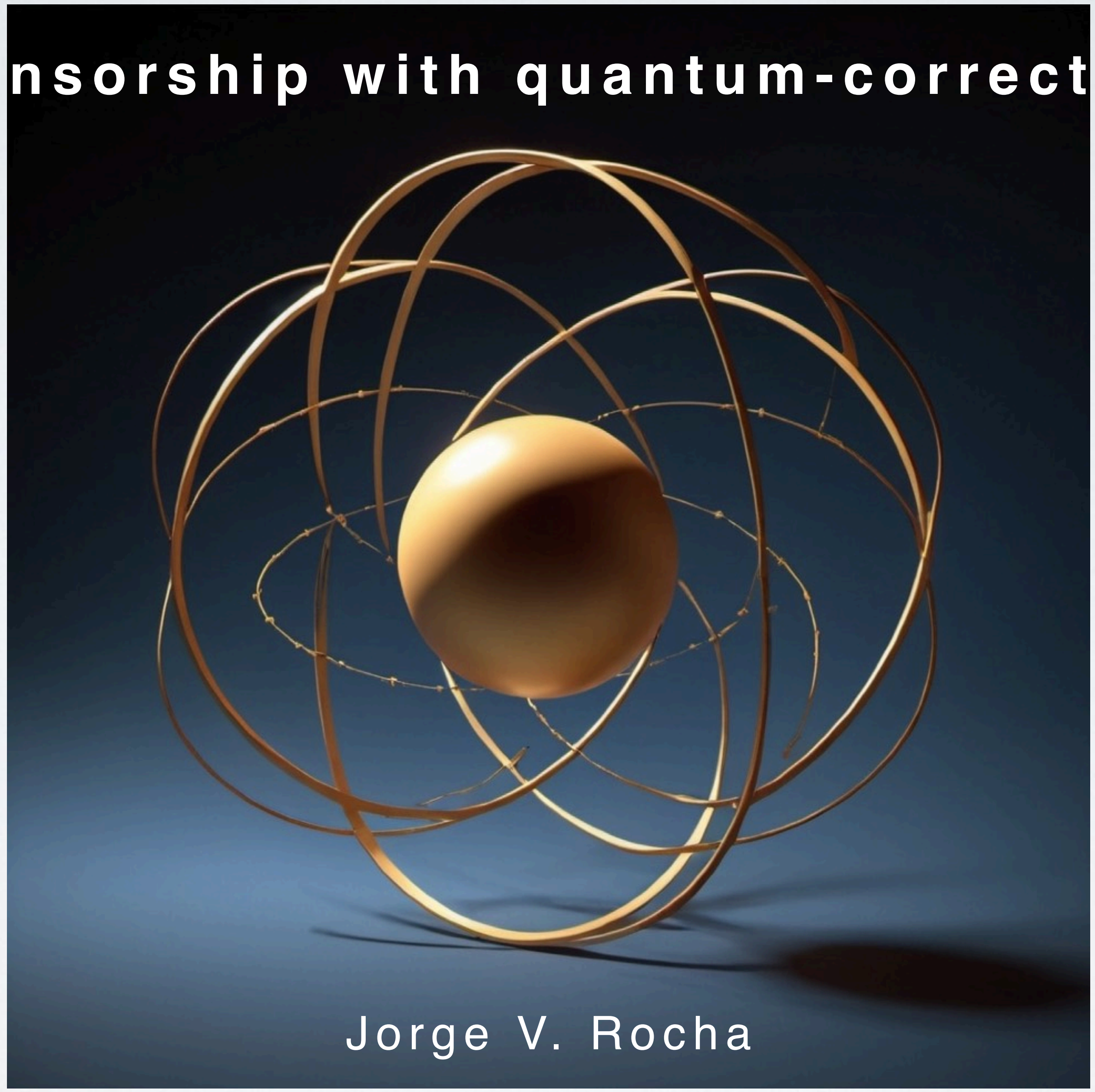
Weak cosmic censorship with quantum-corrected black holes



Jorge V. Rocha

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Outline

1. Cosmic Censorship Conjecture (CCC) and its status
2. The quantum BTZ black hole
3. Testing CCC with quantum-corrected BHs

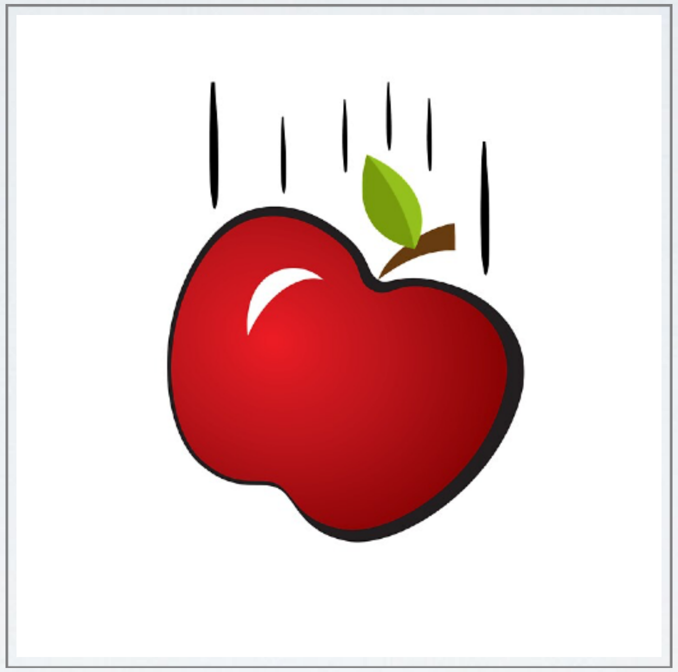
Mainly based on:

❖ **Antonia M. Frassino**, **JVR** and **Andrea P. Sanna**,
JHEP 07 (2024) 226 [arXiv:2405.04597]

The Weak Cosmic Censorship Conjecture

Determinism in classical physics

1. Newtonian physics is deterministic.



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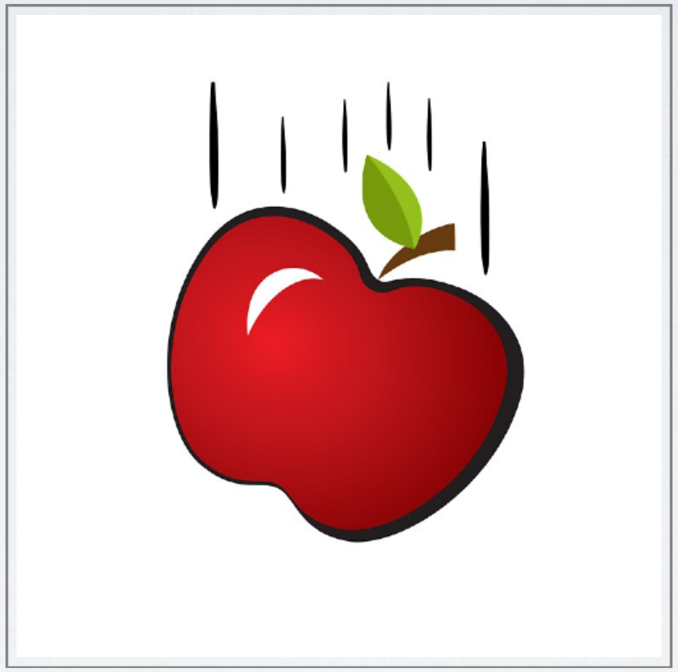

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However, determinism does not carry over automatically…

Gravitational collapse can lead to curvature singularities at which point GR loses its predictive power.

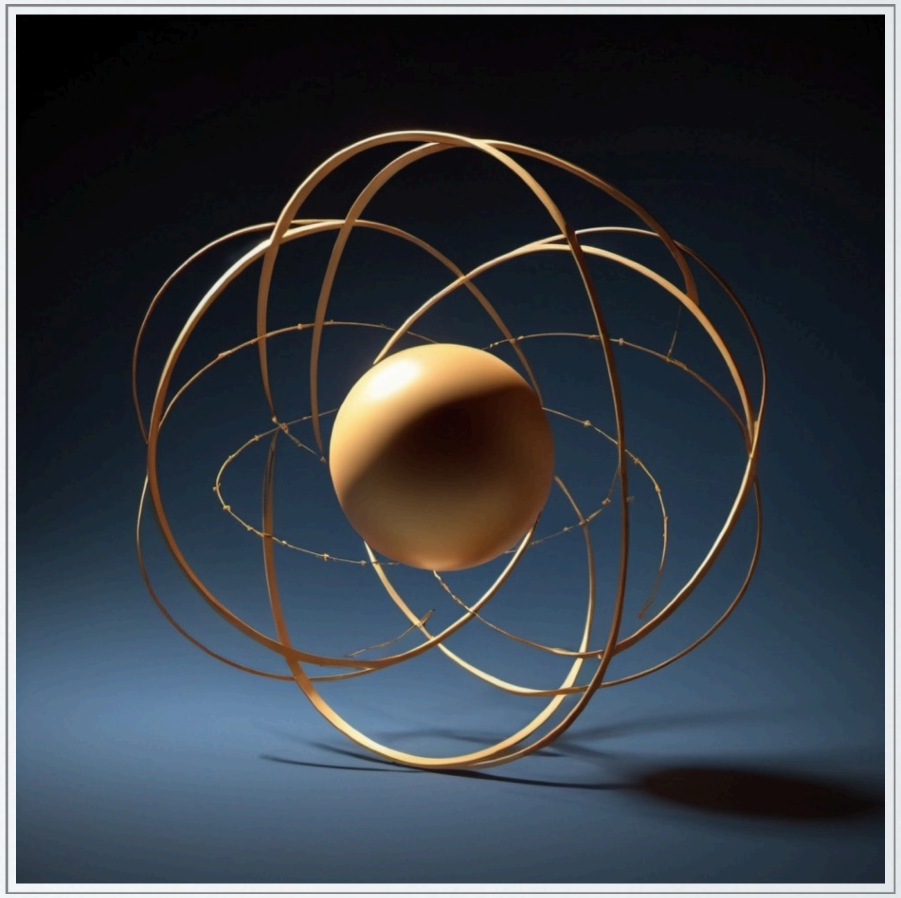
Singularities in GR

1. Formally infinite curvature. Signals breakdown of theory.

2. Commonly 'encountered' in the interior of BHs. (Exception: cosmological singularities)

   If so, damage is contained, because information cannot propagate across the event horizon to the exterior.

