Evolution of creases on the event horizon of a black hole merger

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Based on 2407.07962 with Max Gadioux and Harvey Reall

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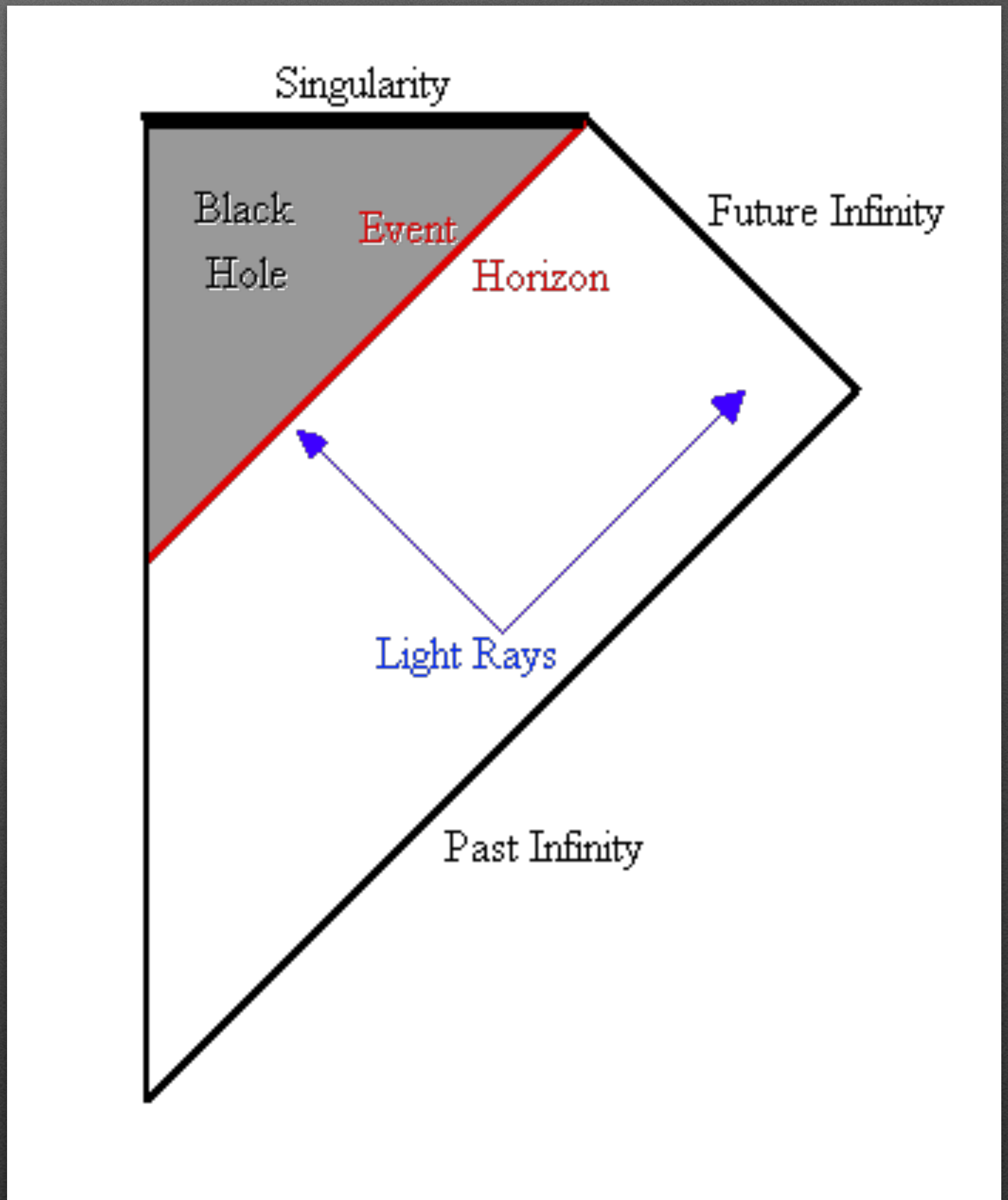
Black Holes

- ***Event horizon:*** Past boundary of future null infinity
- Event horizon ruled by ***generators***: null geodesics which have ***no future endpoints*** but may have past endpoints



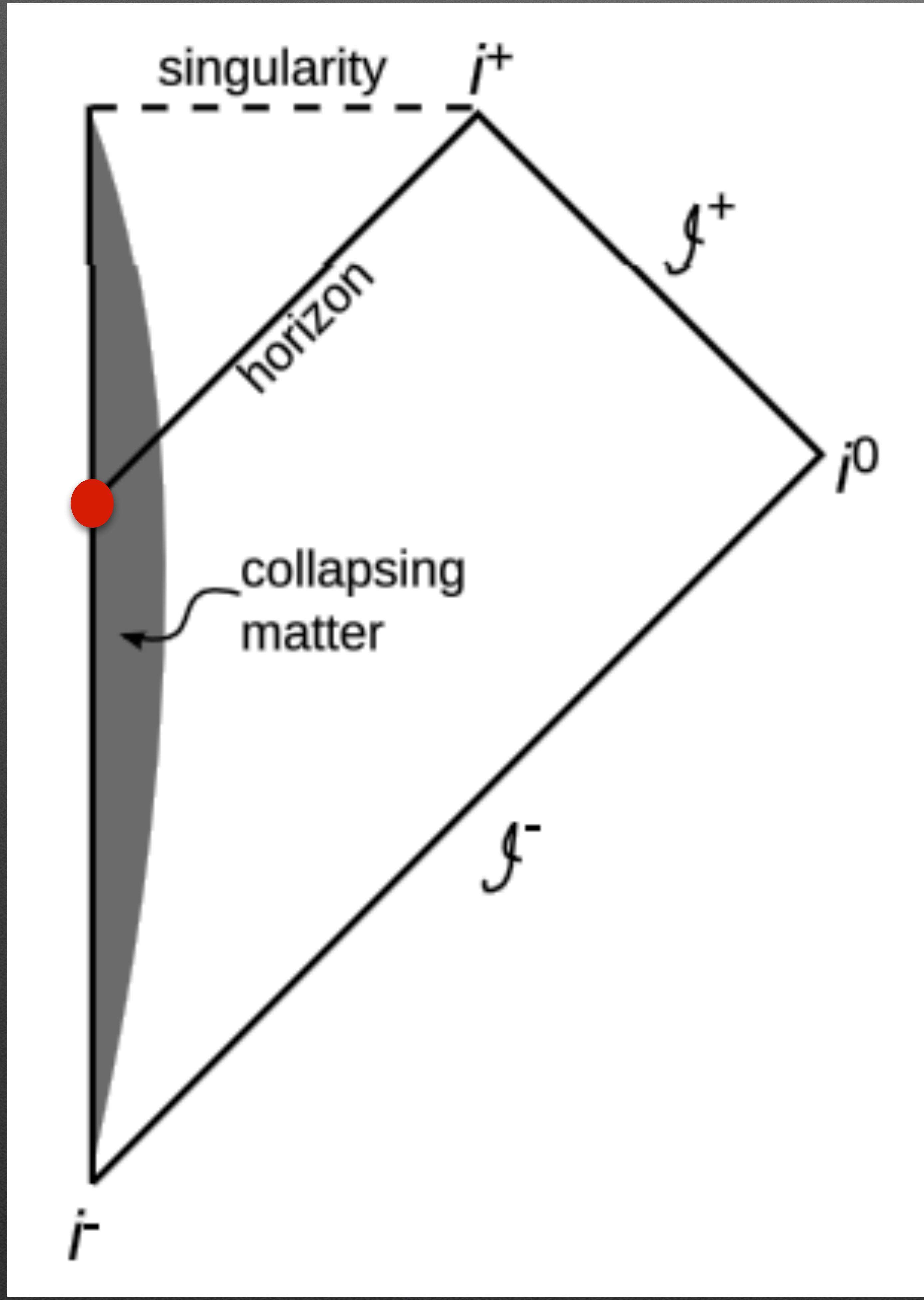
Horizons & Discovery

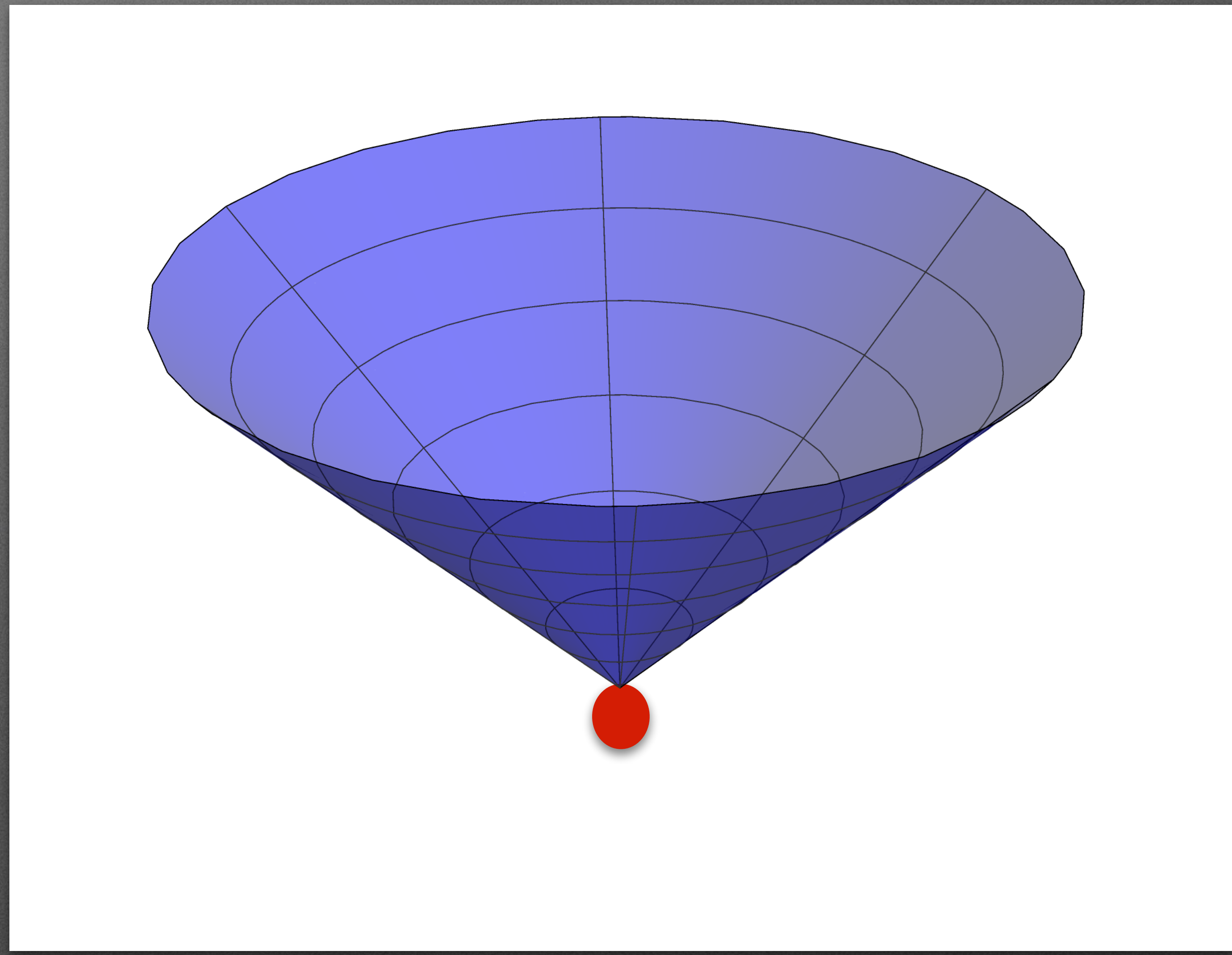
- **Rich mathematical structure** ⇒ can say a lot about their properties
- **Fundamental physics** ⇒ profound connections between gravity, thermodynamics, and quantum physics
- **Tests of GR in the strong field regime** ⇒ GWs, EHT

Dynamical Horizons are Not Smooth

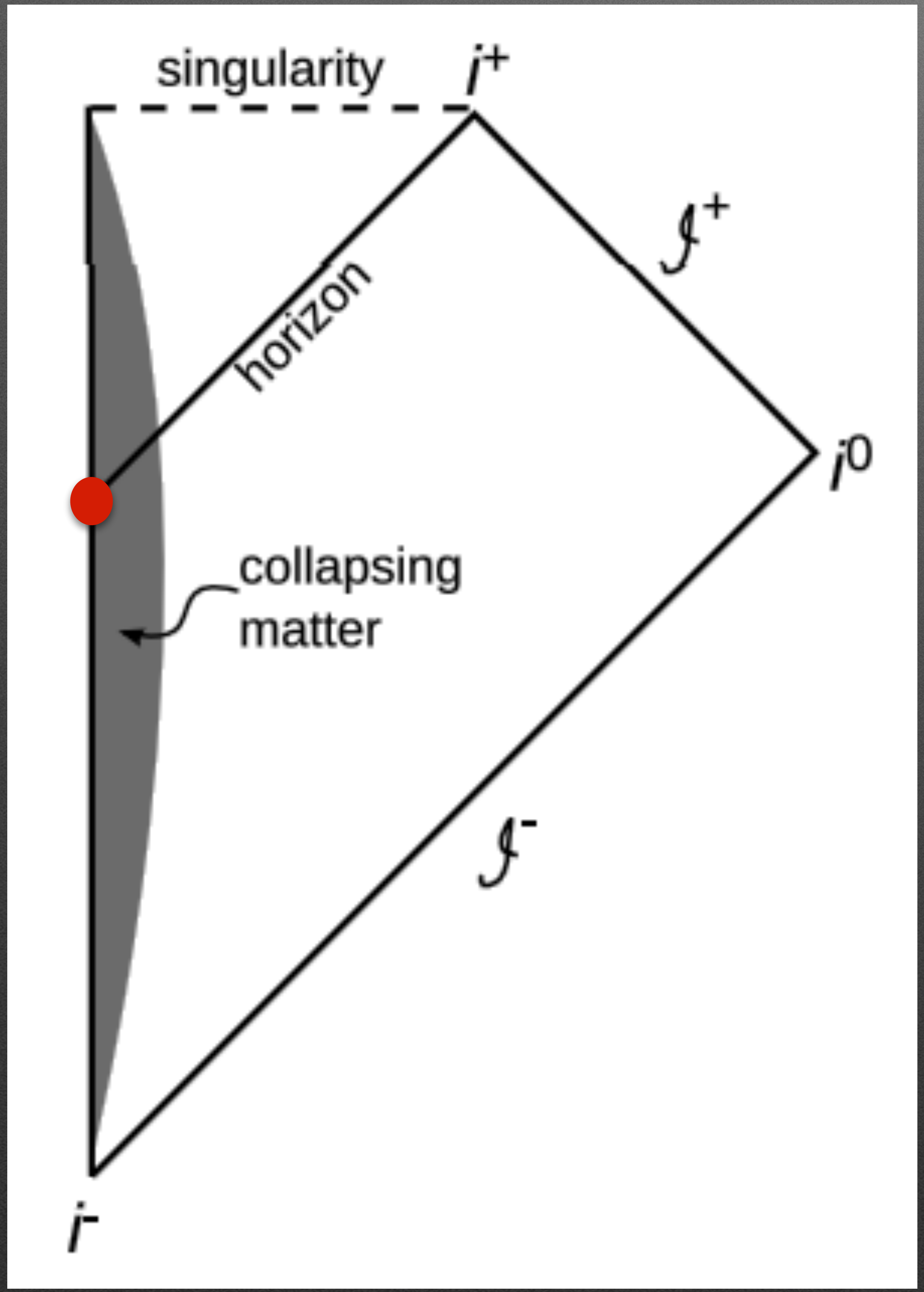
- The event horizon of a ***stationary*** black hole is a ***smooth hypersurface***
- This is ***no longer the case*** for a dynamical event horizon (full spacetime *is* smooth)
- The event horizon is ***nondifferentiable*** at a point p IFF p is a ***past endpoint*** [Beem, Krolak, 1997]

