Black holes in effective field theories
dynamics and new observational signatures

Daniela Doneva

University of Tübingen

In collaboration with:
Stoytcho Yazadjiev, Llibert Aresté Saló, Katy Clough, Pau Figueras, Alex Vañó-Viñuales

Beyond-GR black holes in EFT

Open questions

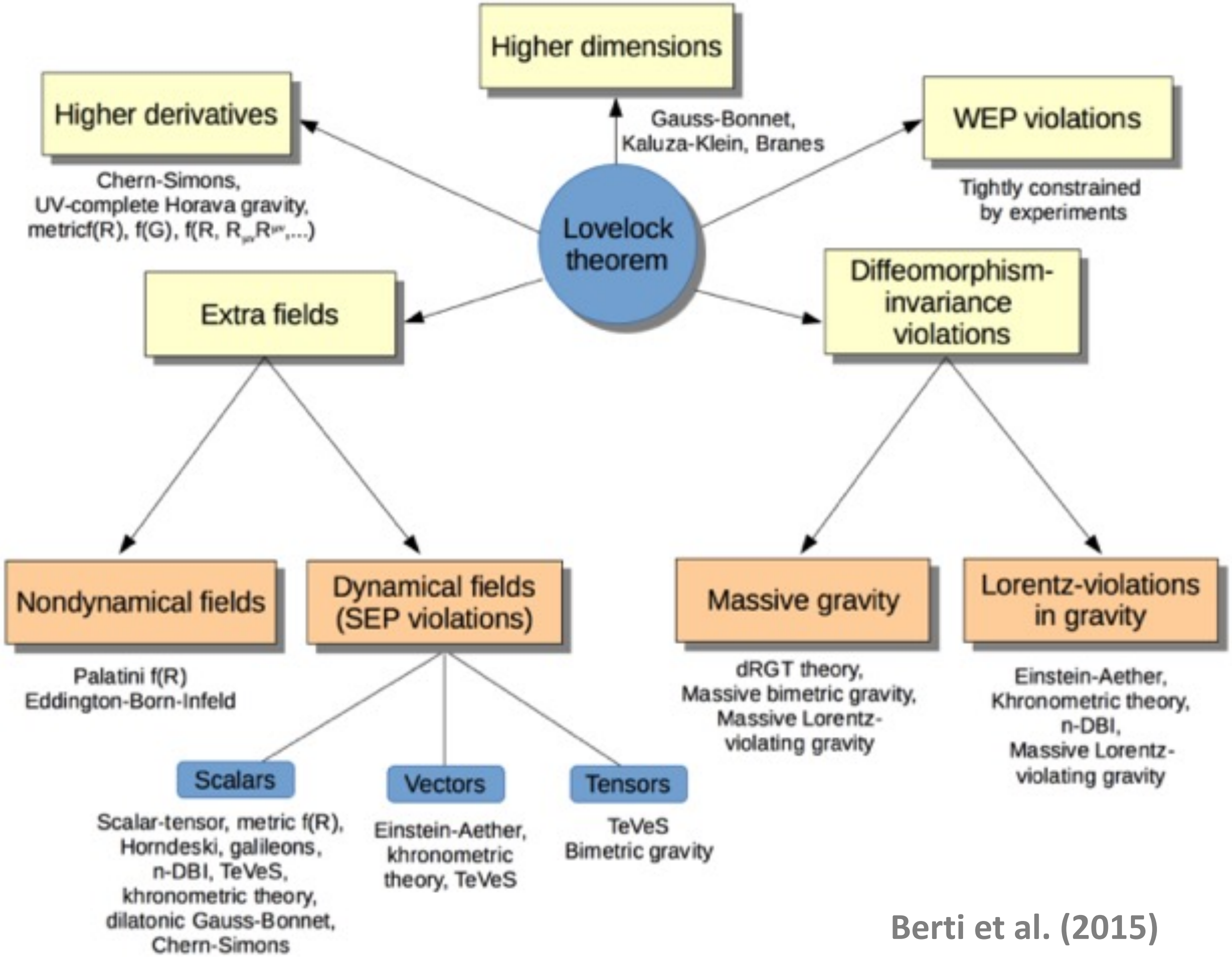
- **Observations**
  - ✓ Strong gravity not well tested yet
  - ✓ Dark matter
  - ✓ Dark energy
  - ✓ Inflation

- **Theory**
  - ✓ Quantum gravity
  - ✓ Theories trying to unify all interactions
  - ✓ New fundamental fields in nature

Lovelock's theorem

Einstein's field equations are **unique** if:

- ✓ we are working in **four dimensions**
- ✓ **diffeomorphism invariance** is respected
- ✓ the **metric** is the **only field** mediating gravity
- ✓ the equations are **second-order differential equations**.



Berti et al. (2015)

Effective field theory

- Modify GR ”**agnostically**” as a residual of some bigger fundamental (not yet known) theory operating at high energies (high curvature)

- Obey some natural **symmetries** (e.g. Lorenz invariance)

- The length (mass) scales involved in the theory are **in agreement with present experiments.**

Effective field theory

GR

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}R$$

Effective field theory

GR **Curvature invariants**

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}R + \left(R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \dots\right)$$

Effective field theory

GR **Curvature invariants** **Fields+couplings**

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}R + \left(R^2, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \dots\right) + (\varphi, A_\mu, \dots)$$

Desired properties:

- In agreement with **known** observations
- Second order field equations (lack of instabilities/ghosts) **?**
- Well-posedness **?**

Second order field equations – Gauss-Bonnet gravity

GR **Gauss-Bonnet invariant R_{GB}^2**

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R + \lambda^2 f(\varphi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right) - 2\nabla_\mu\varphi\nabla^\mu\varphi + V(\varphi)\right]$$

Scalar field

- Equations of motions are of **second order**!
- $\boldsymbol{R_{GB}^2 \sim \frac{1}{r^6}}$: non-negligible only for very compact objects (neutron stars, black holes)
- **GWs** are ideal to **probe such modifications**

scalar-Gauss-Bonnet gravity

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi) + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right].$$

Gauss-Bonnet invariant:

$$\mathcal{R}^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

- Field equations :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi - \frac{1}{2}g_{\mu\nu}V(\varphi),$$

$$\nabla_\alpha\nabla^\alpha\varphi = \frac{1}{4}\frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB},$$

Scalar field coupling $f(\varphi)$

$$\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi} R^2_{GB}$$

Expand $f(\varphi)$ in series around $\varphi = 0$:

$$f(\varphi) = f_0 + f_1\varphi + f_2\varphi^2 + f_3\varphi^3 + f_4\varphi^4 + O(\varphi^5)$$

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- $f_1 \neq 0$: **shift-symmetric** theory, Schwarzschild is not a solution, $|\varphi| > 0$ always

  Kanti et al PRD(1996), Torii et al (1996), Pani&Cardoso PRD (2009)

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Type II:

- $f_1 = 0$, $f_2 > 0$, $R^2_{GB} > 0$: **spontaneous** scalarization, Kerr unstable for **small masses** DD, Yazadjiev PRL (2018), Silva et al PRL (2018), Antoniou et al (2018)
- $f_1 = 0$, $f_2 < 0$, $R^2_{GB} < 0$: **spin-induced** scalarization, Kerr unstable for **large spins**
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Beyond Type II:

- $f_1 = 0$, $f_2 = 0$: $\mu^2_{\rm eff} = 0$, **nonlinear** scalarization, Kerr **linearly stable always, nonlinear scalarized phases can co-exist** DD, Yazadjiev, PRD Lett. (2021)

