Non-Parametric Constraints on the Neutron Star Equation of State with Multi-messenger Data

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- Neutron Stars: Cold, dense nuclear matter laboratories

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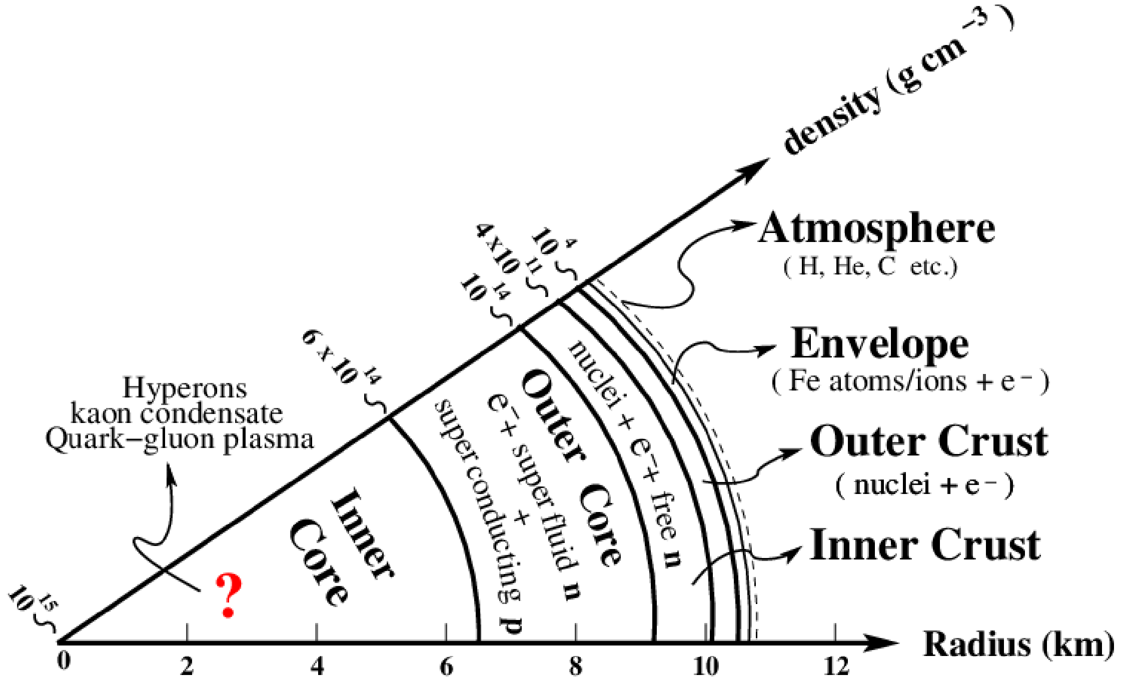

R. Nandi et al, 2013

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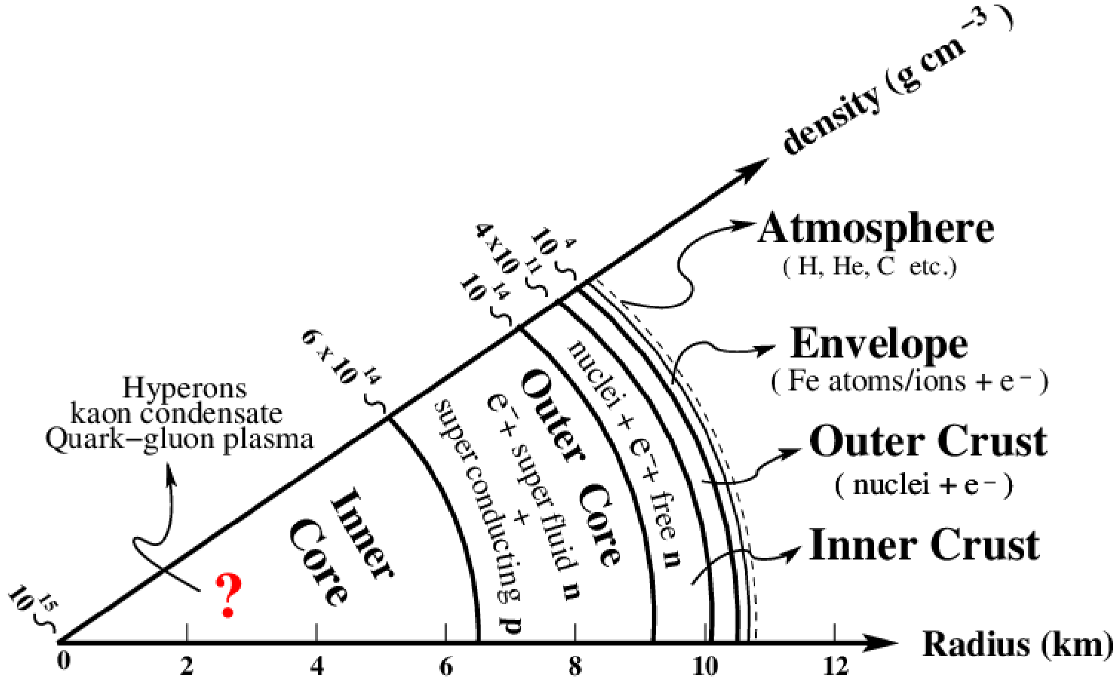
- Equation of State: $P(\rho) = ?$



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Equation of State Parametrization

- Piecewise Polytropes [1]

$$p(\rho) = K_i \rho^{\Gamma_i}$$

- Spectral Decomposition [2]

$$\Gamma(x) = \exp\left(\sum_k \gamma_k x^k\right)$$

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- Small number of parameters
- Model dependent-bias
- Difficulty with phase transitions

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- Novel methodology

Constructing a Gaussian Process EoS

- Gaussian Processes model the EoS as a multivariate Gaussian distribution [3]:

$$f_i \,|\, x_i \sim \mathcal{N}\Big(\langle f_i \rangle \,, \mathrm{Cov}(f_i, f_j) \Big)$$

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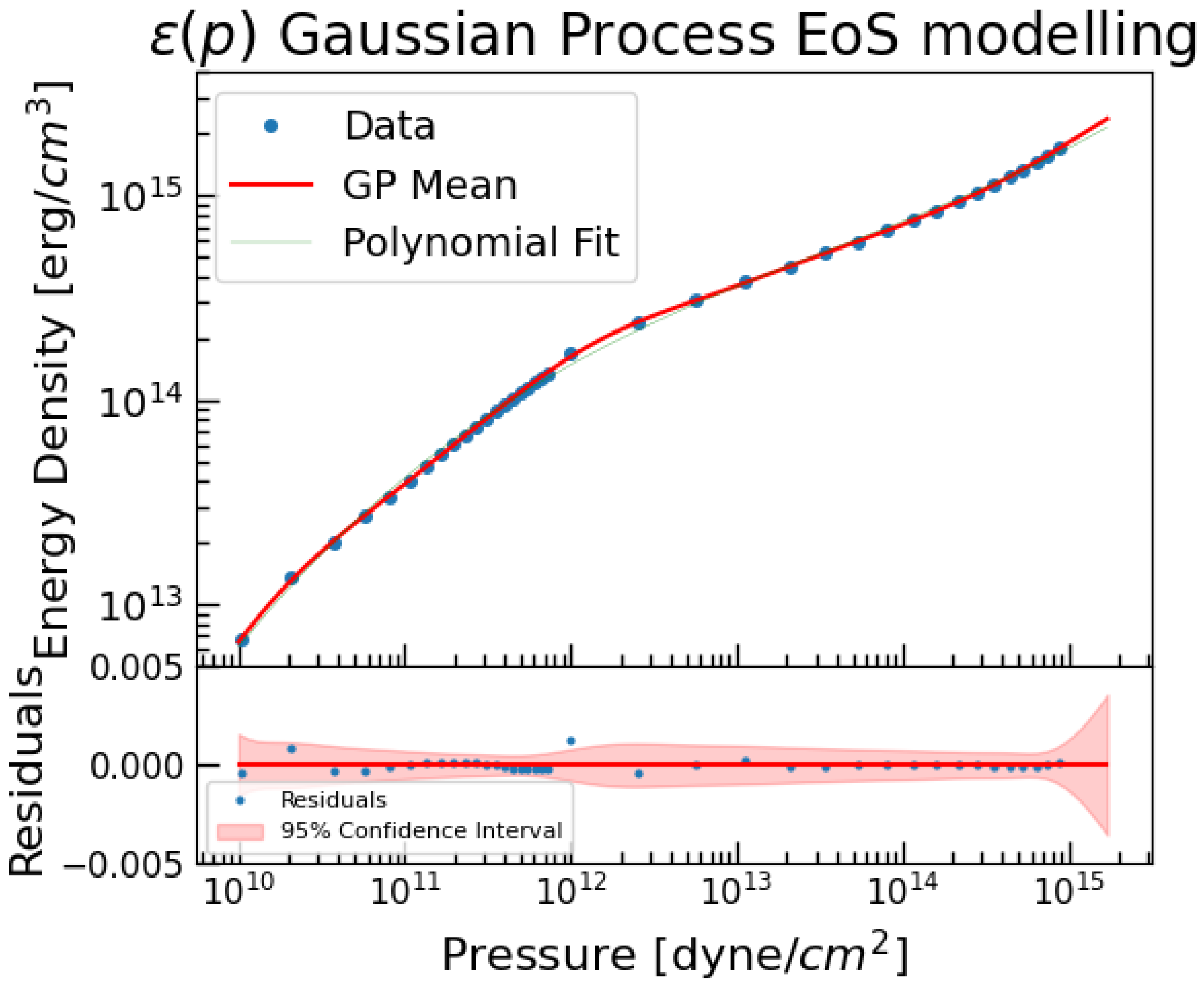

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- GP Regression resamples tabulated EoSs to create a smooth interpolation pre-equipped with error estimates.

