Resonance absorption of gravitational waves and modified gravity in the forthcoming LISA project

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EREP 2024, University of Coimbra - **July 23th, 2024**

Presentation Outline

Stochastic gravitational wave backgrounds

It is customary to express the fraction of the energy density SGWB in terms of the critical energy density at present:

$$\Omega_{\mathrm{i}} = \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{c}}} \ . \tag{1}$$

where $\rho_c = 3H_0^2/(8\pi G)$, is the critical energy density. Similarly, it is convenient to define the spectral energy density as follows:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{c}}} \frac{d\rho_{\mathrm{GW}}(f)}{d\ln f} \ , \tag{2}$$

Some sources of SGWBs:

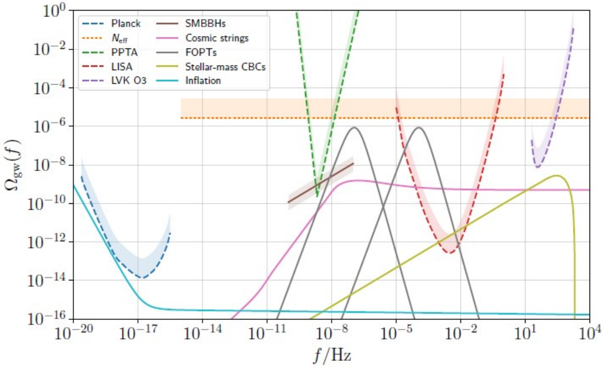


Figure 1: Overview of potential GWB signals across the frequency spectrum (from Renzini, A.I.; Goncharov, B.; Jenkins, A.C.; Meyers, P.M. Galaxies 2022, 10, 34.)

The NANOGrav collaboration

Pulsar Timing Arrays (NANOGrav collaboration) could have detected inspiralling (SMBBHs) with a maximum density of approximately $\Omega_{\mathrm{GW}}(f) \simeq 10^{-8}$ for a frequency around 10^{-7} Hz. This signal exhibits a frequency dependence of the form:

$$\Omega_{\mathrm{GW}}^{\mathrm{CB}}(f) = \Omega_{\mathrm{GW}}^{\mathrm{CB}}(f_{\mathrm{ref}}) \left(\frac{f}{f_{\mathrm{ref}}} \right)^{2/3} , \quad (3)$$

where $\Omega_{\mathrm{GW}}^{\mathrm{CB}}(f_{\mathrm{ref}})$ is the fraction of energy density calculated at a reference frequency.

Perturbation of the orbital elements

A general formalism for this phenomenon was derived by Blas and Jenkins in 2022, PRD 105, 064021 (2022). These authors considered a Langevin equation from which a Fokker-Planck equation was derived for an average over different realizations of the stochastic gravitational background:

$$\dot{P} = \frac{3P^2\gamma}{2\pi}\left[\frac{e\sin\psi\mathcal{F}_r}{1+e\cos\psi}+\mathcal{F}_\theta\right] , \tag{4}$$

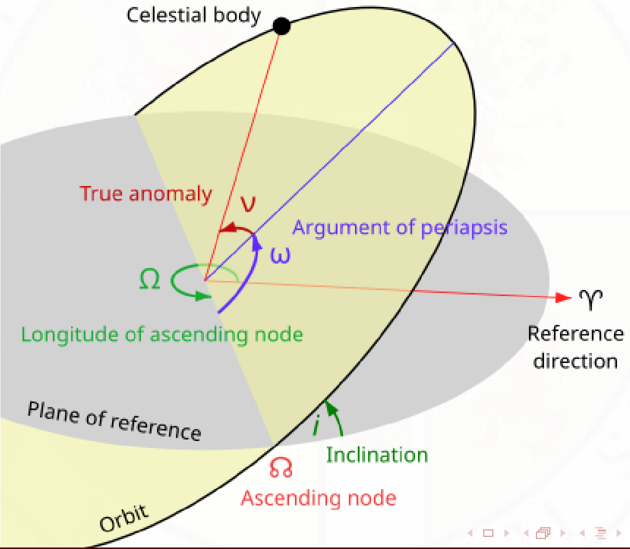
$$\dot{e} = \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5\mathcal{F}_\theta}{2\pi e(1+e\cos\psi)^2} , \tag{5}$$

$$\dot{I} = \frac{P\gamma^3\cos\theta\mathcal{F}_\ell}{2\pi(1+e\cos\psi)^2} , \tag{6}$$

$$\dot{\Omega} = \frac{\tan\theta}{\sin I}\dot{I} , \tag{7}$$

$$\dot{\omega} = \frac{P\gamma^3}{2\pi e}\left[\frac{(2+e\cos\psi)\sin\psi\mathcal{F}_\theta}{(1+e\cos\psi)^2} - \frac{\cos\psi\mathcal{F}_r}{1+e\cos\psi}\right] - \cos I\,\dot{\Omega} . \tag{8}$$

