbol{\mathcal{A}}$



Kerr spacetime

- at the horizon, ZAMO angular velocity coincides with that of the horizon (ZAMO *comoves* with the horizon)

$$\Omega_{\mathrm{Z}}(r_+) = \frac{a}{r_+^2 + a^2} = \Omega_{\mathrm{H}}$$

- by confusing the ZAMOs with observers at rest relative to distant stars, one would conclude that Kerr black holes do not rotate!

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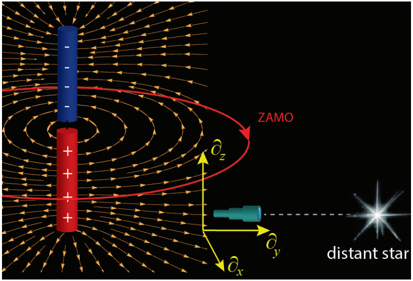

- ZAMOs: $(u_{\rm Z})_\phi = 0$
- have angular velocity

  $$\Omega_{\rm Z} \equiv \frac{u^\phi_{\rm Z}}{u^0_{\rm Z}} = -\frac{g_{0i}}{g_{00}} = \frac{e^{2\Phi}\mathcal{A}_\phi}{g_{\phi\phi}}$$

  relative to asympt. inertial frame
- are *"dragged"* by $\boldsymbol{\mathcal{A}}$
  - artificially large gm potential $\boldsymbol{\mathcal{A}}$ created by the singularities
  - ZAMOs misunderstood as at rest relative to the axis' asymptotic rest frame
  - the velocity curve obtained: $v^\phi_{r{\rm Z}} = -\sqrt{-g^{00}}\Omega_{\rm Z}$ is but minus the velocity of the ZAMOs with respect *to the rigid asymptotic inertial frame*
  - Is the ZAMOs, *not* the dust (static in such frame), that rotates

Conclusions

We have demonstrated that, in light of the experimentally measured galactic rotation curves and gravitational lensing, relativistic effects cannot resolve (or even be relevant) to the missing mass problem

- gravitational lensing rules out the gravitomagnetic field as a player;
- non-linear effects only aggravate the need for dark matter (besides negligible in realistic models)
- general relativistic "galactic" models in the literature originate from pathologies:
  - unphysical singularities, generating artificially large gravitomagnetic fields (ruled out by the observed gravitational lensing);
  - in "exact" models, rotation curves moreover computed relative to unsuitable reference observers — the ZAMOs, being dragged by the singularities
    (e.g. BG model is actually static, does not even rotate!)

References:

Costa-Natário arXiv:2312.12302 (PRD to appear),
Costa et al PRD 108 (2023) 4, 044056 [arXiv:2303.17516]

Thank you!