Universidad Complutense de Madrid
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Beyond dark matter: Constraining hyperconical-relativistic MOND-like model to galaxy cluster RAR observations

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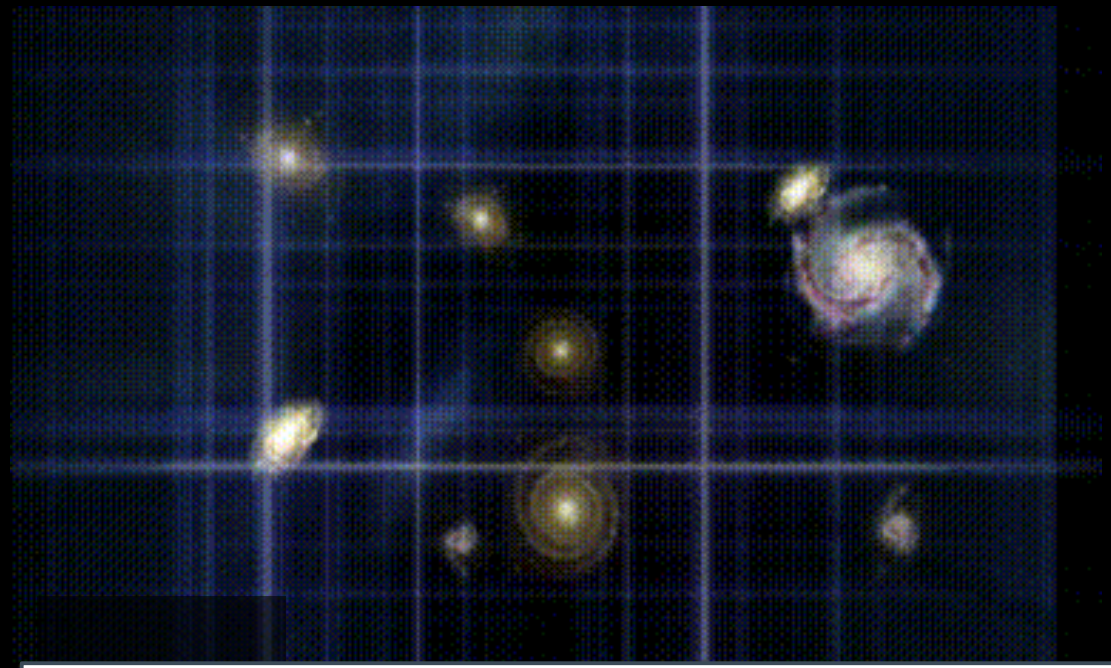
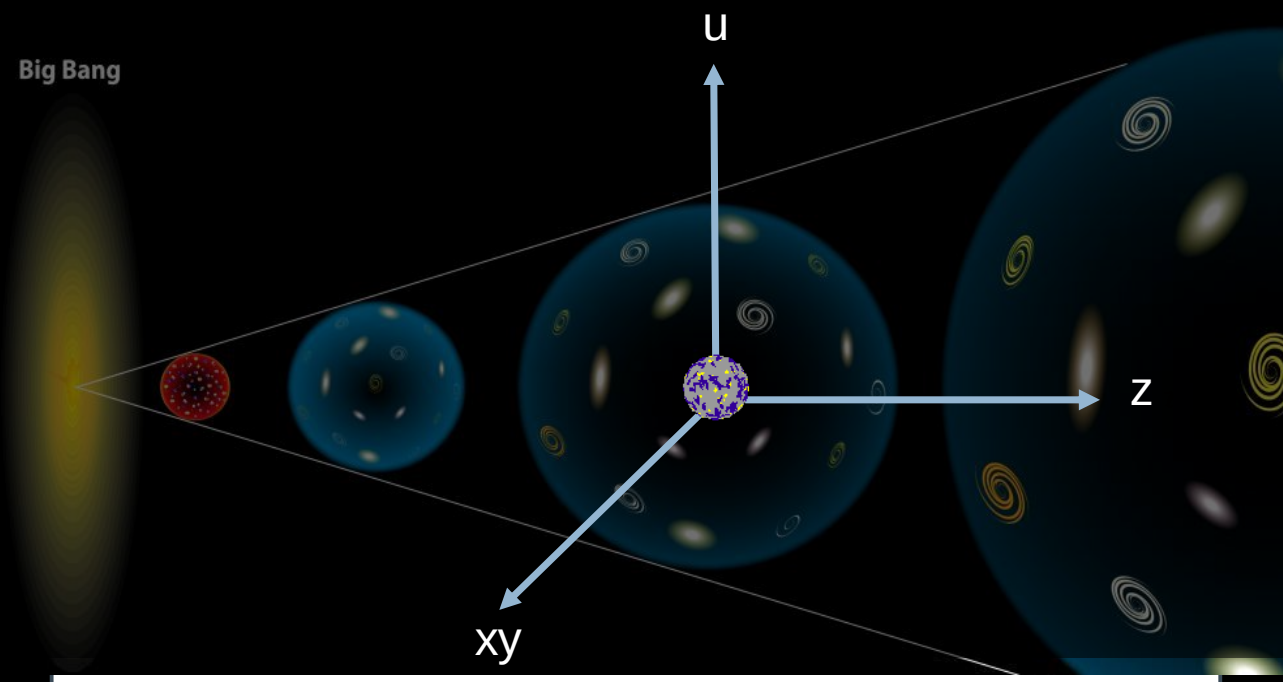
- **Motivation**
  - ✓ Dynamical embedding
  - ✓ Previous results at a cosmic scale
- **Approach**:
  - ✓ Hyperconical modified gravity
  - ✓ Observational data
- **Results**: Fictitious acceleration of...
  - ✓ Galaxies
  - ✓ Clusters
- **Final remarks**

Motivation: Embedding and projection
of Lorentzian manifolds

Intrinsic perspective



Extrinsic perspective



(Pseudo-) Riemannian manifolds

Friedman-Lemaitre-Robertson-Walker metric

Embedding in flat spacetimes $S_t^3 \subset R^{0,4} \subset R^{1,4}$

GR +

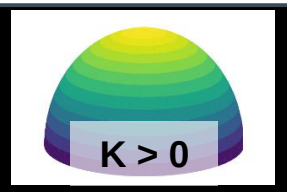
$$g|_{acc} = dt^2 - a(t)^2\left(\frac{dr'^2}{1-Kr'^2} + r'^2\, d\Sigma^2\right)$$

