

Cosmology with the Einstein Telescope

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Universidad de Salamanca

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In collaboration with:
M. Califano – D. Vernieri



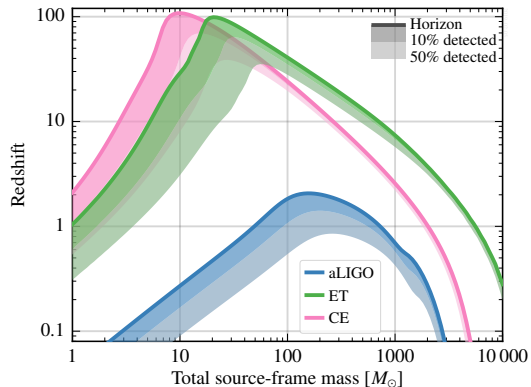
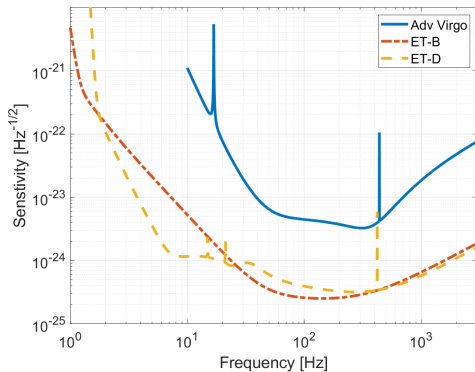
[MNRAS, 518, 3372-3385 \(2023\)](#)
[PRD, 107, 123519 \(2023\)](#)
[arxiv:2407.XXXX](#)
[arxiv:2409.XXXX](#)

Cosmology, Astrophysics and fundamental physics with Gravitational waves

Gravitational Waves (GWs) observatories (e.g., LIGO and Virgo) are playing an important role in testing fundamental physics. Indeed, GW astronomy has generated novel ways to test dark energy and the fundamental properties of gravity. A few clear examples are:

- the tests of gravity in the strong-field regime [[Abbott, et al. PRL, 116, 061101 \(2016\)](#)].
- the observation of the NS-NS binary coalescence GW170817 which solved the long-standing problem of the origin of (at least some) short gamma ray bursts [[Abbott et al., PRL 119, 161101 \(2017\)](#)]
- the first high-precision measurement of the GW speed [[Abbott et al. ApJ, 848, L13 \(2017\)](#)];
- firsts tests of modified theories of gravity [[Ezquiaga and Zumalacárregui, PRL,119, 251304 \(2017\)](#)];
- the observation of tens of BH-BH coalescences which has revealed a previously unknown population of stellar-mass BHs, much heavier than those detected through the observation of X-ray binaries [[Abbott et al., PRX 9, 031040 \(2019\)](#)];
- the tail of the waveform of the first observed event, GW150914, showed oscillations consistent with the prediction from General Relativity for the quasi-normal modes of the final BH [[Abbott et al., PRL 116, 22 \(2016\)](#)];

Cosmology with 3G detectors: Einstein Telescope



Maggiore, M., Van Den Broeck, C., Bartolo, N., et al. JCAP, 2020, 050 (2020)

Cosmology with 3G detectors: Einstein Telescope

- Astrophysics
 - Black hole properties: origin (stellar vs. primordial), evolution, demography.
 - Neutron star properties: interior structure;
 - Multi-messenger astronomy: nucleosynthesis, physics of jets, role of neutrinos.
 - Detection of new astrophysical sources of GWs: core collapse supernovae, isolated neutron stars, stochastic background of astrophysical origin.
- Fundamental physics and cosmology
 - The nature of compact objects: near-horizon physics, tests of no-hair theorem, exotic compact objects.
 - Dark matter: primordial BHs, axion clouds, dark matter accreting on compact objects.
 - Dark energy and modifications of gravity on cosmological scales.
 - Stochastic backgrounds of cosmological origin and connections with high-energy physics (inflation, phase transitions, cosmic strings, ...)

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Luminosity distance from the Gravitational Wave signal

With GWs detectors we can measure the amplitude and the phase evolution of the signal. At quadrupolar order, the GW strain amplitudes of the polarization states can be written as

$$\tilde{h}_+ = \frac{1}{\pi^{2/3}} \left(\frac{5}{24} \right)^{1/2} \frac{c}{d_L} \left(\frac{G\mathcal{M}_c}{c^3} \right)^{5/6} f^{-7/6} \left(\frac{1 + \cos i^2}{2} \right),$$

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where \mathcal{M}_c is the observed chirp mass defined as a combination of individual masses m_1 and m_2 ,
 $\mathcal{M}_c = (1+z) \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}.$

- Binary inspiral allows for a determination of the distance to the source without any reference to the cosmic distance ladder;
- GWs suffer of mass-redshift degeneracy.