Definition of orbital elements:



Drift of orbital parameters

For the deterministic drifts of the period, the eccentricity, the orbital inclination, the argument of the perihelion and the longitude of the ascending node we have:

$$\frac{dP}{dt} = \frac{3P^2}{160} H_0^2 \left(-79\,\Omega_1 + 288\,\Omega_2 - 27\,\Omega_3\right) \;, \tag{9}$$

$$\frac{de}{dt} = \frac{9P}{80e} H_0^2 \left(3\,\Omega_1 - \Omega_3\right) \;, \tag{10}$$

$$\frac{d\iota}{dt} = \frac{3P}{80} H_0^2\, \Omega_2 \, \cot\iota \;, \tag{11}$$

$$\frac{d\omega}{dt} = \frac{\tan\omega}{e^2} \, \frac{H_0^2\, P}{160} \left(-29\,\Omega_1 - 9\,\Omega_3\right) \;, \tag{12}$$

$$\frac{d\Omega}{dt} = 0 \;, \tag{13}$$

where $H_0 \simeq 70$km/s/Mpc is the value of the Hubble parameter at present, and Ω_k is the energy fraction of the stochastic wave background at k times the orbital frequency.

Drift for the Solar system

Given the background originating from the merging of SMBBHs throughout the history of the Universe, which could have been detected by the NANOGrav collaboration, we have the following fraction of energy density:

$$\Omega(f) \simeq 10^{-8.56} \times \left(\frac{10^{-8}}{f}\right)^{2/3} . \tag{14}$$

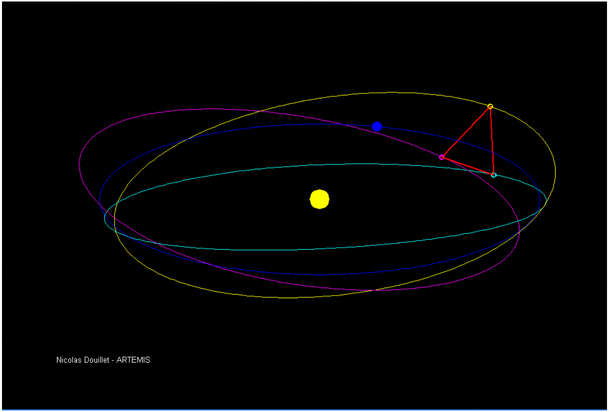
For an SMBBHs background the results are tiny:

- The semi-major axis of the Earth's orbit increases at a rate 1.98×10^{-17} m per year.
- The orbital eccentricity changes at a rate 1.86×10^{-28} per year.
- The orbital inclination also increases at the rate 1.86×10^{-28} per year.

The orbital decay corresponding to the emission of GWs is much larger: 3.50×10^{-13} m per year.

SGWs absorption by LISA

The LISA1, LISA2 and LISA3 spacecraft configuration is an approximate, equilateral triangle. This is also known as the cartwheel configuration.



Secular perturbations on the LISA's arms

The perturbation in orbital frame coordinates can now be calculated:

$$\begin{aligned}
\delta o_x(t) &= \delta a\,(\cos\varepsilon - e) - a\delta e(t) - a\sin\varepsilon(t)\delta\varepsilon(t)\,, \\
\delta o_y(t) &= \sin\varepsilon(t)\left(\delta a(t)\sqrt{1-e^2} - \frac{ae\delta e(t)}{\sqrt{1-e^2}}\right) + a\sqrt{1-e^2}\cos\varepsilon\delta\varepsilon(t)\,, \\
\delta o_z(t) &= 0\,.
\end{aligned} \tag{15}$$

where the perturbation on the eccentric anomaly depends on the secular trends of the semi-major axis and the eccentricity of the LISA spacecraft on their orbits:

$$\delta\varepsilon = \frac{\sin\varepsilon}{1-e\cos\varepsilon}\,\delta e - 3\pi\frac{t}{Pa}\frac{1}{1-e\cos\varepsilon}\,\delta a\,. \tag{16}$$