3. But if some process could lead to the formation of a ***naked singularity*** — a curvature singularity not cloaked by a horizon — we could be in trouble!



Weak Cosmic Censorship Consjecture (wCCC)

1. Formulated by Penrose in 1969 to protect the deterministic picture of classical evolution in gravitational collapse: [Penrose (1969)]

Assuming physically reasonable matter and genericity of initial conditions, a regular configuration cannot develop naked singularities under gravitational collapse with the classical equations of motion, unless they are veiled by event horizons.

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2. This is a respectfully old conjecture, and remains so.
Despite numerous attempts to (dis)prove it.

3. It is a cornerstone of major mathematical developments in GR.

4. Note: Weak Cosmic Censorship ≠ Strong Cosmic Censorship.

Tests of the Weak Cosmic Censorship Conjecture

- Collapse of shells of null dust satisfies Penrose inequalities — which supports wCCC.

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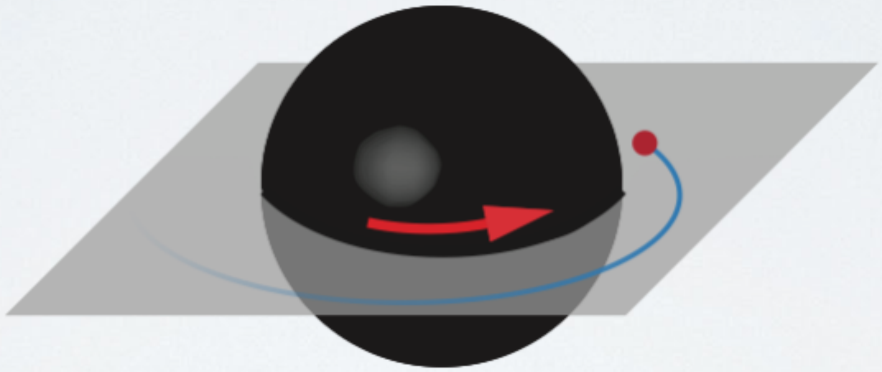
- Wald's thought experiment: attempts to destroy the event horizon of extremal rotating and/or charged BHs with test particles are unsuccessful. [Wald (1974)]



Point particles with dangerously high spin/charge bounce before reaching the black hole, instead of disrupting the horizon.

Tests of the Weak Cosmic Censorship Conjecture

Very long list of variations:

- Different spacetime dimensions.
- Inclusion of cosmological constant.
- Test fields.
- Rotating thin shells.
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All attempts either:

- failed.
- succeeded, but approximation were later understood to be invalid.
- managed to produce naked singularities but scenarios require exotic matter or infinite fine-tuning.

The Quantum BTZ Black Hole

Semi-classical gravity

- In the absence of a complete theory of quantum gravity, it is common to resort to solving the ***semi-classical Einstein equations***:

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi\, G \left\langle T_{\mu\nu}(g_{\alpha\beta}) \right\rangle$$

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renormalized stress tensor of quantum matter fields

- Still, computing $\left\langle T_{\mu\nu} \right\rangle$ and solving the coupled system of equations:

metric + quantum correlators

is often a ***daunting task***.

- Typical approach: assume backreaction to be small and work perturbatively.

Braneworld gravity

In very special cases, alternative non-perturbative approaches are possible.

- One such approach is offered by 'holography':

[Maldacena (1998)]
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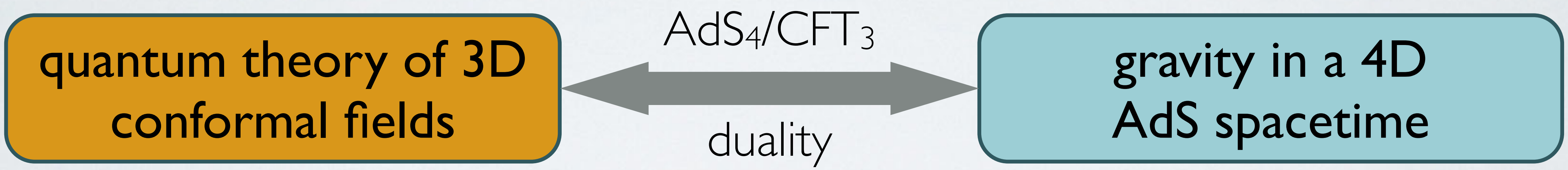

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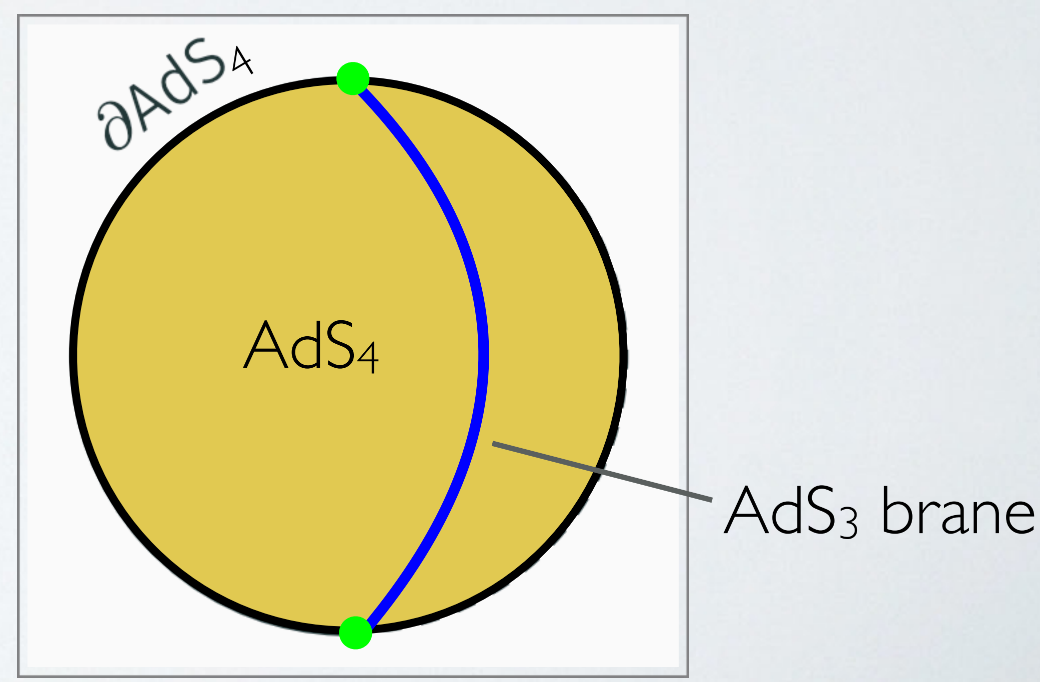
- A consistent description of a Conformal Field Theory in a dynamical spacetime is captured by the Karch-Randall model:

Gravity is recovered on a lower dimensional AdS_3 braneworld supporting also quantum fields.

[Karch, Randall (2001)]



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This braneworld scenario allows the construction of quantum black holes:

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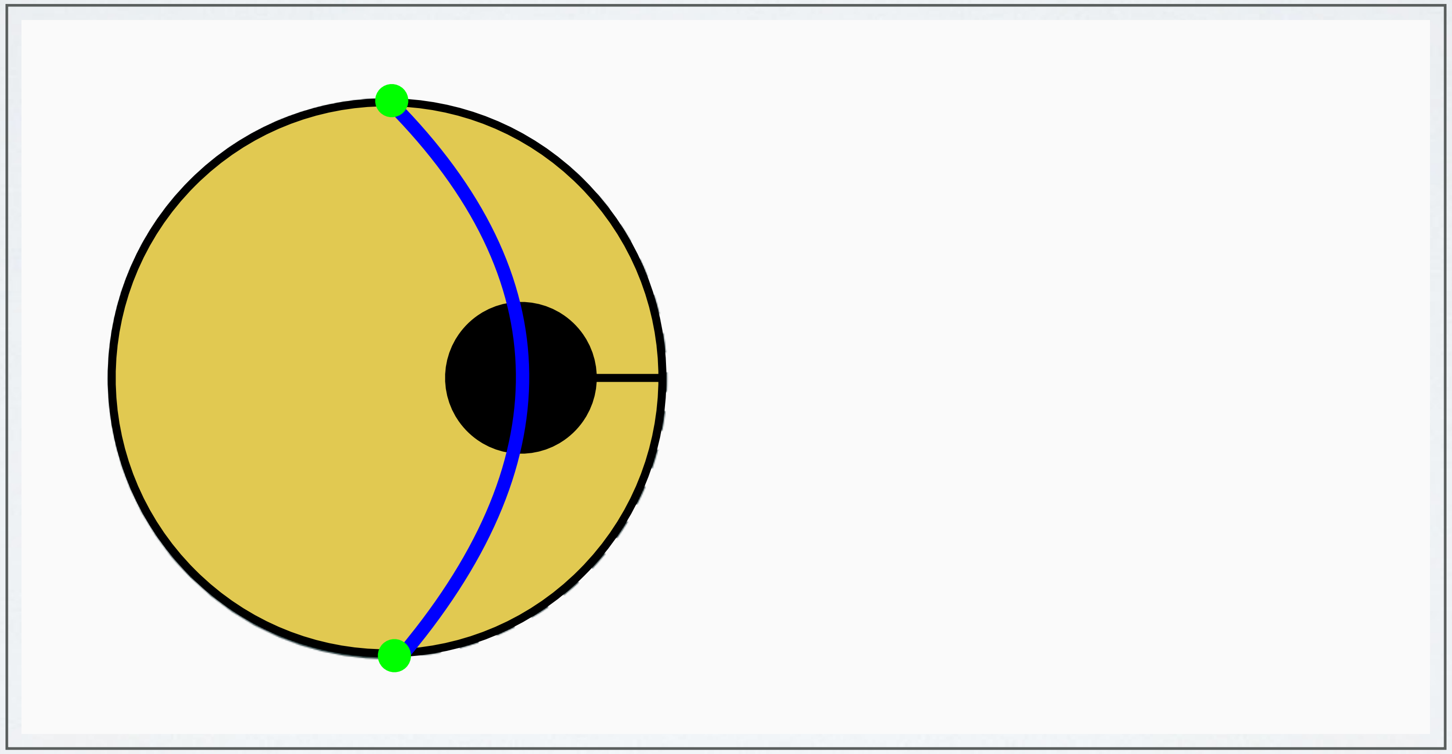

