News tensor on null hypersurfaces

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1. Motivation

2. Foliated null hypersurface

3. Conformal space-time

4. Conclusions

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- There is recent interest in understanding bulk null hypersurfaces in analogy to $\mathscr{I}$: BMS symmetries, WIH structure, Carrollian approaches...

Can one define a **news tensor** on null hypersurfaces in the **bulk**?

2. Foliated null hypersurface

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Set-up

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- Foliation of $\mathcal{N}$ by 2-dimensional leaves $\mathcal{S}_C$. C constant values of a function F

$$\dot{F} := k^a \partial_a F \neq 0 \tag{2.1}$$

$$\ell_b := -\frac{1}{\dot{F}} \partial_b F \ , \quad k^p \ell_p = -1 \ . \tag{2.2}$$

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Set-up. Kinematic quantities

For k^α,

$$\underline{\kappa}_{AB} := \underline{E}^\alpha{}_A \underline{E}^\beta{}_B \nabla_\alpha k_\beta \ , \tag{2.3}$$

$$\underline{\kappa} := \underline{q}^{AB} \underline{\kappa}_{AB} \ , \tag{2.4}$$

$$\underline{\nu}_{AB} := \underline{\kappa}_{AB} - \frac{1}{2} \underline{q}_{AB} \underline{\kappa} \ , \tag{2.5}$$

$$k^\mu \nabla_\mu k^\alpha := \nu k^\alpha \ . \tag{2.6}$$

For ℓ^α,

$$\underline{\theta}_{AB} := \underline{E}^\alpha{}_A \underline{E}^\beta{}_B \nabla_\alpha \ell_\beta \ , \tag{2.7}$$

$$\underline{\theta} := \underline{q}^{AB} \underline{\theta}_{AB} \ , \tag{2.8}$$

$$\underline{\sigma}_{AB} := \underline{\theta}_{AB} - \frac{1}{2} \underline{q}_{AB} \underline{\theta} \ . \tag{2.9}$$

Set-up: connections, curvature

$$v^a \overline{\nabla}_a w^b := \omega_\beta{}^b v^\alpha \nabla_\alpha w^\beta \ , \quad \forall \quad v^\alpha = e^\alpha{}_a v^a \ , \quad w^\alpha = e^\alpha{}_a w^a \ .$$

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$$v^A \underline{\mathcal{D}}_A w^B := \underline{W}_b{}^B v^a \nabla_a w^b \ , \quad \forall \quad v^a = \underline{E}^a{}_A v^A \ , \quad w^a = \underline{E}^a{}_A w^A \ ,$$

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- Notation: curvature tensors $\overline{R}_{abc}{}^d$, $\underline{R}_{ABC}{}^D$.
- Schouten tensor: $S_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta} R$.
- Projections:

$$\overline{S}_{ab} := \frac{1}{2} e^\alpha{}_a e^\beta{}_b S_{\alpha\beta} \ ,$$
$$\underline{S}_{AB} := \underline{E}^a{}_A \underline{E}^b{}_B \overline{S}_{ab} \ .$$

News tensor on a null hypersurface

Consider *conformal rescalings*

$$\left(k'^a, \overline{g}'_{ab}\right) = \left(\lambda^{-1} k^a, \lambda^2 \overline{g}_{ab}\right) \ , \tag{2.10}$$

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$$\left(k'^{a}, \overline{g}'_{ab}\right) = \left(\lambda^{-1} k^{a}, \lambda^{2} \overline{g}_{ab}\right) , \tag{2.10}$$

$$\begin{aligned} \underline{S}'_{AB} =& \underline{S}_{AB} - \lambda^{-1} \underline{\mathcal{D}}_{A} \underline{\lambda}_{B} + 2\lambda^{-2} \underline{\lambda}_{A} \underline{\lambda}_{B} - \frac{1}{2} \lambda^{-2} \underline{q}_{AB} \underline{\lambda}_{M} \underline{\lambda}^{M} \\ & + \lambda^{-1} \left(\underline{\theta}_{AB} k^{\mu} \lambda_{\mu} + \underline{\kappa}_{AB} \ell^{\mu} \lambda_{\mu} \right) + \lambda^{-2} \underline{q}_{AB} k^{\mu} \lambda_{\mu} \ell^{\nu} \lambda_{\nu} \end{aligned} \tag{2.11}$$

