Black-hole ringdown and their progenitors: from numerical relativity to tests of GR

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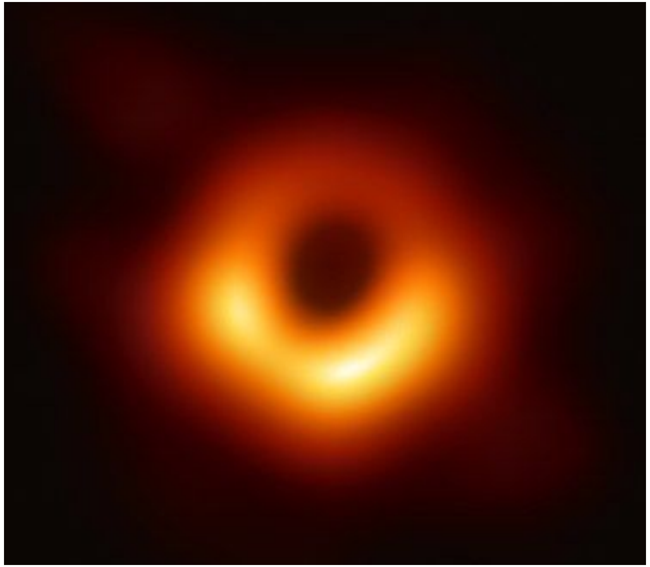
Kerr Black Holes



EHT Collaboration (2019),
arxiv 1906.11238

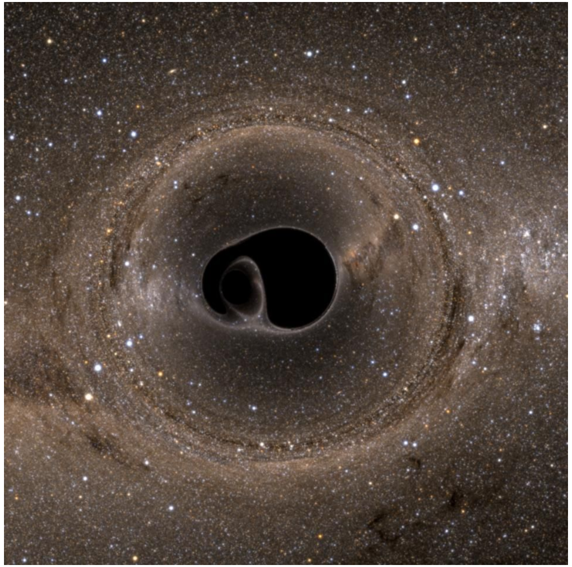
Uniqueness Theorem

Stationary black holes in vacuum in GR are uniquely described by the Kerr spacetime metric

$$
\begin{aligned}
ds^2 \;=\; & -\left(1-\frac{2Mr}{\Sigma}\right)dv^2+2dvdr+\Sigma d\theta^2 \\
& +\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta d\bar{\phi}^2 \\
& -2a\sin^2\theta drd\bar{\phi}-\frac{4Mra}{\Sigma}\sin^2\theta dvd\bar{\phi}\,.
\end{aligned}
$$

Roy P. Kerr
Phys. Rev. Lett. 11, 237 (1963)