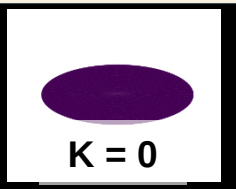
Local **GR** +

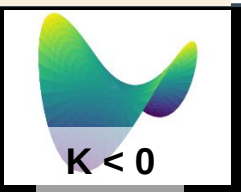
$$g \approx dt^2\left(1-kr'^2\right) - \frac{t^2}{t_0^2}\left(\frac{dr'^2}{1-kr'^2} + r'^2 d\Sigma^2\right) - \frac{2r't}{t_0^2}\frac{dr'dt}{\sqrt{1-kr'^2}},$$

=

a(t) ×







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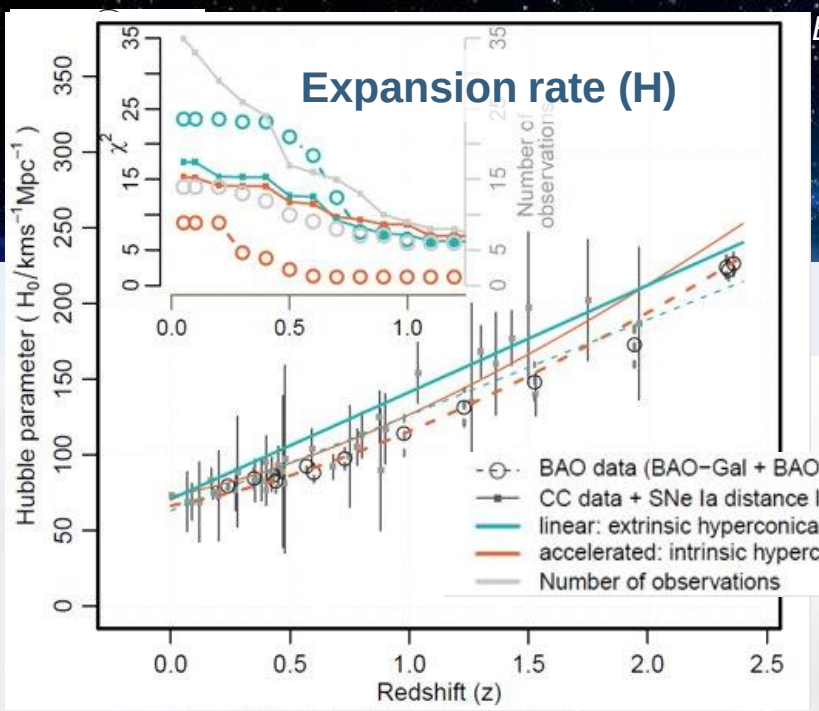
Previous results
at a cosmic scale

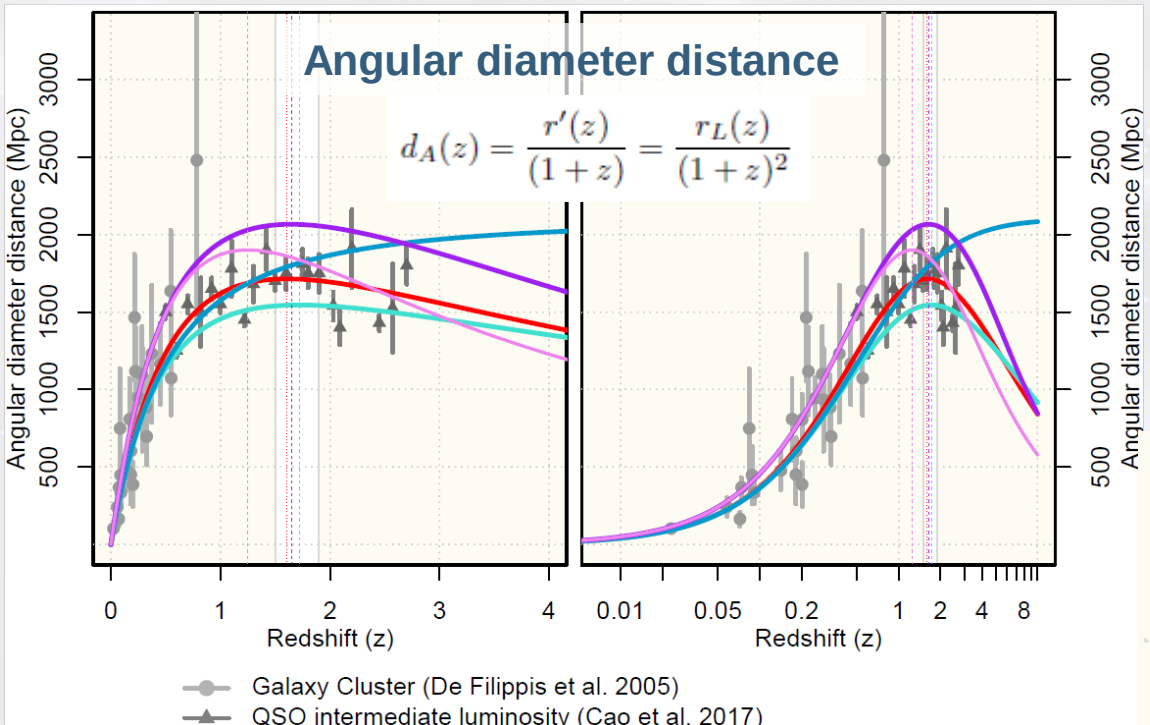
Monjo, R. (2024): ApJ 967, 66, 10.3847/1538-4357/ad3df7