News tensor on a null hypersurface

The news tensor is going to be extracted from the **traceless** and **conformal-invariant** part of

$$\underline{U}_{AB} := S_{AB} + \frac{1}{2}\underline{q}_{AB}\left(\underline{\nu}_{MC}\underline{\sigma}^{MC} - \frac{1}{2}\underline{\kappa}\underline{\theta} + \frac{1}{2}\underline{C}_{MD}{}^{MD}\right) - \frac{1}{2}\left(\theta\underline{\nu}_{AB} + \underline{\kappa}\underline{\sigma}_{AB}\right) \ . \qquad (2.12)$$

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This has the following properties

$$\underline{U}_M{}^M = \underline{K} \ , \tag{2.13}$$

$$\underline{U}'_{AB} = \underline{U}_{AB} - \lambda^{-1}\underline{\mathcal{D}}_A\underline{\lambda}_B + 2\lambda^{-2}\underline{\lambda}_A\underline{\lambda}_B - \frac{1}{2}\lambda^{-2}\underline{q}_{AB}\underline{\lambda}_M\underline{\lambda}^M \ . \tag{2.14}$$

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News tensor on a null hypersurface

A sketched version of a result from FFÁ-Senovilla (Geroch tensor)...

Theorem 1 (The tensor ρ)

If $\mathcal{S}_C$ has $\mathbb{S}^2$-topology, there is a unique symmetric tensor field ρ_{AB} whose behaviour under conformal rescalings (2.10) *is as in* (2.14) *and satisfies the equation*

$$\mathcal{D}_{[C}\rho_{A]B} = 0 \tag{2.15}$$

in any conformal frame. (...) Furthermore, it is given for round spheres by $\rho_{AB} = q_{AB} aK/2$.

This is then generalised for the foliation as $\underline{\rho}_{AB}$ on $\mathcal{N}$.

News tensor on a null hypersurface

Theorem 2 (News tensor for a foliated null hypersurface)

Let $\mathcal{N}$ be a null hypersurface foliated by leaves with $\mathbb{S}^2$-topology and with F the defining function (2.1)*. Then, there is a one-parameter family (depending on F) of symmetric, traceless, conformal-invariant tensor fields*

$$\underline{N}_{AB} := \underline{U}_{AB} - \underline{\rho}_{AB} \ , \qquad (2.16)$$

that satisfies the conformal-invariant equation

$$\underline{\mathcal{D}}_{[A}\underline{U}_{B]C} = \underline{\mathcal{D}}_{[A}\underline{N}_{B]C} \ , \qquad (2.17)$$

Besides, $\underline{N}_{AB}$ is unique with these properties.

3. Conformal space-time

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Conformal completion

Now let $\left(M, g_{\alpha\beta}\right)$ be the conformal completion à la Penrose of a physical space-time

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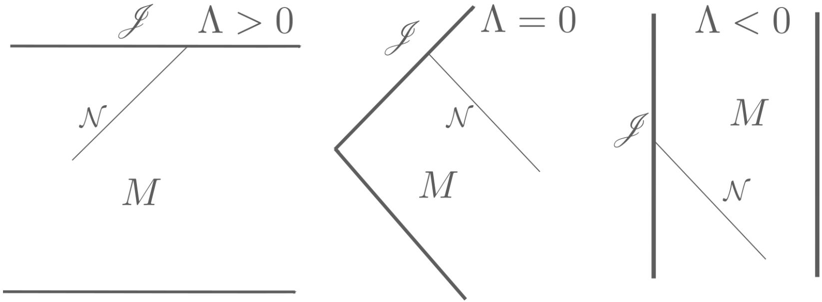
up to *conformal gauge freedom* $\Omega \to \omega\Omega$.