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Redshift information

- Host galaxy identification [**Schutz, Nature, 323, 310 (1986)**]
 - Source sky localization error.
 - Luminosity distance error.
- Cross-Correlation [**Mukherjee and Wandelt arxiv:1808.06615**]
 - Overlapping sky area between GW sources and galaxy surveys.
 - The accurate redshift estimation of galaxies.
- Coincident short GRB
 - Only 0.1% of GW events could have a detected counterpart.
- Tidal deformation [**Messenger and Read, PRL, 108, 091101 (2012)**]
 - We need high precision in the signal analysis.
 - It depends on neutron star equation of state.

Bright Sirens

Dark Sirens

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Building mock catalogues for Einstein Telescope

To build the mock catalogues of cosmological distances, we set as a *fiducial* cosmological model the Λ CDM with the following observational constraints:

$$H_0 = 67.66 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad \Omega_{k,0} = 0.0, \quad \Omega_{\Lambda,0} = 0.6889.$$

Within the Λ CDM cosmology the theoretical luminosity distance will be

$$d_L(z) = S_k \left(\sqrt{|\Omega_{k,0}|} \frac{d_c}{D_H} \right) (1+z), \quad \text{where} \quad d_c(z) = D_H \int_0^z \frac{1}{E(z)} dz,$$

where $D_H = \frac{c}{H_0}$ is the Hubble distance and

$$S_k \left(\sqrt{|\Omega_{k,0}|} \frac{d_c}{D_H} \right) = \begin{cases} \frac{d_H}{\sqrt{|\Omega_{k,0}|}} \sinh \left(\sqrt{|\Omega_{k,0}|} \frac{d_c}{D_H} \right) & \text{if } \Omega_{k,0} > 0 \\ d_c & \text{if } \Omega_{k,0} = 0 \\ \frac{d_H}{\sqrt{|\Omega_{k,0}|}} \sin \left(\sqrt{|\Omega_{k,0}|} \frac{d_c}{D_H} \right) & \text{if } \Omega_{k,0} < 0 \end{cases}$$

Building mock catalogues for Einstein Telescope

We need to know the redshift at which we have to place the GW events. The normalized probability distribution of GW events is defined as [**Regimbau and Hughes, PRD, 79, 062002 (2009)**]

$$p(z) = \frac{R_z(z)}{\int_0^{z_{max}} R_z(z) dz}.$$

$R_z(z)$ is the merger rate density per unit of redshift in the observer frame

$$R_z(z) = \frac{R_m(z)}{1+z} \frac{dV(z)}{dz},$$

where $dV(z)/dz$ is the comoving volume element. Finally, the merger rate takes the following form [**Vitale et al., ApJ, 886, L1 (2019)**]

$$R_m^*(z) = \int_{t_{min}}^{t_{max}} R_f[t(z) - t_d] P(t_d) dt_d .$$

- The time delay distribution can be set to two different functional forms: (i) *power law* form $P(t_d) \propto t_d^{-1}$, or (ii) *exponential* form $P(t_d) \propto \tau^{-1} \exp(-t_d/\tau)$ with an e-fold time of $\tau = 100$ Myr for the time delay distribution.
- $R_f(t)$ is the formation rate of massive binaries and may be set to (i) the *Vangioni model* or to (ii) the *Madau–Dickinson model*.

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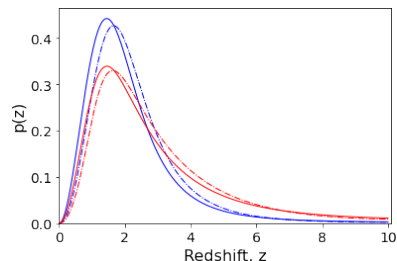
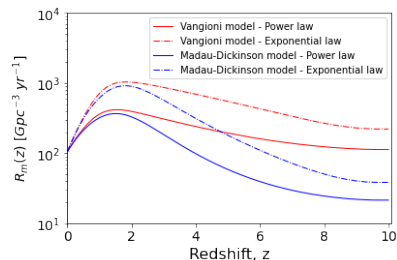
Astrophysical modelling