Quasi-normal modes



Simulation by The SXS (Simulating eXtreme Spacetimes) Project

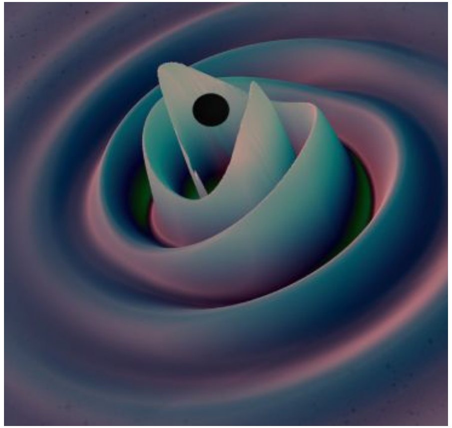
Black-hole perturbations

Linear perturbations about black holes are described by Teukolsky's equations

Black-hole spectrum

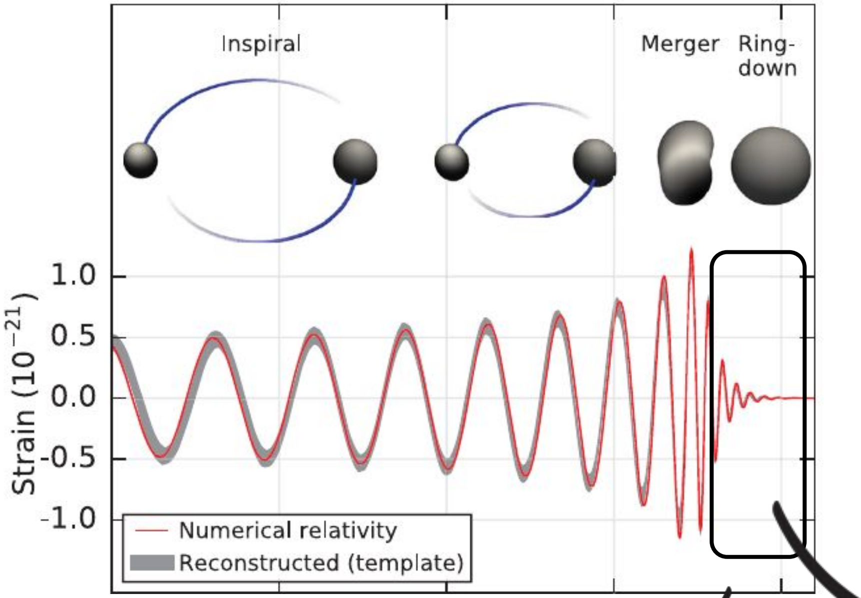
A perturbed black hole emits gravitational waves with a characteristic frequency spectrum

Teukolsky 2014, arxiv 1410.2130
Berti+ (2009), arxiv 0905.2975
Pani (2013), arxiv 1305.6759



Simulation by Georgia Tech, MAYA Collaboration

Quasi normal modes and the no-hair theorem



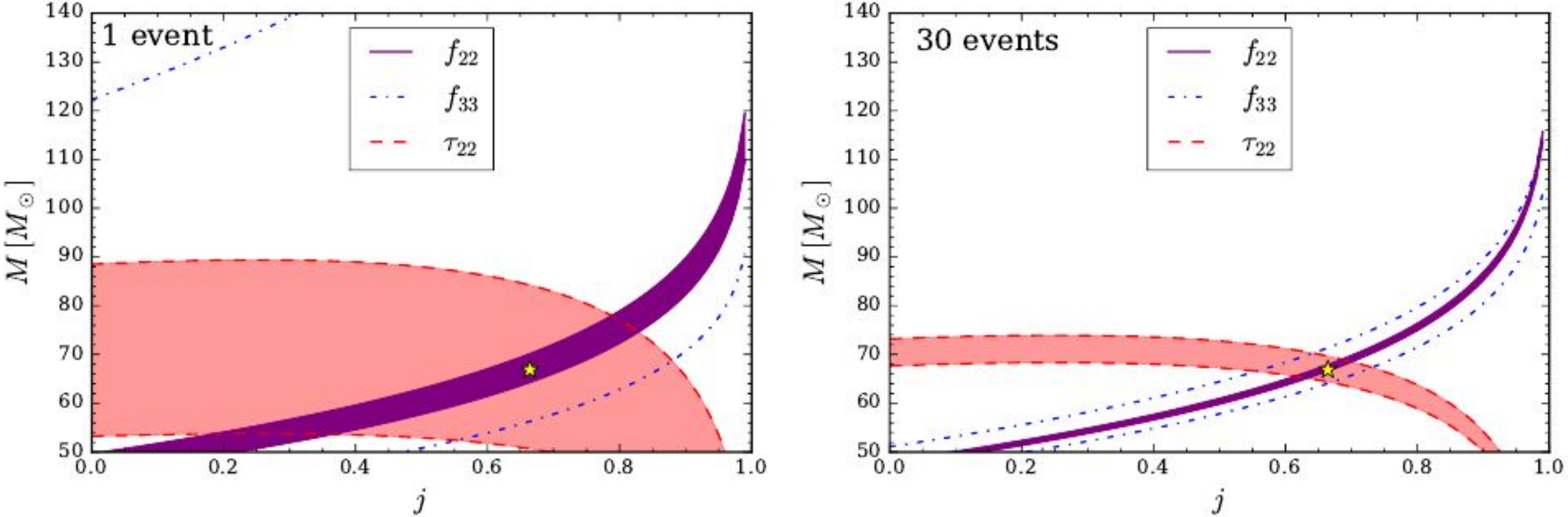
As a consequence of the no-hair theorem, **the quasi normal modes depend only on the mass and spin** of the BH remnant