Scalar field coupling $f(\varphi)$

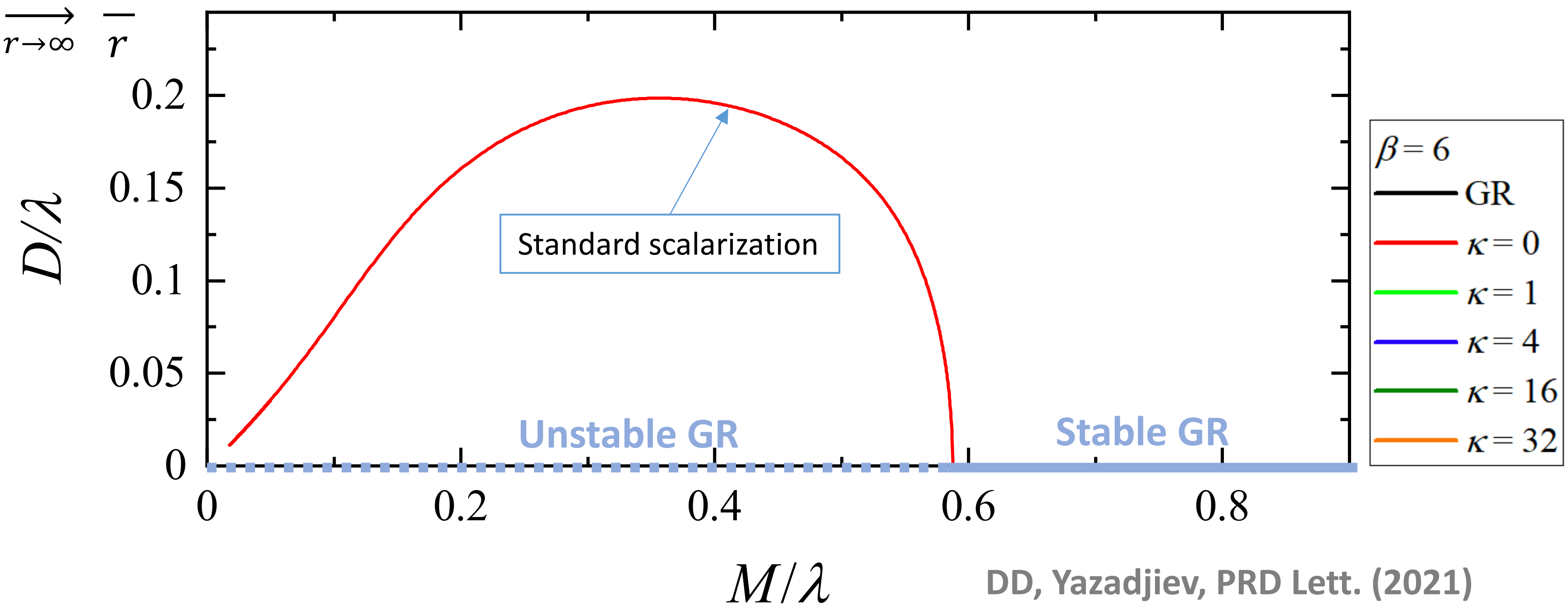
- Standard + nonlinear scalarization

$$f(\varphi) = \beta\varphi^2 + \kappa\varphi^4 + \cdots \quad \left(\text{full form: } \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)}\right)\right)$$

Standard scalarization with φ^2

$$f(\varphi) = \beta\varphi^2 + \cancel{\kappa\varphi^4} + \cdots \quad \left(\text{full form: } \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2 + \cancel{\kappa\varphi^4})}\right)\right)$$

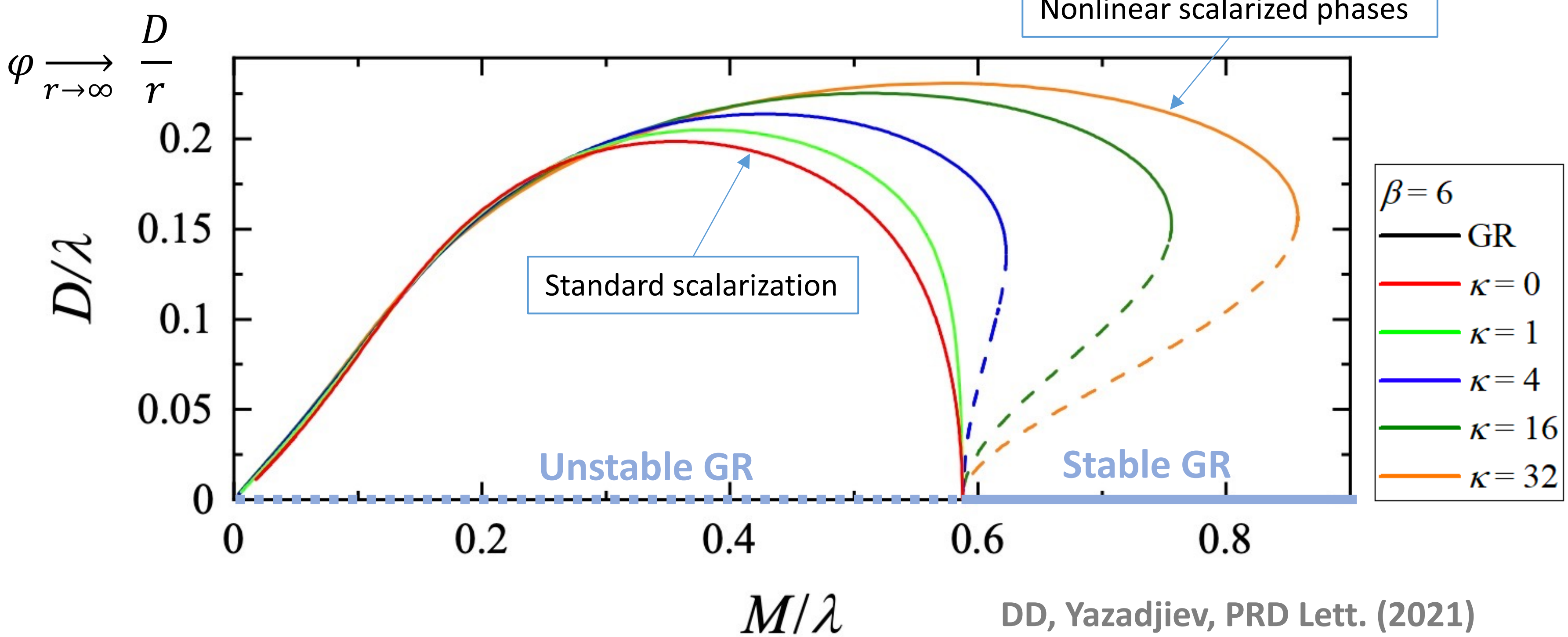
$$\varphi \xrightarrow[r\to\infty]{} \frac{D}{r}$$



DD, Yazadjiev, PRD Lett. (2021)

Standard + nonlinear scalarization

$$f(\varphi) = \beta\varphi^2 + \kappa\varphi^4 + \cdots \quad \left(\text{full form: } \frac{1}{2\beta}\left(1 - e^{-\beta(\varphi^2+\kappa\varphi^4)}\right)\right)$$



DD, Yazadjiev, PRD Lett. (2021)

- Transition from **stable scalarized to GR** happens with a **jump**
- For a similar effect for charged BH see Blázquez-Salcedo et al. PLB (2020)

