I-Love-Q, but δM too

Eneko Aranguren with the collaboration of

José Antonio Font, Nicolas Sanchis-Gual, Raül Vera

Semi-analytical perturbative approach

Background configuration + Perturbations

We consider two scenarios:

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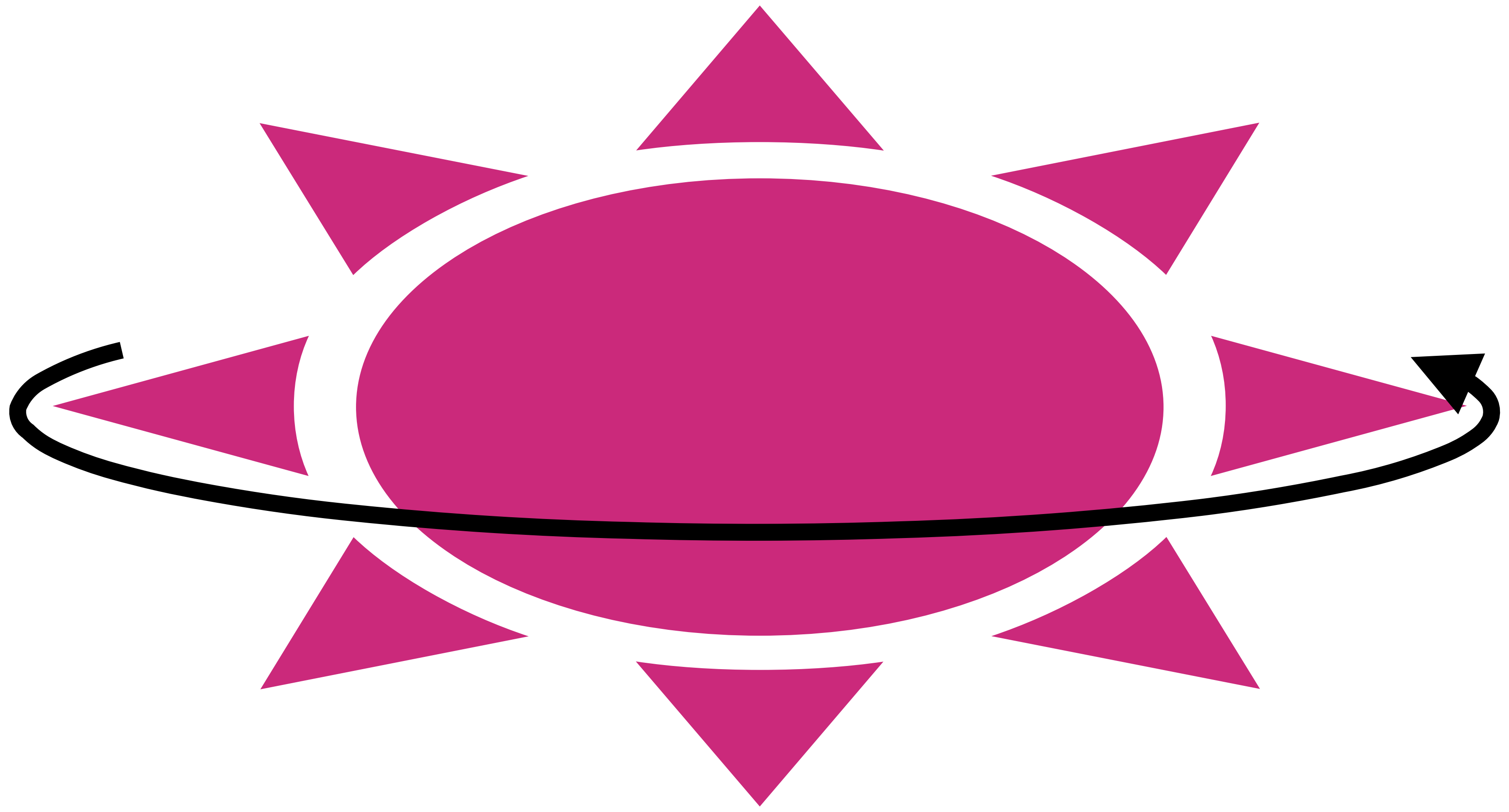
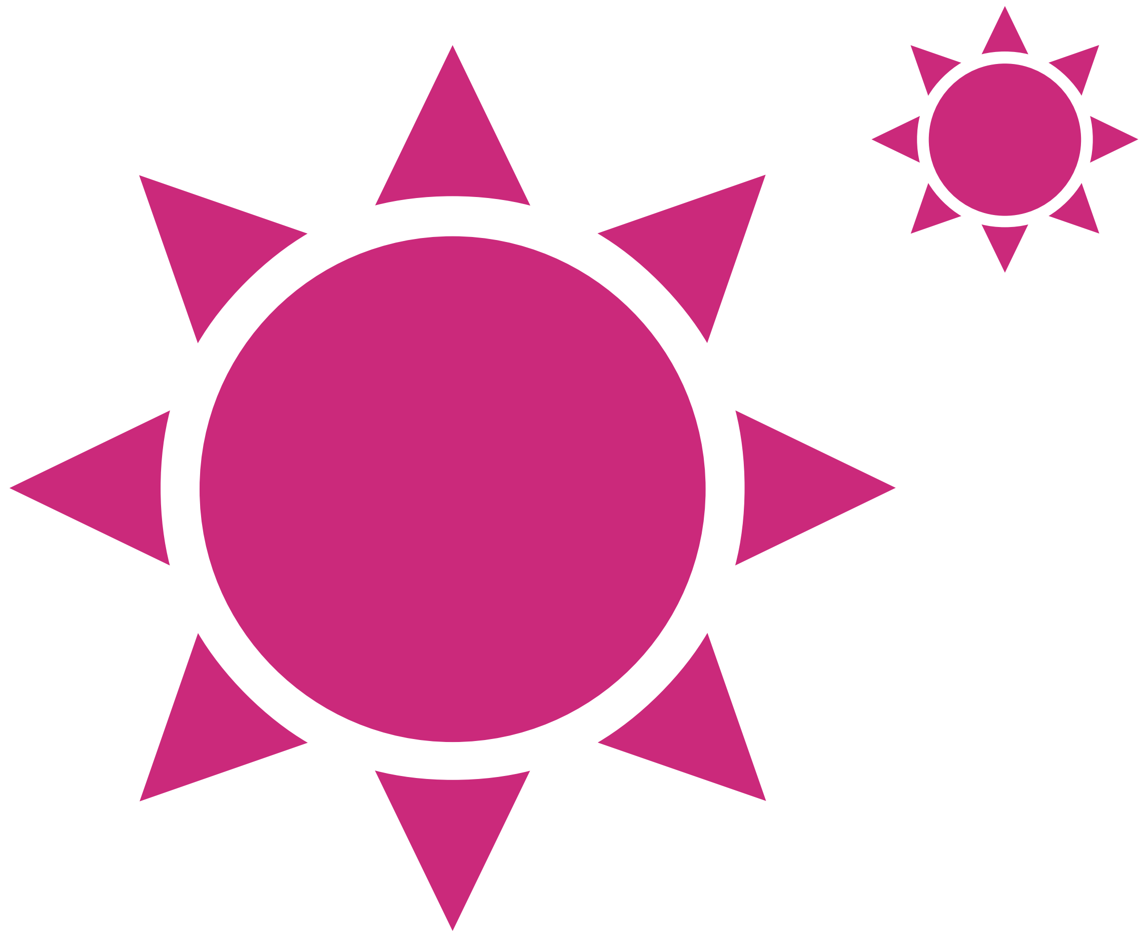
Background configuration + Perturbations

We consider two scenarios:

$\boxed{\text{Isolated \& Rotating}}$ to 2^{nd} order



$\boxed{\text{Tidal Field}}$ to 1^{st} order



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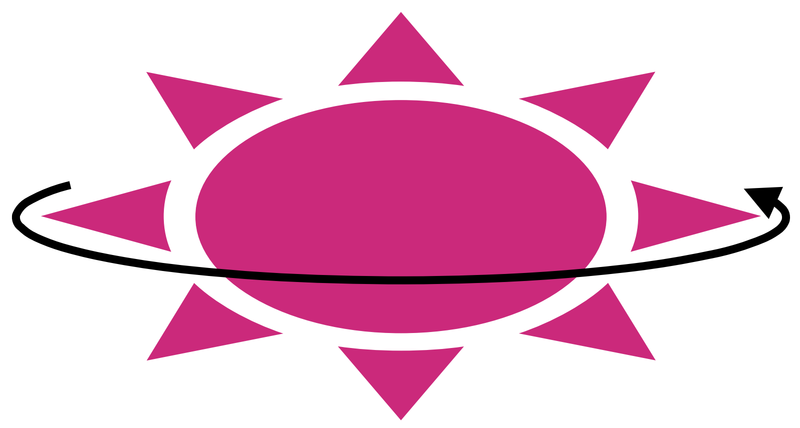
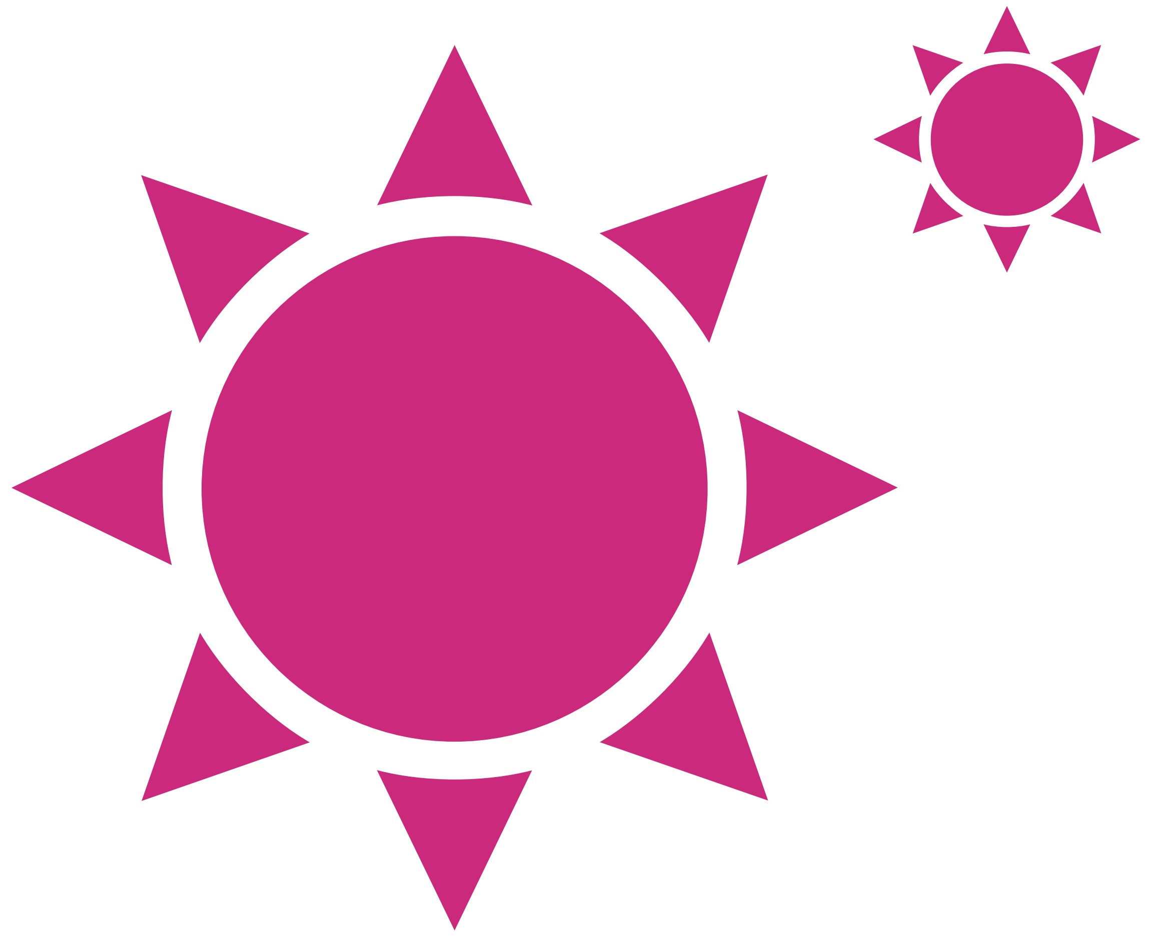


M_0 (Background)

Tidal Field to 1^{st} order



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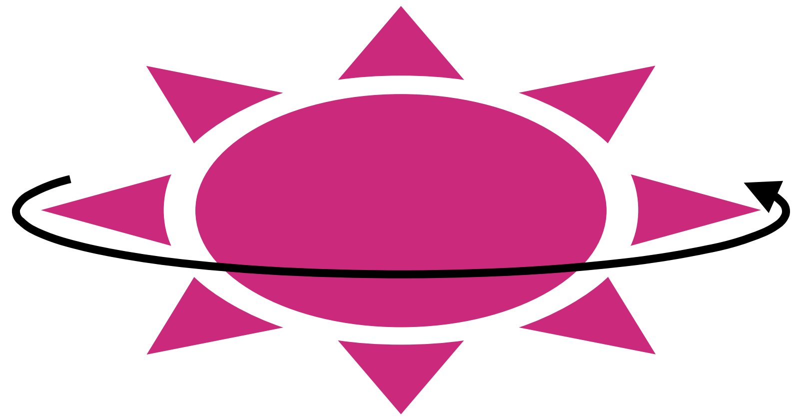
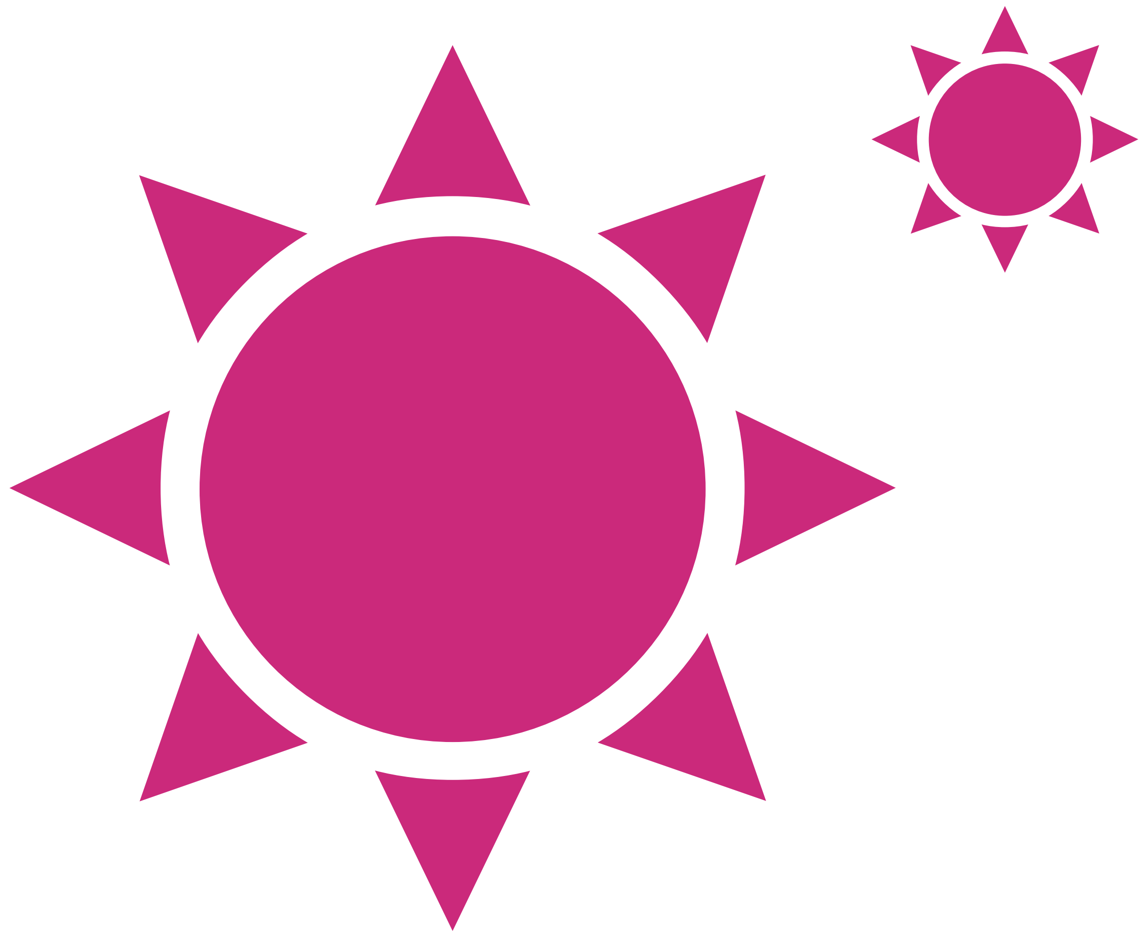


Tidal Field to 1$^{\text{st}}$ order



M_0

I_S (1$^{\text{st}}$ order)

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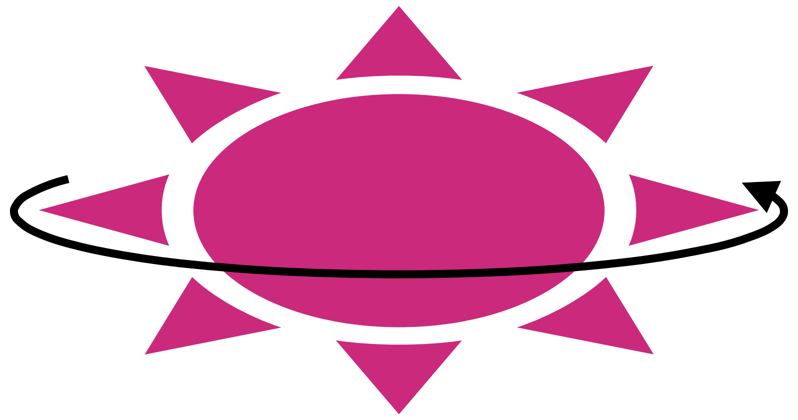
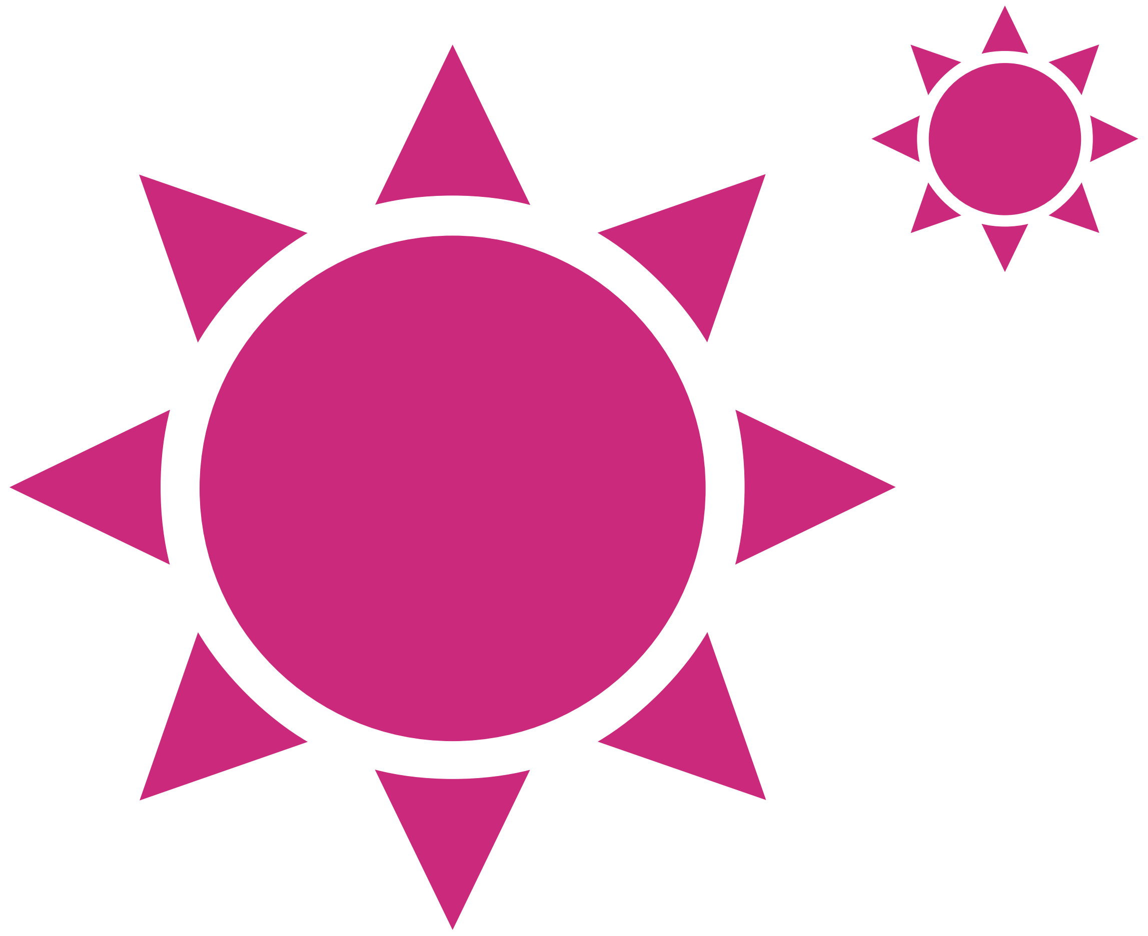


M_0 δM (2^{nd} order)

I_S

Tidal Field to 1^{st} order



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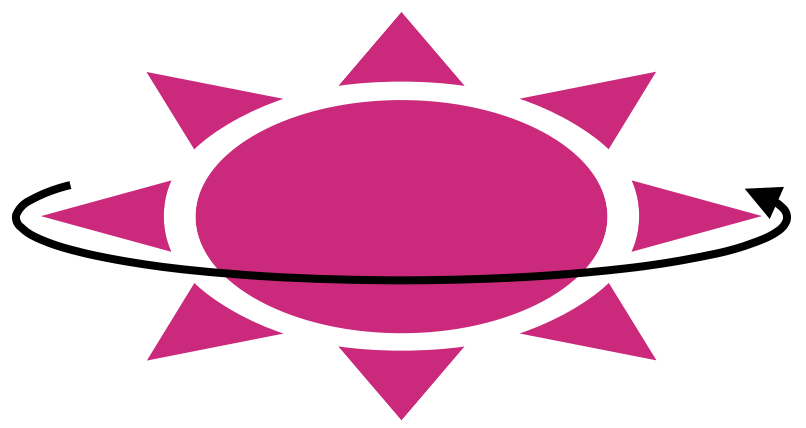

$M_S = M_0 + \Omega_S^2\, \delta M$ (2^{nd} order)

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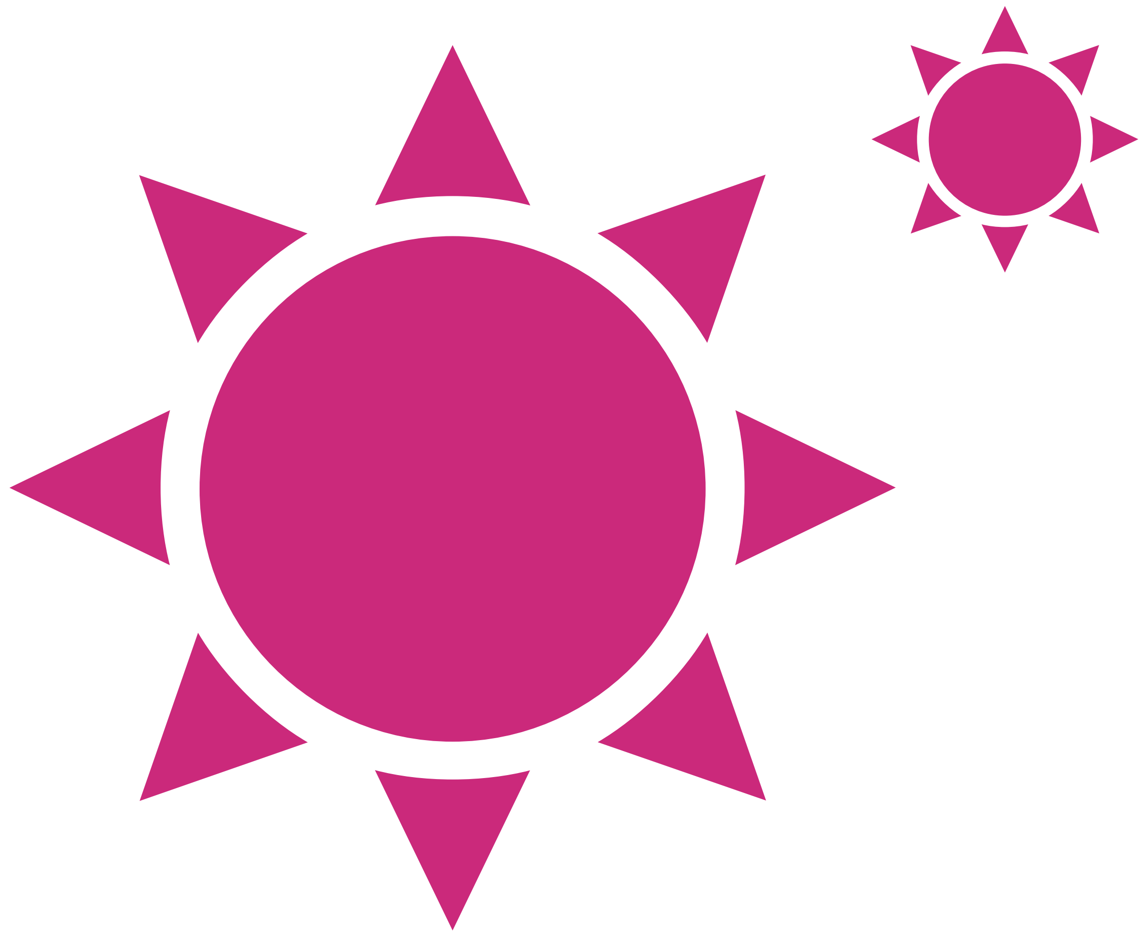

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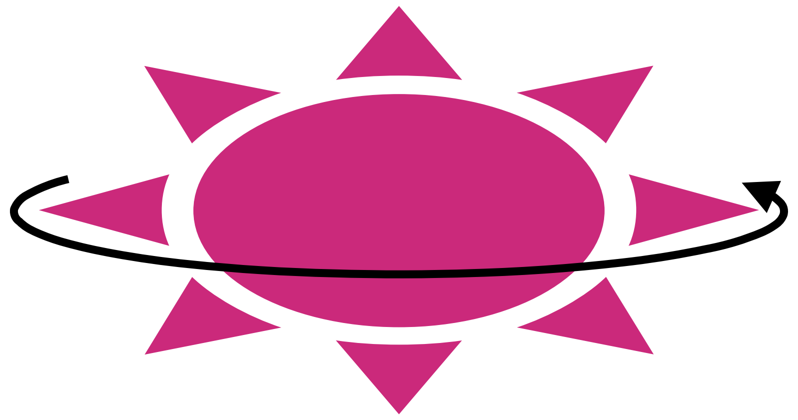