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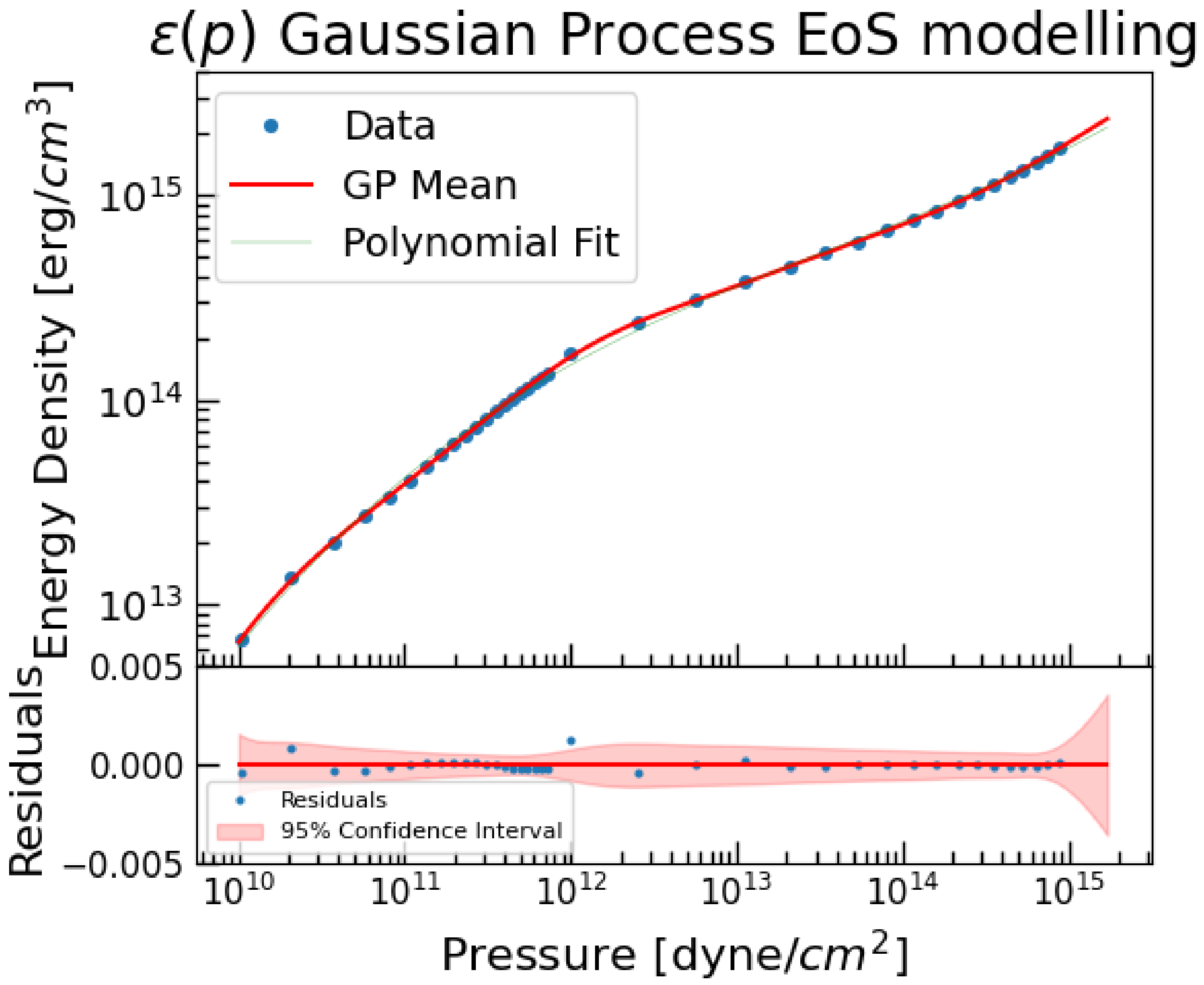
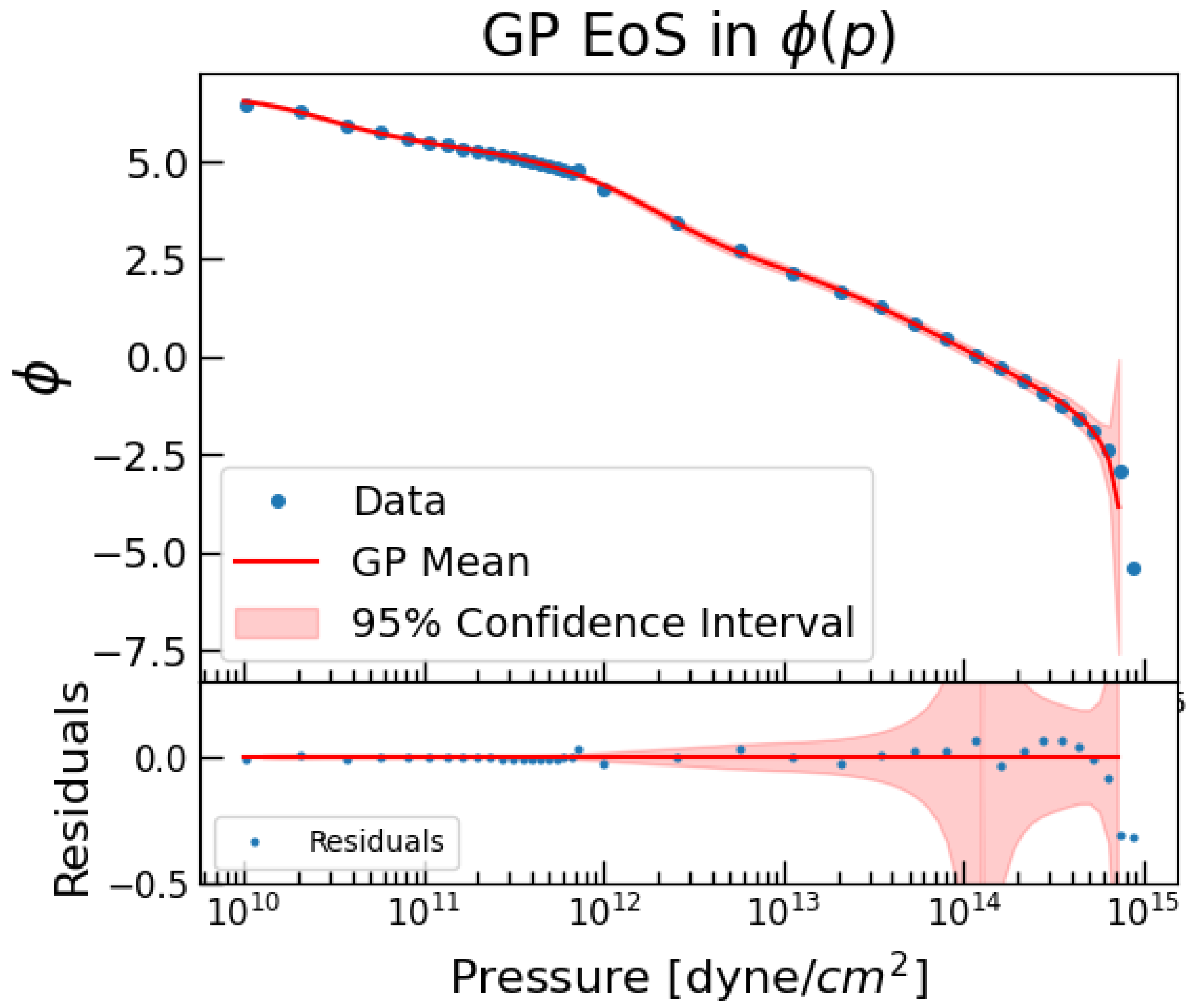
$$\phi = \log\left(c^2 \frac{d\mu}{dp} - 1\right)$$





Constructing a Gaussian Process EoS

$$\frac{dP}{d\mu} > 0$$

$$c_s = \sqrt{\frac{dP}{d\mu}} < c$$

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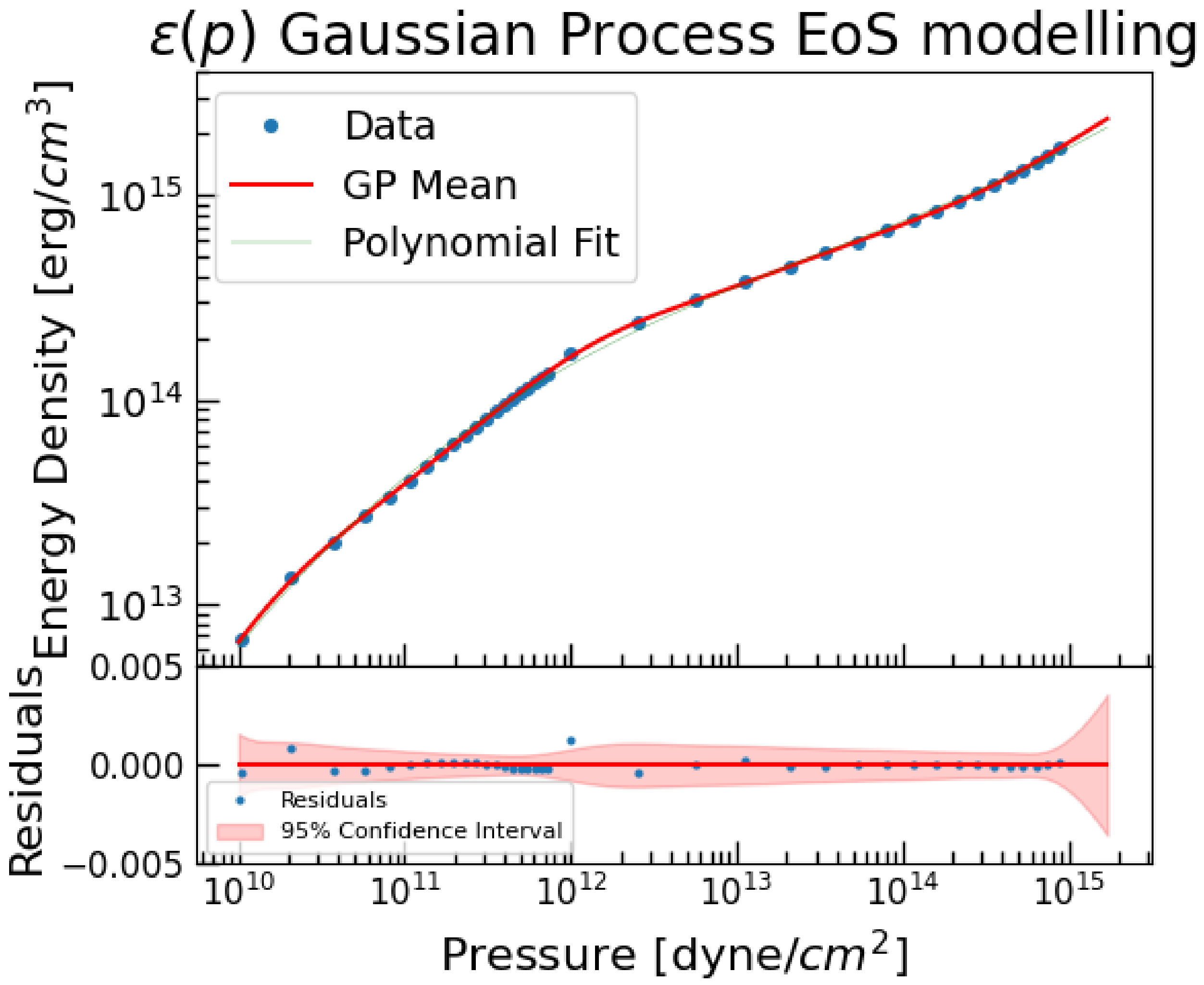

