Entanglement of fields in evaporating black holes

[arXiv:2407.03031]

$(G = \hbar = c = k_B = 1)$

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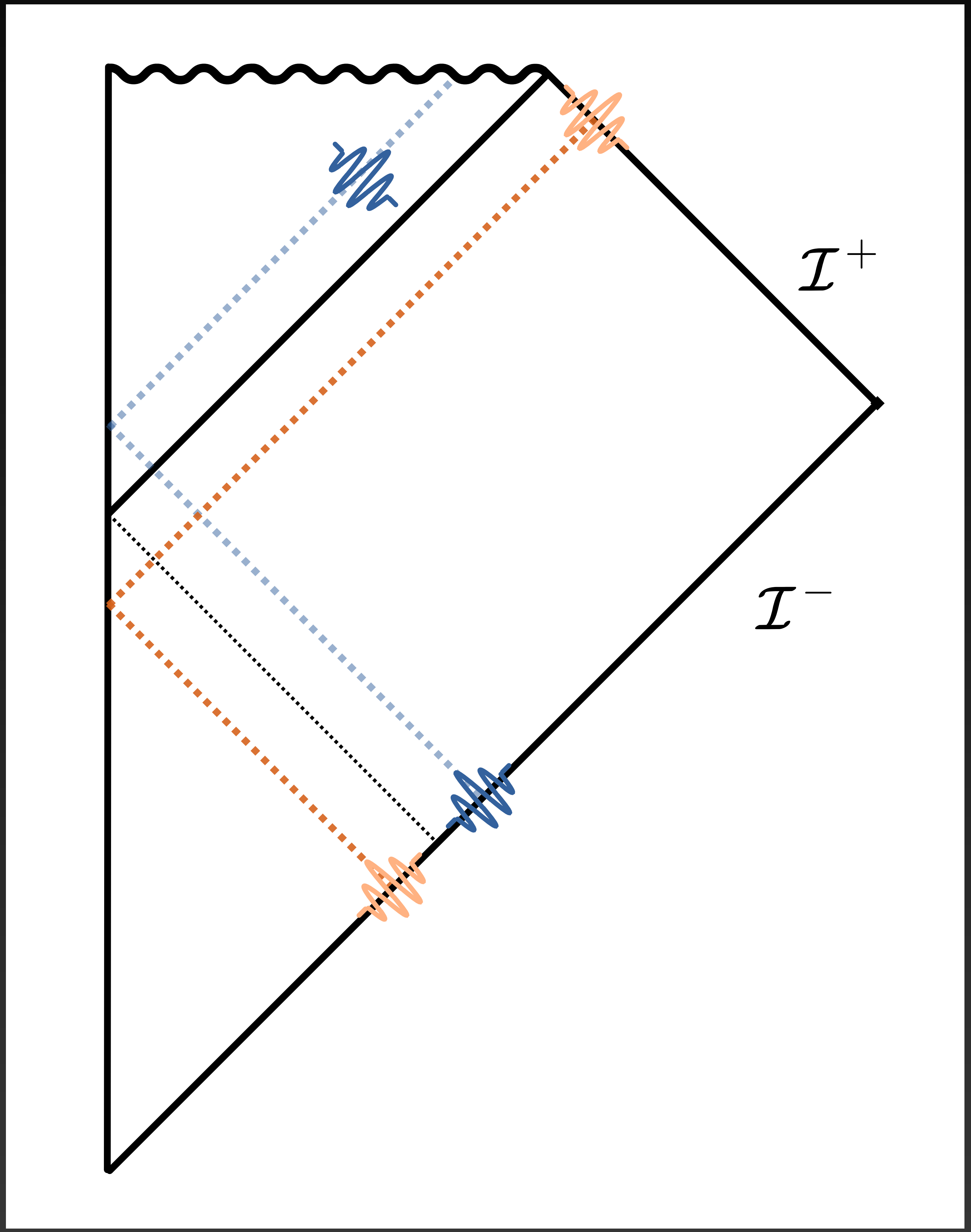
In collaboration with Ivan Agullo and Paula A. Calizaya Cabrera

