Modeling Relativistic Stars within Einstein-Vlasov-Boltzmann Theory

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Einstein-Vlasov-Boltzmann system

- Einstein field equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

- Vlasov-Boltzmann equation

$$p^\mu \partial_\mu f - \Gamma^i{}_{\mu\nu} p^\mu p^\nu \partial_{p^i} f = C[f] \qquad f = f(t, x^i, p^i)$$

- Energy-momentum tensor

$$T^{\mu\nu} = -c \int p^\mu p^\nu f \sqrt{-g} \frac{d^3p}{h^3 p_t}$$

Construction of
Spherical Static Solutions

Spherical Static Solution

- Cartesian Schwarzschild-like coordinates

$$\mathrm{g}_{\mu\nu}\big|_{y=z=0} = \begin{pmatrix} -e^{2\nu} & 0 & 0 & 0 \\ 0 & \frac{1}{1-2m/x} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\partial_t g_{\mu\nu} = 0$$

Spherical Static Solution

- Einstein field equations

$$\partial_x m = -4\pi x^2 T^t{}_t$$

$$\partial_x \nu = \frac{4\pi T^x{}_x x^3 + m}{x^2 - 2mx}$$

Local Equilibrium

- Equilibrium distribution function
  [F. Jüttner - Z. Phys. 47 (1928) 542]

$$f_{\mathrm{eq}} = \frac{g_s}{h^3} \frac{1}{e^{-(\mu_E + U_\nu p^\nu)/k_B T} + \varepsilon}$$

g_s - multiplicity

h - Planck constant

T - temperature

U^μ - average four velocity

ε - fermions: $\varepsilon = 1$ bosons: $\varepsilon = -1$

μ_E - chemical potential

k_B - Boltzmann constant

Local Equilibrium

- Equilibrium distribution function
  [F. Jüttner - Z. Phys. 47 (1928) 542]

$$f_{\mathrm{eq}} = \frac{g_s}{h^3} \frac{1}{e^{-(\mu_E + U_\nu p^\nu)/k_B T} + \varepsilon}$$

- In equilibrium the collision term vanishes

$$C[f_{\mathrm{eq}}] = 0$$

Global Equilibrium

- Ansatz [N.A. Chernikov - Acta Phys. Pol. 26 (1964) 1069]