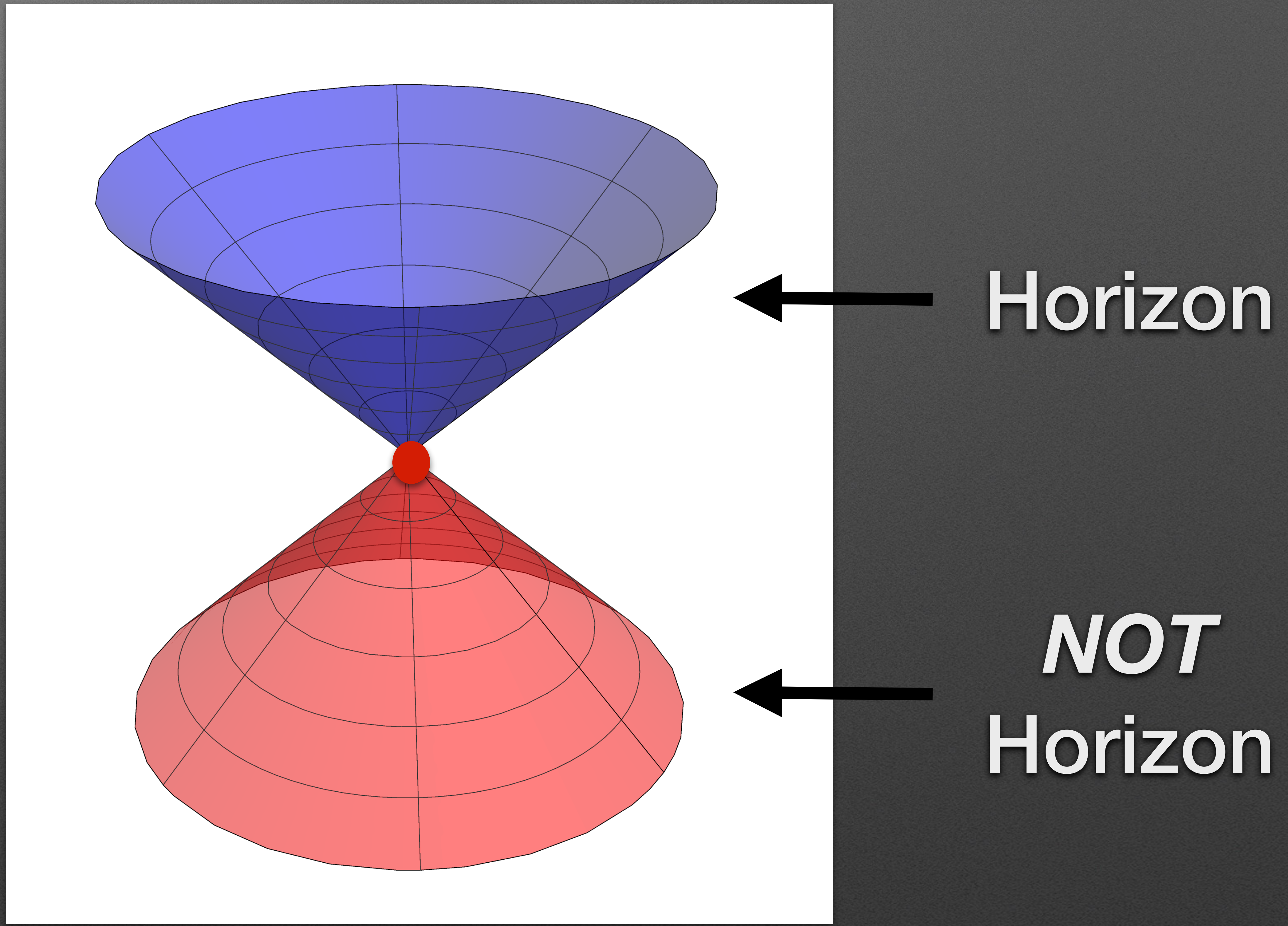
Horizon Past Endpoints





Horizon Past Endpoints





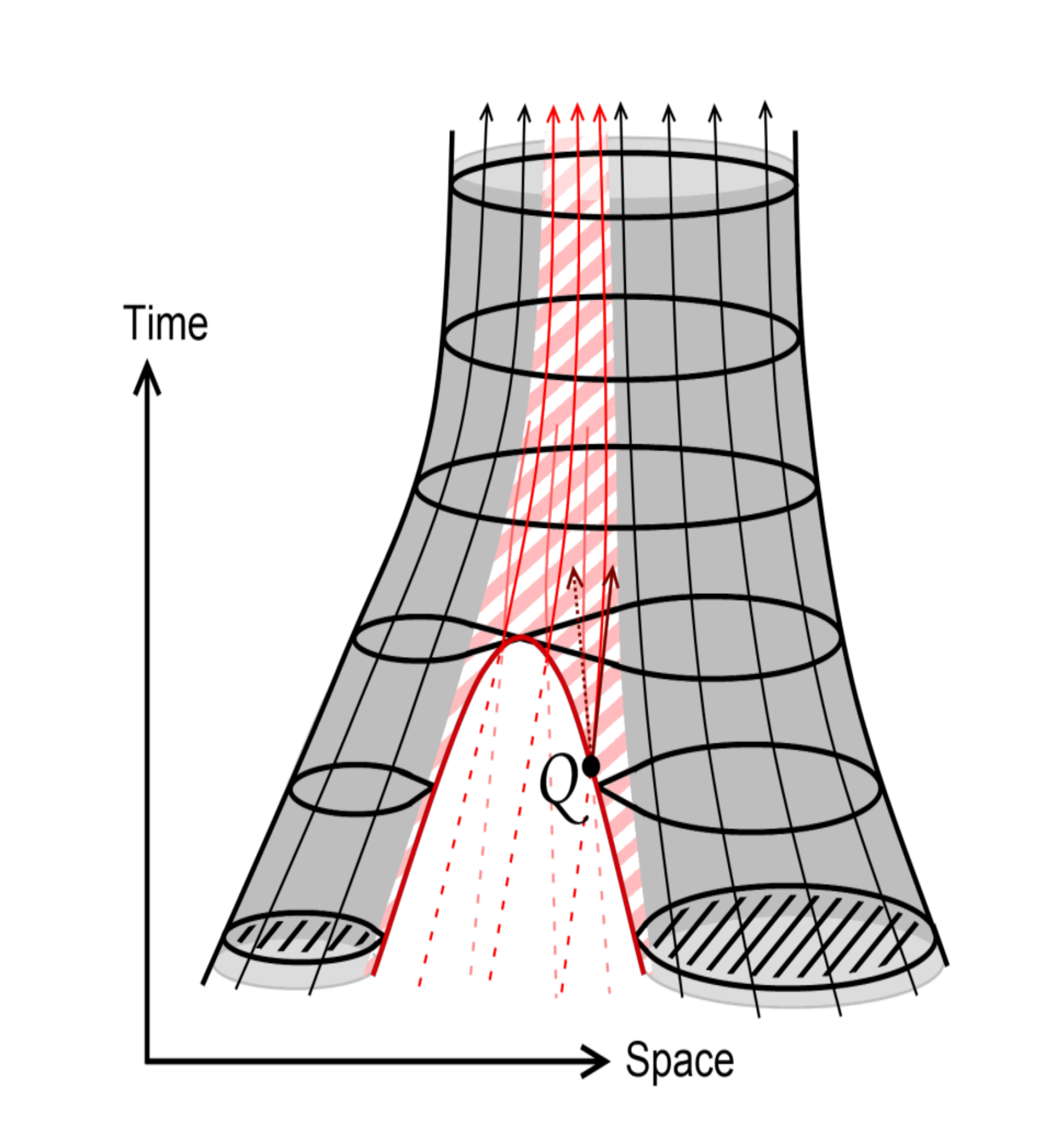
Properties of Nonsmooth Structures

- Two main types of nonsmooth structures:
- *Crease set*: set of points on the horizon where two generators meet. Generically a (D-2) dimensional submanifold
- ***Caustic points***: form the ***boundary*** of the crease set

M. Gadioux, H. Reall, PRD, 2023

Properties of Nonsmooth Structures

- The crease set can undergo a sudden change: "*Perestroika*"
- At a perestroika, possible to construct *exact local model* of the event horizon
- e.g. near the moment of merger in a BBM



M. Gadioux, H. Reall, PRD, 2023

Image: Hammerly, Chen 2010

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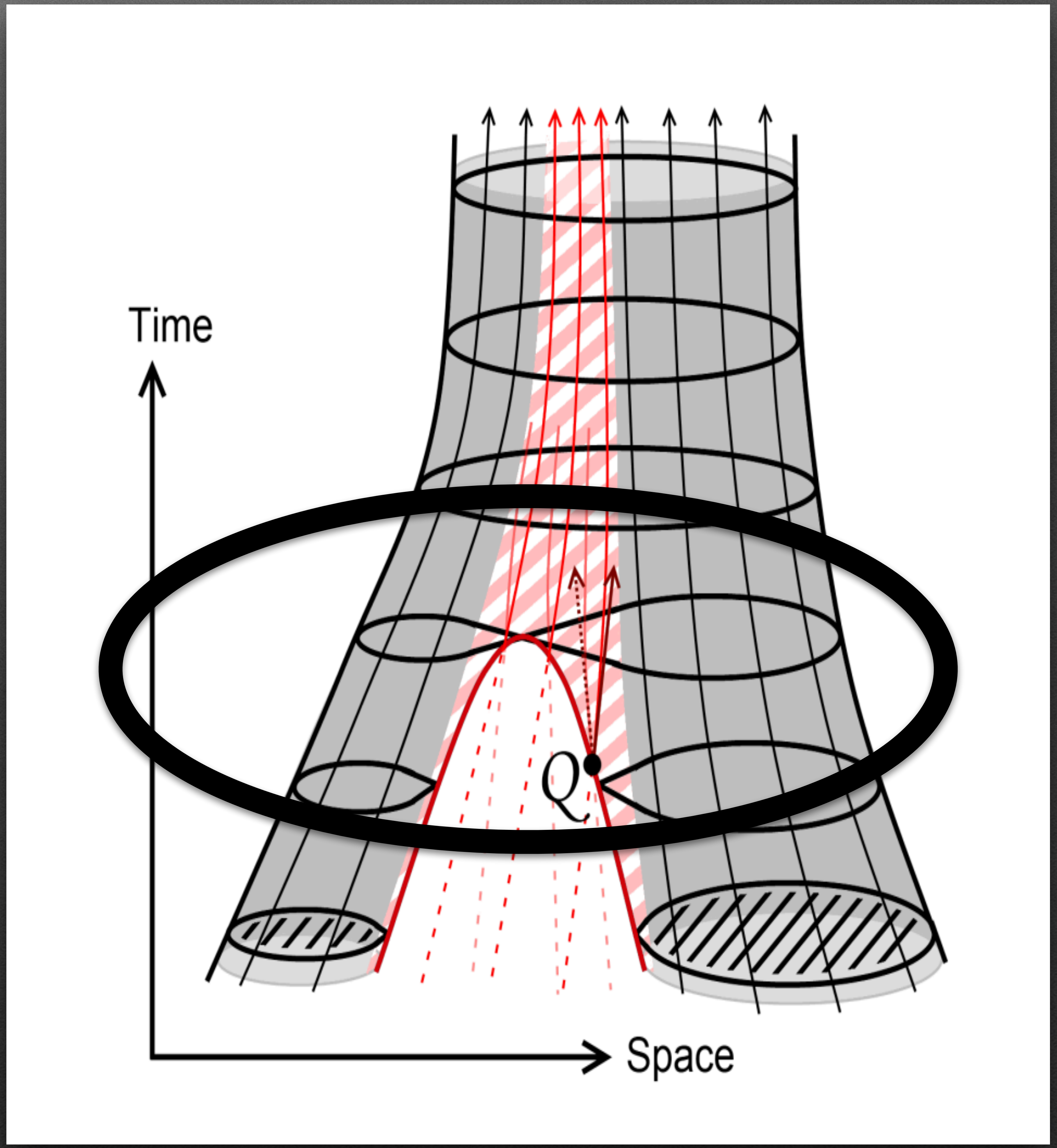