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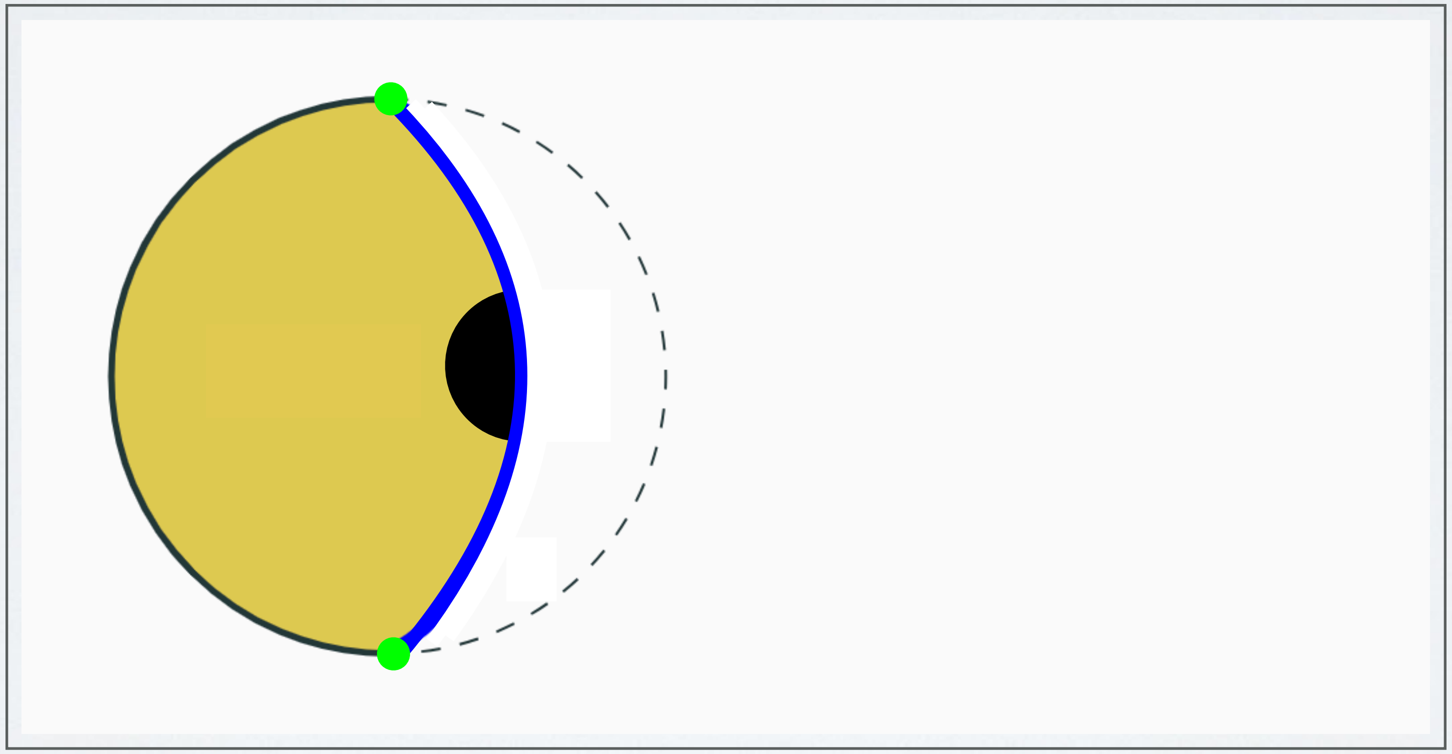

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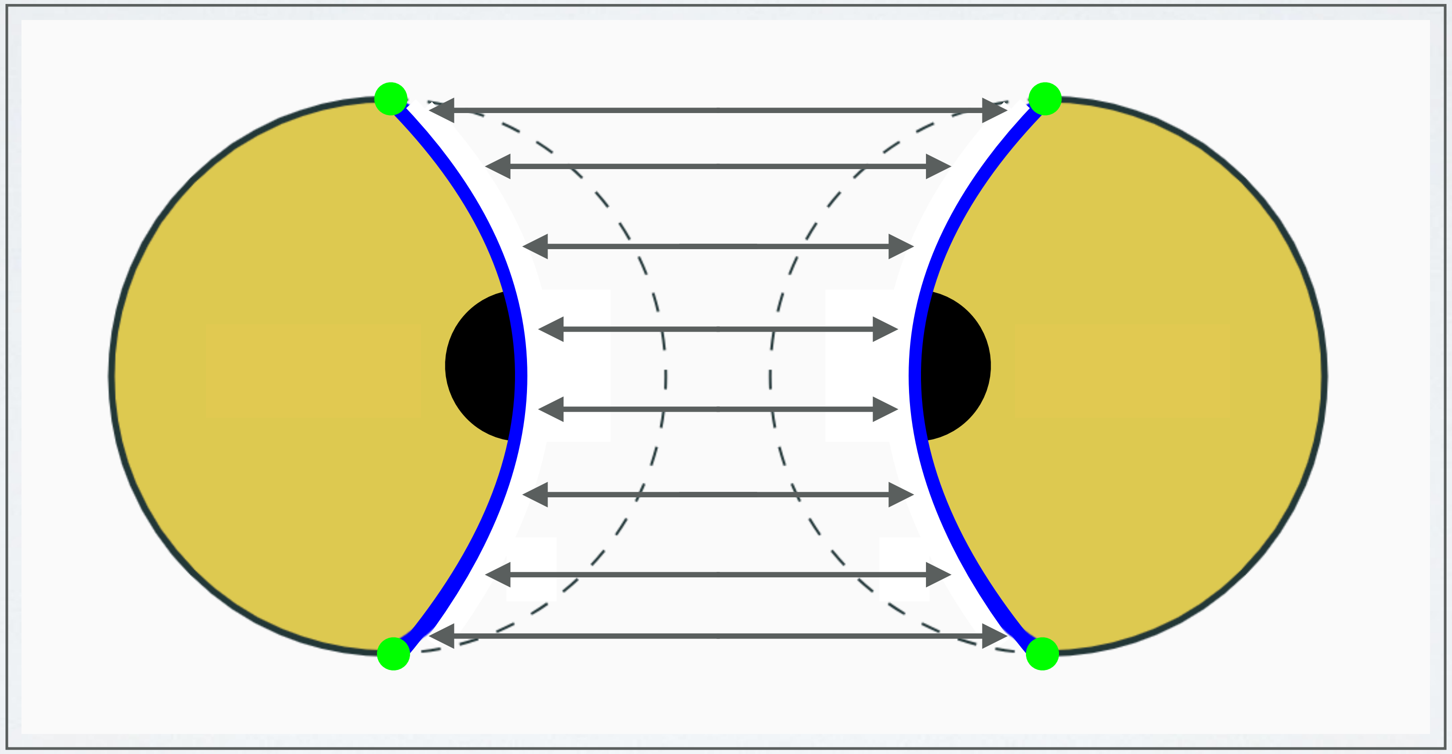

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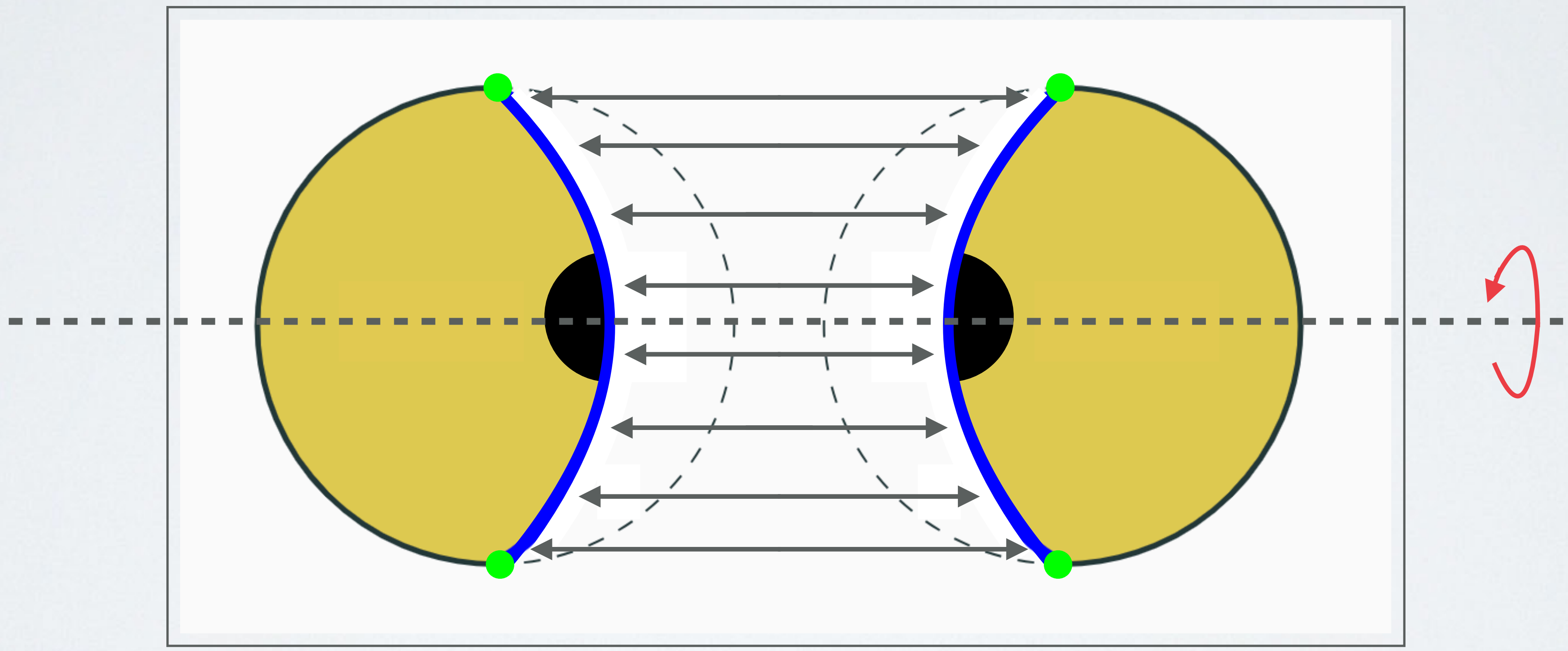