$$f = \frac{1}{e^{-(\mu_E(x^i)+U_\nu(x^i)p^\nu)/k_B T(x^i)} + \varepsilon}$$

- We want a static solution and we chose $g_{it} = 0$

$$U_\mu(x^i) = (-e^\nu, 0, 0, 0)$$

Global Equilibrium

- Vlasov-Boltzmann equation is then satisfied for

$$\alpha_f = \frac{\mu_E}{k_B T} = const.$$

$$k_B T e^{\nu} = const.$$

α_f - fugacity

m_p - particle mass

Global Equilibrium

- Vlasov-Boltzmann equation is then satisfied for

$$\alpha_f = \frac{\mu_E}{k_B T} = const.$$

$$k_B T e^{\nu} = const.$$

← **Tolman law**

α_f - fugacity

m_p - particle mass

Setup

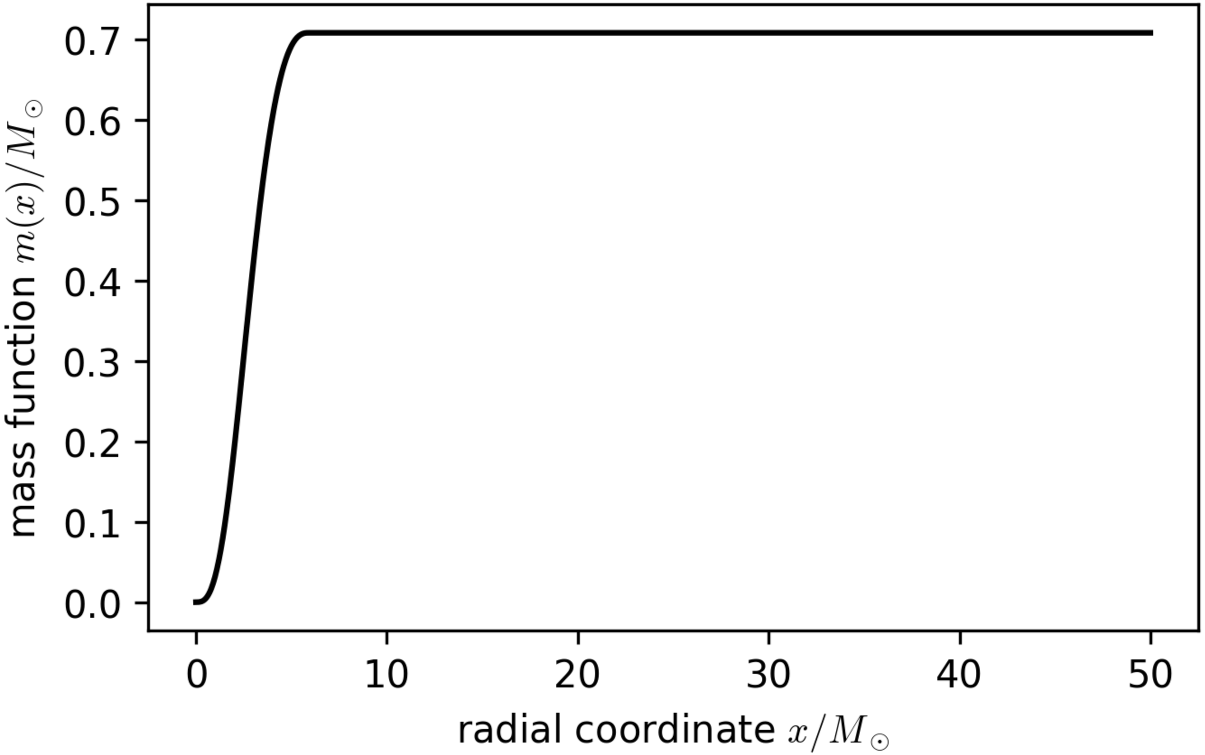
- Choice of particles
  - Only neutrons
  - $m_p = m_n = 8.42\ 10^{-58}\ M_\odot$
    $= 937\ MeV$ (neutron mass)
  - Fermions: $\varepsilon = 1$