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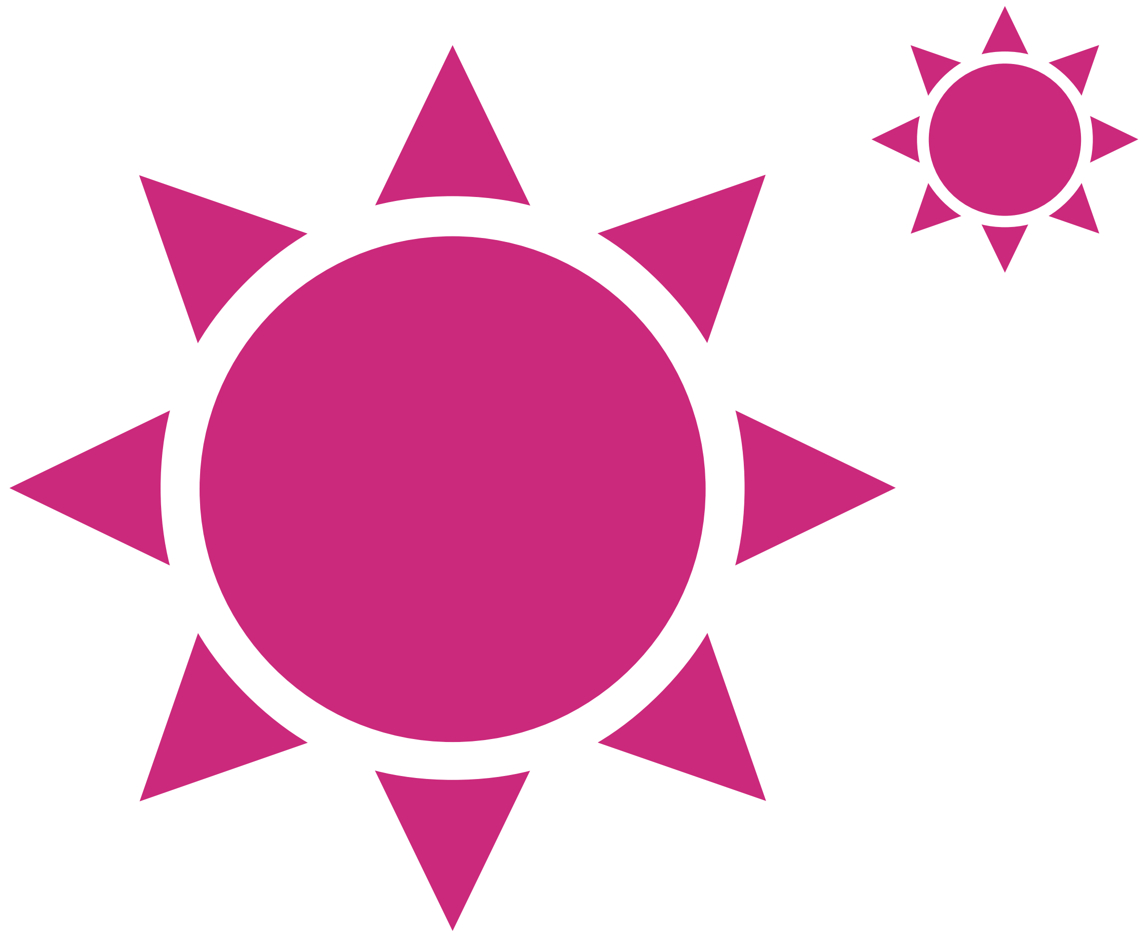

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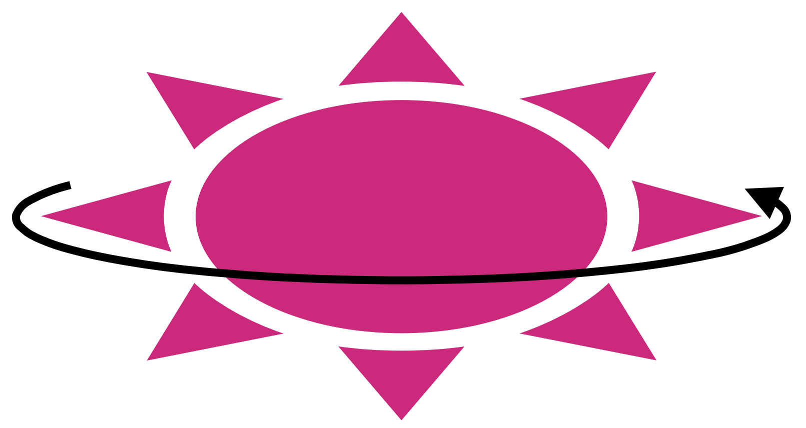
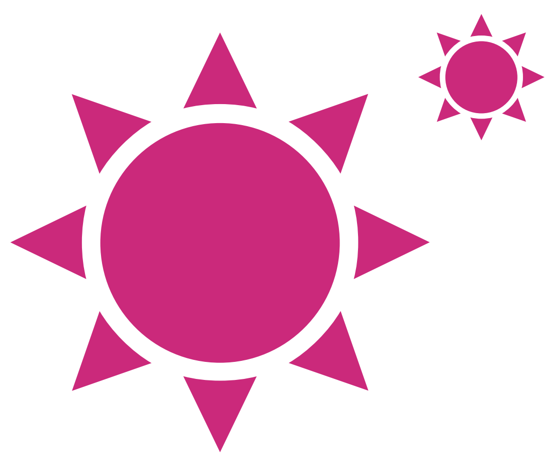
Isolated & Rotating 

Tidal Field 

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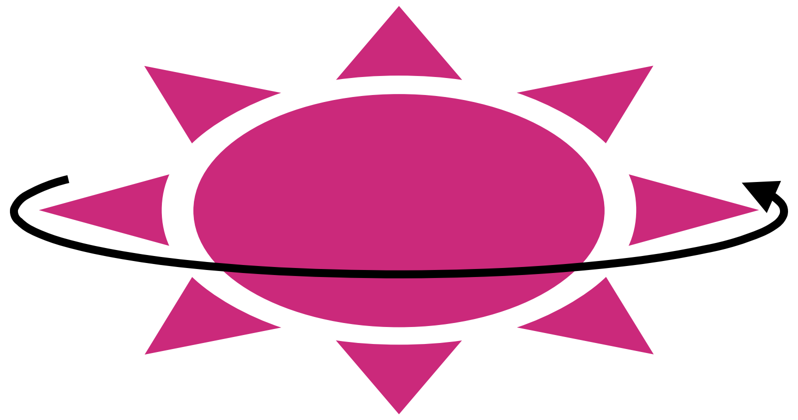

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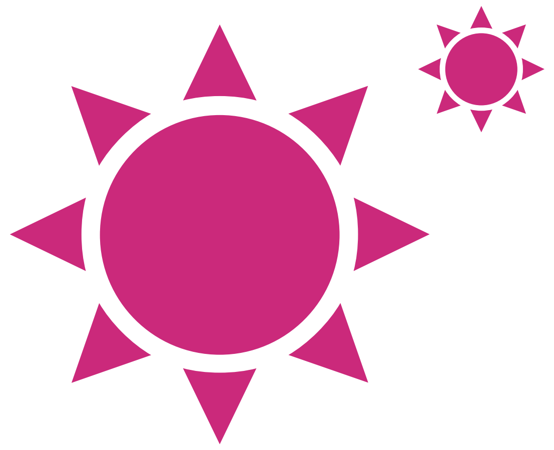

$$M_0$$

λ_S $\quad$ (1^{st} order)

Standard I-Love-Q relations

$$\bar{I} := \frac{I_S}{M_0^3}$$

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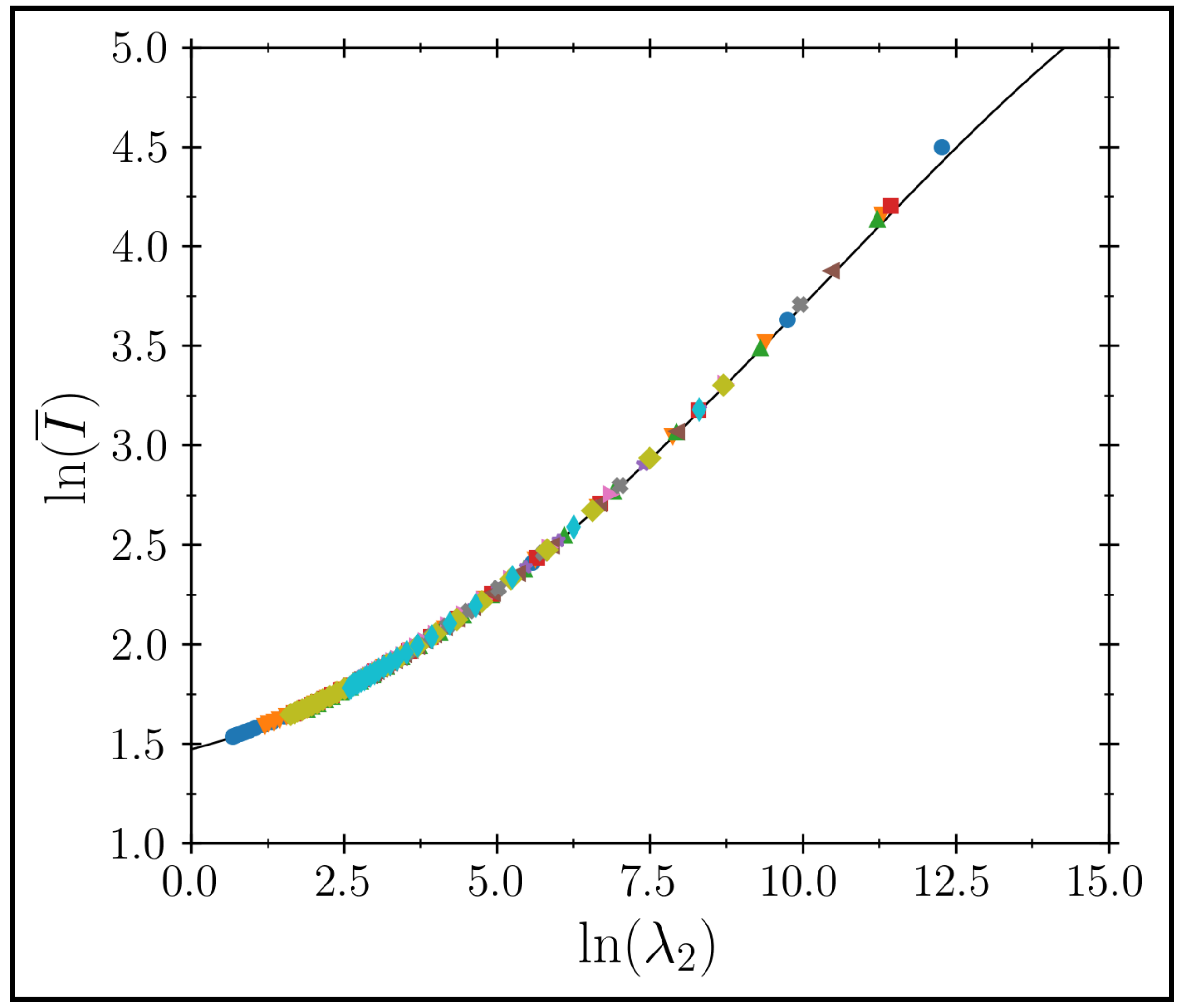
[Yagi & Yunes (2014), …]



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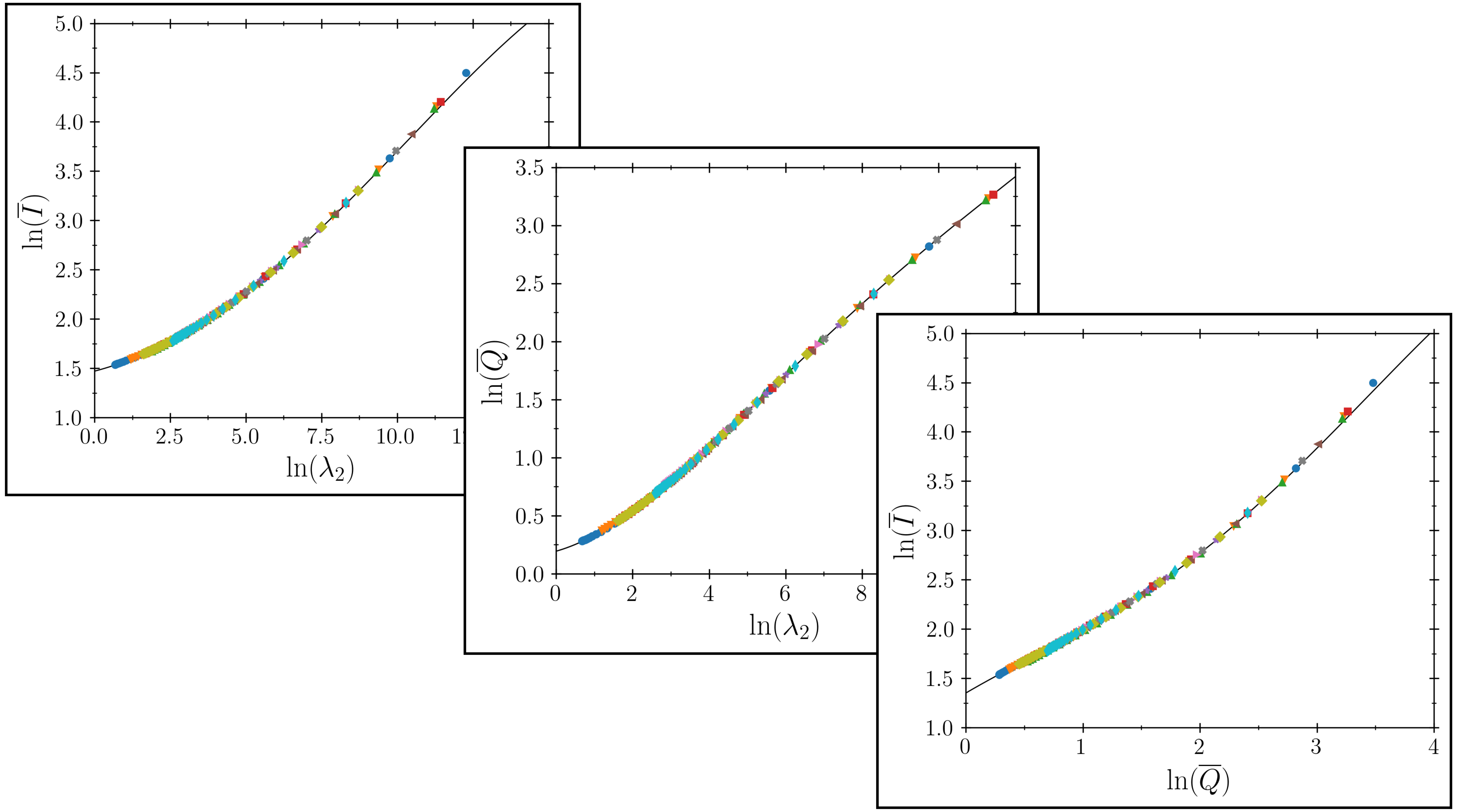

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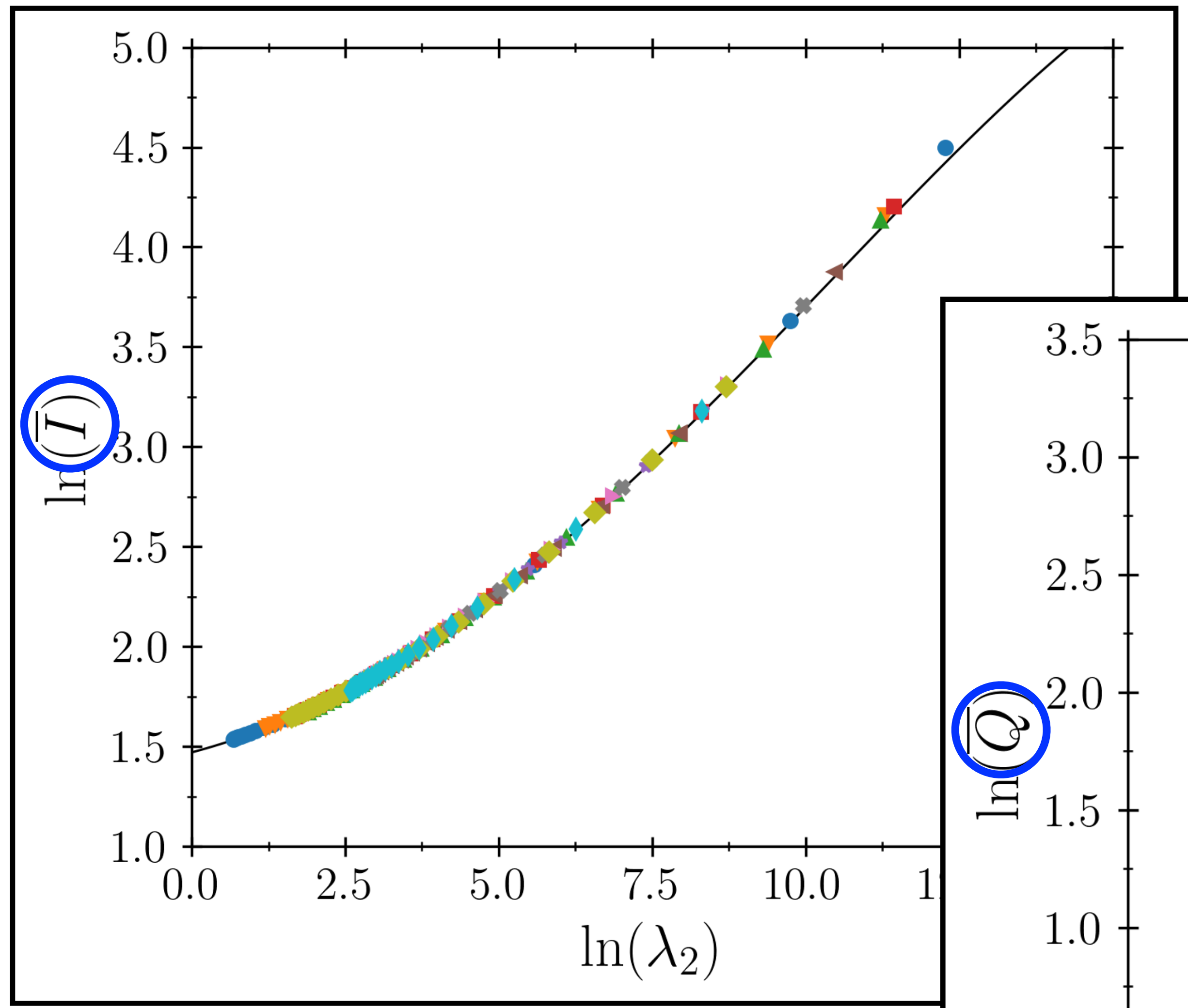
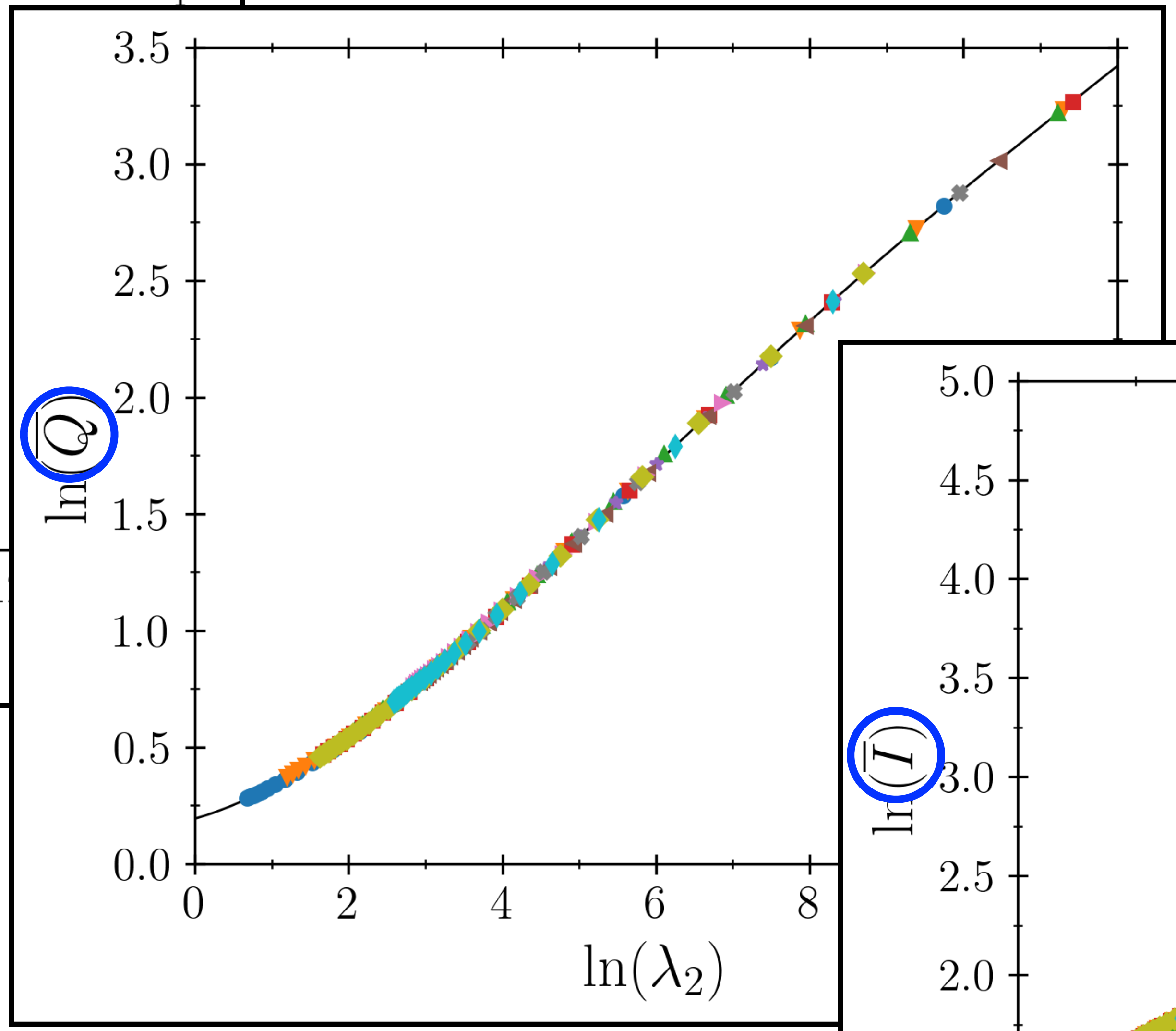
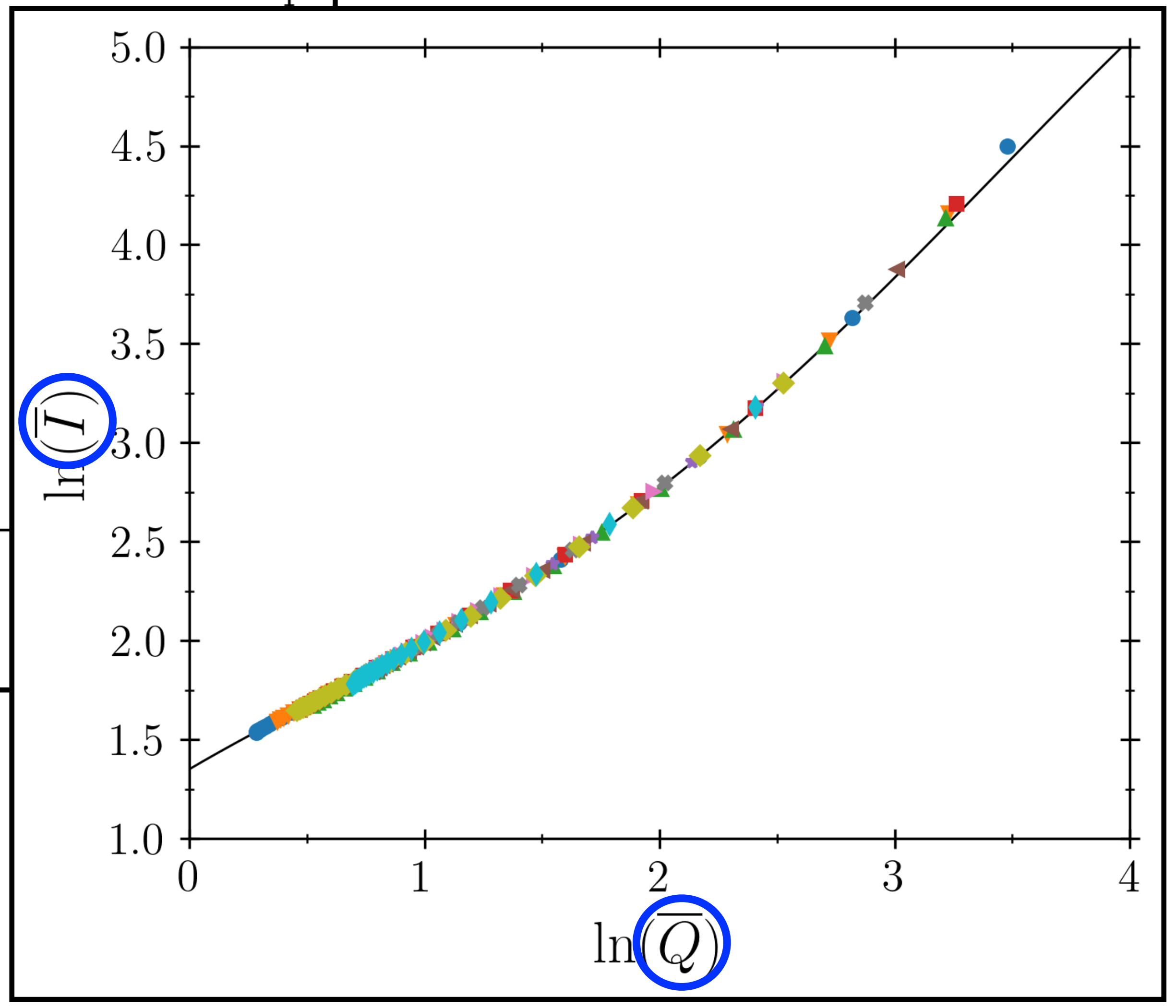