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The quantum BTZ black hole (qBTZ)

The resulting geometry on the 3D brane is a quantum backreacted version of the (classical) BTZ black hole: [Bañados, Teitelboim, Zanelli (1992)]

$$\mathrm{d}s^2 = g_{tt}\,\mathrm{d}t^2 + g_{\phi\phi}\,\mathrm{d}\phi^2 + 2g_{t\phi}\,\mathrm{d}t\,\mathrm{d}\phi + g_{rr}\,\mathrm{d}r^2$$

$$g_{tt} = -\frac{8\sqrt{1-\tilde{a}^2}\,\nu\,\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^3\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}} + \frac{16\tilde{a}^2-4\left(\tilde{a}^2+1\right)\kappa x_1^2}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2} - \frac{r^2}{\ell_3^2},$$

$$g_{\phi\phi} = r^2 - \frac{8\tilde{a}^2\sqrt{1-\tilde{a}^2}\nu\ell_3^3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^3\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}},$$

$$g_{t\phi} = -\frac{4\tilde{a}\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)}{\left(3-\tilde{a}^2-\kappa x_1^2\right)^2}\left(1+\frac{2\sqrt{1-\tilde{a}^2}\nu\ell_3}{\left(3-\tilde{a}^2-\kappa x_1^2\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}\right),$$

$$g^{rr} = H(r) = \frac{r^2}{\ell_3^2} - \frac{8\left(1-\tilde{a}^2\right)^{3/2}\nu\ell_3\left(\tilde{a}^2-\kappa x_1^2+1\right)\sqrt{\frac{4\tilde{a}^2\ell_3^2\left(\kappa x_1^2-2\right)}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}+r^2}}{r^2\left(3-\tilde{a}^2-\kappa x_1^2\right)^3}$$
$$+\frac{16\tilde{a}^2\ell_3^2\left(\tilde{a}^2-\kappa x_1^2+1\right)^2}{r^2\left(\tilde{a}^2+\kappa x_1^2-3\right)^4} + \frac{4\left[\left(\tilde{a}^2+1\right)\kappa x_1^2-4\tilde{a}^2\right]}{\left(\tilde{a}^2+\kappa x_1^2-3\right)^2}.$$

[Emparan, Frassino, Way (2020)]

Comments on qBTZ

1. The qBTZ metric is parametrized by $\{\nu\,,\ \ell_3\,,\ x_1\,,\ \tilde{a}\,,\ \kappa\}$.

- ν controls the quantum backreaction. Classical BTZ is recovered when $\nu \to 0$.
- ℓ_3 is the AdS_3 length. (Inverse of the cosmological constant on the brane.)
- x_1 and $\tilde{a}$ are related to the physical mass and angular momentum of the BH:

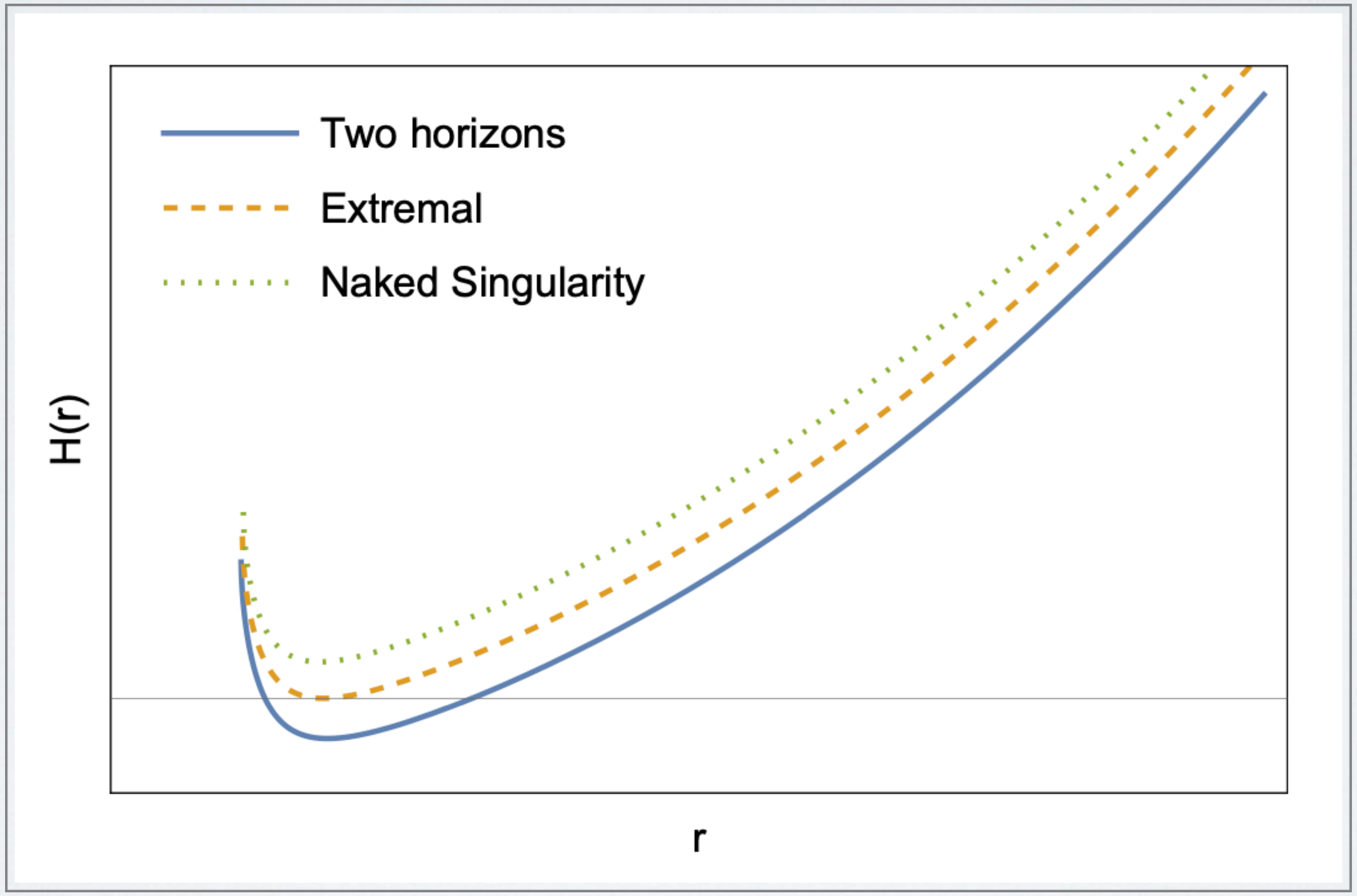
$$M = \frac{1}{2\mathcal{G}_3}\frac{-\kappa x_1^2 + \tilde{a}^2(4-\kappa x_1^2)}{(3-\kappa x_1^2-\tilde{a}^2)^2}\,, \qquad J = \frac{\ell_3}{\mathcal{G}_3}\frac{\tilde{a}(1-\kappa x_1^2+\tilde{a}^2)}{(3-\kappa x_1^2-\tilde{a}^2)^2}\,.$$

- $\kappa = \pm 1\,, 0$ is a discrete parameter.

Comments on qBTZ

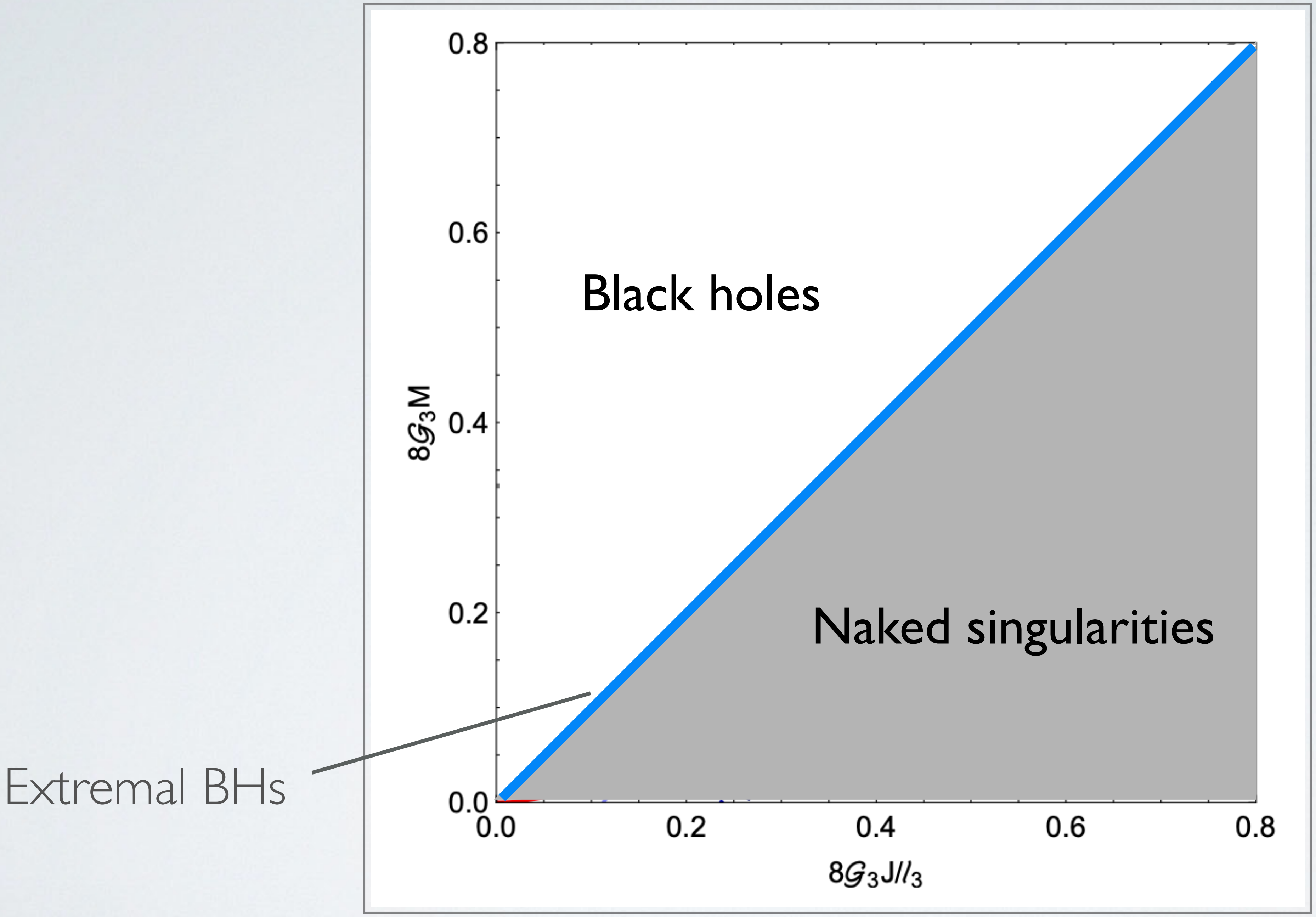
2. The qBTZ geometry still features a singularity at $r = 0$.