Black Hole Spectroscopy

$$f_{lmn} \equiv f_{lmn}(M,\chi)$$
$$\tau_{lmn} \equiv \tau_{lmn}(M,\chi)$$

Black hole spectroscopy

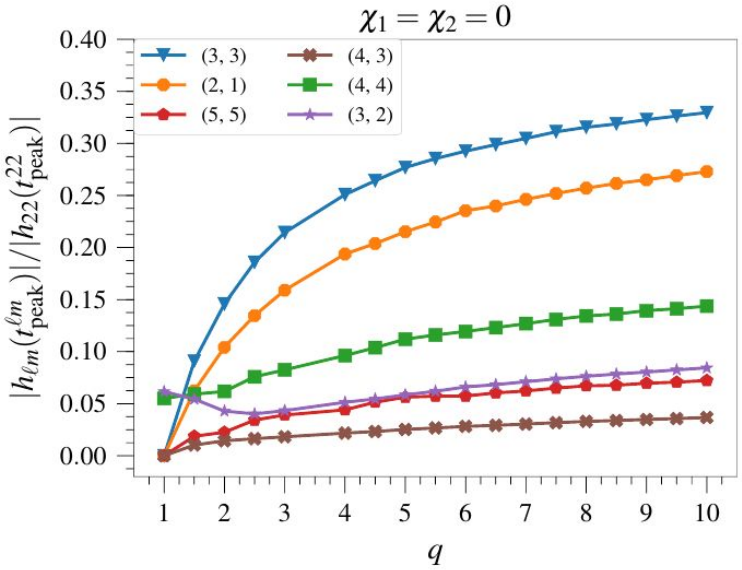
Measure **at least two** quasi-normal modes
Check that they are consistently inverted into mass and spin



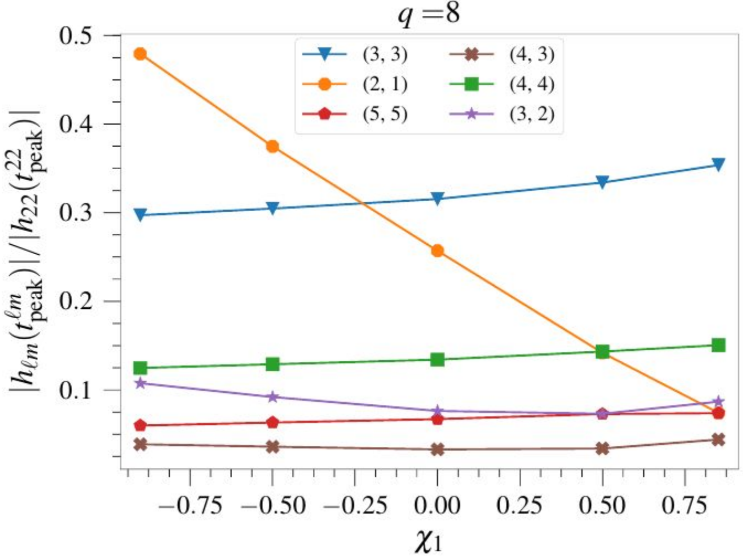
Brito+ (2018) 1805.00293

Ringdown excitation amplitudes

The activation of QNMs is determined by their amplitudes



Cotesta+ (2018) 1803.10701



Amplitudes depend on the intrinsic parameters of the progenitors

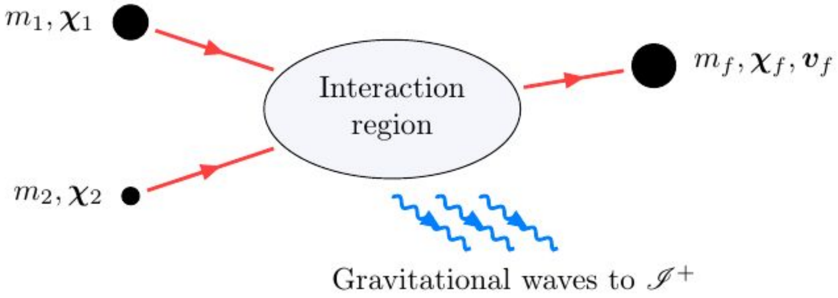
Kamaretsos+ (2012) 1207.0399

Surrogates for ringdown amplitudes

Multipole fits for the ringdown amplitudes

Kamaretsos+ (2012) [1207.0399](1207.0399)
London (2018) [1801.08208](1801.08208)
Forteza+ (2022) [2205.14910](2205.14910)
Cheung+ (2023) [2310.04489](2310.04489)

The are all in closed form



Gaussian Process Regression (GPR)