QFT & black holes

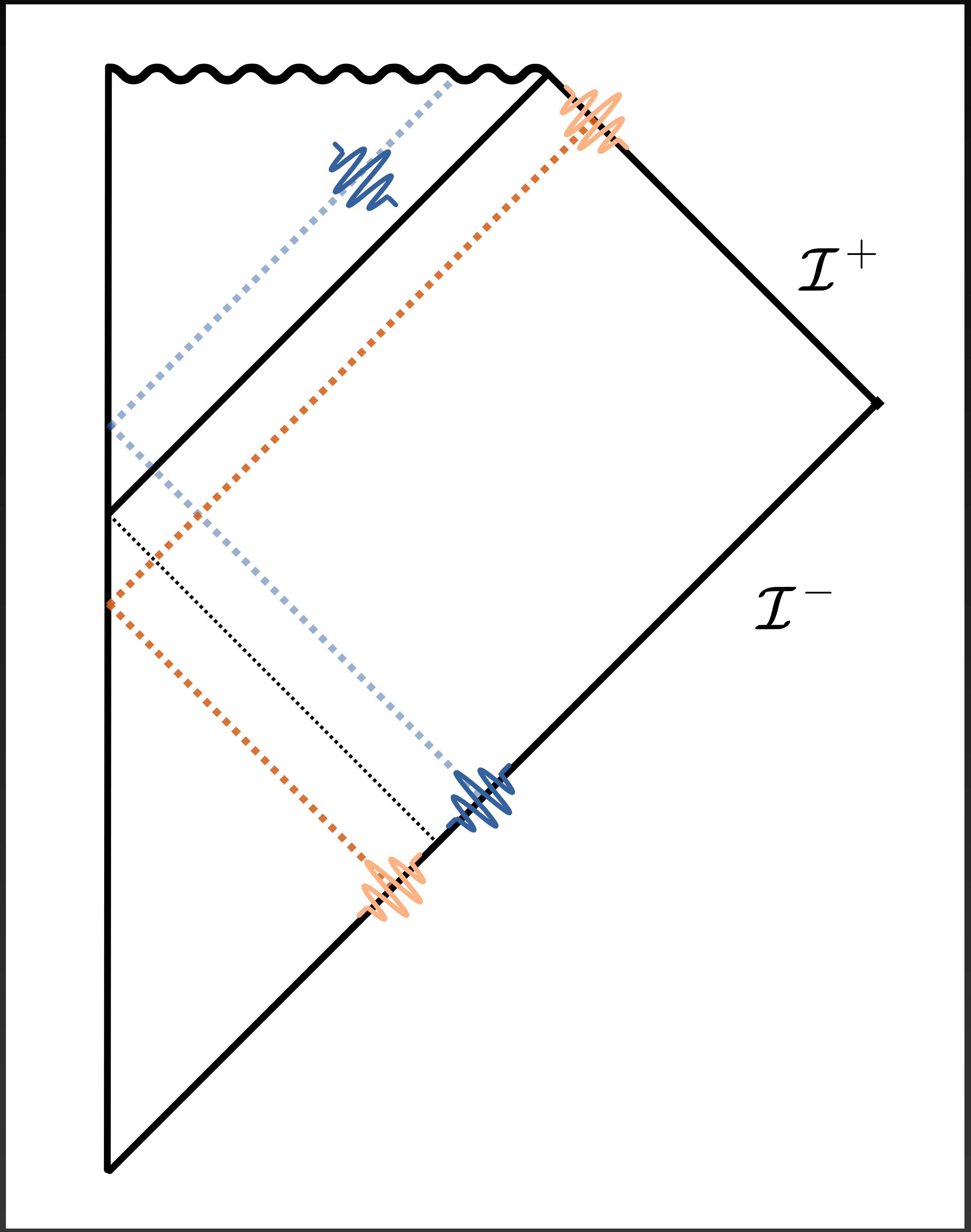
Source: Kurzgesagt

The Hawking effect



- Massless test field in gravitational collapse.
- Hawking modes: Inertial particles at $\mathscr{I}^+$.
- Natural vacuum at $\mathscr{I}^-$ shows [Hawking1975, Wald1975]
  - ✺ Hawking excitations in thermal state.