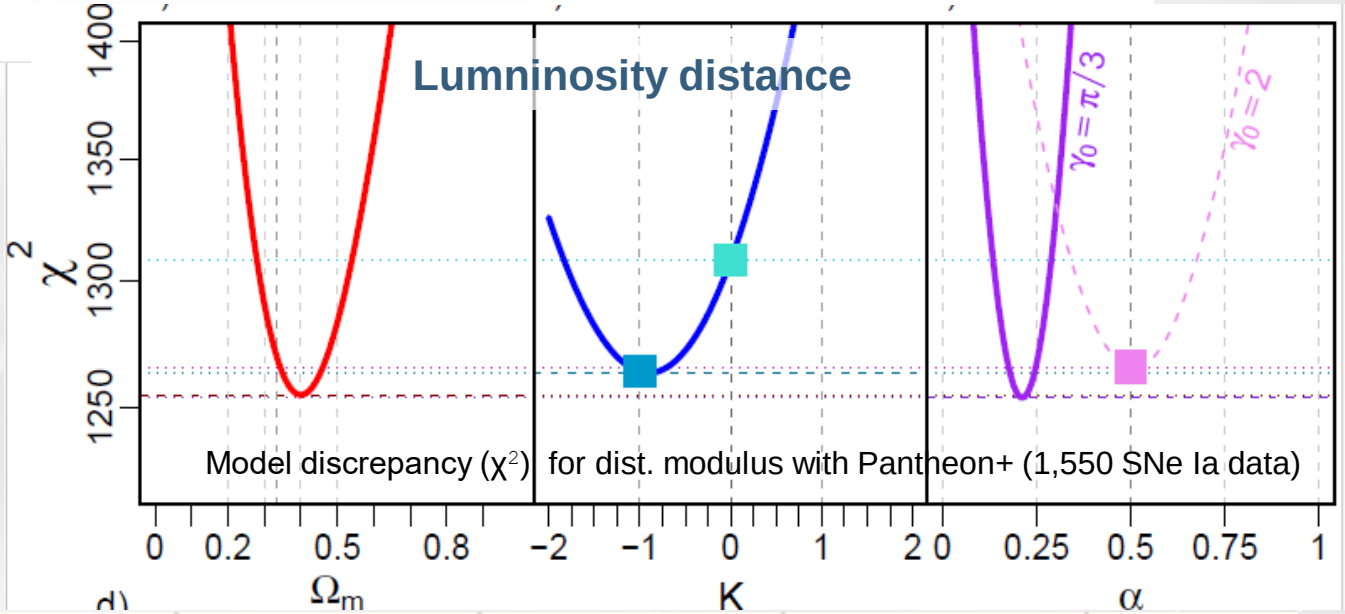


$$r_L^{\Lambda CDM} = \frac{1+z}{H_0}\,\sin_K \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_K(1+z')^2+\Omega_\Lambda}}$$

$$r_L^{coast} = \frac{1+z}{H_0}\sin_K\left(\ln(1+z)\right) \qquad r_L^{hyp} = \frac{1+z}{H_0} f_{\gamma_0}\left(\mathfrak{sn}_k\left(\ln(1+z)\right)\right)$$

$$r_L^{R_h=ct} = \frac{1+z}{H_0}\ln(1+z). \qquad \mathfrak{sn}_k^{-1}(\gamma) := \int_0^\gamma \frac{\sqrt{1-k^{-1}(1-\cos\gamma')^2}}{1-2k^{-1}(1-\cos\gamma')}d\gamma'$$





1−par. flat ΛCDM model [accel. hom., $\Omega_K := 0$, $\Omega_\Lambda = 1 - \Omega_m$]
1−par. curved coasting model [linear FLRW, $-2 < K < 1$]
0−par. Milne coasting model [linear FLRW, $K := -1$]
0−par. $R_h = ct$ coasting model [linear FLRW, $K := 0$]
1−par. hyperconical model [linear inhom., $K := 1$, $\gamma_0 = \pi/3$, $\alpha \neq 1/2$]
0−par. local hypercon. model [linear inhom., $K := 1$, $\gamma_0 = 2$, $\alpha = 1/2$]

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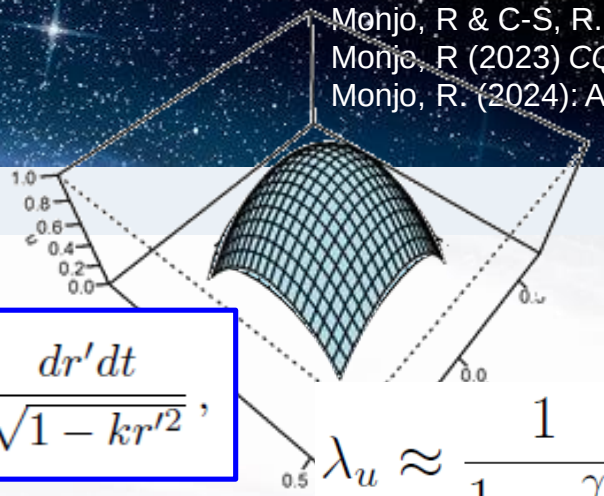
HMG model

Monjo, R & C-S, R. (2023) *CQG* **40**, 195006. 10.1088/1361-6382/aceacc
Monjo, R (2023) *CQG* **40**, 235002. 10.1088/1361-6382/aceacc
Monjo, R. (2024): ApJ 967, 66, 10.3847/1538-4357/ad3df7



Hyperconical metric

$$g \approx dt^2\left(1-kr'^2\right) - \frac{t^2}{t_0^2}\left(\frac{dr'^2}{1-kr'^2} + r'^2 d\Sigma^2\right) - \frac{2r't}{t_0^2}\frac{dr'dt}{\sqrt{1-kr'^2}},$$

$$\lambda_u \approx \frac{1}{1-\frac{\gamma}{\gamma_0}}$$

Cosmic projection

$$\hat{r}' = \lambda_u^{\alpha} r' \approx \left(1+\frac{\alpha r'}{\gamma_0 t_0 c}\right) r'$$

$$\hat{t} = \lambda_u t \approx \left(1+\frac{r'}{\gamma_0 t_0 c}\right) t$$

Hyperconical modified gravity (HMG)

$$h_{\mu\nu}\big|_{Schw} :\approx \hat{g}_{\mu\nu} - \hat{g}_{\mu\nu}\big|_{M\to 0}$$