M. Gadioux, H. Reall, PRD, 2023

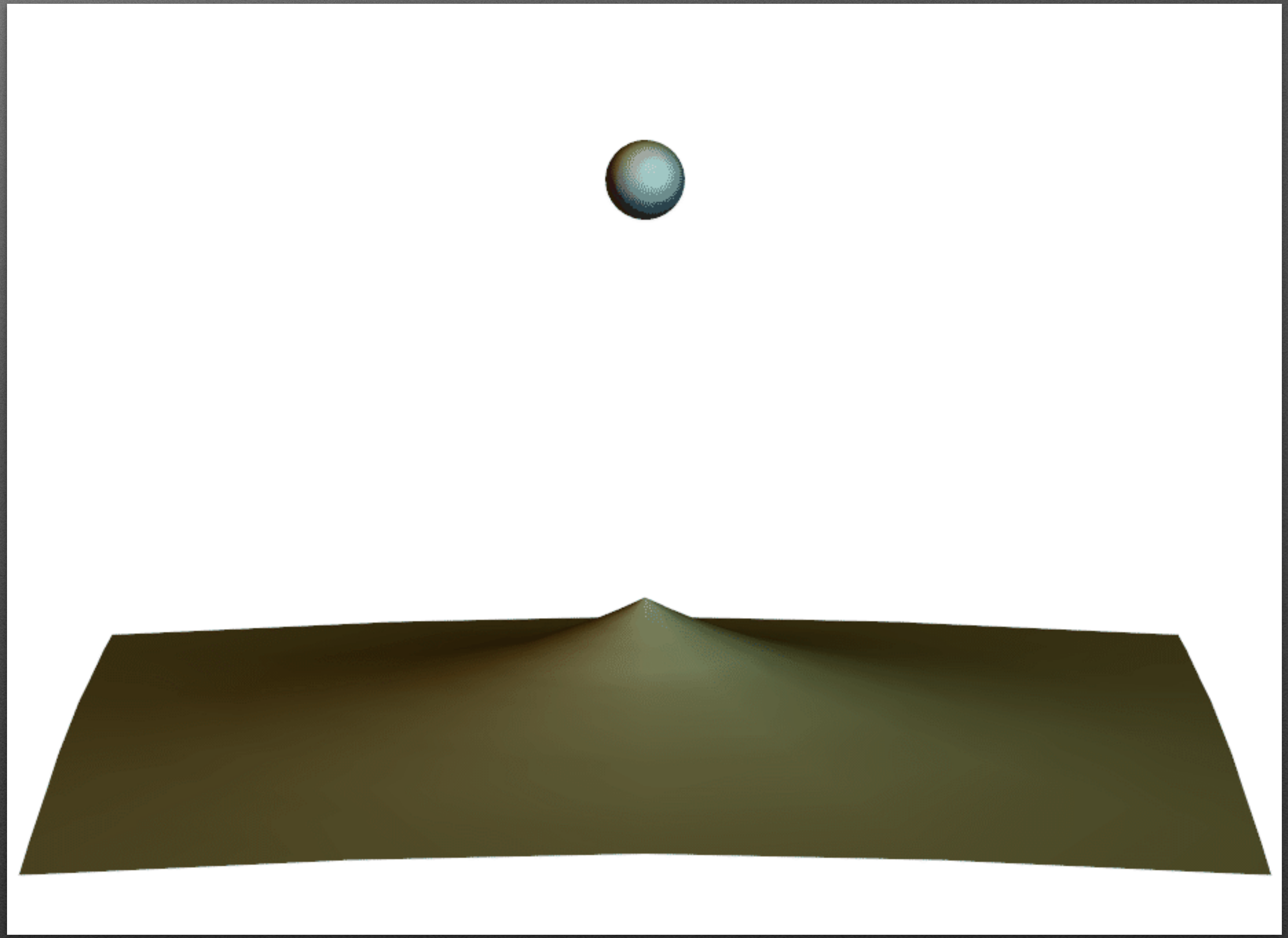
Image: Hammerly, Chen 2010

Our Work (2407.07962):

Construct explicit example of the crease set for a merger and study its properties

Black Hole Mergers

- ***Detailed*** understanding ***requires*** numerics
- In extreme mass ratio merger, event horizon can be obtained ***exactly, analytically!***

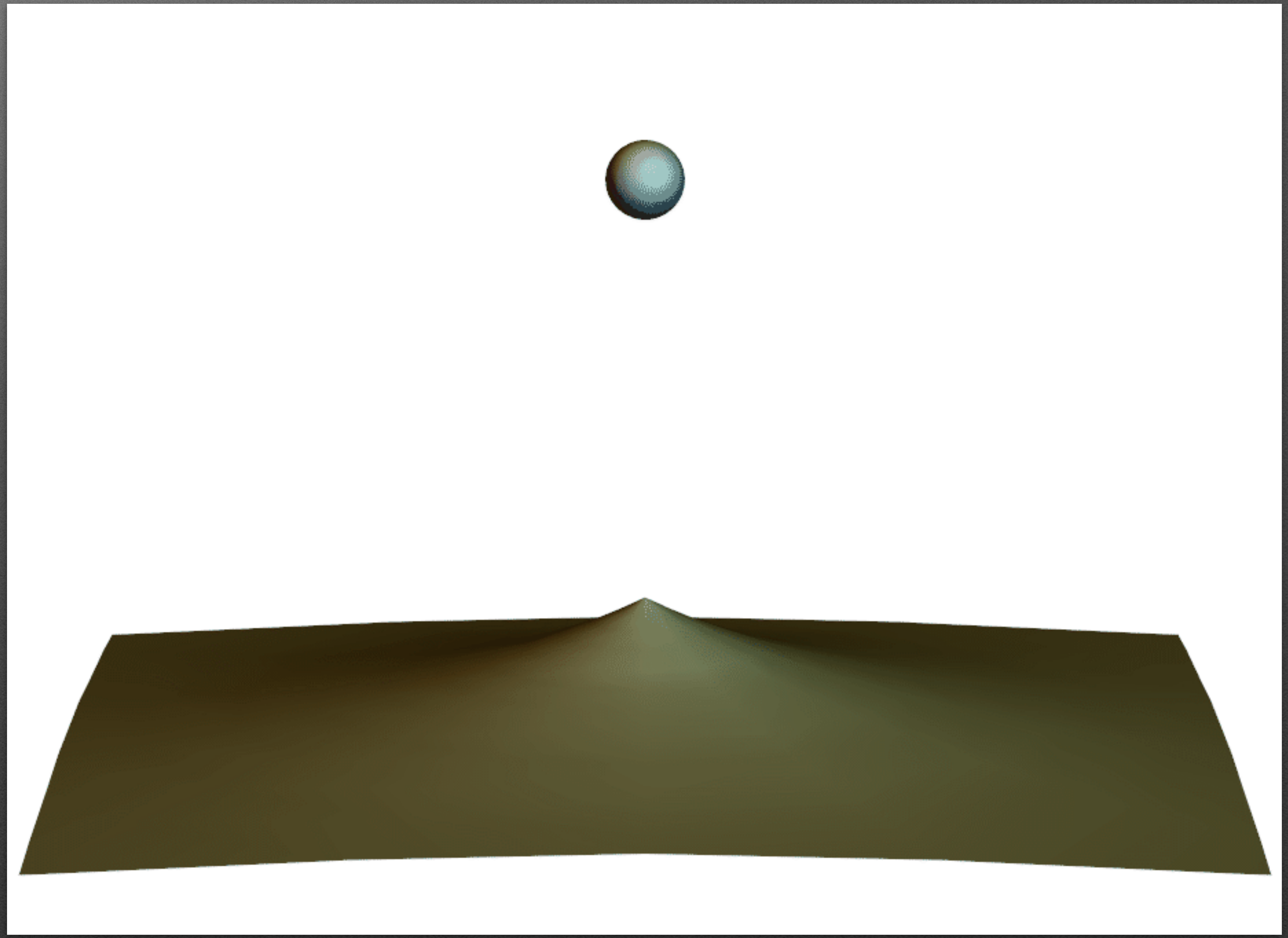


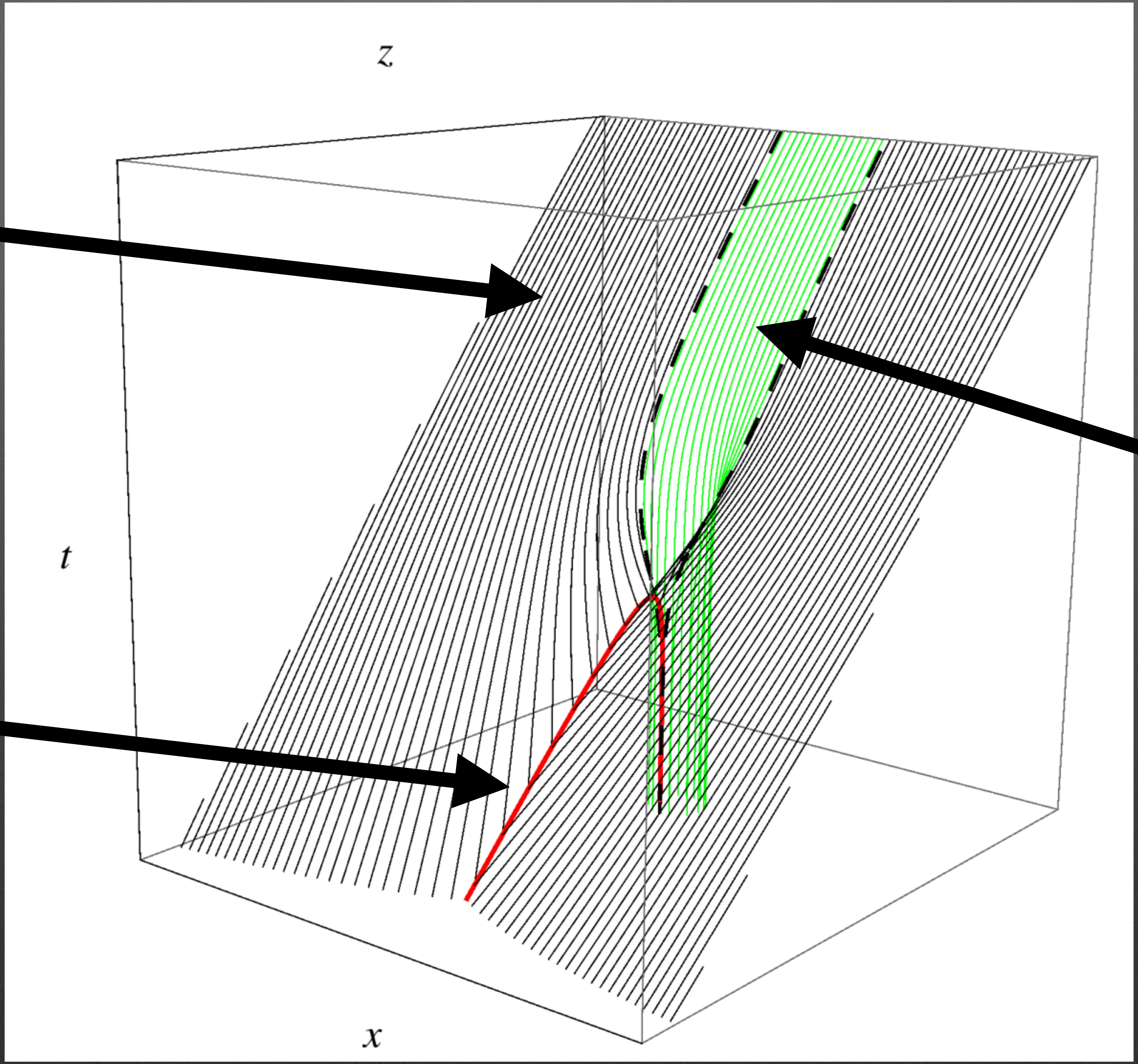
R. Emparan, M. Martínez CQG (2016)
R. Emparan, M. Martínez, M. Zilhão PRD (2017)

(See also Hammerly, Chen 2010)

Black Hole Mergers in EMR Limit

- **Idea:** Large black hole horizon is a null hypersurface in the spacetime of the small black hole
- **Process:** Identify a late-time Rindler horizon; trace null geodesics backward in time to obtain the full event horizon





Large BH generators (black)

Small BH generators (green)

Line of Caustics (red)

