Ridges due to latent heat in rotating neutron stars, in GR and f(R) gravity

Pablo Navarro Moreno, Aneta Wojnar,
Felipe J. Llanes-Estrada and Eva Lope-Oter

Universidad Complutense de Madrid

Spanish and Portuguese Relativity Meeting
EREP 2024

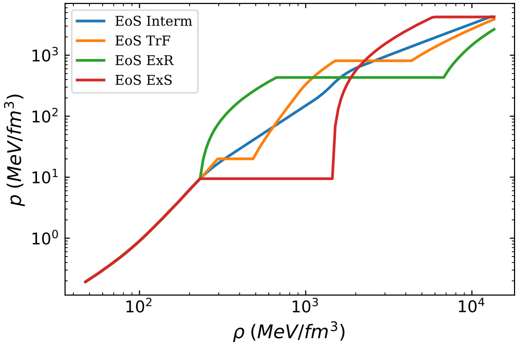
Contents

Main statement

non-analyticities in observables $\Longrightarrow$ L of NSs
Seidov limit of GR exceeded $\Longrightarrow$ modified gravity

Contents

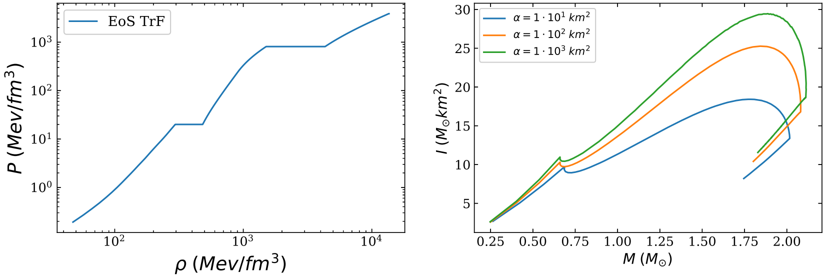
Equations of State (EoS)



[1] E. Lope-Oter and F. J. Llanes-Estrada,Eur. Phys. J. A **58** (2022) no.1, 9 doi:10.1140/epja/s10050-021-00656-9.

Equations of State

Moment of inertia in R^2-gravity



Non-analyticities lead to kinks in the observables!

Latent Heat

- Latent Heat = Intensity of the phase transition
- Several ways of defining it

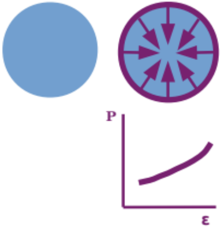
$$L = p_1 \frac{\rho_2 - \rho_1}{\rho_2 \rho_1}$$

- Phase transition takes place at $p = p_1$.

Maximum Latent Heat

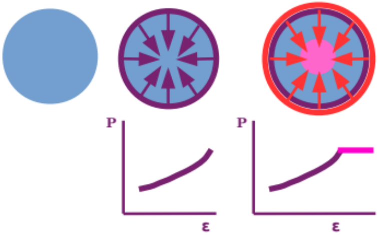
Long phase transition $\Longrightarrow$ gravitational collapse

Maximum Latent Heat



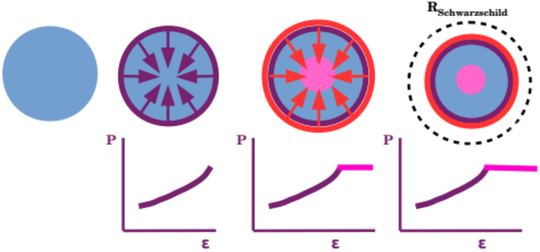
Long phase transition $\Longrightarrow$ gravitational collapse

Maximum Latent Heat



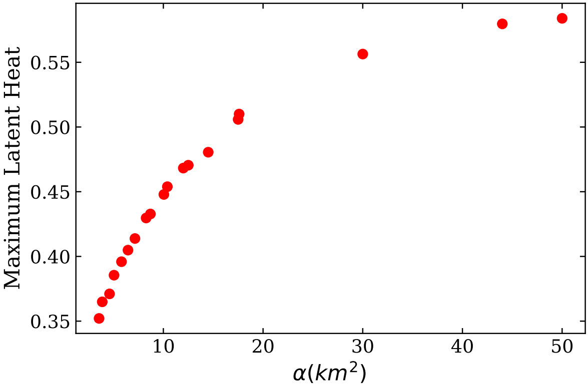
Long phase transition $\Longrightarrow$ gravitational collapse