The Hawking effect



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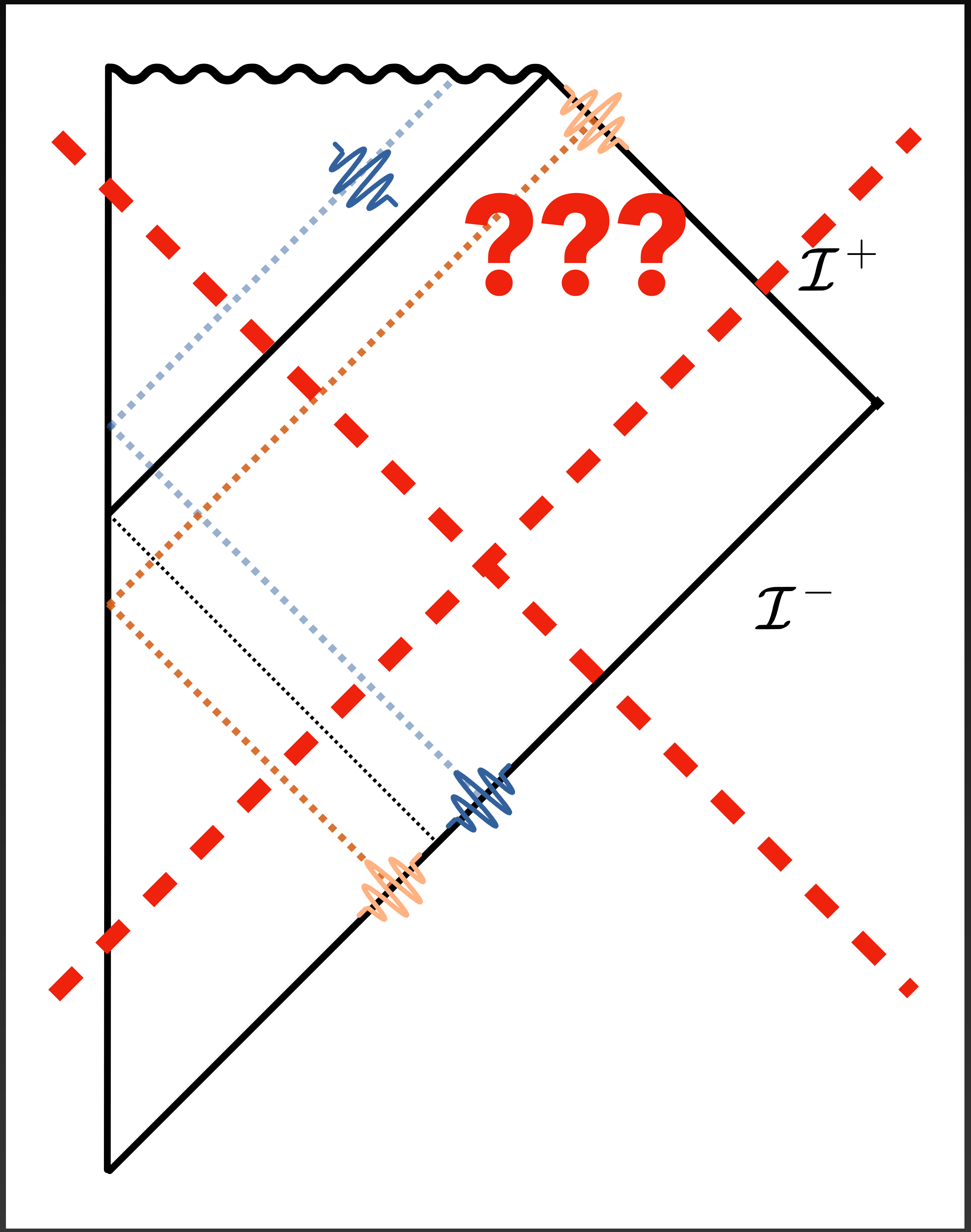
- Natural vacuum at $\mathscr{I}^-$ shows [Hawking1975, Wald1975]
  - Hawking excitations in thermal state.
  - Pairwise entanglement with "partners".
- Partners: Reflection across **event** horizon at $\mathscr{I}^-$.

