Radiation reaction in magnetized black holes

Can the tail term be ignored?

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TÉCNICO LISBOA

Introduction

Magnetized black holes

- **Why? — Motivation**
    - Theoretical interest — production of cosmic rays
    - Astrophysical relevance

R. D. Blandford and R. L. Znajek, MNRAS 179, 433 (1977)
R. Penrose, Riv. Nuovo Cim. 1, 252 (1969)
R. A. Daly, Astrophys. J. 886, 37 (2019)
M. Liska *et al*, MNRAS 487, 550–561 (2019)

Magnetized black holes

- **What? — Setup**
  - Schwarzschild BH + asymptotically uniform test magnetic field (no backreaction)

$$A_\mu = \frac{B_0}{2} r^2 \sin^2\theta \, \delta^\phi_\mu$$

  - Calculate the effects of **radiation reaction** on **circular orbit** motion

- **Charged particle motion -** circular orbits in the equatorial plane
  - Broke symmetry $\hat{z} \to -\hat{z}$ ➡ 2 distinct circular orbit configurations

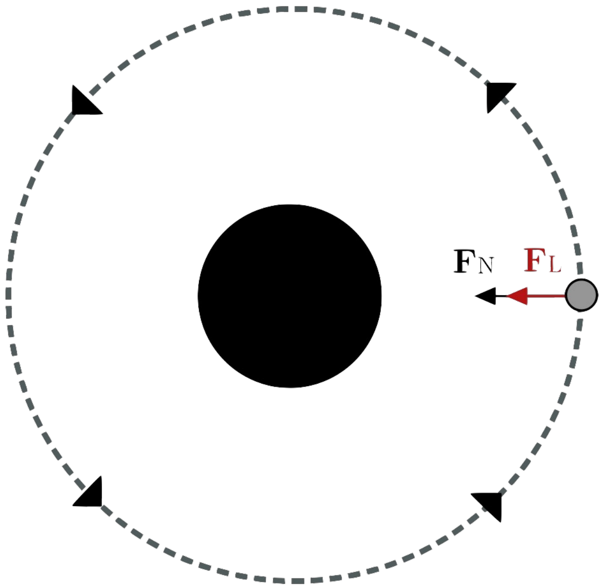
R. M. Wald, PRD **10**, 1680 (1974)
V. P. Frolov and A. A. Schoom, PRD **82**, 084034 (2010)
M. Kolos *et al*, CQG **32**, 165009 (2015)