Maximum Latent Heat



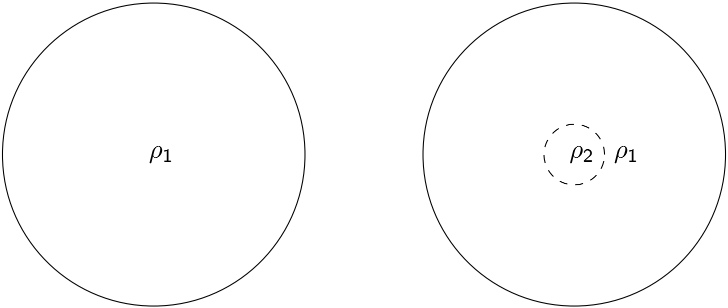
Long phase transition $\Longrightarrow$ gravitational collapse

Maximum Latent Heat



Long phase transition $\Longrightarrow$ gravitational collapse

Small core approximation



Star with no phase transition   Star with small core in a new phase

[2] Z. Seidov, Sov. Astron., **15** 347 (1971).

Small core approximation

Perturbation theory in equilibrium equations $\Longrightarrow$ critical $q = \rho_2/\rho_1$ $\Longrightarrow$ loss of stability.

- In GR

$$q_{crit} = \frac{3}{2}\left(1 + \frac{p_1}{\rho_1}\right)$$

Small core approximation

Perturbation theory in equilibrium equations $\Longrightarrow$ critical $q = \rho_2/\rho_1 \Longrightarrow$ loss of stability.

- In GR

$$q_{crit} = \frac{3}{2}\left(1 + \frac{p_1}{\rho_1}\right)$$

- In R^2-gravity

$$q_{crit} = \frac{3}{4}\left[1 - \frac{p_1}{\rho_1} - x + \sqrt{3}\sqrt{3 + 18\frac{p_1}{\rho_1} + 27\left(\frac{p_1}{\rho_1}\right)^2 + 2x + 6\frac{p_1}{\rho_1}x + 3x^2}\right]$$

$$x = \frac{\phi_c^2}{24\pi\alpha\rho_1}$$

Contents

NS in General Relativity

Static star

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + g^{\mu\nu}p$$

NS in General Relativity

Static star

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + g^{\mu\nu} p$$

Tolman-Oppenheimer-Volkoff (TOV) equations

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dp(r)}{dr} = -(\rho(r) + p(r))\frac{4\pi r^3 p(r) + m(r)}{r^2(1 - 2m(r)/r)}$$

$$\frac{d\nu}{dr} = -\frac{1}{\rho(r) + p(r)}\frac{dp(r)}{dr}$$

NS in General Relativity

Slowly rotating star (Hartle & Thorne)

$$ds^2 = -e^{\nu}(1+2h)dt^2 + e^{\lambda}\left[1 + \frac{2m}{r-2M}\right]dr^2 + r^2(1+2k)(d\theta^2 + \sin^2\theta(d\phi - \omega dt)^2)$$

NS in General Relativity

Slowly rotating star (Hartle & Thorne)

$$ds^2 = -e^{\nu}(1+2h)dt^2 + e^{\lambda}\left[1+\frac{2m}{r-2M}\right]dr^2 + r^2(1+2k)(d\theta^2+\sin^2\theta(d\phi-\omega dt)^2)$$

$$\frac{d}{dr}\left(r^4 j(r)\frac{d\varpi}{dr}\right) + 4r^3\frac{dj}{dr}\varpi = 0$$

$$\frac{dm_0}{dr} = \dots$$

$$\frac{dp_0^*}{dr} = \dots$$

$$\dots$$

[3] Hartle J B 1967 Astrophys. J. 150 1005–1029.

[4] Hartle J B and Thorne K S 1968 Astrophys. J. 153 807.