3. Event horizon is located at the largest root of $g^{rr} = H(r)$, if it exists:

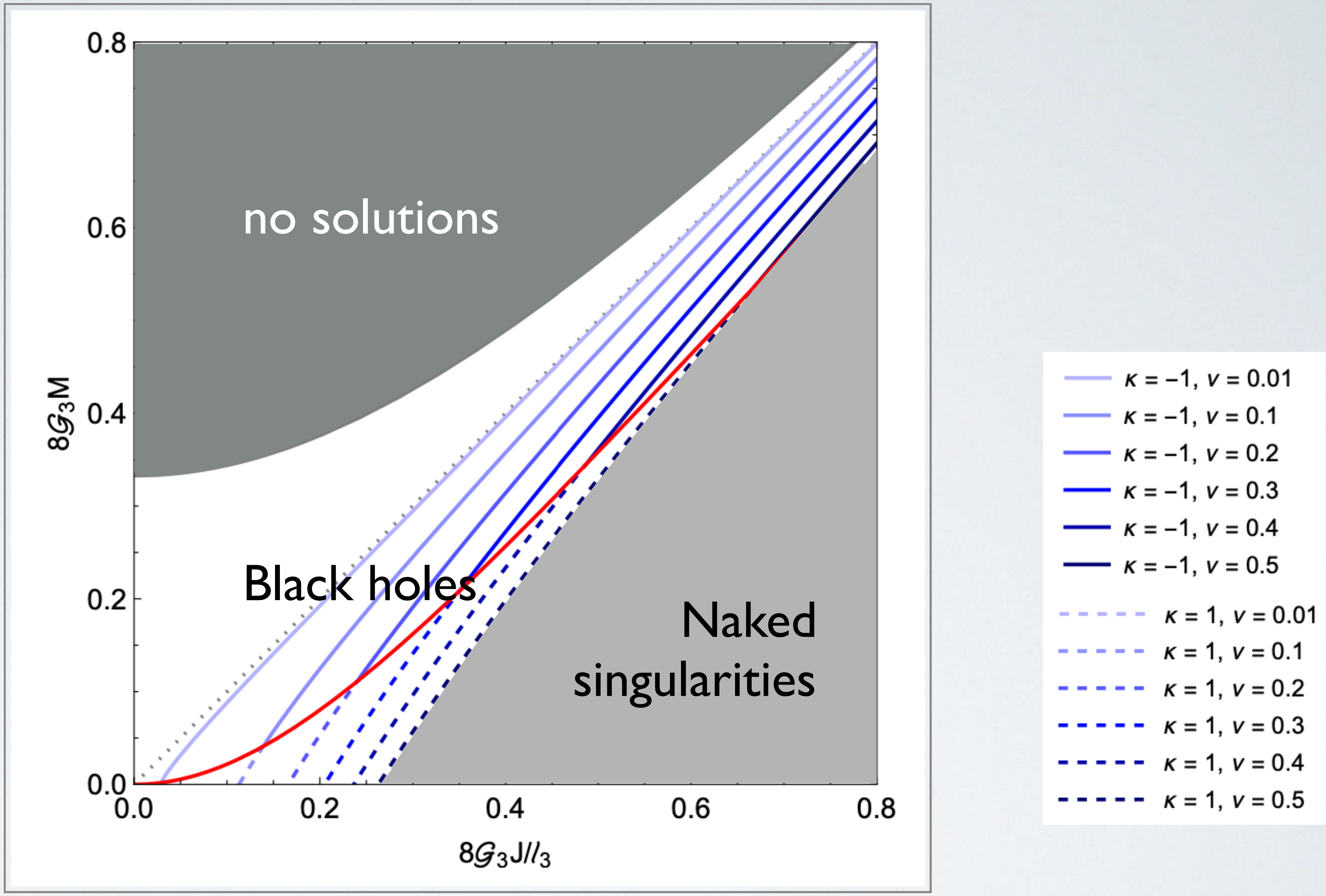


Testing Cosmic Censorship with Quantum Black Holes

Mass vs. spin diagram of the classical BTZ spacetime



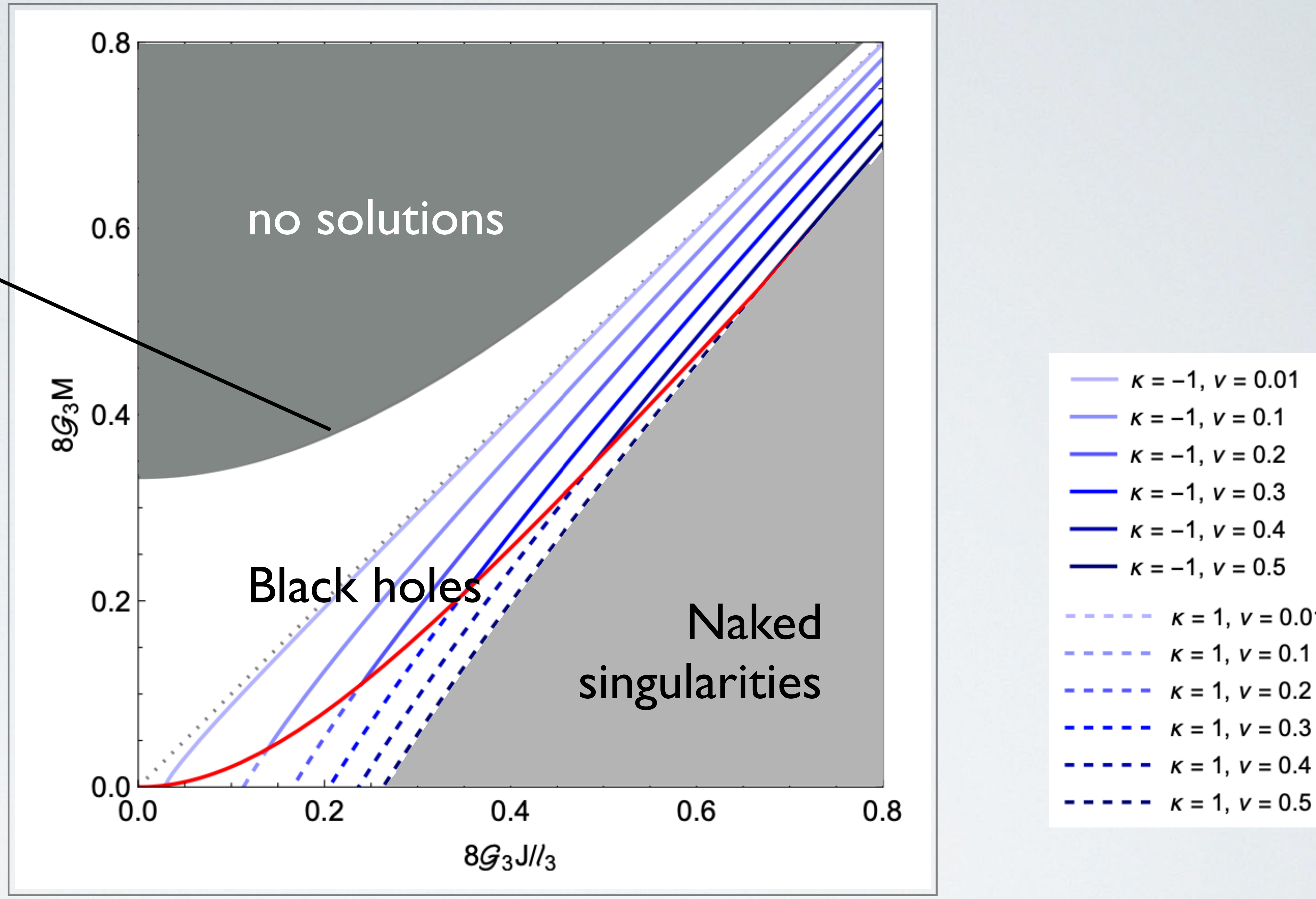
Mass vs. spin diagram of the qBTZ spacetime



- Curve of extremal qBTZ spacetimes depends on the backreaction parameter ν.
- Region of parameter space occupied by BHs grows with backreaction.

Mass vs. spin diagram of the qBTZ spacetime

Upper bound
on mass of BHs
(from holographic derivation)



