# True and coordinate singularity resolution in the planar AdS black hole

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based on arXiv:2409.XXXX
(coming out after the summer!)

LEVERHULME TRUST

$\langle G|\hbar\rangle$

# Outline

I. The classical spacetime

II. Quantisation and Unitarity

III. Results and conclusions

# I. The classical spacetime

**The planar AdS black hole** (BH) is a solution of the Einstein's equation with planar symmetry. The line element of the interior of the black hole can be written as [1]

$$\mathrm{d}s^2 = -N^2\mathrm{d}r^2 + v^{2/3}\left(e^{4k/3}\mathrm{d}t^2 + e^{-2k/3}(\mathrm{d}x^2 + \mathrm{d}y^2)\right) \qquad \Lambda < 0$$

where N, k, and v are functions of r. (In the interior of the BH the radial coordinate is timelike).

- v → "volume".
- k → Anisotropy variable (relative streching of $g_{tt}$ with respect to $g_{xx}$).
- N→ Lapse function.

[1]: S. Hartnoll, JHEP 23 (2023) 2208.04348

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We work in **Parametrised UniModular gravity** (or Henneaux and Teitelboim Unimodular gravity).

$$S_{PUM} = \frac{1}{\kappa}\int \mathrm{d}^4x \left[\sqrt{-g}\left(\frac{1}{2}R - \Lambda\right) + \Lambda\partial_\mu\mathcal{T}^\mu\right] + \text{ bound. terms}$$

The cosmological constant is now a dynamical variable (which happens to be a constant of motion)

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# I. The classical spacetime

With this metric Ansatz, the action becomes

$$S_{PUM} = \frac{1}{\kappa}\int \mathrm{d}r \left[\frac{(k')^2 v^2 - (v')^2}{3Nv} - \Lambda N v + \Lambda T'\right]$$

where ' means derivative with respect to r.

[2]: S. Gielen and LMP, Class. Quant. Grav 37 and 39 (2020 and 2021), 2005.05357, 2109.02660

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Hence, the **Hamiltonian** and constraint become (after a suitable unit choice for κ):

$$\mathcal{H} = \frac{\sqrt{3}}{2} N v \left(\frac{\pi_k^2}{v^2} - \pi_v^2 + \Lambda\right) \implies \mathcal{C} = \frac{\pi_k^2}{v^2} - \pi_v^2 + \Lambda \approx 0$$

This Hamiltonian is exactly **the same as the Hamiltonian of a flat homogeneous and isotropic cosmology** with a free massless scalar field and dark energy [2]!

This toy model has 3 canonically conjugated pairs $\{T, \Lambda\} = \{k, \pi_k\} = \{v, \pi_v\} = 1$

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# I. The classical spacetime

One can deparametrise the theory using T to find

$$v(T) = \sqrt{-\frac{\pi_k^2}{\Lambda} + 4\Lambda(T - T_0)^2}$$

$$k(T) = \frac{1}{2}\log\left|\frac{\pi_k - 2\Lambda(T - T_0)}{\pi_k + 2\Lambda(T - T_0)}\right| + k_0$$

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This metric has two singularities:

- $v \to 0$ and $k \to \infty$ at $T=T_-=T_0-\pi_k/2\Lambda$, this one is a **true singularity** (Kretschmann scalar diverges).
- $v \to 0$ and $k \to -\infty$ at $T=T_+=T_0+\pi_k/2\Lambda$, this one is a **coordinate singularity** (Kretschmann is finite).

$T_-$ corresponds to the **BH singularity** and $T_+$ to the **BH horizon**.

# II. Quantisation and Unitarity

The constraint can be written as $\mathcal{C} = g^{AB}\pi_A\pi_B + \Lambda$ where $g_{AB} = \begin{pmatrix} v^2 & 0 \\ 0 & -1 \end{pmatrix}$

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This leads to the **Wheeler-DeWitt equation**

$$(\Box + \Lambda)\Psi = \left(-\frac{1}{v^2}\partial_k^2 + \partial_v^2 + \frac{1}{v}\partial_v + \Lambda\right)\Psi = 0$$

Where **“box” is the Laplace-Beltrami operator associated with g**.