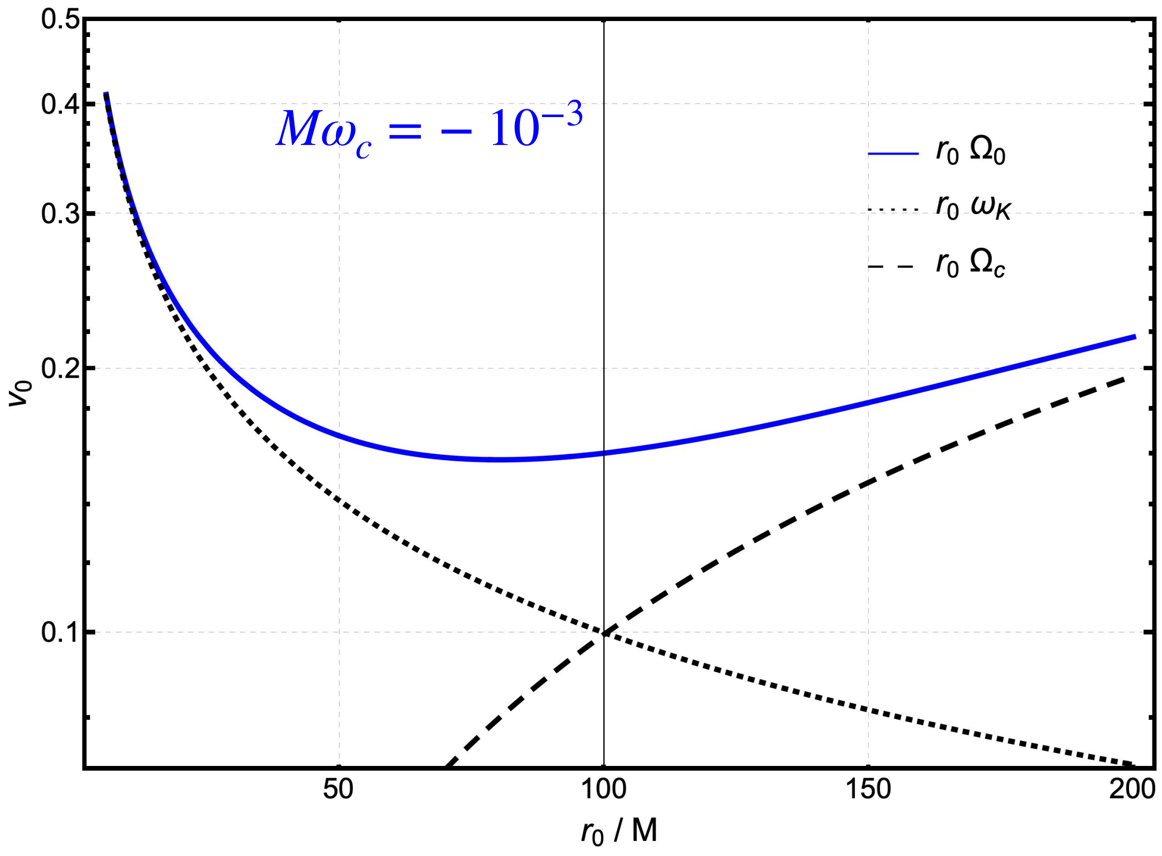
Magnetized black holes

- **Minus configuration**

$$\Omega_0 \equiv \frac{d\phi}{dt} > 0 \qquad\qquad \omega_c = \frac{qB_0}{m} < 0$$

  - Attractive Lorentz force; exists in flat space





$$\Omega_0 \sim \omega_K = \sqrt{\frac{M}{r_o^3}} \qquad (r_0 \to 2M)$$

$$\Omega_0 \sim \Omega_C = \frac{\omega_c}{\sqrt{1 + r_0^2\omega_c^2}} \qquad (r_0 \to \infty)$$

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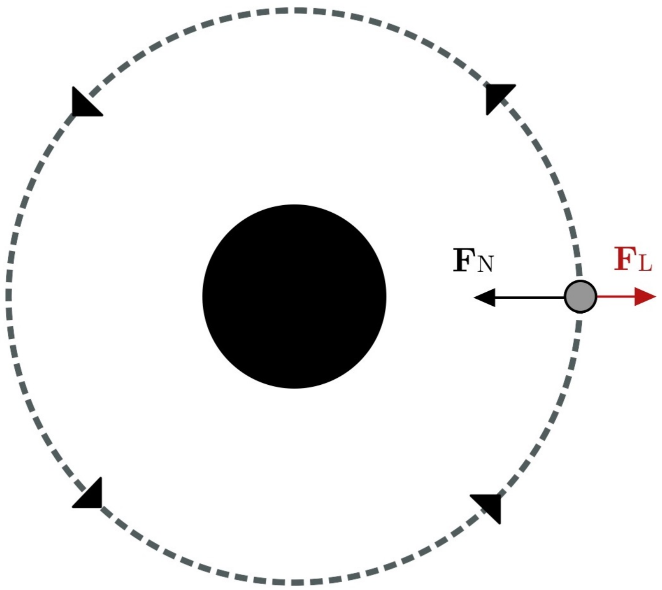
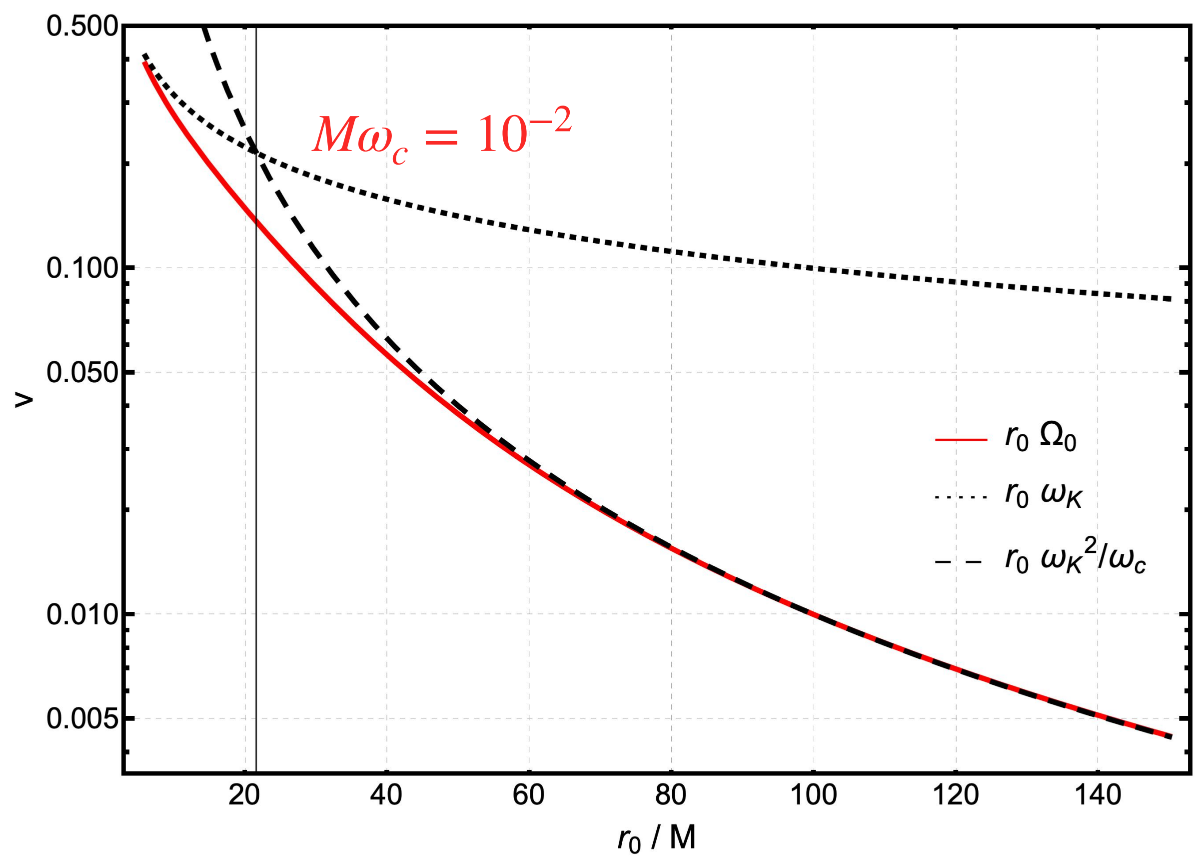
Magnetized black holes