[Wald1975]

With back-reaction…



- At $\mathscr{I}^+$, thermal radiation with $T = (8\pi M)^{-1}$.
- Energy considerations ⟶ Mass loss over time.

[Hawking1975; Page1976; Christensen&Fulling1977,…]

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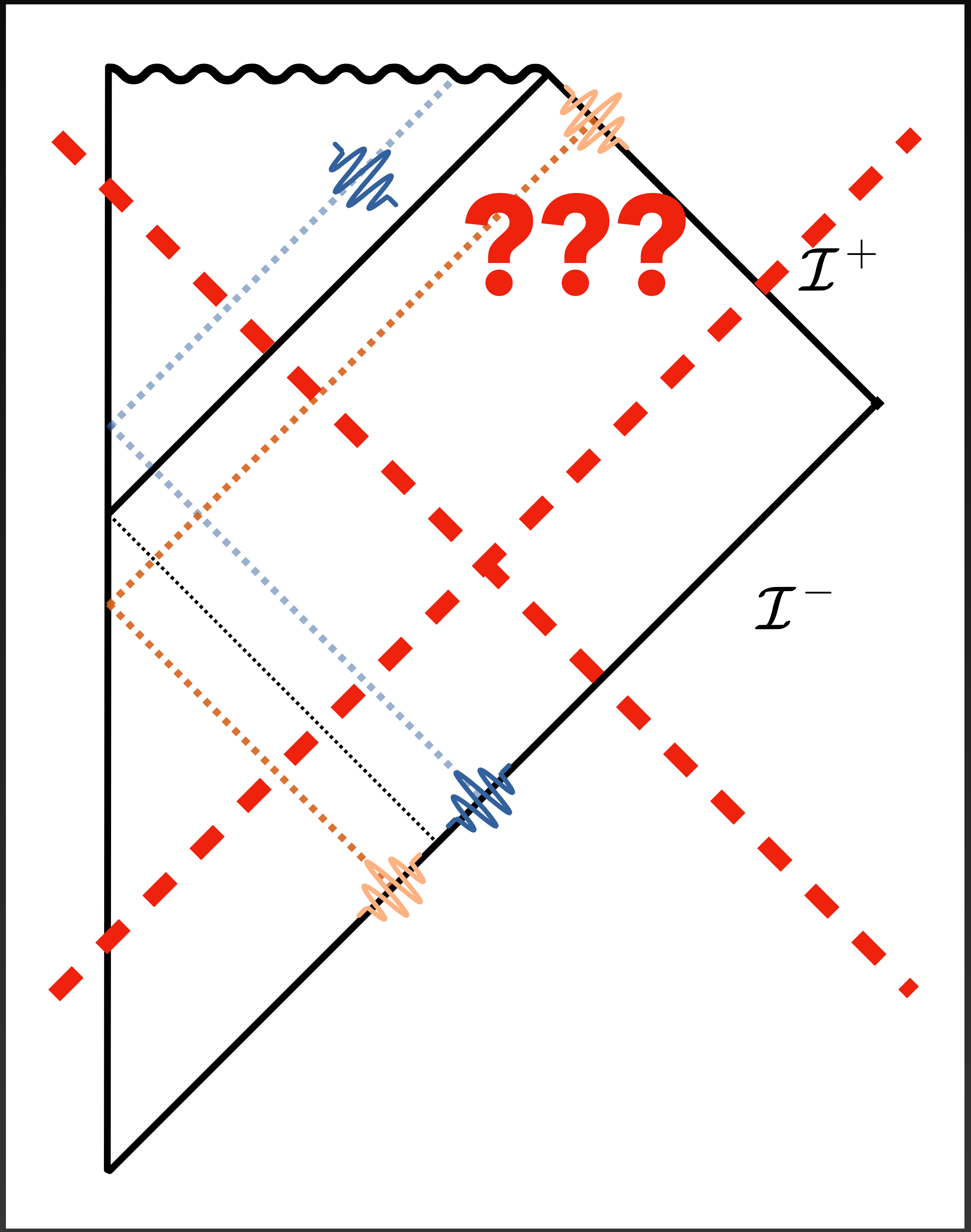

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- Energy considerations → Mass loss over time.

Information (loss)?!

