Recent Results in Conformal Killing Gravity (CKG)

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What is CKG?

- A 3^{rd} order metric theory of gravitation due to Harada (2023) with field equations (HFEs): $\tilde{G}_{(ab;c)} = \tilde{T}_{(ab;c)}$ where $\tilde{G}_{ab} \equiv G_{ab} - \frac{1}{6} G g_{ab}$ and $\tilde{T}_{ab} \equiv T_{ab} - \frac{1}{6} T g_{ab}$.

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where C_{ab} is a conformal Killing tensor. Hence the name conformal Killing gravity.

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- Actually $K_{ab} \equiv \tilde{G}_{ab} - \tilde{T}_{ab}$ is a Killing tensor and $C_{ab} = K_{ab} + \frac{1}{2} K g_{ab}$.

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Yet another theory – why study CKG?

- Harada considered the evolution of the scale factor $a(t)$ of the Friedmann-Robertson-Walker-Lemaître (FRWL) metric:

$$ds^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right).$$

- He obtained a third order ODE which has the first integral:

$$2\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a} + \frac{2k}{a^2} = \frac{4\pi G}{3}(5\rho + 3p) + \frac{\Lambda}{3}.$$

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- He showed that even in the case of a matter-dominated universe ($p = 0$) with $\Lambda = 0$ there could be a transition from deceleration to accelerating expansion.

- Thus removing the need to assume the existence of dark energy.

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Static black hole solutions

- Harada(2023) obtained a metric for a static spherically symmetric vacuum field in CKG:

$$\mathrm{d}s^2 = \mathrm{e}^{a(r)}\mathrm{d}t^2 - \left(\mathrm{e}^{-a(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right) \qquad (1)$$

where $\mathrm{e}^{a(r)} = 1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2 - \frac{1}{5}\lambda r^4$ where m, Λ and λ are integration constants. Generalised Schwarzschild-Kottler solution.

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- However (1) is not the most general form for a spherically symmetric static metric which is

$$\mathrm{d}s^2 = \mathrm{e}^{a(r)}\mathrm{d}t^2 - \left(\mathrm{e}^{b(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right). \qquad (2)$$

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The general case

- The HFEs for the metric(2) with $T_{ab} = 0$ lead to 2 ODEs of third order for $a(r)$ and $b(r)$. However, writing $b = 2f - a$ one may deduce a 2nd order ODE $rf'' - 2rf'^2 - f' = 0$. Hence $\mathrm{e}^b = \mathrm{e}^{-a}/(c + dr^2)$ where c and d are constants.

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- Eliminating b the remaining field equation may be linearised by writing $y = \mathrm{e}^a$ to yield

$$(c+dr^2)r^3y''' - (2c-dr^2)r^2y'' - (2c+dr^2)ry' + 8cy = 8. \quad (3)$$

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- If $d = 0$, c may be set to 1 by a rescaling of the t-coordinate. Similarly if $c = 0$, d may be set to 1. In either case the ODE becomes homogeneous and easy to solve. Otherwise we may set $c = \pm 1$. For Lorentzian signature $c + dr^2 > 0$.

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- For $d = 0$ generalised Schwarzschild-Kottler and Reissner-Nordström solutions are obtained.
- For $c = 0$ there are strange vacuum & electrovac solutions (Barnes 2023):

$$y = e^a = \lambda - \Lambda r^2/3 + m \log r - 1/(2r^2) + q^2/(4r^4), \quad e^b = e^{-a}/r^2.$$

- Physical Interpretation ???!!!

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The solutions 2

- For $c > 0$ and $d \neq 0$, Barnes(2023) obtained solutions involving power series with radius of convergence $1/\sqrt{|d|}$.
  $y = e^a = 1+(1+2dr^2)q^2/r^2-2mp_1(r)/r-\lambda p_2(r)r^4/5-\Lambda r^2/3$
  $p_1(r) = 1 + dr^2/2 - d^2r^4 \ldots$
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- If, in addition, $q = \Lambda = 0$ we have the static Einstein Universe which is a vacuum solution in CKG!

The solutions 3

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- In these the scale factor a satisfies $\dot{a}^2 - \alpha a^4 - \frac{\Lambda}{3} a^2 + k$. The solutions in general involve Jacobi elliptic functions, but are elementary if $\alpha = 0$ or $\frac{\Lambda^2}{9} + 4k\alpha = 0$.

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- Open question: does CKG admit spherically symmetric matter distributions emitting gravitational waves?

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pp-waves in CKG

- Defining pp-waves as fields which admit a covariantly constant null bivector: $W_{ab;c} = 0$ where $W_{ab} = p_{[a}k_{b]}$ with $p_a p^a = -1$ and $k_a k^a = 0$, it follows that $k_{a;b} = 0$ and the metric may be written as
$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v + 2H(u,x,y)\mathrm{d}u^2 - \mathrm{d}x^2 - \mathrm{d}y^2.$$

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- The CKG solutions differ from the GR ones only by a term $+c(x^2 + y^2)$ added to H where c is an arbitrary constant leading to an extra constant circularly polarized mode in addition to those in GR.

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- The CKG solutions differ from the GR ones only by a term $+c(x^2 + y^2)$ added to H where c is an arbitrary constant leading to an extra constant circularly polarized mode in addition to those in GR.
- if $H = c(x^2 + y^2)$ the metric is a non-flat vacuum conformally flat field in CKG.

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- Thus different matter sources may produce the same metric. The metric does not uniquely determine the energy-momentum tensor.
- Thus, if $\tilde{T}_{ab}$ & $\tilde{T}'_{ab}$ differ by the Killing tensor $\lambda g_{ab}/3$, (λ constant) then T_{ab} & T'_{ab} differ by a dark energy term λg_{ab}. Hence dark energy does not gravitate in CKG.

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Multiple Matter Sources 2

- If a metric admits symmetries generated by Killing vectors $\xi_I^a, I = 1 \ldots N$, then $K_{ab} = kg_{ab} + \sum_{I=1}^N \sum_{J=1}^N k_{IJ} \xi_{I(a} \xi_{|J|b)}$, where k and the k_{IJ} are constants, is a Killing tensor.

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- All known exact solutions in CKG admit Killing vectors: Static spherically symmetric 4KVs, pp-waves at least 1KV plane waves at least 5KVs, FRWL metrics 6KVs Einstein Universe 7KVs, Minkowski, de Sitter & ADS 10 KVs. Thus there are a plethora of possible sources in these cases.

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- Even with no symmetries there is the ubiquitous dark energy ambiguity.

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Conclusions

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- Thank you for listening if you have been! Questions?

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