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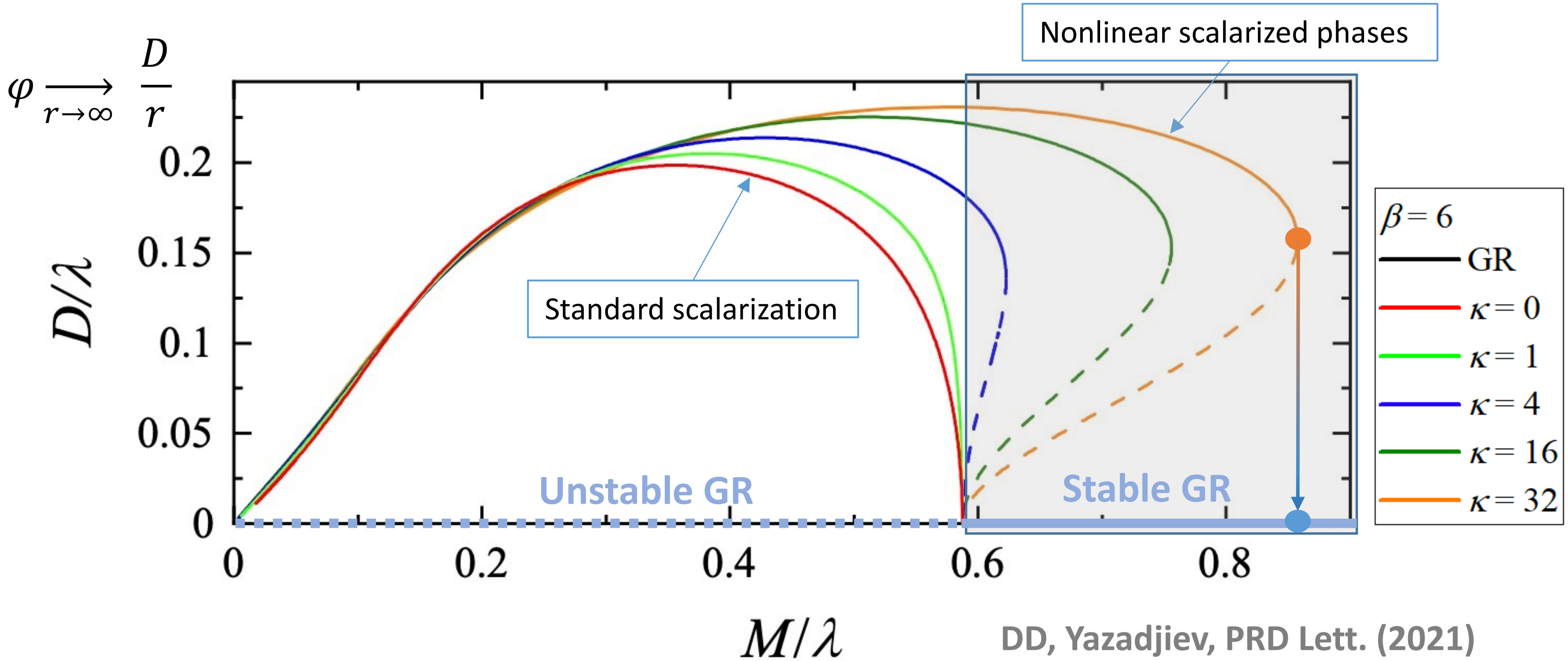

DD, Yazadjiev, PRD Lett. (2021)

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Well-posedness

- A solution exists;
- The solution is unique;
- It changes continuously with changes in the data.

Gauss-Bonnet equations – significatly more complicated

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Gamma_{\mu\nu} = \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi - \frac{1}{4} g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi - \frac{1}{2} g_{\mu\nu} V(\varphi)$$

$$\nabla_\alpha\nabla^\alpha\varphi = \frac{dV(\varphi)}{d\varphi} - \frac{\lambda}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{GB},$$

$$\Gamma_{\mu\nu} = -\frac{1}{2} R\Omega_{\mu\nu} - \Omega^\alpha_\alpha \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right) + 2 R_{\alpha(\mu}\Omega^\alpha_{\nu)} - g_{\mu\nu} R^{\alpha\beta}\Omega_{\alpha\beta} + R^\beta_{\mu\alpha\nu}\Omega^\alpha_\beta$$

$$\Omega_{\mu\nu} = \lambda\nabla_\mu\nabla_\nu f(\varphi).$$

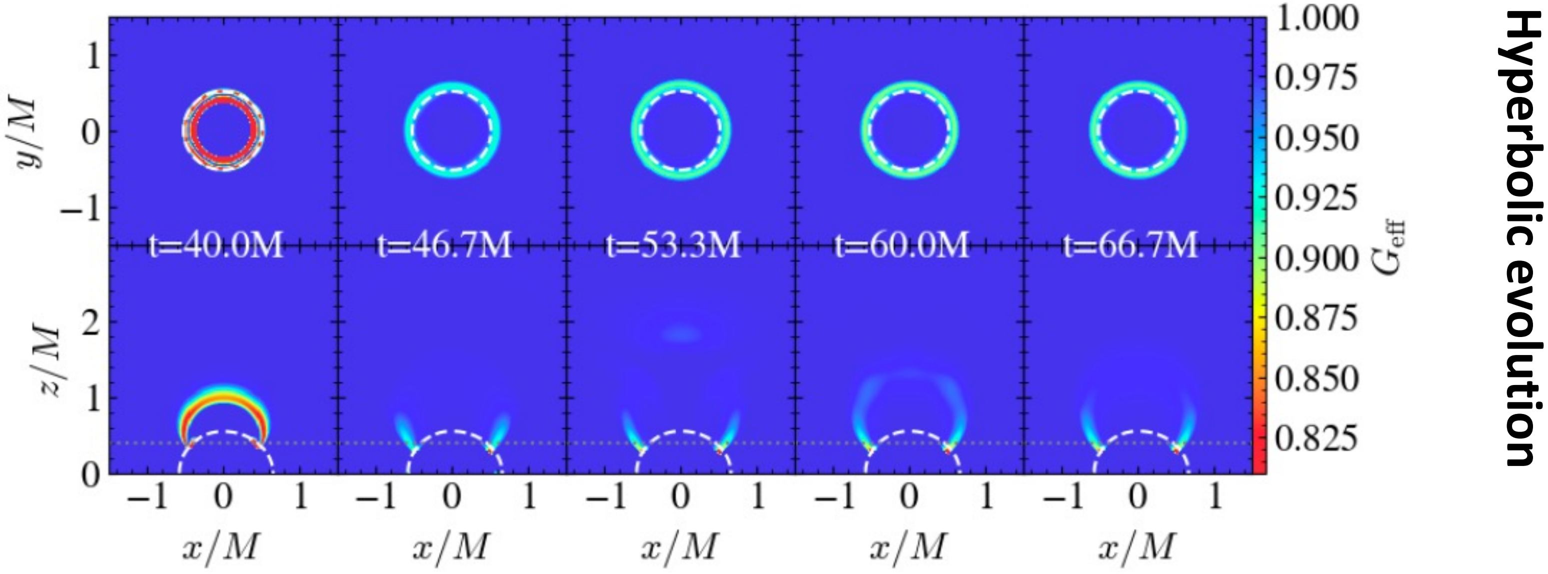
Studying hyperbolicity

- **Principle symbol** – a matrix assembled by the coefficients in front of the leading (2nd) order derivative in the differential equation
- The contribution from **the Gauss-Bonnet sector to the principal part** of the evolution equations **only affects the physical modes** (purely gravitational modes and mixed scalar-gravitational ones).
- The modes of the purely gravitational sector lie on **the null cone of the effective metric** Real PRD (2021), Areste-Salo et al PRL (2022), PRD (2022)

$$g_{\text{eff}}^{\mu\nu} = g^{\mu\nu} - \Omega^{\mu\nu}$$
$$\Omega_{\mu\nu} = \lambda \nabla_\mu \nabla_\nu f(\varphi)$$