Stellar Profile

- Mass function

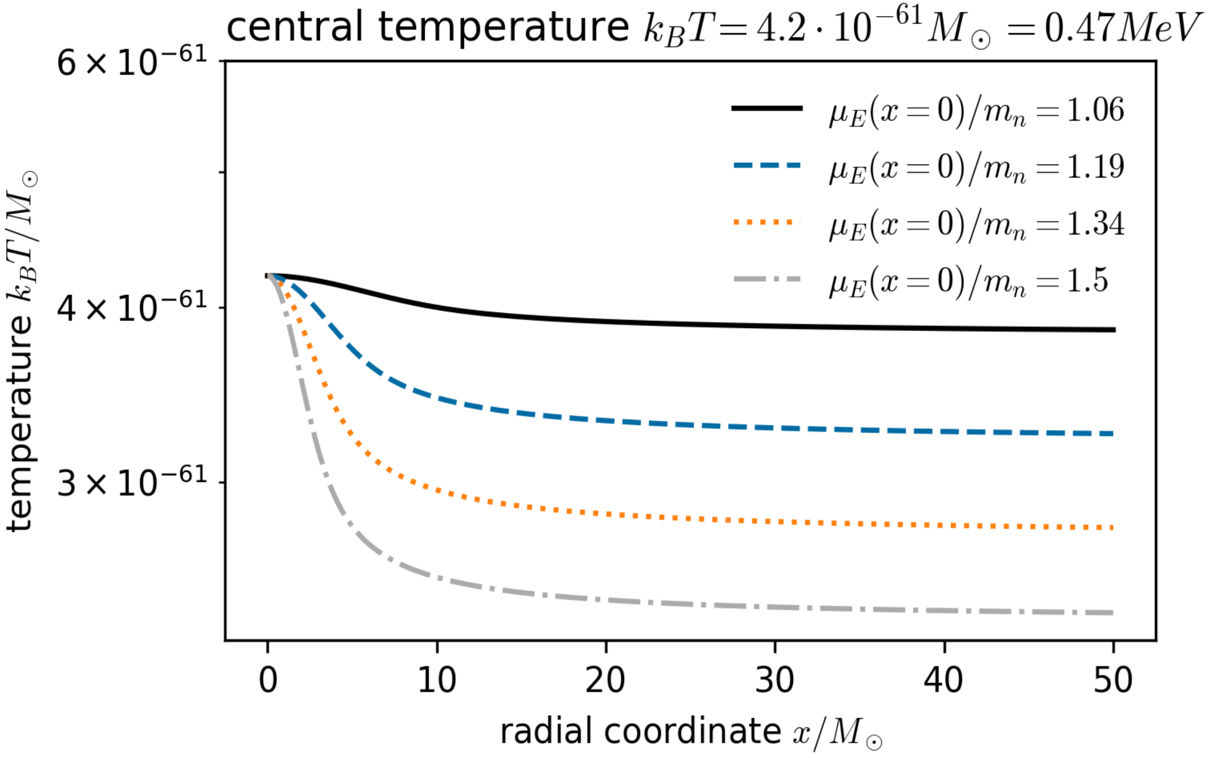
$$\mu_E(x=0) = 1.34 m_n$$

$$k_B T = 0.0005 m_n$$



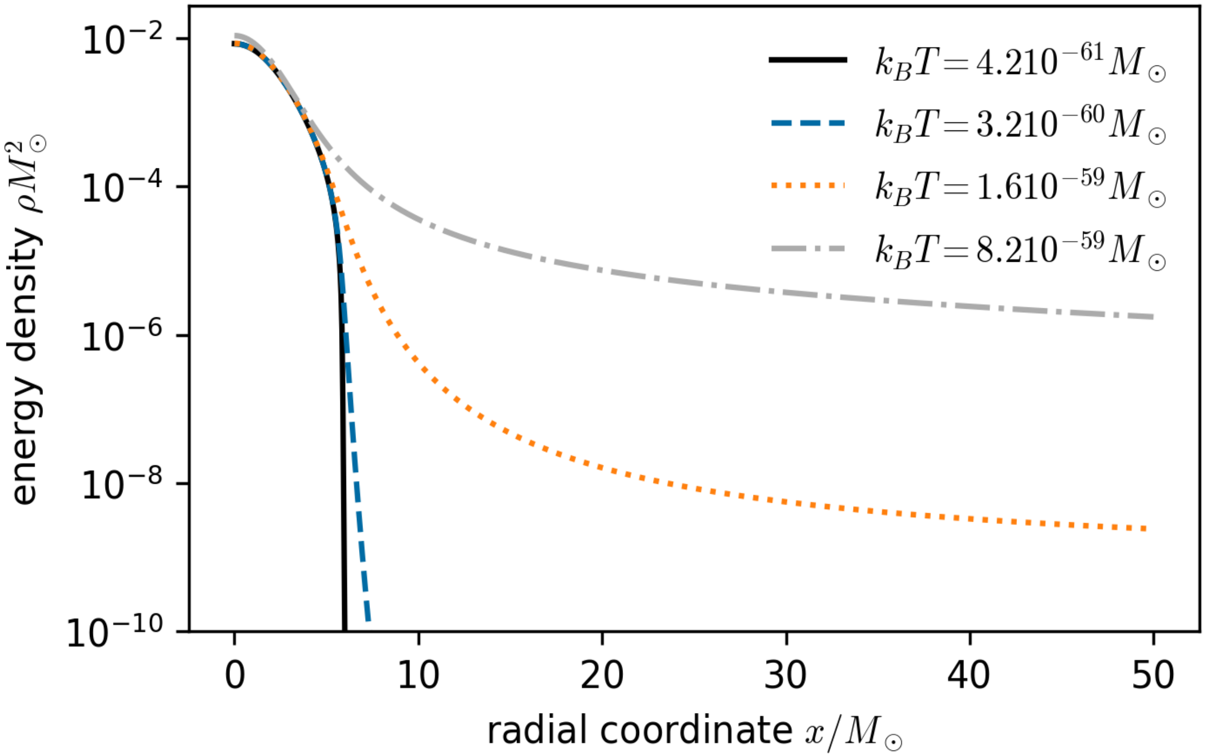