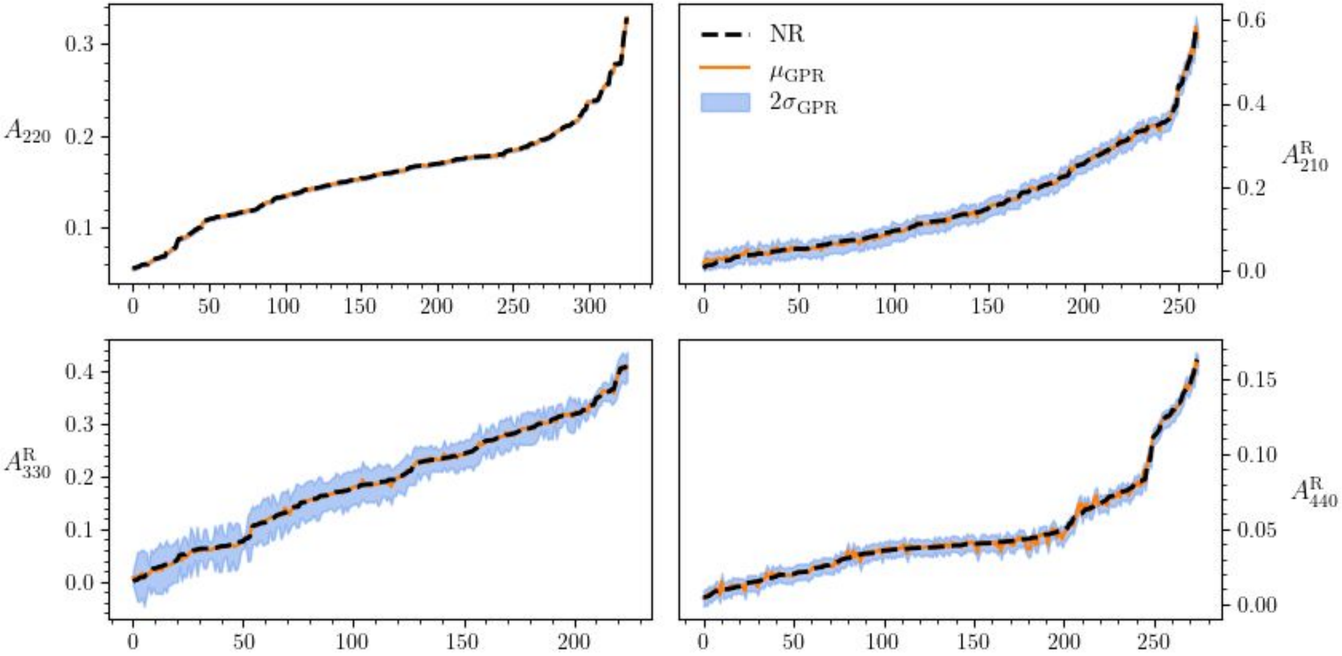
- Parametric-free: does not did ansatze
- Predicts a distribution of function
- Outputs also uncertainties over predictions

GPR used to build the most accurate surrogate fon final mass and spin

Varma+ (2018) [1809.09125](1809.09125)
Boschini+ (2023) [2307.03435](2307.03435)

We extend the surrogate to ringdown amplitudes of **non-precessing quasi-circular** binaries

