Imperial College London

Parallel session 5

23rd July 2024

New windows onto nHz Gravitational Wave science with astrometry

Giorgio Mentasti

Outline

- Gravitational waves and detectors
- Stochastic Gravitational Waves Backgrounds (SGWB).
- The low frequency SGWB
  - Pulsar Timing Arrays
  - Astrometry
- Present and future of GW astrometry

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Gravitational Wave interferometry

- 2014: First direct detection of a binary BH merger by LIGO-Virgo (m ~ 30 M_{sun}, f ~ 100Hz, d ~ 400 Mpc)

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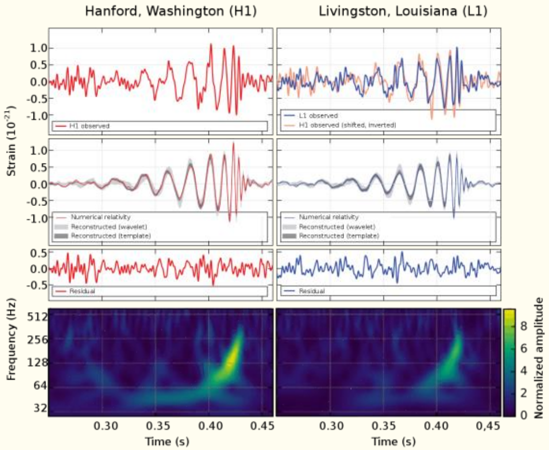

LIGO GW150914 discovery event (2014)

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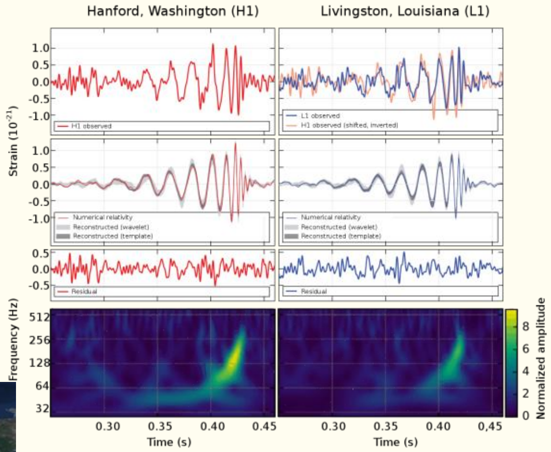
Gravitational Wave interferometry



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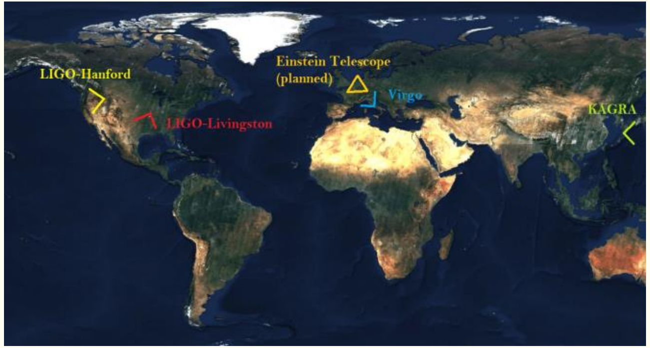

The network of ground based detectors

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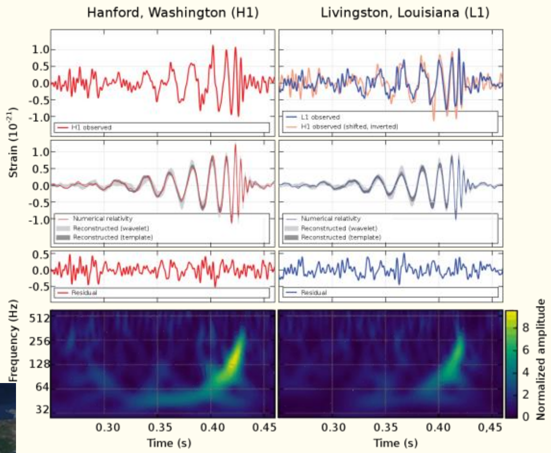

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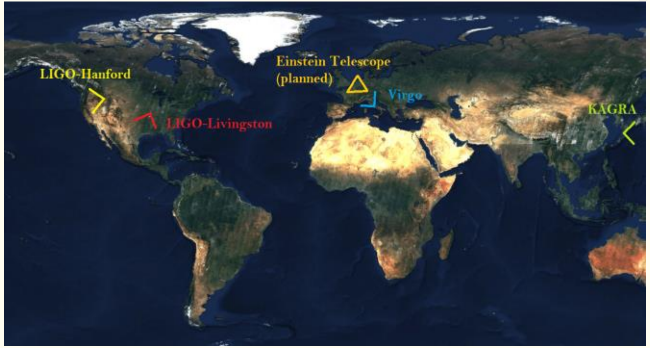
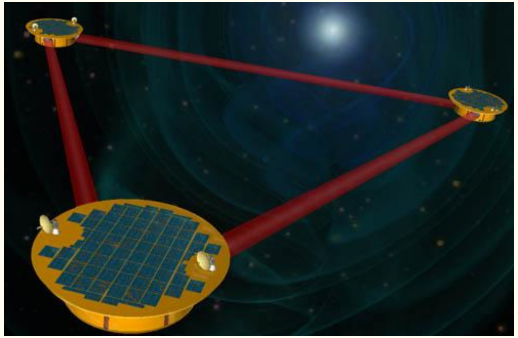


The network of ground based detectors

- ~ 2035: Space based (LISA) and future ground based instruments (ET, CE…)