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R) P_2(\cos\theta)$$

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R)P_2(\cos\theta)$$

$$M_T = m(R) + m_0(R) + \frac{J^2}{R^3}$$

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R)P_2(\cos\theta)$$

$$M_T = m(R) + m_0(R) + \frac{J^2}{R^3}$$

$$J = \frac{R^4}{6}\left(\frac{d\varpi}{dr}\right)_R$$

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R)P_2(\cos\theta)$$

$$M_T = m(R) + m_0(R) + \frac{J^2}{R^3}$$

$$J = \frac{R^4}{6}\left(\frac{d\varpi}{dr}\right)_R$$

$$I = \frac{J}{\Omega}$$

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R) P_2(\cos\theta)$$

$$M_T = m(R) + m_0(R) + \frac{J^2}{R^3}$$

$$J = \frac{R^4}{6}\left(\frac{d\varpi}{dr}\right)_R$$

$$I = \frac{J}{\Omega}$$

$$\chi = \frac{J}{M^2}$$

NS in General Relativity

Observables

$$r_p(\theta) = R + \xi_0(R) + \xi_2(R)P_2(\cos\theta)$$

$$M_T = m(R) + m_0(R) + \frac{J^2}{R^3}$$

$$J = \frac{R^4}{6}\left(\frac{d\varpi}{dr}\right)_R$$

$$I = \frac{J}{\Omega}$$

$$\chi = \frac{J}{M^2}$$

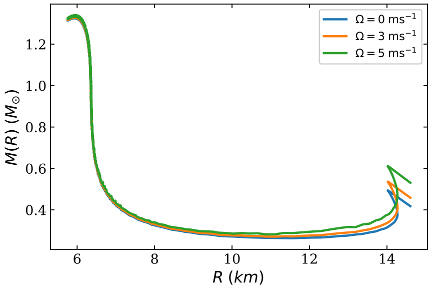
$$e = \sqrt{1 - \left(\frac{r_{\mathrm{polar}}}{r_{\mathrm{eq}}}\right)^2}$$

NS in GR

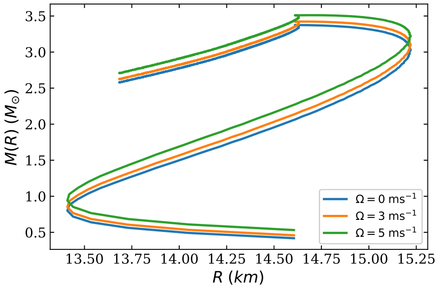
Results

NS in GR

Mass-radius diagram

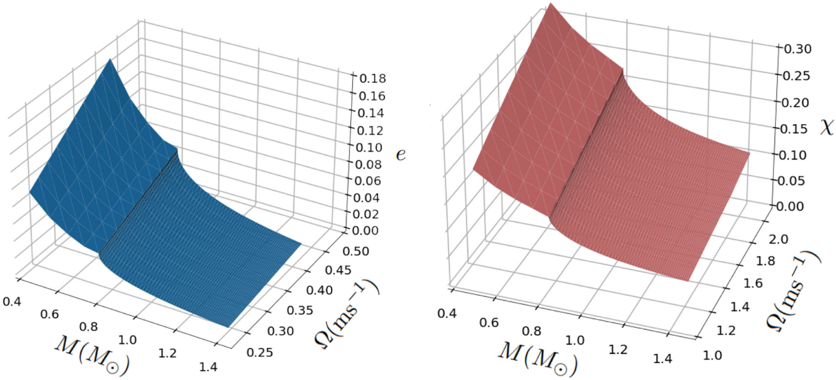


EoS ExS



EoS ExR

NS in GR



[5] P. N. Moreno, F. J. Llanes-Estrada and E. Lope-Oter, Annals Phys. **459** (2023), 169487 doi:10.1016/j.aop.2023.169487.

Contents

NS in R^2-gravity

$$\text{Jordan frame} \xrightarrow[\text{transformation}]{\text{Conformal}} \text{Einstein frame}$$

$$g^*_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}, \qquad \Omega^2(\phi) = e^{\frac{2\phi}{\sqrt{3}}}$$

NS in R^2-gravity

Jordan frame $\xrightarrow[\text{transformation}]{\text{Conformal}}$ Einstein frame

$$g^*_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}, \qquad \Omega^2(\phi) = e^{\frac{2\phi}{\sqrt{3}}}$$

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g^*} \left(R^* - 2g^{*\mu\nu} \nabla^*_\mu \phi \nabla^*_\nu \phi - V(\phi) \right) + S_M(\Omega^{-2} g^*_{\mu\nu}, \chi)$$

$$V(\phi) = \frac{1}{4\alpha} \left(1 - e^{-\frac{2}{\sqrt{3}}\phi} \right)^2$$

NS in R^2-gravity

TOV equations in the Einstein frame

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\lambda}\right)\right]=\frac{8\pi}{\Omega^4}\rho+e^{-2\lambda}\phi'^2+\frac{1}{2}V(\phi)$$