Extract “barred” quantities





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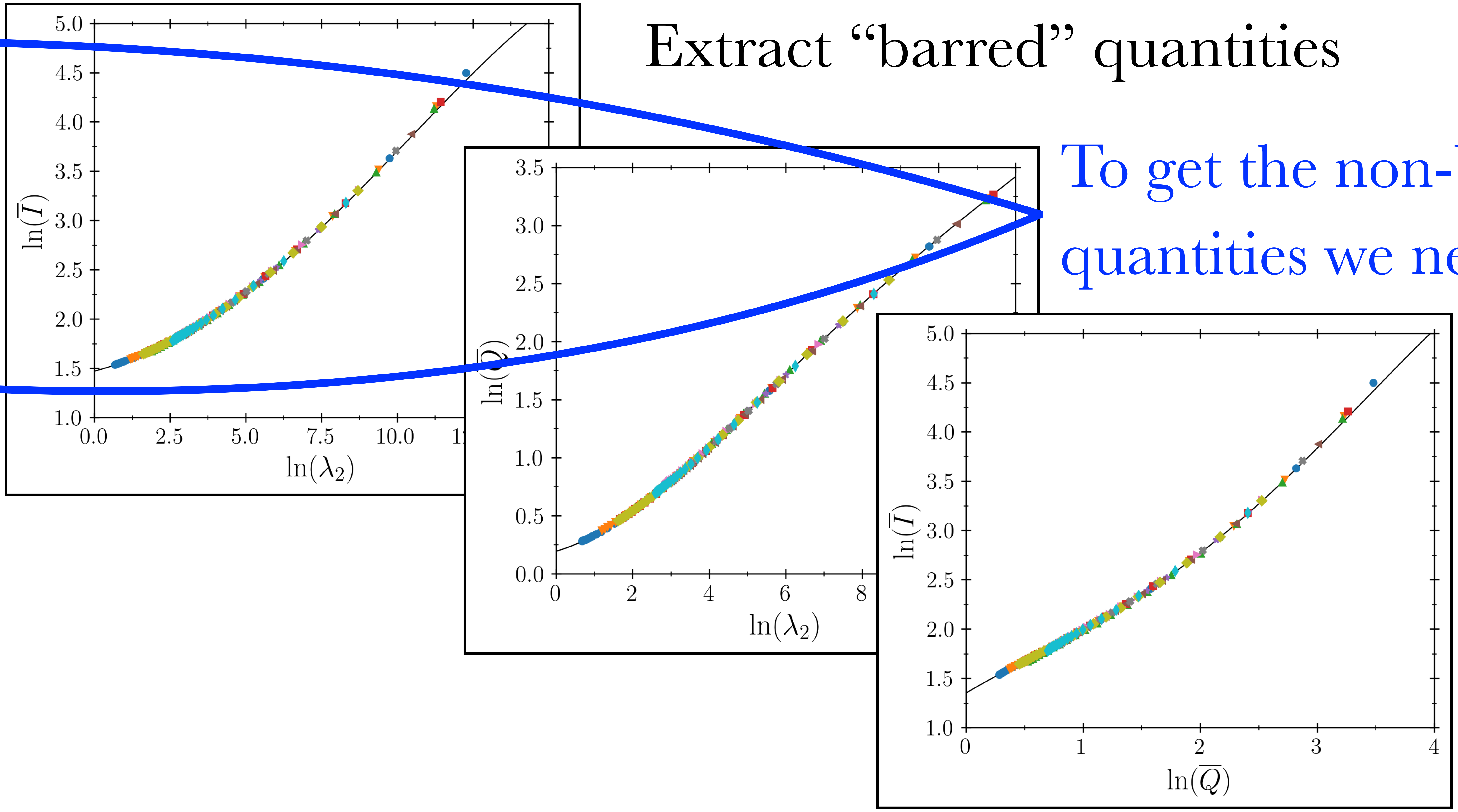

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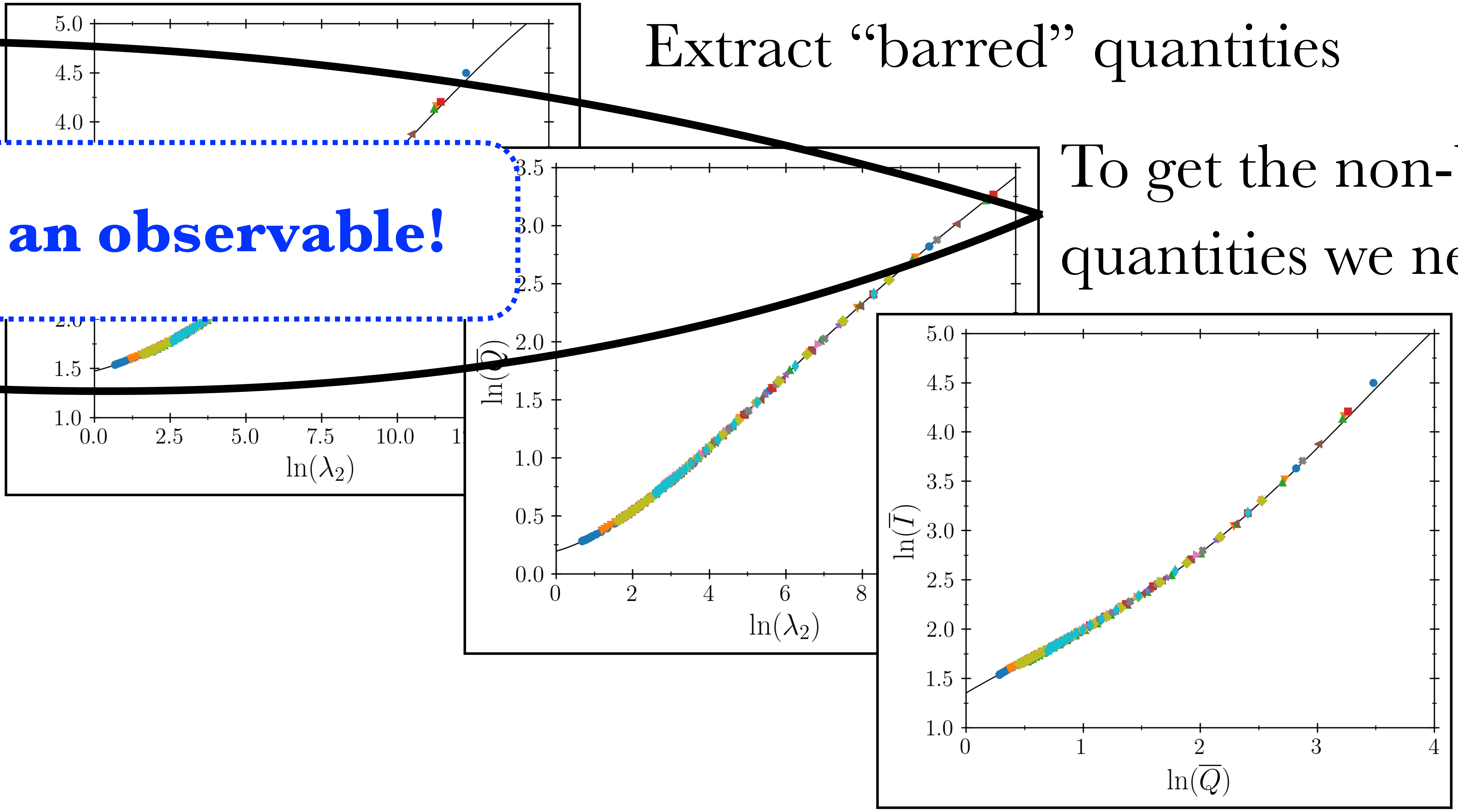

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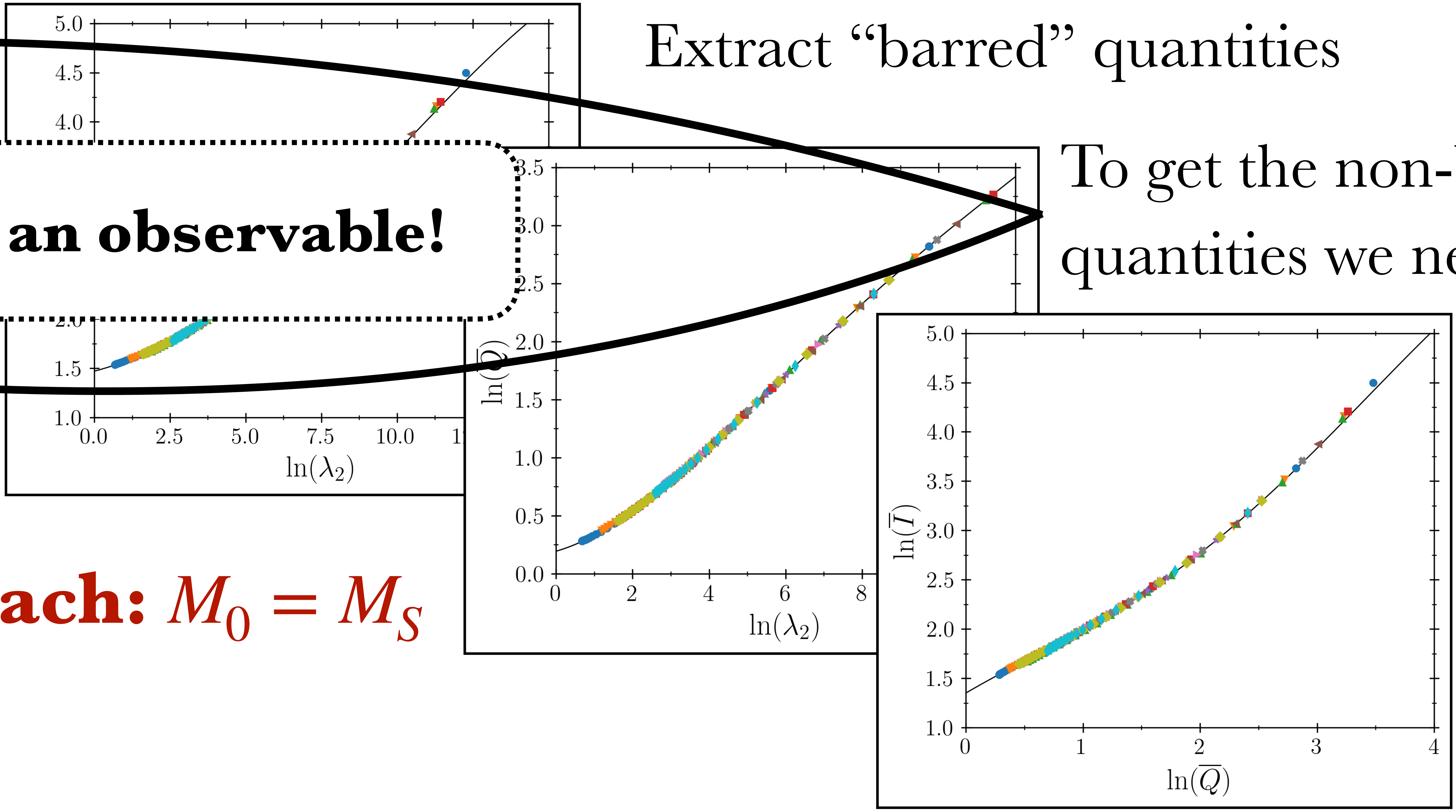

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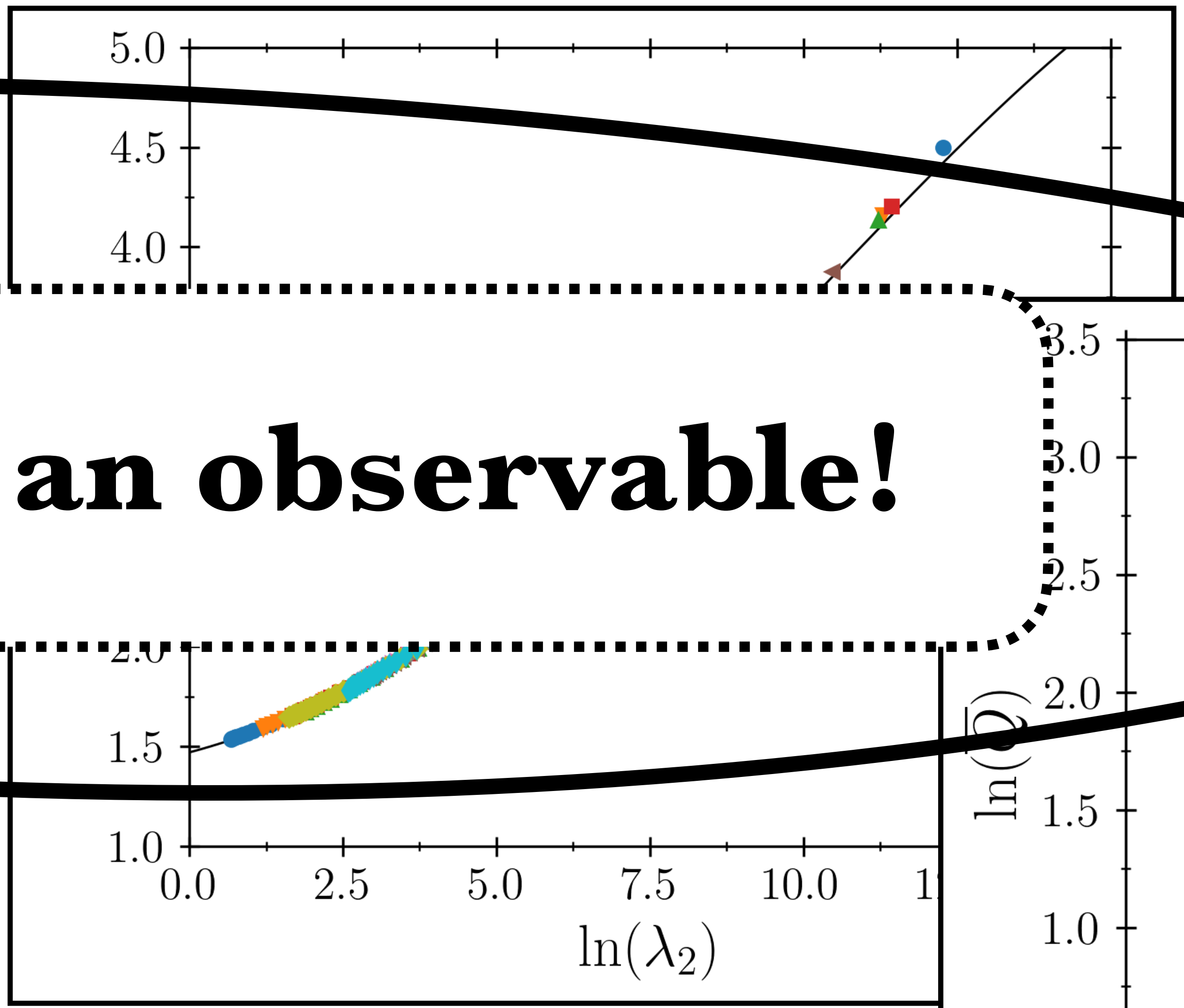

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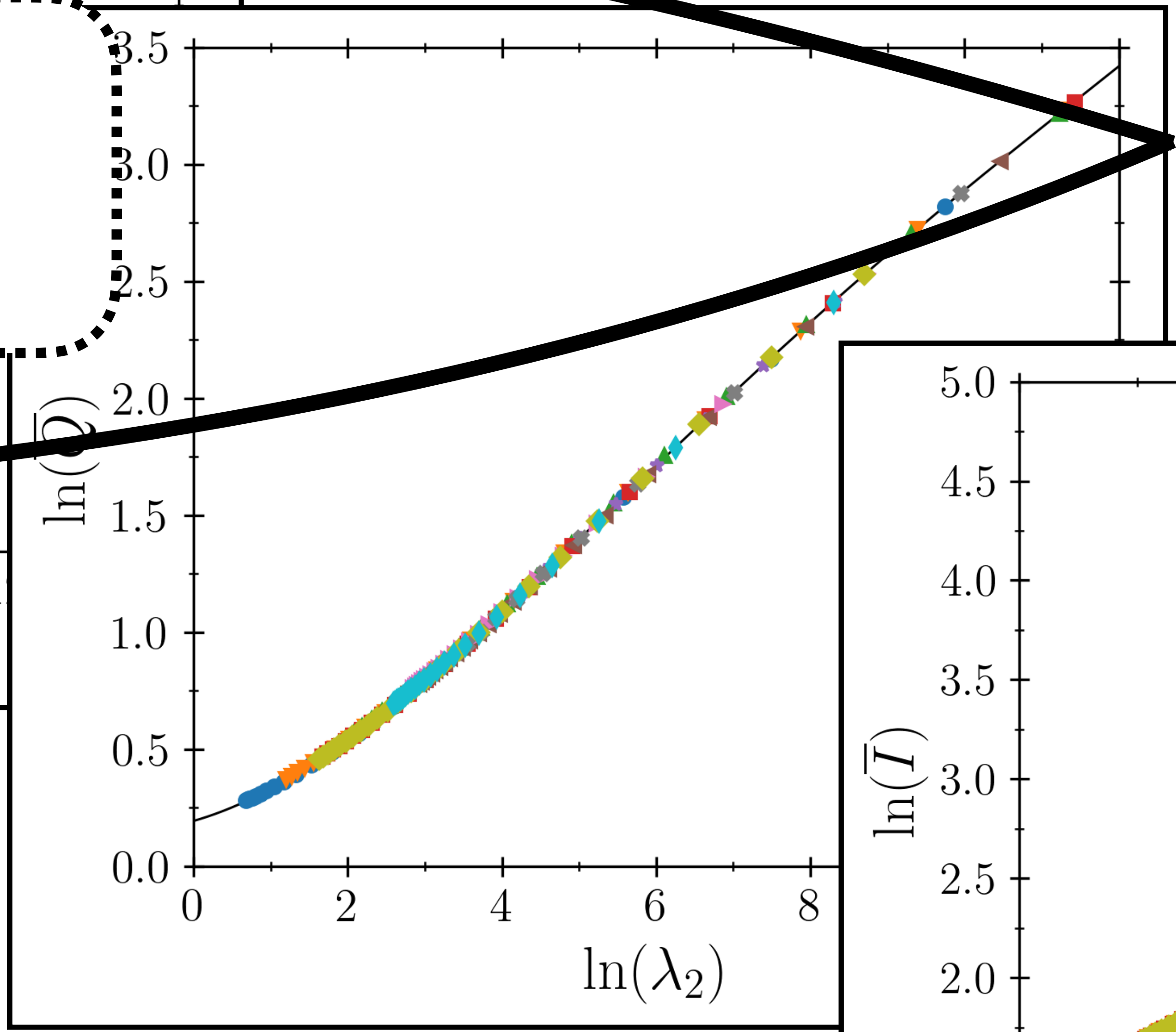

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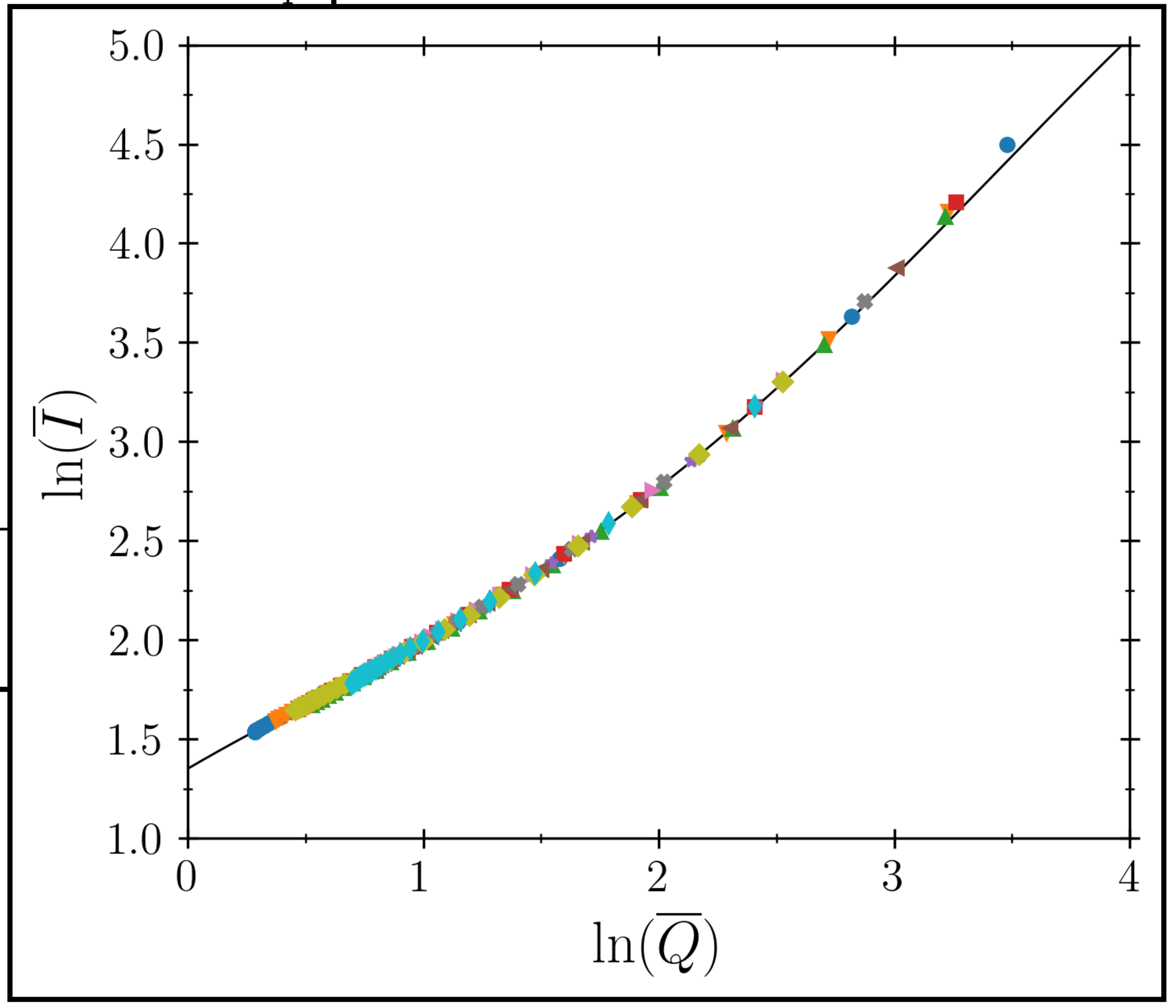

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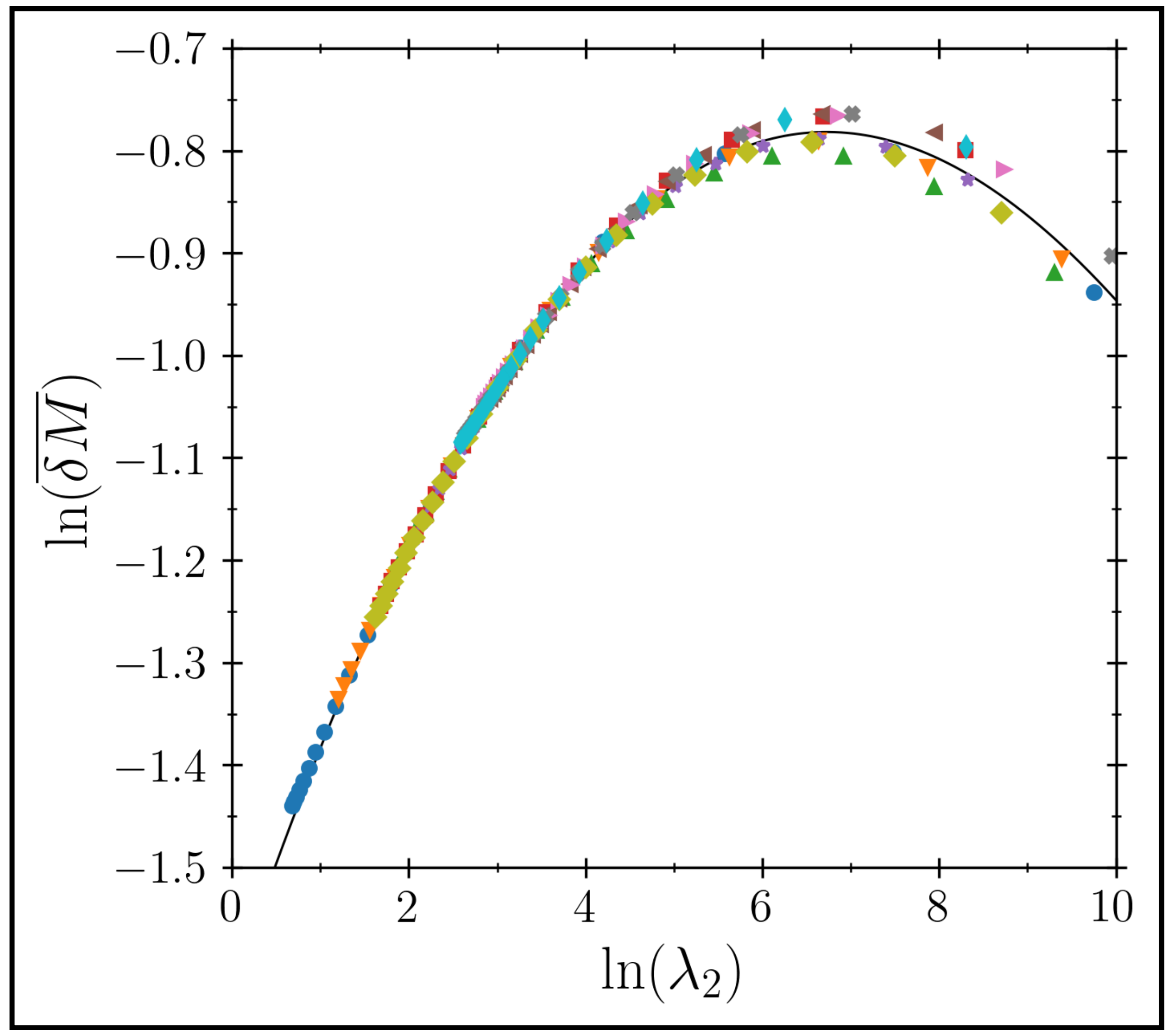
Breaks universality [Doneva et al. (2014)]

Extension: I-Love-Q-δM

"Universal" relations for $\overline{\delta M}$ [Reina, Sanchis-Gual, Vera, Font (2017)]



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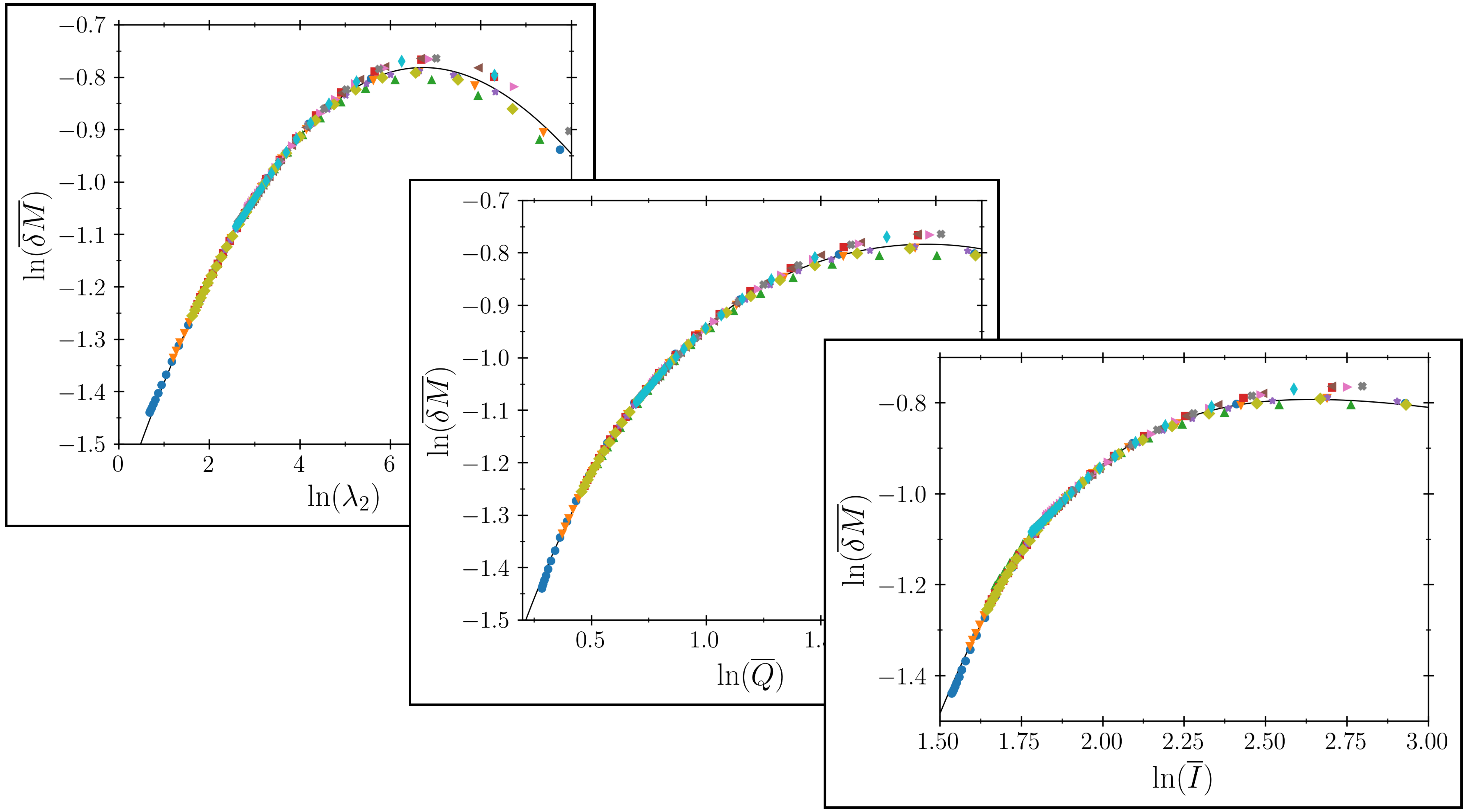