Stellar Profile

- Temperature



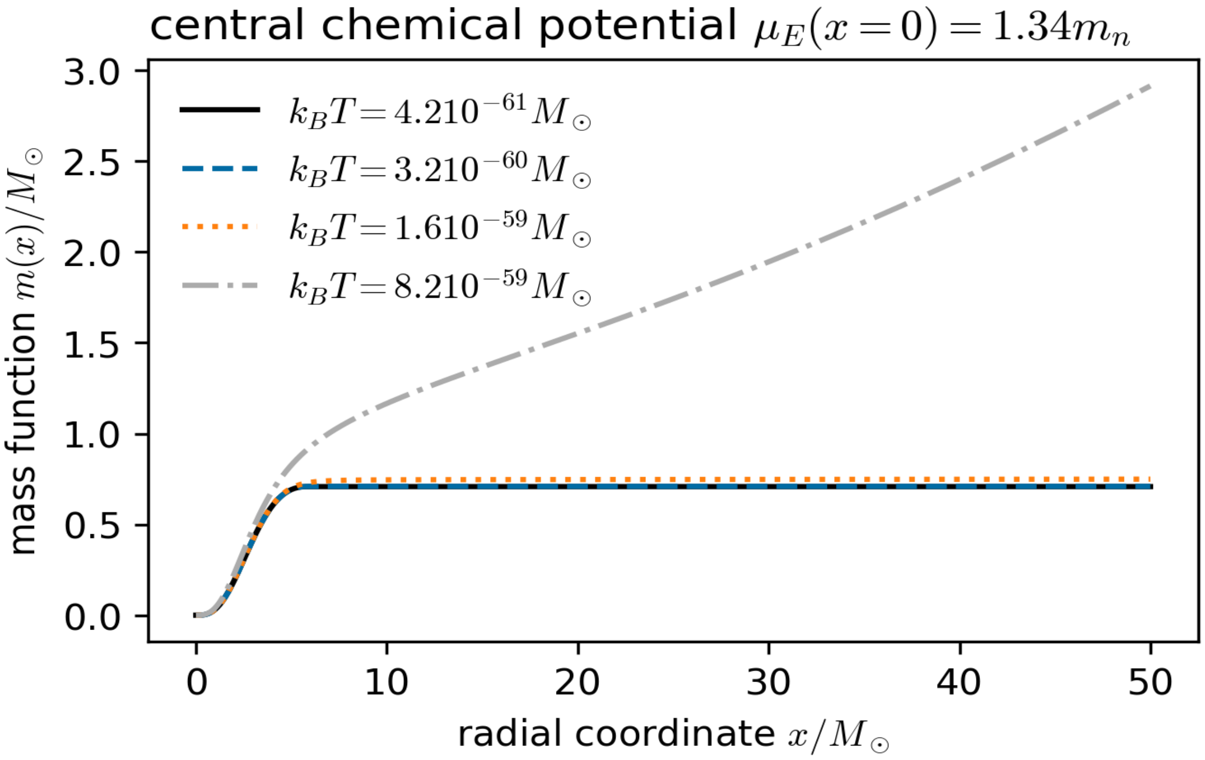
Stellar Profile

- Energy density

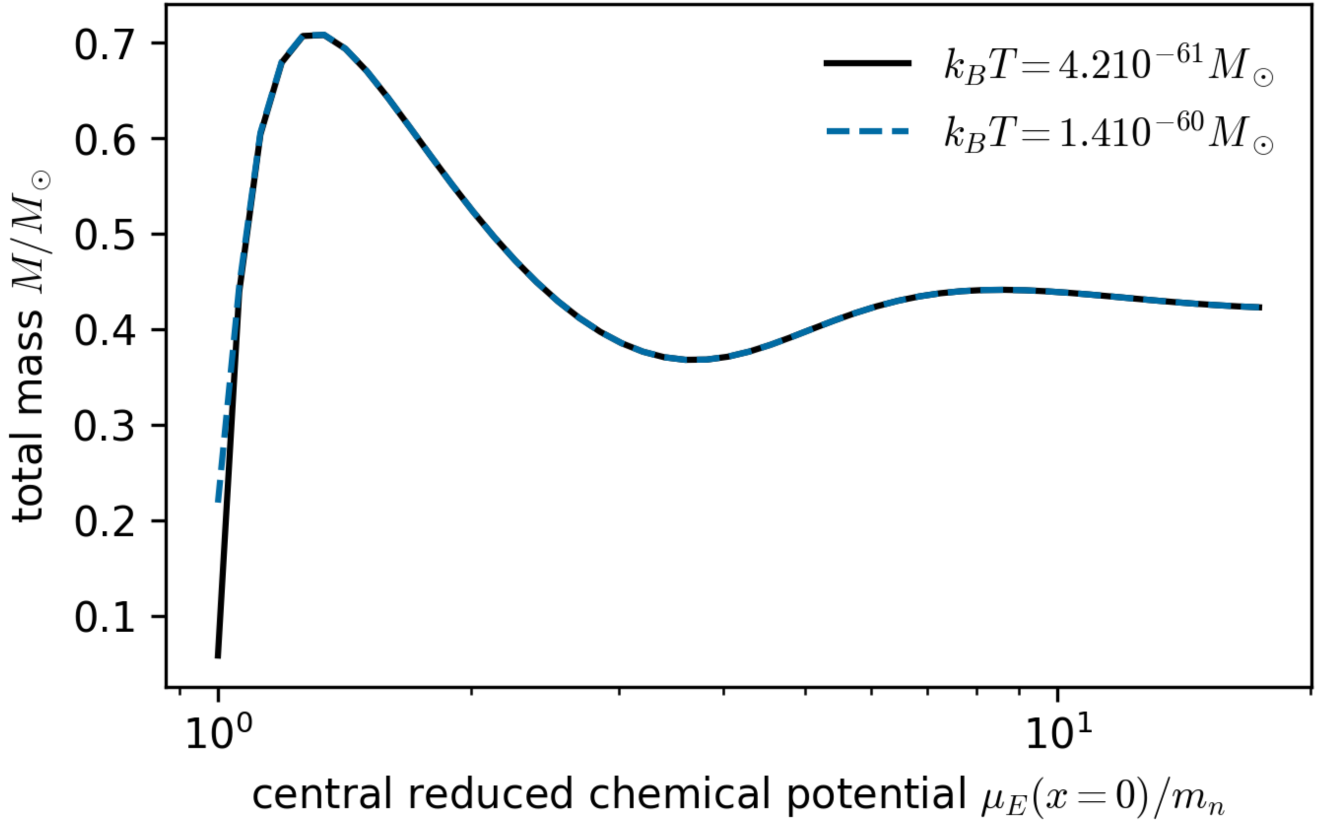