Conformal completion



rescaled Weyl tensor

$$\Omega d_{\alpha\beta\gamma}{}^{\delta} := C_{\alpha\beta\gamma}{}^{\delta}$$

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The radiative parts (ϕ_3 and ϕ_4) are encoded in

$$ {}^{N}D^{ab} := \omega_{\alpha}{}^{a}\omega_{\beta}{}^{b}N^{\mu}N^{\nu}d^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu}, \tag{3.1}$$

$$ {}^{N}C^{ab} := \omega_{\alpha}{}^{a}\omega_{\beta}{}^{b}N^{\mu}N^{\nu}\,{}^{*}d^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu} \tag{3.2}$$

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or, for convenience, in

$$\overset{N}{\underline{C}}{}^{A} := \ell_b \underline{W}_a{}^{A}\,\overset{N}{C}{}^{ab}\ , \quad (3.3)$$

$$\overset{N}{\underline{D}}{}_{AB} := \underline{W}_a{}^{A}\underline{W}_b{}^{B}\,\overset{N}{D}{}^{ab}\ . \quad (3.4)$$

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adapted $\mathcal{N}$

$$
\begin{aligned}
{}^{N}\underline{C}_A = \underline{\epsilon}^{SR}\Big[& \underline{\mathcal{D}}_R\underline{N}_{SA} - \underline{\mathcal{D}}_R\underline{L}_{SA} - \underline{\kappa}_{AR}(\underline{\mathcal{D}}_M\underline{\sigma}_S{}^M - \frac{1}{2}\underline{\mathcal{D}}_S\underline{\theta} - \frac{1}{4}\underline{\theta}\underline{\varphi}_S \\
& + \frac{1}{2}\underline{\varphi}^M\underline{\sigma}_{SM} + {}^{\ell}\underline{d}_S) - \underline{\theta}_{AR}\left(\underline{\mathcal{D}}_M\underline{\nu}_S{}^M - \frac{1}{2}\underline{\mathcal{D}}_S\underline{\kappa} + {}^{N}\underline{d}_S\right)\Big] + \Omega^*\underline{y}_A \ , \\
{}^{N}\underline{D}_{AB} = & \pounds_{\vec{N}}\underline{N}_{AB} + \pounds_{\vec{N}}\underline{\rho}_{AB} - \pounds_{\vec{N}}\underline{L}_{AB} - \underline{\kappa}^E{}_A\Big[-\frac{1}{2}S_{\mu\nu}N^\mu\ell^\nu\underline{q}_{EB} \\
& + \pounds_{\vec{N}}\underline{\sigma}_{EB} + \frac{1}{2}\underline{\nu}_{EB}\underline{\theta} - \frac{1}{2}\underline{\sigma}_{EB}\underline{\kappa} - 2\underline{\nu}_{C(E}\underline{\sigma}_{B)}{}^C - \frac{1}{2}\underline{L}^M{}_M\underline{q}_{EB} + k^d\varphi_d\underline{\sigma}_{EB} \\
& - \underline{\varphi}_E\underline{\varphi}_B - \underline{\mathcal{D}}_{(B}\underline{\varphi}_{E)} + 2\underline{\varphi}_{(E}\underline{\mathcal{D}}_{B)}\ln\dot{F} - \frac{1}{2}\underline{q}_{EB}\Big(2\underline{\varphi}^M\underline{\mathcal{D}}_M\ln\dot{F} - \underline{\varphi}_C\underline{\varphi}^C) \\
& - \underline{\mathcal{D}}_C\underline{\varphi}^C\Big) + F_{EB} + \frac{1}{2}\underline{q}_{EB}\underline{K}\Big] - 2\,{}^{N}\underline{S}_{(B}\left(\underline{E}^m{}_{A)}N^pH_{pm} - \underline{\varphi}_{A)}\right) \\
& + {}^{N}\underline{S}_B\underline{\varphi}_A - \underline{\mathcal{D}}_A{}^{N}\underline{S}_C + \overline{S}_{ab}N^aN^b\underline{\theta}_{AC} + \Omega\underline{y}_{AB} \ .
\end{aligned}
$$

Infinity $\mathscr{J}$ with $\Lambda = 0$

Several simplifications take place:

1. The Weyl tensor $C_{\alpha\beta\gamma}{}^{\delta}$ vanishes but $d_{\alpha\beta\gamma}{}^{\delta} \overset{\mathscr{J}}{\neq} 0$.