LISA, the planned space-based interferometer

Low frequency gravitational waves

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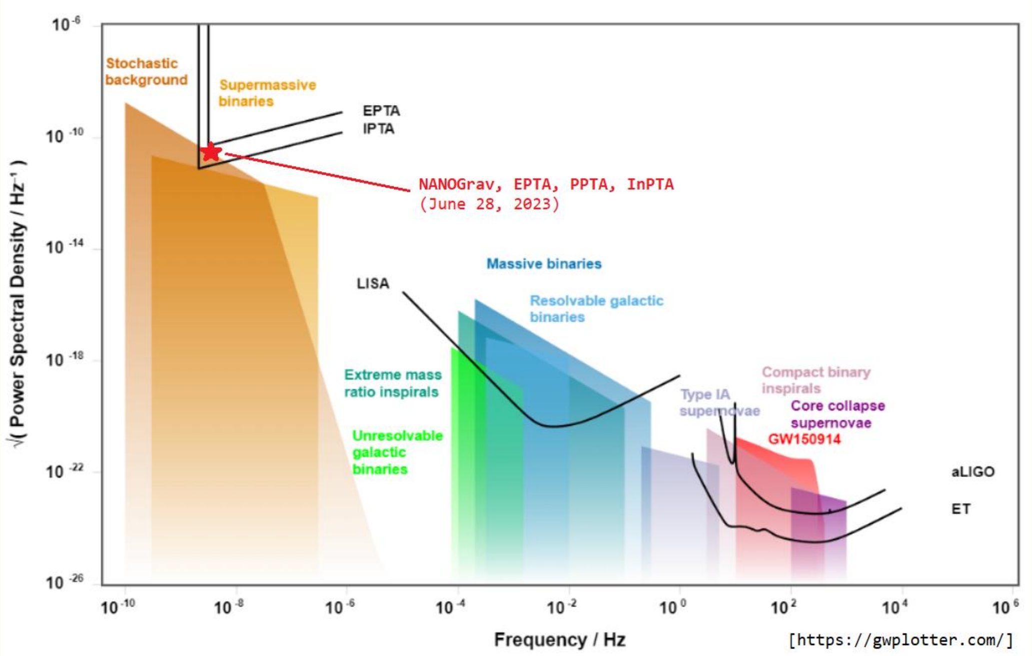
- GW interferometry: f ≥ uHz
- Pulsar Timing Array and astrometry probe the nHz band

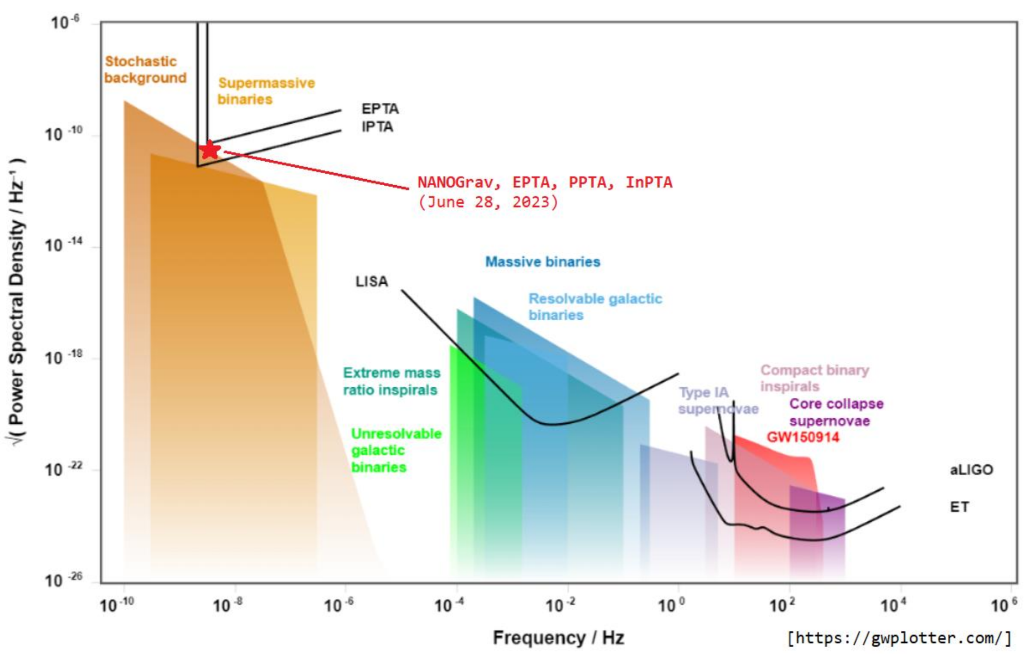


[https://gwplotter.com/]

Frequency band of the gravitational wave sources and detectors

Low frequency gravitational waves

- GW interferometry: f ≥ uHz
- Pulsar Timing Array and astrometry probe the nHz band
- Many expected sources of nHz gravitational waves (supermassive BHs, phase transitions, ultralight DM…)



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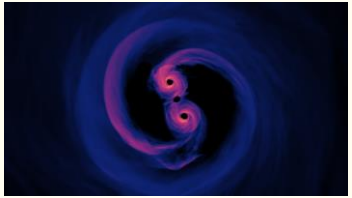
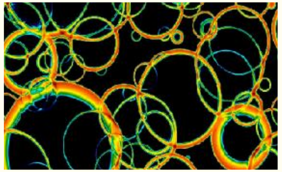
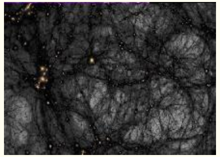


SMBH binary inspiral



1st order phase transitions



Ultralight Dark Matter

$$g_{ab}(t,\vec{x}) = \eta_{ab} + h_{ab}(t,\vec{x})$$

Coherent and stochastic searches

- **Coherent** search: a deterministic template for the GW signal

$$h_{ab}(t,\vec{x}) = \underbrace{e^{2\pi i f(t-\hat{n}\cdot\vec{x})}}_{\substack{\text{Planar}\\ \text{wave}}} \sum_{\lambda} \underbrace{h_{\lambda}(f,\hat{n})}_{\text{Amplitude}} \underbrace{e^{\lambda}_{ab}(\hat{n})}_{\substack{\text{Polarization}\\ \text{tensors}}}$$

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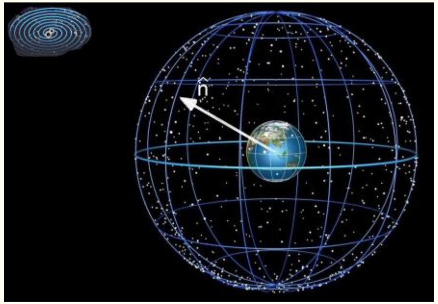

