TDIFF SCALAR FIELDS: SYMMETRY RESTORATION AND MODEL SELECTION

D. Jaramillo-Garrido, A. L. Maroto, PMM, arxiv: 2307.14861, JHEP 2024.
D. Jaramillo-Garrido, A. L. Maroto, PMM, arxiv: 2402.17422, PRD (*accepted*).

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OUTLINE

- Why and how to break Diff invariance?
- TDiff matter
  - Gravity of a TDiff scalar field.
  - Conservation of the EMT tensor.
  - Potential and kinetic regimes.
  - Interesting kinetic models.
- Symmetry restoration
  - Gravity in the Diff frame.
  - Model selection: the effective speed of sound.
- Discussion and further comments.

WHY AND HOW TO BREAK DIFF INVARIANCE?

- GR + dark components is our best description of gravitational phenomena.
  - ➢ GR is invariant under diffeomorphisms (Diff).

- Unimodular gravity reconsiders Diff invariance (non-dynamical field fixed, $g = 1$)
  - ➢ Invariance under transverse diffeomorphisms (TDiff) and Weyl rescallings (-> WTDiff).

A. Einstein, Math. Phys. 1919.

- More general theories breaking Diff to TDiff (coordinate transformation with Jacobian $J = 1$)

E. Álvarez, D. Blas, J. Garriga, E. Verdaguer, Nucl. Phys. B 2006.

- **Breaking Diff to TDiff just in the matter sector:**
  - We do not know the symmetries of the dark sector.
  - It can be interpreted as non-usual minimal coupling of matter to gravity.
  - GR vacuum solutions.

TDIFF MATTER

- Physics laws are invariant under coordinates transformations such that $J = \det\left(\frac{\partial y^\mu}{\partial x^\nu}\right) = 1.$

  - The allowed infinitesimal coordinates transformations $y^\mu = x^\mu + \xi^\mu$ satisfy $\partial_\alpha \xi^\alpha = 0.$
  - Tensor densities become tensors.

We consider that Diff invariance breaks to TDiff only in the matter sector: $g = |\det(g_{\mu\nu})|$

$$d^4x\sqrt{|g|} \to d^4x\, f(g)$$

We can arbitrarily fix one component of the metric less than in GR.

Einstein-Hilbert action for gravity: $S = S_{EH} + S_m$ ➡ $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$

- Due to the Bianchi identities the EMT is still conserved.

$$T_{\mu\nu} = \frac{2}{\sqrt{|g|}}\frac{\delta S_m}{\delta g^{\mu\nu}}$$

E. Álvarez, A. F. Faedo, and J. J. López-Villarejo, JCAP 2009. A. L. Maroto, JCAP 2024.

GRAVITY OF A TDIFF SCALAR FIELD

$$S_m = \int d^4x \left\{ \frac{f_k(g)}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - f_v(g) V(\psi) \right\}$$

- EoM of the field: $\partial_\mu \left(f_k(g) \partial^\mu \psi \right) + f_v(g) V'(\psi) = 0$
- Einstein equations $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$

We do not focus only on FLRW, but assume a time-like $\partial_\mu \psi$ (+,-,-,-)

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$$

Perfect fluid EMT

$$\rho = \frac{2}{\sqrt{g}} \left\{ \frac{1}{2} f_k (\partial\psi)^2 + g \left[f_v' V - \frac{1}{2} f_k' (\partial\psi)^2 \right] \right\}$$

$$u^\alpha \equiv \frac{\partial^\alpha \psi}{\sqrt{(\partial\psi)^2}}$$

$$p = -\frac{2g}{\sqrt{g}} \left[f_v' V - \frac{1}{2} f_k' (\partial\psi)^2 \right]$$

D. Jaramillo Garrido, A. L. Maroto and PMMD, JCAP 2024.

CONSERVATION OF THE EMT TENSOR

Before solving directly Einstein equations, we can extract some information from the conservation of the EMT.

This information will emphasize the particular characteristics of this theory with respect to GR.

Diff is a symmetry of the whole theory →

- There are 4 gauge degrees of freedom.
- The EMT is automatically conserved
  (equivalent to taking into account the field equation).

TDiff is a symmetry of the whole theory →

- There are 3 gauge degrees of freedom.

Diff invariance is only broken in the dark sector →

- The EMT is still conserved
  (the conservation equation leads to an equation for the degree of freedom that is no longer free).
  - We get a constraint in g.

D. Jaramillo Garrido, A. L. Maroto and PMMD, JHEP 2024.

$$S_m = \int d^4x \left\{ \frac{f_k(g)}{2} g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - f_v(g)V(\psi) \right\}$$

Kinetic domination regime

$$w = \frac{p}{\rho} = \frac{g f_k'}{f_k - g f_k'} .$$

- EoM: there is a conserved shif-symmetric current.
- The conservation of the EMT implies a constraint on g.