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- The formation rate of massive binaries may be set to
 - (i) the Vangioni model [Vangioni et al., *MNRAS*, 447, 2575 (2015)]

$$\text{SFR}(z) = \frac{\nu a \exp(b(z - z_m))}{a - b + b \exp(a(z - z_m))},$$

- (ii) the Madau–Dickinson model [Madau and Dickinson, *ARA&A*, 52, 415 (2014)]

$$\text{SFR}(z) = \frac{(1+z)^\alpha}{1 + [(1+z)/C]^\beta},$$



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- We select sky angles θ and ϕ from an isotropic distribution, and orientation angle i and the polarization ψ from uniform distribution.
- We use Fisher Matrix to estimate the SNR ρ for the ET, and retain an event if $\rho > \rho_{thr}$.
- We add a Gaussian noise component, $\mathcal{N}(d_L^{fid}, \sigma_{d_L})$, to our estimations of the luminosity distances d_L^{fid} based on the *fiducial* cosmological model. The variance σ_{d_L} accounts for different sources of uncertainties [Cutler and Flanagan, PRD, 49, 2658 (1994); Dalal et al., PRD, 74, 063006 (2006); Hirata et al., PRD, 81, 124046 (2010); Tamanini et al., JCAP, 2016, 002 (2016); Kocsis et al., ApJ, 637, 27 (2006)]
- We select events with a measured electromagnetic counterpart (THESEUS):
 - We need to estimate the flux [Mészáros et al., A&A, 529, A55 (2011)]
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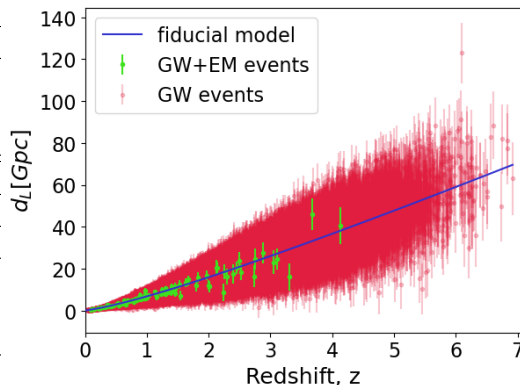
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Selecting the Electromagnetic Counterpart

GW events						
Yr	SNR = 9		SNR = 12		SNR = 15	
	#	z_{max}	#	z_{max}	#	z_{max}
1	52309	6.23	29594	3.97	17235	2.65
5	260088	6.77	146512	4.03	85595	2.86
10	521552	6.91	294284	4.05	171270	2.86

GW+EM events						
Yr	SNR = 9		SNR = 12		SNR = 15	
	#	z_{max}	#	z_{max}	#	z_{max}
1	10	2.66	8	1.18	5	1.49
5	55	3.00	41	1.95	32	2.15
10	110	4.13	83	2.60	68	2.19

Table: Numbers of GW events detected by ET.



Methodology

Statistical analysis

- Monte Carlo Markov Chain (MCMC) algorithm (emcee);
- We fit the luminosity distance for **bright** and **dark** sirens.
- We produce forecast for the (non-flat) Λ CDM, ω CDM and interacting dark matter-energy models;
- We choose uniform priors on the cosmological parameters.

Bright Sirens

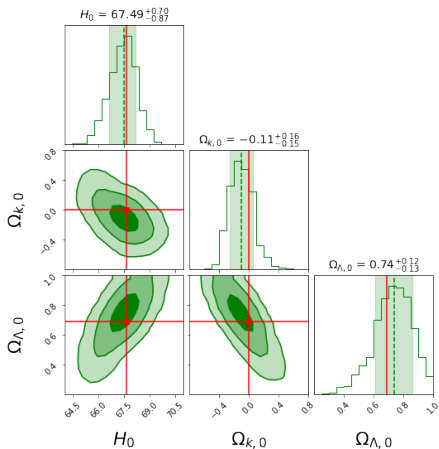
We include in the single-event likelihood the **selection effects** $\rho > \rho_{thr}$, $F(\theta_V) > F_{min}$ [[Del Pozzo, Phys. Rev. D, 86, 043011 \(2012\)](#); [Mandel et al., MNRAS, 486, 1086 \(2019\)](#); [Ye and Fishbach, Phys. Rev. D, 104, 043507 \(2021\)](#)]