- **Plus configuration**

$$\Omega_0 \equiv \frac{d\phi}{dt} > 0 \qquad \omega_c = \frac{qB_0}{m} > 0$$

  - Repulsive Lorentz force; don't exist in flat space





$$\Omega_0 \sim \omega_K = \sqrt{\frac{M}{r_o^3}} \qquad (r_0 \to 2M)$$

$$\Omega_0 \sim \frac{\omega_K^2}{\omega_c} = \frac{M}{\omega_c r_0^3} \qquad (r_0 \to \infty)$$

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Radiation reaction in curved space

- Particles follow the DeWitt—Brehme equation

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + \frac{2q^2}{3m}\left(\frac{D^2u^\mu}{d\tau^2} + u^\mu u_\nu \frac{D^2u^\nu}{d\tau^2}\right) + \frac{2q^2}{m} f^{\mu\nu}_{\text{tail}} u_\nu + \frac{q^2}{3m}\left(R^\mu{}_\nu u^\nu + R^\nu{}_\alpha u_\nu u^\alpha u^\mu\right)$$

$= 0$ (vacuum)

- Self-force is now: Abraham—Lorentz—Dirac + **tail term** + ~~curvature~~
- In Tursunov *et al* (2018): **neglect the tail**
  - “Orbit widening” in plus configuration!

B. S. DeWitt and R. W. Brehme, Ann. Phys. **9**, 220 (1960)
E. Poisson *et al*, Living Rev. Rel. **14**, 7 (2011)
A. Tursunov *et al*, ApJ **861**, 2 (2018)

Electromagnetic perturbations of a Schwarzschild black hole

The Teukolsky equation

- For different (ψ, s, T) it describes **scalar**, **electromagnetic**, and **gravitational** perturbations

- Spin-weighted Fourier-harmonic decomposition

$$\psi = \int_{-\infty}^{\infty} d\omega \sum_{\ell,m}^{\infty} R_{\omega\ell m}(r)\, {}_sY_{\ell m}(\theta,\phi) e^{-i\omega t} \qquad T = \int_{-\infty}^{\infty} d\omega \sum_{\ell,m}^{\infty} T_{\omega\ell m}(r)\, {}_sY_{\ell m}(\theta,\phi) e^{-i\omega t}$$