R. Emparan, M. Martínez CQG (2016)

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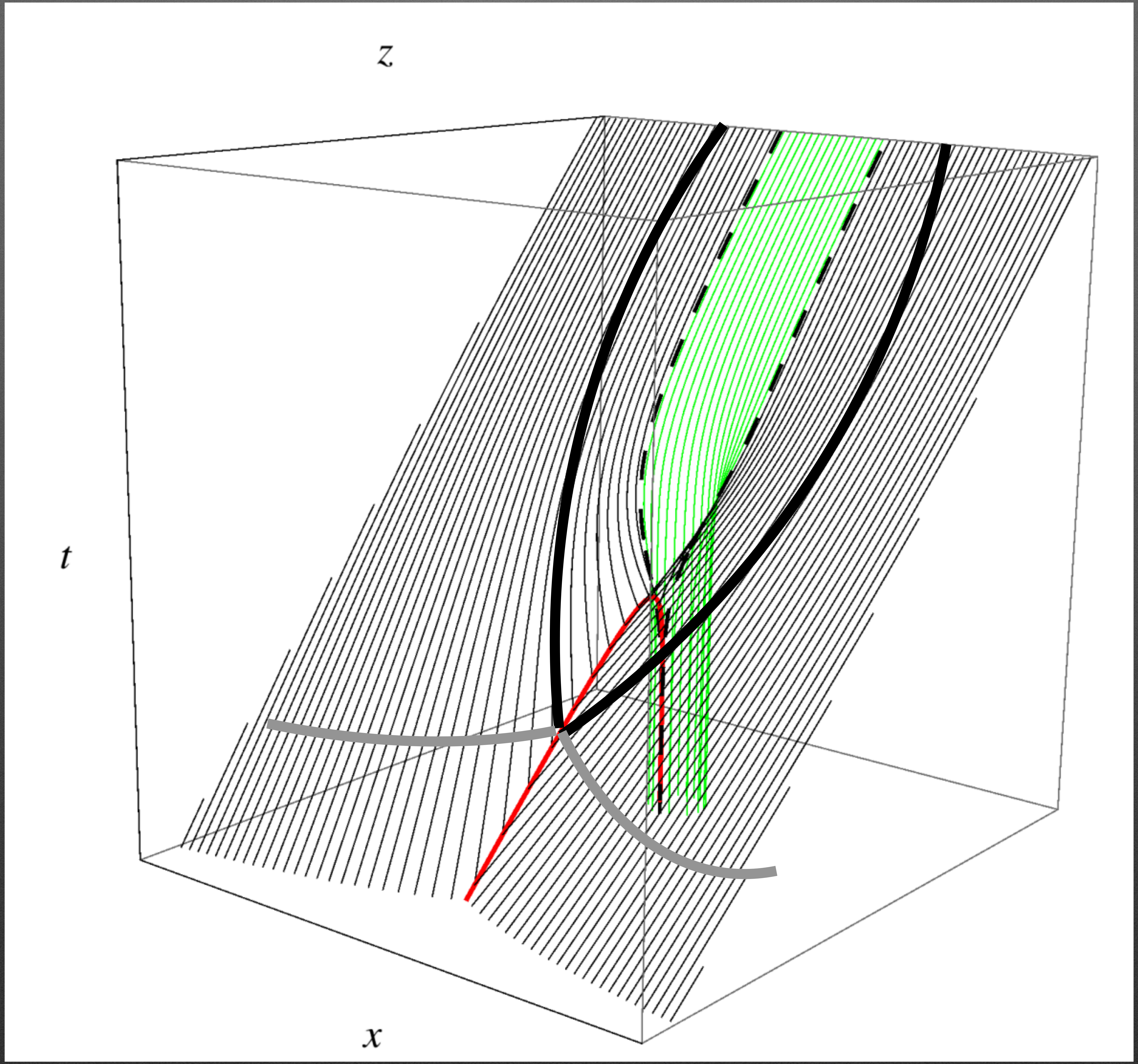
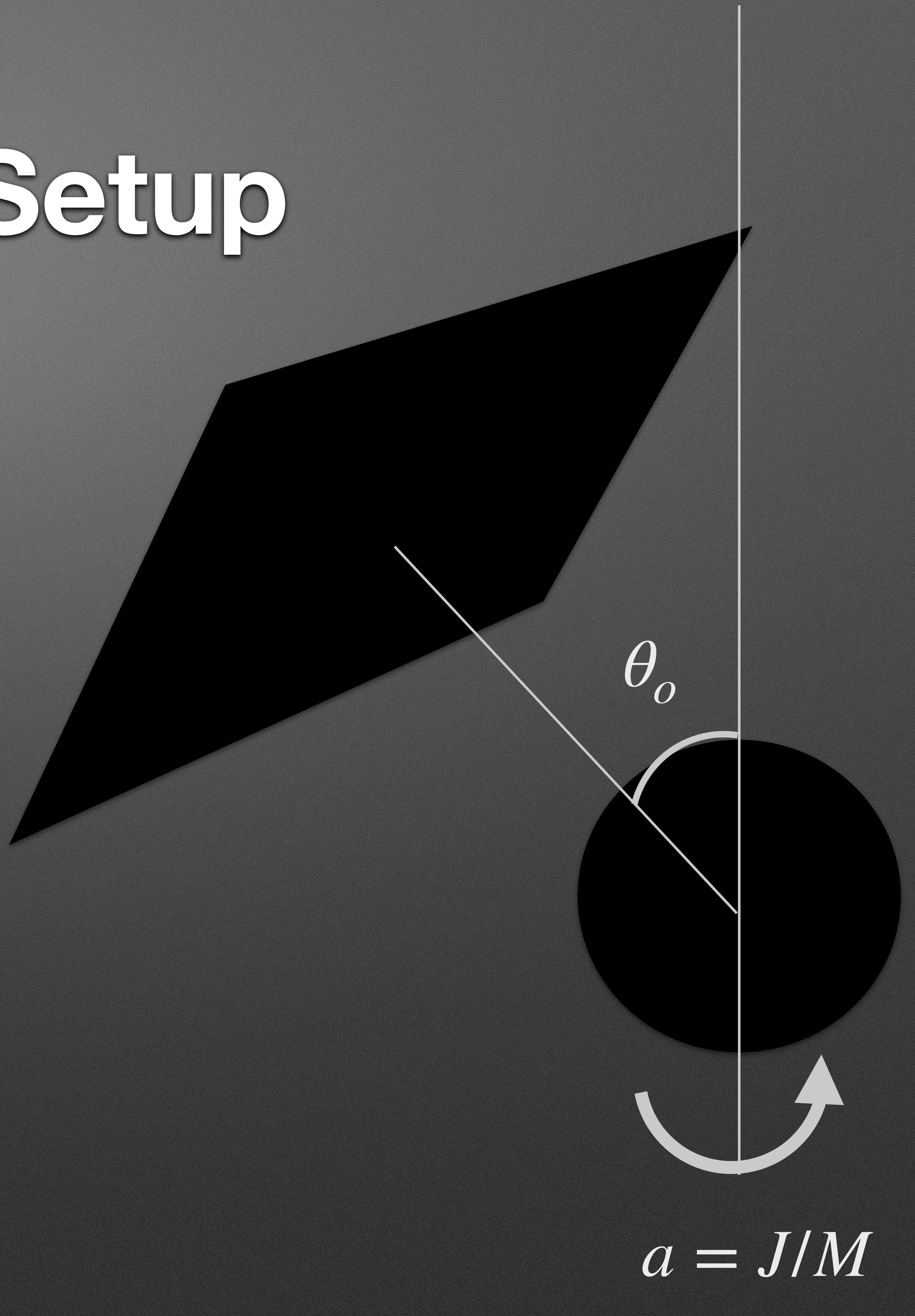


Small BH generators (green)

R. Emparan, M. Martínez CQG (2016)

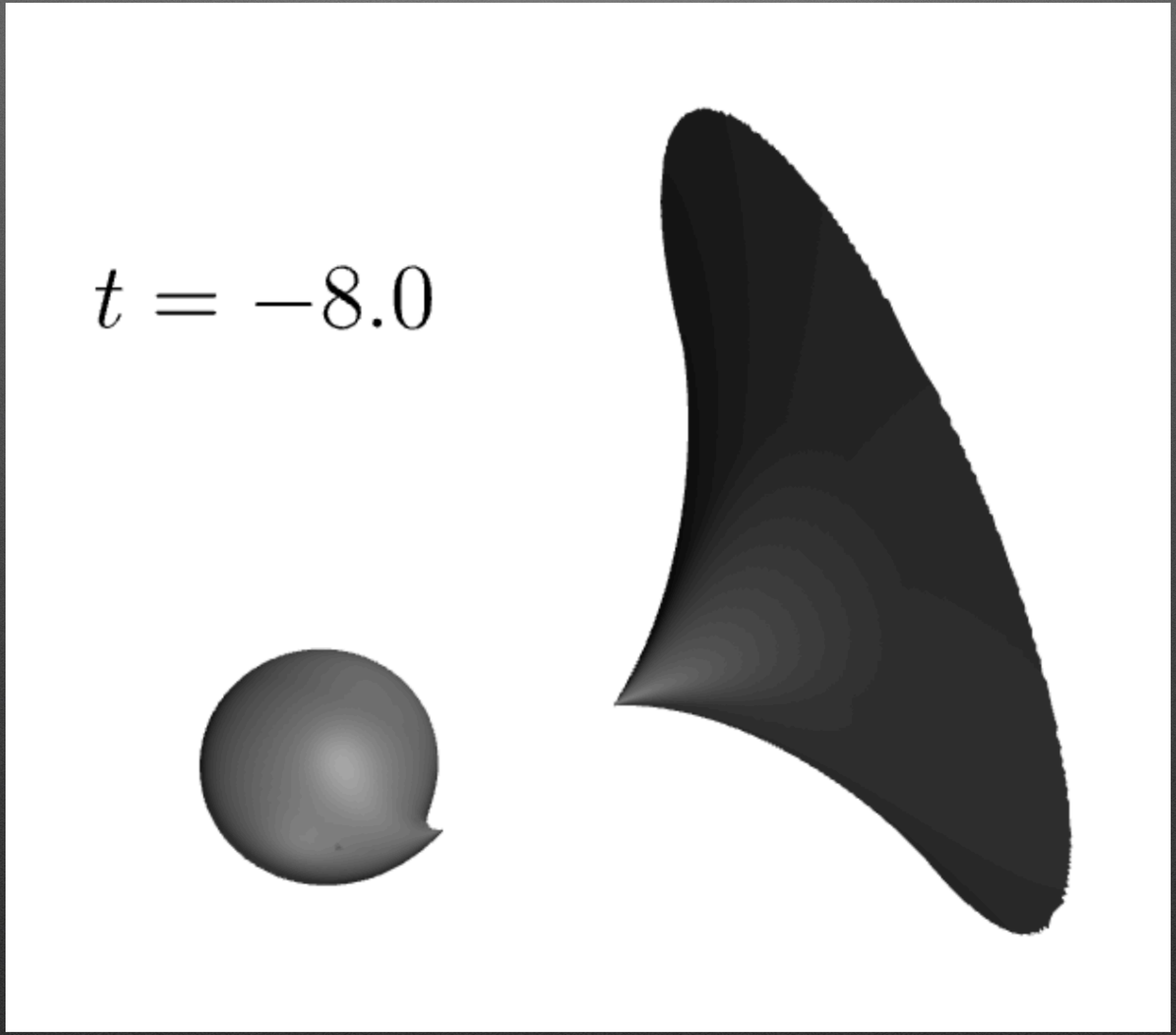
Problem Setup

- Small black hole described by Kerr metric
- Large black hole asymptotically a Rindler horizon
- Event horizon found by tracing null geodesics in the Kerr spacetime (numerically and perturbatively)