- **Hyperbolicity loss:** the normalized determinant of the effective metric (with respect to GR) < 0 (East&Ripley PRL (2021), Areste-Salo et al PRL (2022), Hegade et al PRD (2023)))

Normalized determinant – spin-induced black hole



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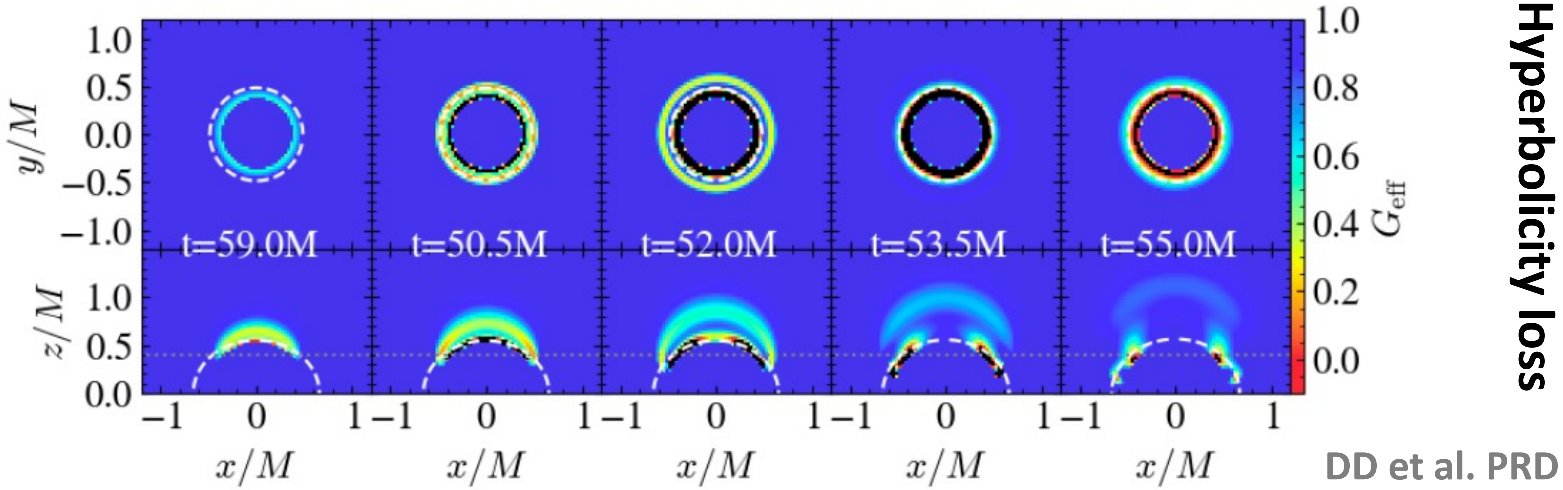


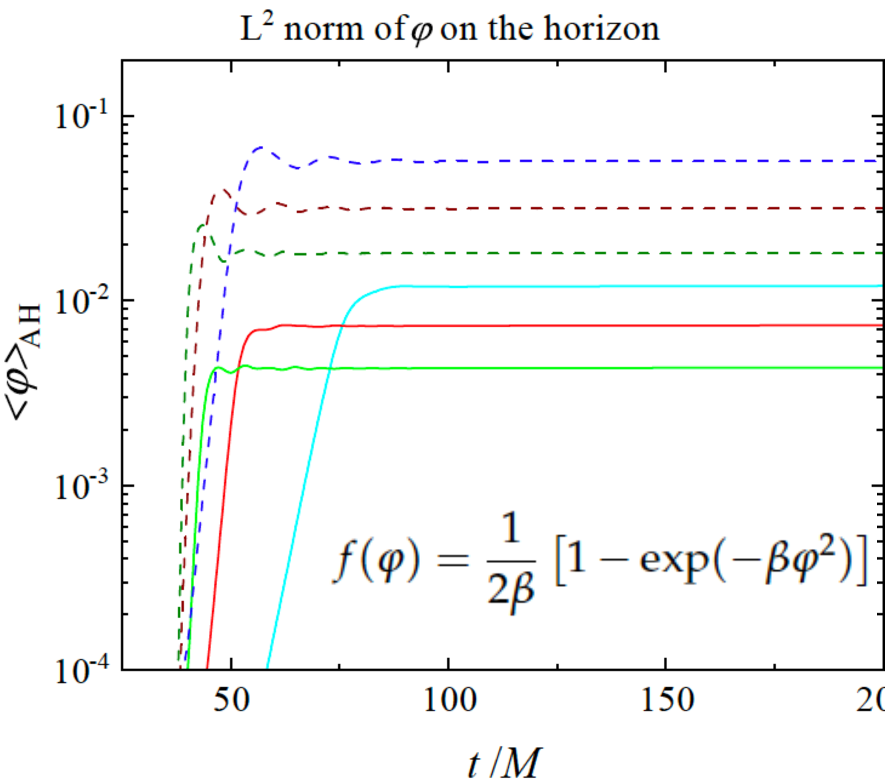
DD et al. PRD (2023)

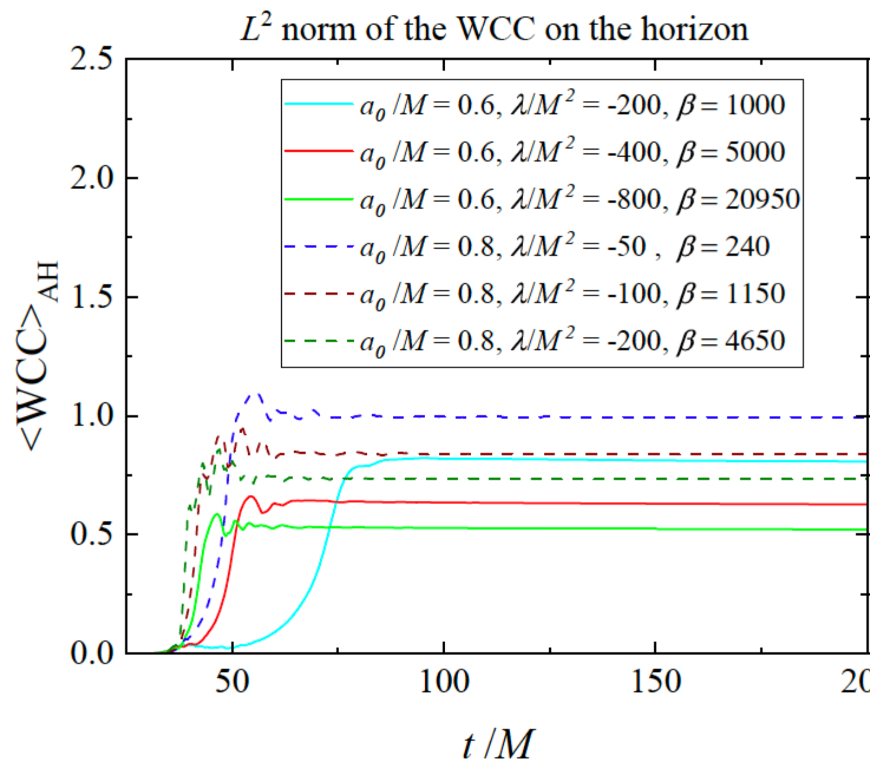
Limitting models and weak coupling condition