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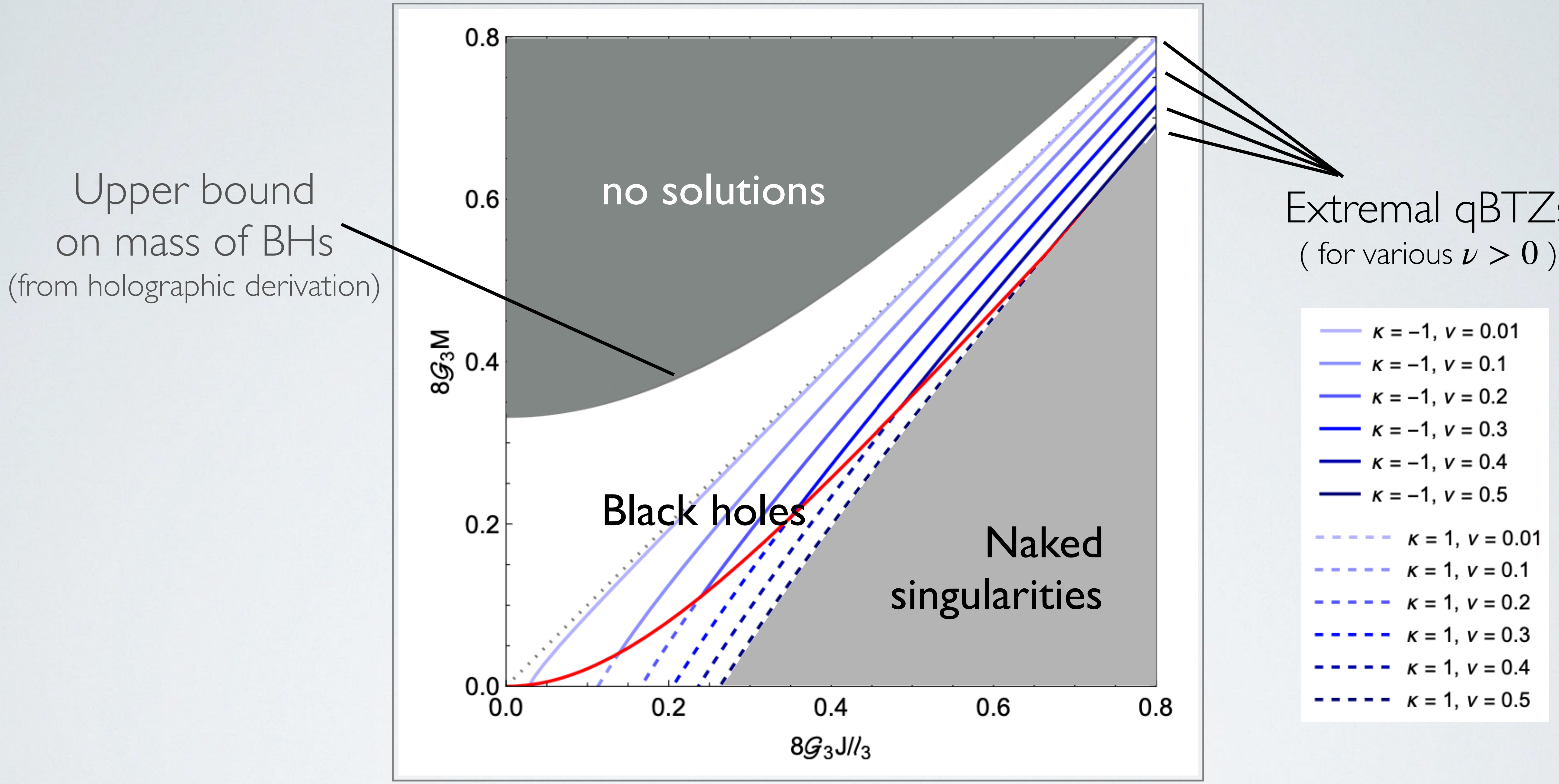

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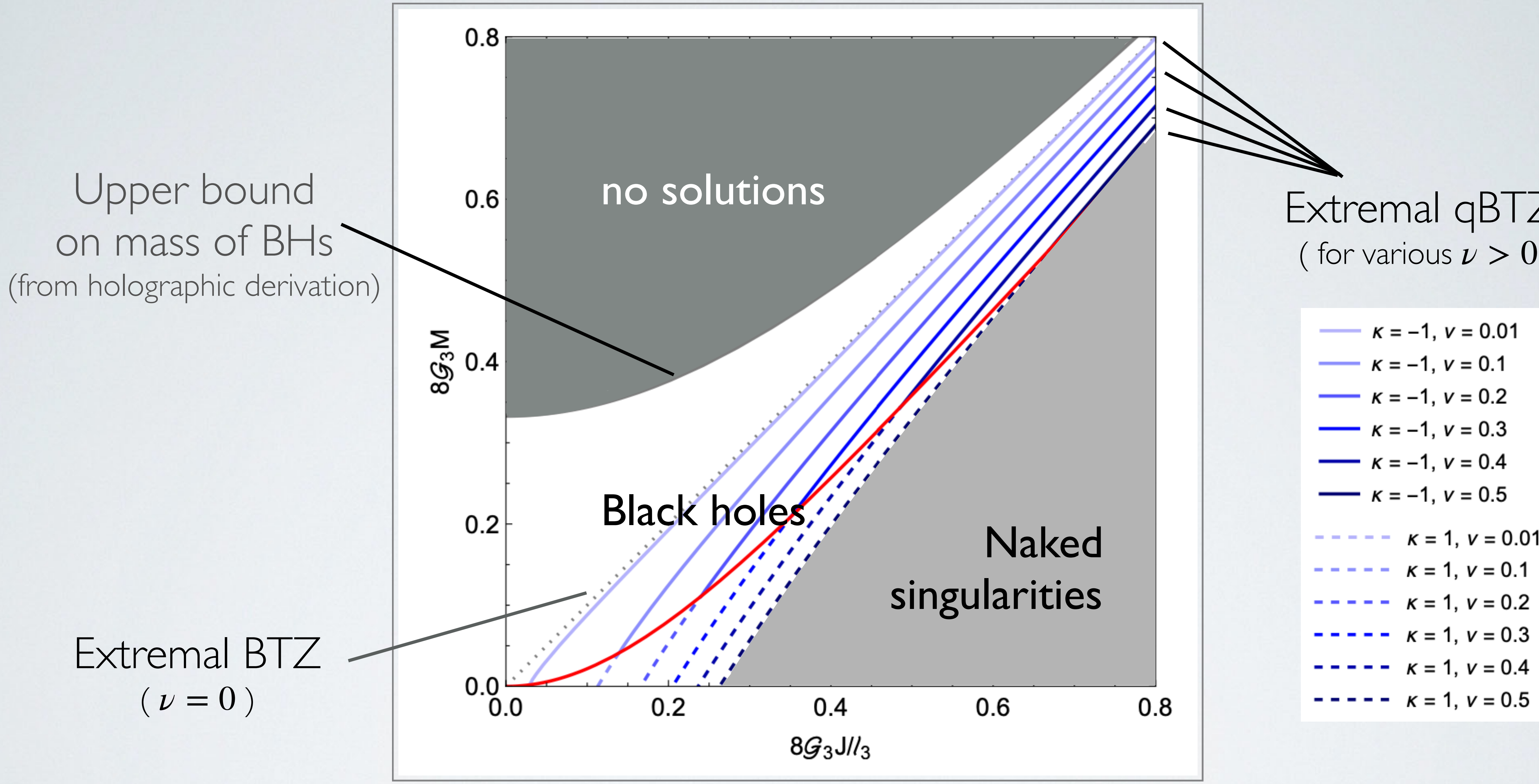

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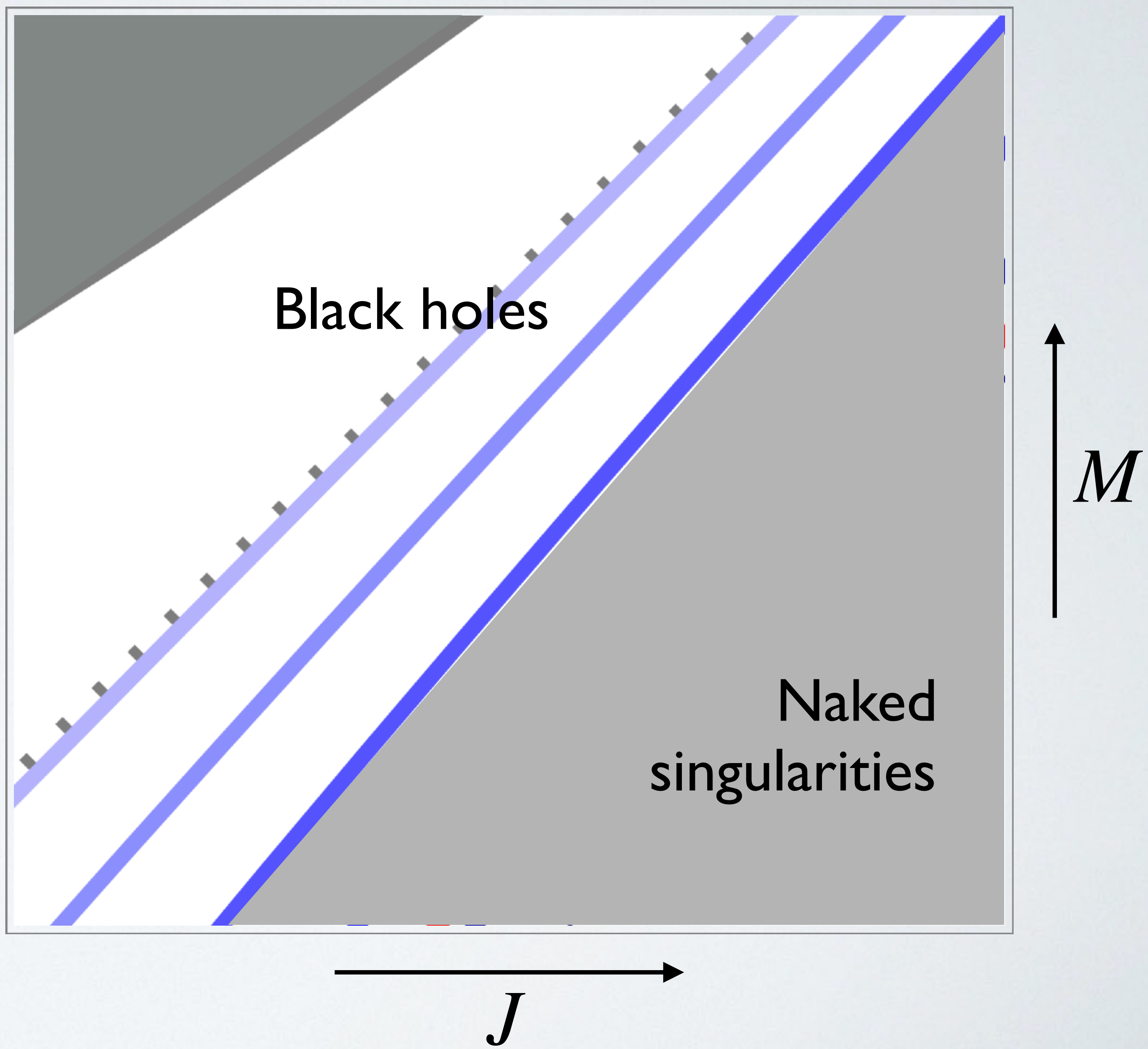
Strategy ‘à la’ Wald

- Work in the test particle approximation, with the absorbed particle imparting linear perturbations on the mass and angular momentum of the BH.