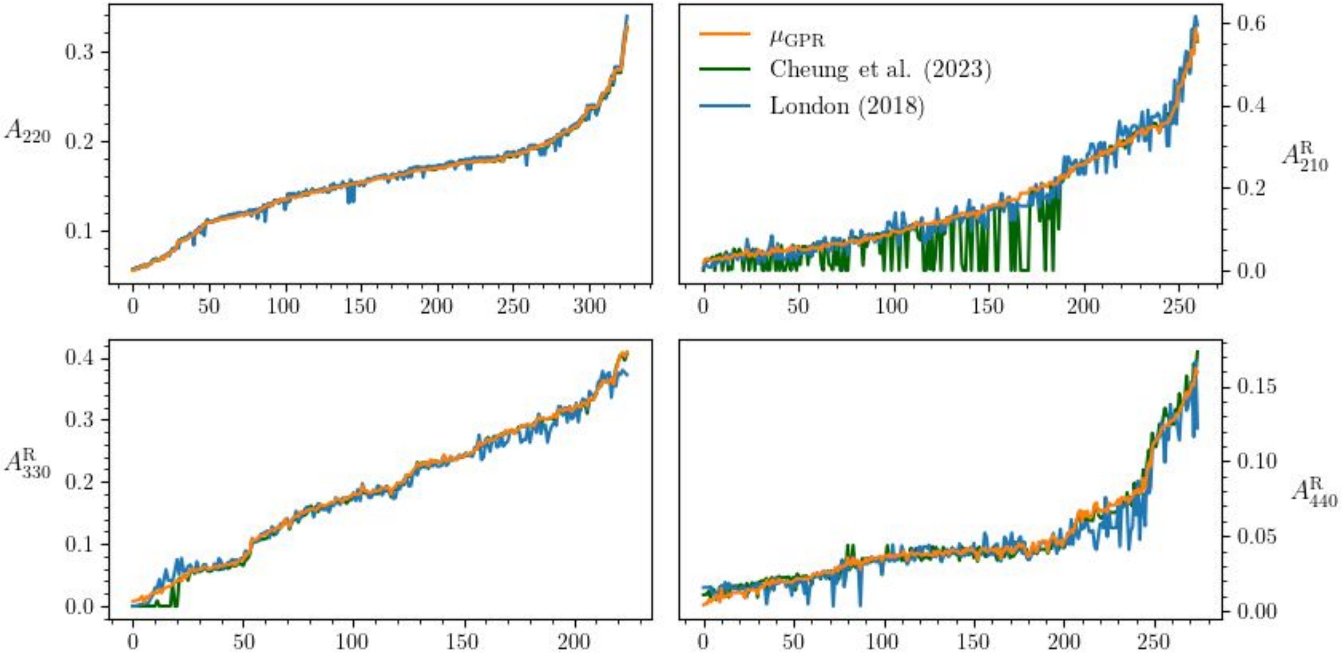
Results: comparison with NR data



Deviation of mean GPR predictions from NR data is order 10e-3

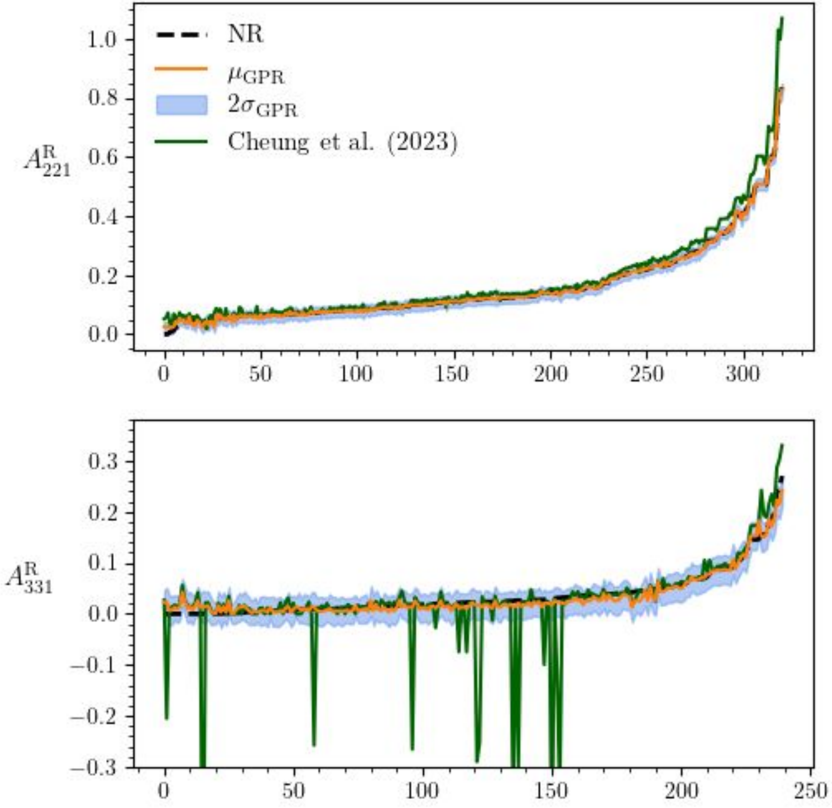
NR data are comprised within ~1 GPR standard deviations

Results: comparison with previous fits (I)



Overall agreement with the most updated fits: London (2018) and Cheung et al. (2023)