- **Weak coupling condition**

$$\sqrt{|\lambda f'(\varphi)|}/L \ll 1$$

$$L^{-1} = \max\{|R_{ij}|^{1/2}, |\nabla_\mu \varphi|, |\nabla_\mu \nabla_\nu \varphi|^{1/2}, |\mathcal{R}^2_{GB}|^{1/4}\}$$





DD et al. PRD (2023)

Resolving the problem

- **Gauge change** – not very likely to help, hyperbolicity loss due to eigenvalues of physical modes becoming imaginary **Areste-Salo et al PRL (2022), PRD (2022), DD et al. PRD (2023)**

- **Fixing approach** **Franchini et al PRD (2022), Cayuso at al PRL (2023), Lara et al (2024)**
  - ✓ A prescription to control the high frequency behaviour of an EFT
  - ✓ Modify in an *ad hoc* way the higher-order contributions to the field equations
  - ✓ Add a driver equation to let the solution relax to its correct value

- **Addition interactions** in the action can mitigate the hyperbolicity loss

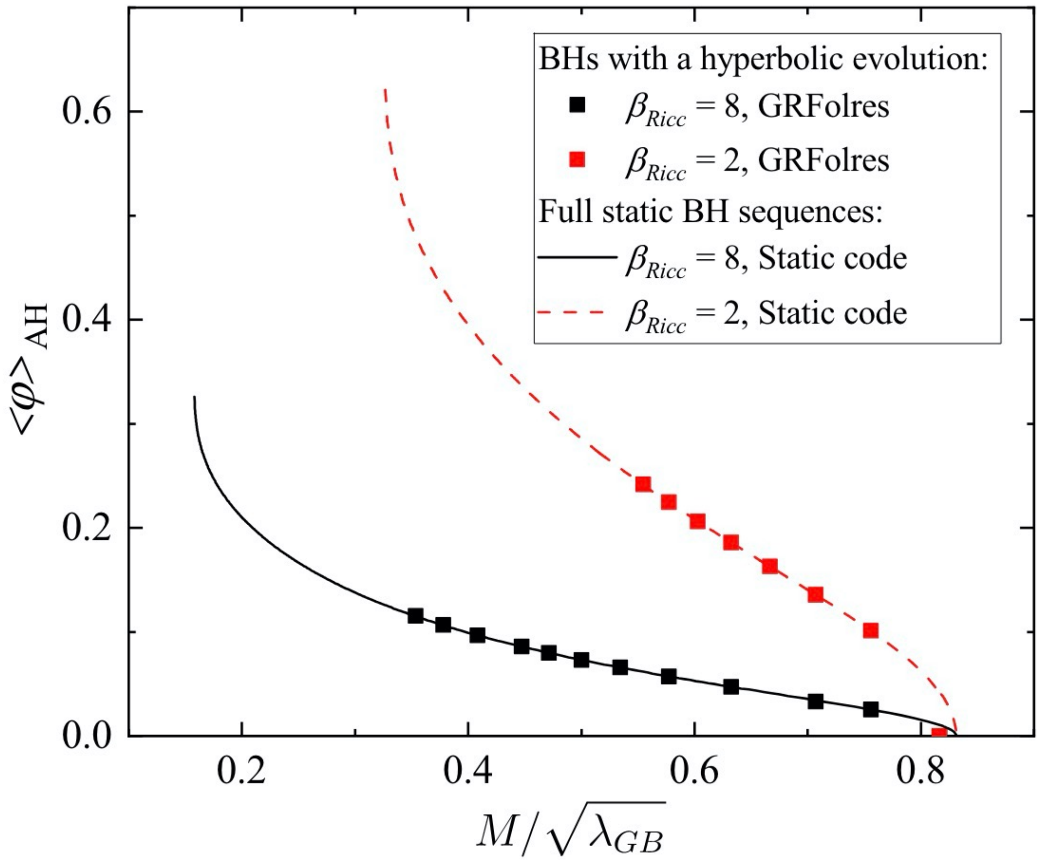
$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi + \frac{1}{4} \lambda(\varphi) f(\varphi) R^{GB} - \beta(\varphi) R \right]$$

Ricci scalar coupling

  - ✓ Previously unstable solution **turn stable** **Antoniou et al PRD (2021)**
  - ✓ Loss of hyperbolicity is **mitigated** in 1D simulation **Thaalba et al (2023)**

Ricci scalar coupling $\lambda(\varphi) = \lambda_{\rm GB}\,\varphi^2 \quad \beta(\varphi) = \beta_{\rm Ricc}\,\varphi^2$

- Effective metric: $g^{\mu\nu}_{\rm eff} = g^{\mu\nu}(1-\beta(\varphi)) - \Omega^{\mu\nu}$
- **3+1 simulations** – hyperbolicity maintained only for "weak" scalar field



DD, Areste-Salo, Yazadjiev PRD (2024)

Binary mergers

Decoupling limit vs Solving the full equations

- The **full field equations**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu \varphi \nabla_\nu \varphi - g_{\mu\nu} \nabla_\alpha \varphi \nabla^\alpha \varphi - \frac{1}{2} g_{\mu\nu} V(\varphi)$$

$$\nabla_\alpha \nabla^\alpha \varphi = \frac{1}{4} \frac{dV(\varphi)}{d\varphi} - \frac{\lambda(\varphi)^2}{4} \frac{df(\varphi)}{d\varphi} R^2_{GB}$$

- **Initial data**?

Decoupling limit vs Solving the full equations

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- **Initial data**?

- **Decoupling limit** – follow only the scalar field dynamics

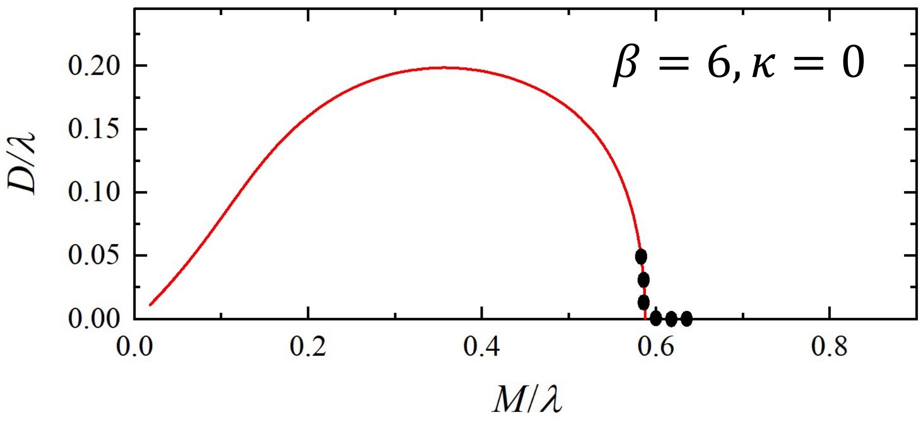
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0$$

$$\nabla_\alpha \nabla^\alpha \varphi = \frac{1}{4} \frac{dV(\varphi)}{d\varphi} - \frac{\lambda(\varphi)^2}{4} \frac{df(\varphi)}{d\varphi} R^2_{GB}$$