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We use **T as relational time**, so “box” becomes the Hamiltonian of the system hence the **physical inner product** is:

$$\langle\Phi|\Psi\rangle = \int_0^\infty \mathrm{d}v\ v \int_{-\infty}^\infty \mathrm{d}k\ \Phi^*\Psi$$

# II. Quantisation and Unitarity

The solutions to the Wheeler-DeWitt equation are **Bessel functions in a superposition of cosmological constants**

$$\Psi(v,k,\Lambda) = \int \frac{\mathrm{d}p}{2\pi} e^{ipk} \left( \alpha(p,\Lambda) J_{ip}(\sqrt{\Lambda} v) + \beta(p,\Lambda) J_{-ip}(\sqrt{\Lambda} v) \right)$$

However, **this theory is not unitary** (the operator "box" is not self-adjoint), but **Unitarity can be restored** by **the boundary condition**

$$\lim_{v \to 0} \left( \Phi^* \partial_v \Psi - \Psi \partial_v \Phi^* \right) = 0$$

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This boundary condition ensures that there is no probability flux “past” v=0.
The solutions to the Wheeler-DeWitt equation and the boundary condition are

$$\Psi(v,k,T) = \int \frac{\mathrm{d}p}{2\pi} e^{ipk} \sum_{n\in\mathbb{Z}} e^{i\Lambda_n^p T} \sqrt{\frac{-2\Lambda_n^p \sinh(|p|\pi)}{|p|\pi}} \alpha(p,\Lambda_n^p) K_{i|p|} \left( \sqrt{-\Lambda_n^p} v \right)$$

# II. Quantisation and Unitarity

where $\Lambda_n^p = -e^{-\frac{(2n+1)\pi}{|p|}+\theta(p)}$

Parametrises the self-adjoint extensions

The parameter θ(p) is left up to choice. This will influence quantitatively the dynamics and could have applications. We choose

$$\theta(p) = \pi/|p|$$

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Close to v → 0, the Bessel functions look like plane waves

$$K_{i|p|}\left(\sqrt{-\Lambda_n^p}v\right) \xrightarrow[v\to 0]{} \frac{1}{2}\left(\Gamma(-i|p|)e^{i|p|\log\left(\frac{\sqrt{-\Lambda}}{2}v\right)} + \Gamma(i|p|)e^{-i|p|\log\left(\frac{\sqrt{-\Lambda}}{2}v\right)}\right)$$

Hence, close to the coordinate singularity and the BH singularity, the wave functions look like a **superposition of plane waves both incoming and outgoing from the singularities**.

# III. Results and conclusions

To analyse singularity resolution, we calculate **expectation values of observables**

$\langle\Psi| v |\Psi\rangle = \langle v\rangle (T)$, where Ψ is a suitable semiclassical state