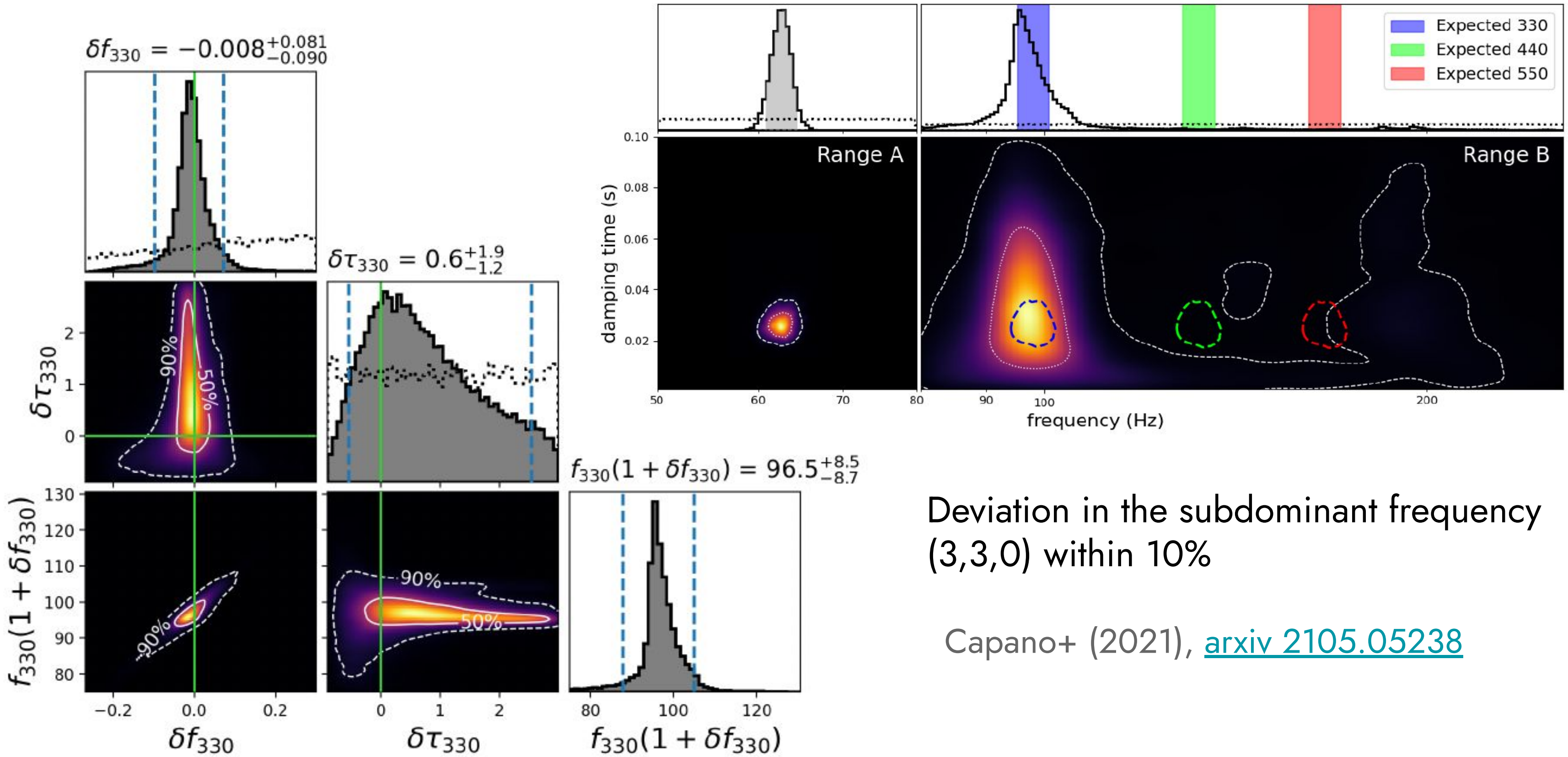
Results: comparison with previous fits (II)