$$\Delta R_{00} - \frac{1}{2}\Delta R\, g_{00} = 8\pi \mathrm{G} P_{00}^m$$

$$\Delta R_{ij} - \frac{1}{2}\Delta R\, g_{ij} = 8\pi \mathrm{G} P_{ij}^m$$

Local GR + cosmic projection

$$\tilde{h}_{tt} \approx -\frac{2\mathrm{G}M}{rc^2}\left(1-\frac{\alpha r}{\gamma_0 t c}\right) + \frac{2}{\gamma_0 c}\left(\frac{r}{t}+\frac{t}{t_0}\dot{r}'\right)$$

$$\frac{v^4}{c^4} \approx \left(\frac{\mathrm{G}M}{rc^2}\right)^2 + \frac{2\mathrm{G}M}{\gamma_0 t c^3}$$

$$\frac{2c}{\gamma_0 t} := a_0$$

Milgromian acceleration

but now with:

$$\gamma_0^{-1} = \gamma_{sys}^{-1}\cos\gamma_{sys}$$

Baryonic Tully-Fisher relation (BTFR) and the mass-discrepancy acceleration relation (MDAR)

1-parameter HMG model: γ_0 = constant
2-parameter HMG model: γ_0 = *f*(geometry of the system, that is: γ_{center} and ε_H)

Observational data

10 galaxy clusters: <u>Radial Acceleration Relation</u>

- Range: 0.033 < z < 0.090
- Rotation curves (distances; observed and Keplerian speeds)
- Source: **HIghest X-ray FLUx Galaxy Cluster Sample (HIFLUGCS; Li et al. 2023)**

A0085 A1795 A2029 A2142 A3158 A0262 A2589 A3571 A0576 A0496

60 high-quality rotation curves

- Range: 0.00022 < z < 0.028
- Radial acceleration relationship (RAR)
- Source: **Spitzer Photometry and Accurate Rotation Curves (SPARC; McGaugh et al. 2007, 2016)**
- Features: Galaxies filtered to well-measured intermediate radii (Lelli et al. 2019).

U2885	N5907	N4157	N4088	N4051	N3949	N4085	F563−V2	U6446	N247
N5533	N5371	N2903	N2683	U5005	N2403	N4183	N1003	N300	N1560
N6674	N2998	N4013	N6946	U6973	F568−V1	F563−1	U6917	U6667	DDO170
N3992	N5033	N4100	N4138	U5999	N3972	U6930	M33	N5585	N3109
N7331	N3953	N3893	N3877	U1230	N4010	N6503	N7793	U6399	IC2574
N801	N3521	N4217	N3726	N3917	N3769	U6983	U2259	N55	DDO154

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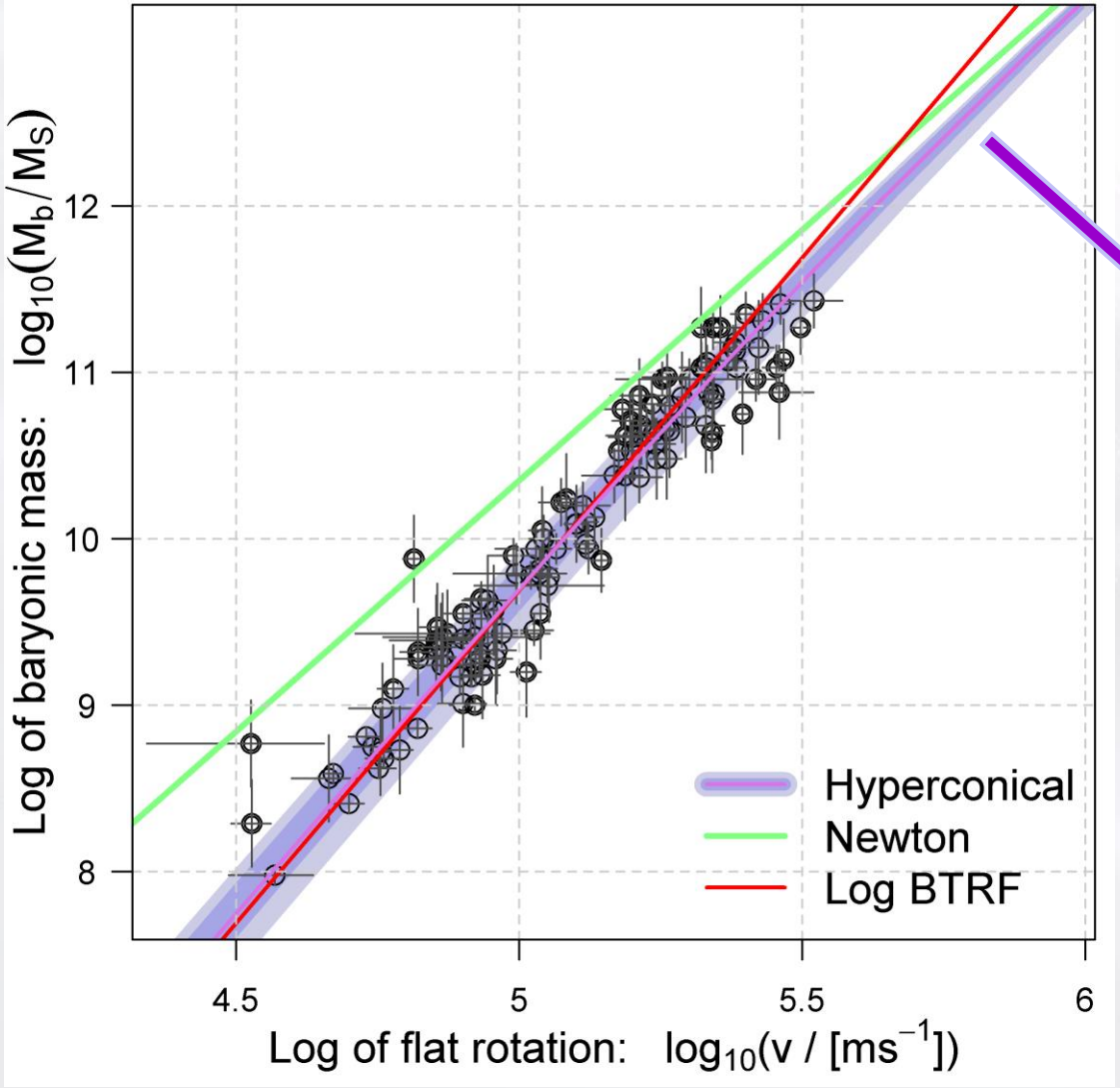
Results for galaxies