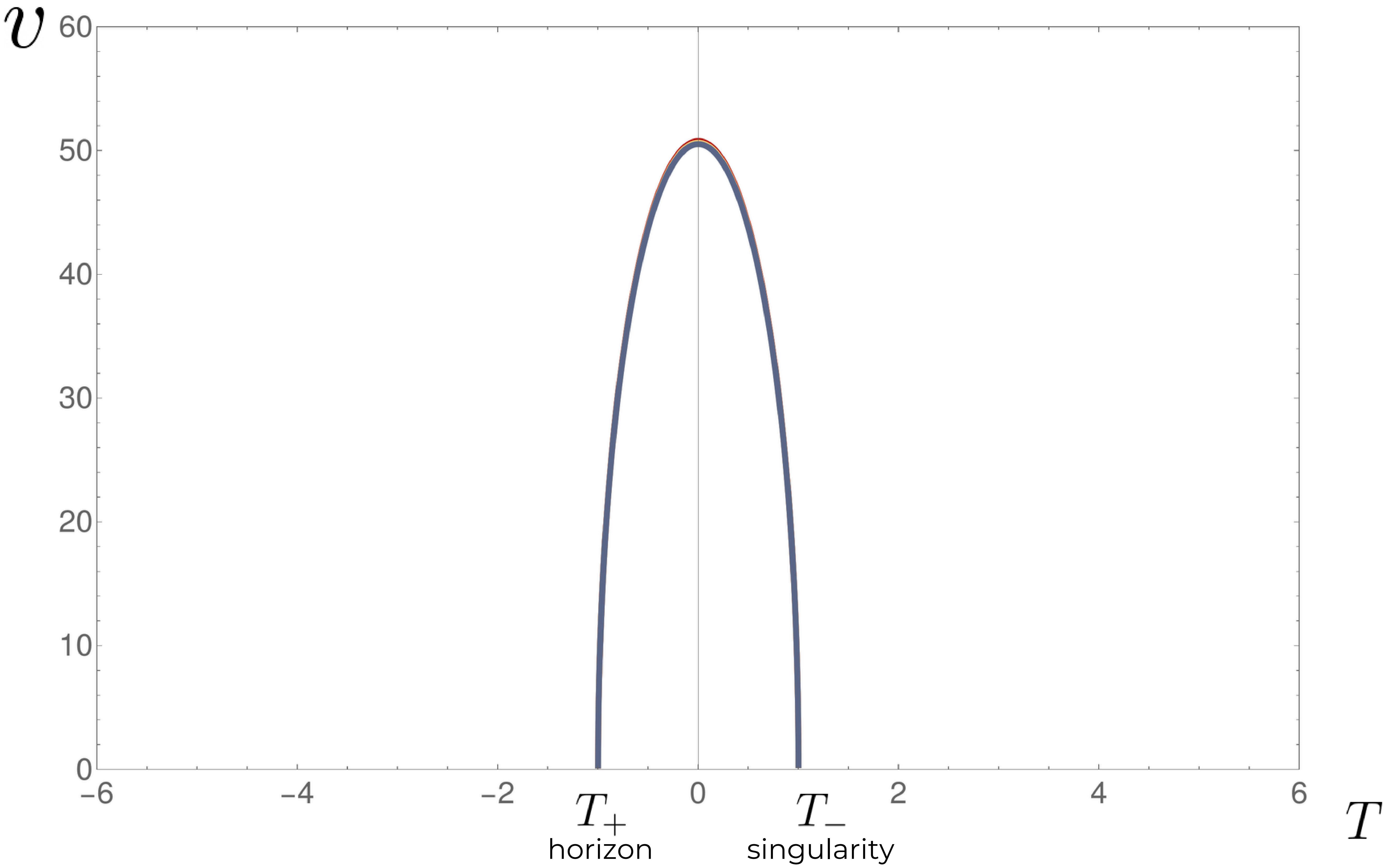
$$\alpha(p, \Lambda_n^p) = e^{-(p-p_0)^2/(2\sigma_p^2)-(\Lambda_n^p-\Lambda_0)^2/(2\sigma_\Lambda^2)}$$