Stellar Profile

- Mass function

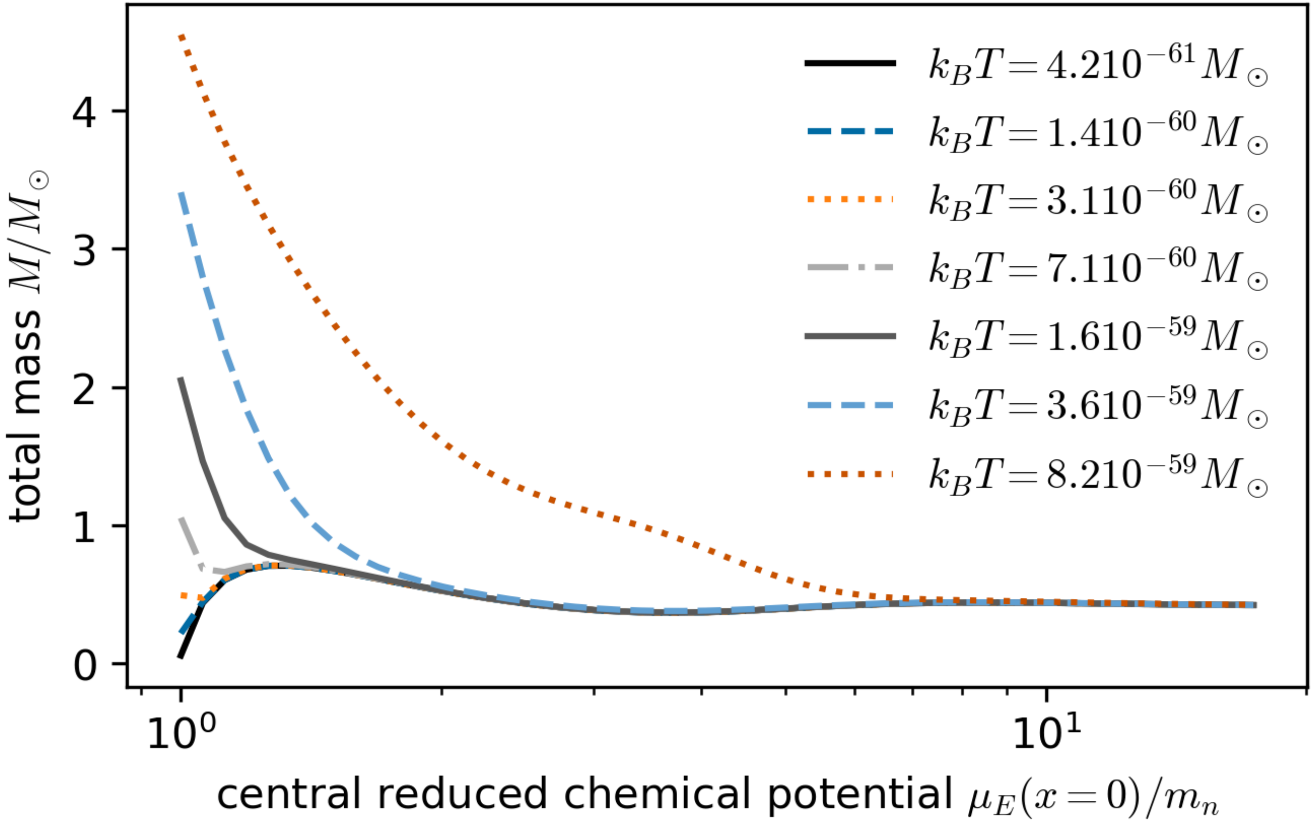


Total Mass



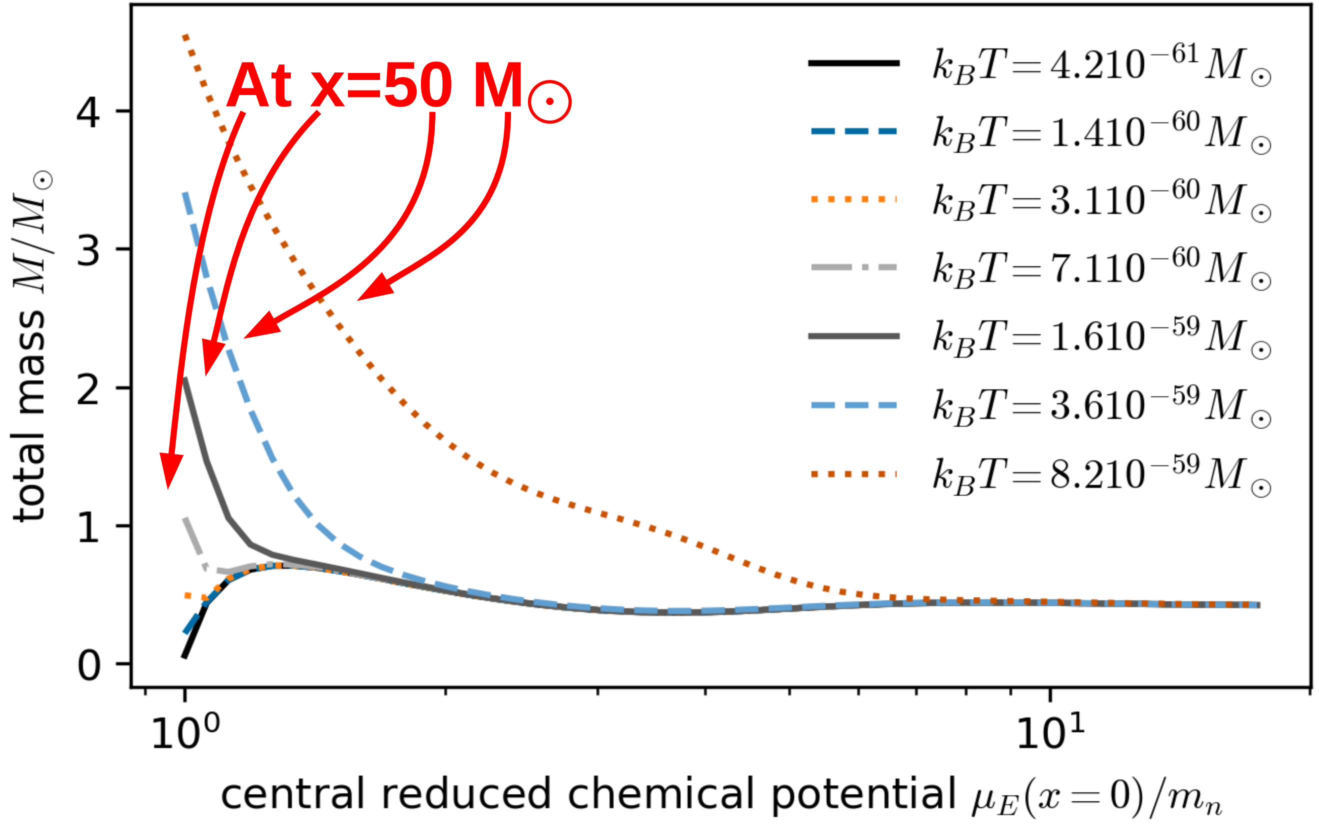
0.7
0.6
0.5
0.4
0.3
0.2
0.1
total mass $M/M_\odot$
10^0
10^1
central reduced chemical potential $\mu_E(x=0)/m_n$
$k_B T = 4.210^{-61} M_\odot$
$k_B T = 1.410^{-60} M_\odot$