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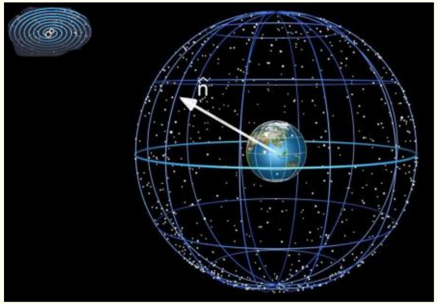

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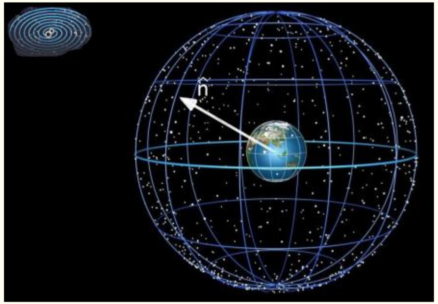
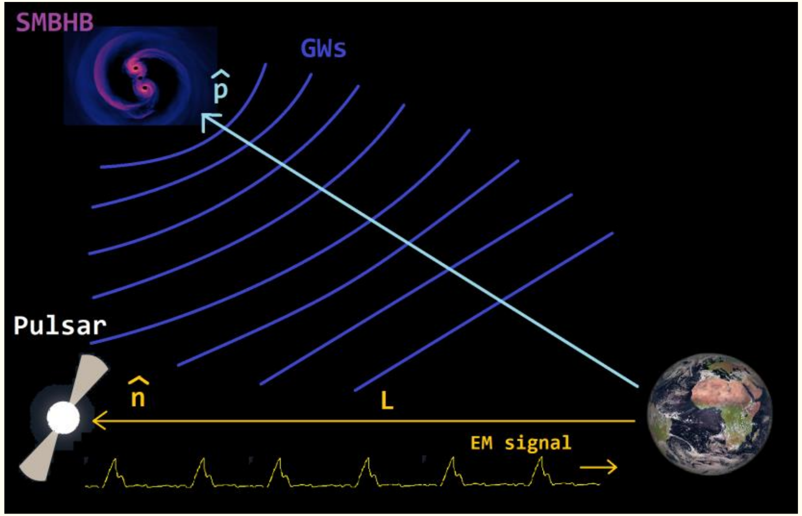
- Power spectrum

$$\langle h^{*}_{\lambda}(f,\hat{n})\, h_{\lambda'}(f,\hat{n})\rangle = \delta_{\lambda\lambda'}\delta(f-f')\delta(\hat{n}-\hat{n}')\mathcal{H}_{\lambda}(|f|,\hat{n})$$



Pulsar Timing Array

- Redshift measurement



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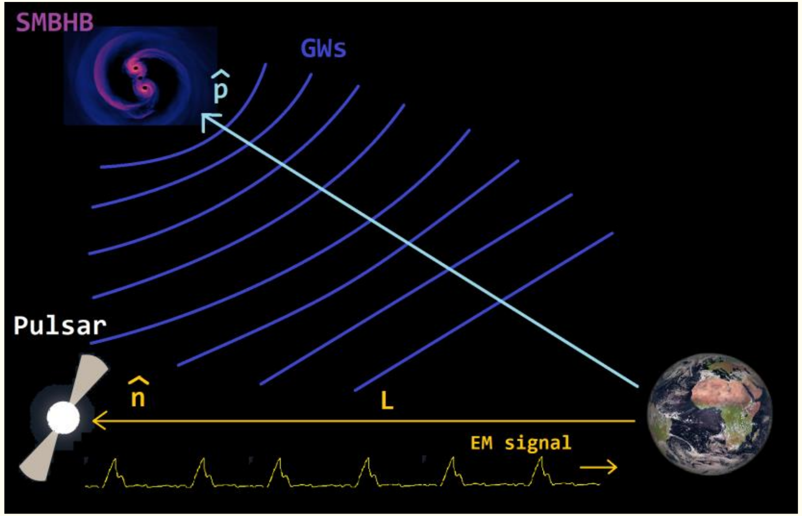
$$z(t) \equiv \frac{f_s - f_o(t)}{f_s}$$



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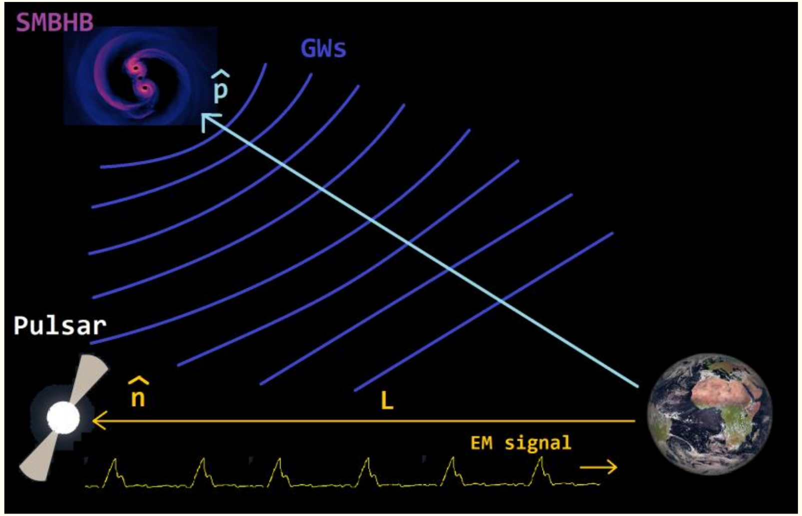
$$z(t) \equiv \frac{f_s - f_o(t)}{f_s} \propto \frac{1}{2}\frac{\hat{n}^i\hat{n}^j}{1+\hat{n}\cdot\hat{p}}h_{ij}(t)$$



