

# REGULAR BLACK HOLES FROM PURE GRAVITY

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EREP 2024 — UNIVERSIDADE DE COIMBRA  
JULY 23<sup>rd</sup> 2024



Based on:

- [PB, Pablo A. Cano, Robie A. Hennigar]  
*arXiv:2403.04827*

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  - ⇒ They occur in the interior of black holes
- Expected to be an artifact of an incomplete description...
  - ⇒ Fundamental question: how do they get resolved?

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$$f(r) \stackrel{r \rightarrow \infty}{\equiv} 1 - \frac{m}{r^{D-3}} + \dots$$

- ...but the curvature singularity gets replaced by a de Sitter core

$$f(r) \stackrel{r \rightarrow 0}{\equiv} 1 - \frac{r^2}{\alpha} + \dots$$

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$$\mathcal{L} = R - \frac{12}{\sigma} \frac{(\sigma \mathcal{F})^{3/2}}{(1 + (\sigma \mathcal{F})^{3/4})^2}, \quad \mathcal{F} \equiv F_{ab}F^{ab} \quad \sigma \equiv \frac{2q^3}{m}$$

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⇒ regular black holes are not the general solutions of these theories



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- Perhaps those would resolve singularities somehow
- Understanding such effects is in general completely out of reach...

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- The result is a generic resolution of the Schwarzschild singularity!
- Regular black holes arise as (the unique) exact solutions of Einstein gravity + infinite towers of higher-curvature terms

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$$\mathcal{Z}_1 = + R ,$$

$$\mathcal{Z}_2 = + R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} ,$$

$$\begin{aligned} \mathcal{Z}_3 = & + (D-4)R_a{}^b{}_c{}^d R_b{}^e{}_d{}^f R_e{}^a{}_f{}^c + \frac{3(3D-8)}{8(2D-3)} R_{abcd}R^{abcd}R - \frac{3(3D-4)}{2(2D-3)} R_a{}^c R_c{}^a R - \frac{3(D-2)}{(2D-3)} R_{abcd}R^{acb}{}_e R^{de} + \frac{3D}{(2D-3)} R_{acbd}R^{ab}R^{cd} \\ & + \frac{6(D-2)}{(2D-3)} R_a{}^c R_c{}^b R_b{}^a + \frac{3D}{8(2D-3)} R^3 , \end{aligned}$$

$$\begin{aligned} \mathcal{Z}_4 = & - \frac{384(D-8)R_a{}^b{}_c{}^d R_b{}^c{}_a{}^d R_c{}^d{}_a{}^b}{(D-2)^5(D^3-8D^2+48D-96)} - \frac{1152R_{ab}R^{ab}R_{cd}R^{cd}}{(D-2)^5(D^3-8D^2+48D-96)} - \frac{64(D^3-10D^2+40D+24)RR_a{}^c{}_c{}^b R_b{}^a}{(D-1)(D-2)^5(D^3-8D^2+48D-96)} \\ & + \frac{24(D^4-6D^3+20D^2+104D-64)R^2 R_{ab}R^{ab}}{(D-1)^2(D-2)^5(D^3-8D^2+48D-96)} - \frac{(D^5+6D^4-64D^3+416D^2+176D-480)R^4}{(D-1)^3(D-2)^5(D^3-8D^2+48D-96)} - \frac{96(D+2)RR^{ab}R^{cd}W_{acbd}}{(D-1)(D-2)^4(D-3)(D-4)} \\ & - \frac{6(2D^5-D^4-31D^3+20D^2+20D-16)R^2 W_{abcd}W^{abcd}}{(D-1)^2(D-2)^3(D-3)(D-4)(2D^4-17D^3+49D^2-48D+16)} + \frac{96(2D^4-7D^3-7D^2+18D-8)RR_b{}^a{} W_{ac}{}^{de} W_{de}{}^{bc}}{(D-1)(D-2)^3(D-3)(D-4)(2D^4-17D^3+49D^2-48D+16)} \\ & + \frac{384R_a{}^c{}_c{}^b R^{de} W_{bdce}}{(D-2)^4(D-3)(D-4)} - \frac{48(7D^2-10D+4)R^{ab}R^{cd}W_{ac}{}^{ef} W_{bdef}}{(D-2)^3(D-3)(2D^4-17D^3+49D^2-48D+16)} \\ & - \frac{8(2D^4-15D^3+26D^2+27D-58)RW_{ab}{}^{ef} W^{abcd}W_{cdef}}{(D-1)(D-2)^2(D-3)(D-4)(D^2-6D+11)(D^3-9D^2+26D-22)} - \frac{48(7D^2-10D+4)R_a{}^c{}_c{}^b W_b{}^{def} W_{cdef}}{(D-2)^3(D-3)(2D^4-17D^3+49D^2-48D+16)} \\ & + \frac{96R^{ab}W_a{}^{cde} W_{bc}{}^{fg} W_{defg}}{(D-2)^2(D-3)(D-4)(D^2-6D+11)} - \frac{3(3D-4)W_{ab}{}^{cd} W_{cd}{}^{ef} W_{ef}{}^{gh} W_{gh}{}^{ab}}{(D-2)(D-3)(D^5-14D^4+79D^3-224D^2+316D-170)} , \end{aligned}$$