Combining the constraint on g with the field equation

$$\Rightarrow \quad \rho = \frac{c_\rho}{(w-1)\sqrt{g}} .$$

- As $w(g)$ and $\rho(g)$, **adiabatic fluid** perturbations: $\delta p = c_s^2 \delta\rho$

$$c_s^2 = -\frac{g f_k (f_k' + 2g f_k'')}{f_k^2 + (2g f_k')^2 - g f_k (5 f_k' + 2g f_k'')} .$$

Potential domination regime

$$T_{\mu\nu} = 2\sqrt{g} f_v' V g_{\mu\nu}$$

- EoM: field at an extremum of the potential $V = V_0$
- The conservation of the EMT implies:
  - $f_v = A\sqrt{g} + B$
  - Or g =constant (**cosmological constant**).

D. Jaramillo Garrido, A. L. Maroto and PMM, JHEP 2024.

INTERESTING KINETIC MODELS

- **Constant equation of state models**

$f_k(g) = C|g|^{↵}$ $w = \frac{↵}{1-↵}$ $c_s^2 = w$

candidate for dark radiation.

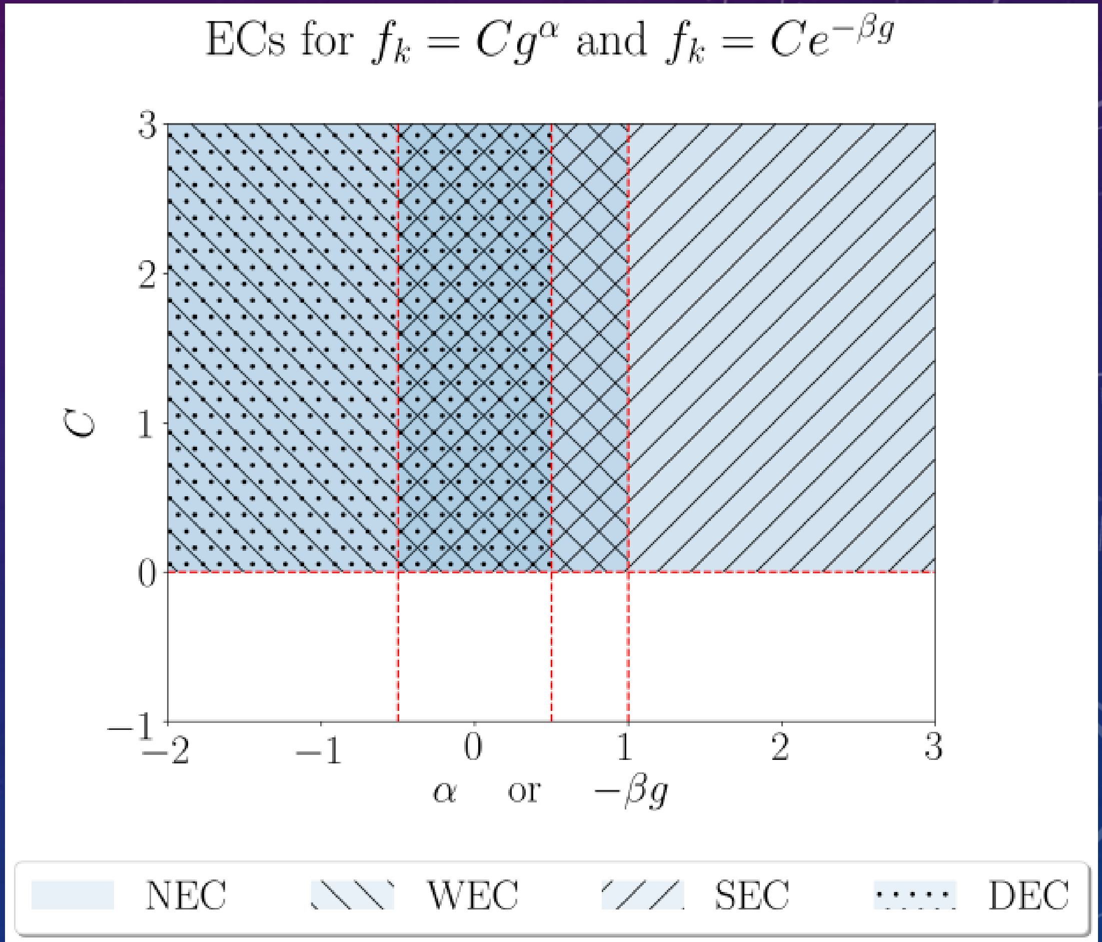
➢ **Candidate for dark matter ($\alpha = 0$).**

$$c_s^2 = 0\,.$$

- **Models with different gravitational domains**

$$f_k(g) = Ce^{-\beta g} \qquad w = -\frac{\beta g}{1+\beta g}$$

$$c_s^2 = \frac{\beta g(1-2\beta g)}{1+5\beta g+2\beta^2 g^2}\,.$$



D. Jaramillo Garrido, A. L. Maroto and PMMD, JHEP 2024.

Cosmological case: A. L. Maroto, JCAP 2024 D. Alonso, J. De Cruz Pérez, A. L. Maroto PRD 2024.