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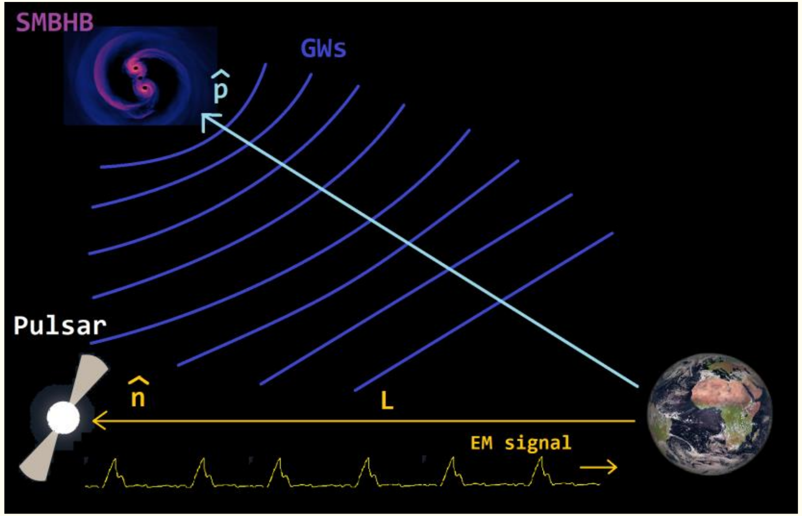
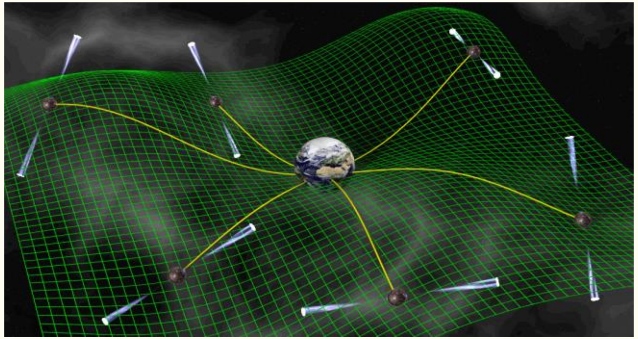
- Correlated signals (GW passing throung many pulsar locations)





[David Champion, MPIRA]

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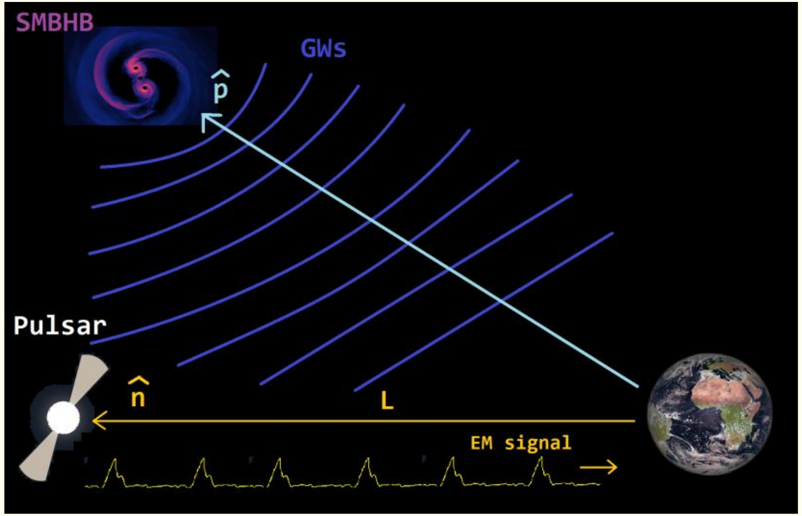

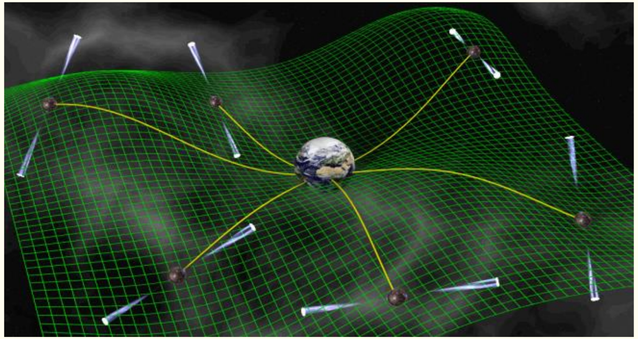

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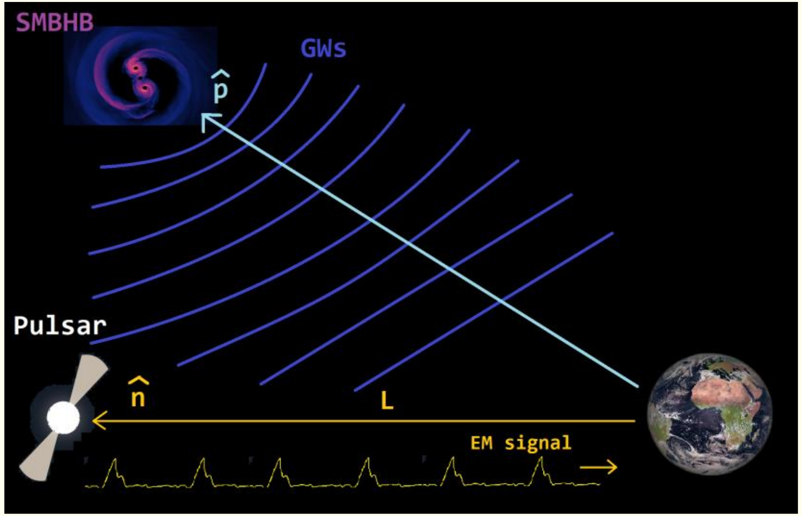