Total Mass



$k_B T = 4.210^{-61} M_\odot$
$k_B T = 1.410^{-60} M_\odot$
$k_B T = 3.110^{-60} M_\odot$
$k_B T = 7.110^{-60} M_\odot$
$k_B T = 1.610^{-59} M_\odot$
$k_B T = 3.610^{-59} M_\odot$
$k_B T = 8.210^{-59} M_\odot$
total mass $M/M_\odot$
0
1
2
3
4
10^0
10^1
central reduced chemical potential $\mu_E(x=0)/m_n$

Total Mass


At x=50 M⊙
$k_BT = 4.210^{-61} M_\odot$
$k_BT = 1.410^{-60} M_\odot$
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Outlook

Multiple Particle Species

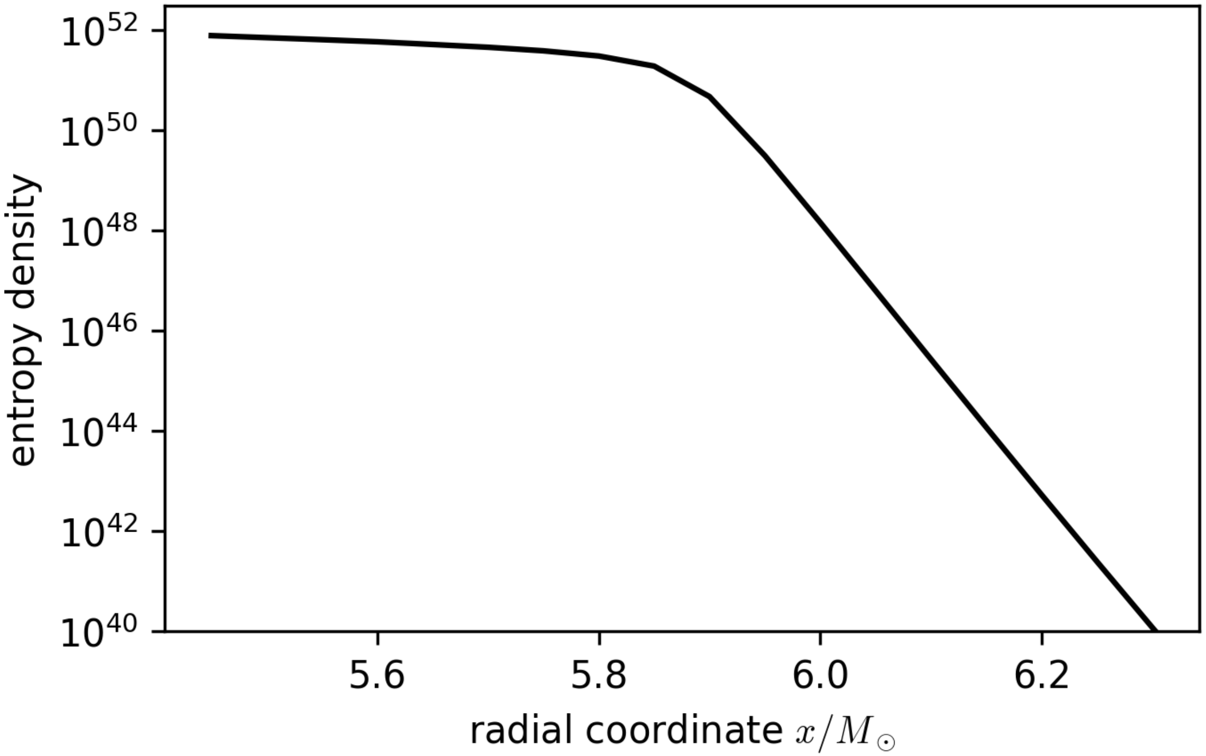
- Non-interacting species
  - For example dark matter particles
  - Ansatz carries over
- Interacting particle species
  - Scattering term $C[f]$ no longer vanishes for Fermi-Dirac distribution