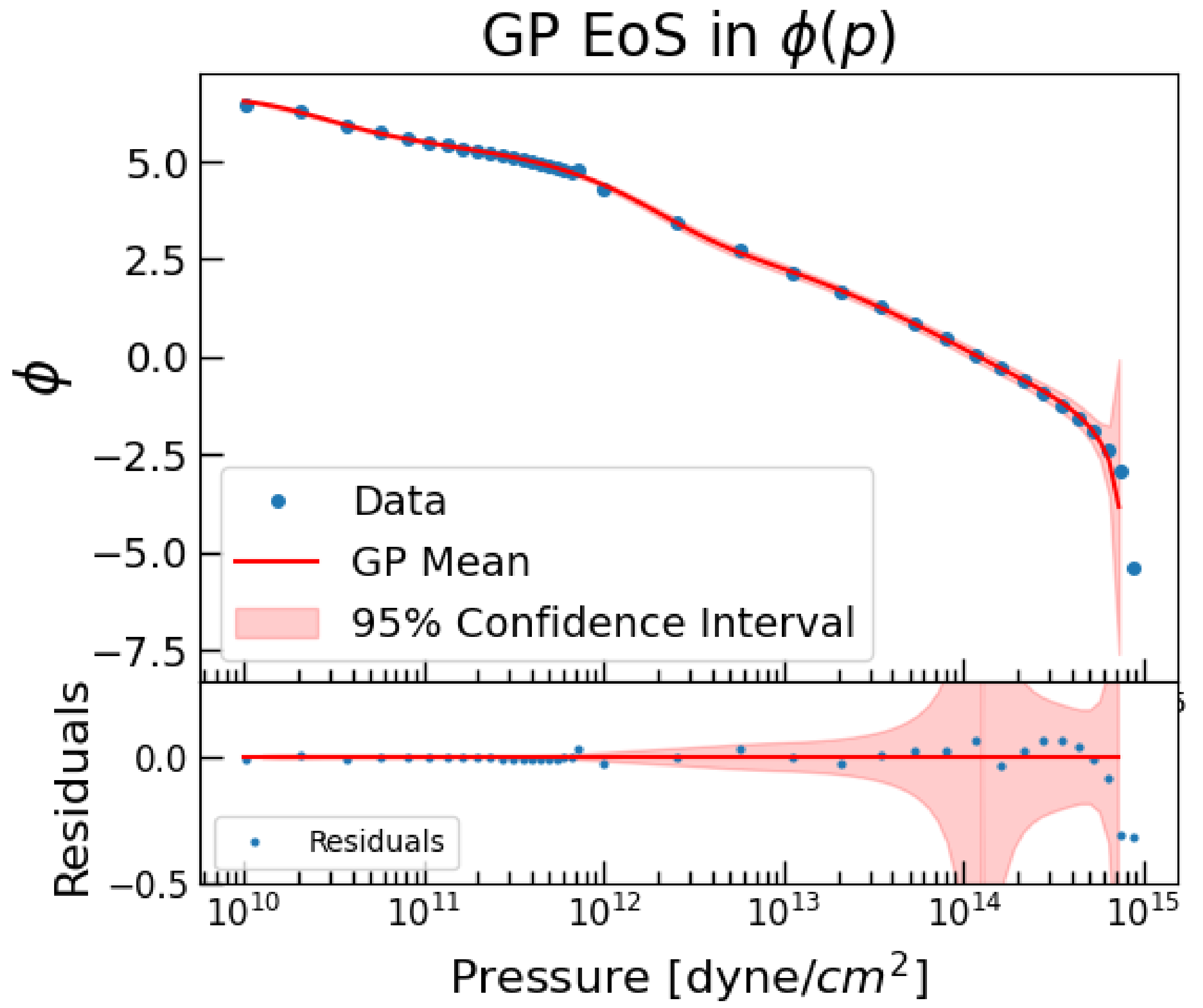

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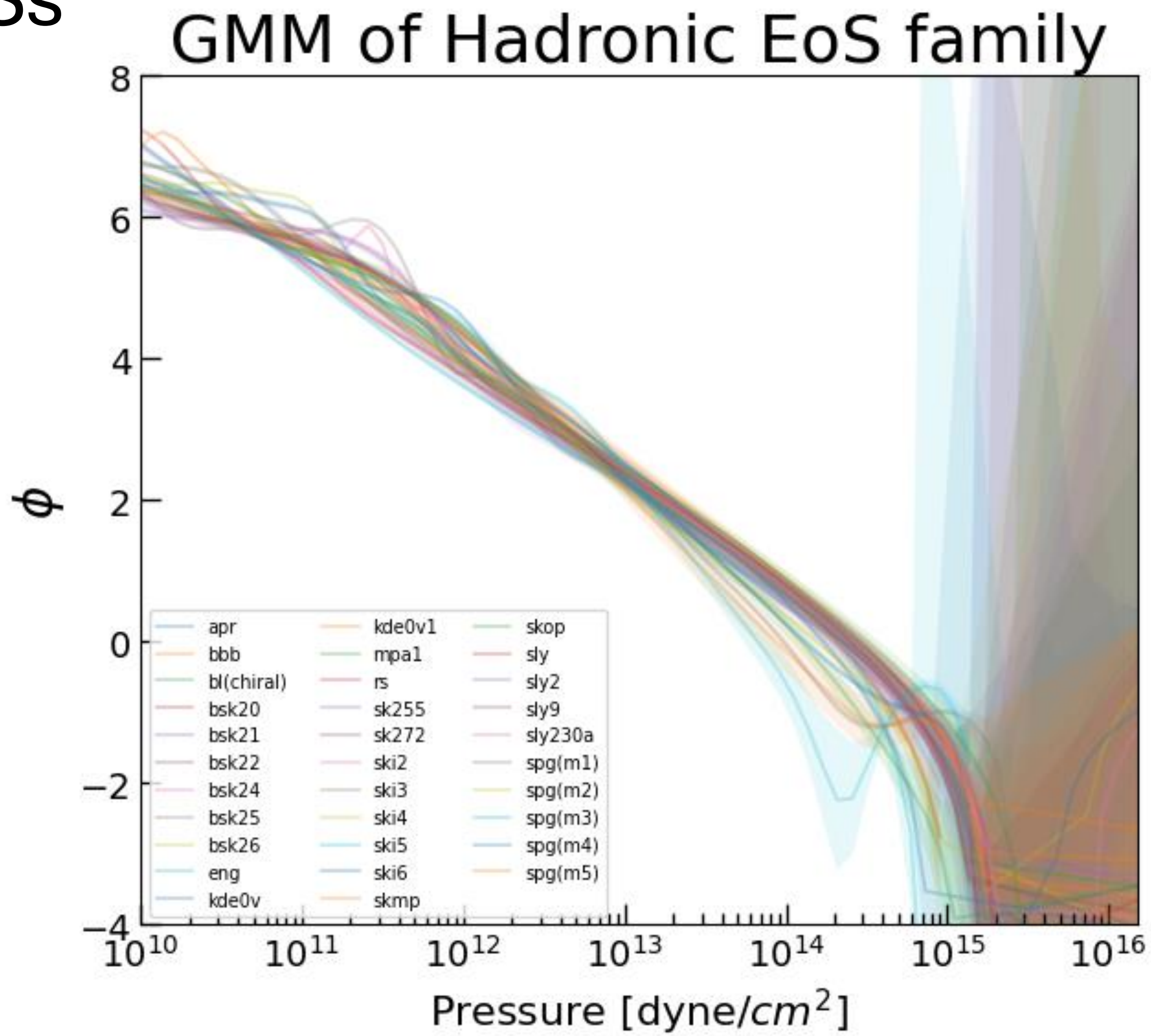