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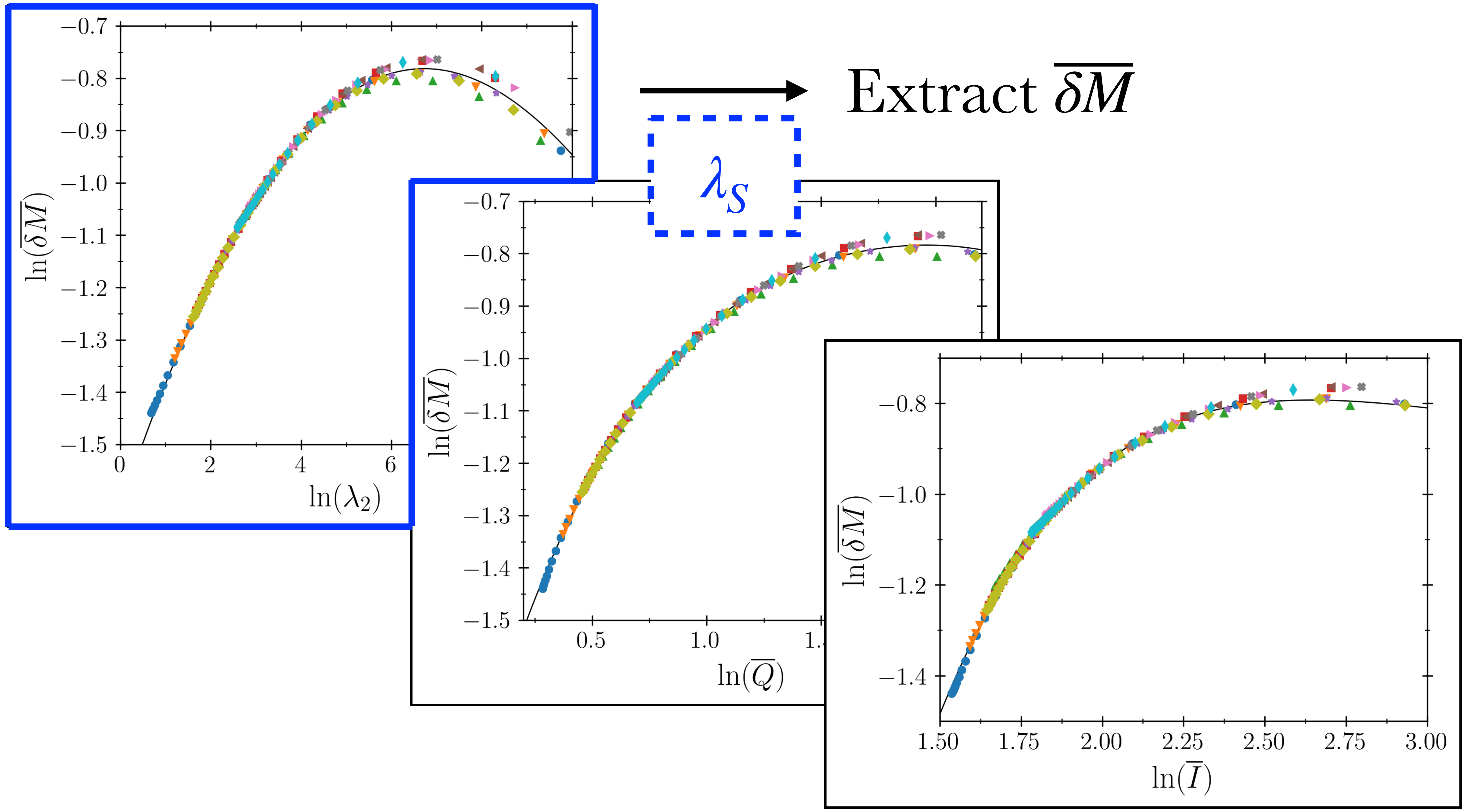

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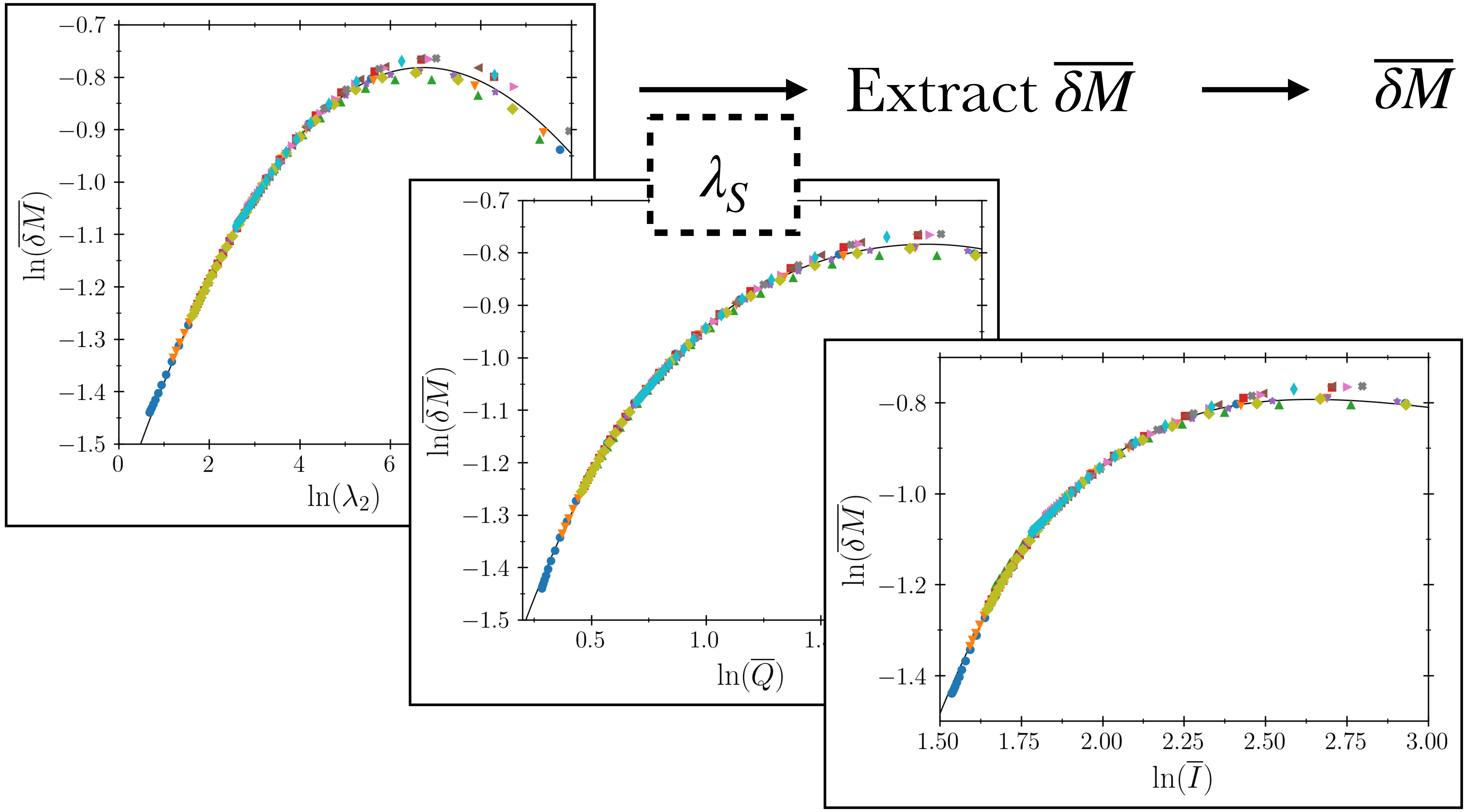

$\longrightarrow$ Extract $\overline{\delta M}$ $\longrightarrow$ $\overline{\delta M} := \dfrac{M_S - M_0}{\Omega_S^2\, \bar{I}^2\, M_0^3}$

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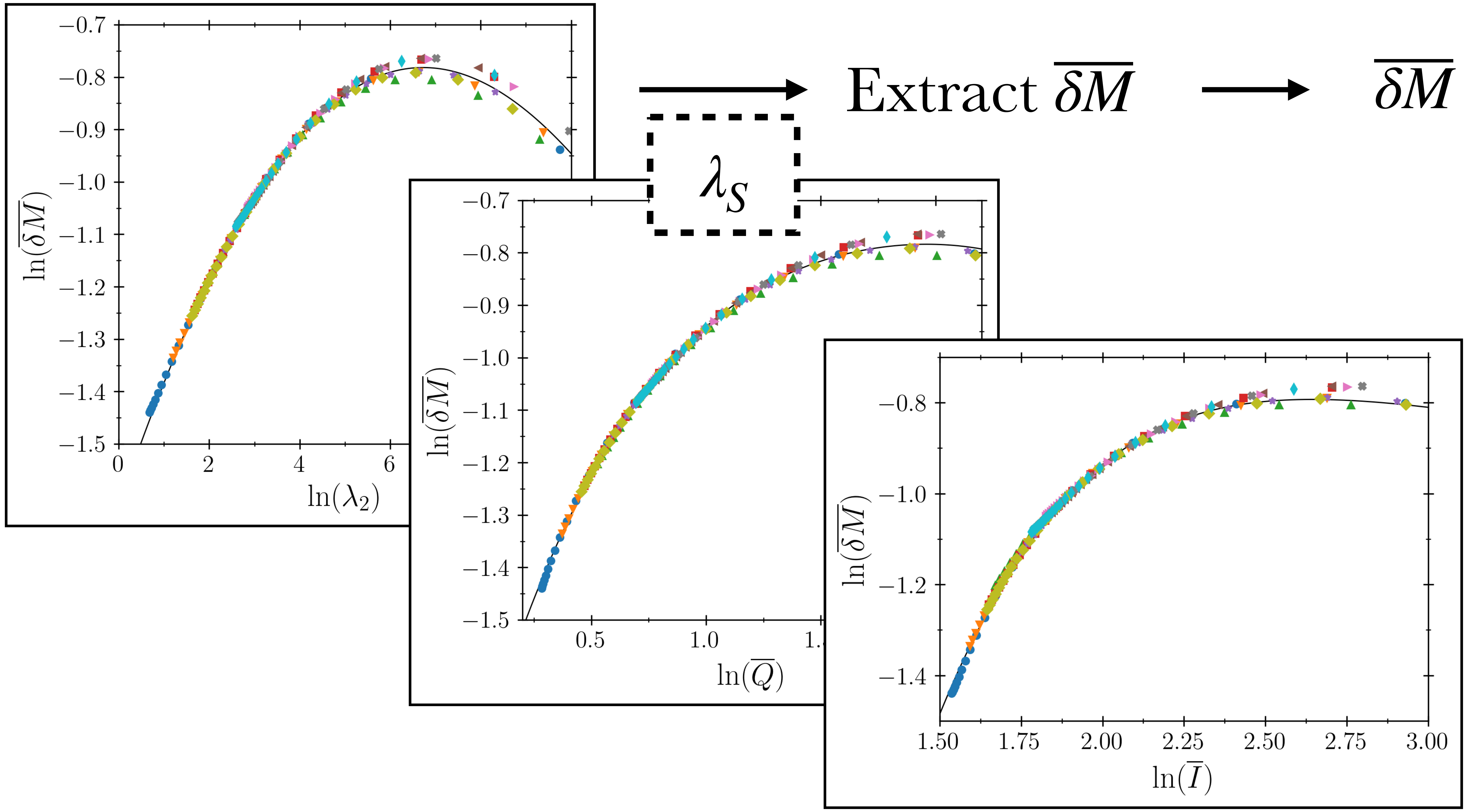

$$\xrightarrow{\boxed{\lambda_S}} \text{Extract } \overline{\delta M} \longrightarrow \overline{\delta M} := \frac{M_S - M_0}{\Omega_S^2\, \overline{I}^2\, M_0^3} \xrightarrow{\boxed{M_S,\, \Omega_S}} M_0$$

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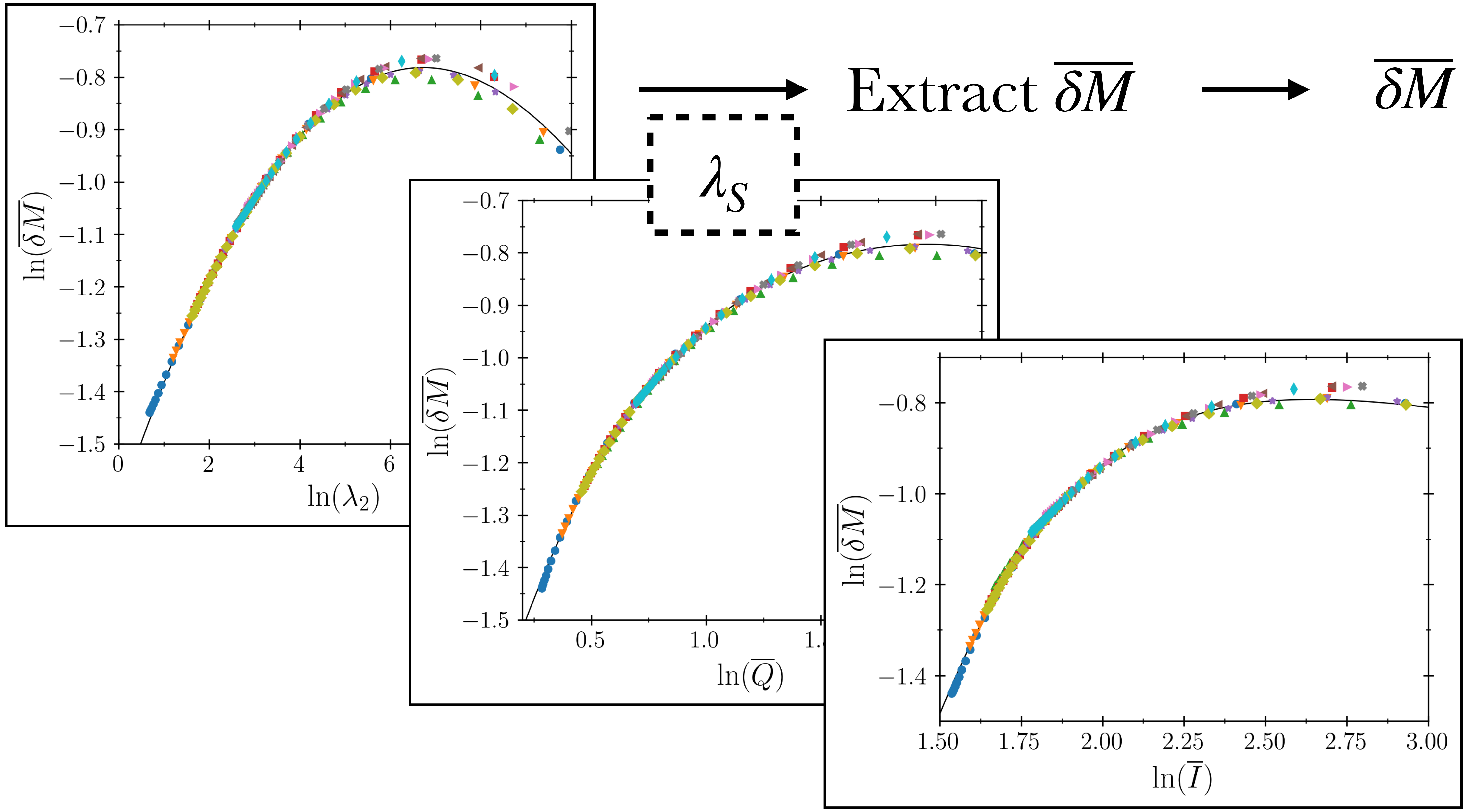

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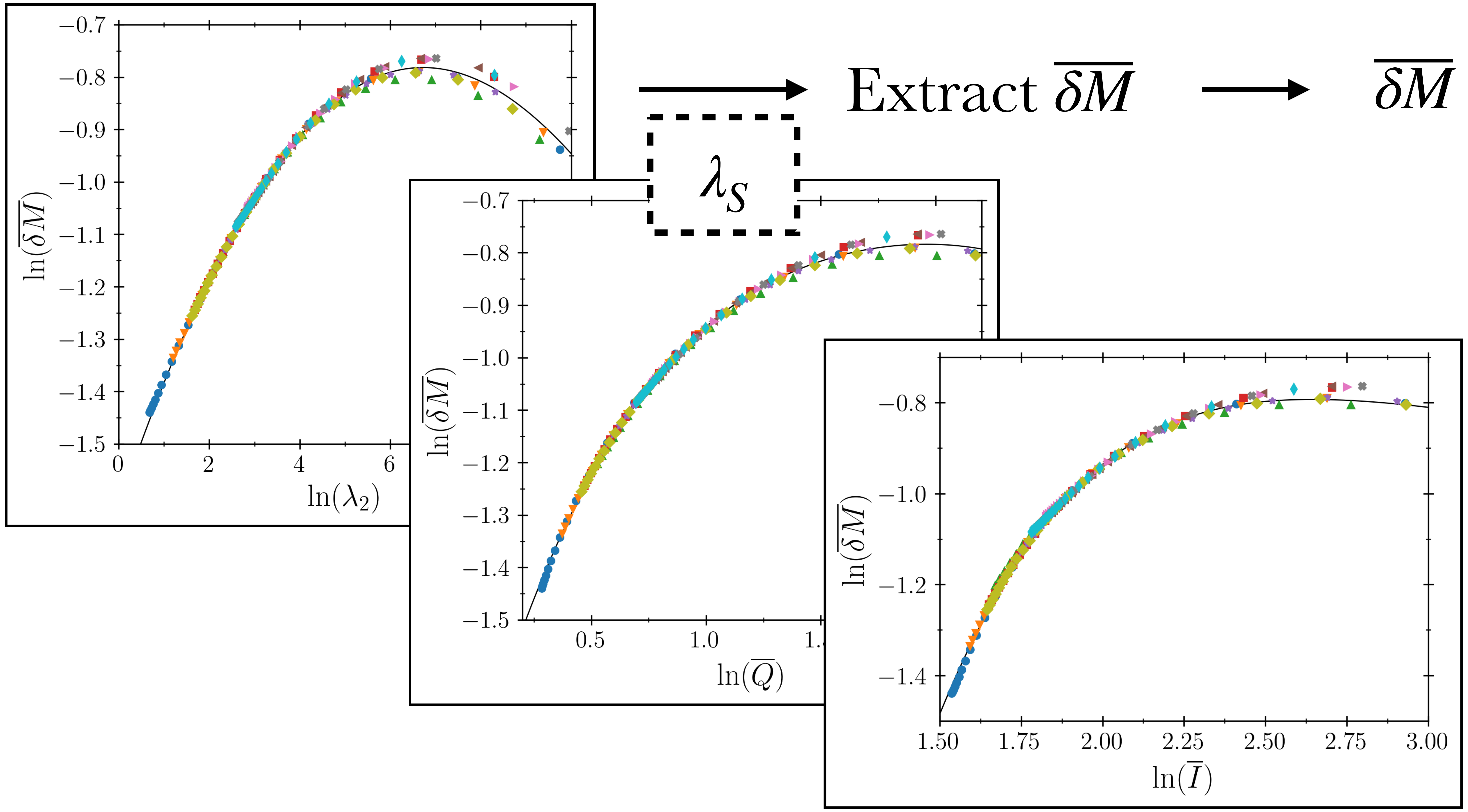

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The extended approach is more precise, but… how much?

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Polytropic EoS:

$$P = K\rho^{\gamma}$$

Spin-parameter:

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$K = 100 \quad \gamma = 2$ Vary $P_c \ \chi_S$

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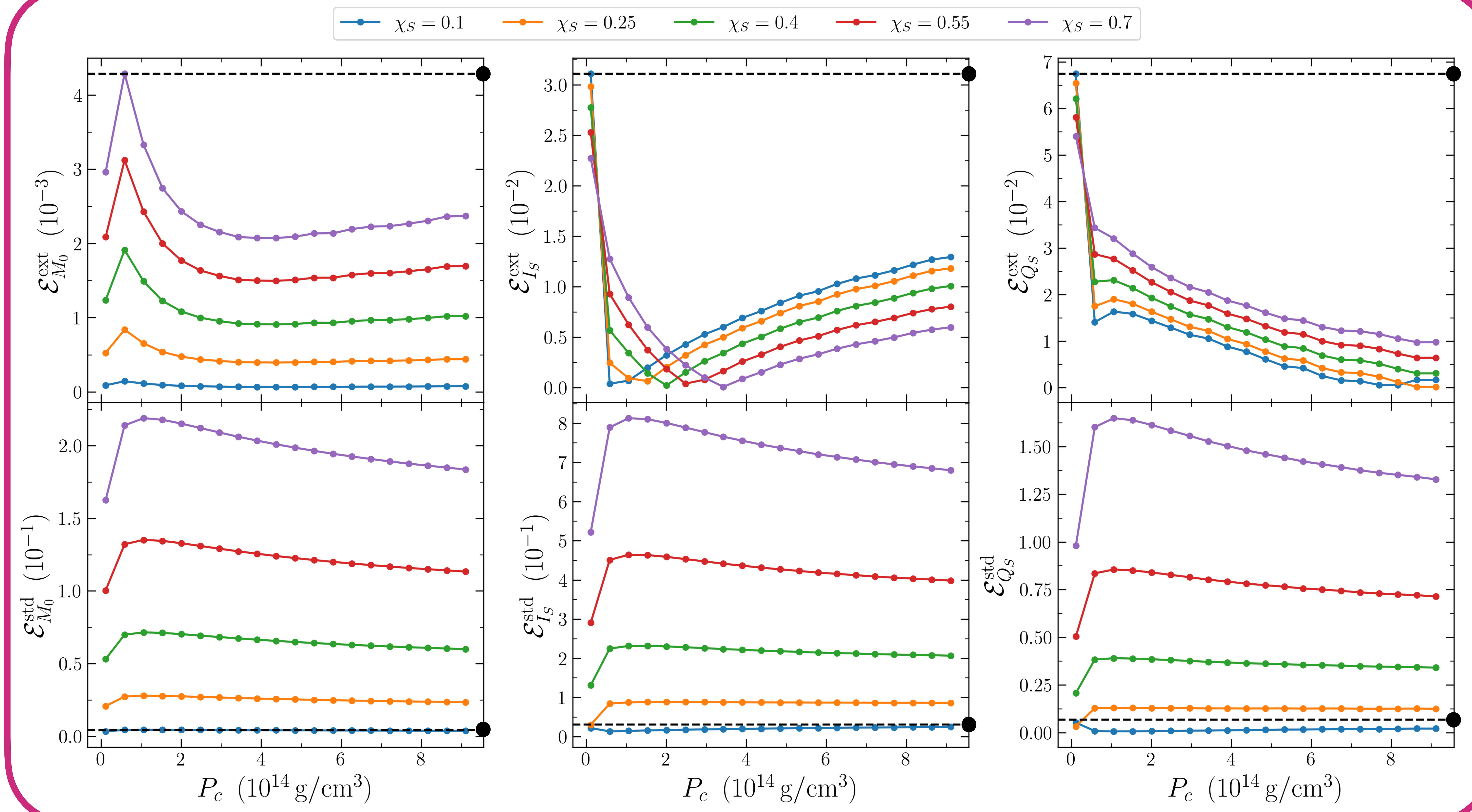

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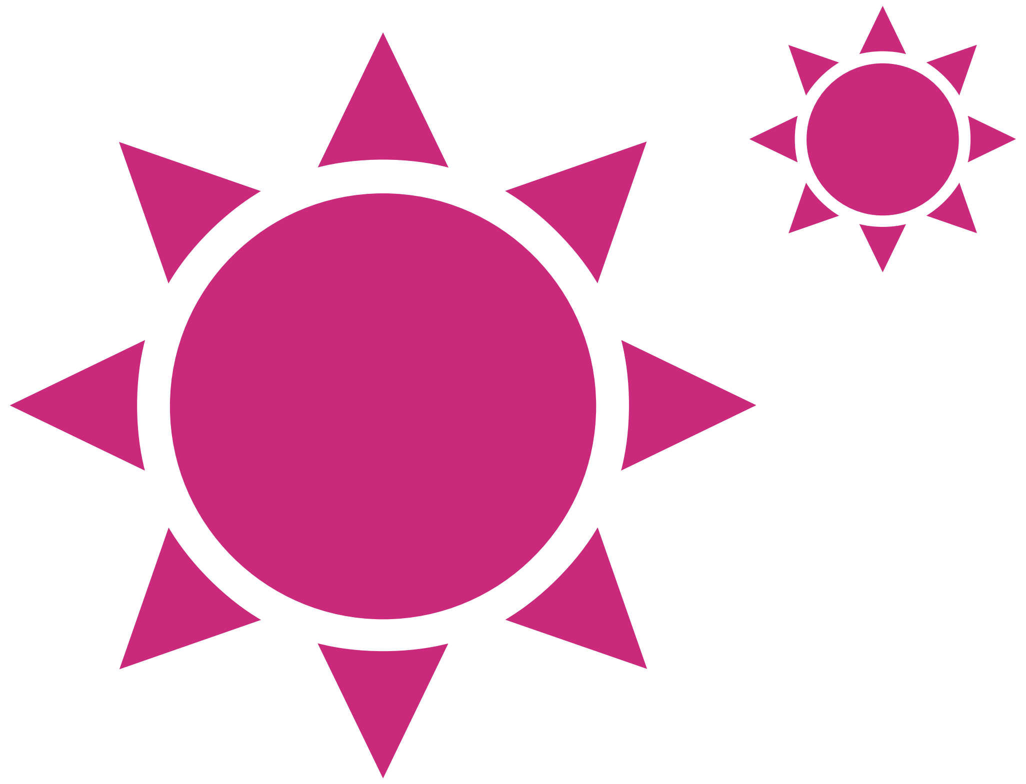
Inferring the EoS