- **No gravitational waveform** can be extracted

Decoupling limit – jump in the solutions

$$f(\varphi) = \frac{1}{2\beta}\left(1 - e^{-\beta\left(\varphi^2 + \kappa\varphi^4\right)}\right)$$



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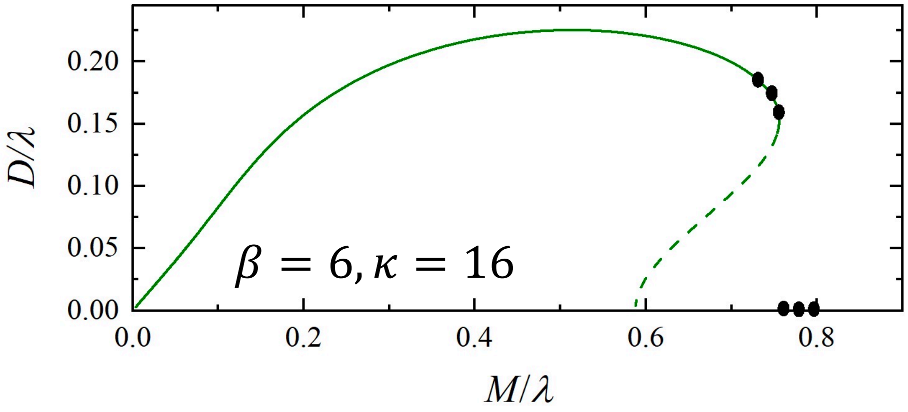
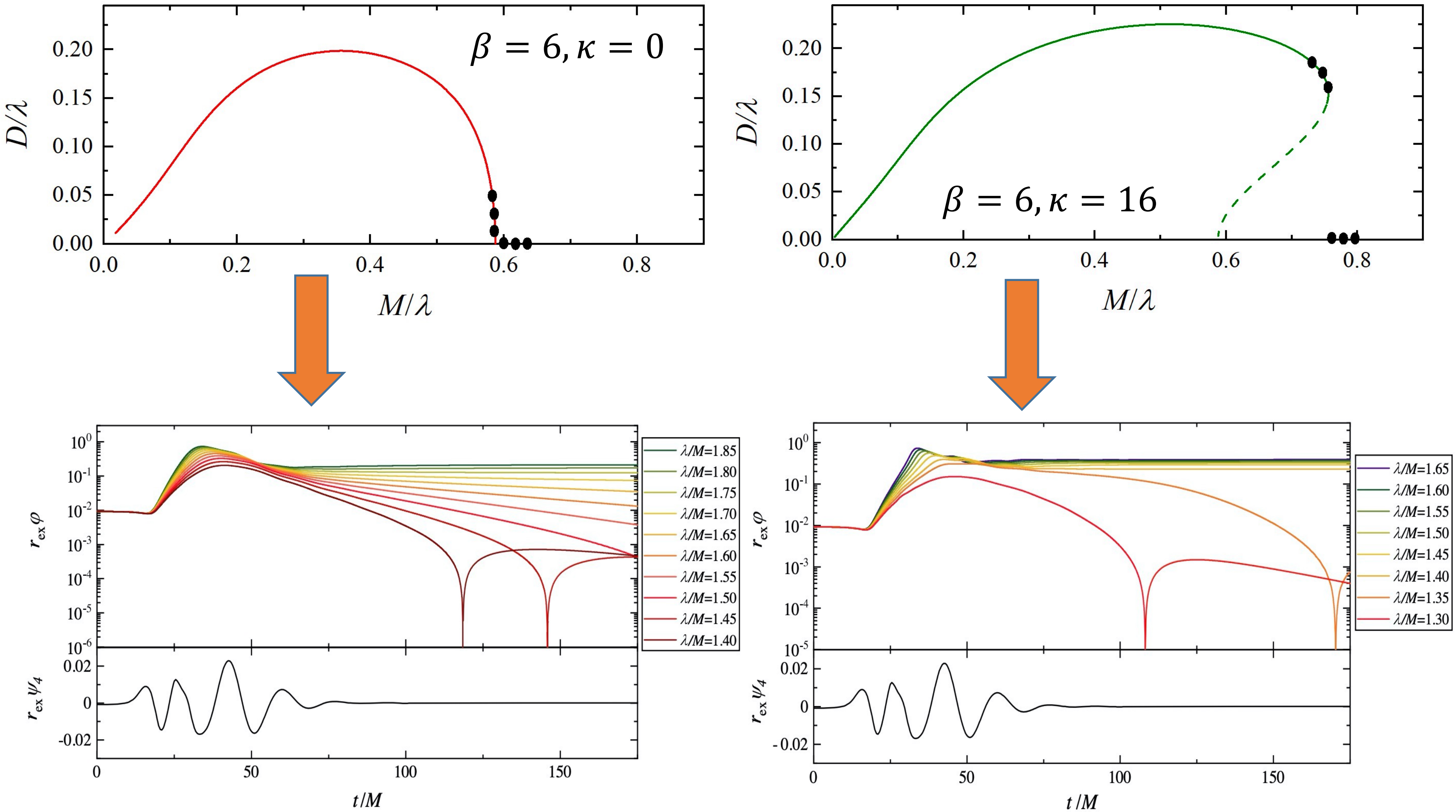


Head on collision: Jump vs. No Jump

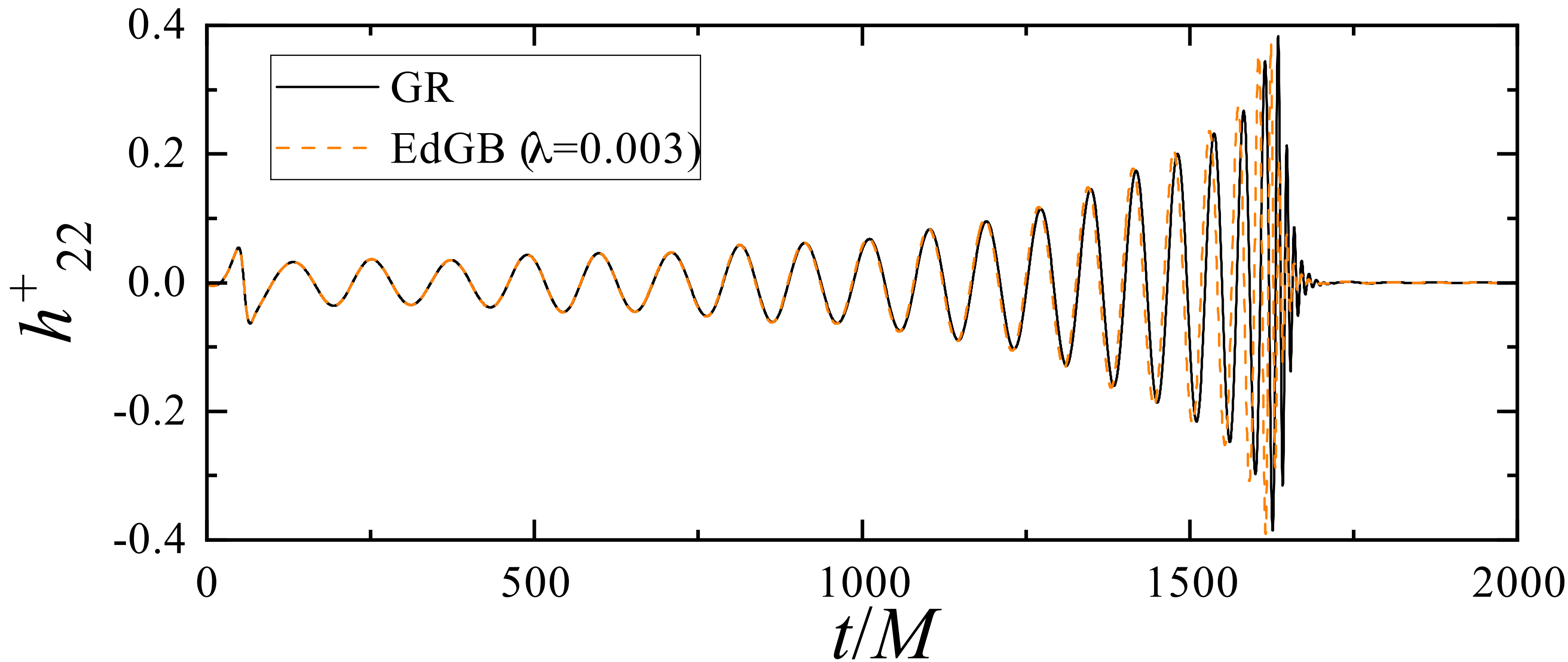


DD, Vano-Vinuales, Yazadjiev (2022)

- IMR (in)consistency in modified gravity?