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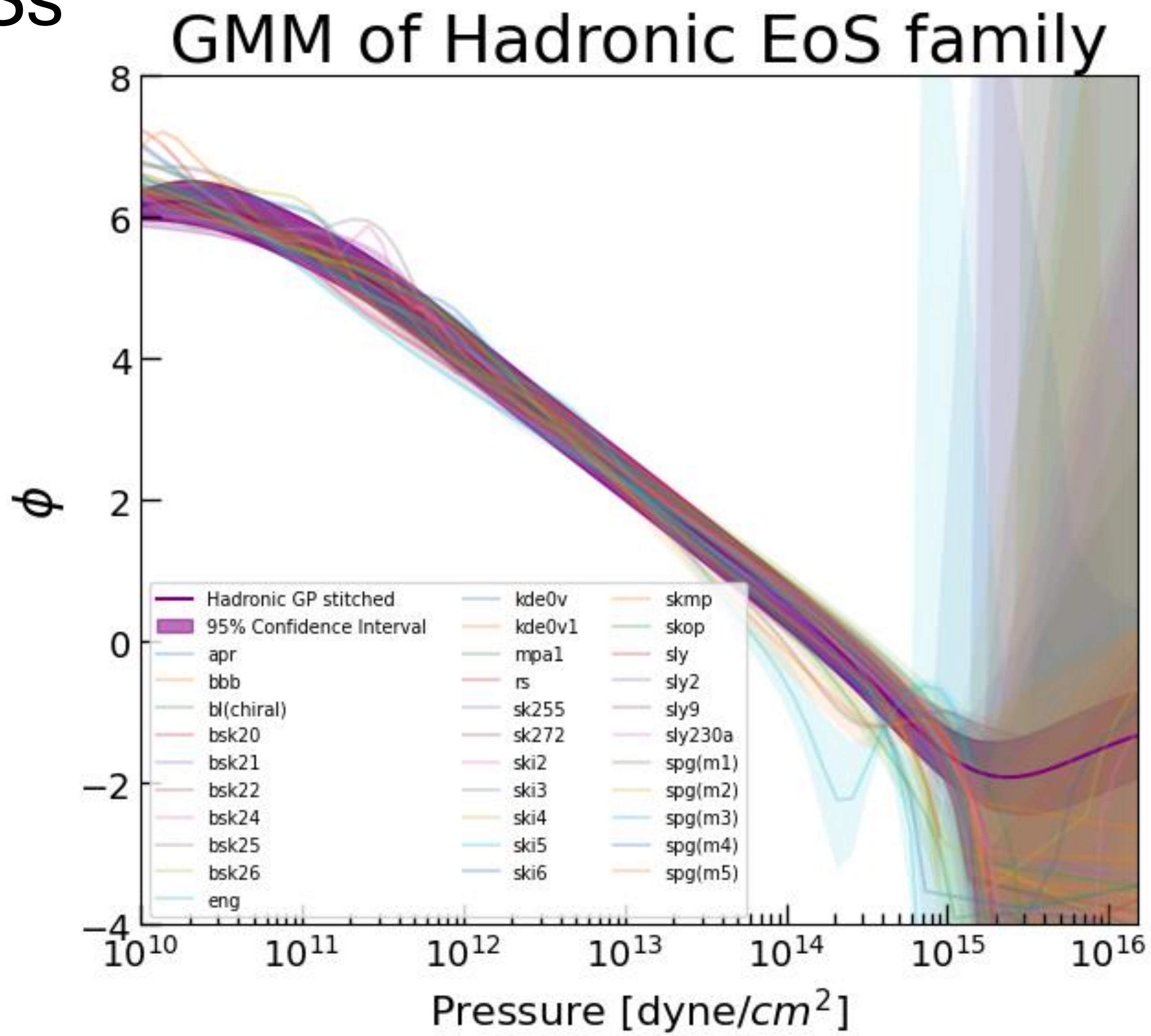
- GMM can be stitched unto known models at low pressure



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Sampling a Gaussian Mixture

- EoS Prior = GMM Draws

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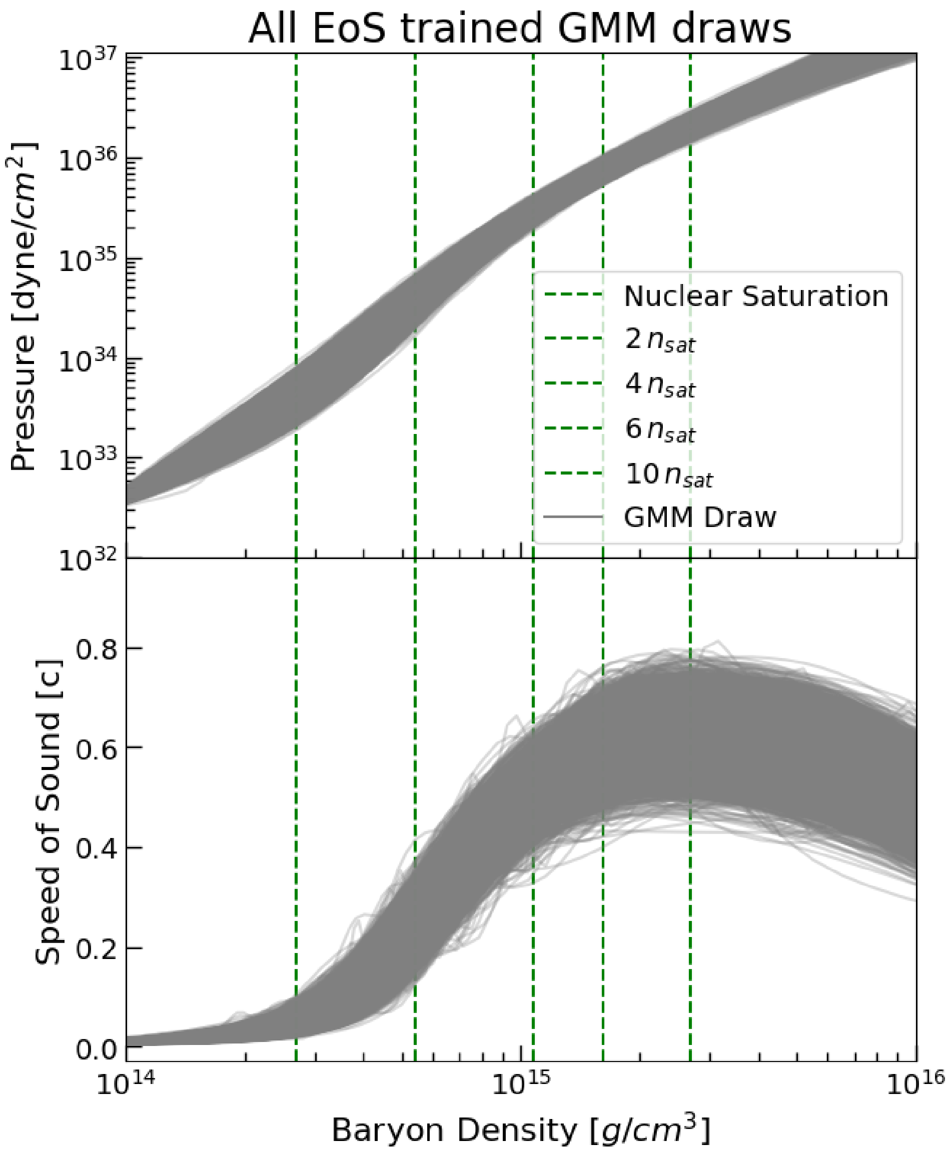
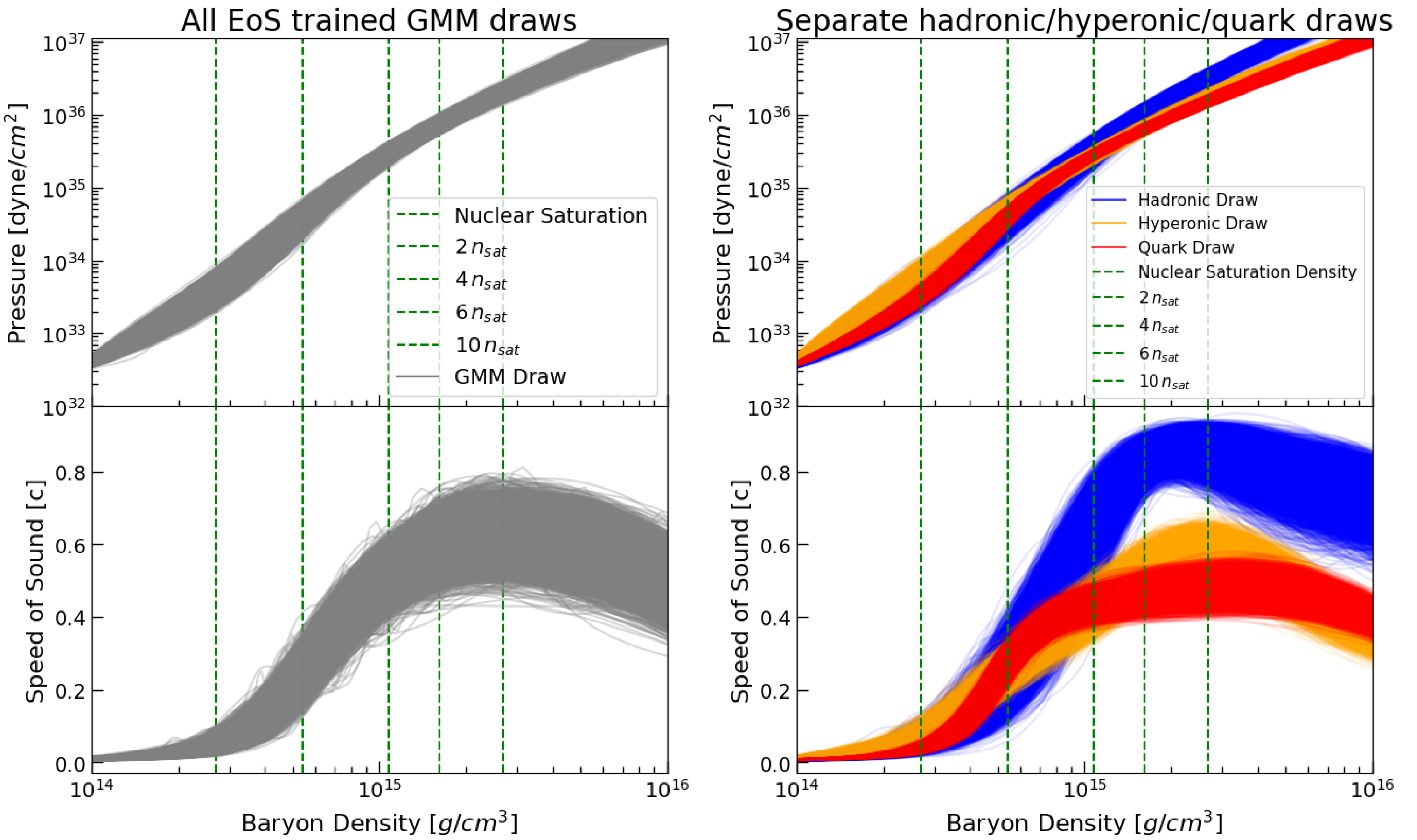