Event horizons are global (teleological)…

What are Hawking partners "without" them?

Modeling evaporation

Importance of null rays

- Asymptotically flat spacetime, $v, u \rightarrow$ affine parameters at $\mathscr{I}^-, \mathscr{I}^+$.

- Null rays naturally define a map $v = p(u)$ between $\mathscr{I}^-$ and $\mathscr{I}^+$.

    ✹ Long believed to determine particle content of quantum fields at $\mathscr{I}^+$.

    e.g. [Hajicek 1987; Hu 1996; Visser 2003; Barceló, Liberati, Sonego, Visser 2011; Frolov, Zelnikov 2018]

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- **Important observation**: Any map that locally satisfies

$$\dot{p}(u) \approx A_\star e^{-\kappa_\star u},$$

around $u = u_\star$ leads to Hawking radiation at $\mathscr{I}^+$ with temperature $\kappa_\star$.

Evaporating black holes

- Our physical hypotheses:

  1. Global dynamics of fields is ruled by $v = p(u)$, up to back-scattering.

  2. There is "time-dependent" Hawking radiation at $\mathscr{I}^+$ for $u \in [u_0, u_{Pl}]$:

$$T(u) \approx \frac{1}{8\pi M(u)}, \qquad \dot{M}(u) = -\frac{\alpha}{M(u)^2}, \qquad \alpha \sim 10^{-4}$$

value of α : [Page2013]

- Mathematically, this means that we locally require

$$\dot{p}(u) \approx A_\star e^{-\kappa_\star u}, \quad \kappa_\star = \frac{1}{4M(u_\star)}$$

on intervals $[u_\star - \Delta u, u_\star + \Delta u]$ such that $M(u_\star) \ll \Delta u \ll M(u_\star)^2/\sqrt{\alpha}$.

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Similar quantity already introduced in [Barceló, Liberati, Sonego, Visser 2011]

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"Instantaneous would-be horizon"

$$\dot{p}(u) \approx A_\star e^{-\kappa_\star u}, \quad \kappa_\star = \frac{1}{4M(u_\star)} \longrightarrow p(u) \approx v_\star^{(H)} - \kappa_\star^{-1} \dot{p}(u_\star) e^{-\kappa_\star (u - u_\star)}$$

on intervals $[u_\star - \Delta u, u_\star + \Delta u]$ such that $M(u_\star) \ll \Delta u \ll M(u_\star)^2/\sqrt{\alpha}$.

Hawking partners

Partners in general

e.g. [Hotta, Schützhold, Unruh 2015; Trevison, Yamaguchi, Hotta 2019; Hackl, Johnson 2019]

- Let $|0\rangle$ be the "inertial" vacuum of a massless scalar at $\mathscr{I}^-$.

- Single-mode subsystem: Algebra generated by any pair $(\hat{a}_A, \hat{a}_A^\dagger)$ s.t.