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3. The expansion $\underline{\kappa}$, Friedrich scalar f and the acceleration scalar ν coincide $2\nu = 2f = \underline{\kappa}$.

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3. The expansion $\underline{\kappa}$, Friedrich scalar f and the acceleration scalar ν coincide $2\nu = 2f = \underline{\kappa}$.
4. The projection $\underline{\varphi}_A$ of φ_a to the leaves vanishes.

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Infinity $\mathscr{I}$ with $\Lambda = 0$

$$ {}^{N}\!\underline{D}_{AB} \overset{\mathscr{I}}{=} \pounds_{\vec{N}} \underline{N}_{AB} \ , \qquad (3.5) $$

$$ {}^{N}\!\underline{C}_{A} \overset{\mathscr{I}}{=} -\underline{\epsilon}^{RS} \underline{\mathcal{D}}_{R} \underline{N}_{SA} \ . \qquad (3.6) $$

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\underline{N}_{AB} \overset{\mathscr{J}}{=} 0 \Longleftrightarrow \underline{C}_{A} \overset{\mathscr{J}}{=} 0 \overset{\mathscr{J}}{=} \underline{D}_{AB} \Longleftrightarrow \phi_3 \overset{\mathscr{J}}{=} 0 \overset{\mathscr{J}}{=} \phi_4
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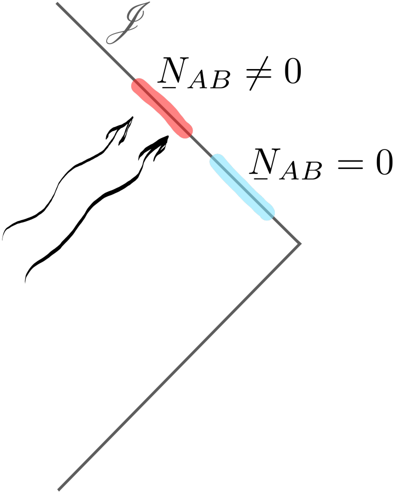

Infinity $\mathscr{J}$ with $\Lambda = 0$

General expression in terms of the time derivative of the shear

$$\underline{N}_{AB} \overset{\mathscr{J}}{=} \pounds_{\vec{N}} \underline{\sigma}_{AB} - \frac{1}{2} \underline{\sigma}_{AB} \underline{\kappa} - \underline{\rho}_{AB} + \frac{1}{2} \underline{q}_{AB} \underline{K} + F_{AB}$$

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4. Recall that the radiative structure of $\mathscr{J}$ is very special, without any gauge fixing ($\mathscr{J}$ can *expand*).

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4. Recall that the radiative structure of $\mathscr{J}$ is very special, without any gauge fixing ($\mathscr{J}$ can *expand*).

Future work:

Use this news tensor on null hypersurfaces that 'touch' $\mathscr{J}$ and analyse its asymptotic behaviour for any Λ. Also, take the limit on horizons that 'almost touch' $\mathscr{J}$.

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No more simplifications occur, in general.

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One imposes $\underline{k} = 0$ (non-expanding condition).

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$$ \underline{N}_{AB} = \frac{1}{2}\underline{q}_{AB}\underline{K} - \underline{\rho}_{AB} \ . \tag{5.2} $$

Only vanishes if the leaves are round!