- Trained on all EoSs directly

Sampling a Gaussian Mixture



Trained on all EoSs directly

or

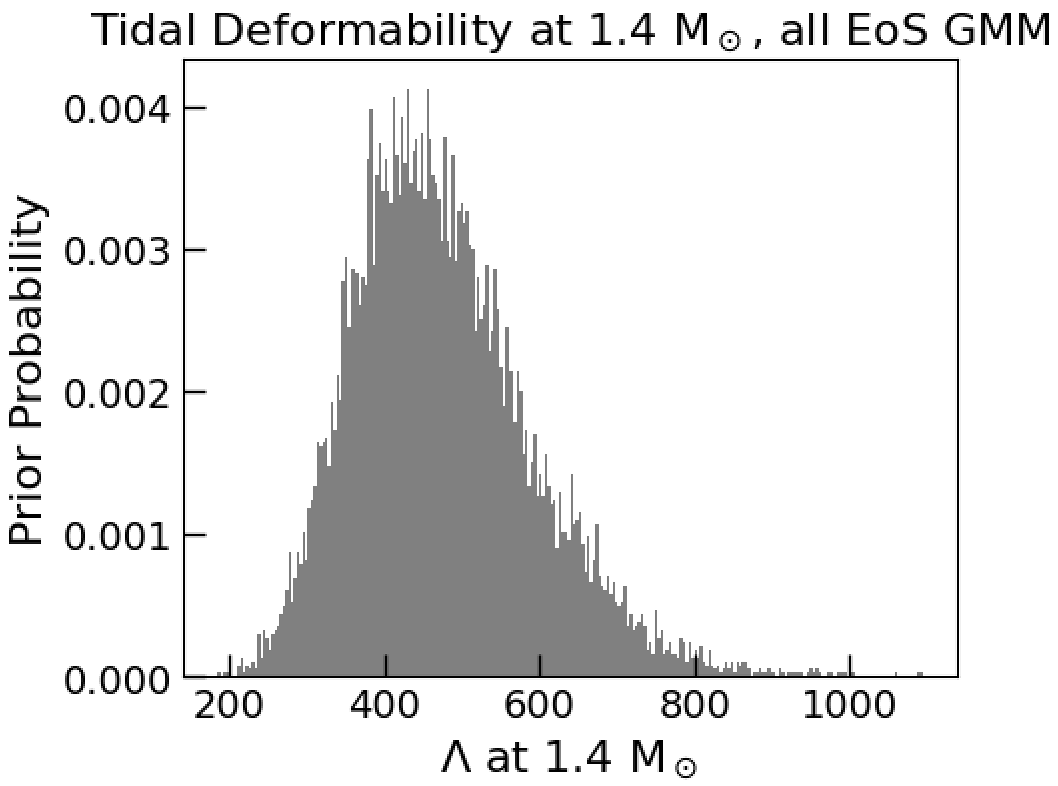
3 Separate GMMs based on:

- 32 Hadronic (Blue)
- 17 Hyperonic (Orange)
- 26 Quark (Red)

Bayesian Updating and Prior Sorting

1. Order distribution before drawing
   e.g. by predicted tidal deformability

2. Nested Sampling with some data-stream
   e.g. GW170817 BNS merger



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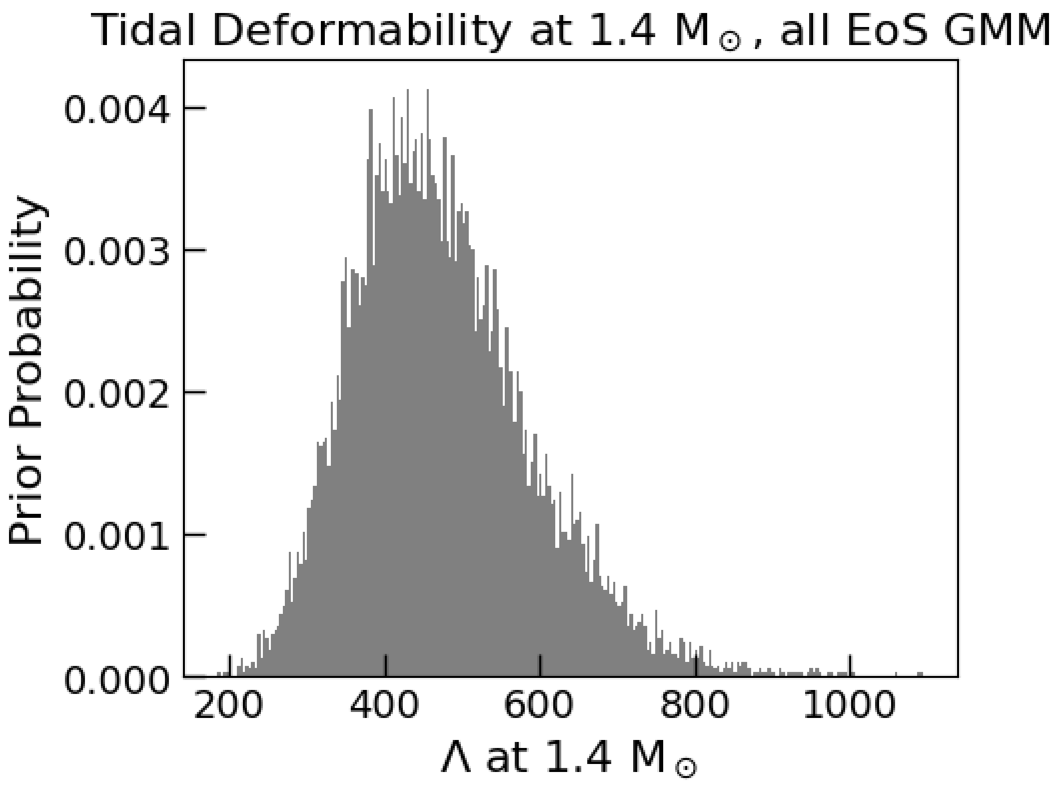

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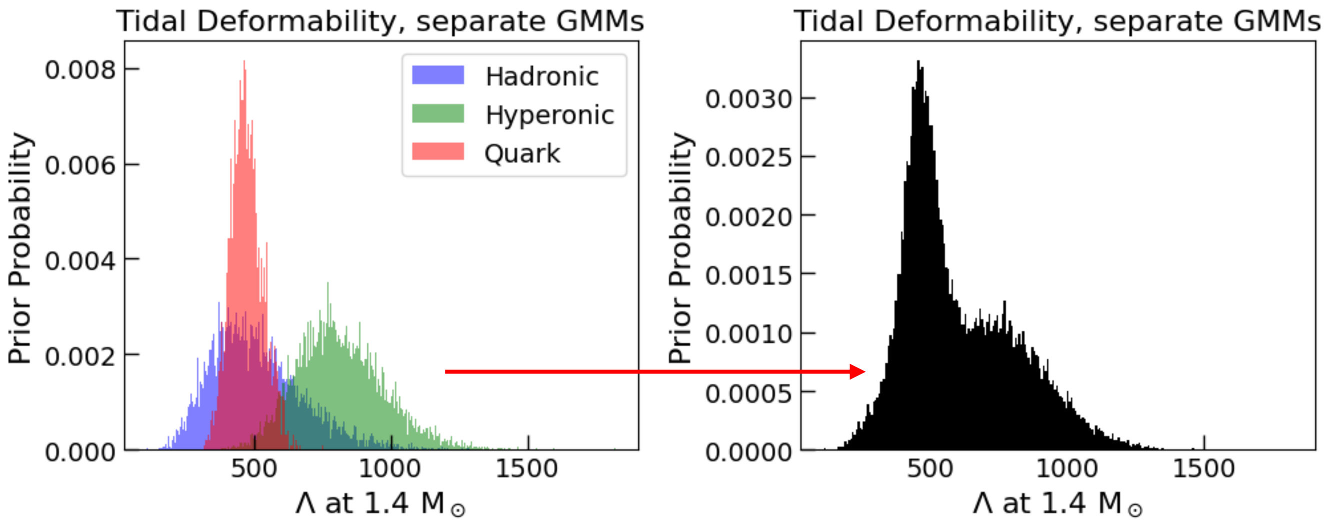
3. Reorder resulting posterior

4. Bayesian updating (prior becomes new posterior)

Repeat with new likelihoods!



Gravitational Wave Likelihood

- Complete analysis of GW170817
  - 17 parameter space (orbital, location etc.)
  - EoS index/category is but one of them
  - Ordered by tidal deformability:

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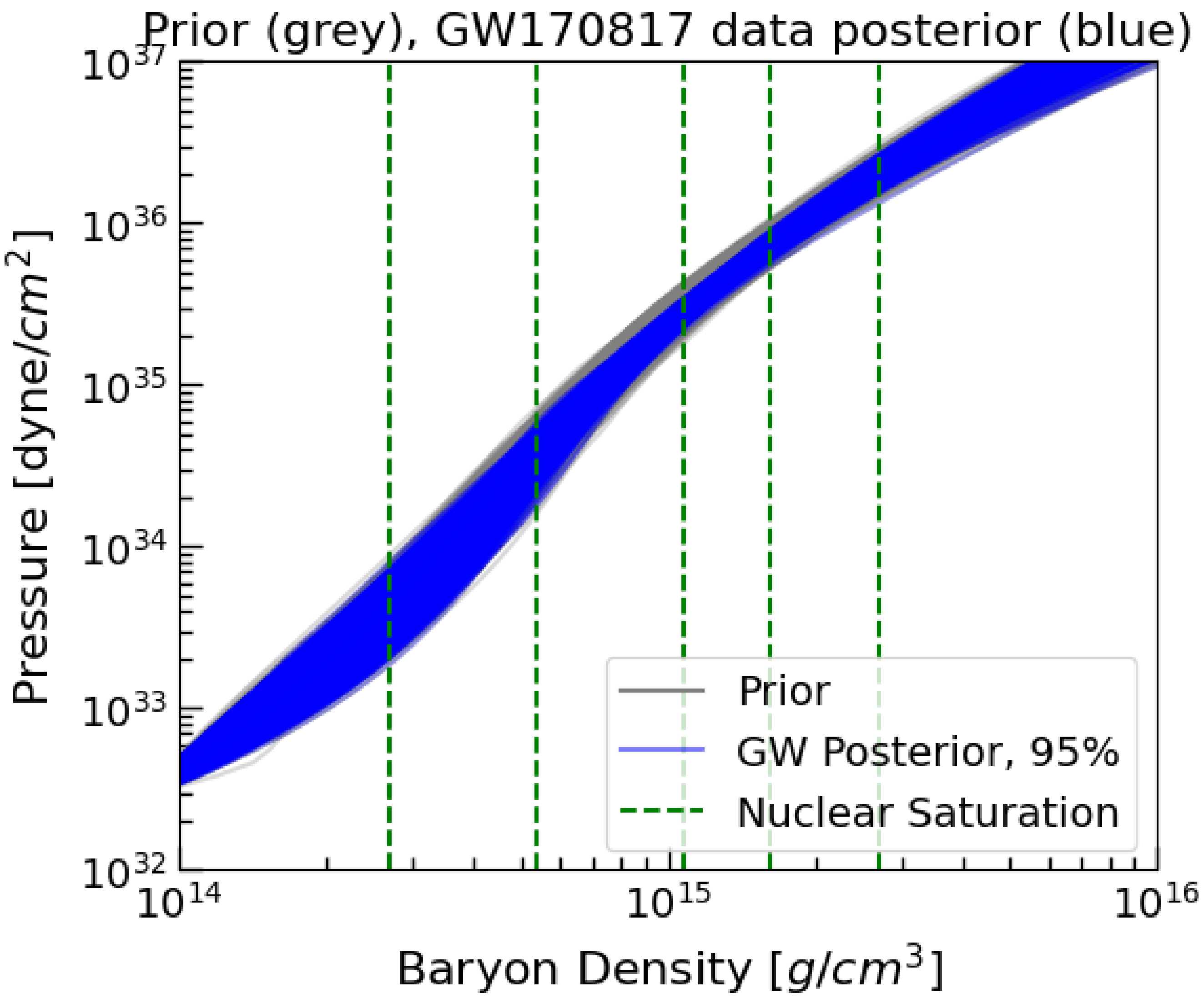

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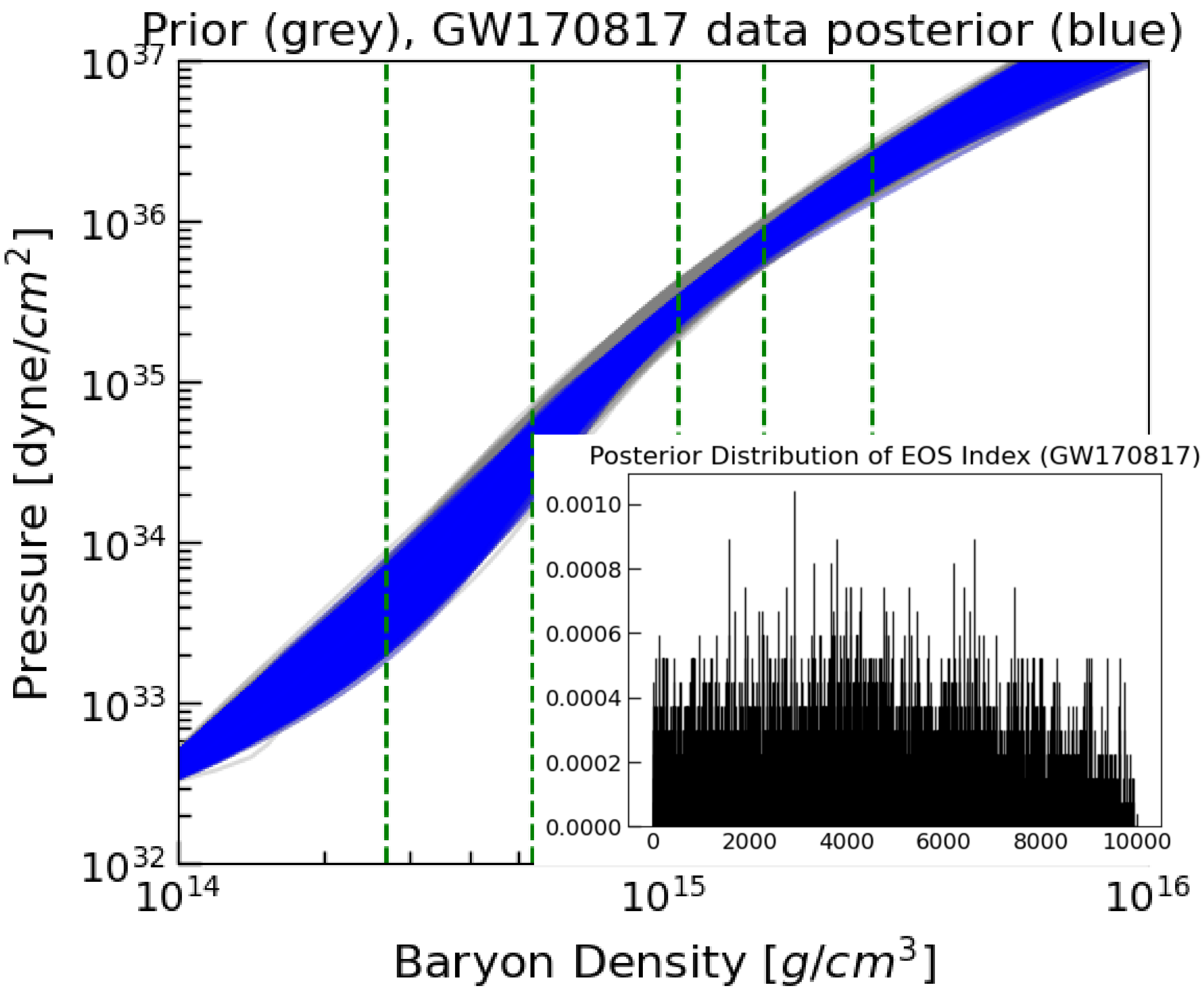

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  - Ordered by tidal deformability:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}.$$

$$\Delta\Psi_{5PN}^{tidal} = -\frac{117}{256}\frac{M^2}{m_1 m_2}\tilde{\Lambda}\left(\frac{v}{c}\right)^5.$$

Bayesian Updating with NS observations

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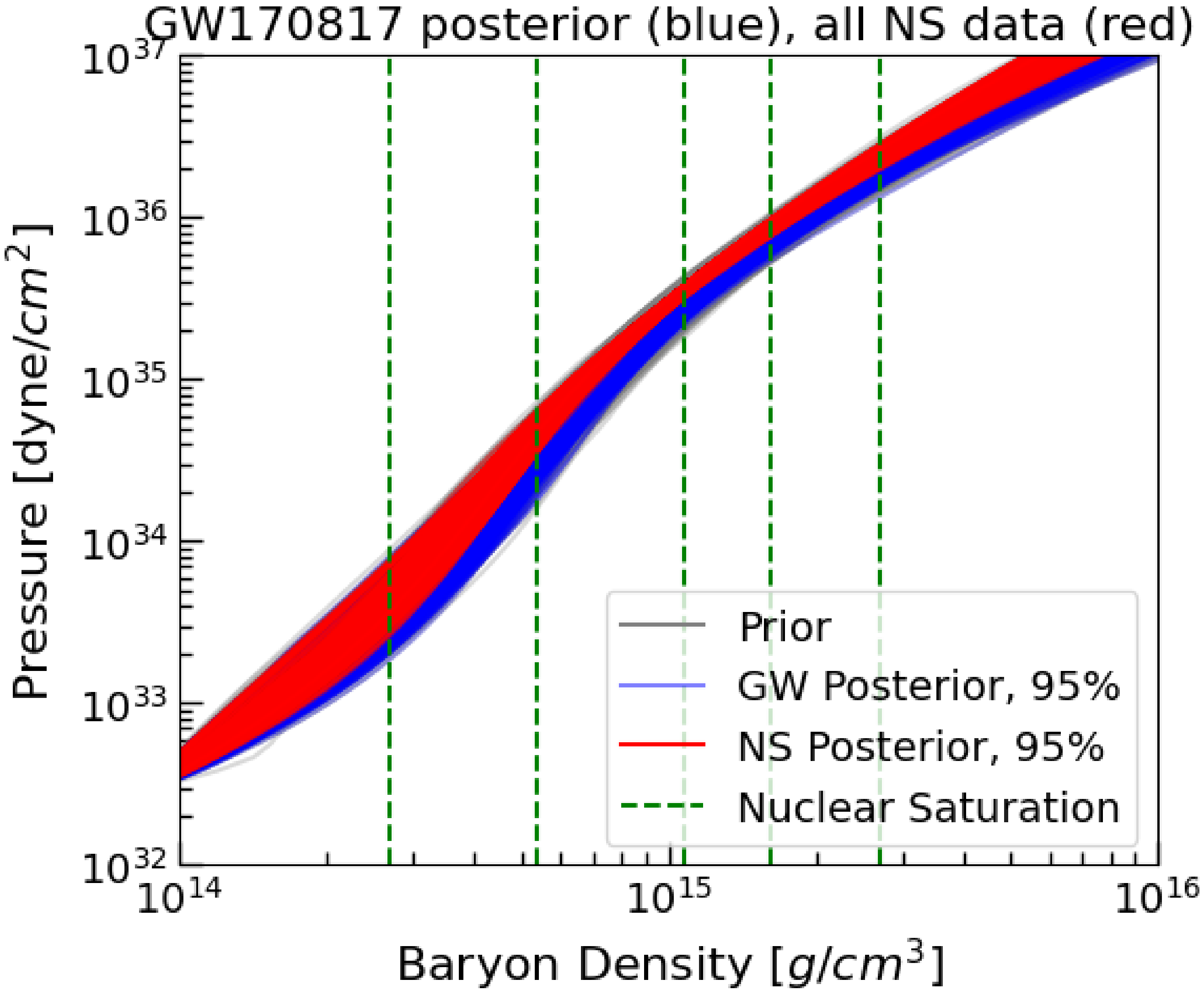

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$$\mathcal{L}_{M_j}(\vec{\alpha}) = \int_0^{M_{\max}(\vec{\alpha})} P(M_j)dM$$