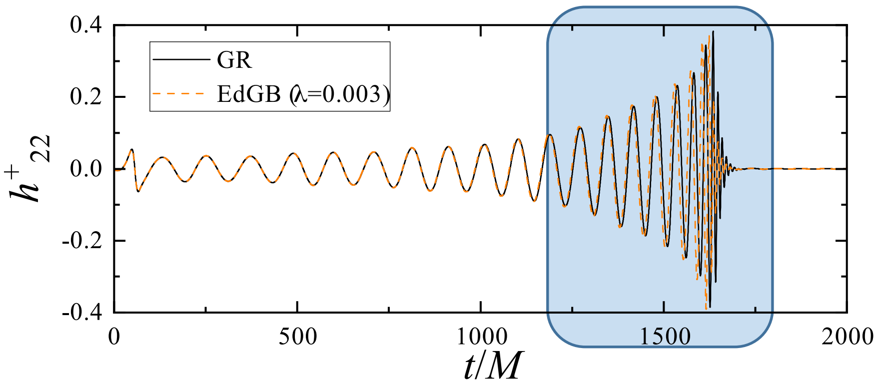
Full problem – inspiral of two equal mass BHs

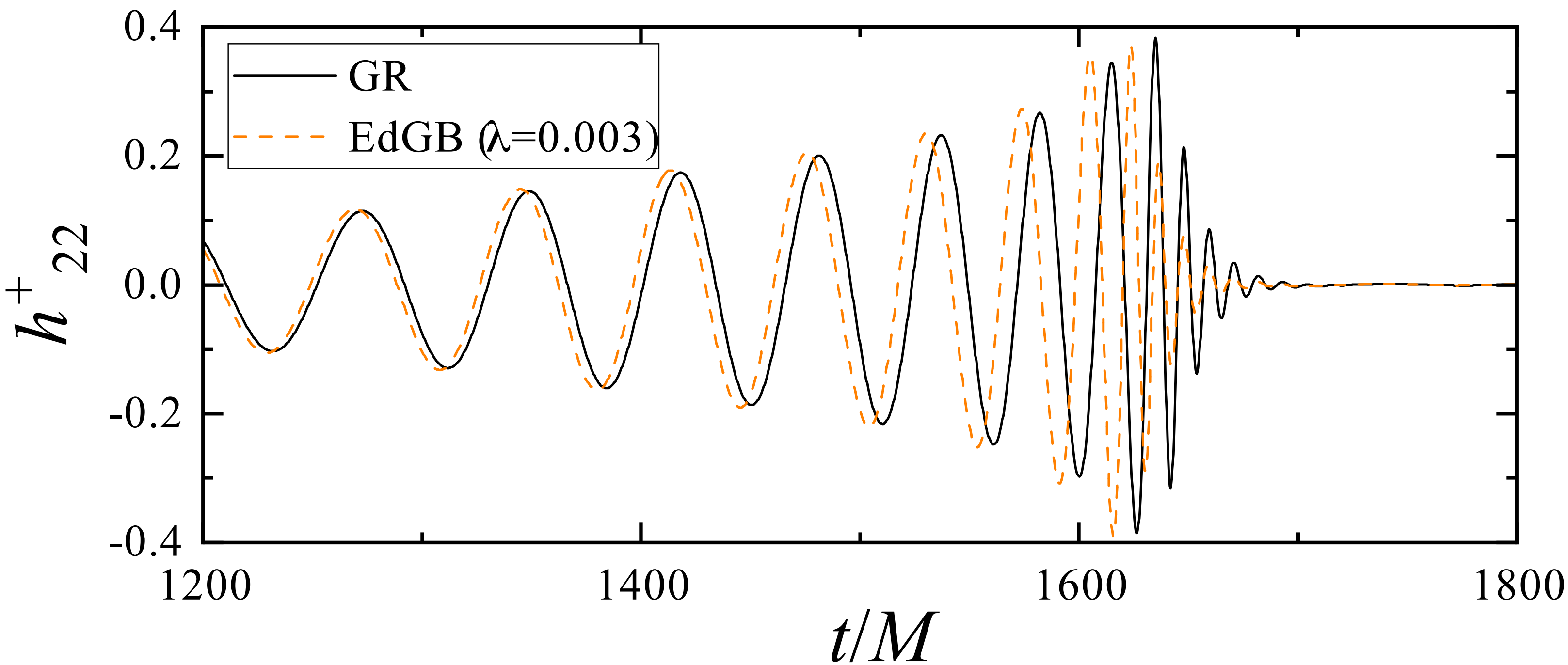
- Solving the full problem with **GRFolres + GRChombo**
- **Initial data**: two GR equal mass BHs (MAYA catalog)
- **EdGB gravity** (no scalarization)
- **Scalar dipole radiation** – **zero** for equal mass binary