Absorption of SGWs by LISA:

To evaluate the upper and lower limits of the perturbation of the spacecraft distance to one another, we have taken into account that the stochastic gravitational wave background detected by the NANOGrav collaboration corresponds to an energy density in the reference frequency of 1 year^{-1} given by:

Fraction of energy density for the SGWB (NANOGrav):

$$\Omega_{\mathrm{gw}} = 9.3^{+5.8}_{-4.0} \times 10^{-9} \,, \tag{17}$$

Results for the LISA1-LISA2 arm:

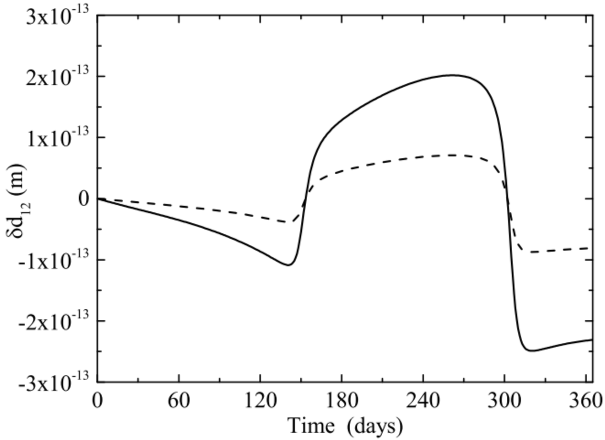


Figure 3: Perturbation on the distance among LISA1 and LISA2 spacecraft.

Other perturbationds (Modified Gravity):

There is a plethora of modified gravity models:

- Brans-Dicke theory with a gravitational parameter mediated by a scalar field.
- $f(R)$ models.
- Phenomenological models.
- Among the countless theories on the literature...

We consider the extra radial force:

$$\mathcal{R} = \kappa \, H_0 \, \dot{r} \ , \tag{18}$$

where κ is a constant, H_0 is the Hubble constant and $\dot{r}$ is the radial velocity (Iorio, L.: On the anomalous increase of the lunar eccentricity. In: Gravitational Waves and Experimental Gravity, pp. 255 to 258 (2011)).

Why this phenomenological proposal of a "fifth-force"?:

This force was proposed to provide an idea for the observed anomalous increase of the Earth-Moon distance.

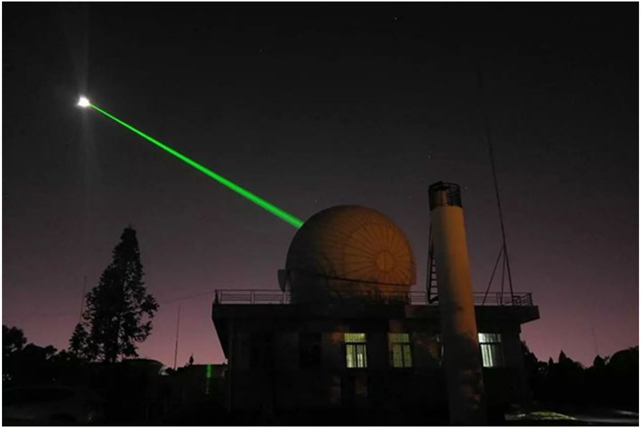


Figure 4: Lunar Laser Ranging from a telescope in China.

Anomalous increase of the Moon's orbital eccentricity:

Iorio's phenomenological "fifth-force" predicts:

$$\frac{de}{dt} = (9 \pm 3) \times 10^{-12} \text{ per year.}$$

This discrepancy has not been possible with models of the Lunar core and tidal forces.

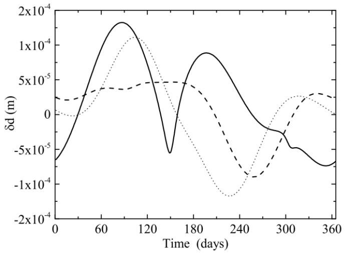


Figure 5: Perturbations on the distances among the LISA spacecraft.

Conclusions

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- We find that the effects are totally negligible for the planets, although it could reach the threshold of detectability with better experimental designs.
- The chance of finding the effect of modified gravity on the distances, among the LISA cartwheel spacecraft, is much larger than the detection of an SGWB.

A gravitational wave detector?
A Pandora's box for MG?