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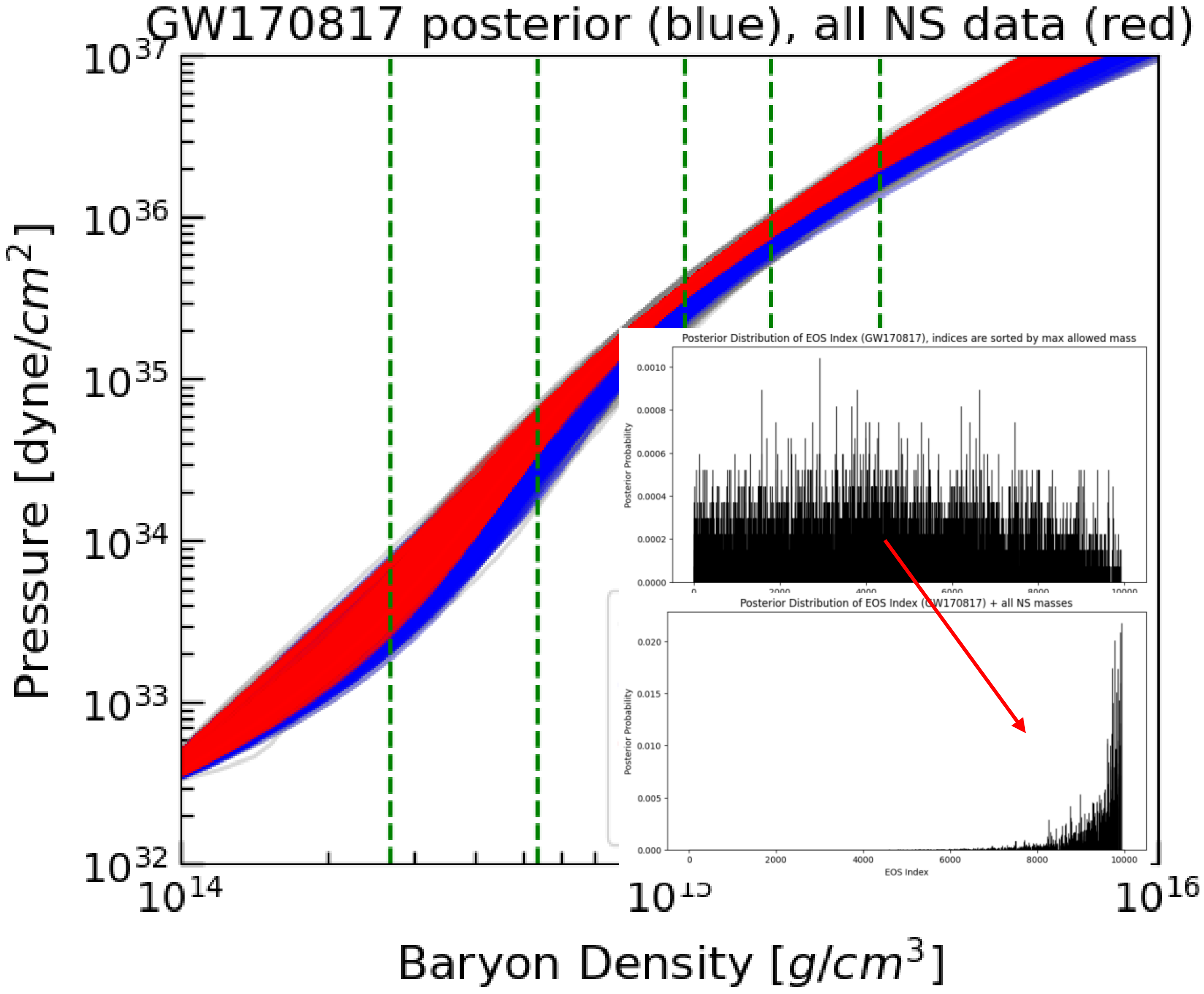
$$\mathcal{L}_R(\vec{\alpha}) = \int dM q(M)\mathcal{L}_l(M, R(M, \vec{\alpha}))$$



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Introducing theoretical constraints

- Perturbative QCD informs the EoS at ~40 nuclear saturation density [7]:

$$P(\mathrm{QCD}\,|\,\mathrm{EoS}) = \int d\vec{\beta}_H P(\vec{\beta}_H)\mathbf{1}_{[\Delta p_{\min},\Delta p_{\max}]}(\Delta p)$$

- Affected mainly by: $P(10\rho_{sat})$

- Nuclear symmetry energy constraints (χEFT) [8]:

$$S = \epsilon(n_s)/n_s - E_{\mathrm{sym}}$$

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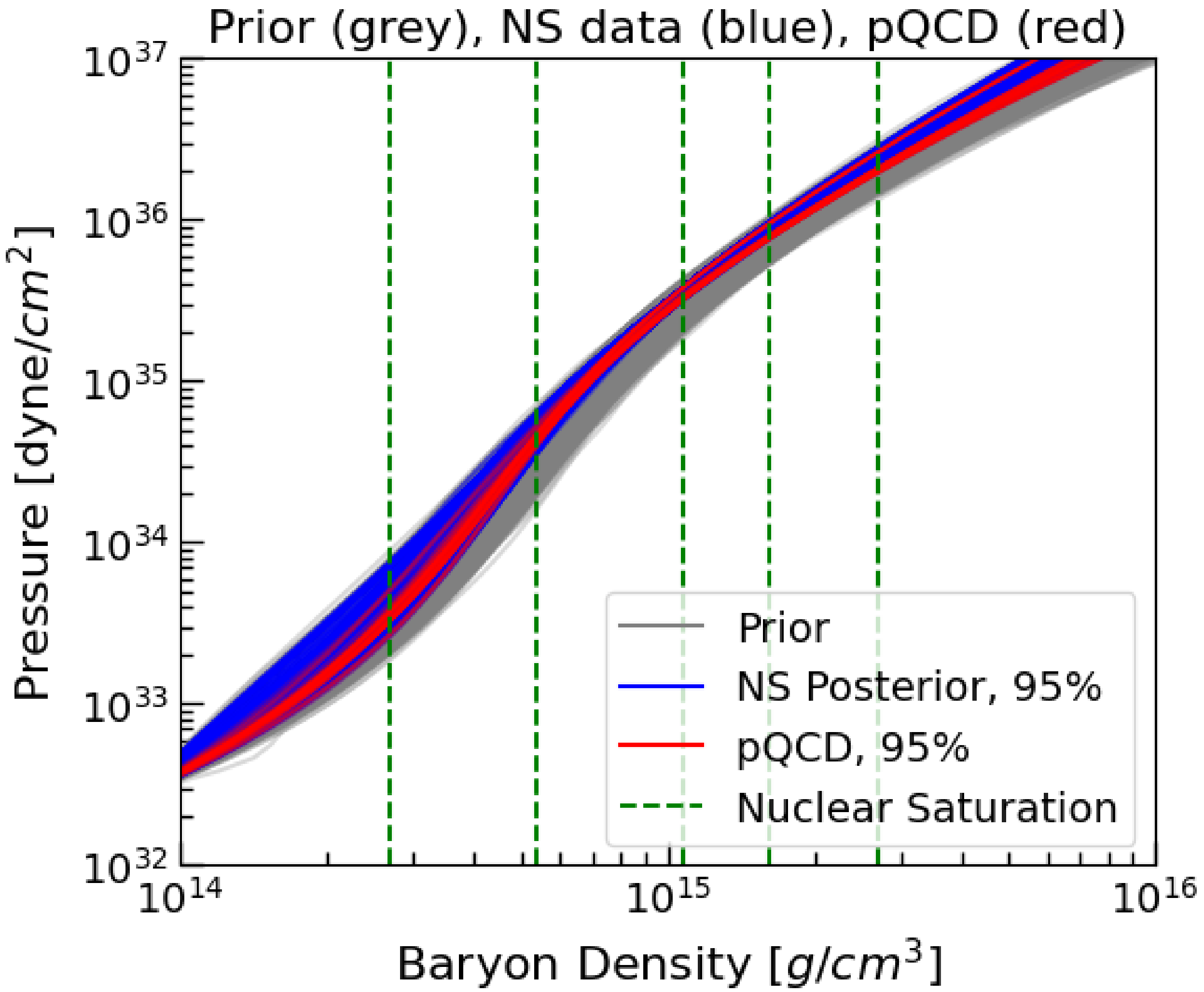