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   Fix ℓ_3 , x_1 , $\tilde{a}$.
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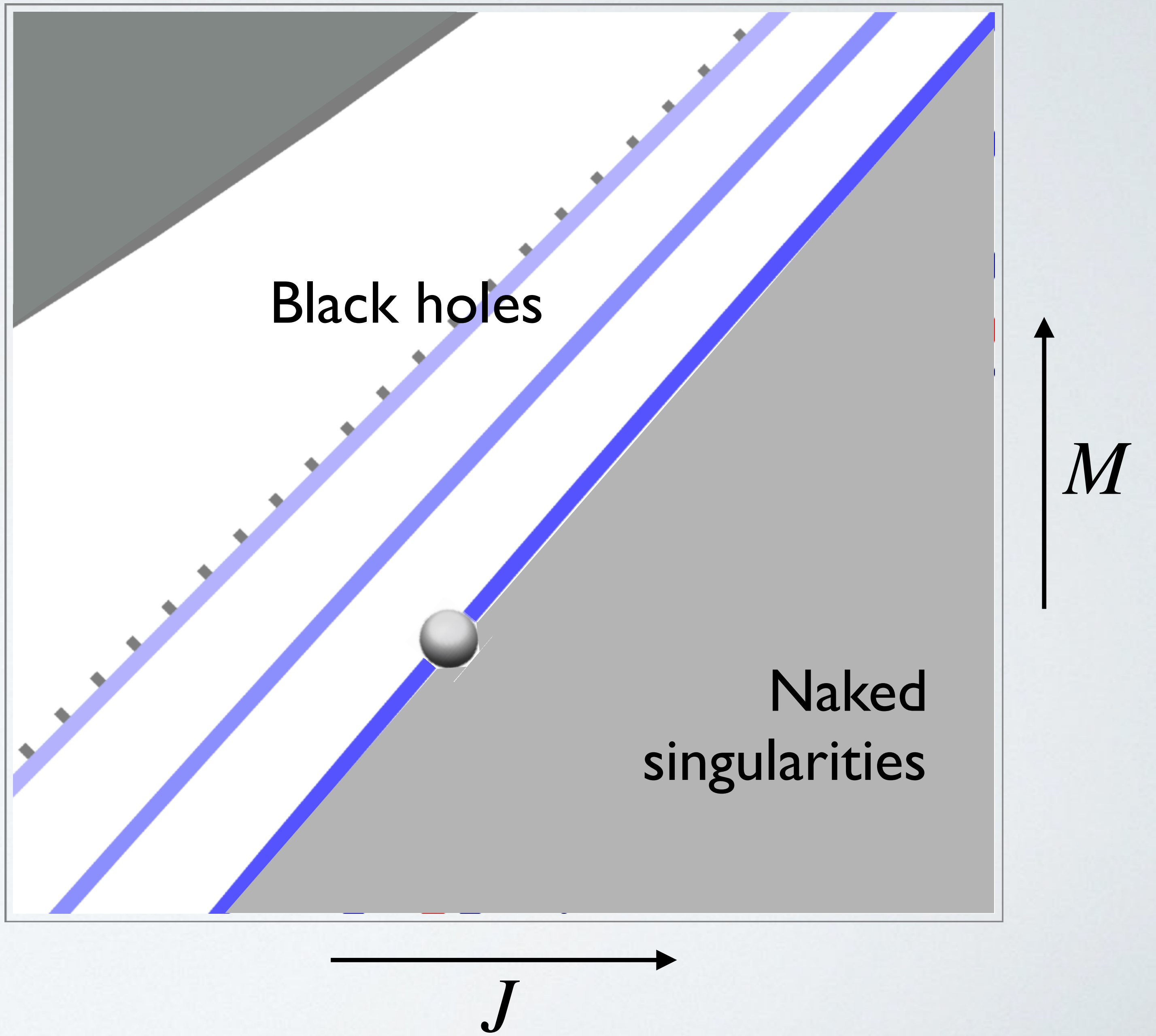
2. Translate to physical quantities of the initial BH, M_0 and J_0.



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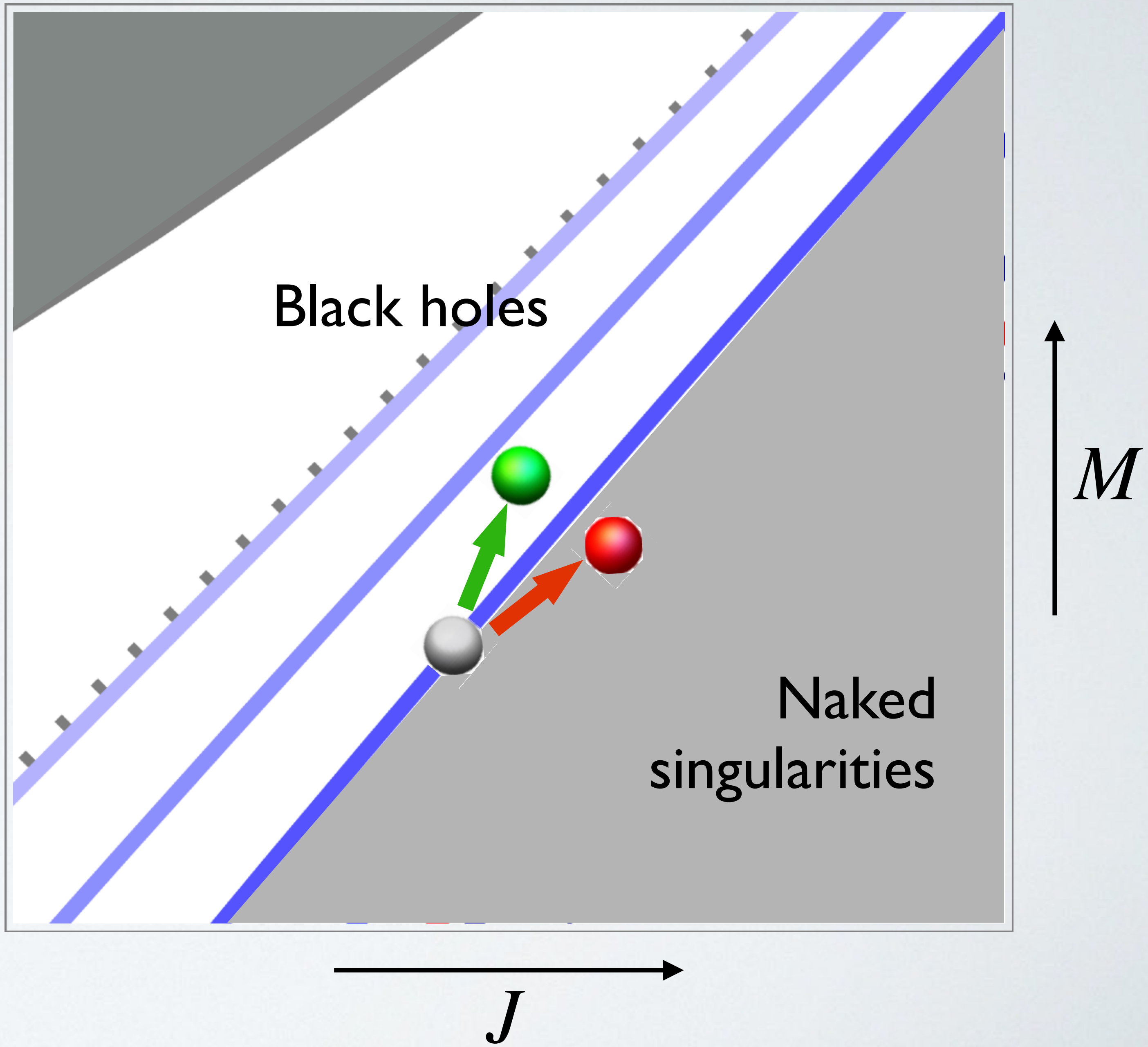
2. Translate to physical quantities of the initial BH, M_0 and J_0.
3. Variations δM and δJ can also be converted to variations of the BH parameters δx_1 and $\delta \tilde{a}$.
4. Finally, determine sign of $\delta H_{min} = [\ldots]\delta M + [\ldots]\delta J$.



Testing the resilience of the event horizon of the qBTZ

Since $\delta H_{min} = [\ldots]\delta M + [\ldots]\delta J$, we still need a relation between δJ and δM .

Only particles with sufficiently low angular momentum are absorbed:

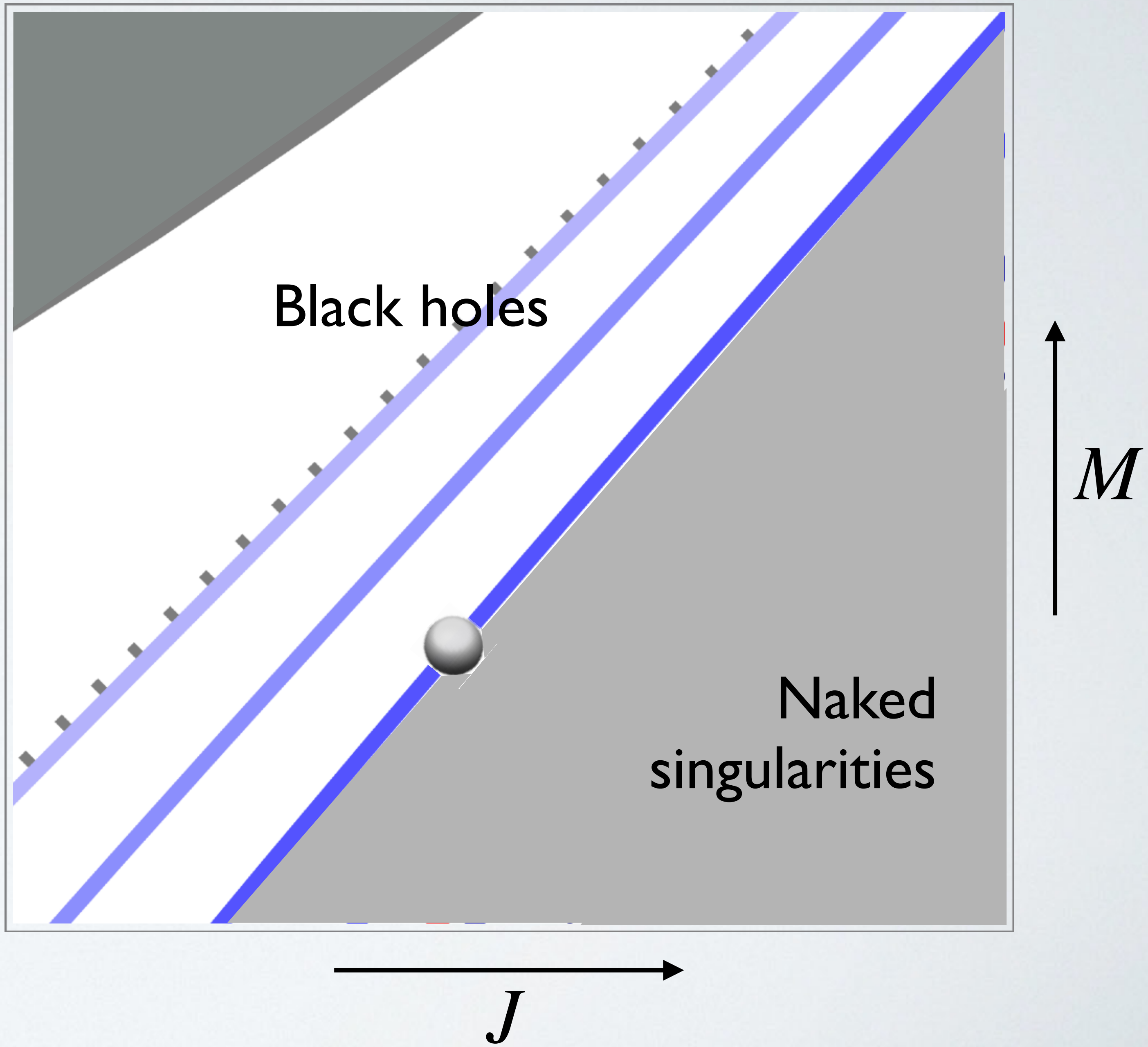
$$\frac{\delta J}{\delta M} < \ell_{max}$$