# Quasi-topological Gravities

$$\begin{aligned}
 \mathcal{L}_5 = & \frac{512(-64 - 12D + D^2)R_a{}^c R^{ab} R_b{}^d R_c{}^e R_{de} + 5120(4 + D)R_{ab}R^{ab}R_c{}^e R^{cd}R_{de}}{(-4 + D)(-2 + D)^6(-128 + 32D + D^3)} + \frac{640(4098 + 27136D - 2880D^2 + 464D^3 - 126D^4 + 7D^5)R_a{}^e R^{ab} R_b{}^d R_{cd}R}{(-4 + D)(-2 + D)^6(-1 + D)(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
 & - \frac{1920(-768 - 320D + 280D^2 - 58D^3 + 11D^4)R_{ab}R^{ab}R_{cd}R^{cd}R}{(-4 + D)(-2 + D)^6(-1 + D)(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
 & - \frac{100(4096 - 27136D + 7168D^2 + 160D^3 - 64D^4 + 116D^5 - 16D^6 + D^7)R_a{}^e R^{ab} R_{bc}R^b{}^d R^c{}^e}{(-4 + D)(-2 + D)^6(-1 + D)^2(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
 & + \frac{40(30720 - 20992D - 41216D^2 + 17920D^3 - 2784D^4 + 656D^5 + 28D^6 - 8D^7 + D^8)R_{ab}R^{ab}R^b{}^c R^d{}^e}{(-4 + D)(-2 + D)^6(-1 + D)^3(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
 & - \frac{(155648 + 231424D - 530176D^2 + 136384D^3 - 14336D^4 + 4272D^5 + 1296D^6 - 264D^7 + 16D^8 + D^9)R^b{}^c R^d{}^e}{(-4 + D)(-2 + D)^6(-1 + D)^4(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
 & - \frac{240(-80 - 100D + 8D^2 + 7D^3)R^{ab}R^{cd}R^e{}^f W_{abcd}}{(-4 + D)(-3 + D)(-2 + D)^3(-1 + D)^2(96 - 48D + 7D^2)} \\
 & - \frac{10(-128 + 896D - 2552D^2 + 3900D^3 - 2970D^4 + 425D^5 + 710D^6 - 243D^7 - 10D^8 + 8D^9)R^b{}^c R^d{}^e W_{abcd}W^{abcd}}{(-4 + D)(-3 + D)(-2 + D)^4(-1 + D)^3(16 - 48D + 49D^2 - 17D^3 + 2D^4)(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & + \frac{240(64 - 256D + 260D^2 + 292D^3 - 795D^4 + 516D^5 - 35D^6 - 42D^7 + 8D^8)R^{ab}R^b{}^c R^d{}^e W_{abcd}W_{bcde}}{(-4 + D)(-3 + D)(-2 + D)^4(-1 + D)^2(16 - 48D + 49D^2 - 17D^3 + 2D^4)(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
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 & - \frac{240(128 - 704D + 1464D^2 - 1240D^3 + 60D^4 + 503D^5 - 221D^6 + 28D^7)R^{ab}R^{cd}R_{ac}{}^e{}^f W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^4(-1 + D)(16 - 48D + 49D^2 - 17D^3 + 2D^4)(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & - \frac{11520R_a{}^e R^{ab}R_b{}^d R^c{}^e W_{bcde}{}^f}{(-3 + D)(-2 + D)^5(96 - 48D + 7D^2)} \\
 & - \frac{20(3632 - 7644D - 4296D^2 + 23905D^3 - 23526D^4 + 8460D^5 + 560D^6 - 1437D^7 + 478D^8 - 70D^9 + 4D^{10})R^b{}^c R^d{}^e W_{abcd}W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^3(-1 + D)^2(11 - 6D + D^2)(-22 + 26D - 9D^2 + D^3)(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
 & - \frac{240(128 - 704D + 1464D^2 - 1240D^3 + 60D^4 + 503D^5 - 221D^6 + 28D^7)R_a{}^e R^{ab}R_{bcde}{}^f W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^4(-1 + D)(16 - 48D + 49D^2 - 17D^3 + 2D^4)(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & - \frac{13360R_a{}^c R^{ab}R_b{}^d R^c{}^e W_{bcde}{}^f}{(-3 + D)(-2 + D)^5(96 - 48D + 7D^2)} \\
 & + \frac{960(4 - 12D + 11D^2)R_a{}^e R^{ab}R^{cd}W_{bcde}{}^f W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^4(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & + \frac{160(-232 + 550D - 253D^2 - 242D^3 + 221D^4 - 42D^5 + 6D^6)R^{ab}R_{bc}{}^c{}^d W_{ac}{}^e{}^f W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^3(-1 + D)(11 - 6D + D^2)(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