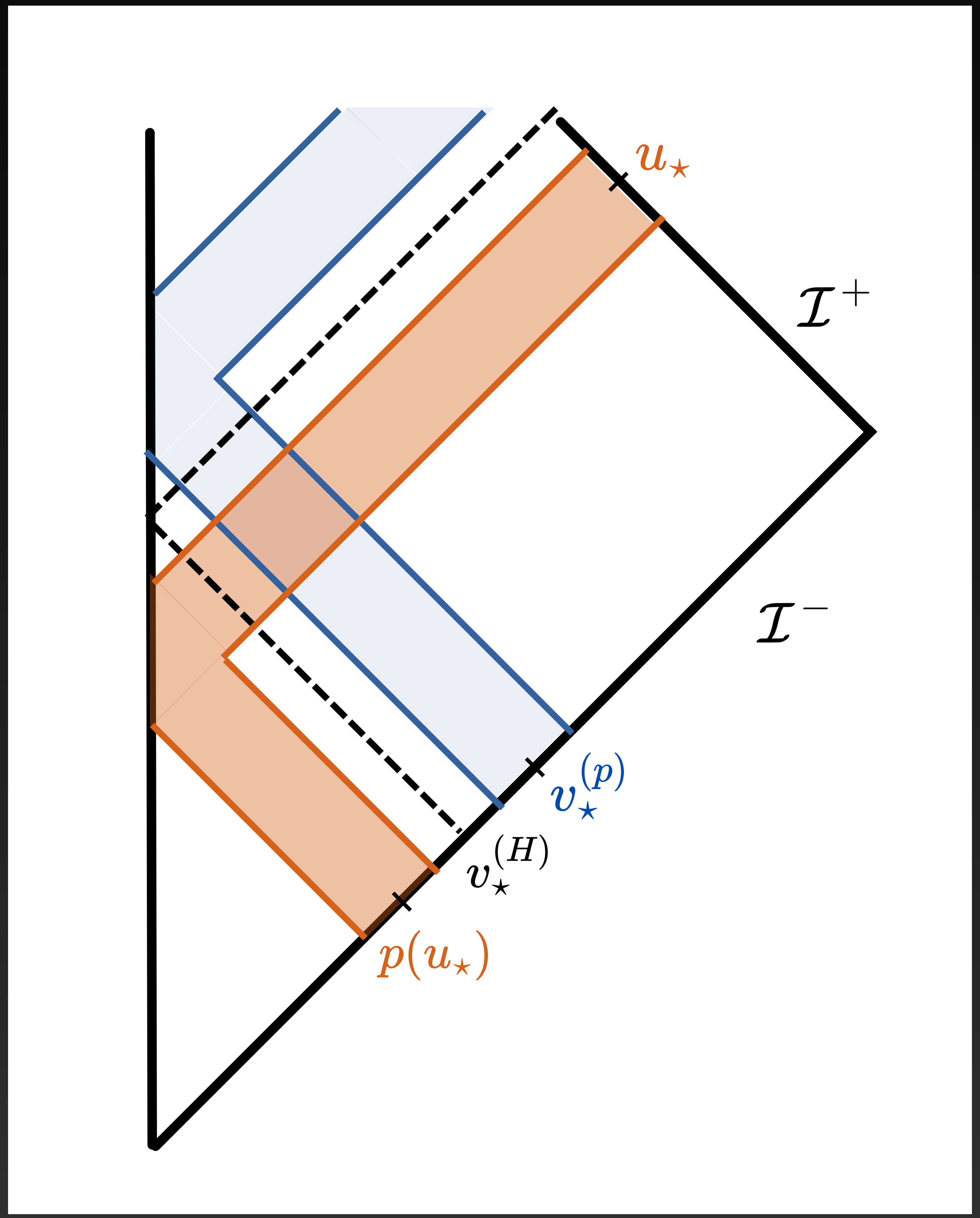
$$\hat{a}_A = \sum_{l,m} \int_0^\infty d\omega \left[\alpha_{\omega lm} \hat{a}_{\omega lm} + \beta_{\omega lm} \hat{a}^\dagger_{\omega lm} \right], \quad \left(\sum_{lm} \int_0^\infty d\omega \left| \beta_{\omega lm} \right|^2 < \infty \right)$$

- "Trace" of $|0\rangle$ over all d.o.f. but one single-mode subsystem A can be mixed.

- If reduced state is mixed, single mode-subsystem that purifies it: **Partner**.

Evaporating black holes

- Our definition of Hawking mode $f_\star$ at $\mathscr{I}^+$:
  - ✹ Truncated "+frequency." wave-packet, C_0^∞ .
  - ✹ Support within exponential approximation.
- $f_\star$ defines a single-mode subsystem.
- Evolution to $\mathscr{I}^-$: Geometric optics.
- Partner $f_{\star_P}$ ≈ Reflection of $f_\star$ across $v_\star^{(H)}$.



Physical consequences

$f_{\star_p} \approx$ Reflection of $f_\star$ across $v_\star^{(H)}$

Where are the partners centered at $\mathscr{I}^-$?