- Rewrite as Schrödinger-like radial ODE for the mode amplitudes

$$\left(r^2 f \frac{d^2}{dr^2} + 2(s+1)(r-M)\frac{d}{dr} - V_s \right) R_{\omega\ell m} = 4\pi r^2 T_{\omega\ell m}$$

S. A. Teukolsky, ApJ **185**, 635 (1973)
S. L. Detweiler, ApJ **225**, 687 (1978)
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Charged particle in a circular orbit

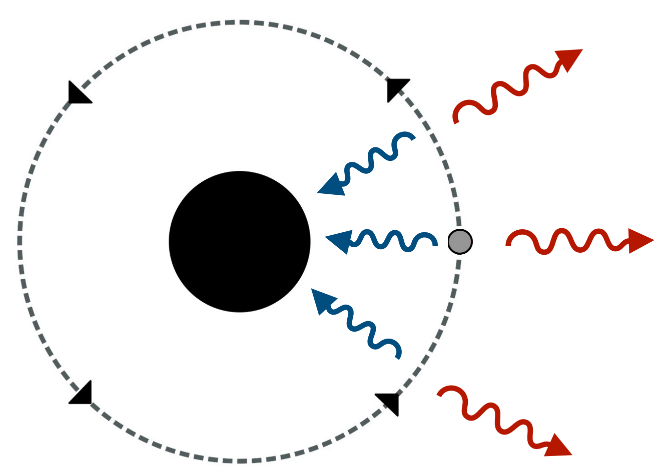
- Focus on **electromagnetic** case, $s = -1$

- Multipolar expansion of **EM energy flux** in at **infinity** and on the **horizon**:

$$\dot{E}^{\infty} \equiv \left.\frac{dE}{dt}\right|_{\infty} = \frac{1}{2\pi} \sum_{\ell,m} \left| Z^{\infty}_{\ell m} \right|^2 \qquad \dot{E}^{H} = \sum_{\ell,m} \frac{32\,(m\Omega_0)^2 M^6 (16\,(m\Omega_0)^2 + 1/M^2)}{\pi \left[\ell(\ell+1)\right]^2} \left| Z^{H}_{\ell m} \right|^2$$

- **Task:** solve the homogeneous radial equation with physical BCs.

  - Purely outgoing at infinity
  - Purely ingoing on the horizon



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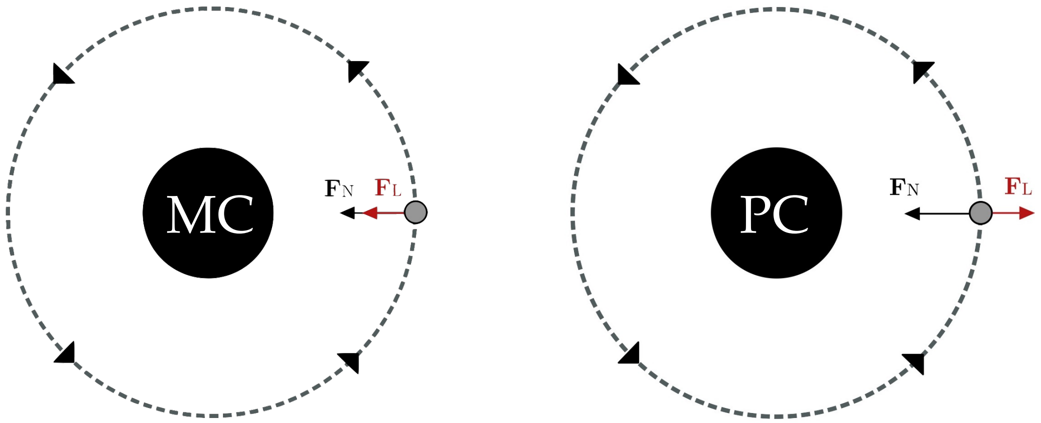
Results

Results

- **Task:** solve the homogeneous radial equation with physical BCs.
  - **Analytically** in the low frequency limit
  - **Numerically** in the general case



Recall

- Can the tail term be ignored?
- Can we see "orbital widening" in PC orbits?
  - Negative energy fluxes?