 & + \frac{320(4 - 12D + 11D^2)R_a{}^e R^{ab}R_b{}^d W_{bcde}{}^f W_{bcde}{}^f}{(-4 + D)(-3 + D)(-2 + D)^4(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & - \frac{80(12 - 28D + 17D^2)R^b{}^c R^d{}^e W_{ac}{}^e{}^f W_{ab}{}^b{}^c W_{efgh}}{(-3 + D)(-2 + D)^3(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
 & - \frac{15(-528 + 482D + 241D^2 - 425D^3 + 194D^4 - 39D^5 + 3D^6)R^{ab}R^c{}^d W_{bc}{}^e{}^f W^{abcd}W_{cd}{}^b{}^c W_{efgh}}{(-4 + D)(-3 + D)(-2 + D)^2(-1 + D)(85 - 99D + 48D^2 - 11D^3 + D^4)(-170 + 316D - 224D^2 + 79D^3 - 14D^4 + D^5)} \\
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 & - \frac{4(-5 + 4D)W_{ab}{}^c{}^d W_{cd}{}^e{}^f W_{bc}{}^g{}^h W_{ef}{}^h{}^i W_{gh}{}^i{}^j}{(-3 + D)(-2 + D)(-1150 + 2954D - 3202D^2 + 1934D^3 - 705D^4 + 155D^5 - 19D^6 + D^7)}.
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 & - \frac{640(4098 + 27136D - 2880D^2 + 464D^3 - 126D^4 + 7D^5)R_a{}^e R^{ab} R_b{}^d R_{cd} R}{(-4 + D)(-2 + D)^6(-1 + D)(-128 + 32D + D^3)(-96 + 48D - 8D^2 + D^3)} \\
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 & - \frac{240(128 - 704D + 1464D^2 - 1240D^3 + 60D^4 + 503D^5 - 221D^6 + 28D^7)R^{ab}R^{cd}R_{wac}{}^e f W_{def}}{(-4 + D)(-3 + D)(-2 + D)^4(-1 + D)(16 - 48D + 49D^2 - 17D^3 + 2D^4)(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & - \frac{11520R_a{}^e R^{ab} R_b{}^d R_c{}^e f W_{bcdf}}{(-3 + D)(-2 + D)^5(96 - 48D + 7D^2)} \\
 & - \frac{20(3632 - 7644D - 4296D^2 + 23905D^3 - 23526D^4 + 8460D^5 + 560D^6 - 1437D^7 + 478D^8 - 70D^9 + 4D^{10})R^2W_{ab}{}^c f W^{abcd}W_{cdef}}{(-4 + D)(-3 + D)(-2 + D)^3(-1 + D)^2(11 - 6D + D^2)(-22 + 26D - 9D^2 + D^3)(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
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 & - \frac{13360R_a{}^c R^{ab} R_b{}^d R_c{}^e f W_{cdef}}{(-3 + D)(-2 + D)^5(96 - 48D + 7D^2)} \\
 & + \frac{960(4 - 12D + 11D^2)R_a{}^e R^{ab} R^{cd} W_{bcde} f^2 W_{cdef}}{(-4 + D)(-3 + D)(-2 + D)^4(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & + \frac{160(-232 + 550D - 253D^2 - 242D^3 + 221D^4 - 42D^5 + 6D^6)R^{ab}R_{wac}{}^cde W_{bcde} f^2 W_{def}}{(-4 + D)(-3 + D)(-2 + D)^3(-1 + D)(11 - 6D + D^2)(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
 & + \frac{320(4 - 12D + 11D^2)R_a{}^e R^{ab} R_b{}^d W_{cde} f^2 W_{def}}{(-4 + D)(-3 + D)(-2 + D)^4(16 - 56D + 69D^2 - 27D^3 + 4D^4)} \\
 & - \frac{80(12 - 28D + 17D^2)R^{ab}R^{cd}W_{ac}{}^e f W_{bcde} W_{efgh}}{(-3 + D)(-2 + D)^3(176 - 600D + 775D^2 - 482D^3 + 161D^4 - 28D^5 + 2D^6)} \\
 & - \frac{15(-528 + 482D + 241D^2 - 425D^3 + 194D^4 - 39D^5 + 3D^6)R_{wbc}{}^cde f W^{abcd}W_{cde}{}^gh W_{efgh}}{(-4 + D)(-3 + D)(-2 + D)^2(-1 + D)(85 - 99D + 48D^2 - 11D^3 + D^4)(-170 + 316D - 224D^2 + 79D^3 - 14D^4 + D^5)} \\
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 & - \frac{4(-5 + 4D)W_{ab}{}^cde f W^{abcd}W_{cde}{}^gh W_{ef}{}^ij W_{ghij}}{(-3 + D)(-2 + D)(-1150 + 2954D - 3202D^2 + 1934D^3 - 705D^4 + 155D^5 - 19D^6 + D^7)}.
 \end{aligned}$$