Monjo, R. (2023): CQG **40**, 235002, 10.1088/1361-6382/ad0422

Fictitious cosmic acceleration for maximum galaxy rotation velocity



MDAR modeling. Fitting of the hyperconical model to the baryonic Tully–Fisher (BTRF) relation with $v(M_b)$ for 123 galaxies

DOI:10.1088/1361-6382/ad0422

$$\frac{v^4}{c^4} \approx \left(\frac{GM}{rc^2}\right)^2 + \frac{2GM}{\gamma_0 t c^3}$$

$$v(M_b;\gamma_M) \approx \sqrt[4]{\left(\frac{GM_b}{r_M}\right)^2 + \frac{2GM_b c}{t}\frac{\cos\gamma_M}{\gamma_M}}$$

Mass-discrepancy acceleration relation (MDAR)

$GM_b a_0$

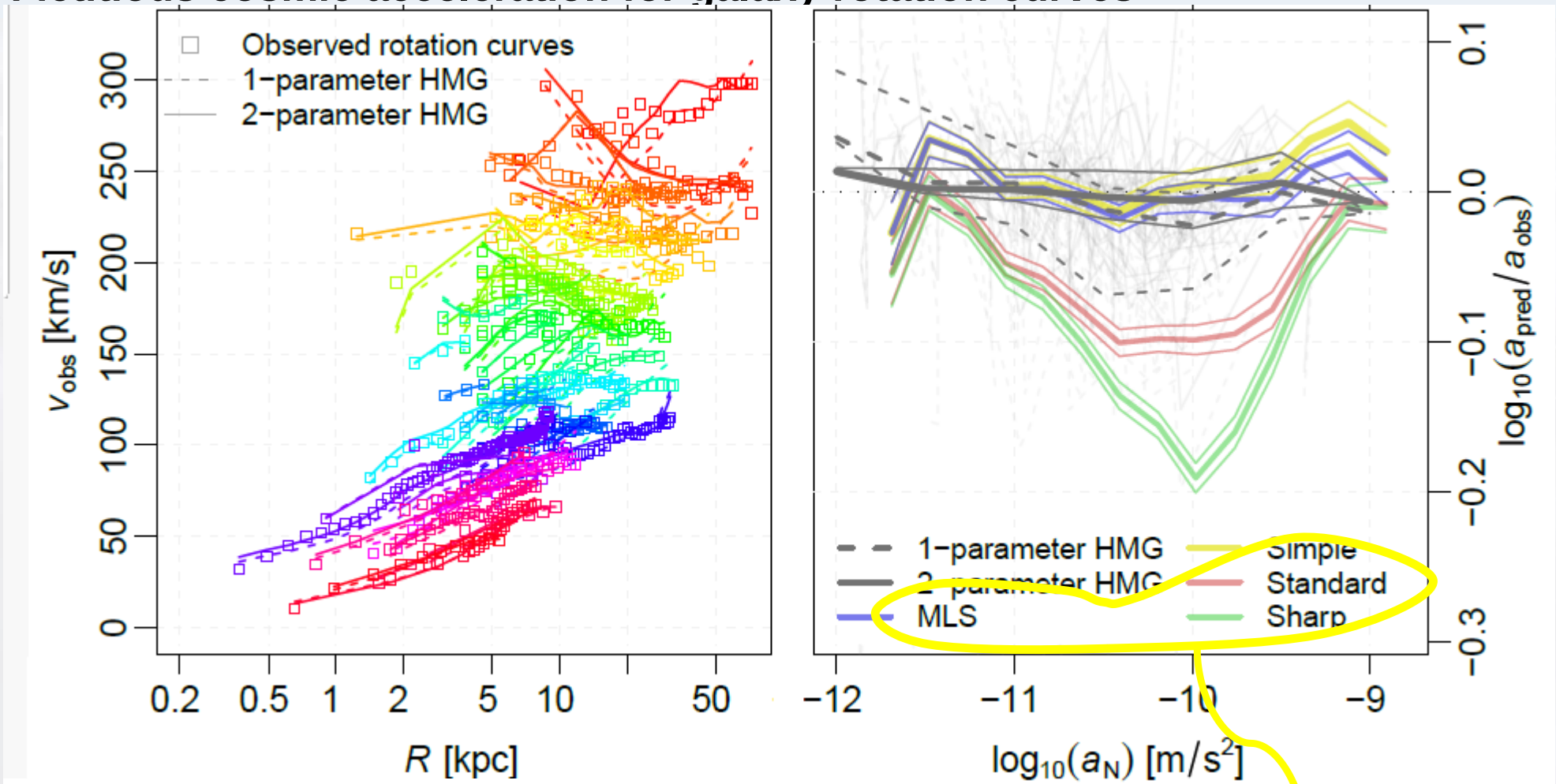
Milgromian acceleration

$a_0 = (2c/t)\cdot\cos\gamma_M/\gamma_M$

Results for galaxies

Monjo, R. (2023): CQG **40**, 235002, 10.1088/1361-6382/ad0422
Monjo, R. & Banik, I. (2024): MNRAS. (under review)

Fictitious cosmic acceleration for galaxy rotation curves



U2885 N5907 N4157 N4088 N4051 N3949 N4085 F563−V2 U6446 N247
N5533 N5371 N2903 N2683 U5005 N2403 N4183 N1003 N300 N1560
N6674 N2998 N4013 N6946 U6973 F568−V1 F563−1 U6917 U6667 DDO170
N3992 N5033 N4100 N4138 U5999 N3972 U6930 M33 N5585 N3109
N7331 N3953 N3893 N3877 U1230 N4010 N6503 N7793 U6399 IC2574
N801 N3521 N4217 N3726 N3917 N3769 U6983 U2259 N55 DDO154

Left: Fitting of 60 galaxy rotation curves according to the HMG model with 1 parameter (dashed lines) or 2 parameters (continuous lines).
Right: Performance of the 1- and 2-parameter HMG model for the ratio of predicted (apred) and observed (aobs) centripetal acceleration for the 60 galaxies (light gray lines) and comparison with MOND interpolation functions (MLS, simple, standard, and sharp functions, fitted to 153 suitable galaxies): as in Fig.23 of Banik et al. (2023)