NS in R^2-gravity

TOV equations in the Einstein frame

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\lambda}\right)\right]=\frac{8\pi}{\Omega^4}\rho+e^{-2\lambda}\phi'^2+\frac{1}{2}V(\phi)$$

$$\frac{2}{r}e^{-2\lambda}\frac{d\nu}{dr}+\frac{\left(e^{-2\lambda}-1\right)}{r^2}=\frac{8\pi}{\Omega^4}p+e^{-2\lambda}\phi'^2-\frac{1}{2}V(\phi)$$

NS in R^2-gravity

TOV equations in the Einstein frame

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\lambda}\right)\right] = \frac{8\pi}{\Omega^4}\rho + e^{-2\lambda}\phi'^2 + \frac{1}{2}V(\phi)$$

$$\frac{2}{r}e^{-2\lambda}\frac{d\nu}{dr} + \frac{\left(e^{-2\lambda}-1\right)}{r^2} = \frac{8\pi}{\Omega^4}p + e^{-2\lambda}\phi'^2 - \frac{1}{2}V(\phi)$$

$$\frac{dp}{dr} = -(\rho+p)\left[\frac{d\nu}{dr} - \frac{1}{\sqrt{3}}\frac{d\phi}{dr}\right]$$

NS in R^2-gravity

TOV equations in the Einstein frame

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\lambda}\right)\right] = \frac{8\pi}{\Omega^4}\rho + e^{-2\lambda}\phi'^2 + \frac{1}{2}V(\phi)$$

$$\frac{2}{r}e^{-2\lambda}\frac{d\nu}{dr} + \frac{\left(e^{-2\lambda}-1\right)}{r^2} = \frac{8\pi}{\Omega^4}p + e^{-2\lambda}\phi'^2 - \frac{1}{2}V(\phi)$$

$$\frac{dp}{dr} = -(\rho+p)\left[\frac{d\nu}{dr} - \frac{1}{\sqrt{3}}\frac{d\phi}{dr}\right]$$

$$\phi'' + \left[\frac{d\nu}{dr} - \frac{d\lambda}{dr} + \frac{2}{r}\right]\phi' = \frac{\kappa}{2\sqrt{3}}e^{2\lambda}\Omega^{-4}(3p-\rho) + \frac{1}{4}e^{2\lambda}\frac{dV}{d\phi}$$

[6] K. V. Staykov, *et al.*, JCAP **10** (2014), 006 doi:10.1088/1475-7516/2014/10/006.

NS in R^2-gravity

Observables

$$R_s = R\Omega^{-1}(\phi(R))$$

NS in R^2-gravity

Observables

$$R_s = R\Omega^{-1}(\phi(R))$$

$$M = \lim_{r\to\infty} \frac{r}{2}\left(1 - e^{-2\lambda(r)}\right)$$

NS in R^2-gravity

Slowly rotating star

$$ds_*^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - 2\omega dt d\phi$$

NS in R^2-gravity

Slowly rotating star

$$ds_*^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - 2\omega dt d\phi$$

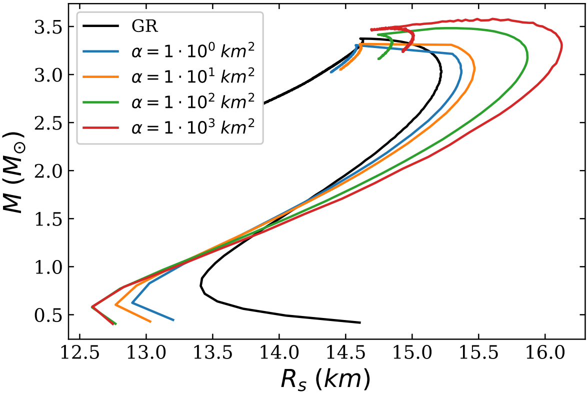
$$\frac{e^{\nu-\lambda}}{r^4}\frac{d}{dr}\left[e^{-\nu-\lambda}r^4\frac{d\varpi(r)}{dr}\right] = 16\pi\Omega^{-4}(\rho+p)\varpi(r)$$