GPR is robust to overfitting outside interpolation region

See negative amplitudes returned from polynomial fits for the 331 overtone

GW190521: a case study for LVK



Deviation in the subdominant frequency (3,3,0) within 10%

Capano+ (2021), arxiv 2105.05238

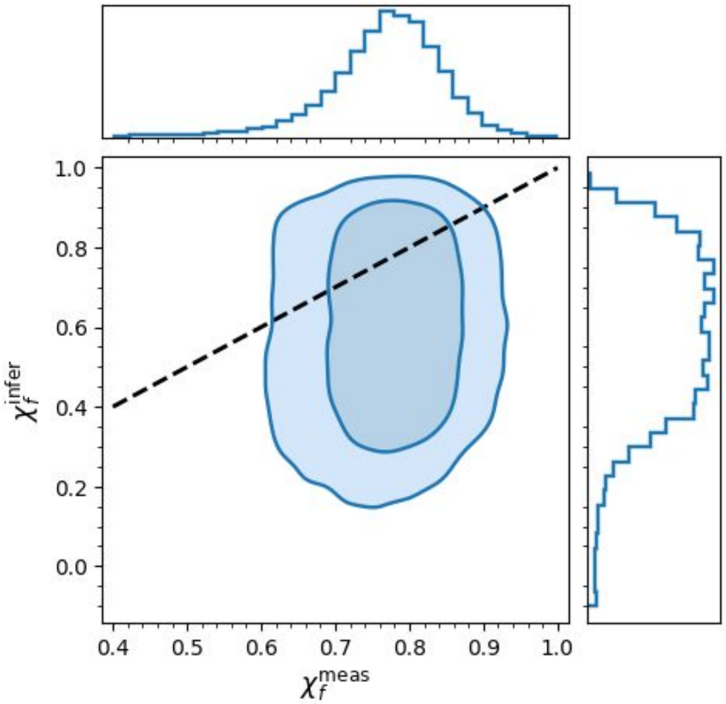
GW190521: amplitude-spin consistency

GR is a deterministic theory

- *Final spin and ringdown amplitudes* are determined by the same initial params
- We *have surrogate fits for both* final spin and ringdown amplitudes

Amplitude-spin consistency

- Measure amplitudes and final spin as independent in ringdown
- Use amplitudes to infer initial params
- Use initial params to infer final spin
- Compare with measured final spin



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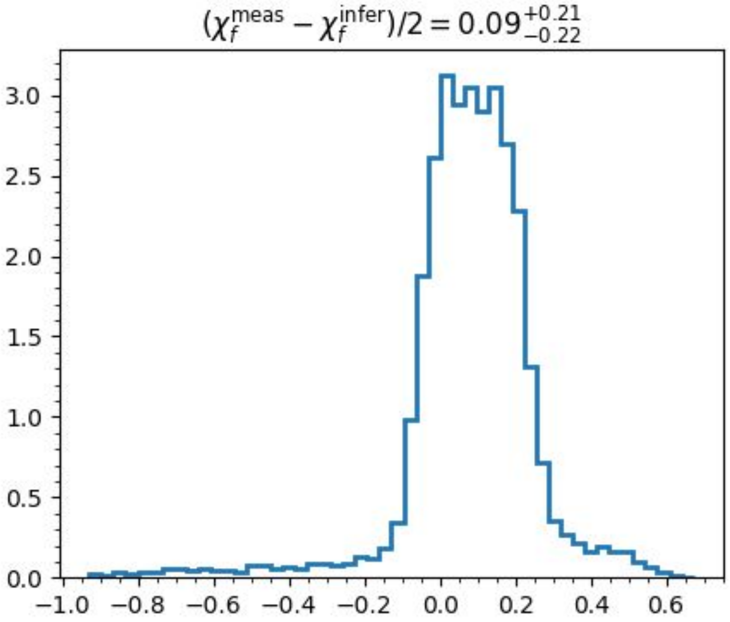
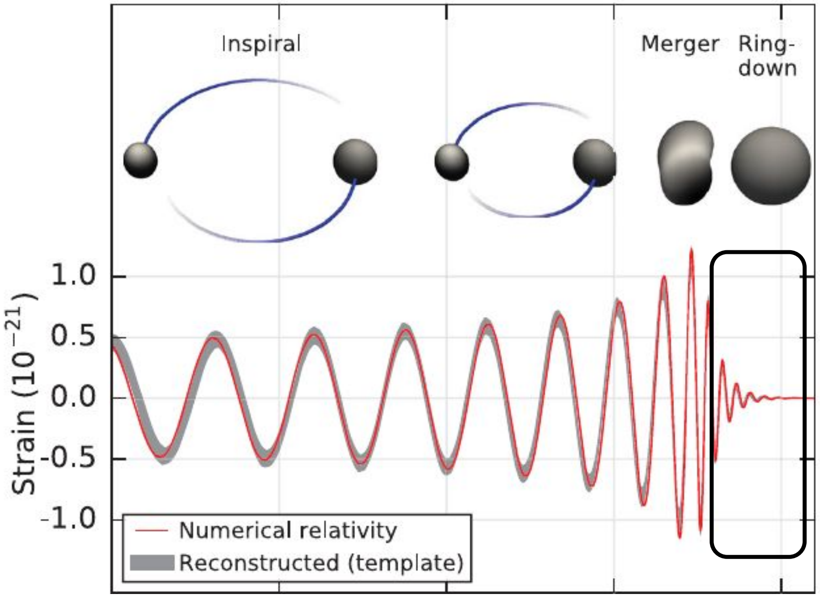


Thank you!