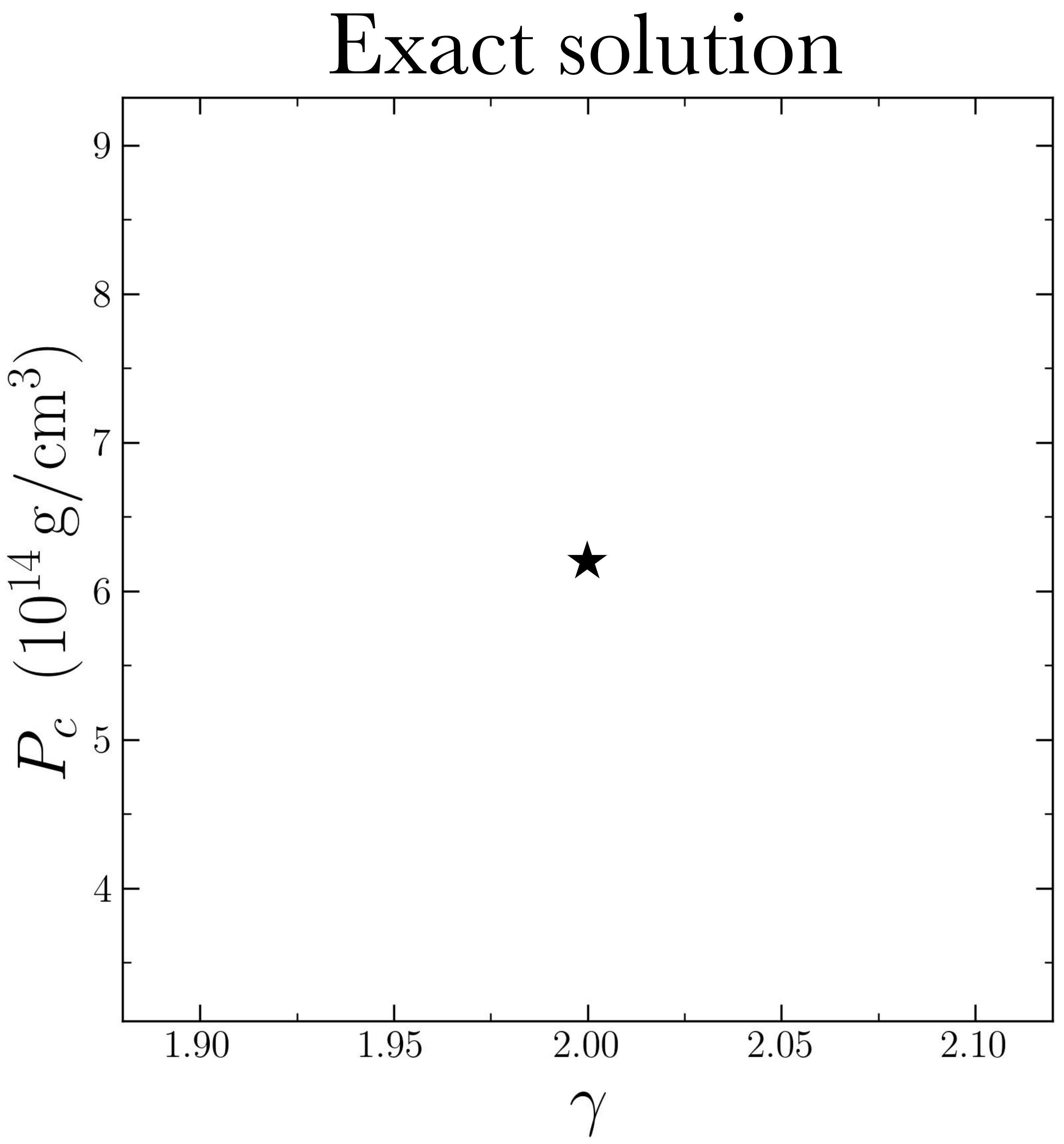
1) Assume the observed star has a polytropic EoS

2) Measure λ_S, M_S and Ω_S

3) Extract I_S, M_0 and Q_S using the universal relations

4) See which combinations of P_c, γ and K provide
$M_S, \lambda_S \ + \ I_S, M_0, Q_S$

Inferring the EoS $K = 100 \quad \chi_S = 0.3 \quad \text{Free } \gamma \; P_c$



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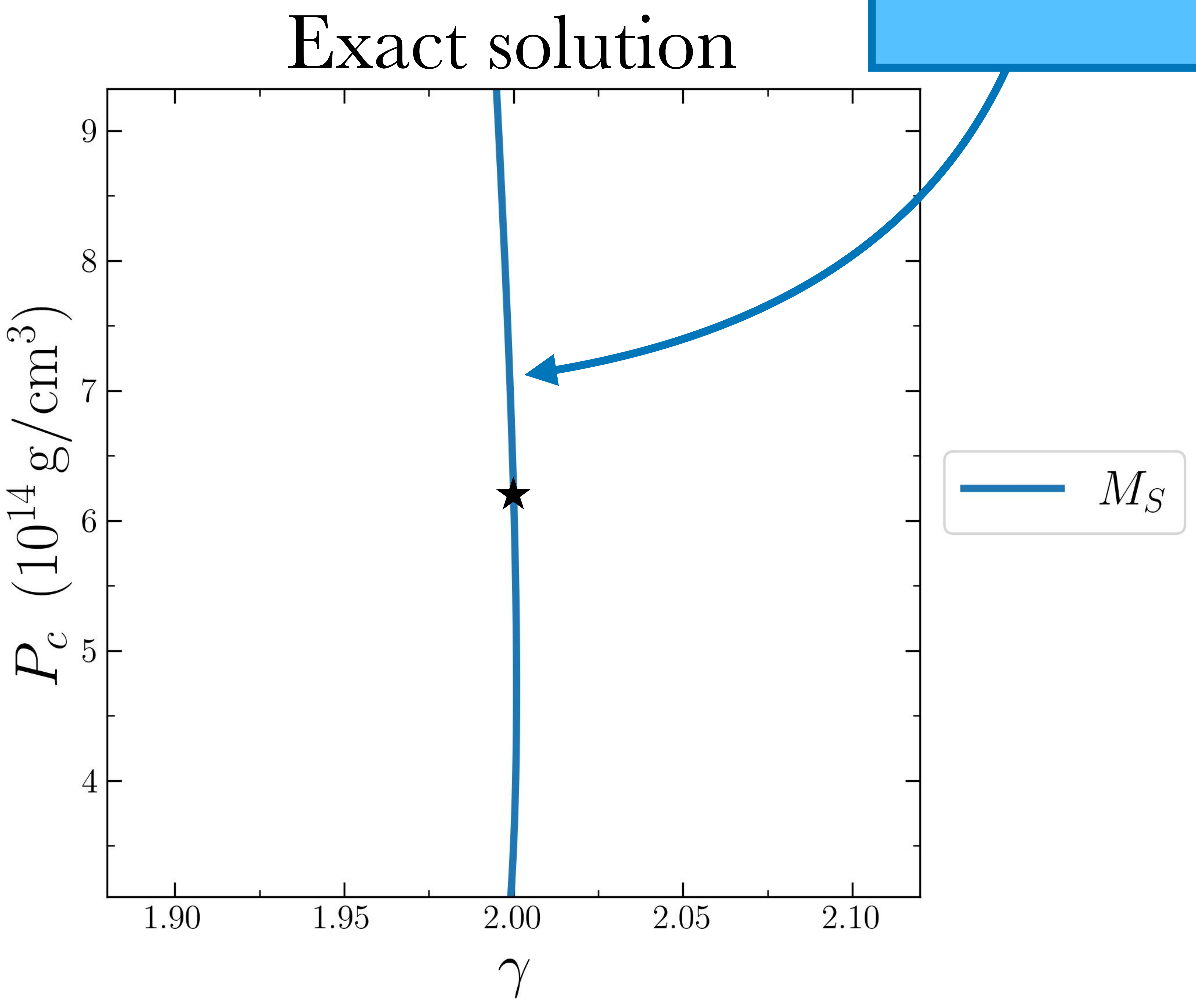
Same value of $M_S^\star$



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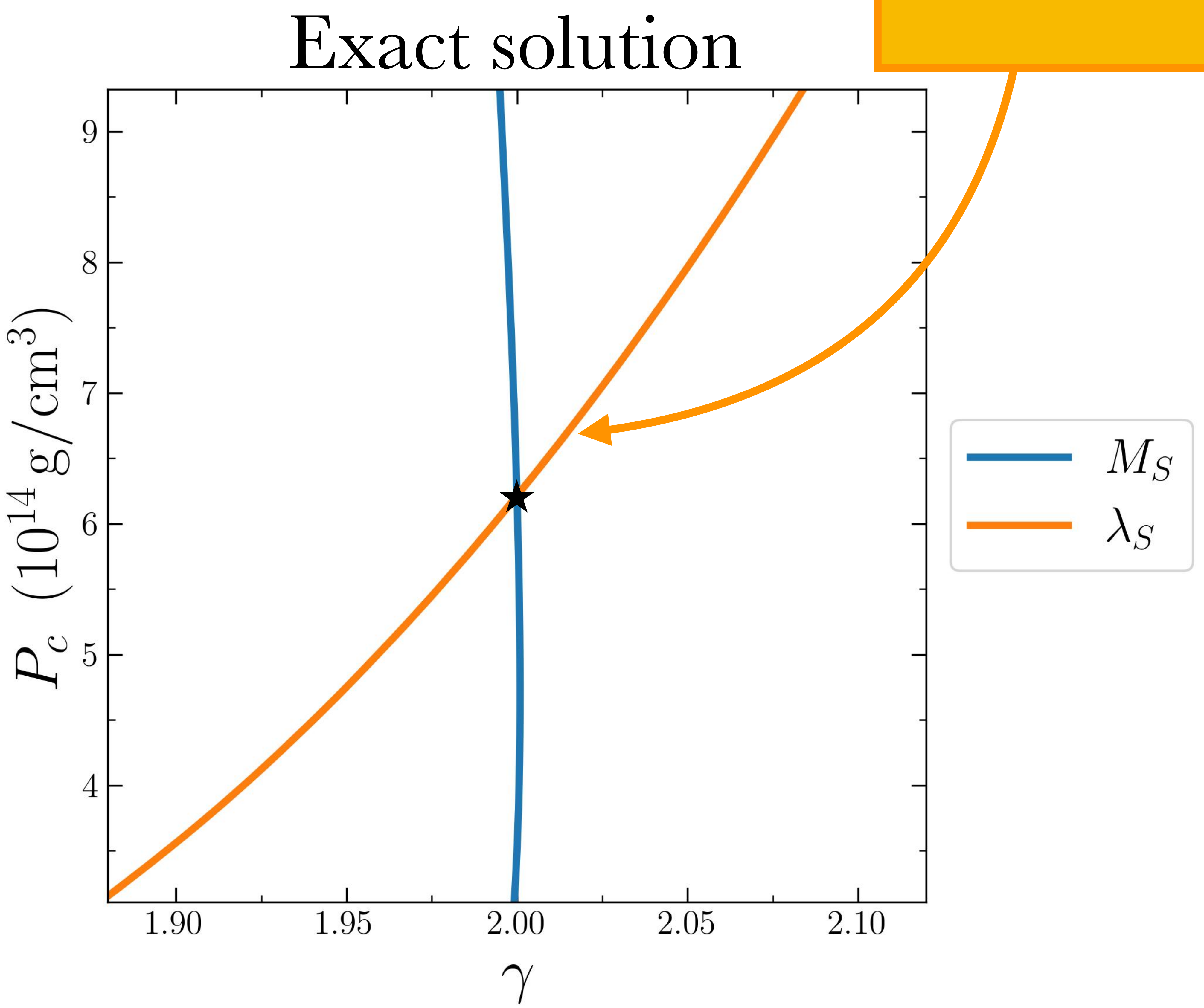
Same value of $\lambda_S^\star$



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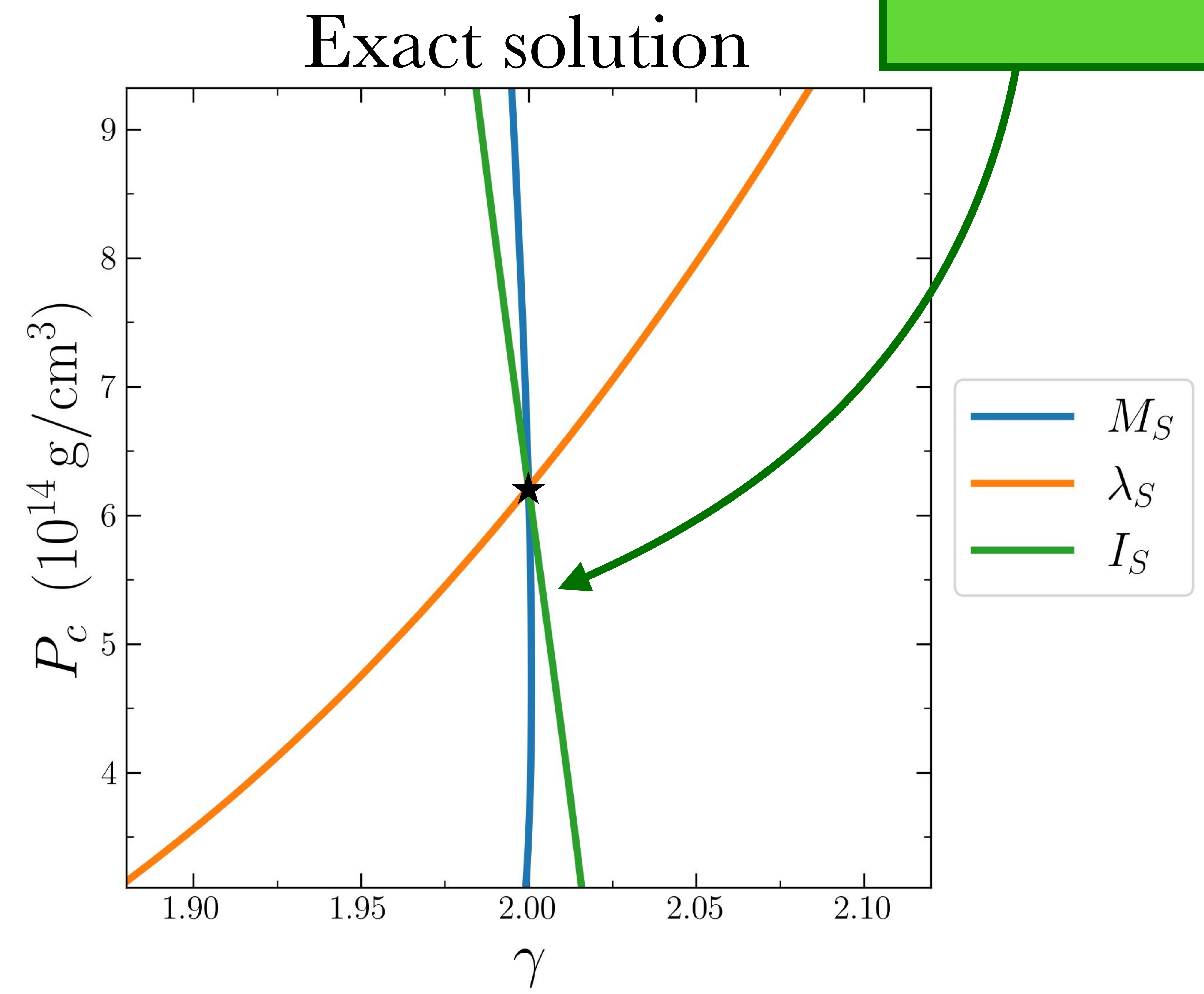
Same value of $I_S^\star$



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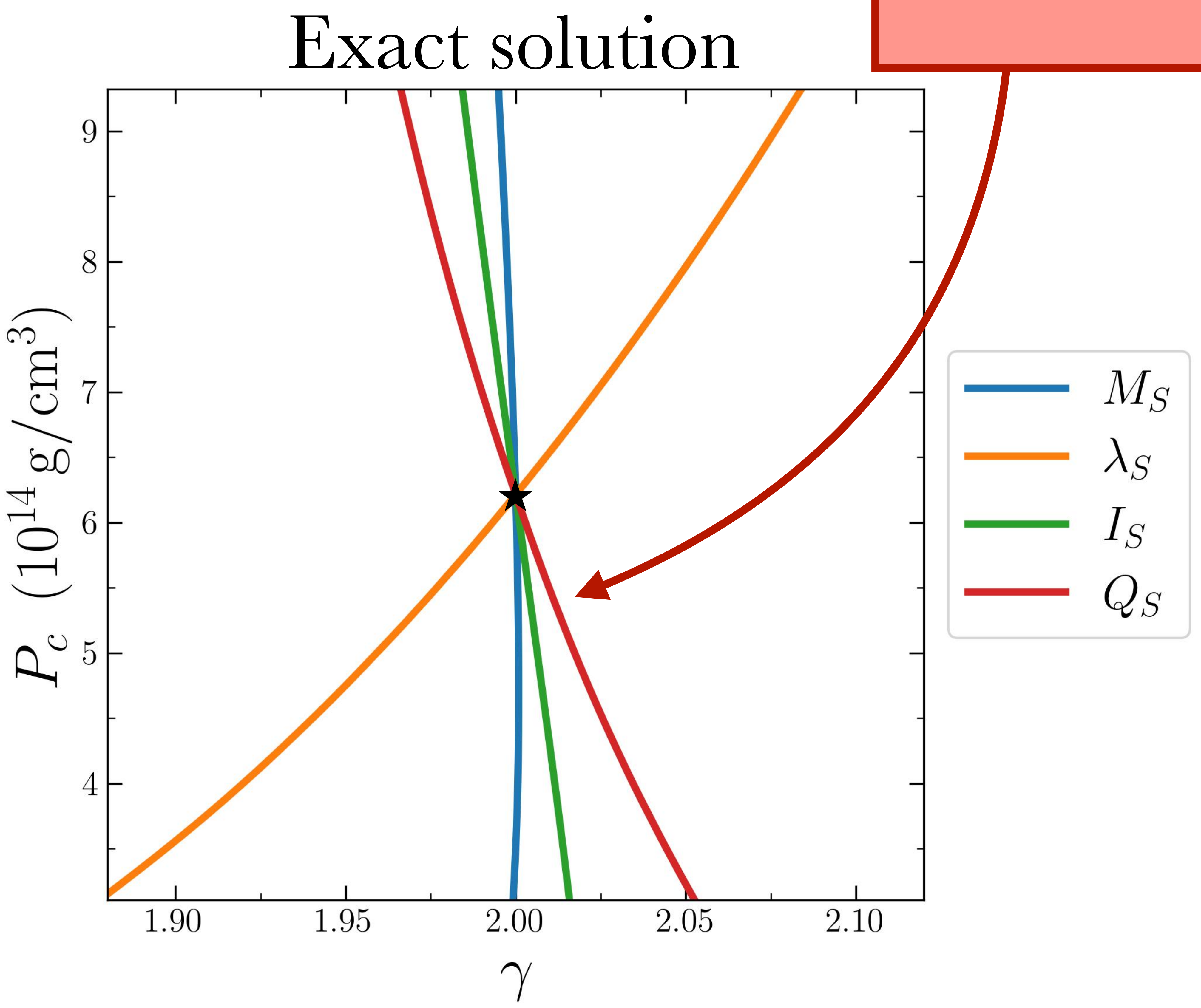
Same value of $Q_S^\star$



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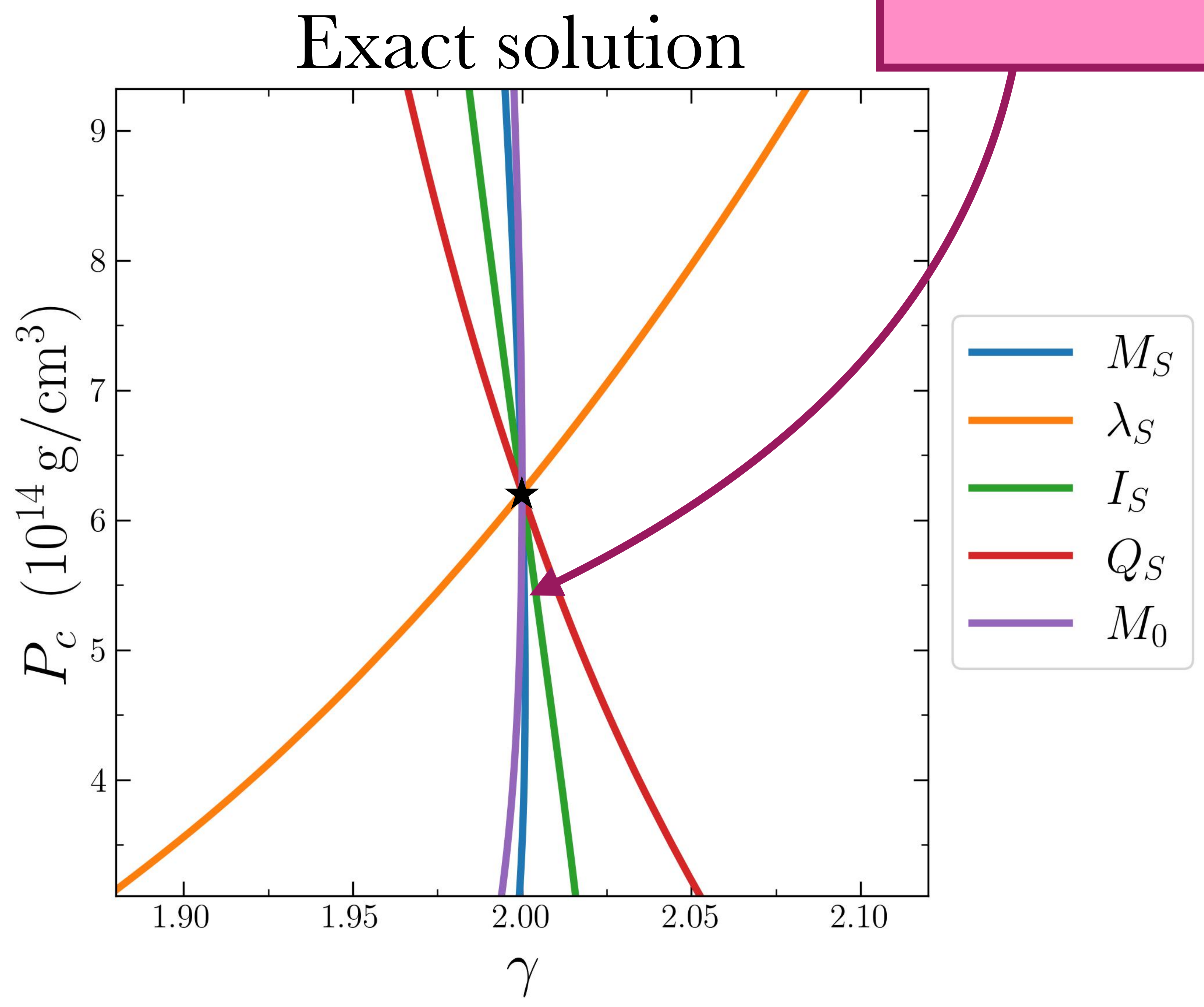
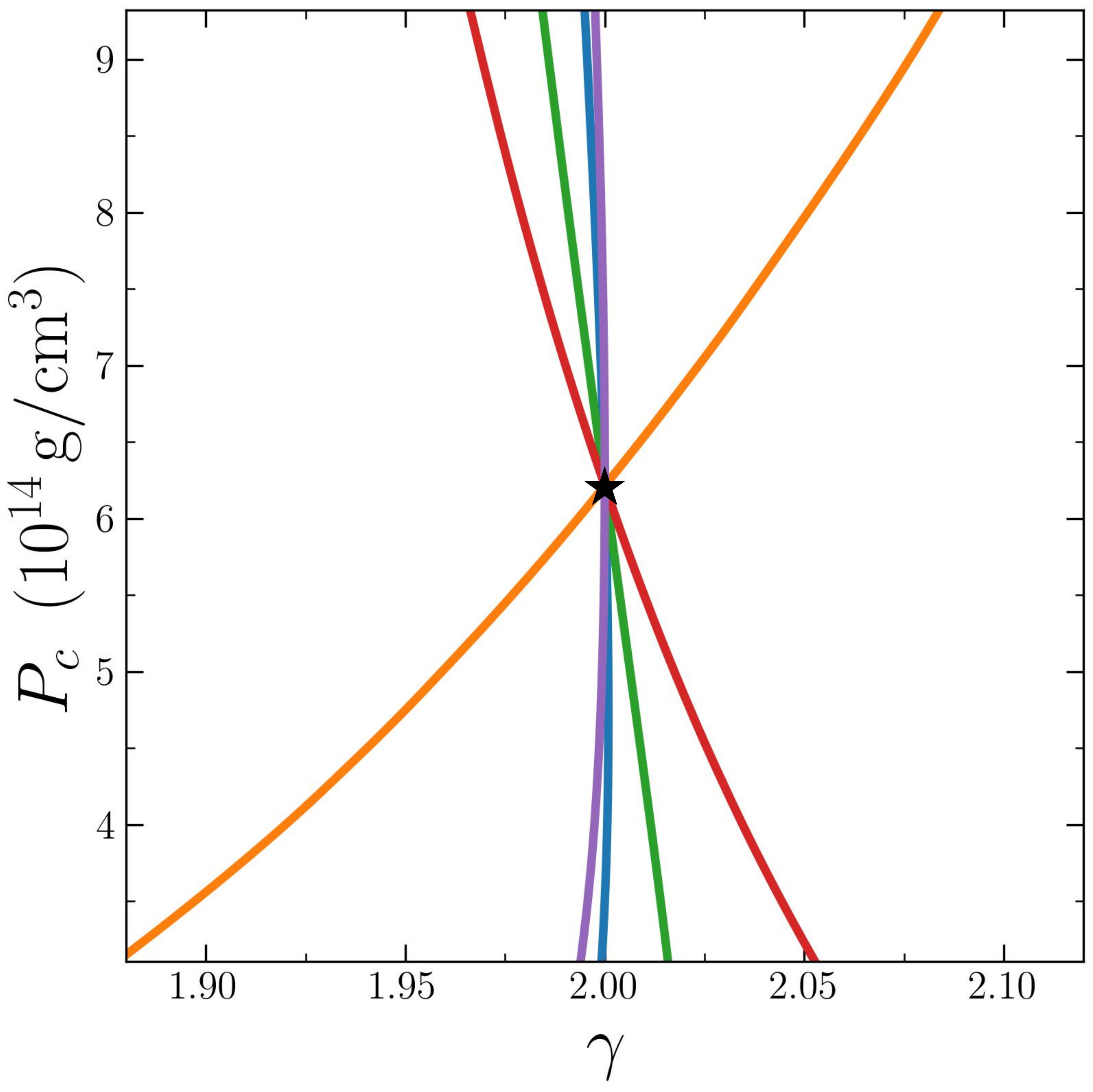
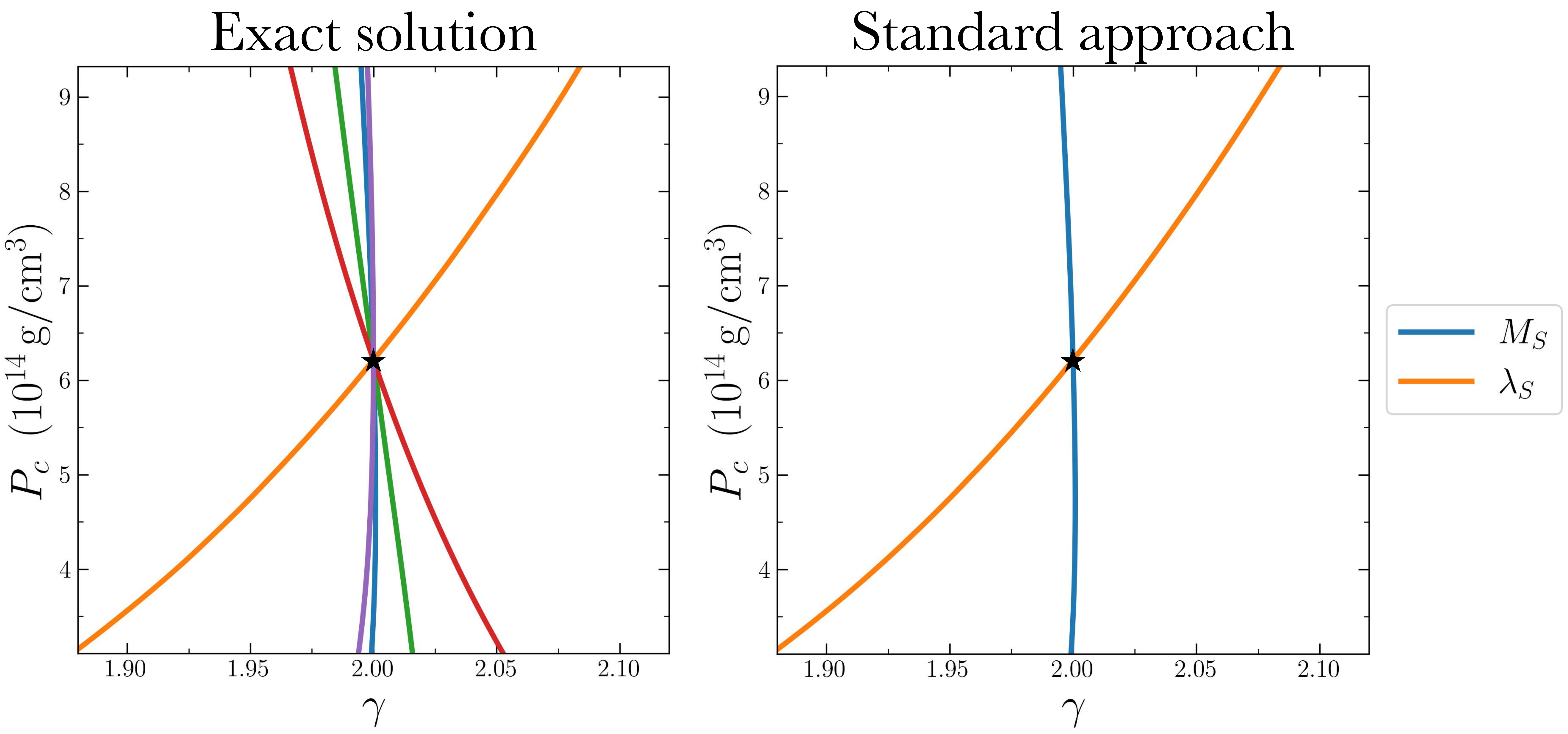