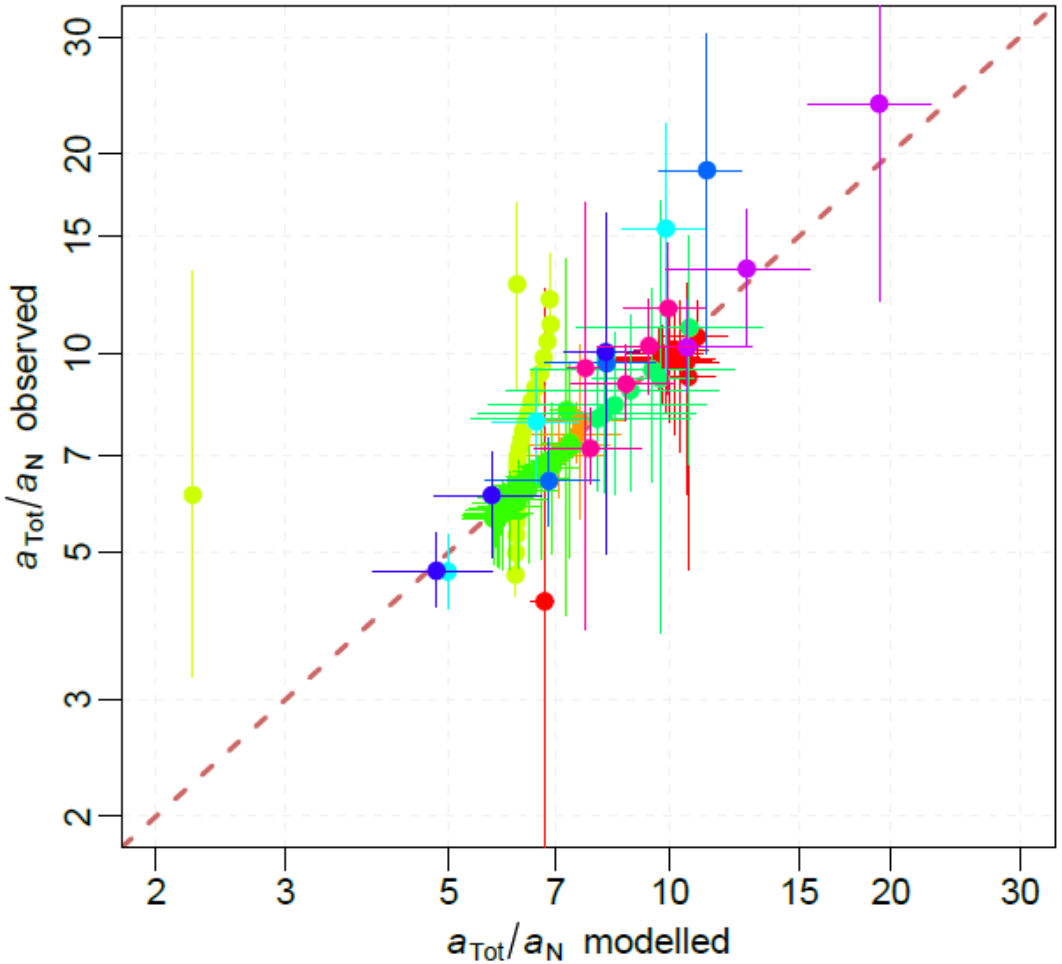
Banik et al. 2023. MNRAS, 527, 4573 10.1093/mnras/stad3393

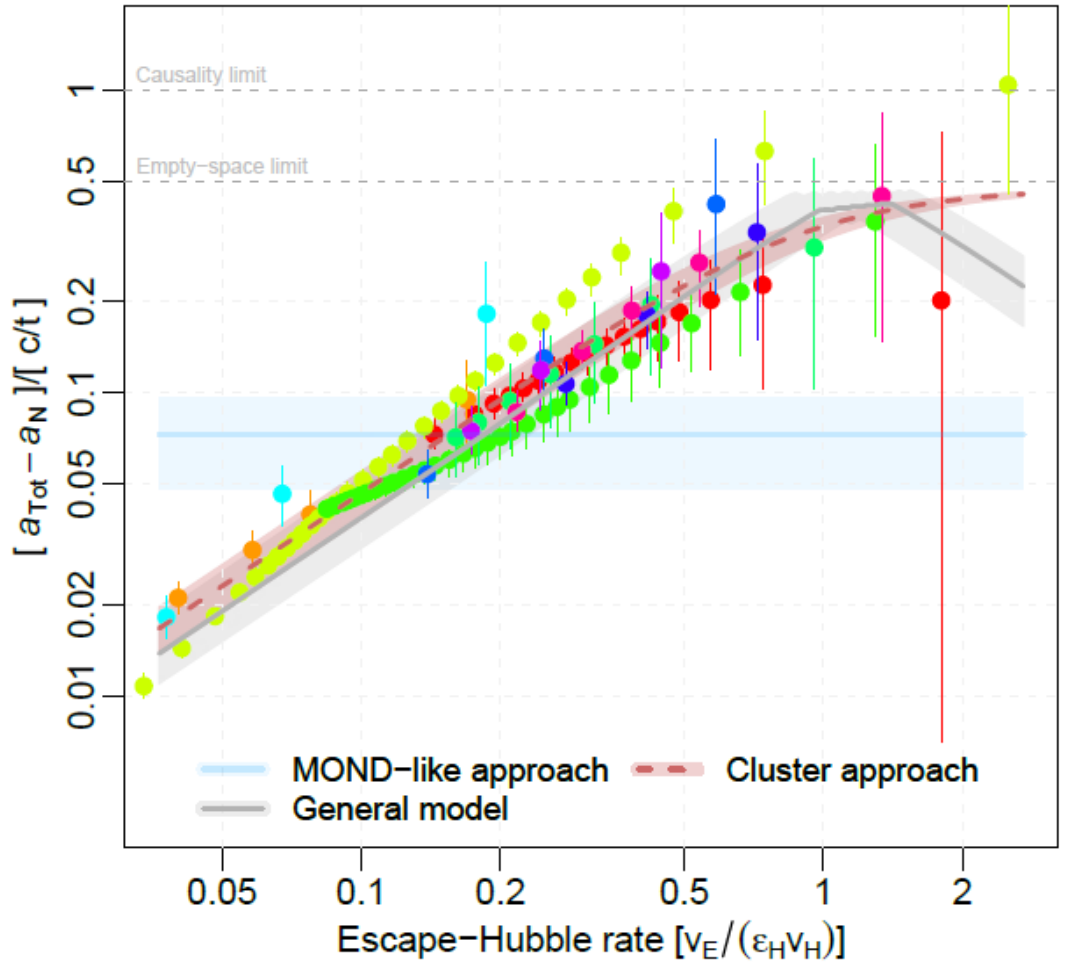
Results for clusters

Monjo, R. & Banik, I. (2024): MNRAS. (under review)