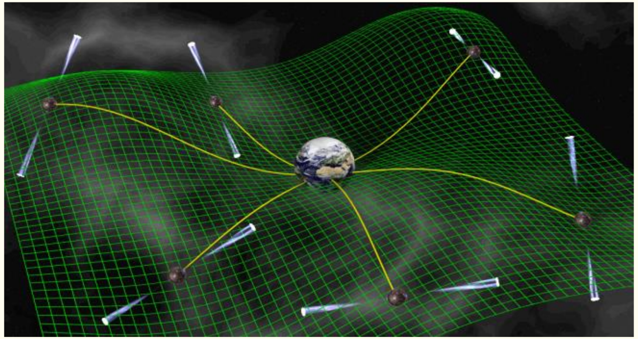

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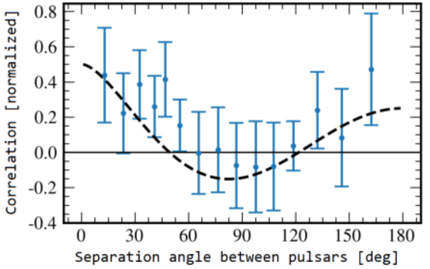
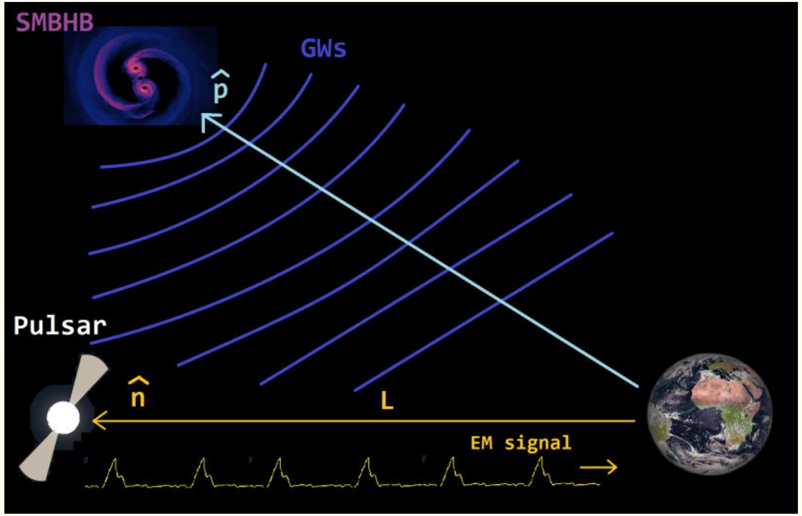
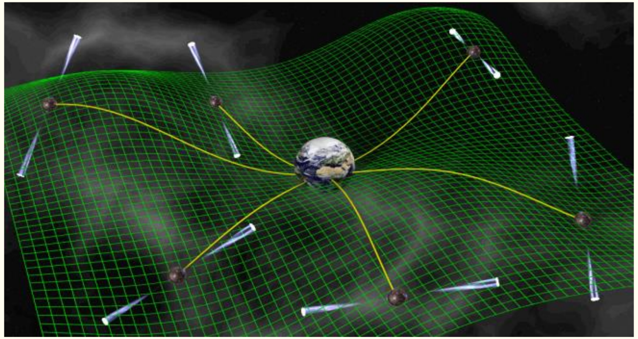


NANOGrav, 2023

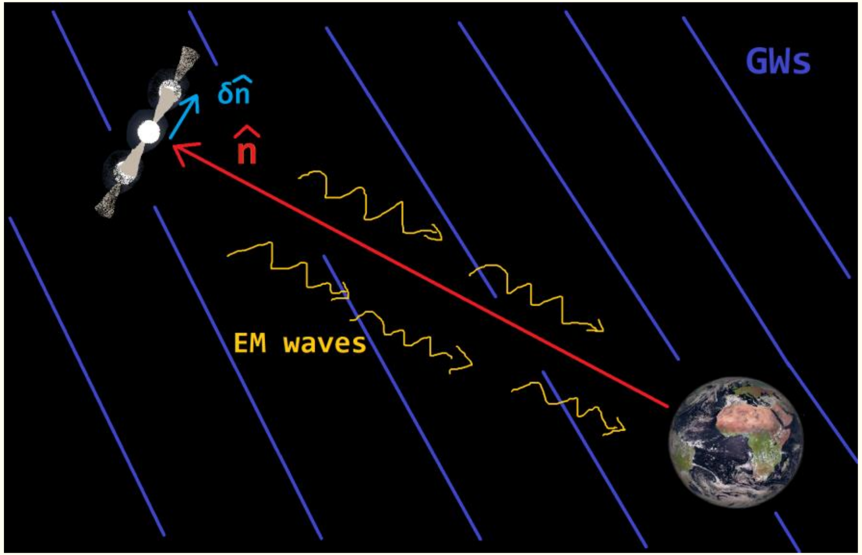




[David Champion, MPIRA]

GW Astrometry

- Light traveling through GWs: geodesics aberrated



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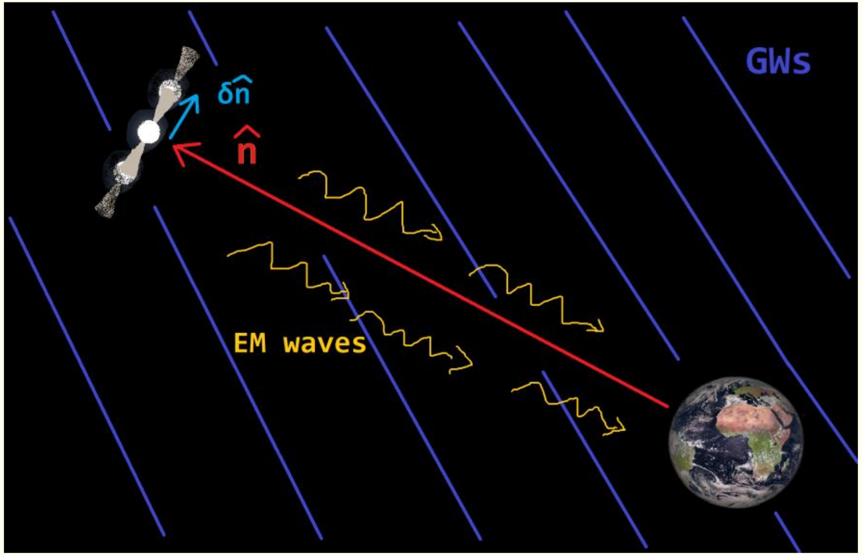
GW Astrometry

- Light traveling through GWs: geodesics aberrated
- The apparent position of objects in the sky varies in time

$$\delta n_i(t) = \frac{1}{2}\left[\frac{n_i - p_i}{1 - p \cdot n} n^j n^k - n^j \delta_i^k\right] h_{jk}(t)$$