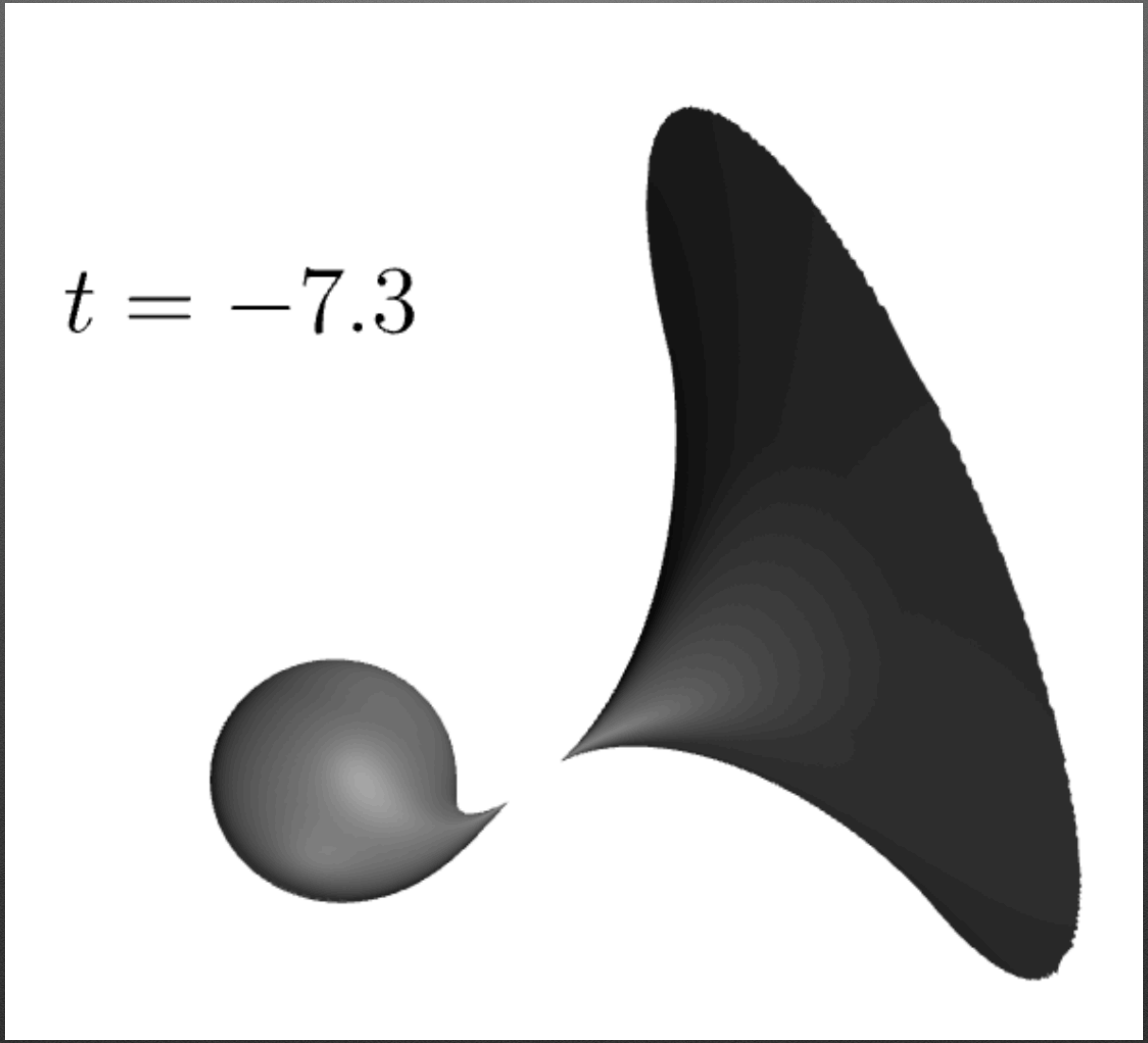


Results

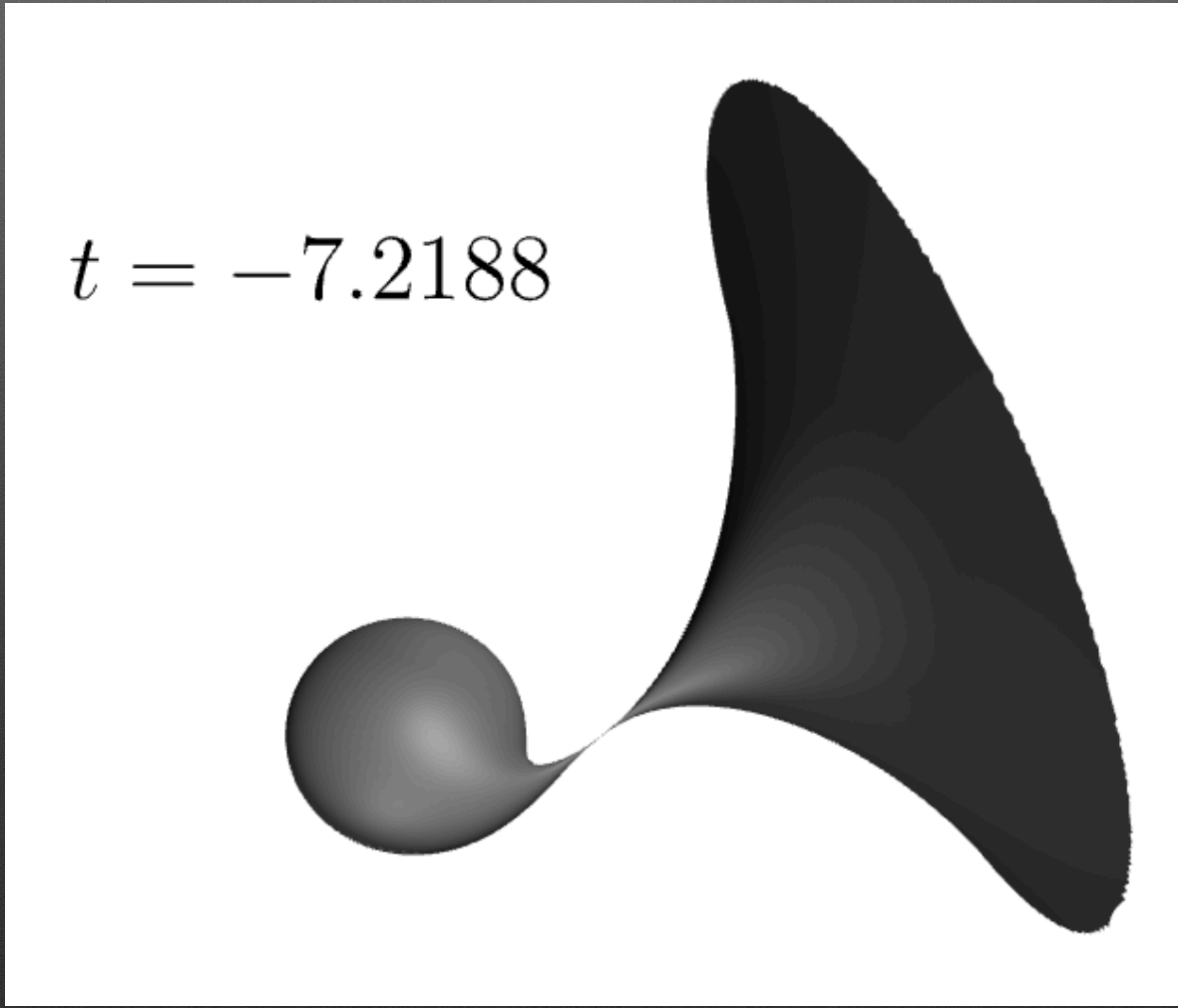
Horizon Cross Sections



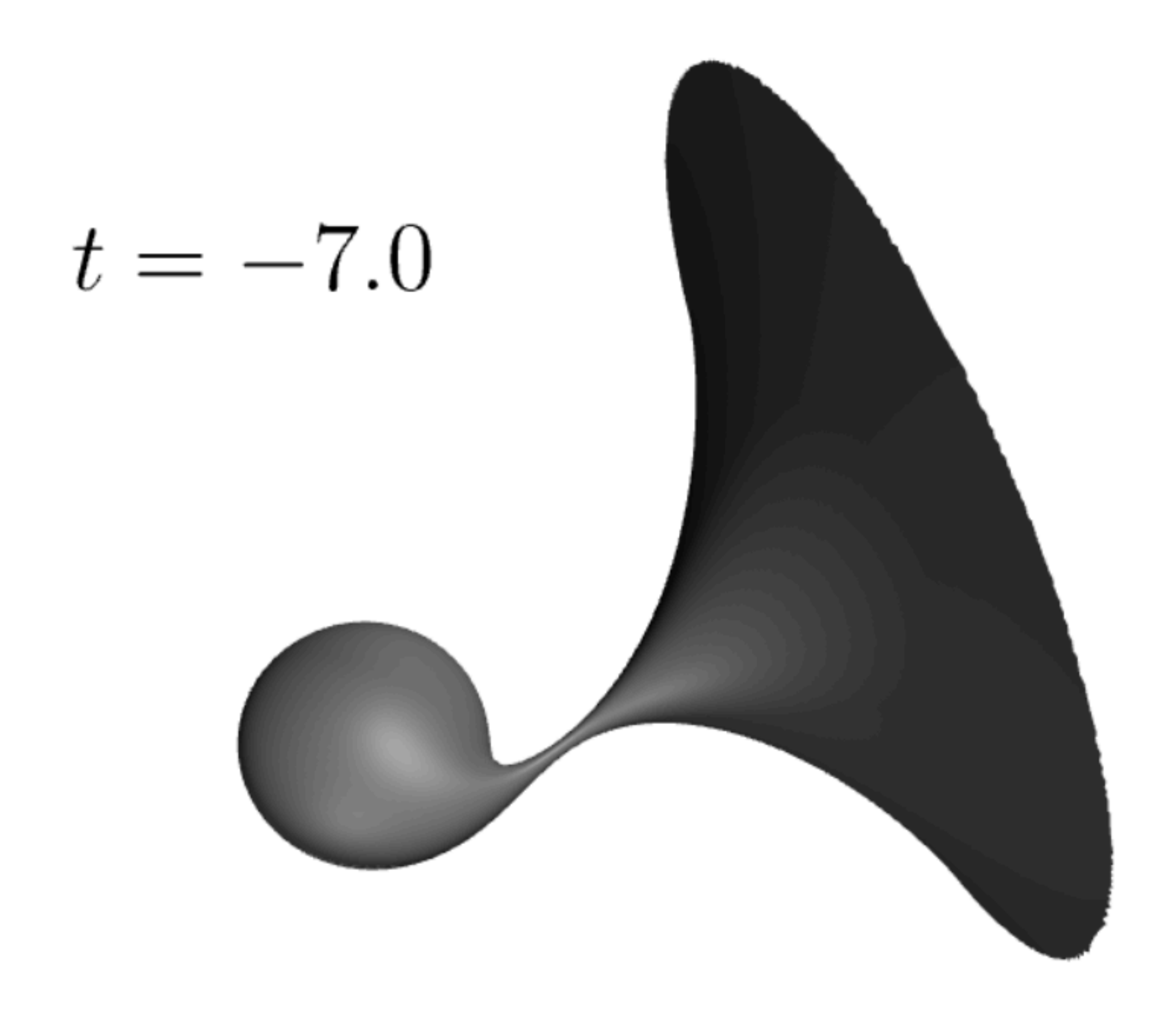
Horizon Cross Sections



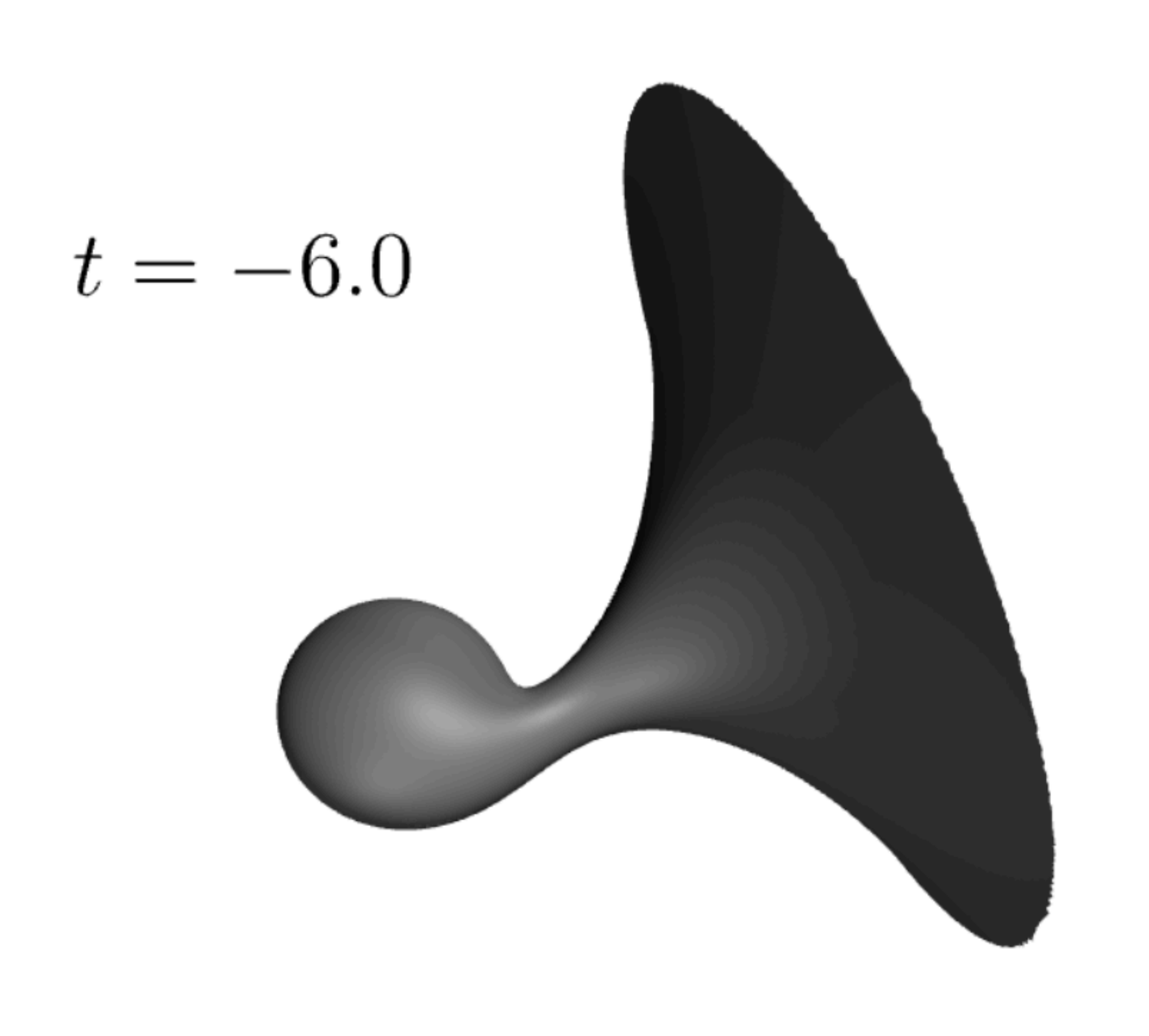
Horizon Cross Sections



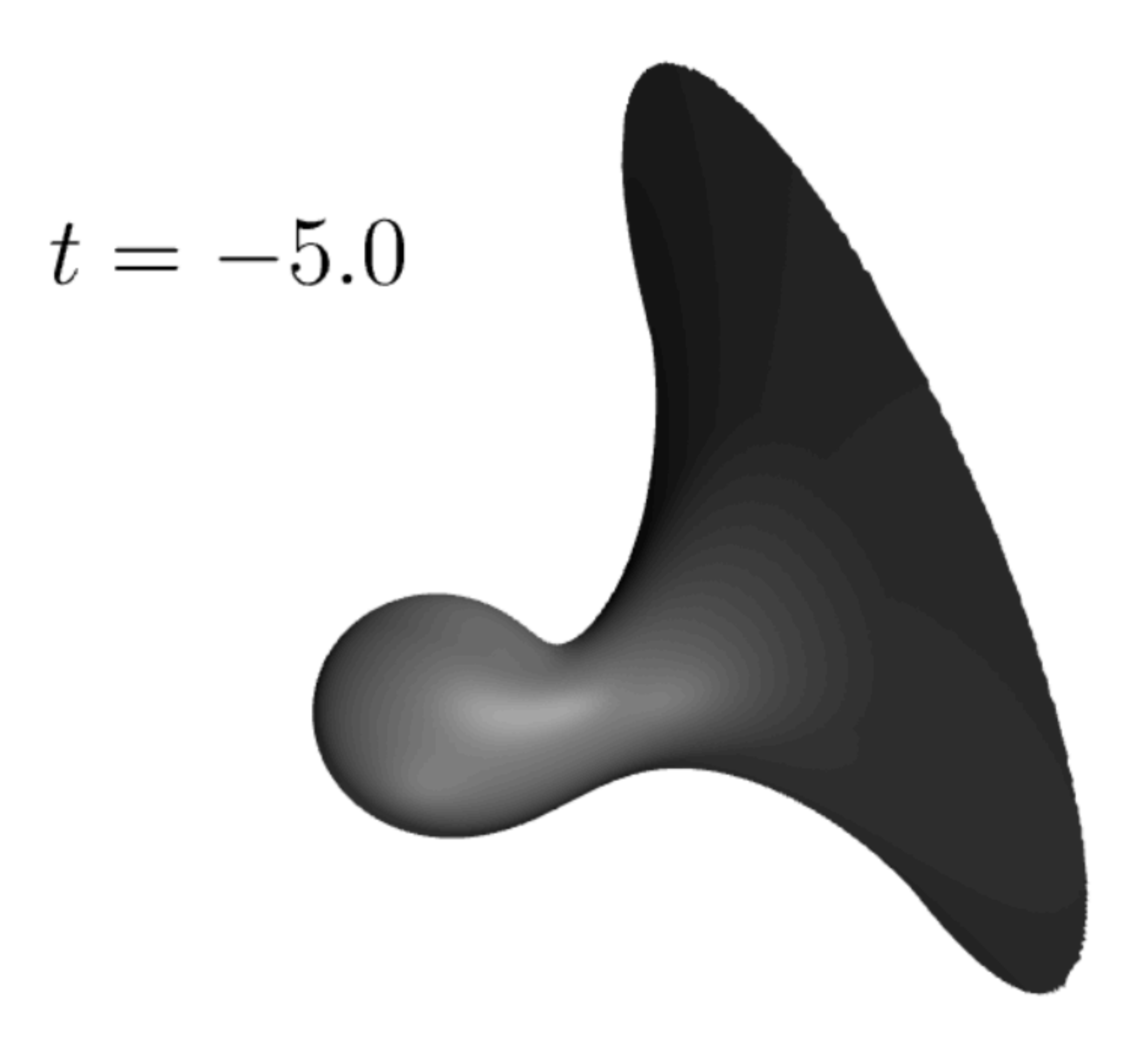
Horizon Cross Sections



Horizon Cross Sections



Horizon Cross Sections



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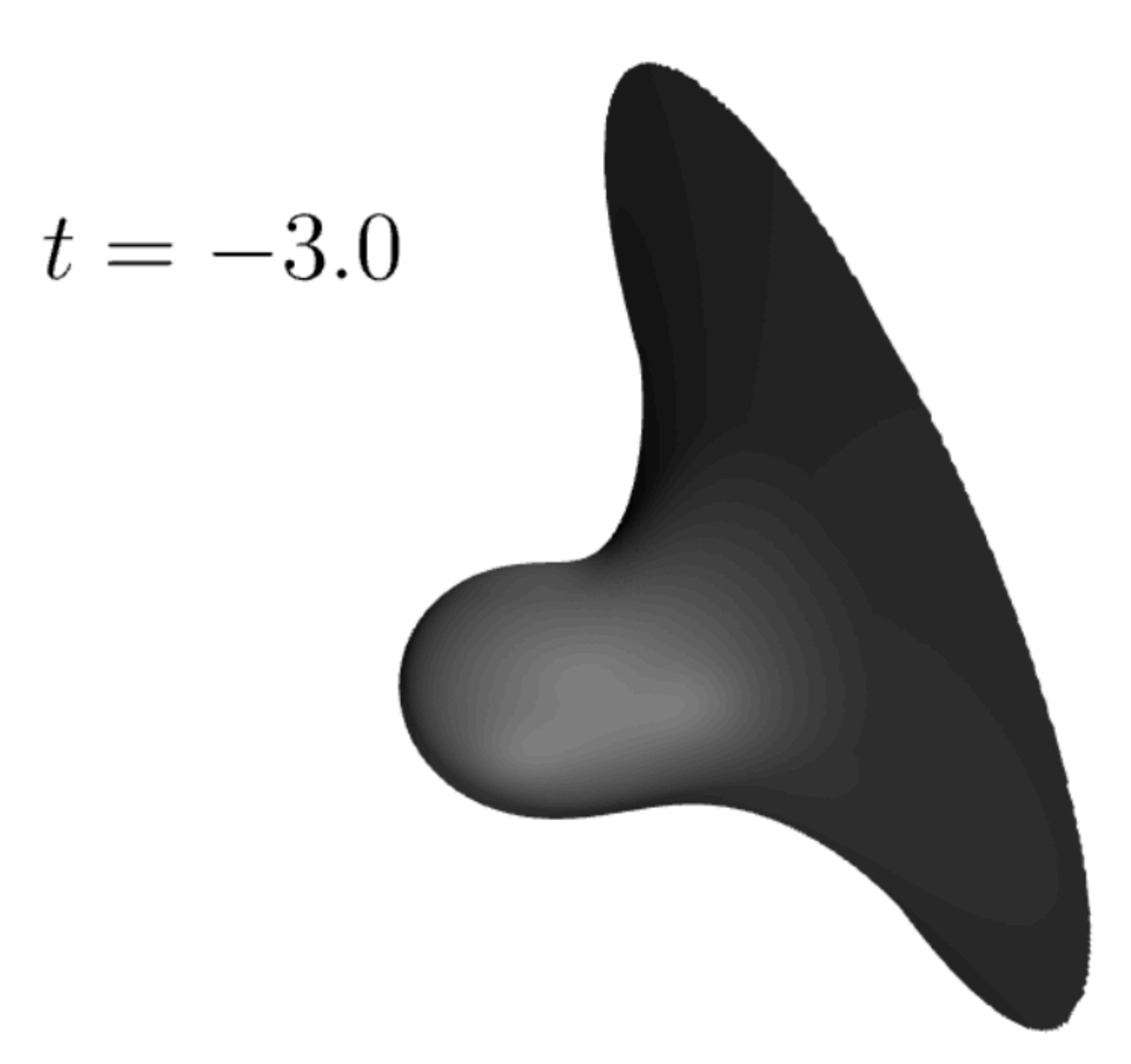