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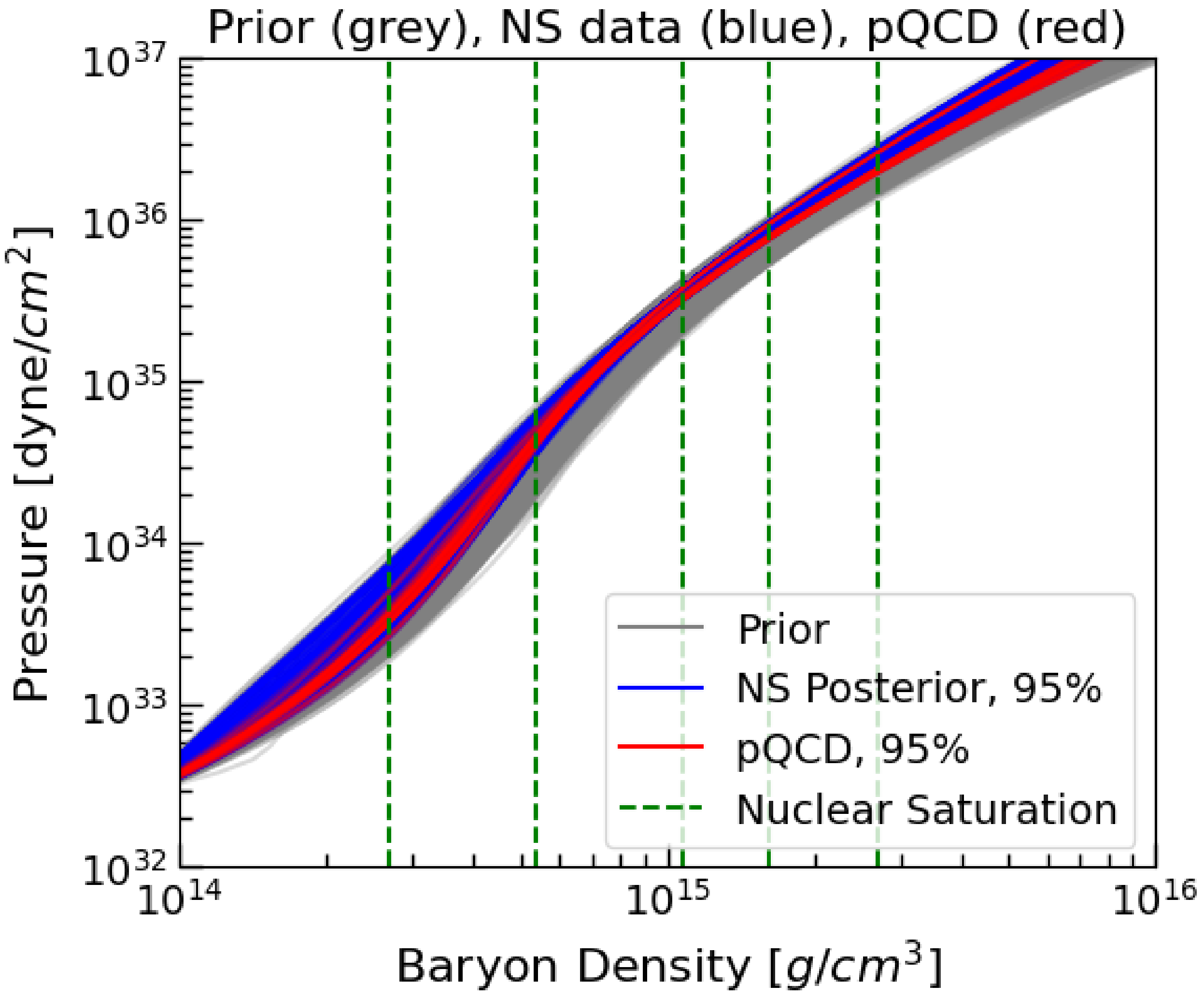
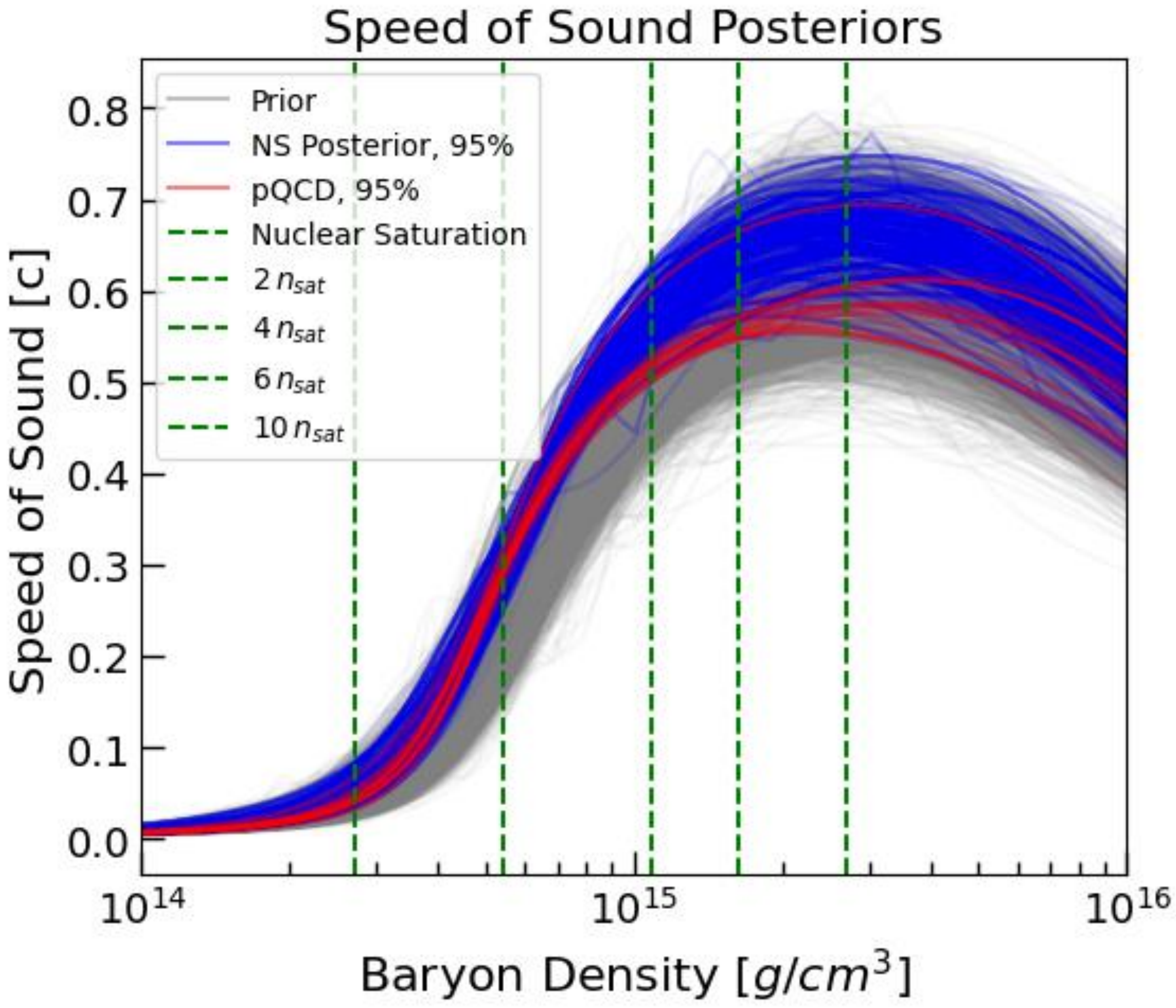
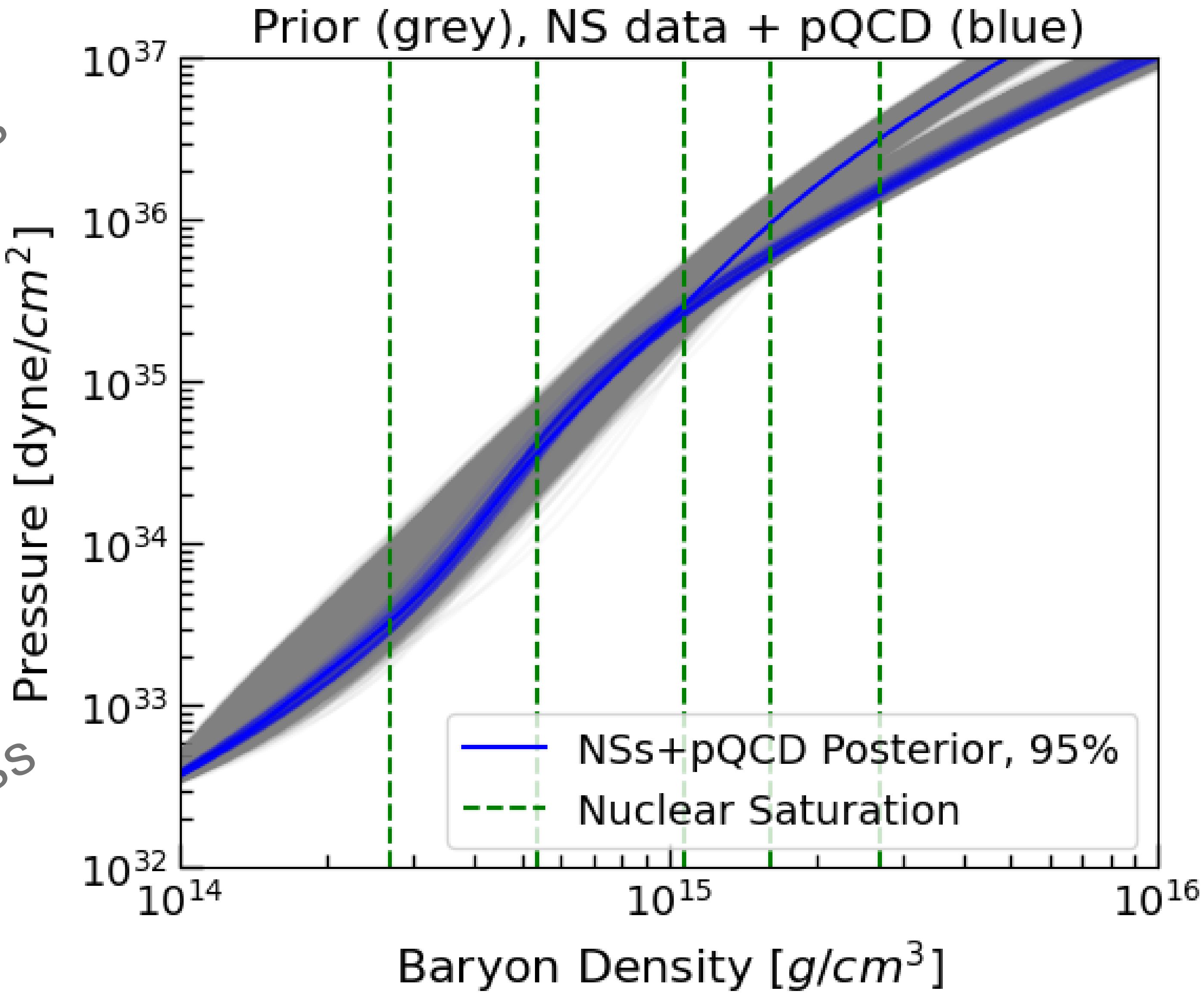
Results for model agnostic overarching GP





Results for separate GMMs prior



Conclusion

- Analysis framework permits fast and scalable EoS inference with minimal assumptions both on standard NS posterior data, but also raw detector data

- Multi-messenger data constrains different EoS regimes, from ~2-4 sat. density by NS observations, pQDC calculations inform ~10 sat. density regime

- Future work should make use of non-parametric EoS constructions to avoid both parametrization bias and facilitate model-agnostic/informed priors