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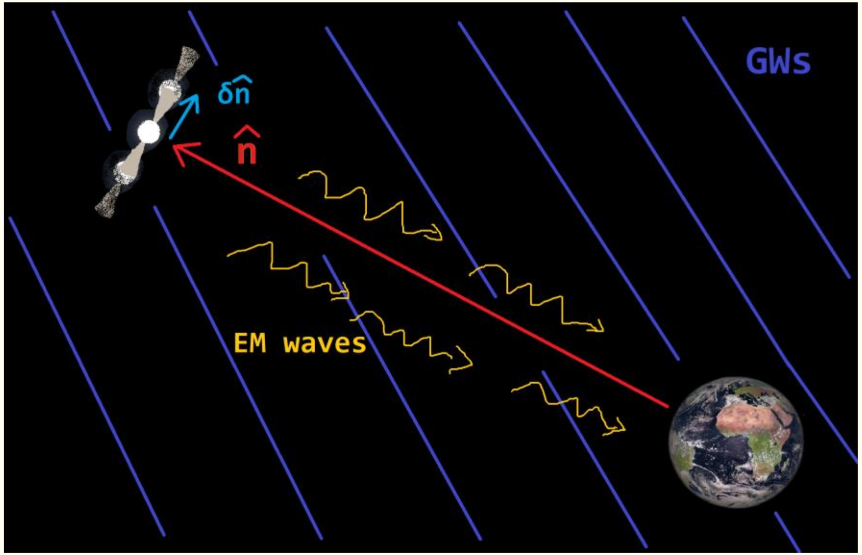

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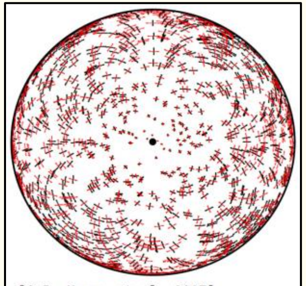
- E.g. Quadrupolar pattern from a single GW along the z-axis



[C.J. Moore et.al, 2017]

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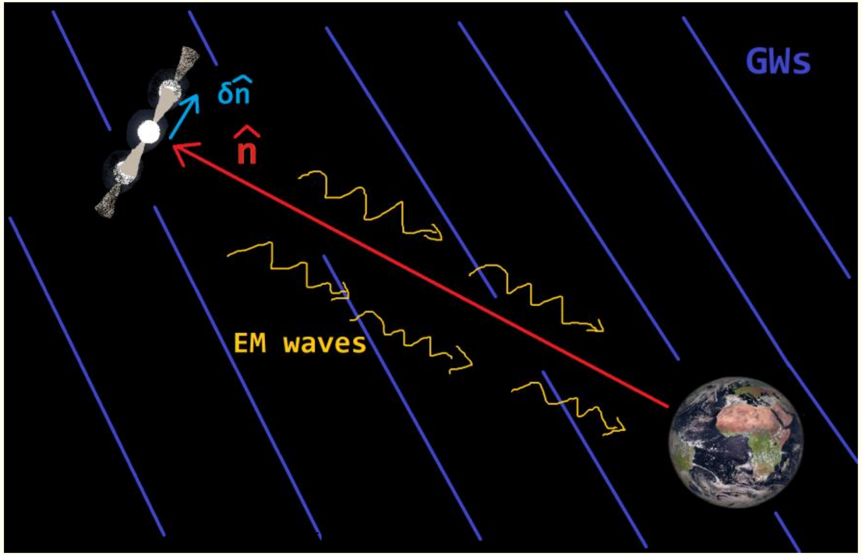

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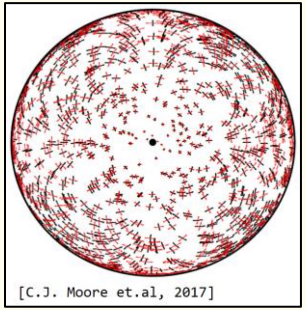

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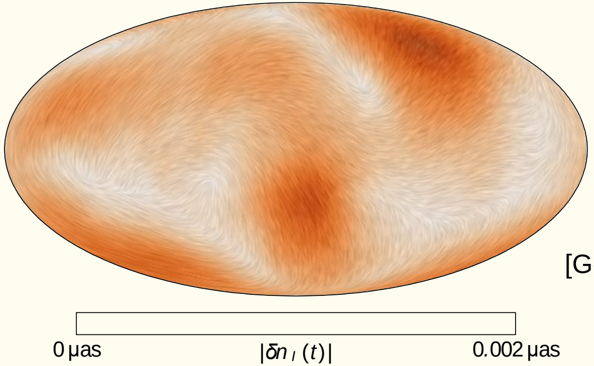

[Golat & Contaldi 2022]

Astrometry with GAIA

- Launched in 2013
- Observation of 10^9 sources with astrometric precision of 10-100 μas.
- Each source is observed 80 times (5-year nominal mission) —10^{-9}-10^{-7} Hz window.
- Extension to 8-10 years.

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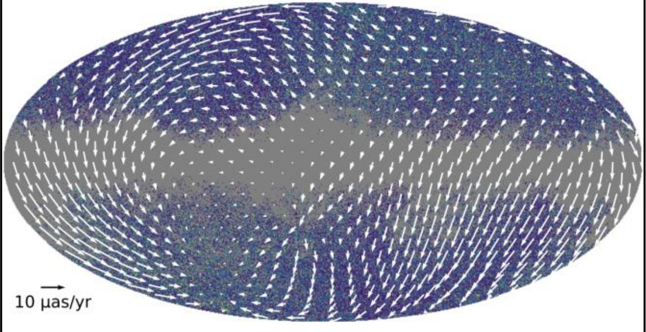
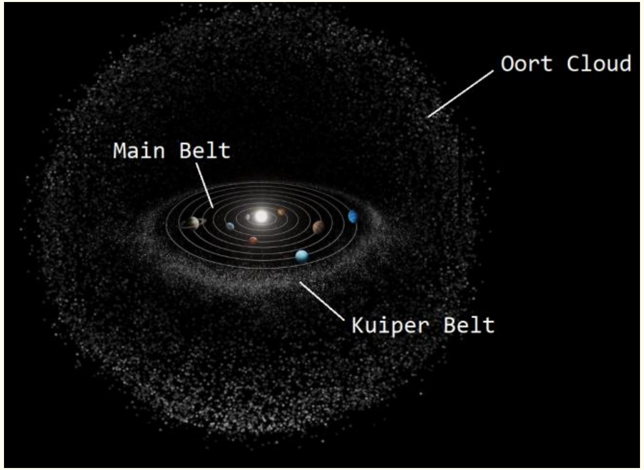
- $\Omega_{GW}<10^{-2}$ constraint on the stochastic GW background (cf. $\Omega_{GW}\sim10^{-8}$ from PTA)