Dark Sirens

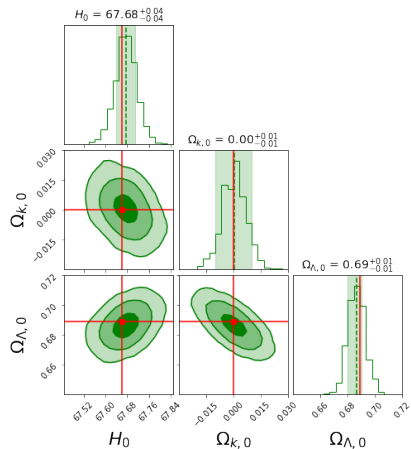
We marginalise over the redshift [[Ding et al., JCAP, 2019, 033 \(2019\)](#)]

Results ΛCDM

Bright Sirens

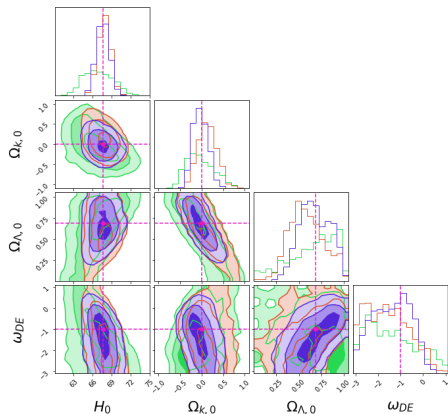


Dark Sirens

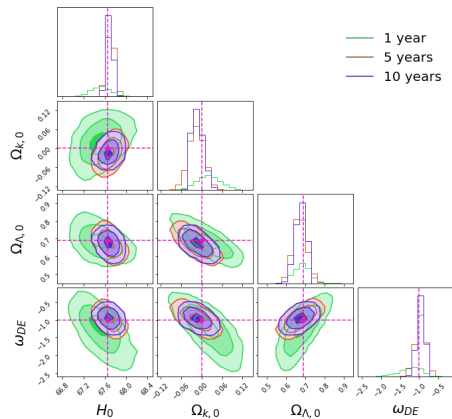


Results ω CDM

Bright Sirens

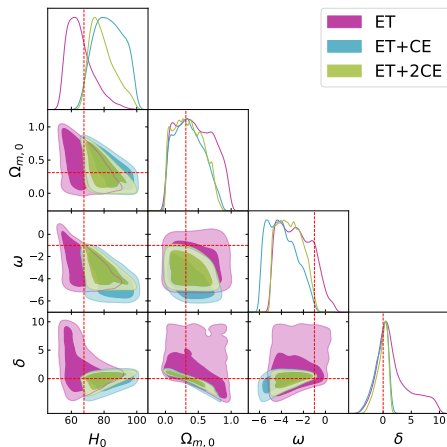


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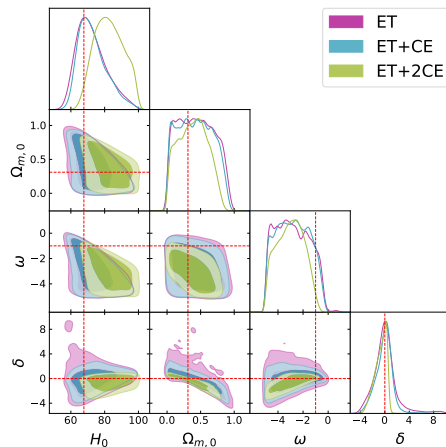


Results Interacting DM-DE

Bright Sirens (prompt)



Bright Sirens (afterglow)



Results Interacting DM-DE

- Results are biased;
- Accuracy on H_0 not comparable to previous example, why?

Degeneration between cosmological parameters could be the main source of the bias.

What happens if we impose a Gaussian prior on Ω_m ?