DD, Aresté Saló, Clough, Figueras, Yazadjiev, in preparation

Inspiral of two equal mass BHs – zoom of the merger





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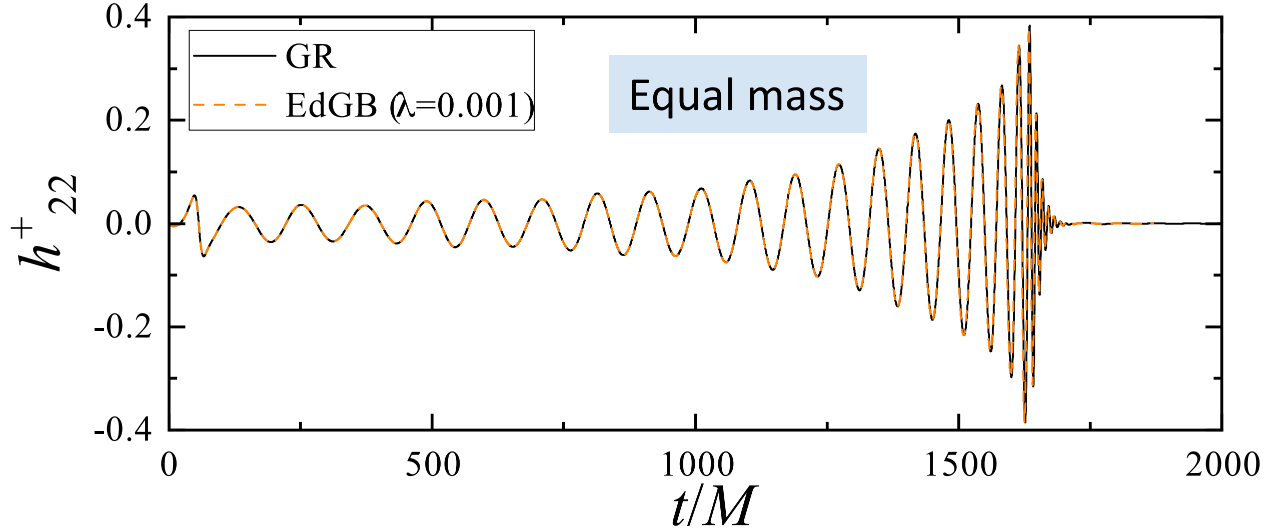
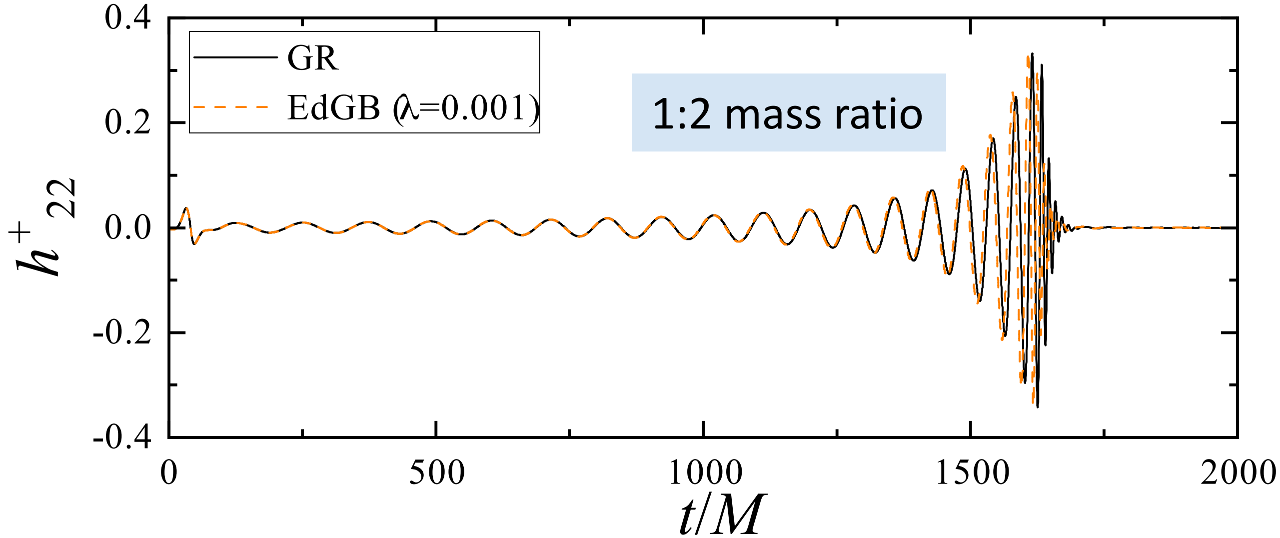
Equal vs unequal mass

- Smaller coupling – no difference in the equal case





Equal vs unequal mass - zoomed

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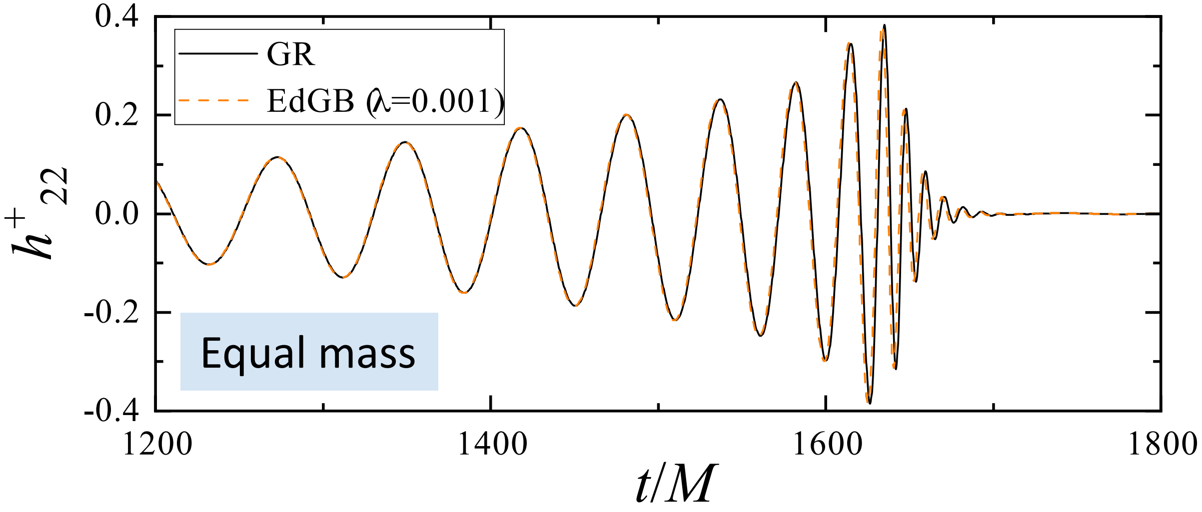

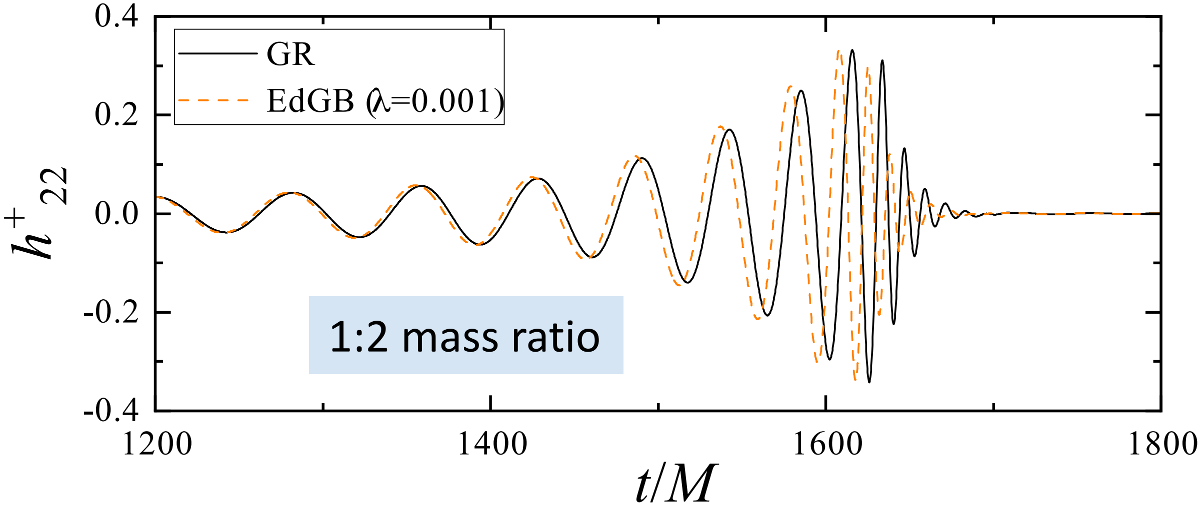

Conclusions

- Black holes in EFT offer a **very interesting venue of exploration**.
- Binary mergers can posses a **qualitatively new phenomenology**
- **Well-posedness** more subtle than in GR
- **Weak coupling condition** should always be obeyed

Outlook

- A deeper investigation **of binary mergers** and their **astrophysics signatures**

THANK YOU!