[S.Jaraba et al., 2023]

Solar system GW Astrometry

- ~10^9 small-sized objects in the solar system, ~10^6 already known.
- Closer (L<< λ_{GW}) but fainter (apparent magnitude m > 9)



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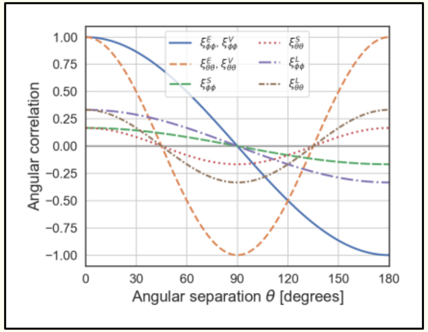
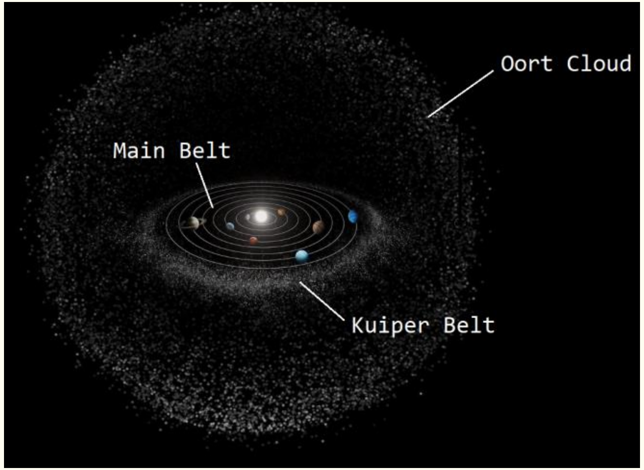


[GM & Contaldi 2024]



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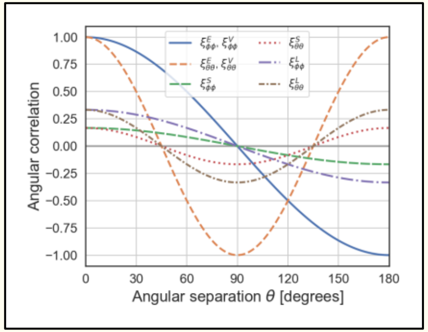

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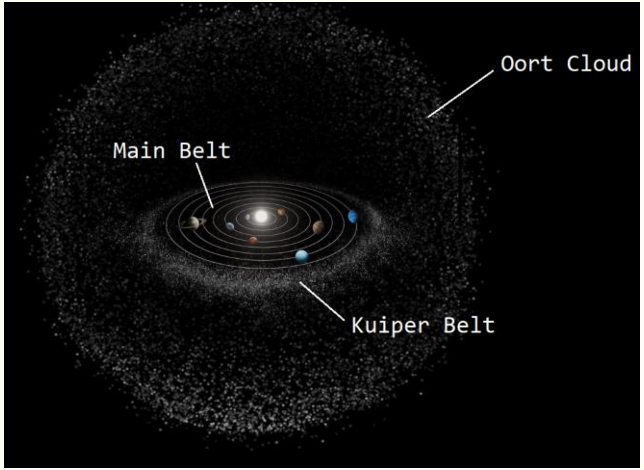

- LSST coming soon! (high accuracy, good cadence, widefield)

Vera C. Rubin (LSST)

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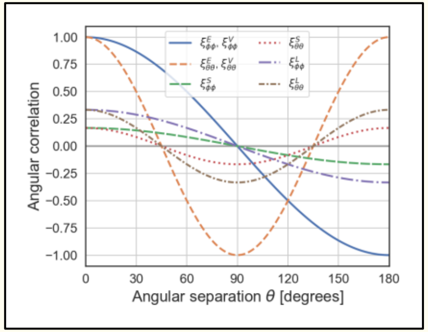
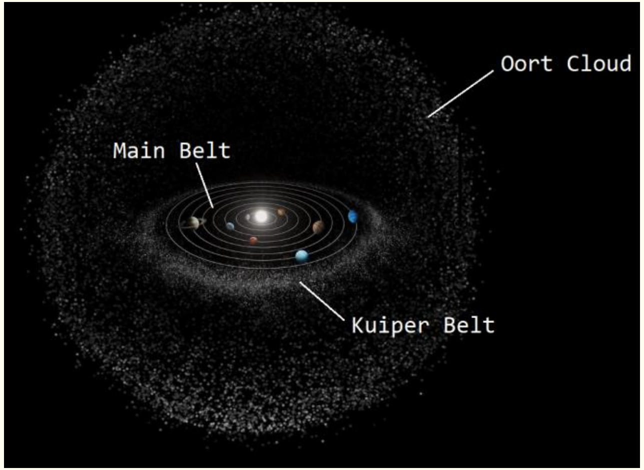
- Closer ($L \ll \lambda_{GW}$) but fainter (apparent magnitude $m > 9$)
- Angular correlation functions (like HD)

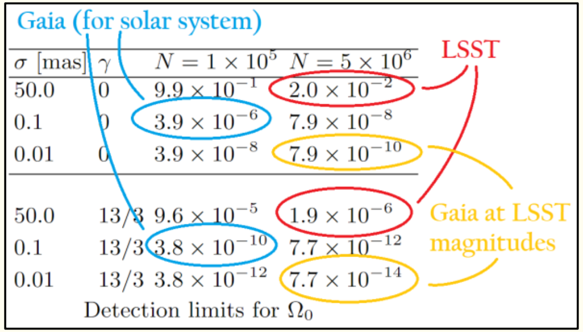


[GM & Contaldi 2024]