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Here we will go beyond the EFT regime...

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to a couple of algebraic equations:

$$N(r) = 1, \quad \boxed{\frac{1-f(r)}{r^2} + \sum_{n=2}^{n_{\text{max}}} \alpha_n \left[ \frac{1-f(r)}{r^2} \right]^n = \frac{m}{r^{D-1}}}$$

where  $m$  is an integration constant related to the mass.

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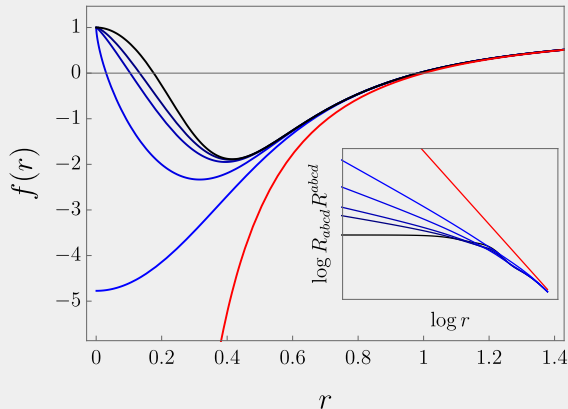
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# Regular Black Holes Bacchanalia

[PB, Cano, Hennigar]

$\alpha_n$	$f(r)$
$\alpha^{n-1}$	$1 - \frac{mr^2}{r^{D-1} + \alpha m}$
$\frac{\alpha^{n-1}}{n}$	$1 - \frac{r^2}{\alpha} \left(1 - e^{-\alpha m/r^{D-1}}\right)$
$n\alpha^{n-1}$	$1 - \frac{2mr^2}{r^{D-1} + 2\alpha m + \sqrt{r^{2(D-1)} + 4\alpha m r^{D-1}}}$
$\frac{(1 - (-1)^n)}{2} \alpha^{n-1}$	$1 - \frac{2mr^2}{r^{D-1} + \sqrt{r^{2(D-1)} + 4\alpha^2 m^2}}$
$\frac{(1 - (-1)^n) \Gamma\left(\frac{n}{2}\right)}{2\sqrt{\pi} \Gamma\left(\frac{n+1}{2}\right)} \alpha^{n-1}$	$1 - \frac{mr^2}{\sqrt{r^{2(D-1)} + \alpha^2 m^2}}$

Hayward, Bardeen-like, Dymnikova-like, etc., black holes as particular cases



### **In sum**

The Schwarzschild black hole singularity gets generically resolved in  $D \geq 5$  by the effect of infinite towers of higher-curvature densities, no tricks involved.

THE END

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- What about  $D = 4$ ?



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