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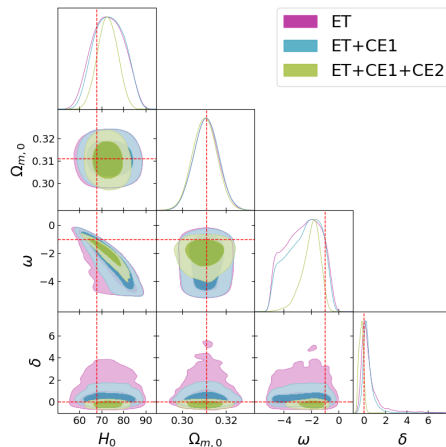
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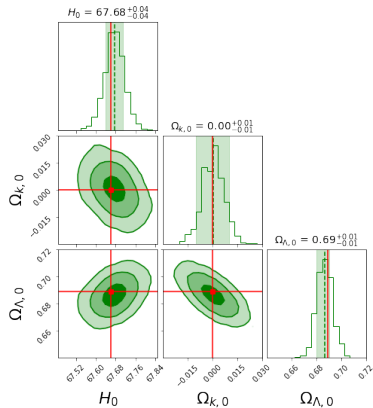
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Dark Sirens: an open issue

Dark Sirens



Marginalize over $P(z)$

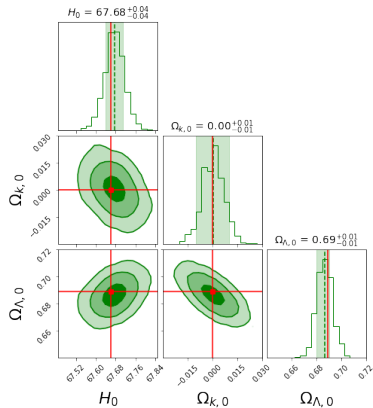
Knowledge of $P(z)$

Fitting of $P(z)$

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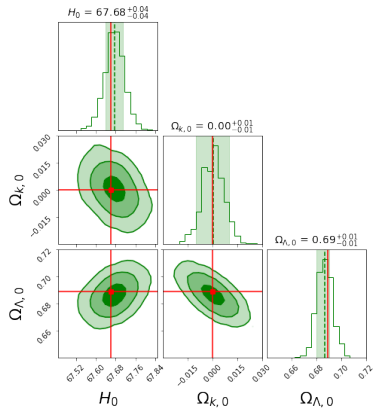
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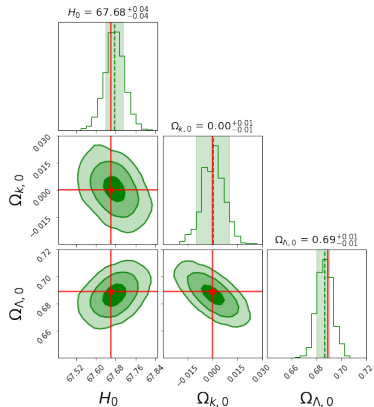
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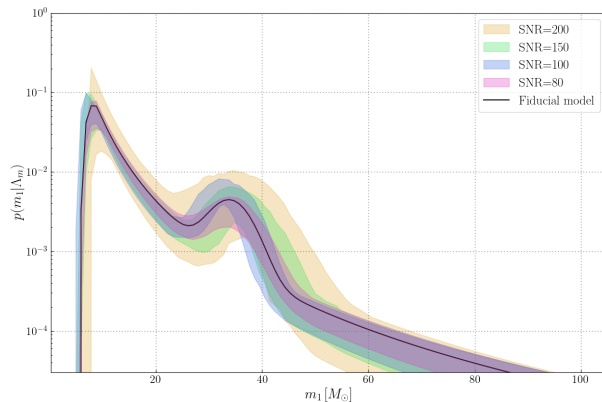
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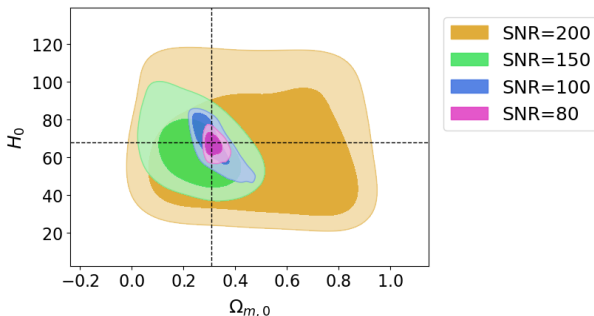


Dark Sirens: an open issue

Fitting of $P(z)$



- We model the source mass and redshift distribution.
- We still use the Fisher Matrix to evaluate the error bars and the SNR.
- Redshift is a nuisance parameter.



Conclusions

- ET will reach an accuracy on the Hubble constant less than 1%
- Good accuracy also on other cosmological parameters
- However, not all cosmological models can be probed with the same precision, it depends on the degeneracies between the parameters. In some cases, external priors are essential.
- Bright vs Dark sirens
 - Astrophysical Modelling;
 - Redshift Localization.

Questions are welcome!