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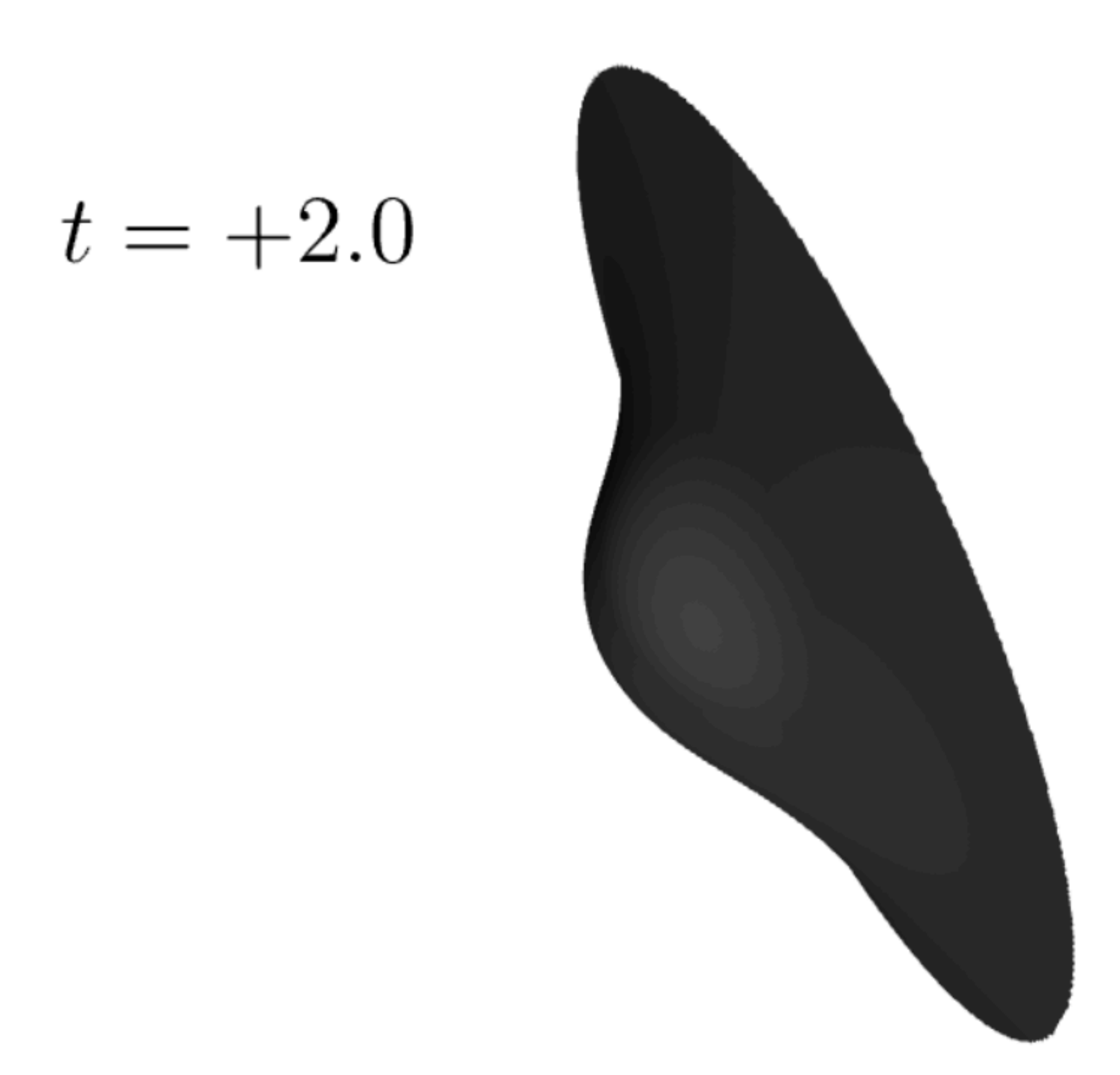

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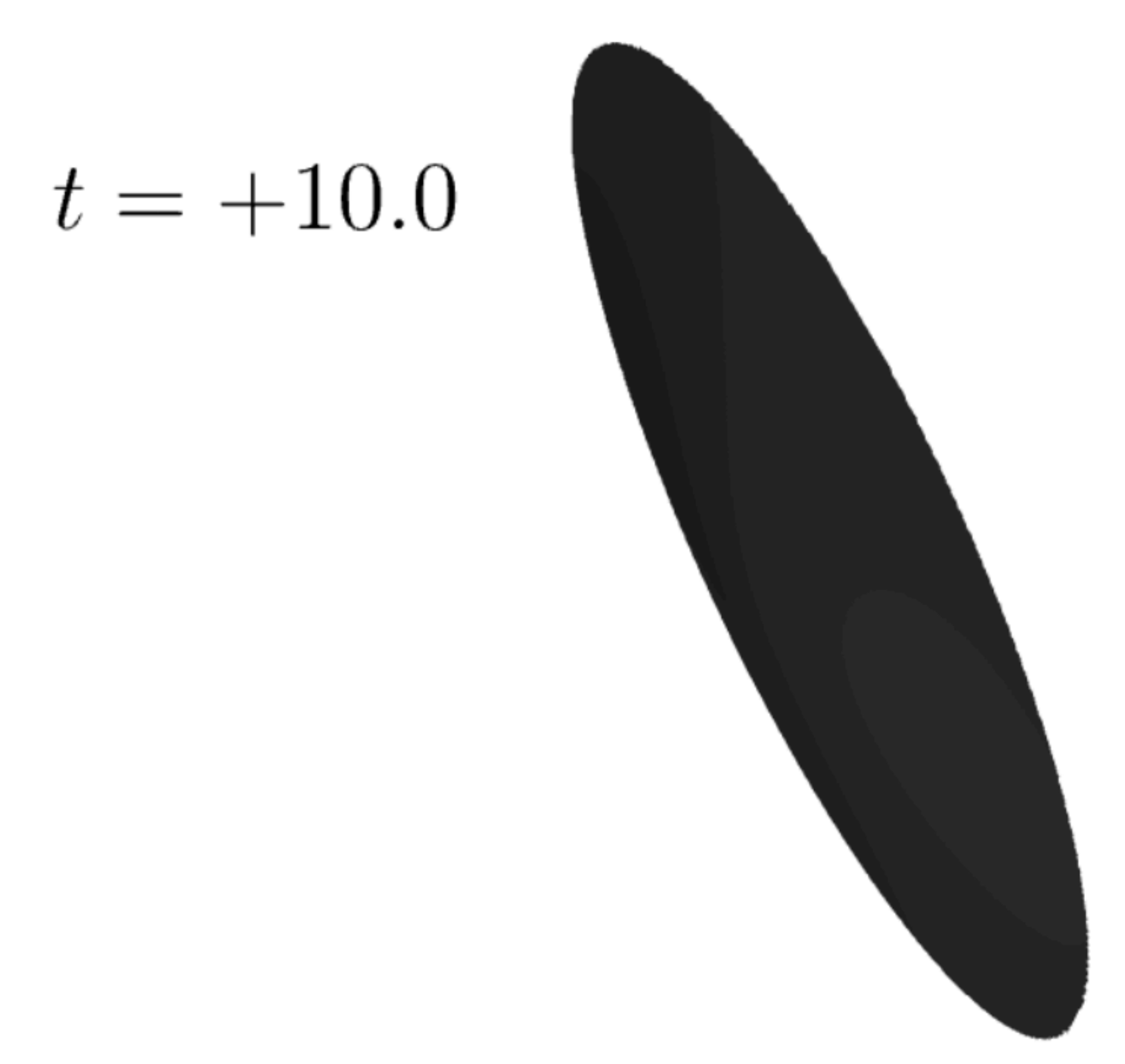
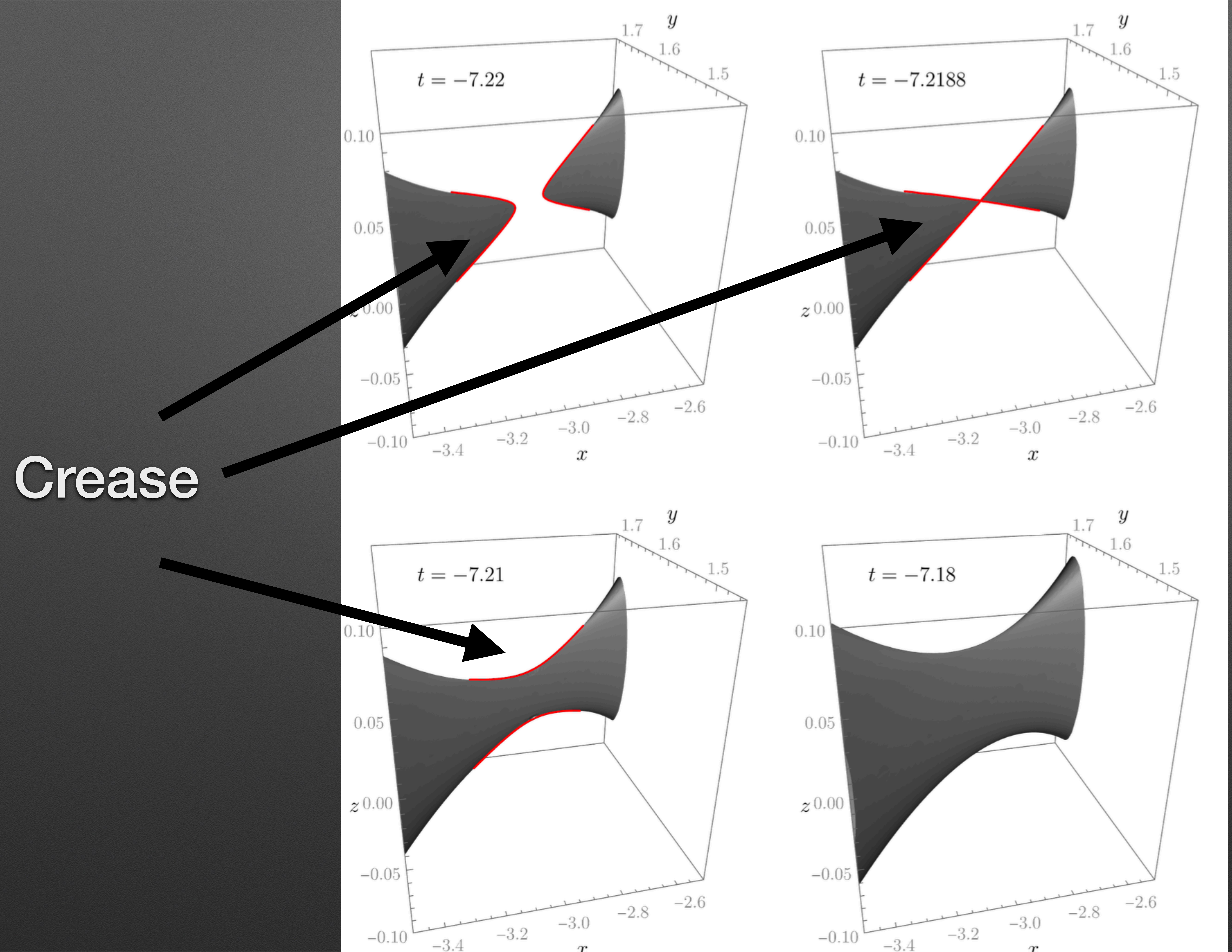
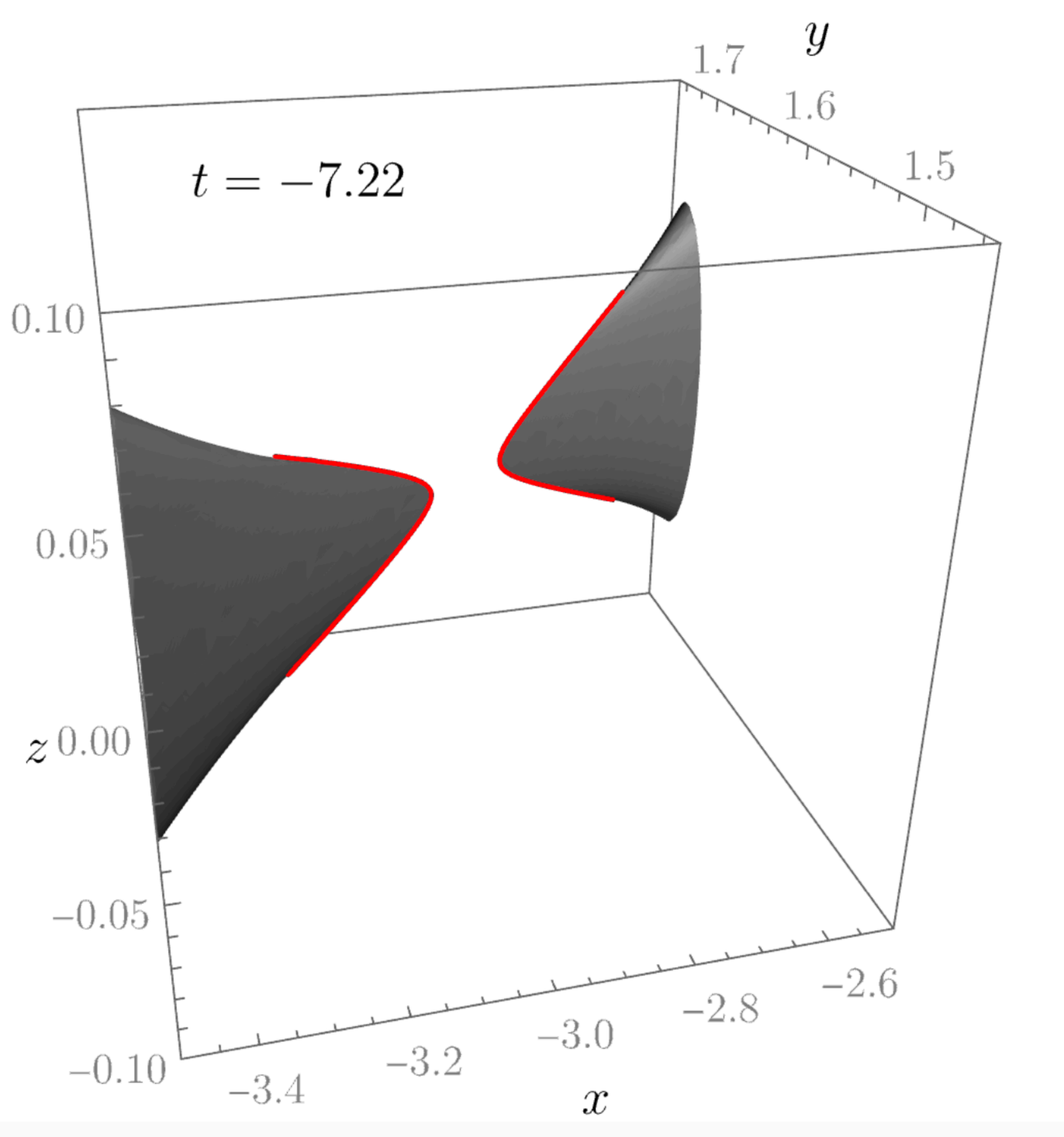