Inferring the EoS $K = 100 \quad \chi_S = 0.3 \quad \text{Free } \gamma \; P_c$

Exact solution

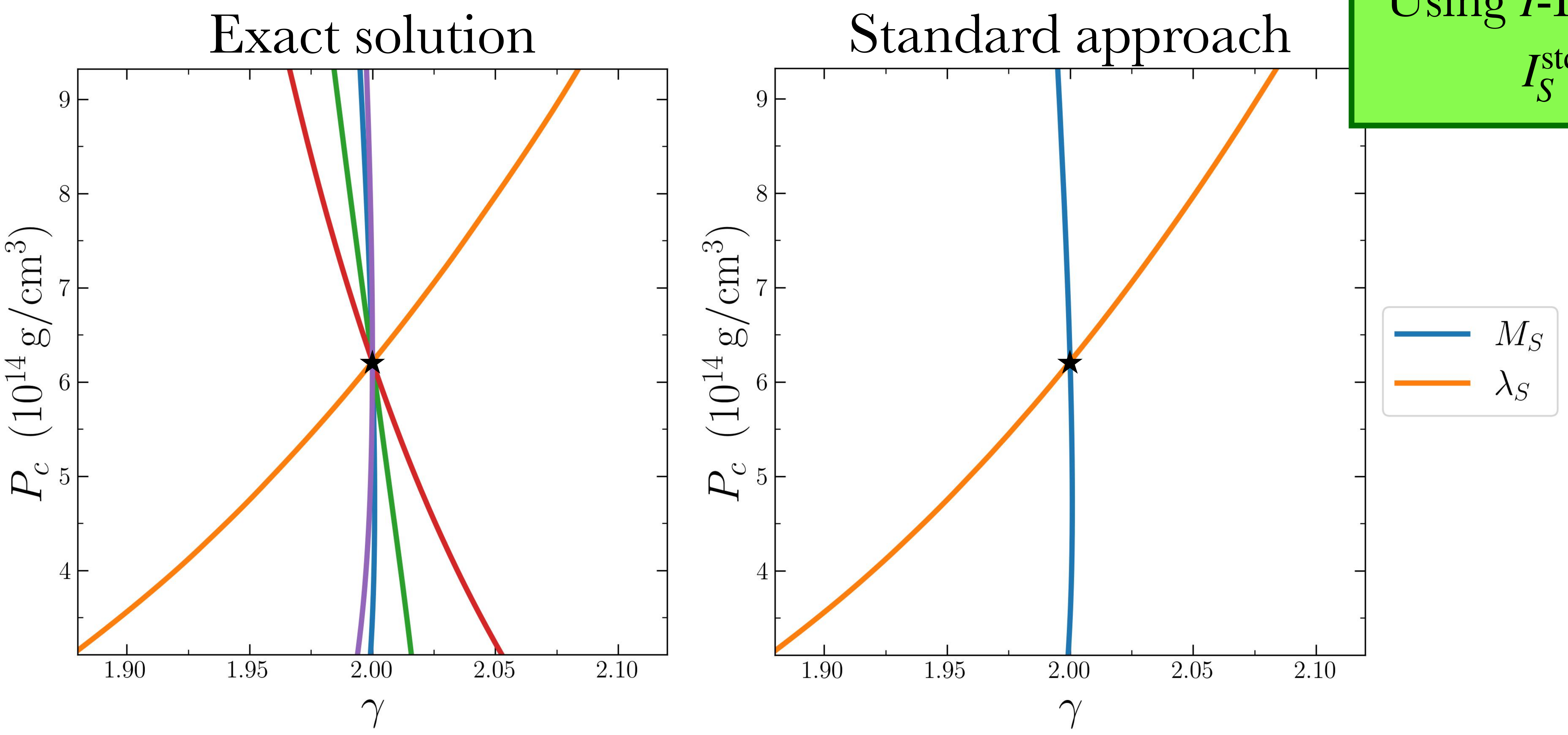


Inferring the EoS $K = 100 \quad \chi_S = 0.3 \quad$ Free $\gamma \ P_c$

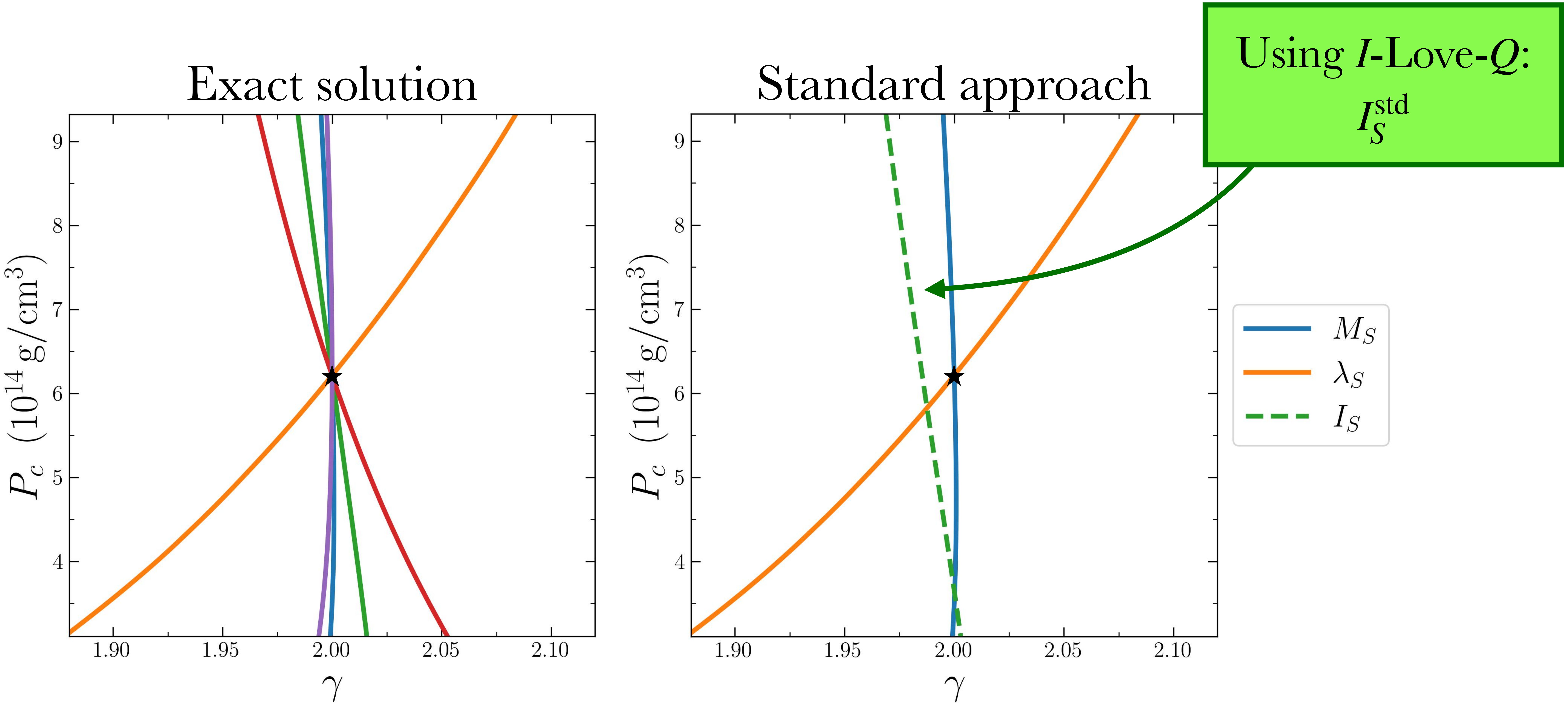


Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c

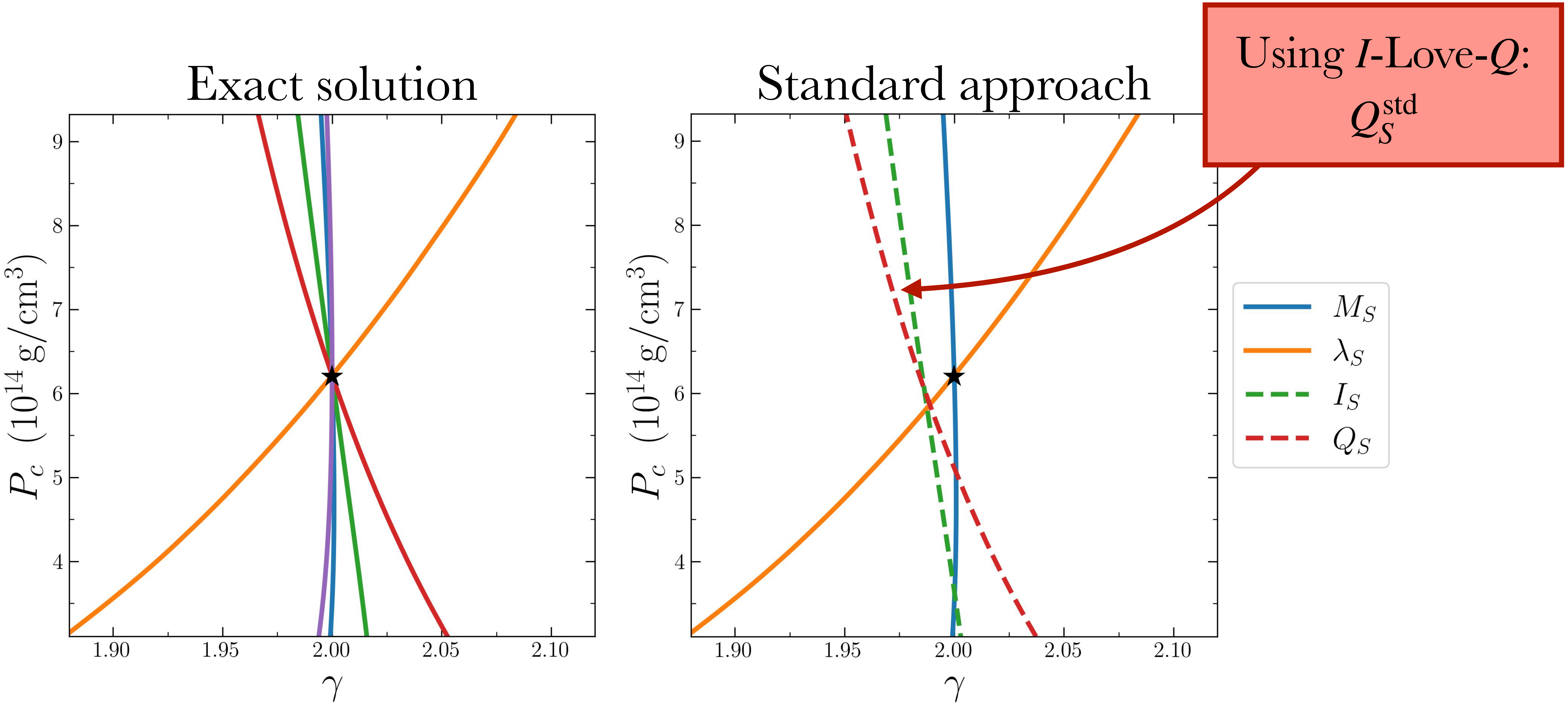
Using I-Love-Q:
$I_S^{\rm std}$



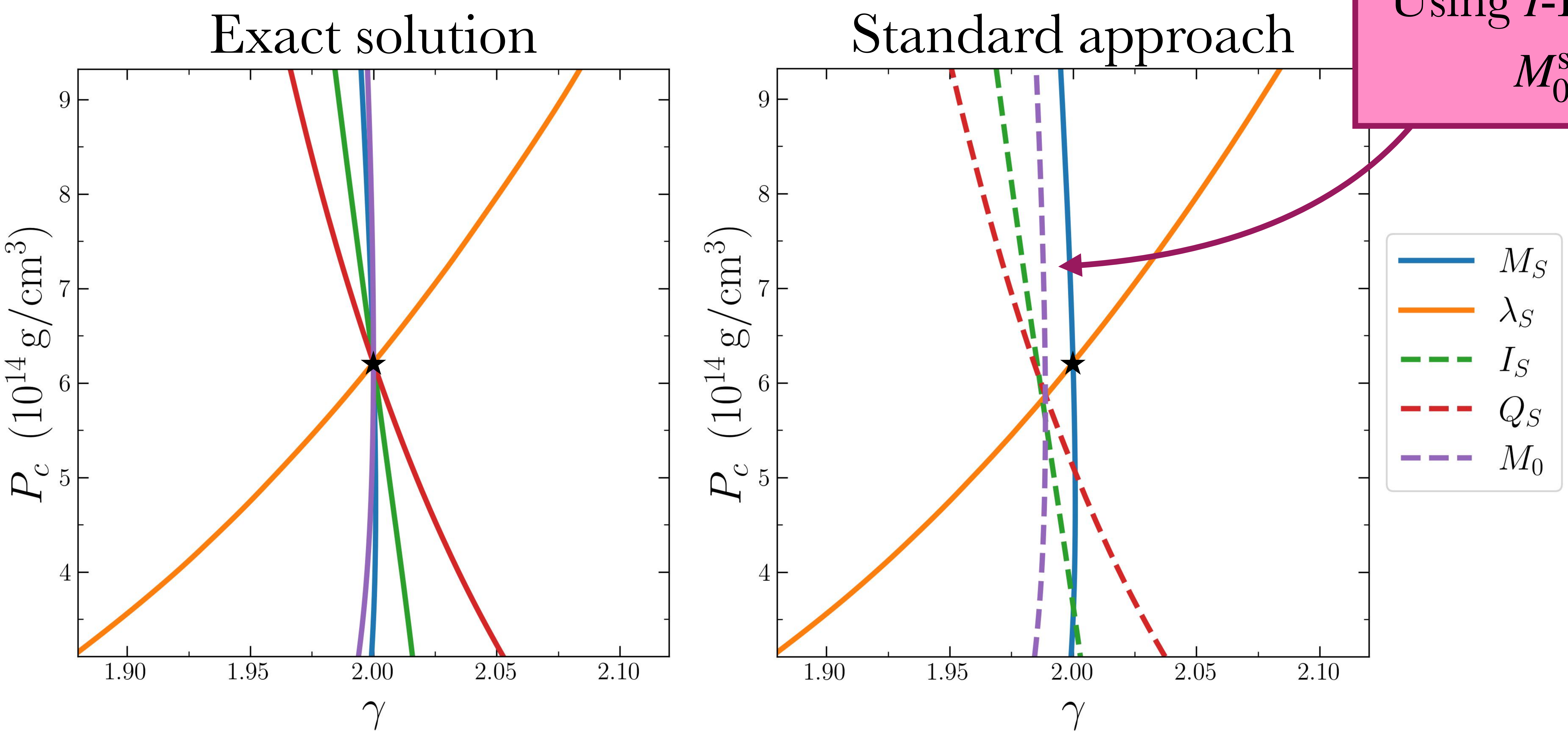
Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c



Inferring the EoS $K = 100 \quad \chi_S = 0.3 \quad \text{Free } \gamma \; P_c$

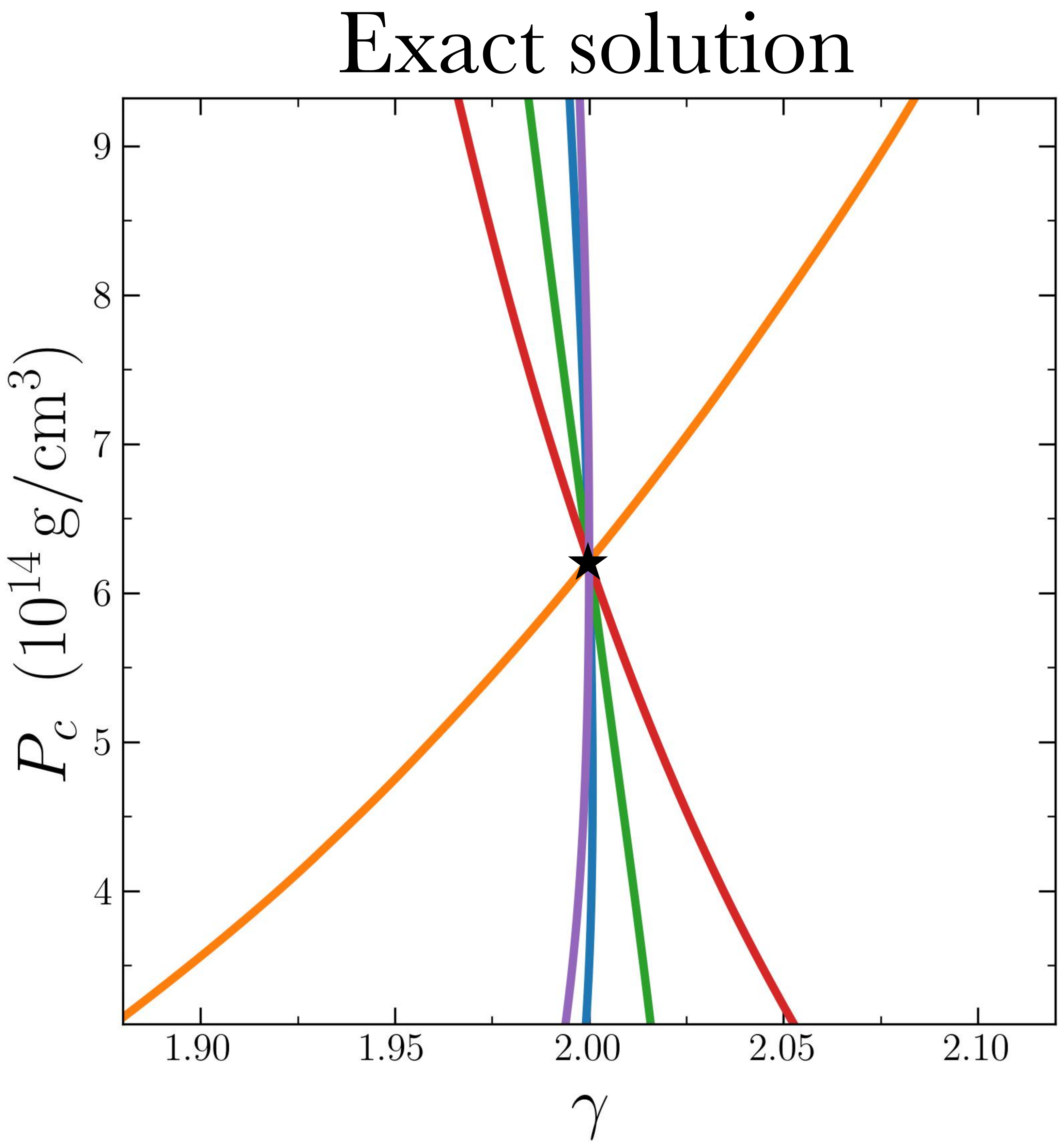


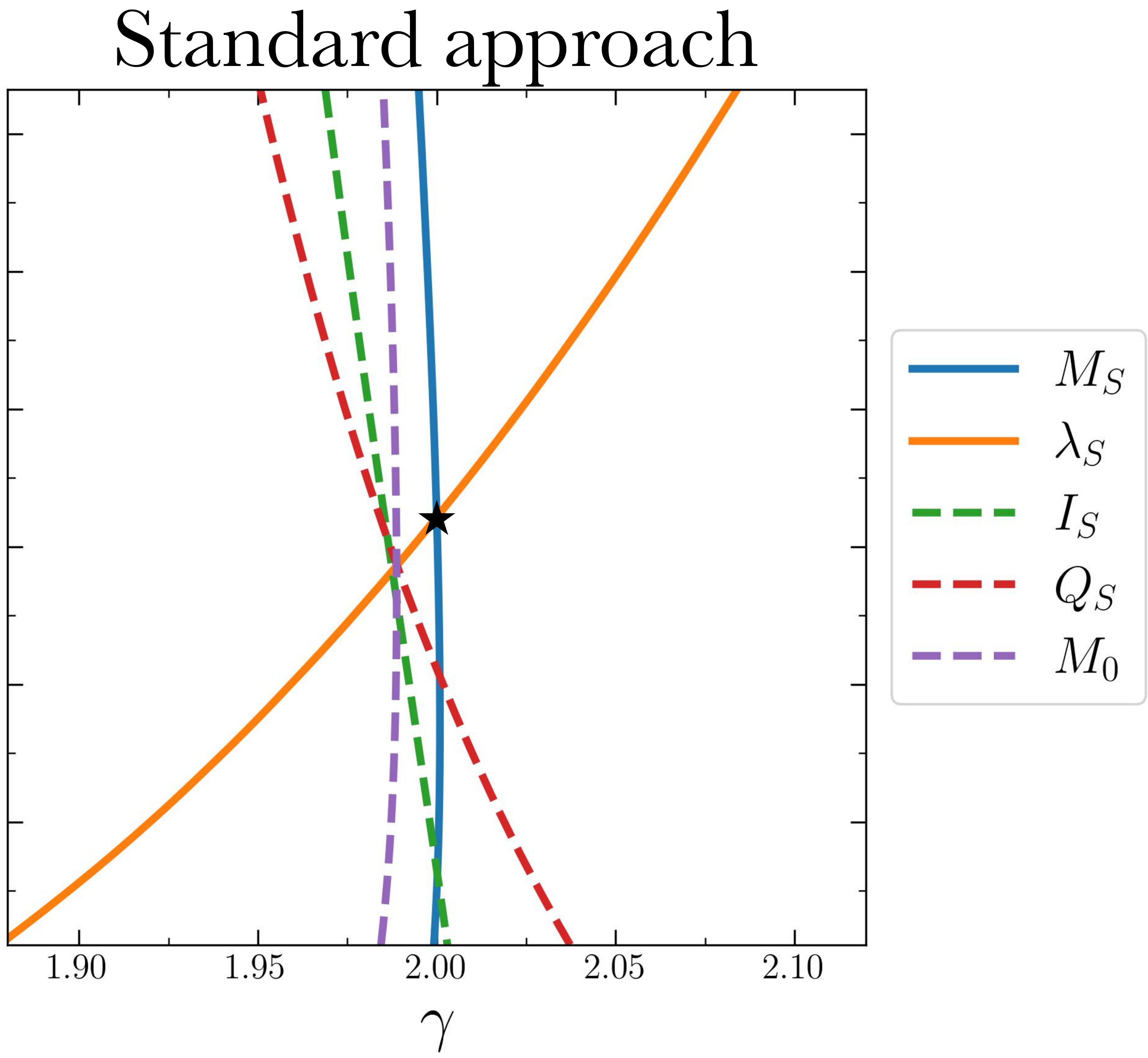
Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c



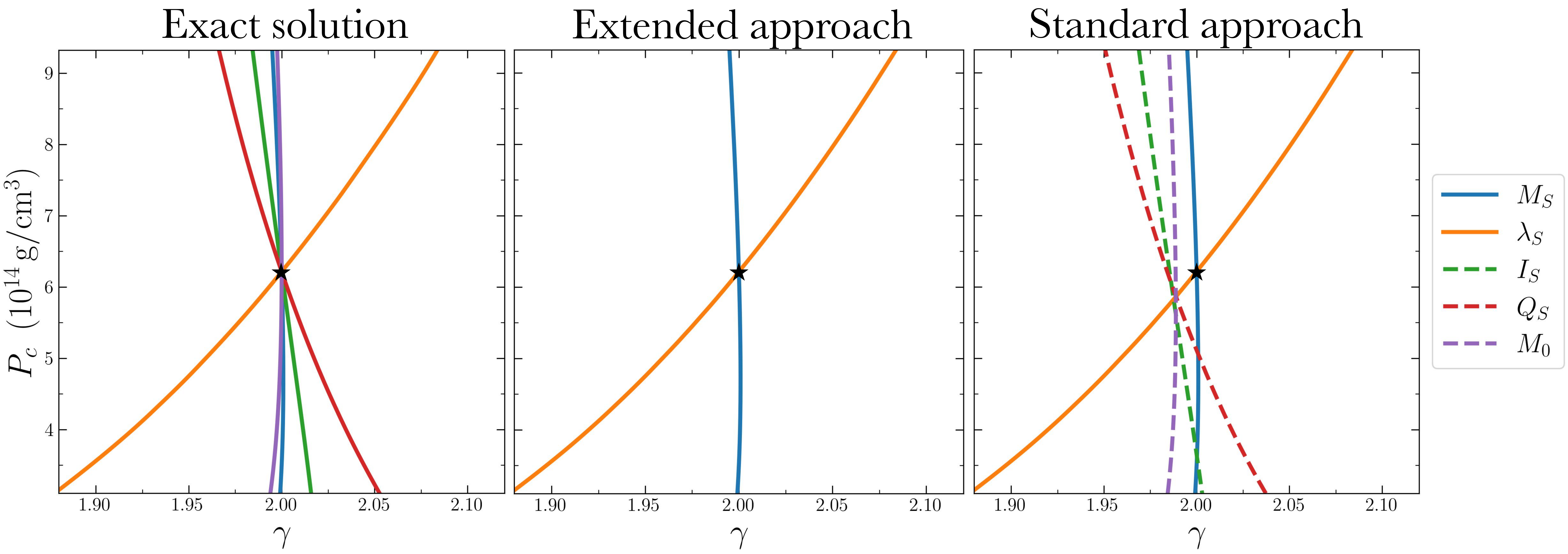
Using I-Love-Q: $M_0^{\rm std}$

Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c





Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c



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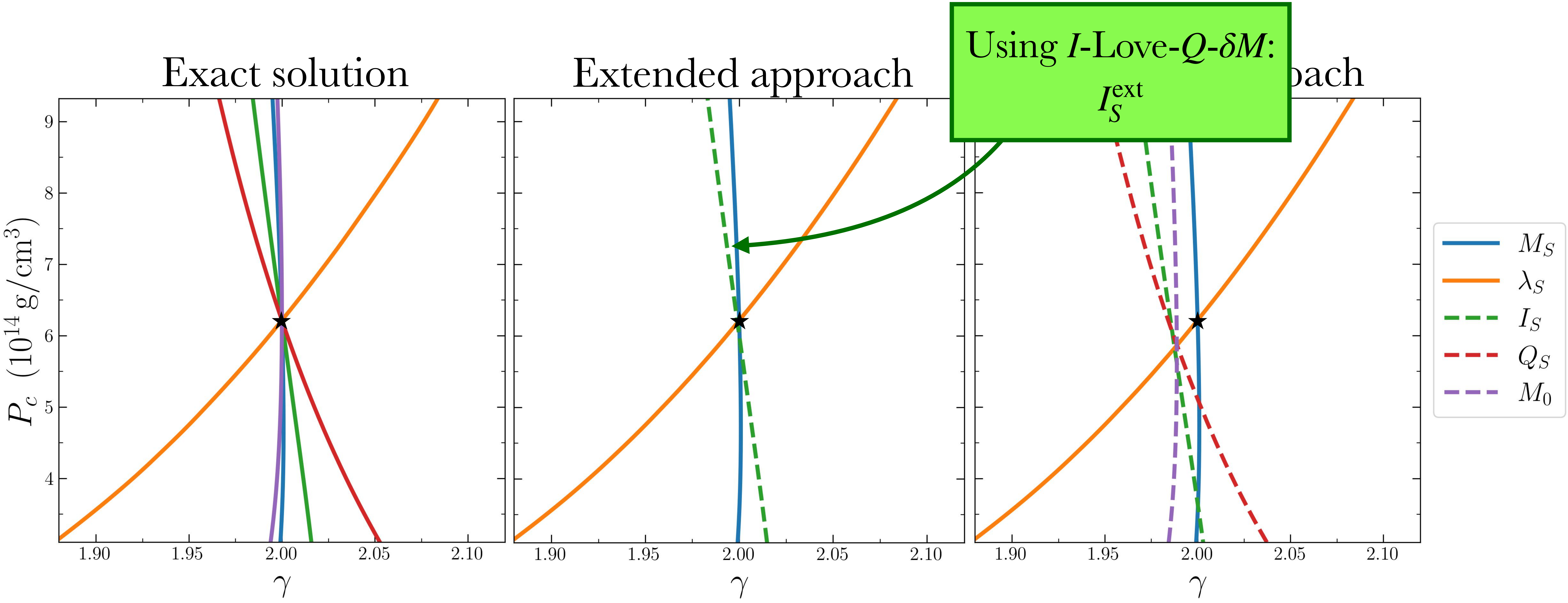