In the most threatening case,

$$\delta J = \ell_{max} \cdot \delta M$$

such perturbation yields, exactly,

$$\delta H_{min} = 0$$



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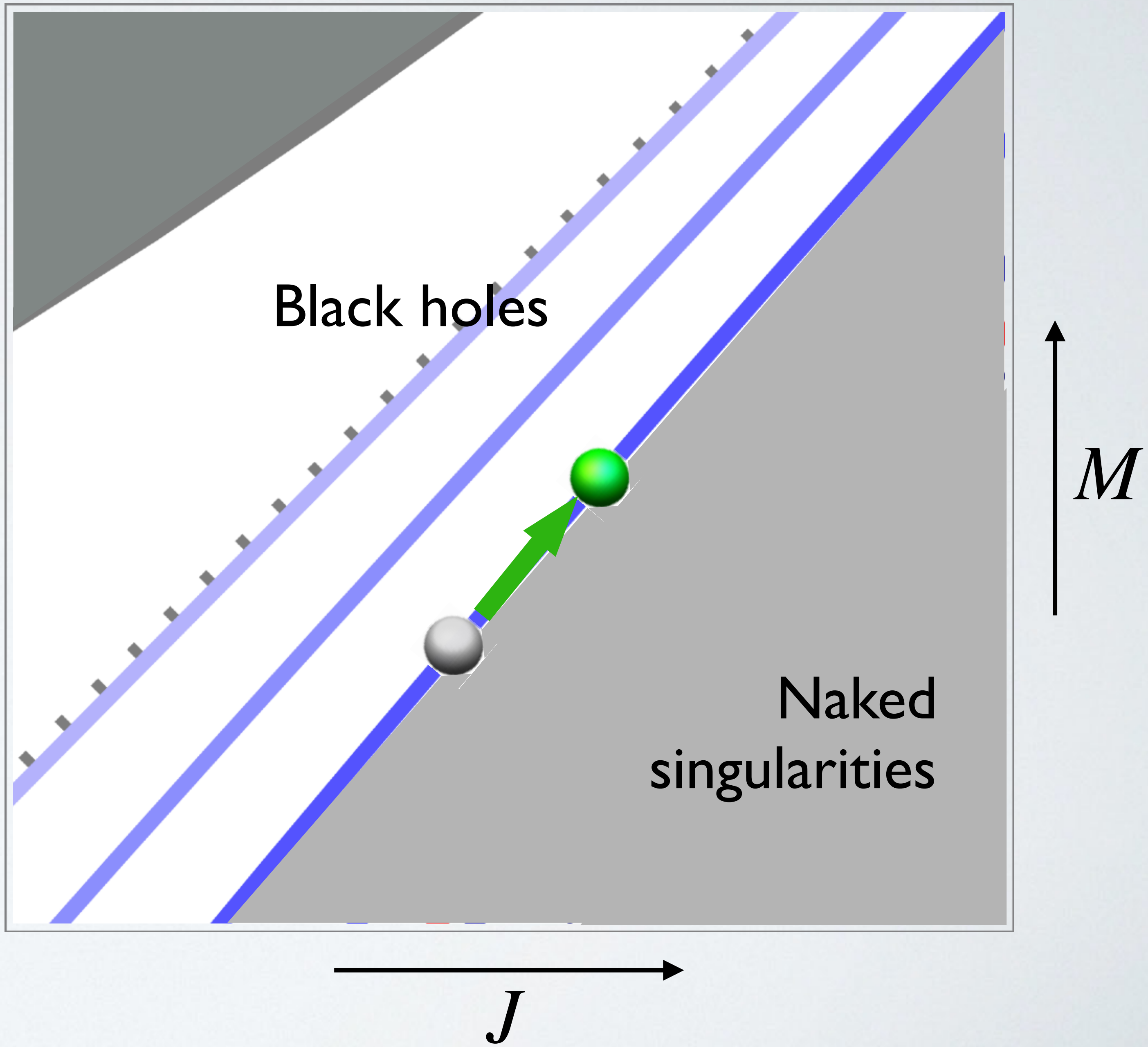
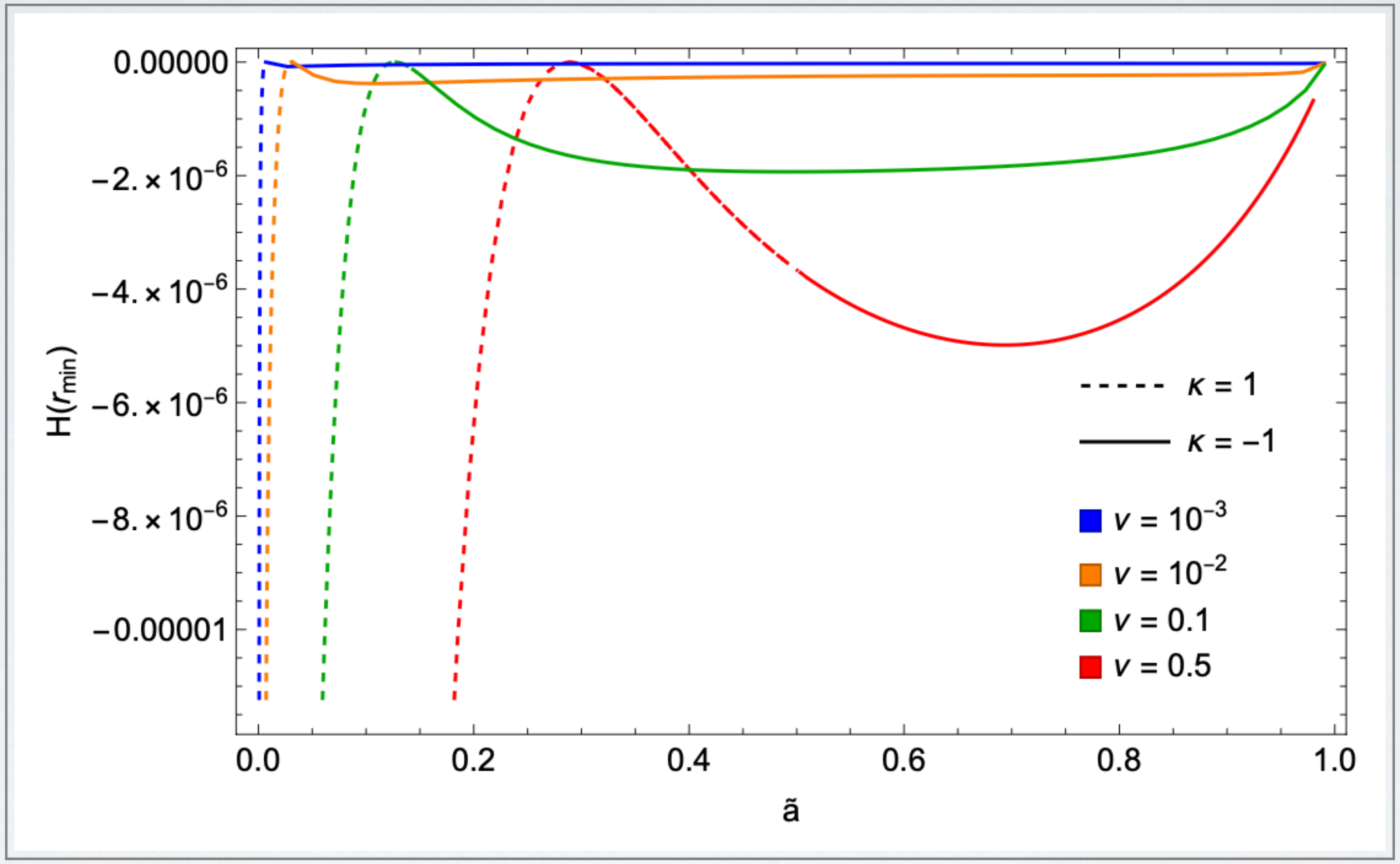
i.e., BH remains extremal.

Event horizon is <u>not</u> destroyed.



Beyond the linear approximation

Numerical assessment, still under the test particle assumption, but considering particles of ***finite*** mass ($\delta M / M \not\ll 1$):



- Cosmic censorship is respected.
- Quantum backreaction typically strengthens cosmic censorship.

Conclusions

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- Weak Cosmic Censorship is key to self-consistency of classical gravity.
- Potential violations of the weak cosmic censorship conjecture would represent an opportunity to learn about quantum gravity.

Take-home message:

Wald's old thought experiment applied to quantum-corrected black holes — including the effects of quantum backreaction in an exact manner — endorses the weak cosmic censorship conjecture.

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Thank you.