Axion-like dark energy: late rather than early

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(Work in progress)

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1. Cosmological problems
   - ΛCDM
   - Quintessence
2. Dynamical system
   - Fixed points
   - Tracking
3. The model
   - Axion-like potential
   - Fixed points and tracking
   - Perturbations
4. Fitting the model
   - Constraining parameters
   - Cosmological tensions
5. Conclusions

ΛCDM

In the context of GR:

$$L = \frac{1}{2k^2}(R - 2\Lambda) + L_{r,m}.$$

- Non dynamical dark energy. ρ_Λ remains constant.
- Coincidence problem. $\Omega_m/\Omega_{DE} \sim 1$ today. How is this sensible to initial conditions?
- Why now? Dark energy seems to be dominant only at late-time, not before.
- Fine-tuning problem. New energetic scale $\rho_\Lambda \approx 10^{-47}\,\mathrm{GeV}^{\,4}$. It is very small compared to other scales.
- Hubble tension and σ_8 tension.

Quintessence

Canonical scalar field minimally coupled to the scalar curvature:

$$L = \frac{1}{2k^2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + L_{r,m}.$$

- ϕ is a dynamical field. ρ_ϕ has a non-trivial evolution.
- **Coincidence problem** can be alleviated via scaling solutions and tracking.
- Some quintessence models allow for a natural explanation of **why now?**
- Can the Hubble tension and the σ_8 tension be alleviated?

Dynamical system

Assuming a spatially flat FLRW metric, $ds^2 = -dt^2 + a^2(t)d\bar{x}^2$, and defining

$$x = \frac{k\dot{\phi}}{\sqrt{6}H}, \quad y = \frac{k\sqrt{V}}{\sqrt{3}H}, \quad \lambda = -\frac{V_{,\phi}}{kV}, \qquad (\Omega_{r,m} = 1 - x^2 - y^2),$$

we obtain an autonomous closed system of equations:

$$x' = -\frac{3}{2}\left[2x + (\omega - 1)x^3 + x(\omega + 1)(y^2 - 1) - \sqrt{\frac{2}{3}}\lambda y^2\right],$$

$$y' = -\frac{3}{2}y\left[(\omega - 1)x^2 + (\omega + 1)(y^2 - 1) + \sqrt{\frac{2}{3}}\lambda x\right],$$

$$\lambda' = -\sqrt{6}f(\lambda)x, \qquad f(\lambda) = \lambda^2[\Gamma(\lambda) - 1], \quad \Gamma = \frac{V_{,\phi\phi}V}{(V_{,\phi})^2}.$$

Exponential potentials and cosmological scaling solutions, E. J. Copeland et al. [arXiv:gr-qc/9711068]
Cosmological Tracking Solutions, P. J. Steinhardt et al. [arXiv:astro-ph/9812313]
General Scalar Fields as Quintessence, A. de la Macorra et al. [arXiv:hep-ph/9909459]
Applications of scalar attractor solutions to Cosmology, S. C. C. Ng et al. [arXiv:astro-ph/0107321]

Fixed points

Point	x	y	λ	Existence	w_{eff}	Accel.	Ω_ϕ
O_λ	0	0	Any	Always	w	No	0
$A_\pm^*$	± 1	0	λ_*	$\forall\ \lambda_*$	1	No	1
B^*	$\frac{\sqrt{3}}{\sqrt{2}}\frac{1+w}{\lambda_*}$	$\sqrt{\frac{3(1-w^2)}{2\lambda_*^2}}$	λ_*	$\lambda_*^2 \geq 3(1+w)$	w	No	$\frac{3(1+w)}{\lambda_*^2}$
C^*	$\lambda_*/\sqrt{6}$	$\sqrt{1-\frac{\lambda_*^2}{6}}$	λ_*	$\lambda_*^2 < 6$	$\frac{\lambda_*^2}{3}-1$	$\lambda_*^2 < 2$	1
D	0	1	0	Always	-1	Yes	1

From *Dynamical systems applied to cosmology: dark energy and modified gravity*, S. Bahamonde et al. [arXiv:1712.03107 [gr-qc]]

where λ_* are roots of $f(\lambda)$. Trajectories $B^* \to C^*$ and $B^* \to D$ can alleviate the coincidence problem (scaling solutions).

Tracking

Unique evolution of ϕ. It does not depend on the initial conditions. Tracking is given by $\omega_\phi \approx const.$, where ω_ϕ is the EOS of the scalar field. The EOM reduces to

$$\Gamma = 1 + \frac{\omega - \omega_\phi}{2(1+\omega_\phi)}, \quad \longrightarrow \quad \omega_\phi = \frac{\omega - 2(\Gamma - 1)}{1 + 2(\Gamma - 1)},$$

where ω is the EOS of the dominating fluid.