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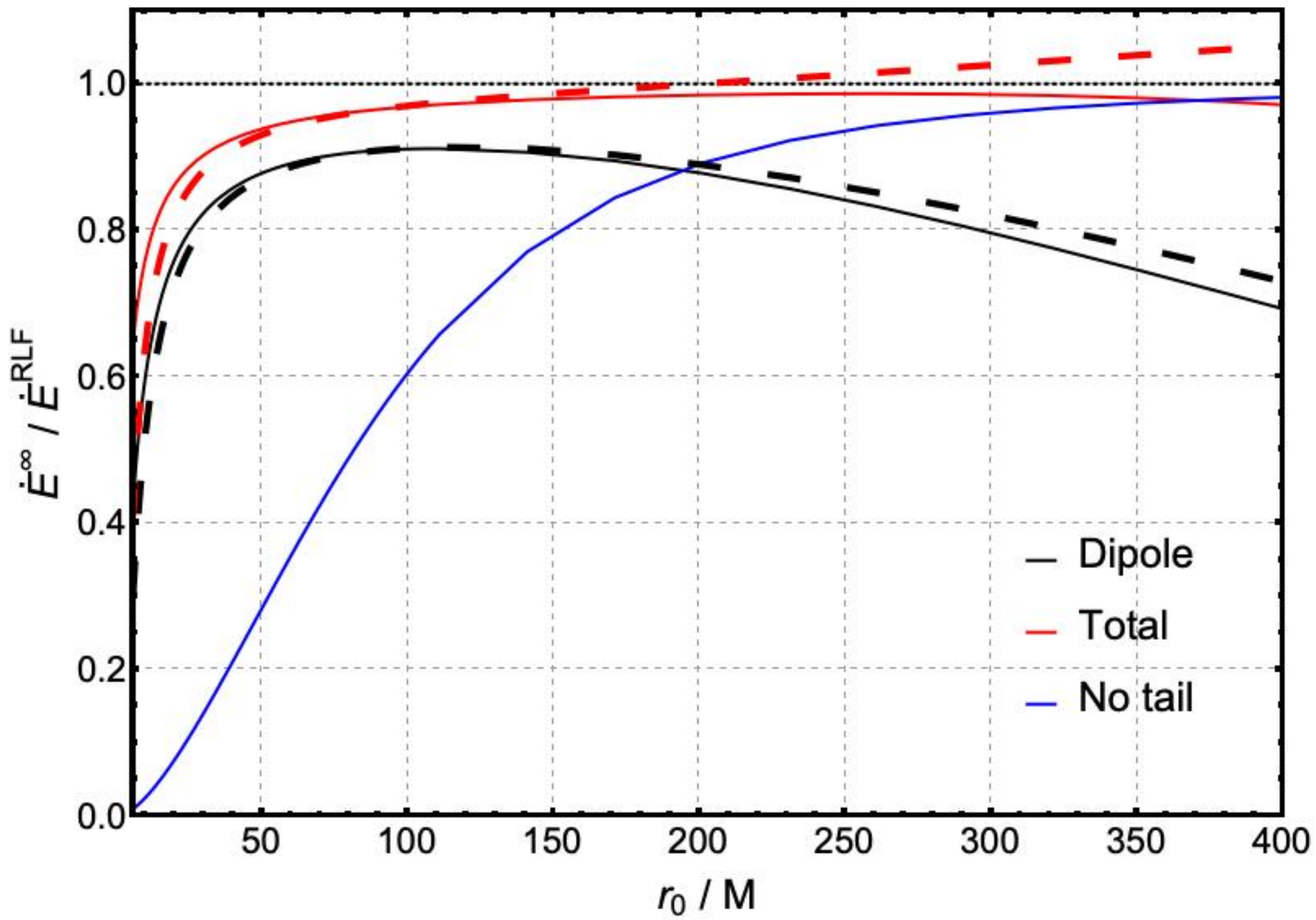
Minus configuration ($M\omega_c = -10^{-3}$)

Energy flux at infinity



- Analytical, numerical and Larmor formula agree for slow orbits
- **The tail term can only be neglected for large radii (magnetic dominated)**