Fictitious cosmic acceleration for galaxy clusters

$$\epsilon_H^2 := \sin^2\gamma_U / \sin^2\gamma_{neigh} - 5/6 \propto t^2/r_{cs}^2 \propto \rho/\rho_{vac}$$





Clusters: A0085 A1795 A2029 A2142 A3158 A0262 A2589 A3571 A0576 A0496

RAR modelling with HMG for total vs. Newtonian acceleration.
Left: Individual fitting with 2 parameters.
Right: Global fitting according to three models:

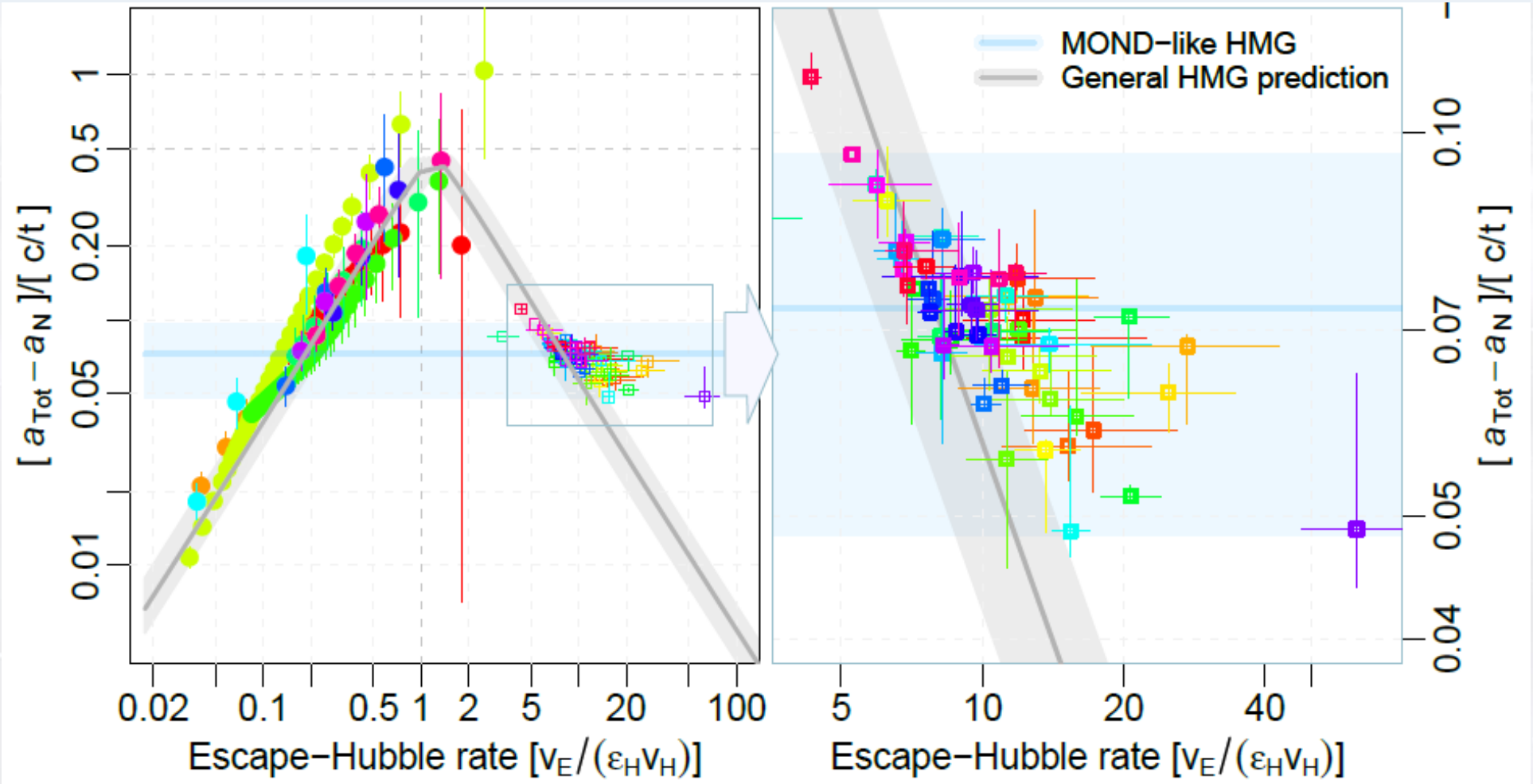
- **MOND-like constant**
- **Fixed 1-par general model**
- **Fixed 2-par Cluster model**

Discussion

Monjo, R. & Banik, I. (2024): MNRAS. (under review)

Fictitious cosmic acceleration for galaxy dynamics

$$\epsilon_H^2 := \sin^2\gamma_U / \sin^2\gamma_{neigh} - 5/6 \propto t^2/r_{cs}^2 \propto \rho/\rho_{vac}$$



Left: Global fitting of the 10 galaxy clusters.