- Tracking with $\omega_\phi > \omega$: it could happen, but it is disregarded (structure formation suppression).
- Tracking with $\omega_\phi = \omega$: $\Gamma = 1$. Scaling solutions. Fixed point B^*.
- **Tracking with $\omega_\phi < \omega$: $\Gamma > 1$ and $\Gamma \approx const.$**

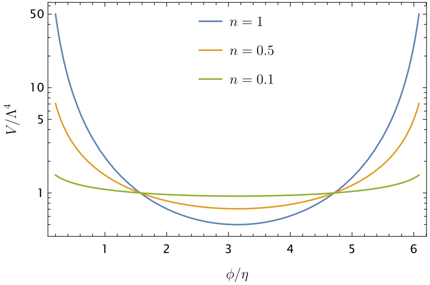
Cosmological Tracking Solutions, P. J. Steinhardt et al. [arXiv:astro-ph/9812313]

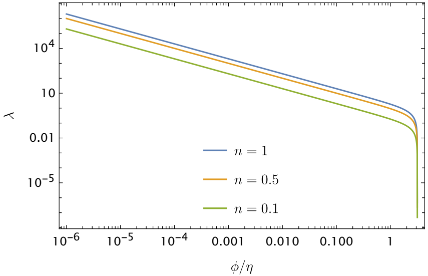
Axion-like potential

$$V(\phi) = \Lambda^4[1 - \cos(\phi/\eta)]^{-n}.$$

Generalization to $-n$

$$\lambda(\phi) = \frac{n}{k\eta}\frac{\sin(\phi/\eta)}{1-\cos(\phi/\eta)}$$





Wave Dark Matter, L. Hui [arXiv:2101.11735 [astro-ph.CO]]
Natural Inflation, K. Freese [arXiv:astro-ph/9310012]
Dark energy from the string axiverse, M. Kamionkowski et al. [arXiv:1409.0549 [hep-ph]]
Early Dark Energy Can Resolve The Hubble Tension, V. Poulin et al. [arXiv:1811.04083 [astro-ph.CO]]
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- Cosmological constant in the limit $n \to 0$.
- Minimum at $\phi/\eta = \pi \to V \approx \frac{\Lambda_{eff}}{k^2} + \frac{1}{2}m^2(\phi - \pi\eta)^2$.
- $\phi_i/\eta \ll 1$ in order to have non-trivial evolution $\to \lambda_i \gg 1$.

Fixed points and tracking

- Fixed points: O_λ and D (minimum of the potential). D is an attractor $\rightarrow$ **Late-time acceleration.**
- $\Gamma(\lambda) = 1 + \frac{1}{2n} + \frac{n}{2k^2\eta^2\lambda^2}$. In the regime $\lambda \gg 1$: $\Gamma \approx 1 + \frac{1}{2n}$ $\rightarrow$ **Tracking** with $\omega_\phi < \omega$. Coincidence problem alleviated!
- Tracking given by $\omega_\phi = -\frac{2(\Gamma-1)}{1+2(\Gamma-1)} \approx -\frac{1}{1+n}$ $\quad (\omega = 0)$:
  - For $n = 1$: $\omega_\phi = -0.5$
  - For $n = 0.1$: $\omega_\phi = -10/11 \approx -0.91$
  - For $n \rightarrow 0$: $\omega_\phi \rightarrow -1$ (flat potential, cosmological constant)
- In addition, $\Omega_\phi \propto t^P$, where $P = \frac{4(\Gamma-1)}{1+2(\Gamma-1)}$. Ω_ϕ increases faster as $\lambda \rightarrow 0$, since Γ diverges. Why now problem alleviated!

Cosmological Tracking Solutions, P. J. Steinhardt et al. [arXiv:astro-ph/9812313]
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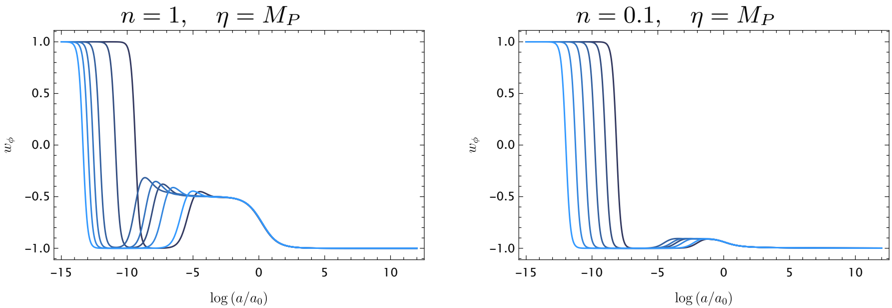
Tracking plot

Evolution of ω_ϕ (each line represents a different set of initial conditions):