Crease
t = −7.22
t = −7.2188
t = −7.21
t = −7.18
x
y
z



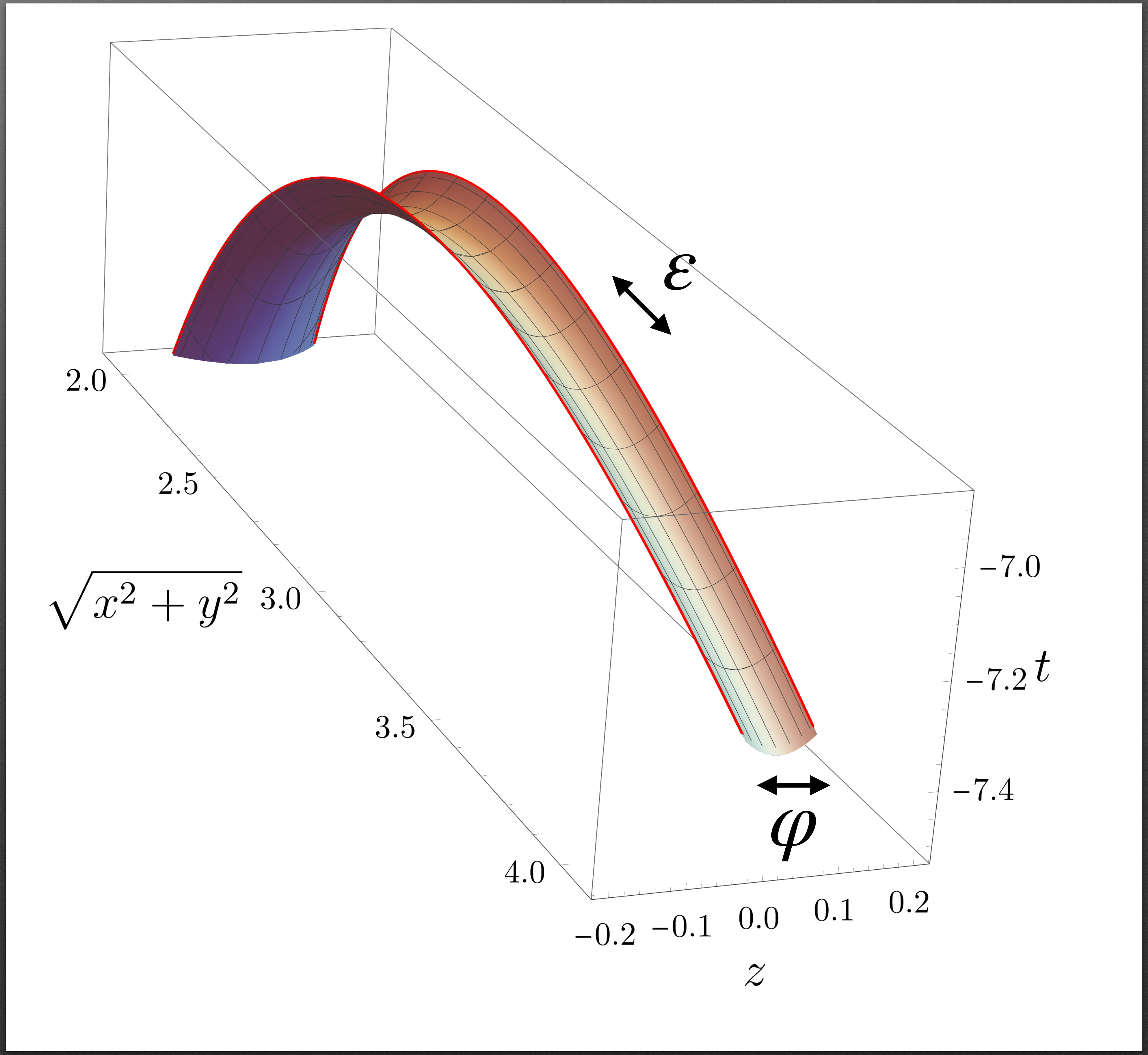
$t = -7.22$
y
1.7
1.6
1.5
0.10
0.05
z 0.00
−0.05
−0.10
−3.4
−3.2
−3.0
−2.8
−2.6
x

Simple Perturbative Expression

$$\frac{t}{M} = -\frac{1}{4\varepsilon^2} - \frac{15\pi}{4}\varepsilon + \left(\frac{225\pi^2}{256} - 8\right)\varepsilon^2$$
$$+\left[-\frac{15\pi\left(375\pi^2+1664\right)}{8192} + \frac{5\pi\chi^2}{32}\left(13 - 6\cos^2\theta_o - 3\sin^2\theta_o\cos^2\varphi\right)\right]\varepsilon^3$$

$$\frac{\sqrt{x^2+y^2}}{M} = \frac{1}{4\varepsilon^2}$$

$$\frac{z}{M} = -\frac{15\pi a^2 \sin^2\theta_o \sin(\varphi)\,\varepsilon^2}{64M^2}$$

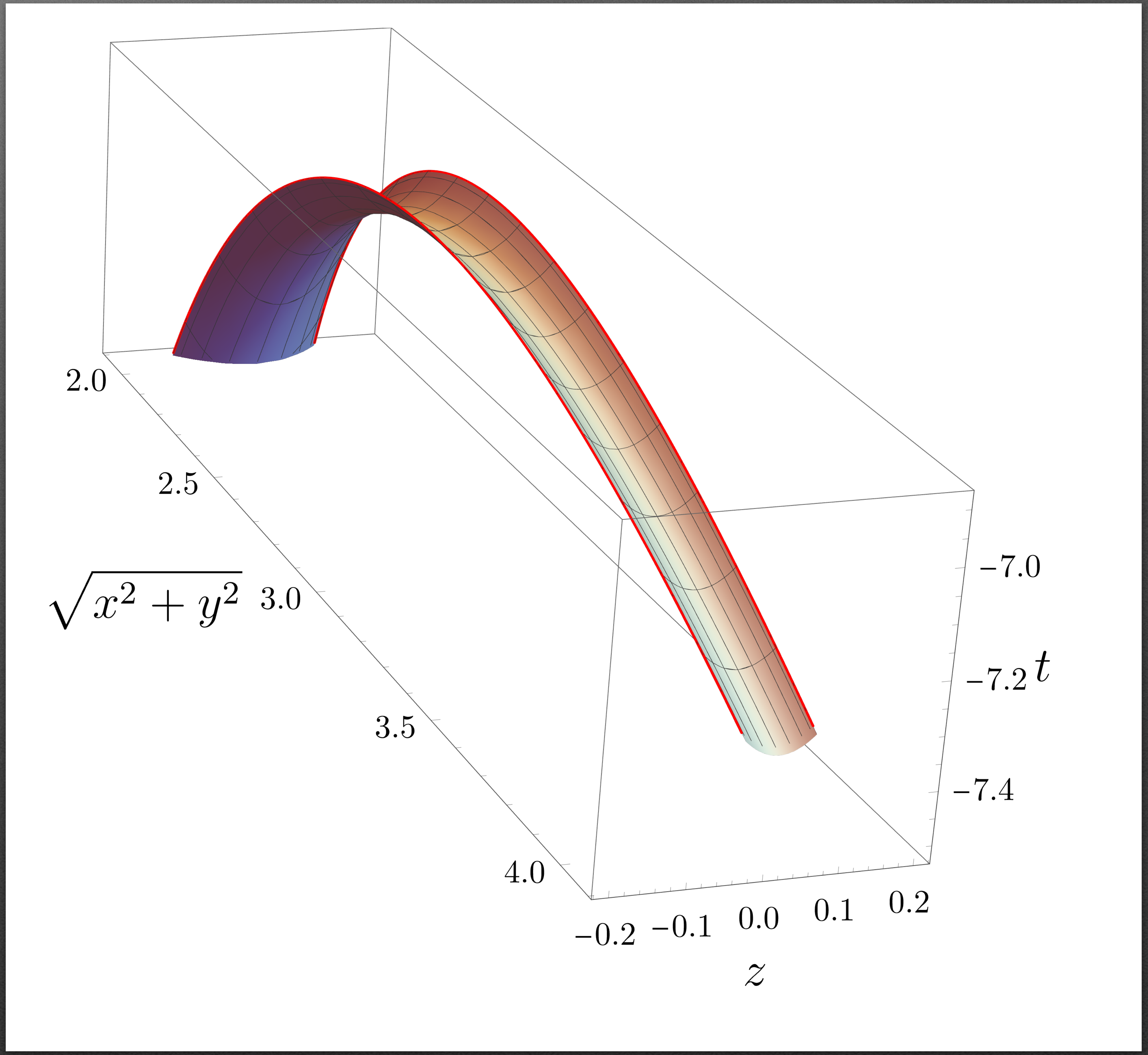


The Crease Set has Finite Area

- In EMR limit, crease set is of infinite extent but has finite area.

$$A_{\text{Crease}} = \frac{15\pi^2 a^2 \sin^2\theta_o}{64}\varepsilon_{\text{Max}} + \mathcal{O}(\varepsilon_{\text{Max}}^2)$$

- About 10% area of small BH at most

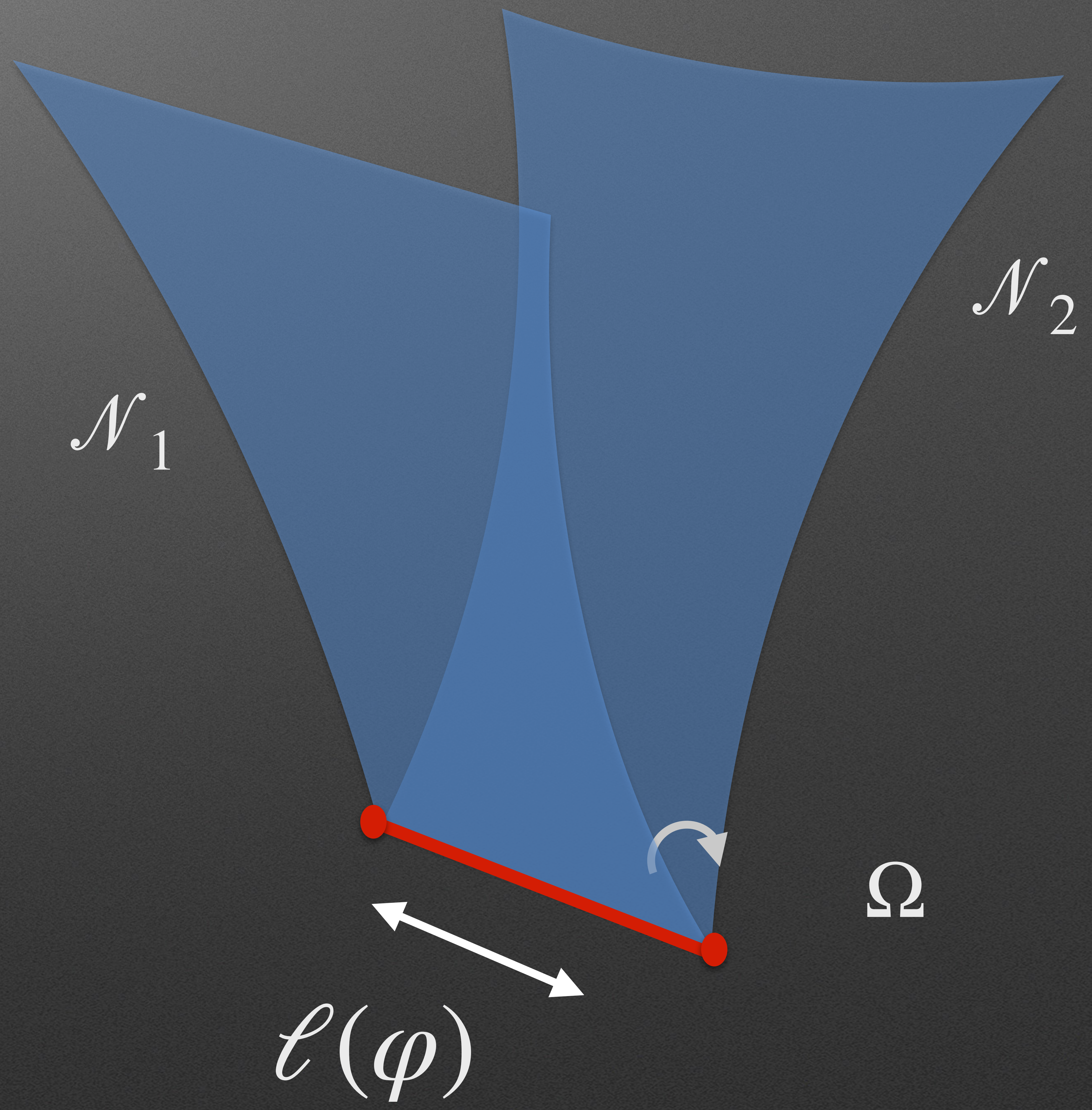


Angle of the Crease

- The angle between the two intersection portions of the horizon goes to zero at the boundary of the crease set