Gaussian superposition

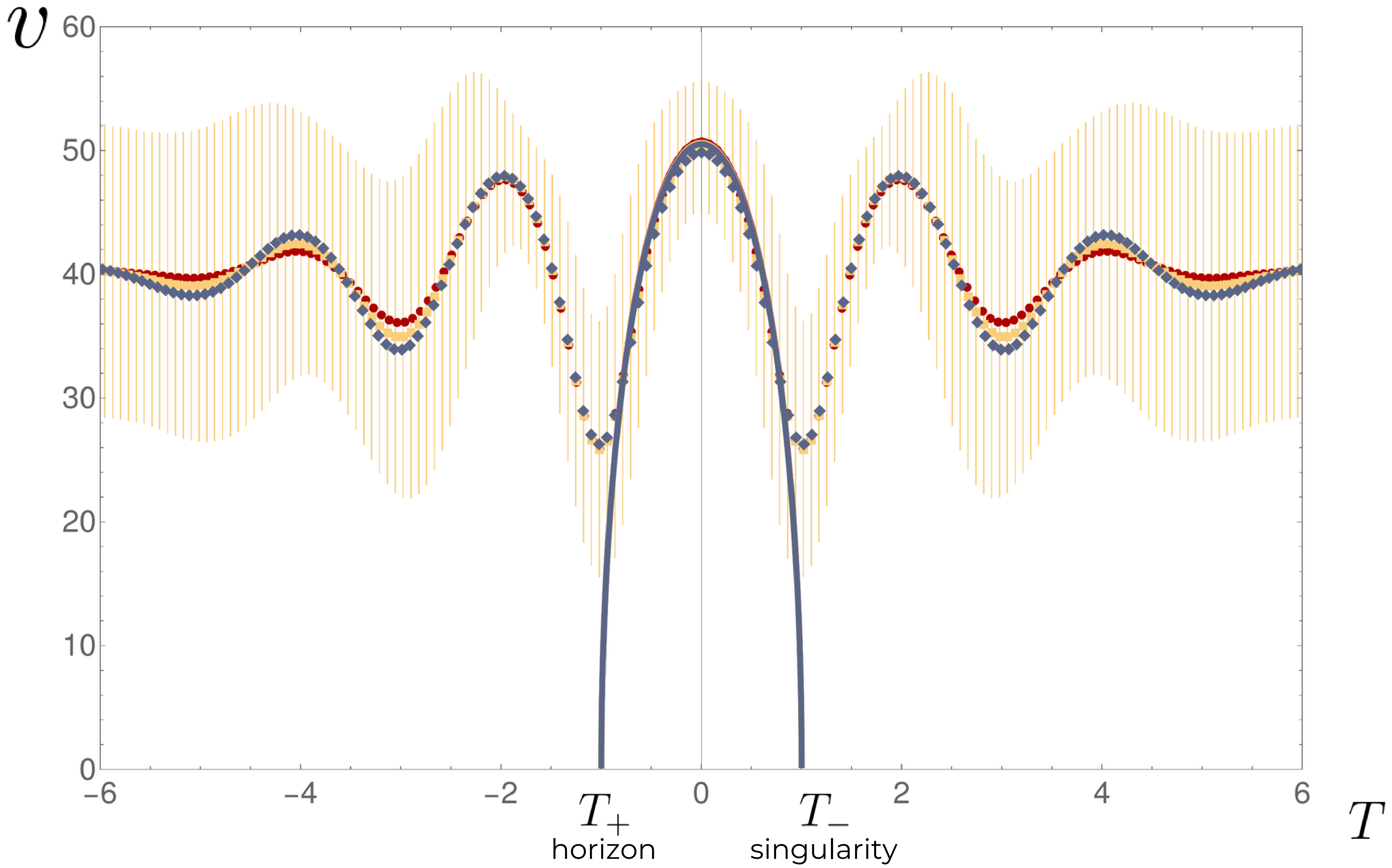
We compare the quantum expectation values with classical trajectories for which

$$\langle\Psi| \Lambda |\Psi\rangle = \Lambda_{eff}, \quad \langle\Psi| p |\Psi\rangle = \pi_{k,eff}$$

# III. Results and conclusions



# III. Results and conclusions



# III. Results and conclusions

Some comments on the figure

- **Both the singularity and the black hole horizon are “resolved”.**
- The state is semiclassical (small variance) for T’s in which the classical solution is well defined but then the variance grows.
- We are unsure the oscillations continue or stop after some point (numerics might not be reliable).
- Different values of the parameters influence the oscillations pattern.

# III. Results and conclusions

Conclusions:

- As it has also been shown in cosmology, **BH singularity resolution can also occur in the WDW theory**.
- Note tha the **resolution of the BH horizon could potentially be avoided** by choosing a different foliation of spacetime.

Possible future directions:

- LQC quantisation of the same spacetime
- Can the exterior of the BH tell us something about the interior? Junction conditions? Choice of self-adjoint extensions?
- How to go past homogeneity?

# Thank you!

write me maybe
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