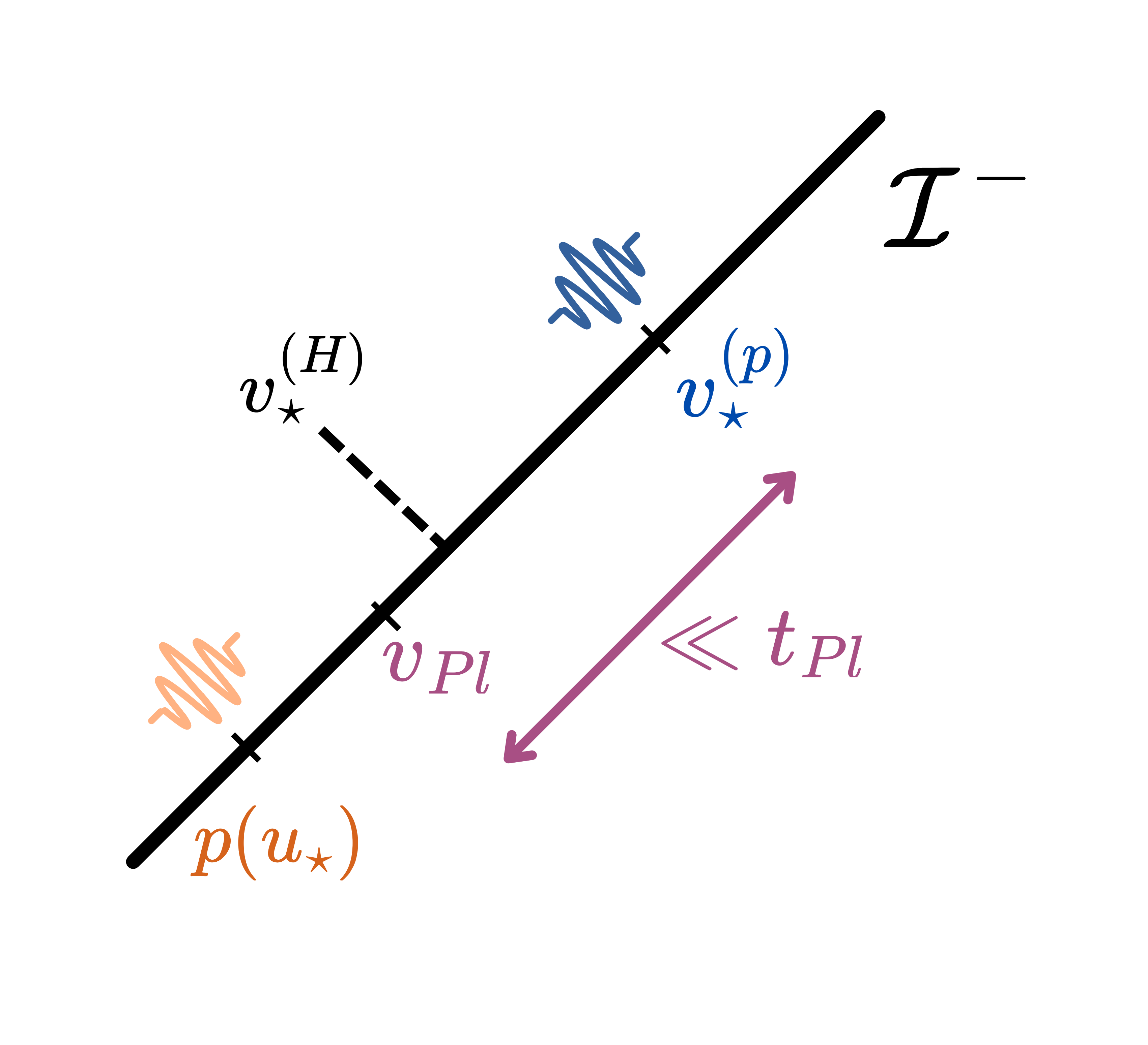
$$v_\star^{(p)} = 2v_\star^{(H)} - p(u_\star)$$