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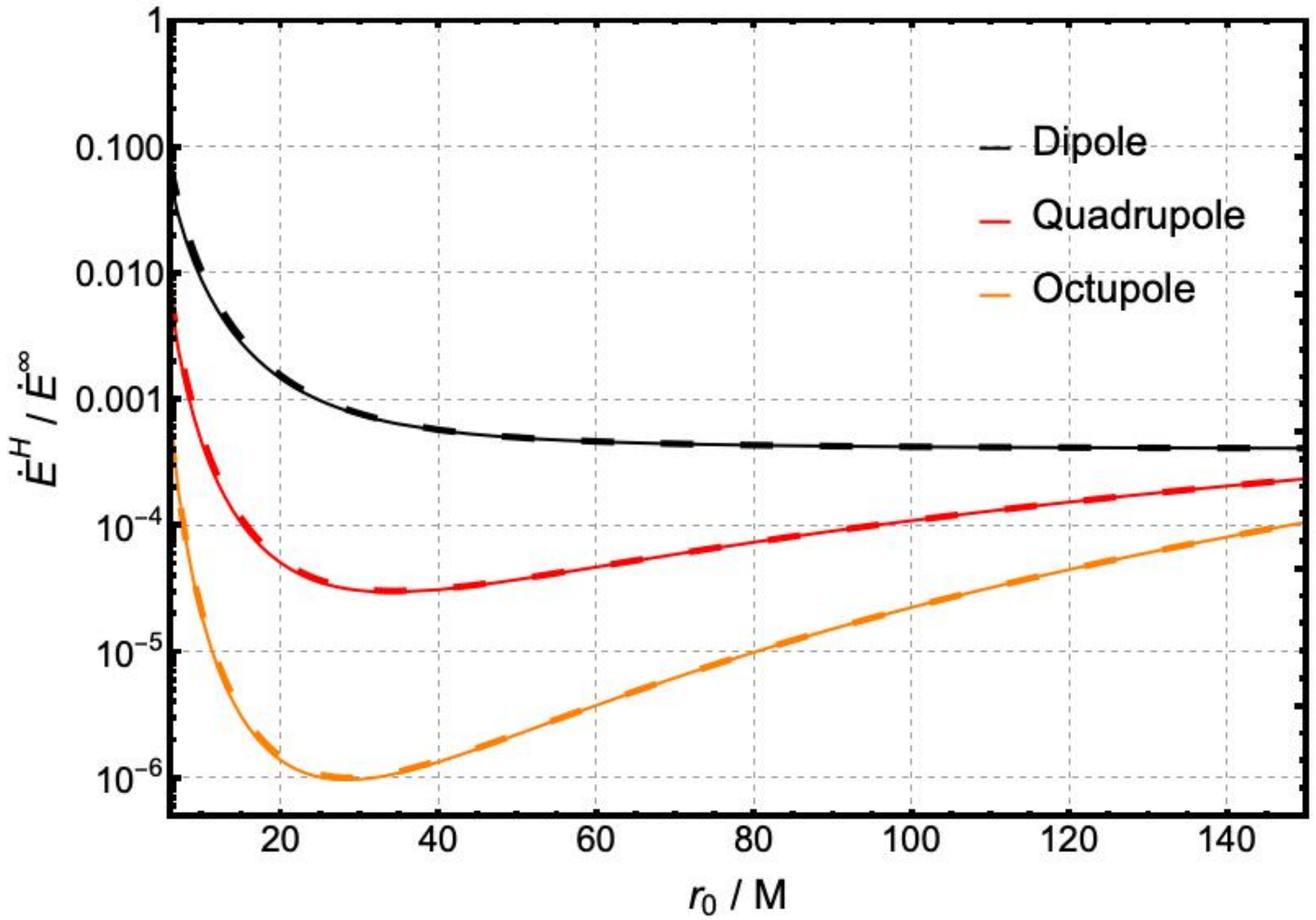
Plus configuration ($M\omega_c = 10^{-2}$)

Ratio of energy fluxes on horizon and at infinity



- Analytical, numerical and Larmor formula agree for slow orbits
- **The tail term can never be neglected**
- All fluxes are positive $\Longrightarrow$ **no "orbital widening"**