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(darkest blue)	$\phi_4 = 5 \times 10^{-7/2} M_P, \quad x_4 = 10^{-5}$
$\phi_1 = 5 \times 10^{-5} M_P, \quad x_1 = 10^{-2}$	$\phi_5 = 5 \times 10^{-3} M_P, \quad x_5 = 10^{-6}$
$\phi_2 = 5 \times 10^{-9/2} M_P, \quad x_2 = 10^{-3}$	$\phi_6 = 5 \times 10^{-5/2} M_P, \quad x_6 = 10^{-7}$
$\phi_3 = 5 \times 10^{-4} M_P, \quad x_3 = 10^{-4}$	(lightest blue)

Structure formation

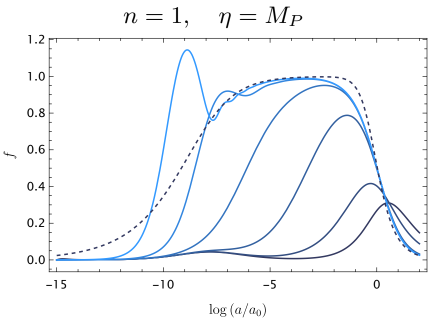
The growth rate function can be parameterized as follows:

$$f = \frac{d \ln \delta_m}{d \ln a} = \Omega_m^\gamma, \text{ valid in the sub-Hubble limit } (\mathrm{k} \gg \mathrm{aH}).$$

- f depends on ω_ϕ through Ω_m. γ is less sensitive.
- Larger values of ω_ϕ increase Ω_ϕ (in the past) $\longrightarrow$ Ω_m decreases as ω_ϕ increases $\longrightarrow$ Larger values of ω_ϕ disfavours structure formation.
- Scaling solutions: $\omega_\phi = \omega$ for a long period. Could be problematic explaining structure formation.
- Tracking with $\omega_\phi < \omega$ could be favoured by measurements.

Growth rate

Evolution of f (solid lines, exact numerical calculations of the model for different modes; dashed line, parametrization Ω_m^γ of ΛCDM, with $\gamma = 0.55$):



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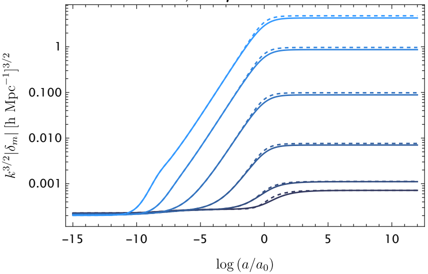
(darkest blue) $k_1 = 10^{-4}\text{h Mpc}^{-1}$
$k_2 = 3.98 \times 10^{-4}\text{h Mpc}^{-1}$
$k_3 = 1.58 \times 10^{-3}\text{h Mpc}^{-1}$
$k_4 = 6.31 \times 10^{-3}\text{h Mpc}^{-1}$
$k_5 = 2.51 \times 10^{-2}\text{h Mpc}^{-1}$
$k_6 = 0.1\text{h Mpc}^{-1}$ (lightest blue)

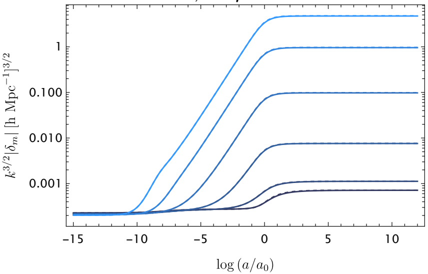
Matter perturbations

Evolution of $\delta_m = \delta\rho_m/\rho_m$ (solid lines, numerical calculations of the model for different modes; dashed lines, calculations of ΛCDM for different modes):

$n = 1, \quad \eta = M_P$



$n = 0.1, \quad \eta = M_P$



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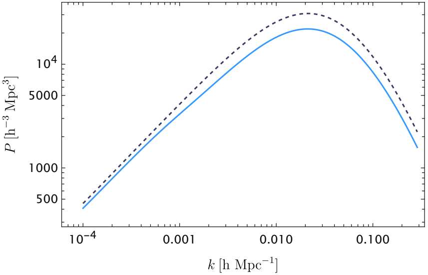
$k_6 = 0.1$h Mpc^{-1} (lightest blue)

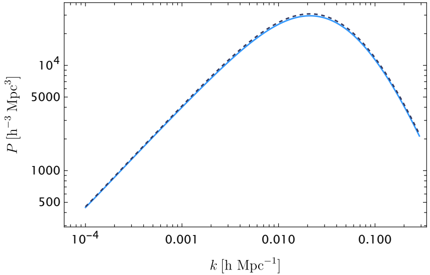
Matter power spectrum

Matter power spectrum suppression (solid line, calculation of the model; dashed line, calculation of ΛCDM):