NS in R^2-gravity

Results

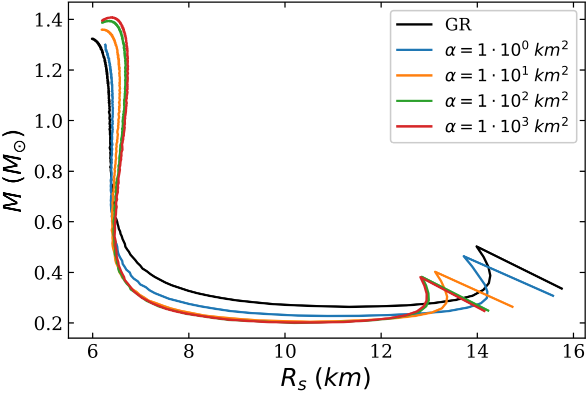
Results: NS in R^2-gravity

Mass-Radius diagram: EoS ExR



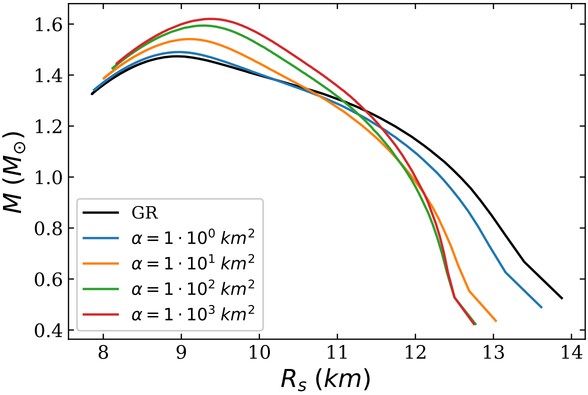
NS in R^2-gravity

Mass-Radius diagram: EoS ExS



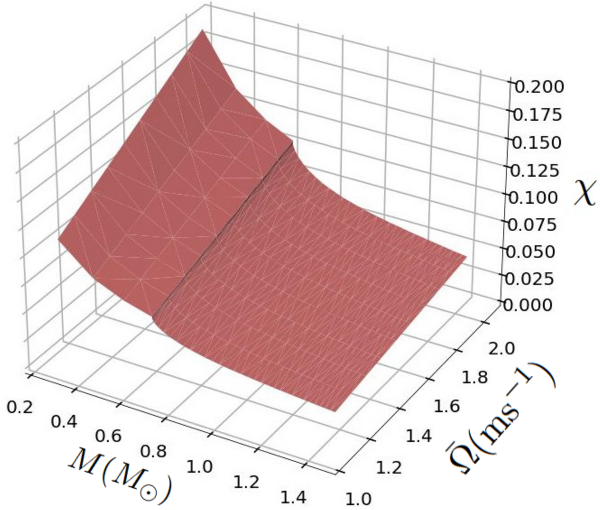
NS in R^2-gravity

Mass-Radius diagram: EoS Interm

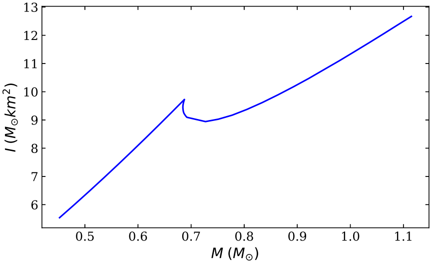


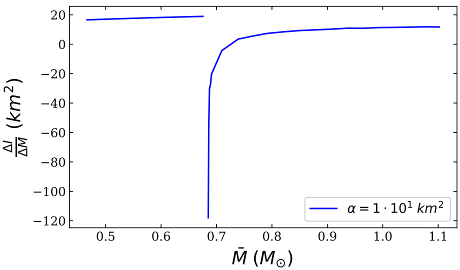
NS in R^2-gravity

Adimensional angular moment χ: EoS TrF



NS in R^2-gravity





Contents

Conclusions

- Non-analyticities in the EoS leave clear kinks and ridges in the observables
- This, as well as the maximum latent heat allowed, might be useful for future observations to constrain the parameter of the theory
- or to obtain information about the matter inside neutron stars
- If GR limit exceeded $\Rightarrow$ modified gravity

References

[1] E. Lope-Oter and F. J. Llanes-Estrada,Eur. Phys. J. A **58** (2022) no.1, 9 doi:10.1140/epja/s10050-021-00656-9.

[2] Z. Seidov, Sov. Astron., **15** 347 (1971).

[3] Hartle J B 1967 Astrophys. J. 150 1005–1029.

[4] Hartle J B and Thorne K S 1968 Astrophys. J. 153 807.

[5] P. N. Moreno, F. J. Llanes-Estrada and E. Lope-Oter, Annals Phys. 459 (2023), 169487 doi:10.1016/j.aop.2023.169487.

[6] K. V. Staykov, *et al.*, JCAP **10** (2014), 006 doi:10.1088/1475-7516/2014/10/006.

...