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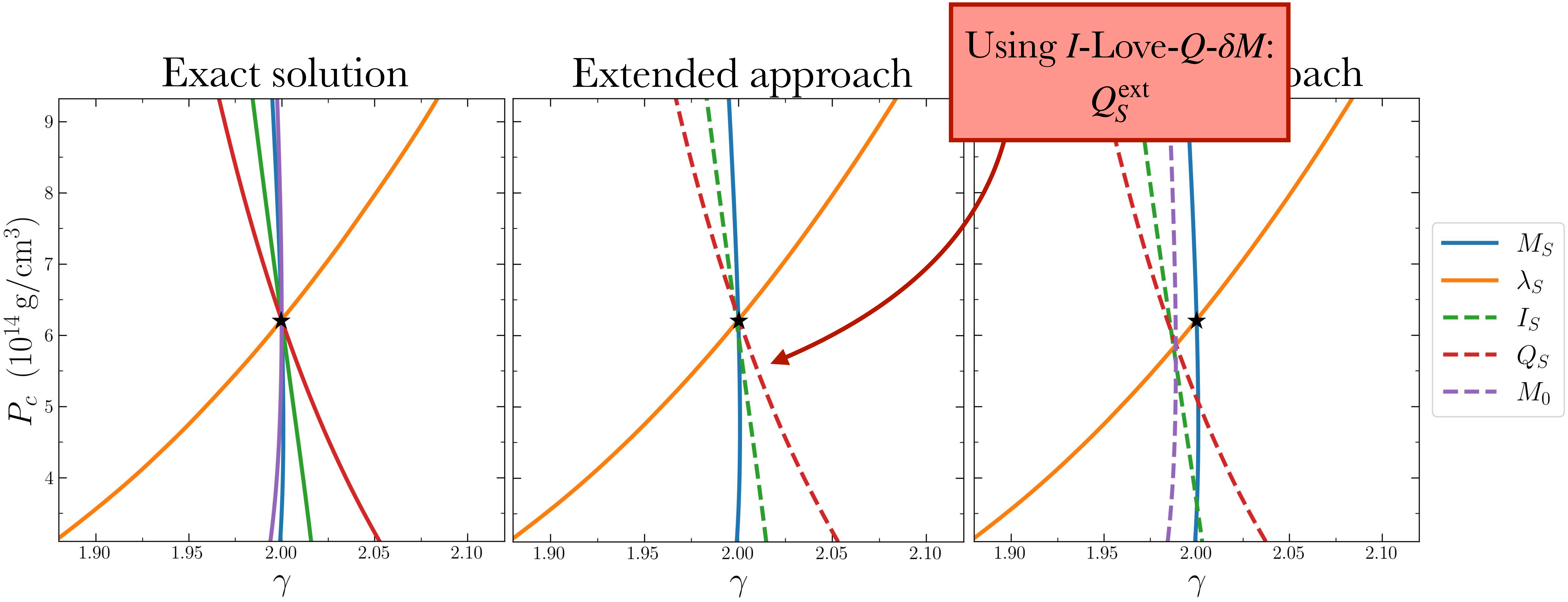
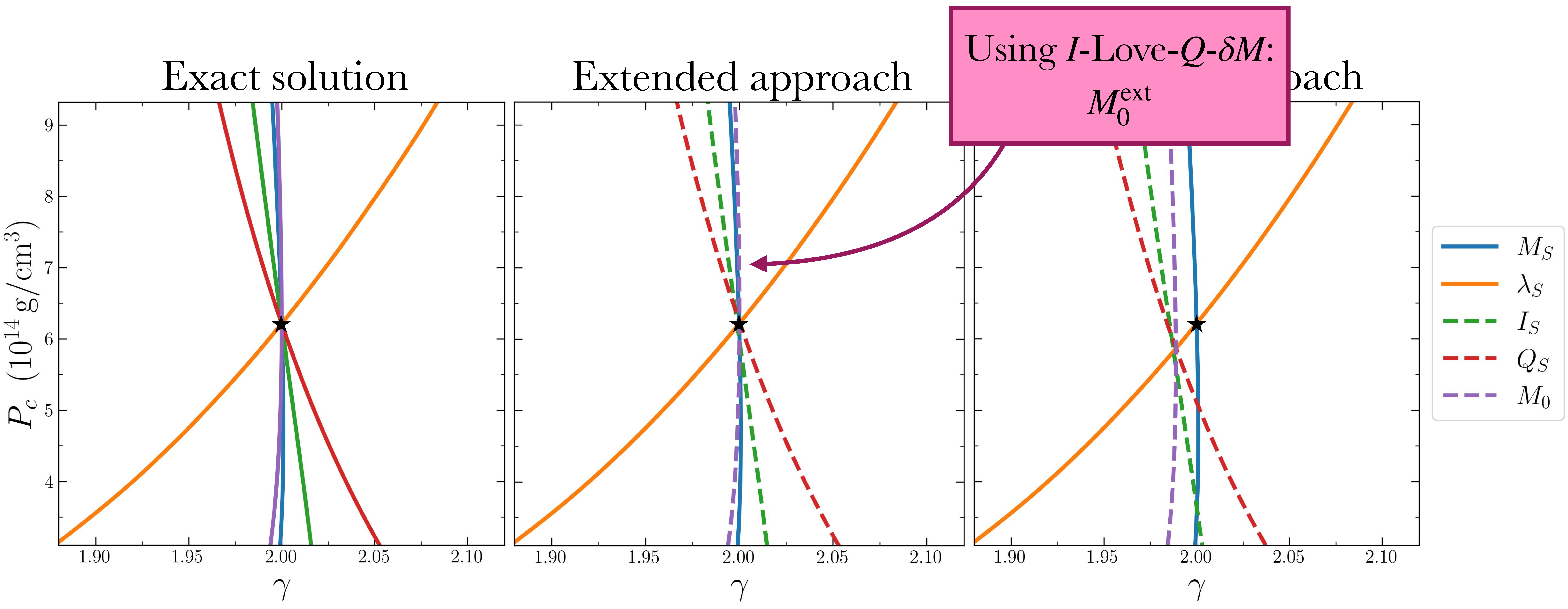


Inferring the EoS

$K = 100$ $\chi_S = 0.3$ Free γ P_c



Inferring the EoS $K = 100$ Free γ P_c

Inferring the EoS $K = 100$ $\chi_S = 0.1$ Free γ P_c

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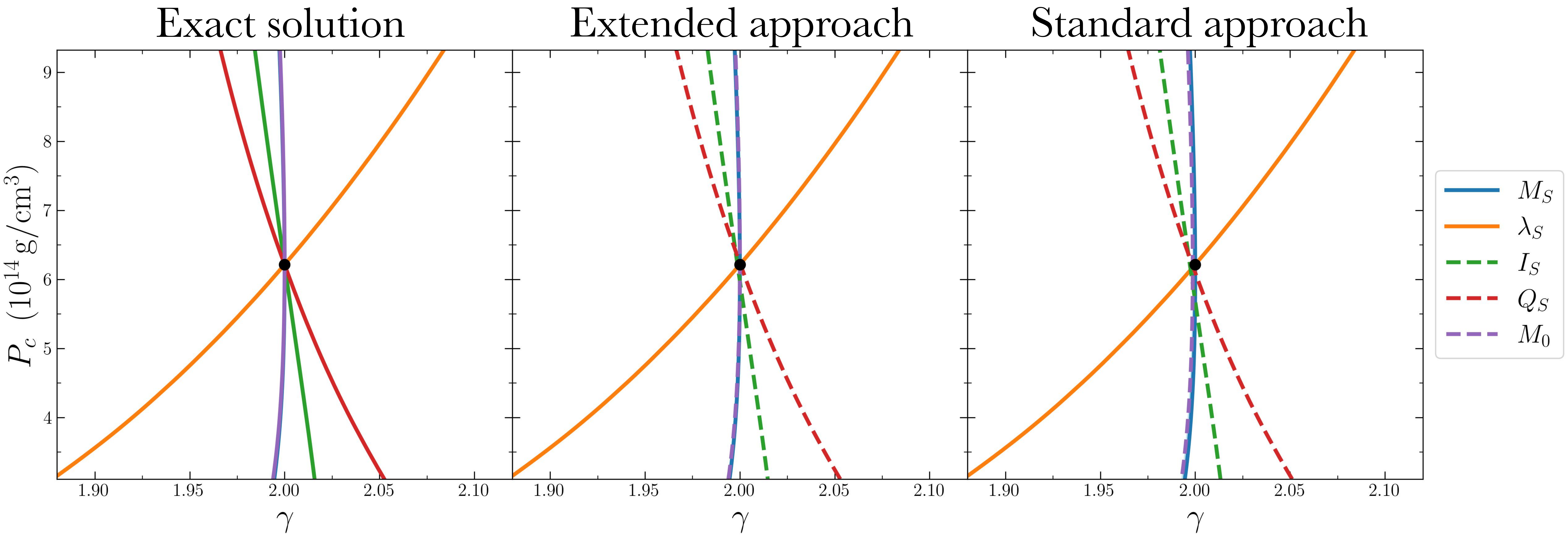
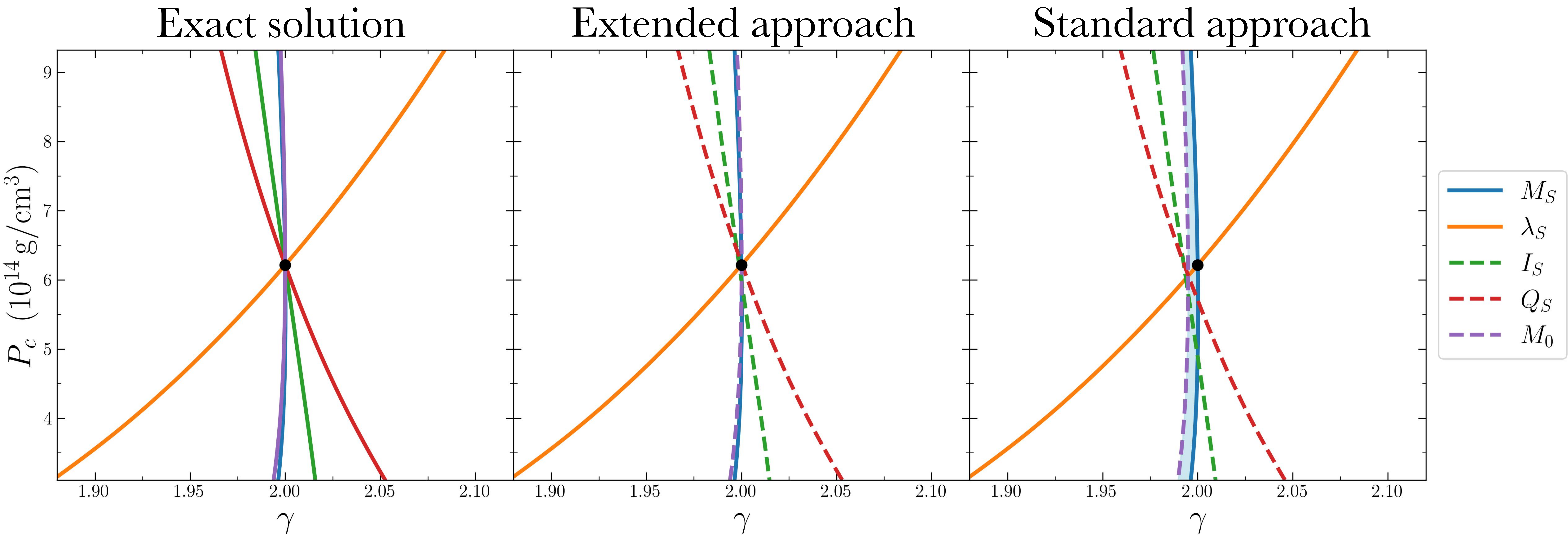
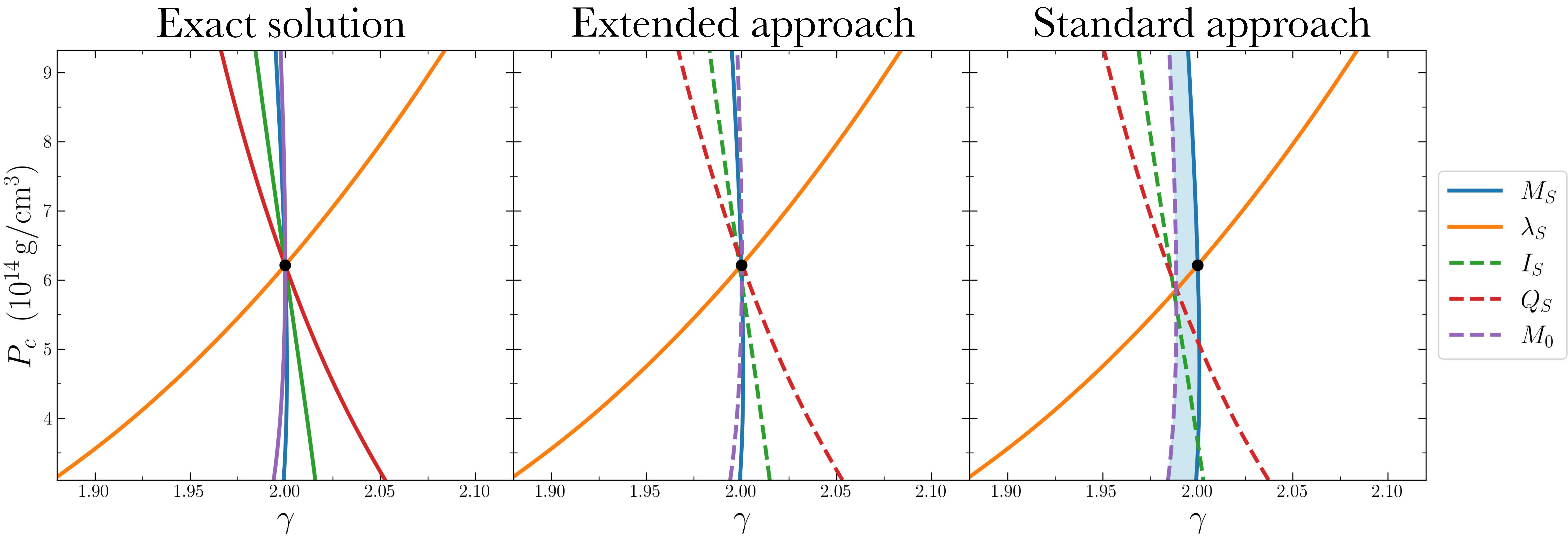


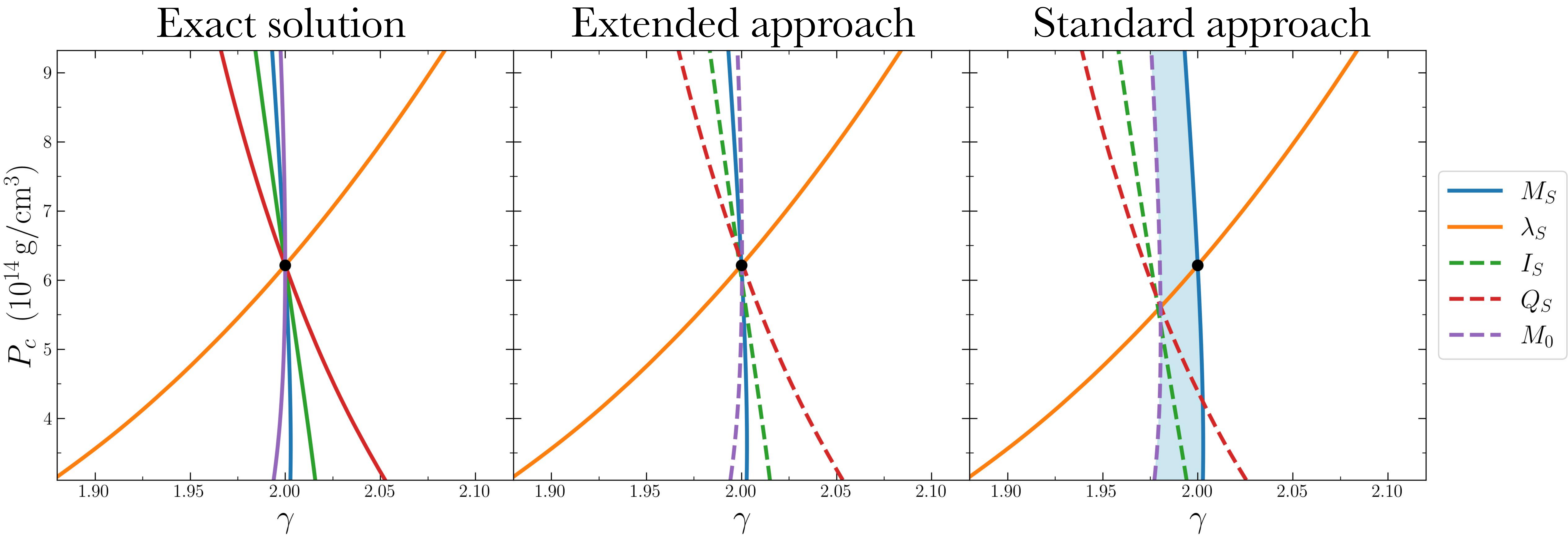
Inferring the EoS $K = 100$ $\chi_S = 0.2$ Free γ P_c



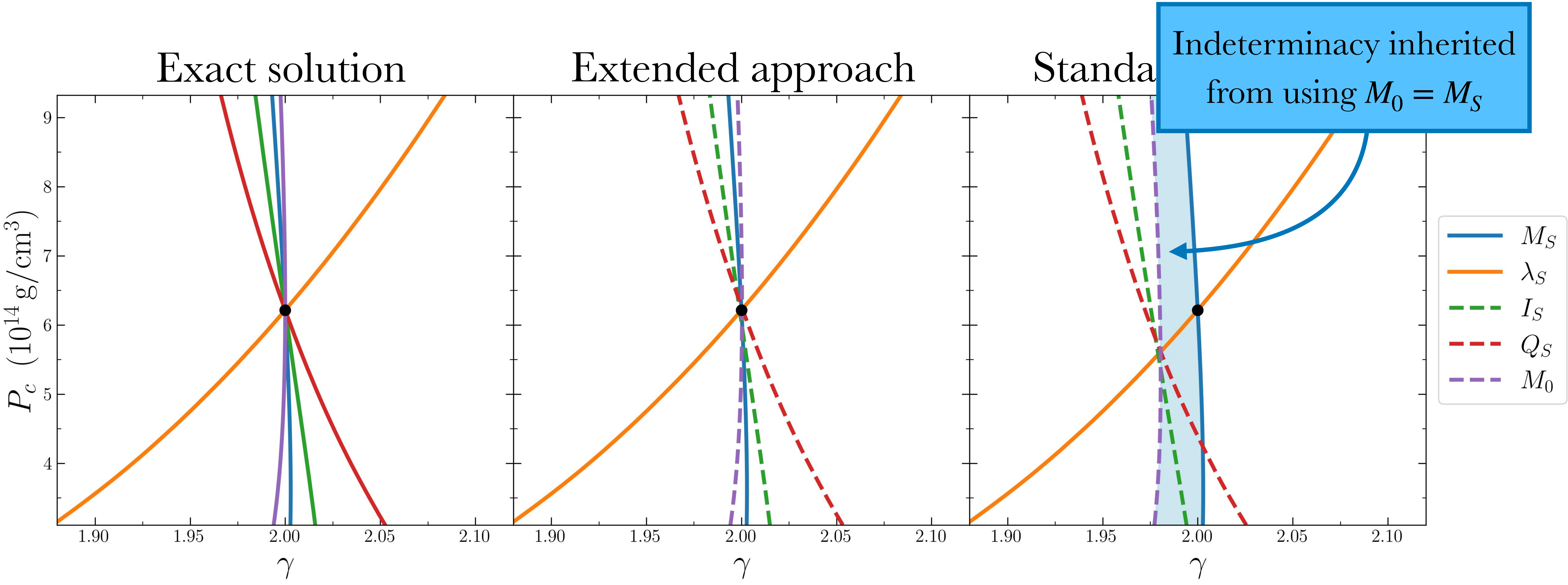
Inferring the EoS $K = 100$ $\chi_S = 0.3$ Free γ P_c



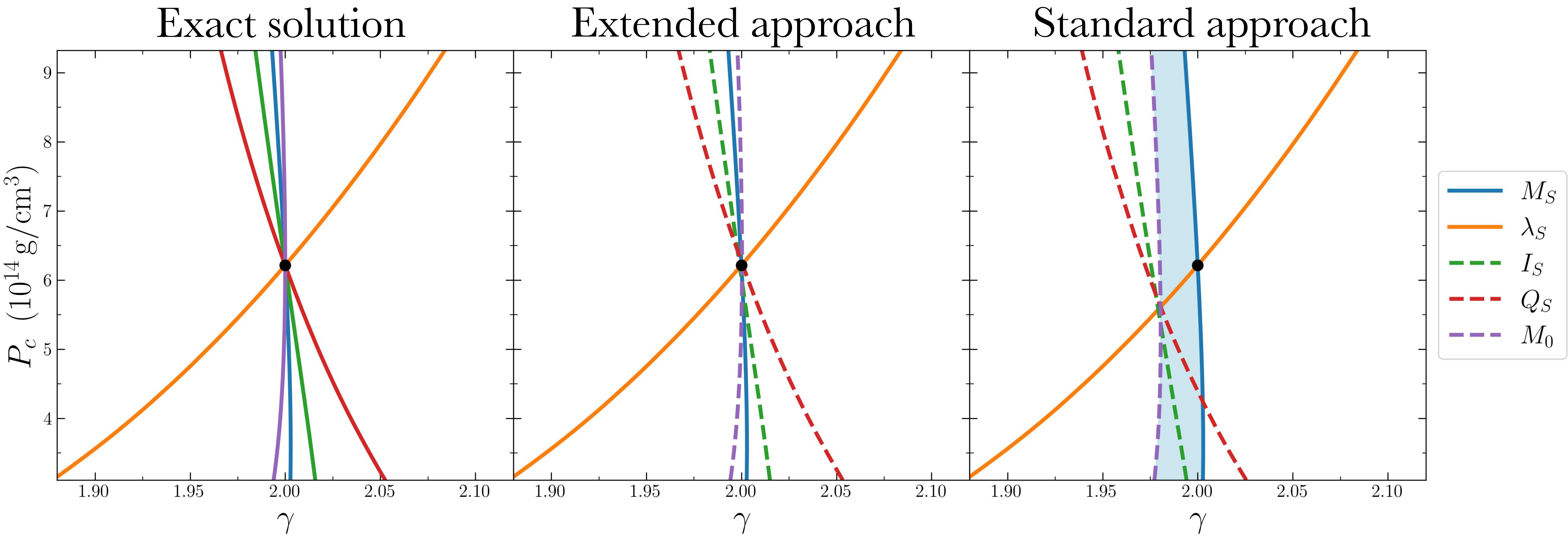
Inferring the EoS $K = 100$ $\chi_S = 0.4$ Free γ P_c



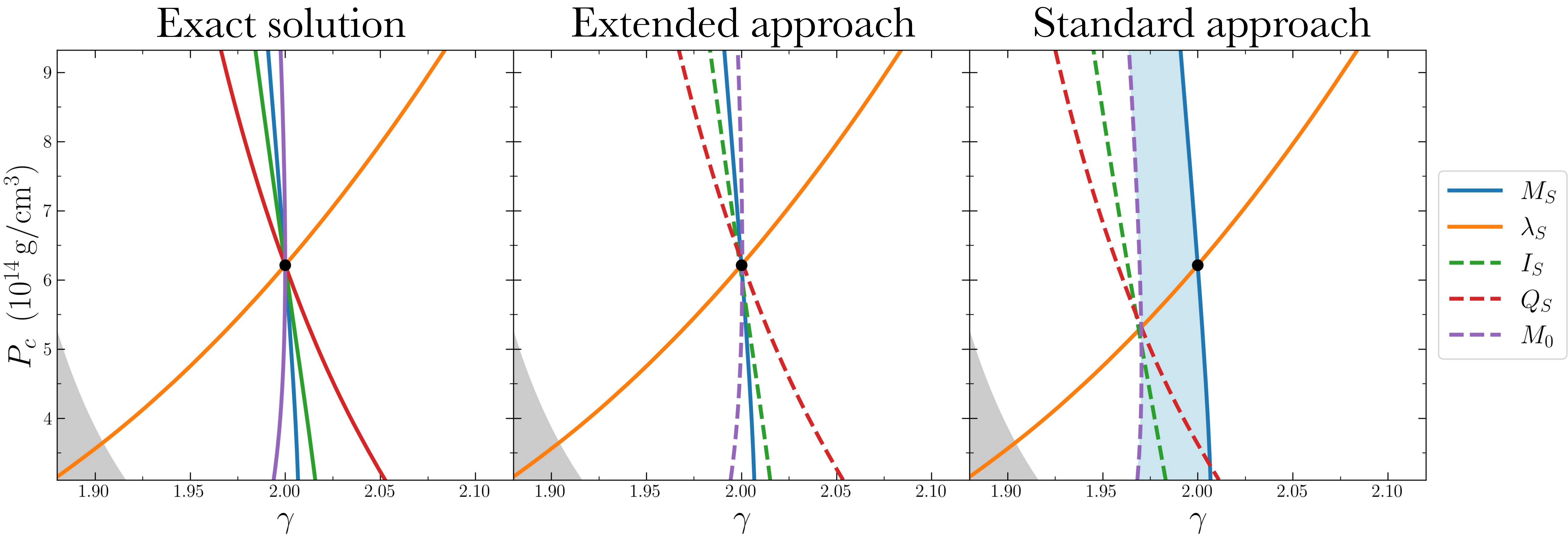
Inferring the EoS $K = 100$ $\chi_S = 0.4$ Free γ P_c