- LSST coming soon! (high accuracy, good cadence, widefield)
- Forecast detectability of the GW background

Vera C. Rubin (LSST)



Gaia (for solar system)

LSST

σ [mas]	γ	$N = 1 \times 10^5$	$N = 5 \times 10^6$
50.0	0	9.9×10^{-1}	2.0×10^{-2}
0.1	0	3.9×10^{-6}	7.9×10^{-8}
0.01	0	3.9×10^{-8}	7.9×10^{-10}
50.0	13/3	9.6×10^{-5}	1.9×10^{-6}
0.1	13/3	3.8×10^{-10}	7.7×10^{-12}
0.01	13/3	3.8×10^{-12}	7.7×10^{-14}

Gaia at LSST magnitudes

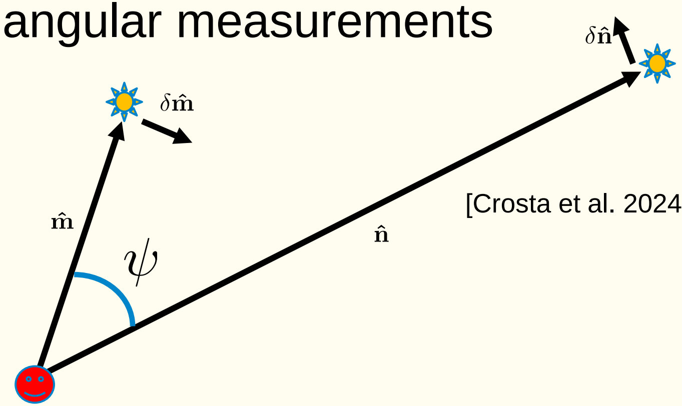
Detection limits for Ω_0

[GM & Contaldi 2024]

Other astrometric techniques

- Measuring absolute angles is difficult -> differential angular measurements

$$\delta\psi = -\frac{1}{\sin\psi}\left[\eta_{ij}(n^i\delta m^j + m^i\delta n^j) + h^{\rm GW}_{ij}n^in^j\right]$$



[Crosta et al. 2024]

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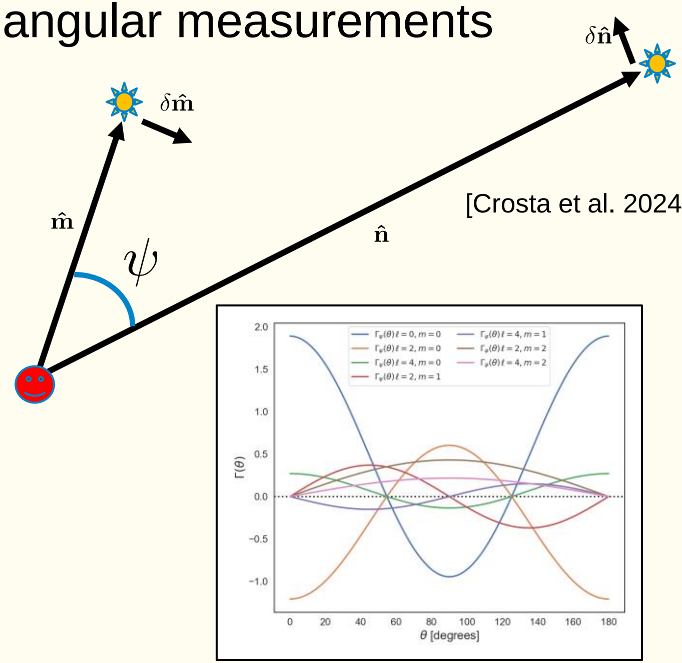
- Cross-correlating differential measurements $\langle \delta\psi(\mathbf{n},t)\delta\psi^*(\mathbf{n}',t')\rangle$



[Crosta et al. 2024]

[GM et al. in prep]

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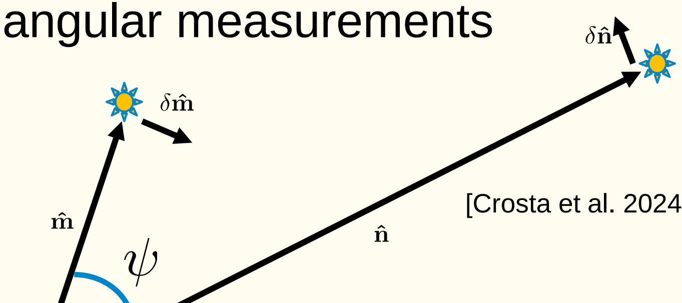
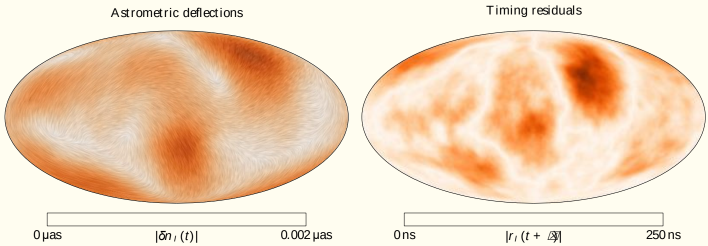
- Cross-correlating differential measurements $\langle\delta\psi(\mathbf{n},t)\delta\psi^*(\mathbf{n}',t')\rangle$.

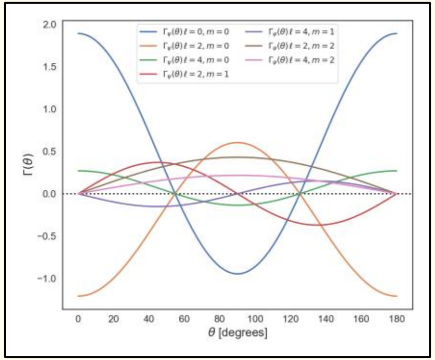
- Astrometry + PTA: independent and richer datasets



[Crosta et al. 2024]



[Golat & Contaldi, 2022]



[GM et al. in prep]

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Summary

- PTA shows evidence of a nHz GW signal.
- Astrometry as a probe of GWs is maturing.
- Astrometry + PTA (& solar + extrasolar astrometry) to mitigate sistematics.
- Data (optical surveys) is there, so use it
- Even more to come…