$n = 1, \quad \eta = M_P$



$n = 0.1, \quad \eta = M_P$



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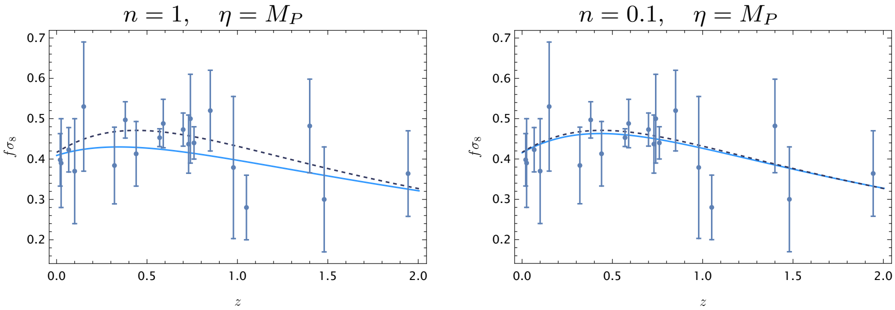
fσ8

fσ8 distribution (solid line, calculation of the model; dashed line, calculation of ΛCDM):



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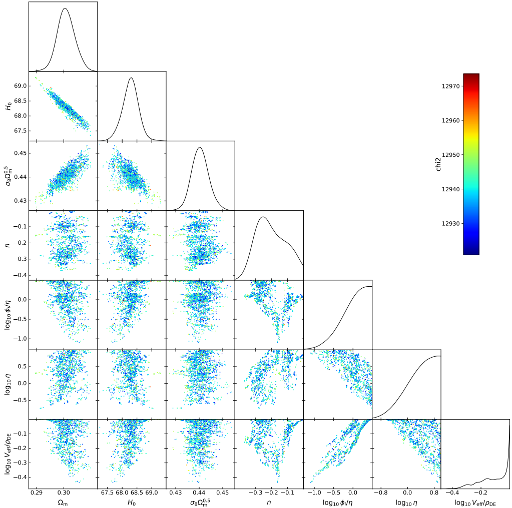
Constraining parameters (I)



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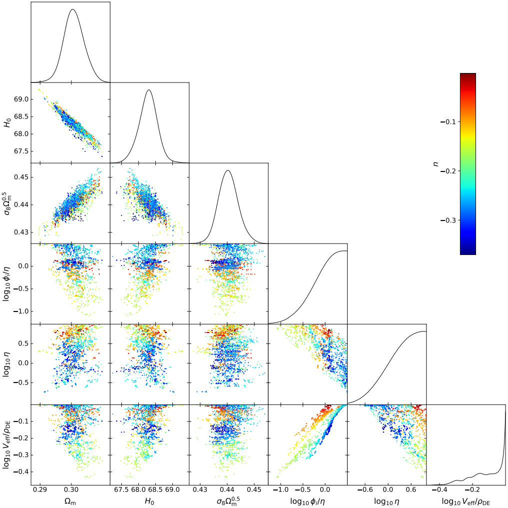
Constraining parameters (II)



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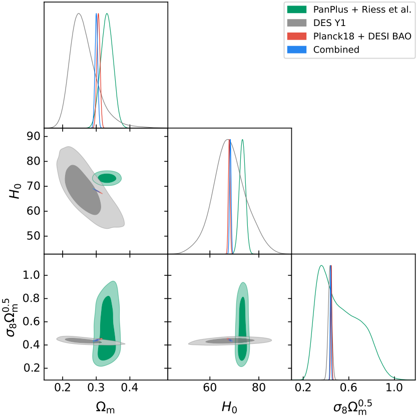
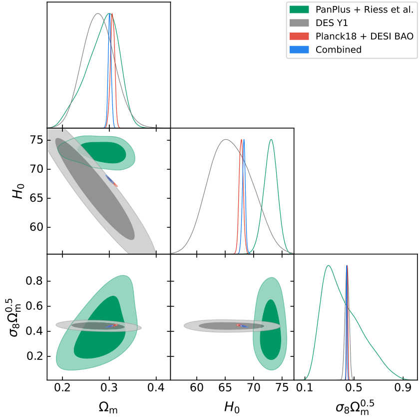
Cosmological tensions

ΛCDM



$$V(\phi) = \Lambda^4[1 - \cos(\phi/\eta)]^{-n}$$



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Conclusions

- Coincidence problem can be alleviated by scaling solutions and tracking.
- The model $V(\phi) = \Lambda^4[1 - \cos(\phi/\eta)]^{-n}$, with $n > 0$, allows for a late-time acceleration and is an example of tracking with $\omega_\phi < \omega$.
- The model also alleviates the why now problem.
- Possible values of n must be constrained in order to correctly explain structure formation.
- We are currently fitting the model. Constraints on the parameters of the theory can be obtained.
- σ_8 tension may be alleviated by the model.

Hubble horizon