Backup

Testing no-hair with black hole spectroscopy



BH spectroscopy: measuring the frequency and damping times of a BH from its GW signal

GR test: check that there is no "third" independent frequency or damping time

Berti, Cardoso, Will (2006), arxiv gr-qc/0512160

Berti+ (2016), arxiv 1605.09286

Baibhav+ (2023), arxiv 2302.03050

Gossan+ (2011), arxiv 1111.5819

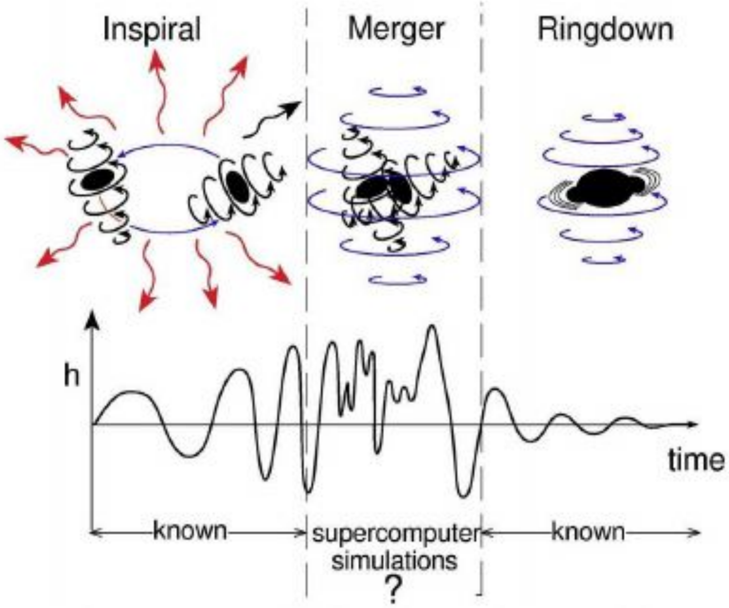
Meidam+ (2014), arxiv 1406.3201

Even if the background is Kerr, no-hair is violated if the perturbation dynamics differ from Teukolsky's

Barausse & Sotriou (2004), arxiv 0803.3433

Tattersall+ (2017), arxiv 1711.01992

Quasi normal modes and the no-hair theorem



If no-hair is violated, you have dependence from (at least one) further scale

Recent progresses in computing modified spectra:
Cano+ (2023) arxiv 2304.02663
Ghosh+ (2023) arxiv 2303.00088

$$f_{lmn} \equiv f_{lmn}(M, \chi, \alpha)$$
$$\tau_{lmn} \equiv \tau_{lmn}(M, \chi, \alpha)$$

Relation to black hole uniqueness

Uniqueness theorem – Stationary black holes are axisymmetric and are described by a member of the Kerr family

Kerr hypothesis – a black hole formed from gravitational collapse or from a binary merger will asymptotically settle to a member of the Kerr family, by emitting all charges except mass and angular momentum (aka the ‘no hair’ theorem) [as reviewed e.g. in Teukolsky 2014 https://arxiv.org/abs/1410.2130]

Assuming the Kerr hypothesis, the quasi-normal mode spectrum follows from Teukolsky’s equation and depends only on the final mass and the final spin of the black hole [as reviewed in e.g. Berti+2009 https://arxiv.org/abs/0905.2975]

$$\omega_{lmn} \equiv \omega_{lmn}\left(M_f, \chi_f\right)$$

What black hole spectroscopy tests (1)

Ringdown tests probe general relativity in the strong field regime

Quasi-normal modes can be approximated by the frequency and damping times of perturbed light rays at the innermost stable circular orbit for null geodesics (aka the ‘photon sphere’) [Cardoso+2009 https://arxiv.org/abs/0812.1806 and ref.s therein]

Higher order WKB methods refine this approximation and reinforce the physical connection between quasi-normal modes and the photon sphere

Black hole spectroscopy tests the BH structure in the vicinity of the photon sphere

What black hole spectroscopy tests (2)

The **uniqueness theorem** holds in several gravitational theories beyond GR. Examples: shift-symmetric Horndeski

However, the dynamics of perturbations is different, hence **the quasi-normal modes are different**

Here you only test the dynamics of the theory but not the metric of the asymptotic solution

Some gravitational theories beyond GR admit black hole solutions **different from Kerr**. Examples: Einstein–dilaton–Gauss-Bonnet, dynamical Chern-Simons

Here, you test the structure of the black hole in the vicinity of the photon sphere* and the dynamics of the perturbations at the same time

*In modified gravity, the connection with photon sphere is still under study, especially for rotating black holes [see Yagi 2022 https://arxiv.org/abs/2201.06186 and ref.s therein]