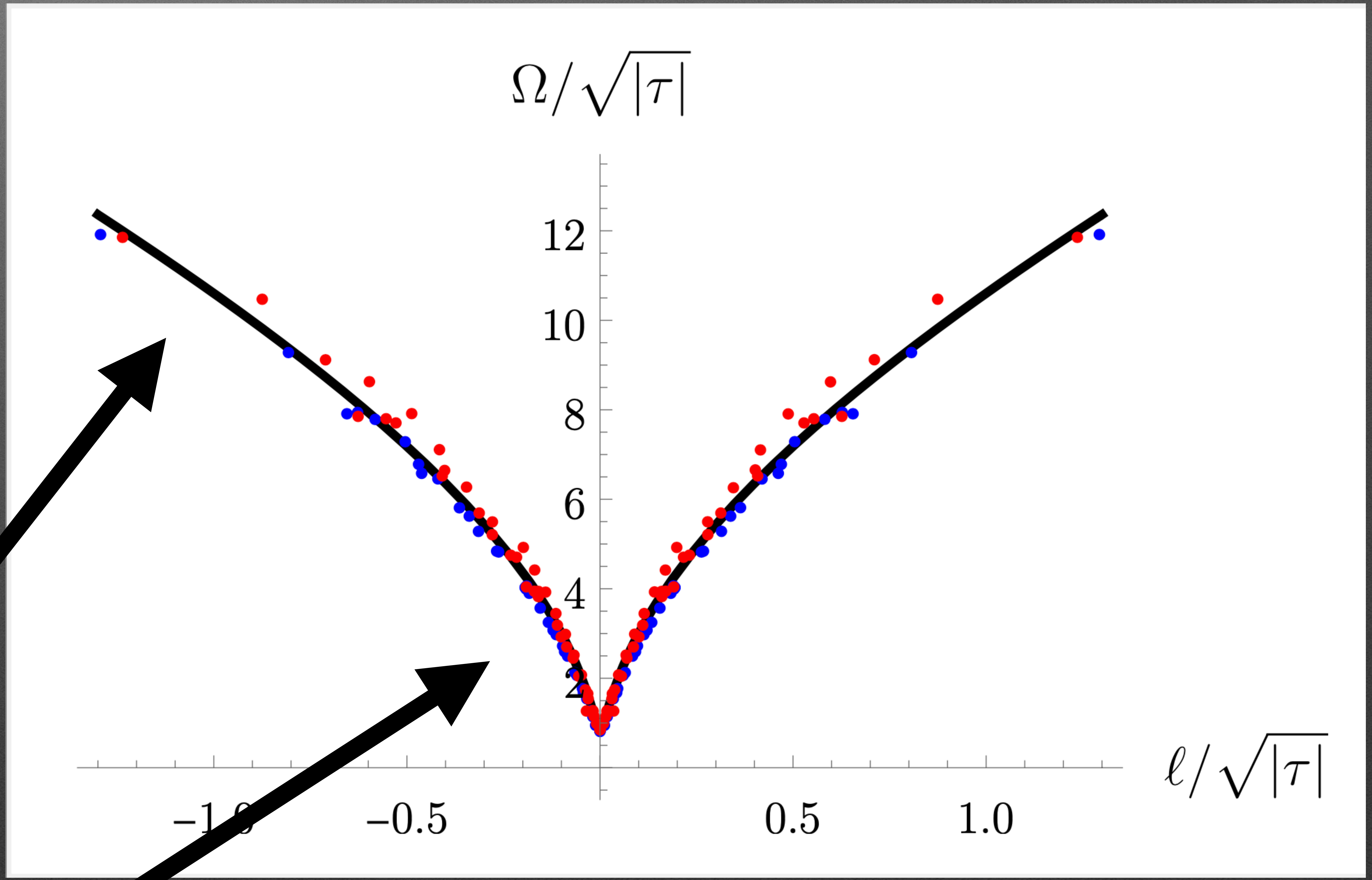
$$\cos(\pi - \Omega) = 1 + \frac{128a}{M} \sin\theta_o \, |\cos\varphi| + \cdots$$

Caustics at $\varphi = \pi/2,\ 3\pi/2$



Angle of the Crease

- Near the moment of merger, compare numerical results with exact local model for generic mergers



Exact local model (black curve)

Numerical data

Summary

- Event horizons of dynamical black holes are *generically* nonsmooth; creases and caustics are the most “important” part
- We’ve constructed the first semi-analytical examples of a crease set in the merger of two Kerr black holes (EMR limit)
    - Our results allow for previous studies of gravitational lensing of Kerr black holes to be extended to much higher order
- We’ve been able to perform a preliminary study of the properties of creases (area; length; angle; entropy)

Final Thoughts

- What physical interpretation does the crease set have?
  - Characterizes "out of equilibrium dynamics"; area vanishes when configuration highly symmetric
  - Crease area provides lower bound for area increase

$$2A_{\text{crease}} \leq A_H$$

  - Thermalization? Latent Heat? Thermodynamic complexity?

Thank You!

Problem Setup

$$ds^2 = -\left(1 - \frac{2\,m\,r}{\Sigma}\right) dt^2 - \frac{4\,m\,a\,r\sin^2\theta}{\Sigma}\,dt\,d\varphi + \frac{\Sigma}{\Delta}\,dr^2 + \Sigma\,d\theta^2 + \left(r^2 + a^2 + \frac{2\,m\,a^2 r\sin^2\theta}{\Sigma}\right)\sin^2\theta\,d\varphi^2,$$

Kerr Metric

$$\int_{\mathrm{P}} \frac{dr}{\pm\sqrt{R}} = \int_{\mathrm{P}} \frac{d\theta}{\pm\sqrt{\Theta}},$$

$$\phi_o - \phi_s = \int_{\mathrm{P}} \frac{a\,(2Mr - a\lambda)\,dr}{\pm\Delta\sqrt{R}} + \int_{\mathrm{P}} \frac{\lambda d\theta}{\pm\sin^2\theta\sqrt{\Theta}},$$

$$t_o - t_s = \int_{\mathrm{P}} \frac{\left[r^2\left(r^2 + a^2\right) + 2aMr\,(a-\lambda)\right]dr}{\pm\Delta\sqrt{R}} + \int_{\mathrm{P}} \frac{a^2\cos^2\theta}{\pm\sqrt{\Theta}} d\theta$$

$$\Delta \equiv r^2 - 2\,m\,r + a^2, \qquad \Sigma \equiv r^2 + a^2\cos^2\theta$$

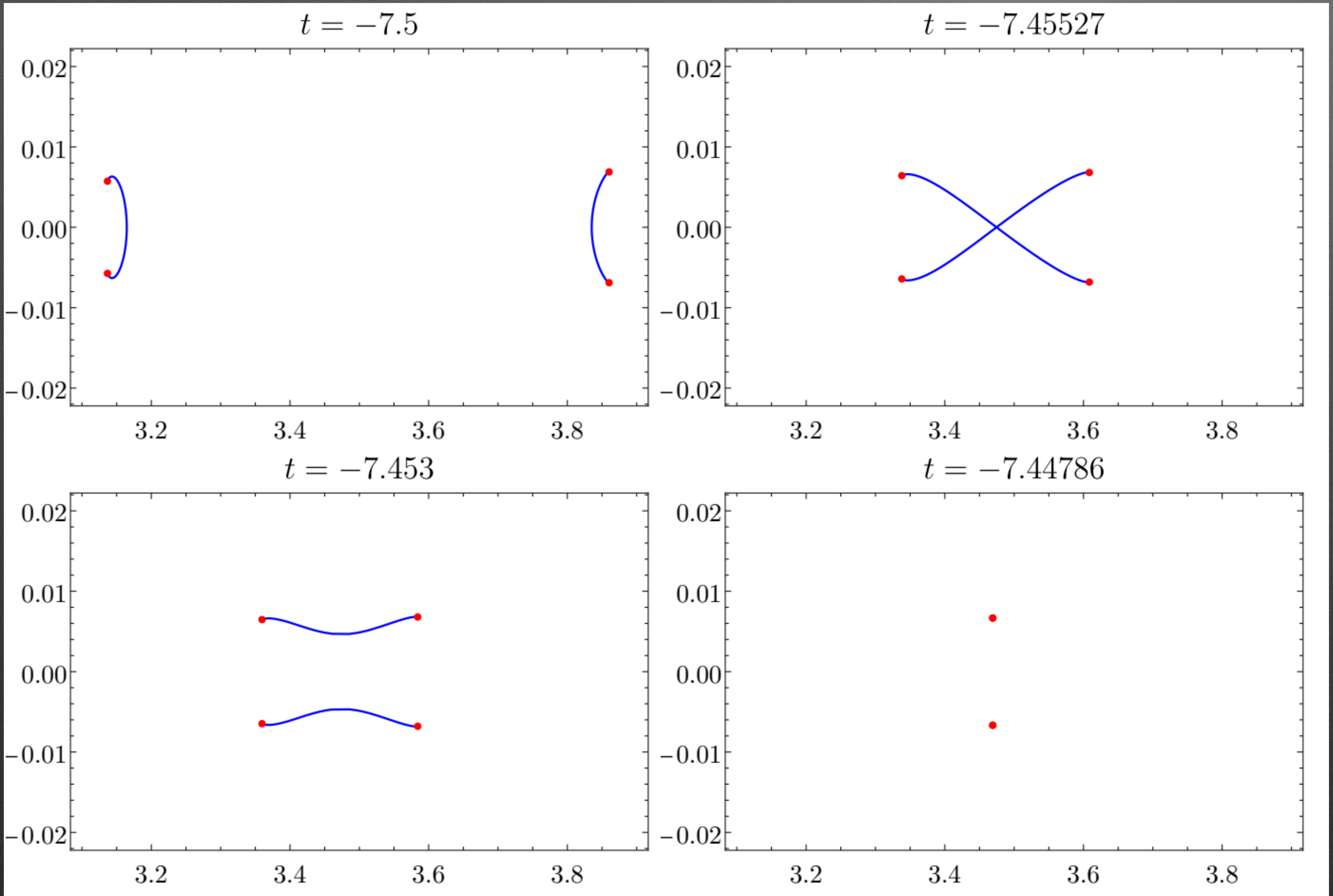
$$R(r) = r^4 + \left(a^2 - \lambda^2 - \eta\right) r^2 + 2M\left[(\lambda - a)^2 + \eta\right] r - a^2\eta$$

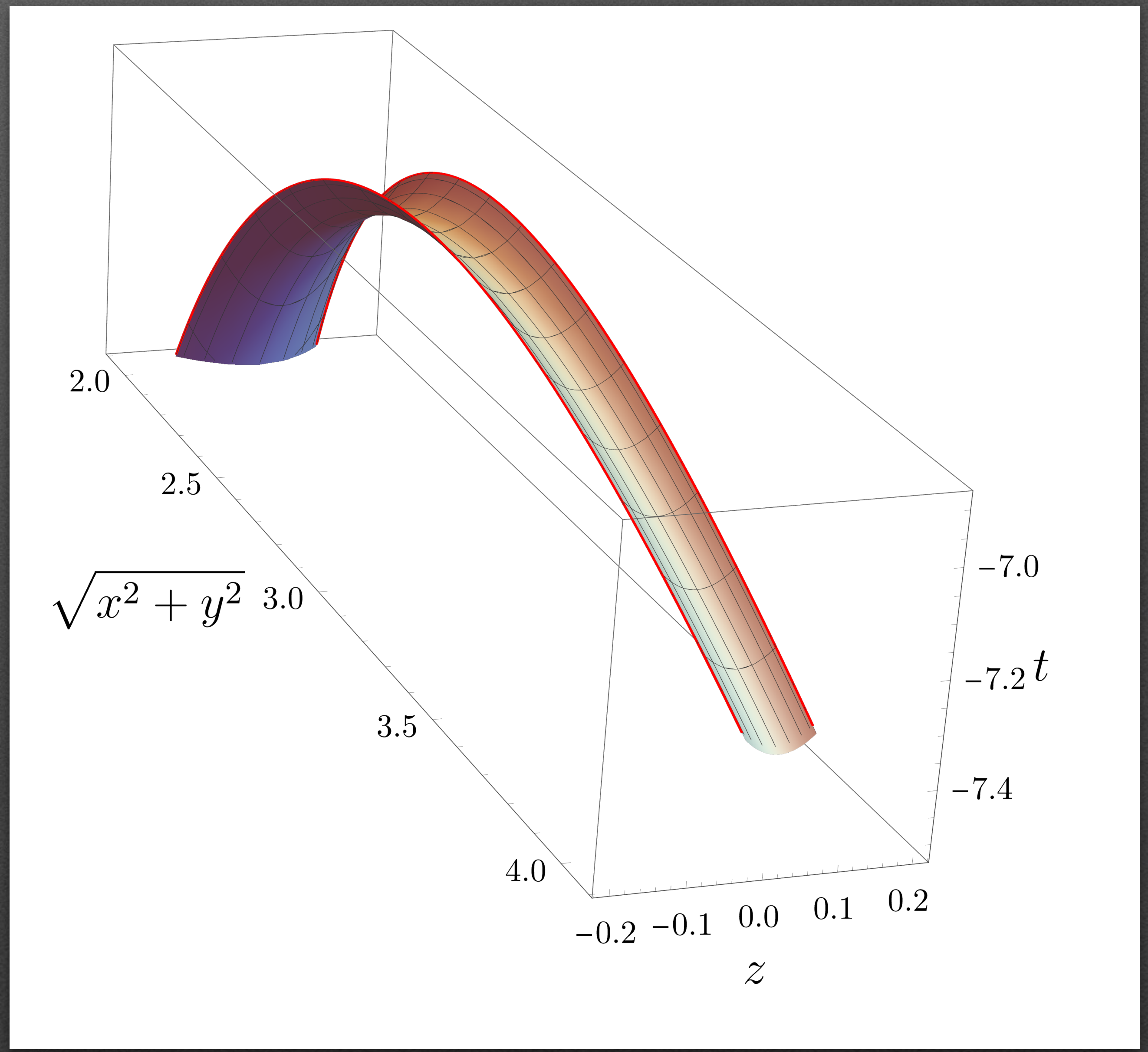
$$\Theta(\theta) = \left(a^2 - \lambda^2\csc^2\theta\right)\cos^2\theta + \eta\,,$$

Kerr Geodesics

Bray 1986
Rauch, Blanford 1994
Mino 2003
Sereno, de Luca 2006, 2008
Gralla, Lupsasca 2020

Perestroika





False Crease