SYMMETRY RESTORATION

Let's introduce a Diff scalar density $\bar{\mu}$ that transforms as $\sqrt{g}$ ➡ $\bar{\mu}\,/\sqrt{g}$ is a Diff scalar.

$$S_{\rm TDiff}[g_{\mu\nu},\Psi] = \int d^4x\, f(g)\,\mathcal{L}\left(g_{\mu\nu},\Psi,\partial_\mu\Psi\right)$$

➡

$$S_{\rm Diff}[g_{\mu\nu},\Psi,\bar{\mu}] = \int d^4x\sqrt{g}\left[\frac{\bar{\mu}}{\sqrt{g}}\,f(g/\bar{\mu}^2)\right]\mathcal{L}\left(g_{\mu\nu},\Psi,\partial_\mu\Psi\right)$$

$\bar{\mu} = 1$ corresponds to the Tdiff coordinate frame.

We introduce a new **vector field** to obtain a local Diff theory (Henneaux and Teitelboim, Phys. Lett. B. 1989)

$$Y = \frac{\bar{\mu}}{\sqrt{g}} = \nabla_\mu T^\mu \qquad H(Y) \equiv Y f(Y^{-2})$$

➡

$$H(Y)\Big|_{\bar{\mu}=1} = \frac{f(g)}{\sqrt{g}}$$

Other choices are posible.

$$S_m = \int d^4x \left\{\frac{f_k(g)}{2} g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - f_v(g)V(\psi)\right\}$$

➡

$$S_m = \int d^4x\sqrt{g}\left[H_k(Y)X - H_V(Y)V(\psi)\right]$$

$$X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi = \frac{1}{2}(\partial\psi)^2$$

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GRAVITY IN THE DIFF FRAME

$$S_{\rm Diff}[g_{\mu\nu},\psi,T^\mu] = S_{\rm EH} + \int d^4x\,\sqrt{g}\,[H_k(Y)X - H_v(Y)V]$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

with

$$T_{\mu\nu} = H_k(Y)\partial_\mu\psi\partial_\nu\psi - [H_k(Y)X - H_v(Y)V]\,g_{\mu\nu} + Y\,[H_k'(Y)X - H_v'(Y)V]\,g_{\mu\nu}\,,$$

- The theory is now Diff invariance ⇒ EMT is now automatically conserved.
- The information from the *g*-constraint is now encoded in the T^μ EoM: $\partial_\nu\,[H_k'(Y)X - H_v'(Y)V] = 0$

Previous results can be easily recovered
$\bar{\mu} = 1 \Rightarrow Y = \nabla_\mu T^\mu = 1/\sqrt{g}$

⇒

- Potential domination regime.
- Kinetic domination regime.

Beyond the simplest cases. ⇒ The *g*-constrain can be easily integrated.

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MODEL SELECTION: THE EFFECTIVE SPEED OF SOUND

The information from the *g*-constraint, encoded in the T^{μ} EoM, leads to $$X = \frac{H_v'(Y)V(\psi) - c_\rho/2}{H_k'(Y)} = X(Y,\psi)$$

➢ The energy density and pressure can be seen as functions of 2 variables: the fluid is not adiabatic in general.

$$\delta p = c_s^2\,\delta\rho + \alpha\,\delta\psi$$

$$c_s^2 = \frac{1}{1 - \frac{B(Y,\psi)}{A(Y,\psi)}}$$

Effective speed of sound of cosmological perturbations:

- Speed of sound in the rest frame.
- Adiabatic speed of sound if the fluid is adiabatic.

$$A(Y,\psi) \equiv H_k\left[V\left(H_k''H_v' - H_k'H_v''\right) - \frac{c_\rho}{2}H_k''\right]$$

$$B(Y,\psi) \equiv 2(H_k')^2\left(H_v'V - \frac{c_\rho}{2}\right) = 2(H_k')^3 X(Y,\psi)$$

- Stability implies $\frac{B(Y,\psi)}{A(Y,\psi)} < 1$
- To avoid superluminal perturbations $\frac{B(Y,\psi)}{A(Y,\psi)} \leq 0$
- For shift-symmetric models these conditions are simplified to: $\frac{H_k'^2}{H_k H_k''} < \frac{1}{2}$ and $H_k'' < 0.$
- There are other adiabatic models (find the complete list in the reference).

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DISCUSSION AND FURTHER COMMENTS

- Breaking Diff invariance to TDiff in the matter sector, we can still consider minimal coupling to gravity and obtain different kinds of interesting models.

  - Shift-symmetric models are adiabatic. Choosing different coupling functions one can get from dark matter to unified dark sector model.
  - Restoring the symmetry provides us with a complementary framework to investigate the gravitational properties of the model.
  - Coupling functions of interest can be found requiring stability of fluid perturbations.

- Note that the total TDiff EMT is conserved. If we had 2 TDiff fields, they would interact as a result of the symmetry breaking.

  - **Don’t miss Diego Tessainer’s talk this afternoon!**

REFERENCES

D. Jaramillo-Garrido, A. L. Maroto, PMM, arxiv: 2307.14861, JHEP 2024.

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