Result on location of partners

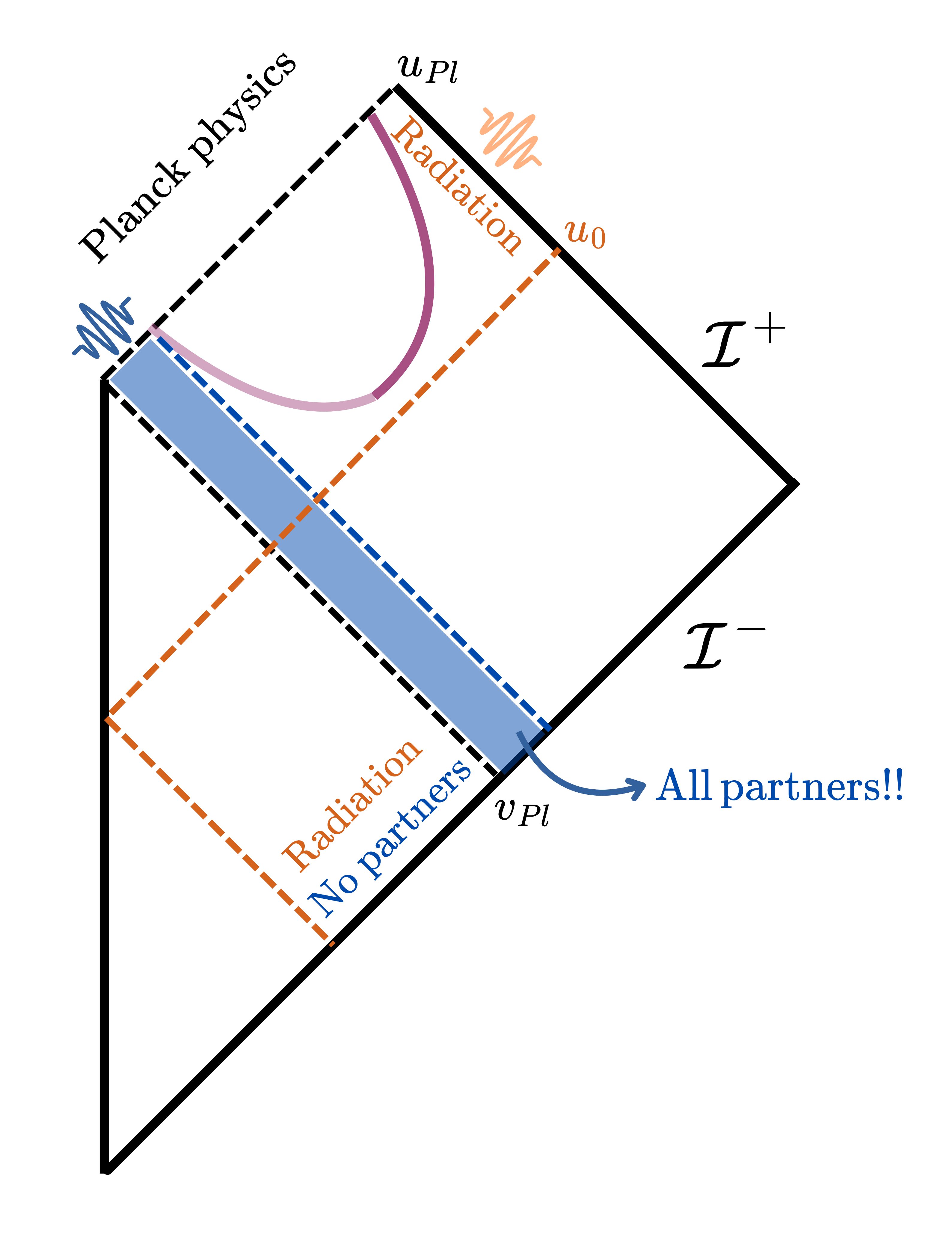
Black hole physics beyond GR may be crucial!

$$0 < v_\star^{(p)} - p(u_{Pl}) \ll t_{Pl}$$

Partners leave $\mathscr{I}^-$ after the last ray that explores semiclassical physics!



Realistic scenario



- Partners cannot leak out semiclassically.
- They must explore the "quantum" black hole if:
  - ❋ Ray $v = p(u_{Pl})$ traverses a trapped region.
  - ❋ Standard GR holds in collapsing region.
  - ❋ Light suffers redshift in collapsing region.
- Reason: Behavior of expansion of null rays.

Conclusions

- General & conservative QFT study.
- Recipe for partners in evap. BHs.
- Info cannot escape semiclassically.
- All partners enter the "quantum" BH.
- Backscattering small but not negligible.

Source: Kurzgesagt

Quantum gravity needed