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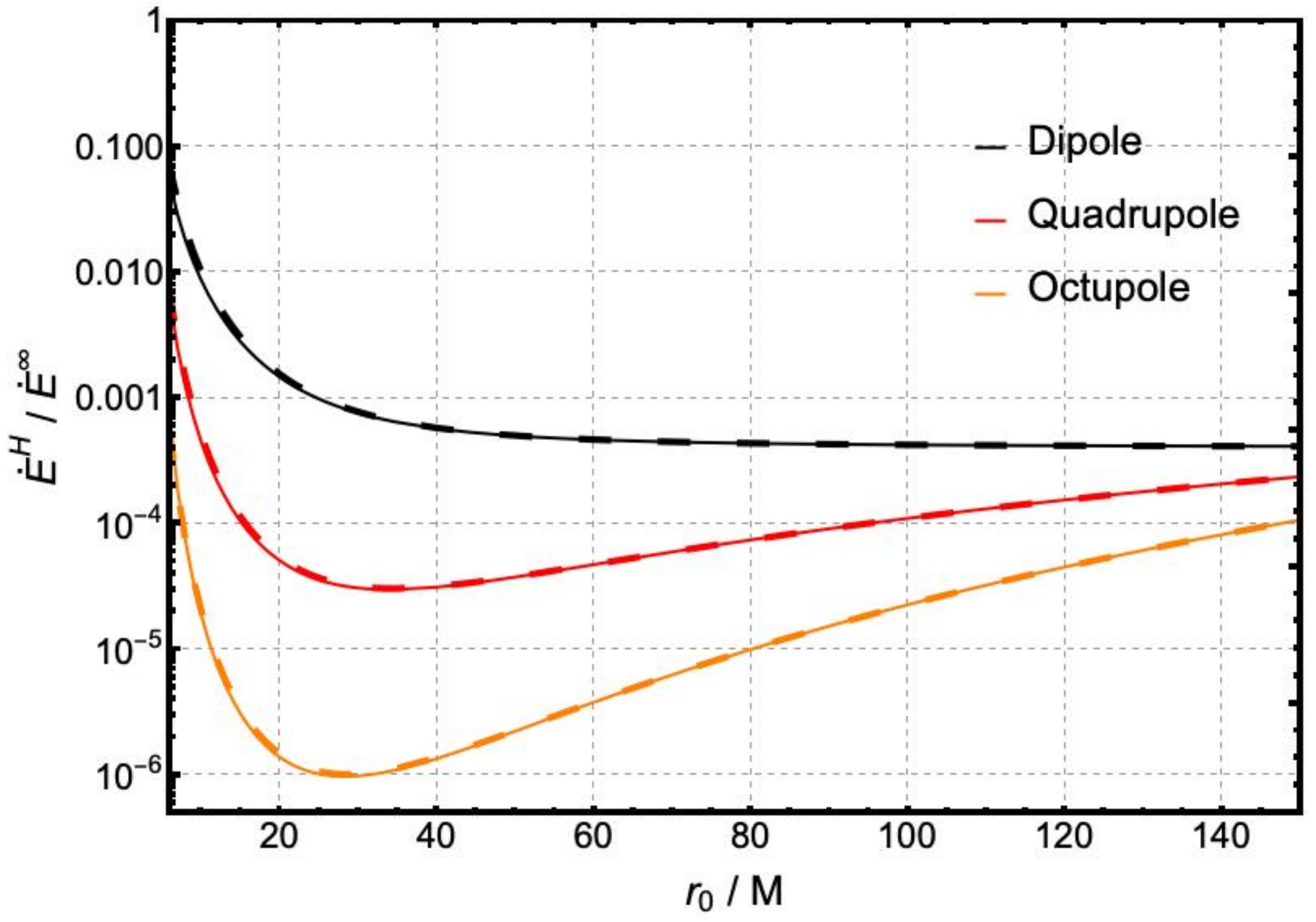
Plus configuration ($M\omega_c = 10^{-2}$)

Ratio of energy fluxes on horizon and at infinity



- **Horizon dominance effect**
  - The energy flux on the horizon cannot be neglected even for arbitrarily wide orbits

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Conclusions

Conclusions

- Circular orbits of charged particles around weakly magnetized Schwarzschild BHs can be split into two configurations: **minus** (attractive Lorentz force) and **plus** (repulsive Lorentz force)
- **Radiation reaction** at the level of the equation of motion introduces a complicated **tail term.**
- We found that **the tail term must be included** in all non—trivial scenarios
  - The tail captures all radiation reaction due to gravitational acceleration
- We also found a horizon dominance effect
  - Floating orbits in Kerr…

Thank you!