Right: Zoom in on the theoretical prediction made for galaxies.

$$\frac{|a_{Tot} - a_N|}{c/t} \approx \frac{1}{\gamma_0} \approx \frac{\cos\gamma_{sys}}{\gamma_{sys}}$$

$\varepsilon_H = 56$

U2885 N5907 N4157 N4088 N4051 N3949 N4085 F563−V2 U6446 N247
N5533 N5371 N2903 N2683 U5005 N2403 N4183 N1003 N300 N1560
N6674 N2998 N4013 N6946 U6973 F568−V1 F563−1 U6917 U6667 DDO170
N3992 N5033 N4100 N4138 U5999 N3972 U6930 M33 N5585 N3109
N7331 N3953 N3893 N3877 U1230 N4010 N6503 N7793 U6399 IC2574
N801 N3521 N4217 N3726 N3917 N3769 U6983 U2259 N55 DDO154

Discussion

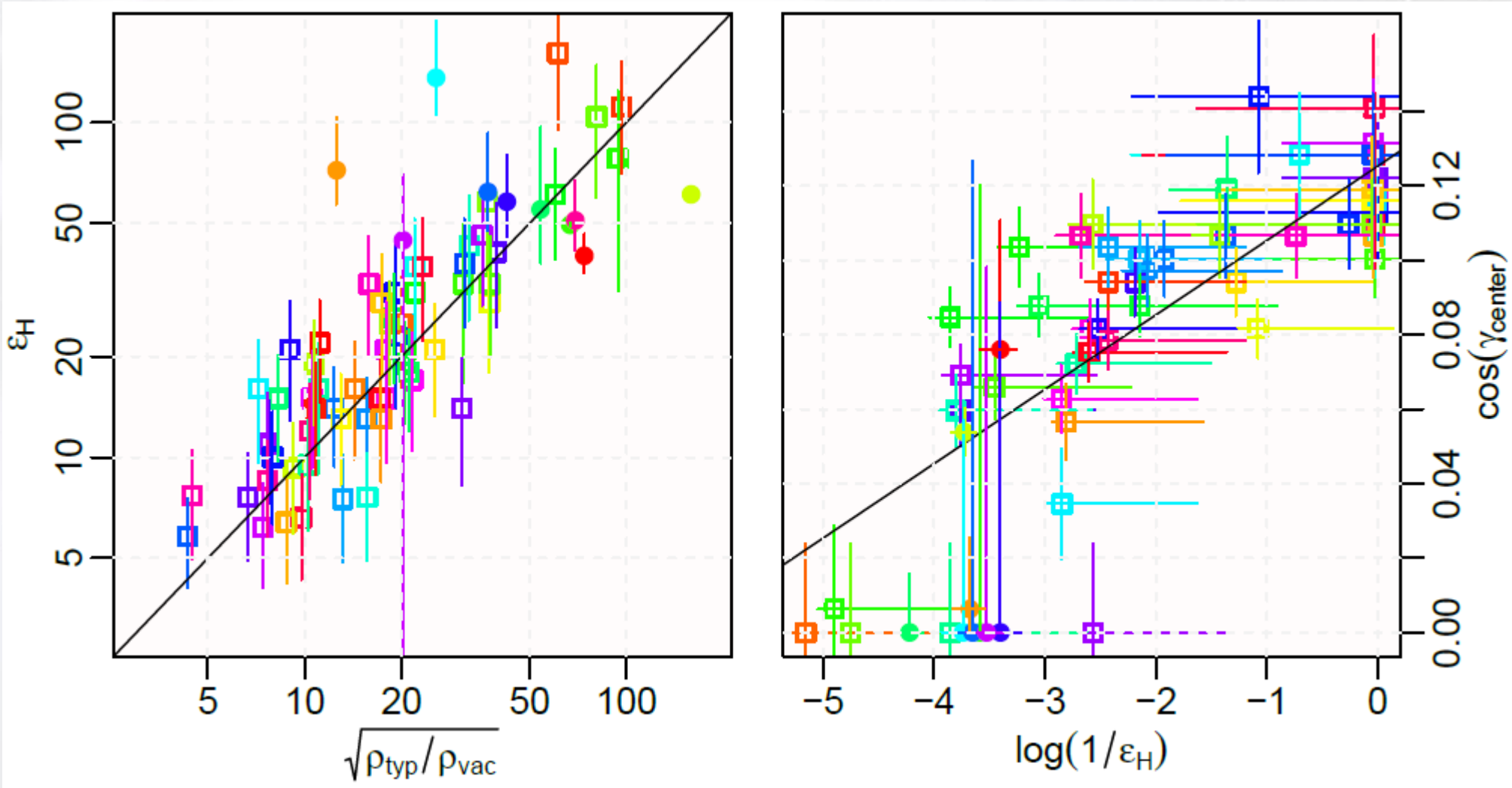
Monjo, R. & Banik, I. (2024): MNRAS. (under review)

Fictitious cosmic acceleration for galaxy dynamics

$$\epsilon_H^2 := \sin^2\gamma_U / \sin^2\gamma_{neigh} - 5/6 \propto t^2/r_{cs}^2 \propto \rho/\rho_{vac}$$

Model parameters fitted to the 60 high-quality galaxy rotation curves (squares), empirical relationships (black lines), and comparison with the fitting to the RAR data of 10 clusters (circles).

$$r_{typ} \sim 50 - 200\,\mathrm{kpc}$$
$$\rho_{vac} := 3/(8\pi G t^2)$$



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Final remarks

Extrinsic perspective **Intrinsic perspective**

(1) **Hypercone** (hypersphere with (c=1)-linear expansion $\Rightarrow$ **Moving frame** (observer) $\Rightarrow$ **Space-time deformation: inhomogeneity** (metrics)

(2) **Projected radially-inhomog. metric** $\cong$ **Apparently accelerated flat metric**

(3) ADM + **conserved Noether currents** lead to the **Hubble flux** and a **"local general relativity"**.

(4) **Prediction** of specific values of **fictitious acceleration** assimilable to cosmic **dark matter-energy**.

(5) **Fictitious acceleration** in galaxies and galaxy clusters: **Hyperconical modified gravity (HMG)**

(6) The **dark matter** of the "galactic halos" can also be explained with our **HMG model**.

(7) **Future works:**

- Modeling **CMB** and **BBN**
- Growth and dynamics of **larger cosmic structures**

Thanks to **Prof. Stacy McGaugh** &
Prof. Pengfei Li for the data

and

Thank you very much for your attention

References:

Monjo, R. & Banik, I. (2024): MNRAS. (under review)
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