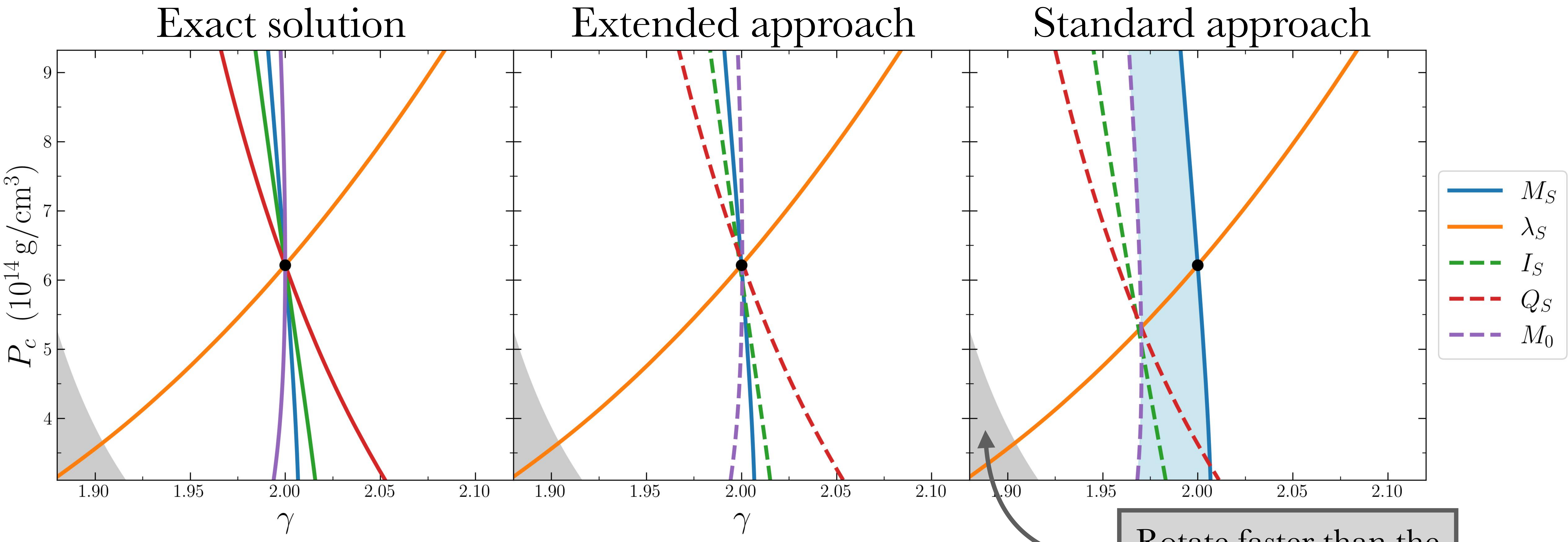
Inferring the EoS $K = 100$ $\chi_S = 0.4$ Free γ P_c



Inferring the EoS $K = 100$ $\chi_S = 0.5$ Free γ P_c

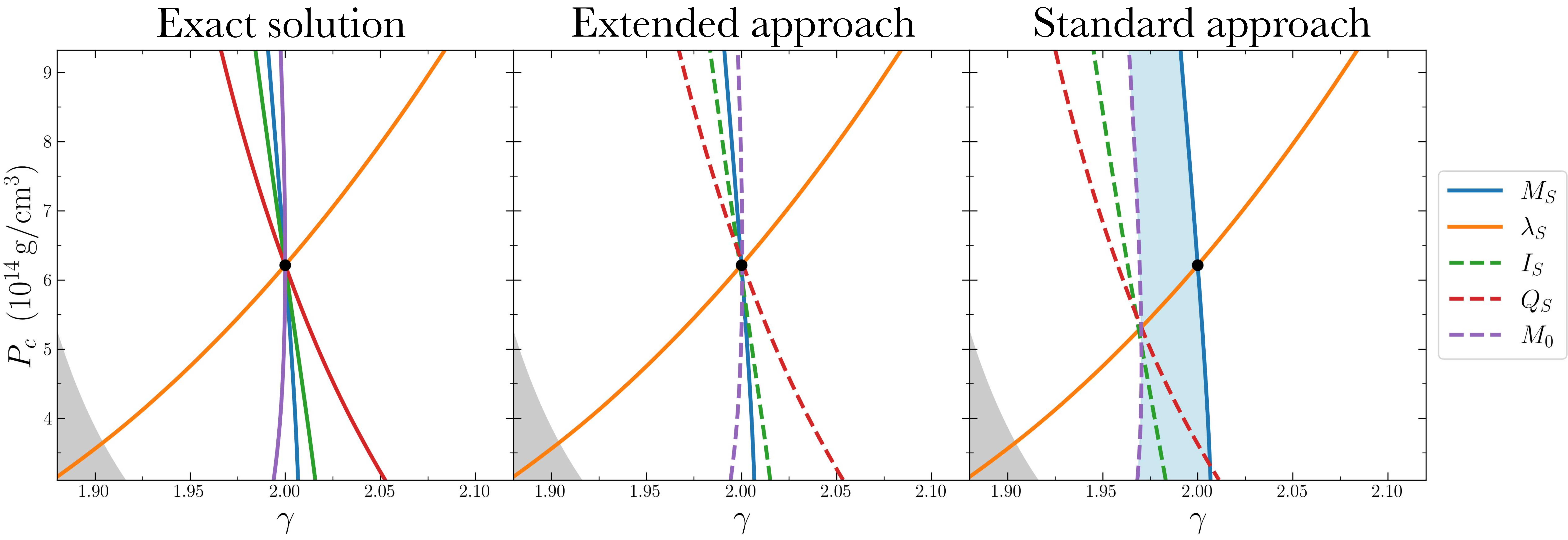


Inferring the EoS $K = 100$ $\chi_S = 0.5$ Free γ P_c

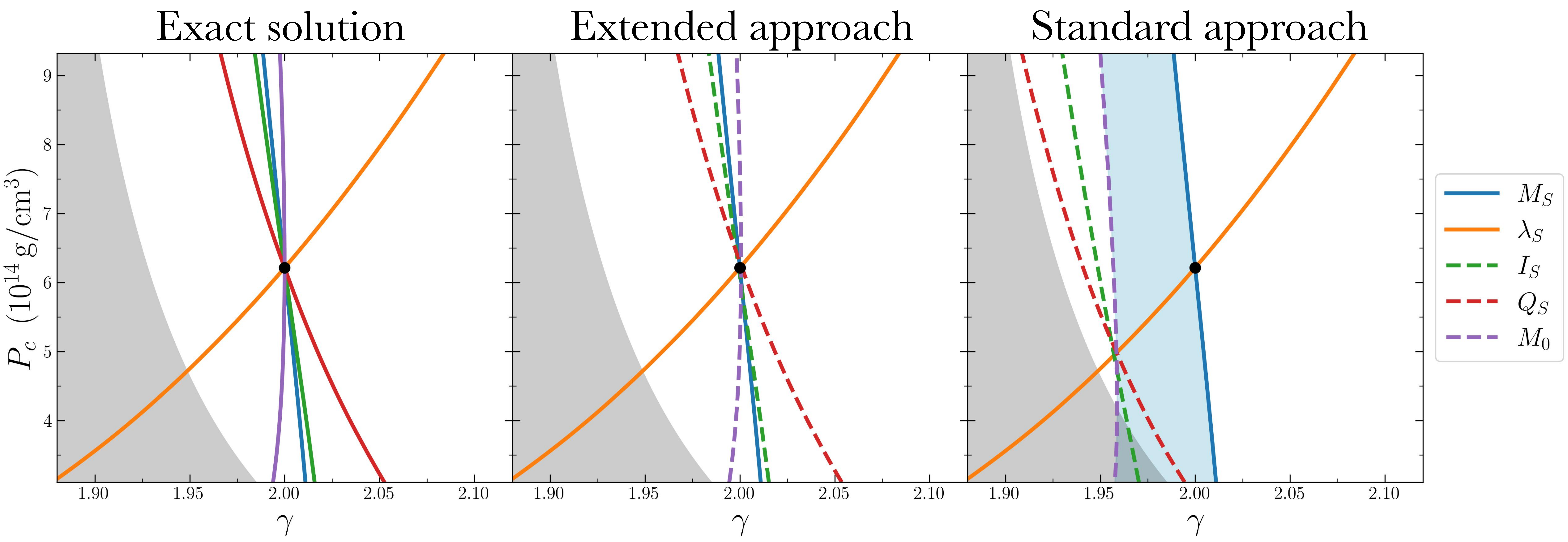


Rotate faster than the mass shedding limit

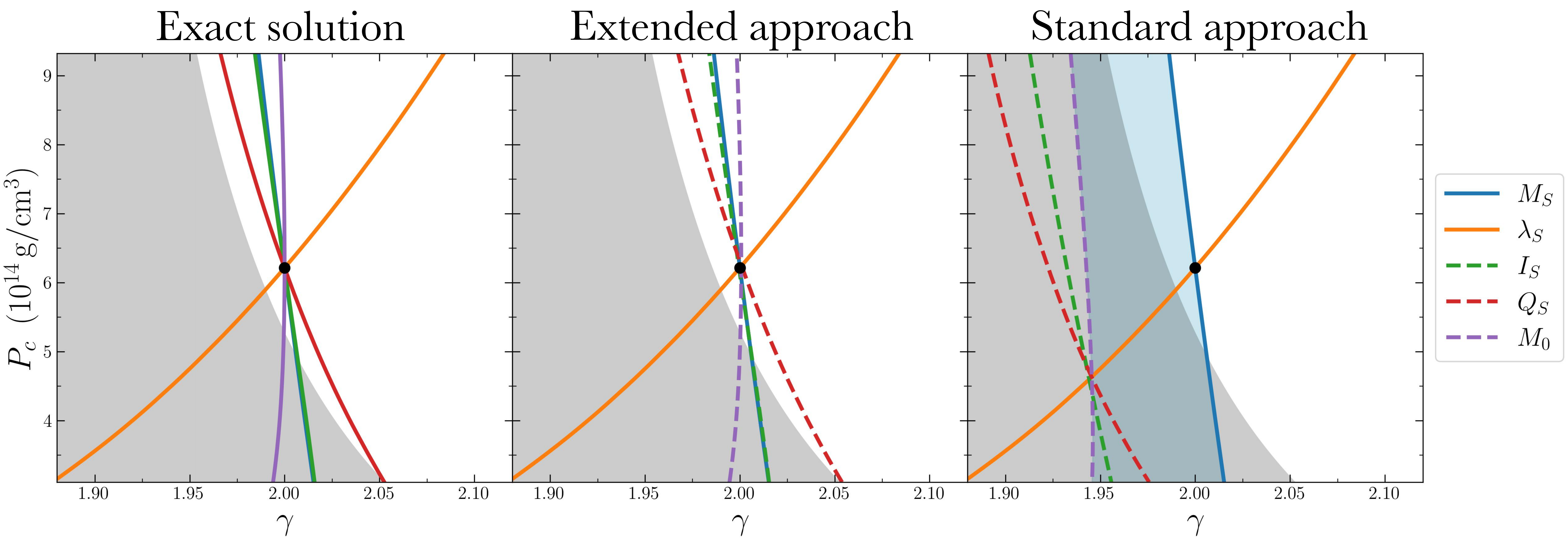
Inferring the EoS $K = 100$ $\chi_S = 0.5$ Free γ P_c



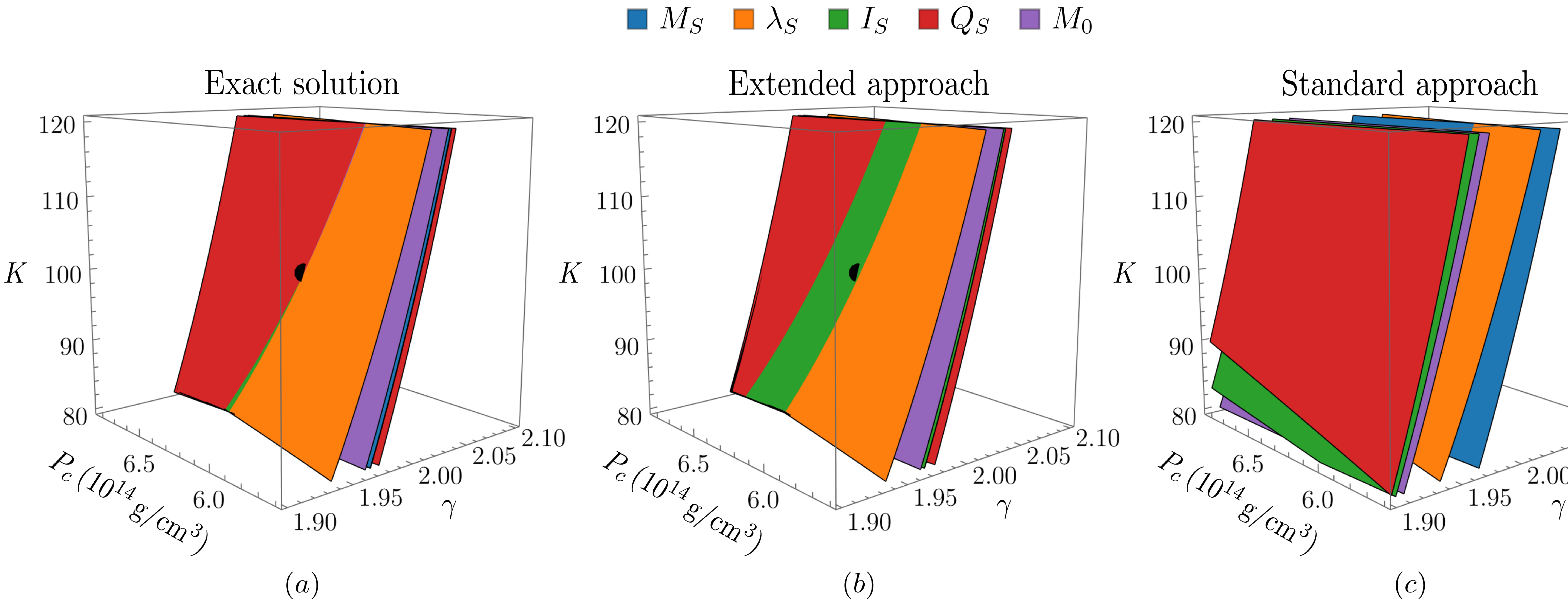
Inferring the EoS $K = 100$ $\chi_S = 0.6$ Free γ P_c



Inferring the EoS $K = 100$ $\chi_S = 0.7$ Free γ P_c



Inferring the EoS $\chi_S = 0.7$ Free γ P_c K



Conclusions

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2. The extended approach enables the inference of 5 (not only 4) quantities of the EoS

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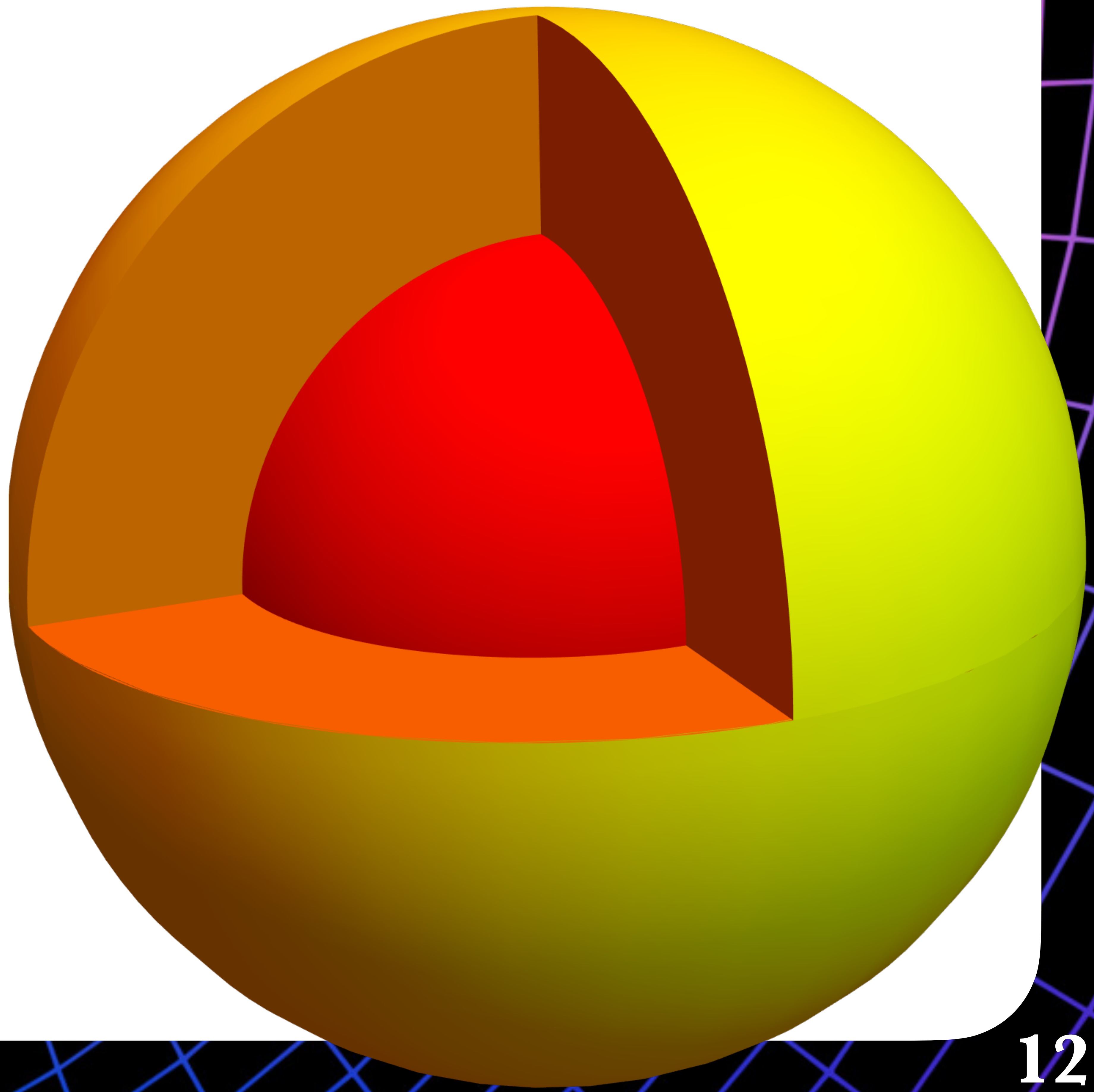
   Example:

   Piecewise polytropic stellar configurations with two regions



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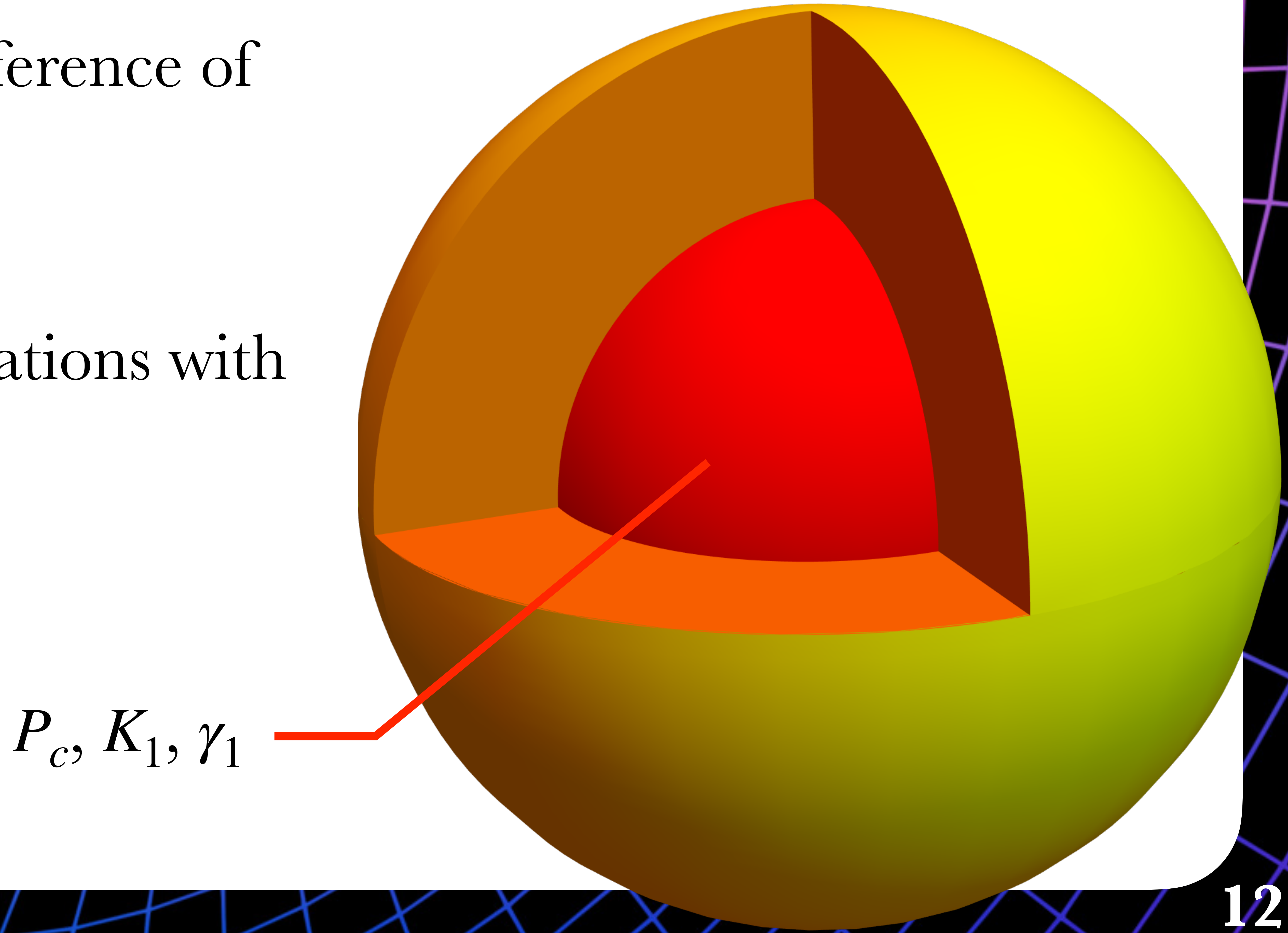

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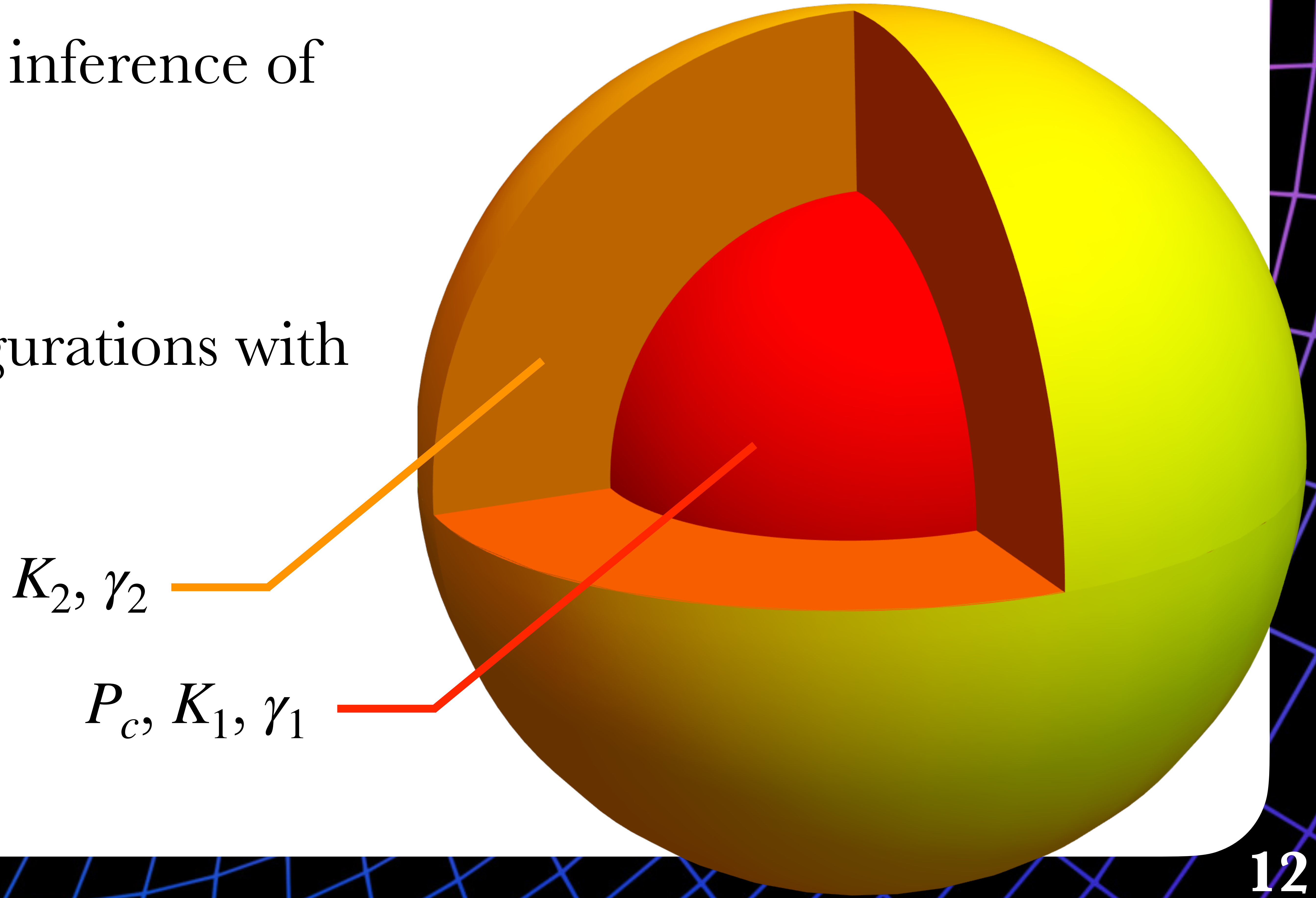

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Standard: $(\lambda_2^\star, M_S^\star) + (I_S^{\mathrm{std}}, Q_S^{\mathrm{std}})$

Extended: $(\lambda_2^\star, M_S^\star) + (I_S^{\mathrm{ext}}, Q_S^{\mathrm{ext}}, M_0^{\mathrm{ext}})$