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$$\overset{N}{\underline{D}}_{AC} = \underline{E}^a{}_A \underline{E}^c{}_C k^b \overline{\nabla}_b \left(\underline{N}_{ac}\right) - \underline{N}_{AC}\left(\nu - \underline{\kappa}\right) \quad ,$$

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If $N_{AB} = 0$, ‘tangential’ radiation (ϕ_1, ϕ_0) and Coulomb term (ϕ_2) still source $\overset{N}{\underline{C}}_A$!

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Only vanishes if the leaves are round!

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- $\{\underline{E}^a{}_A\}$ and $\left\{\underline{W}_a{}^A\right\}$ sets of 2 linearly-independent vector fields and forms. $\underline{E}^a{}_A \ell_a = 0$, $\underline{W}_a{}^A k^a = 0$.

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- Bases on $\mathcal{N}$: $\{\omega_\alpha{}^a\} = \left\{-\ell_\alpha, \underline{W}_\alpha{}^A\right\}$ and $\{e^\alpha{}_a\} = \{k^\alpha, E^\alpha{}_a\}\ .$

$$e^\alpha{}_a \ell_\alpha = \ell_a\ , \quad \ell_\alpha \ell^\alpha = 0\ , \quad \ell^\alpha k_\alpha = -1\ , \quad \ell_\alpha \underline{E}^\alpha{}_A = 0\ , \quad (5.5)$$

$$\underline{W}_\alpha{}^A \underline{E}^\alpha{}_B = \delta^A_B\ , \quad e^\alpha{}_a \omega_\alpha{}^b = \delta^a_b\ , \quad k_\alpha e^\alpha{}_a = 0 = \omega_\alpha{}^a \ell^\alpha\ . \quad (5.6)$$

Conformal completion

Assume the CEFE hold:

$$\nabla_\alpha N_\beta = -\frac{1}{2}\Omega S_{\alpha\beta} + f g_{\alpha\beta} + \frac{1}{2}\Omega^2 \varkappa \grave{T}_{\alpha\beta} \ , \tag{5.7}$$

$$N_\mu N^\mu = \frac{\Omega^3}{12}\varkappa T - \frac{\Lambda}{3} + 2\Omega f \ , \tag{5.8}$$

$$\nabla_\alpha f = -\frac{1}{2} S_{\alpha\mu} N^\mu + \frac{1}{2}\Omega \varkappa N^\mu \grave{T}_{\alpha\mu} - \frac{1}{24}\Omega^2 \varkappa \nabla_\alpha T - \frac{1}{8}\Omega \varkappa N_\alpha T \ , \tag{5.9}$$

$$d_{\alpha\beta\gamma}{}^{\mu} N_\mu + \nabla_{[\alpha}\left(S_{\beta]\gamma}\right) - \Omega y_{\alpha\beta\gamma} = 0 \ , \tag{5.10}$$

$$y_{\alpha\beta\gamma} + \nabla_\mu d_{\alpha\beta\gamma}{}^{\mu} = 0 \ , \tag{5.11}$$

$$R_{\alpha\beta\gamma\delta} = \Omega d_{\alpha\beta\gamma\delta} + g_{\alpha[\gamma} S_{\delta]\beta} - g_{\beta[\gamma} S_{\delta]\alpha} \ . \tag{5.12}$$

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Adapted $\mathcal{N}$

Definition 5.1 (Adapted null hypersurface)

A null hypersurface $(\mathcal{N}, \overline{g}_{ab})$ is said to be adapted in any of the following cases:

1. The conformal factor $\Omega =$ constant$\neq 0$ and $\nabla_\alpha \Omega$ does not vanish at points in $M \setminus \mathscr{J}$ belonging to $\mathcal{N}$,
2. The conformal factor $\Omega = 0$ and $\nabla_\alpha \Omega$ does not vanish at points in $\mathcal{N}$. That is, $\mathcal{N} \equiv \mathscr{J}$ with $\Lambda = 0$.

There is always a family of conformal gauges in which a bulk $\mathcal{N}$ is adapted. Infinity with $\Lambda = 0$ is by definition adapted in any gauge.

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