Summary

- For the temperature profile the Tolman law is recovered.
- Not all combinations of central temperature and density lead to solutions with finite mass

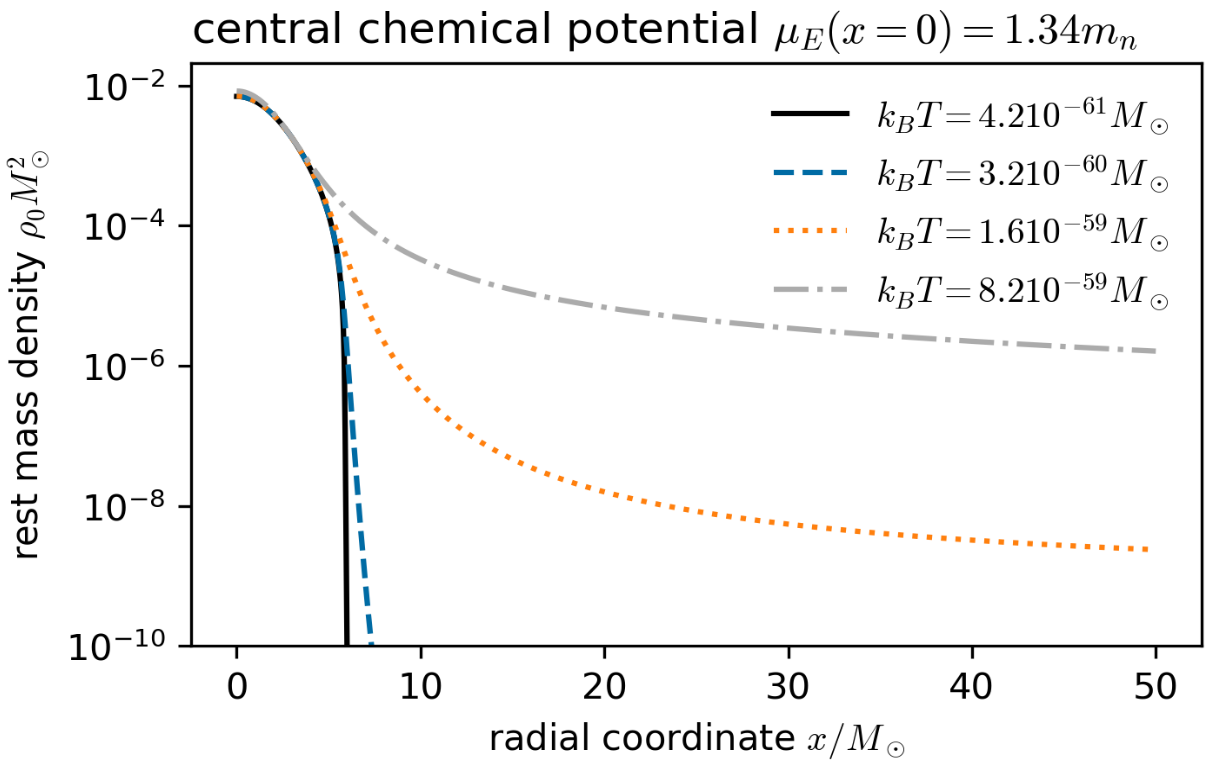
Stellar Profile

- Entropy Density

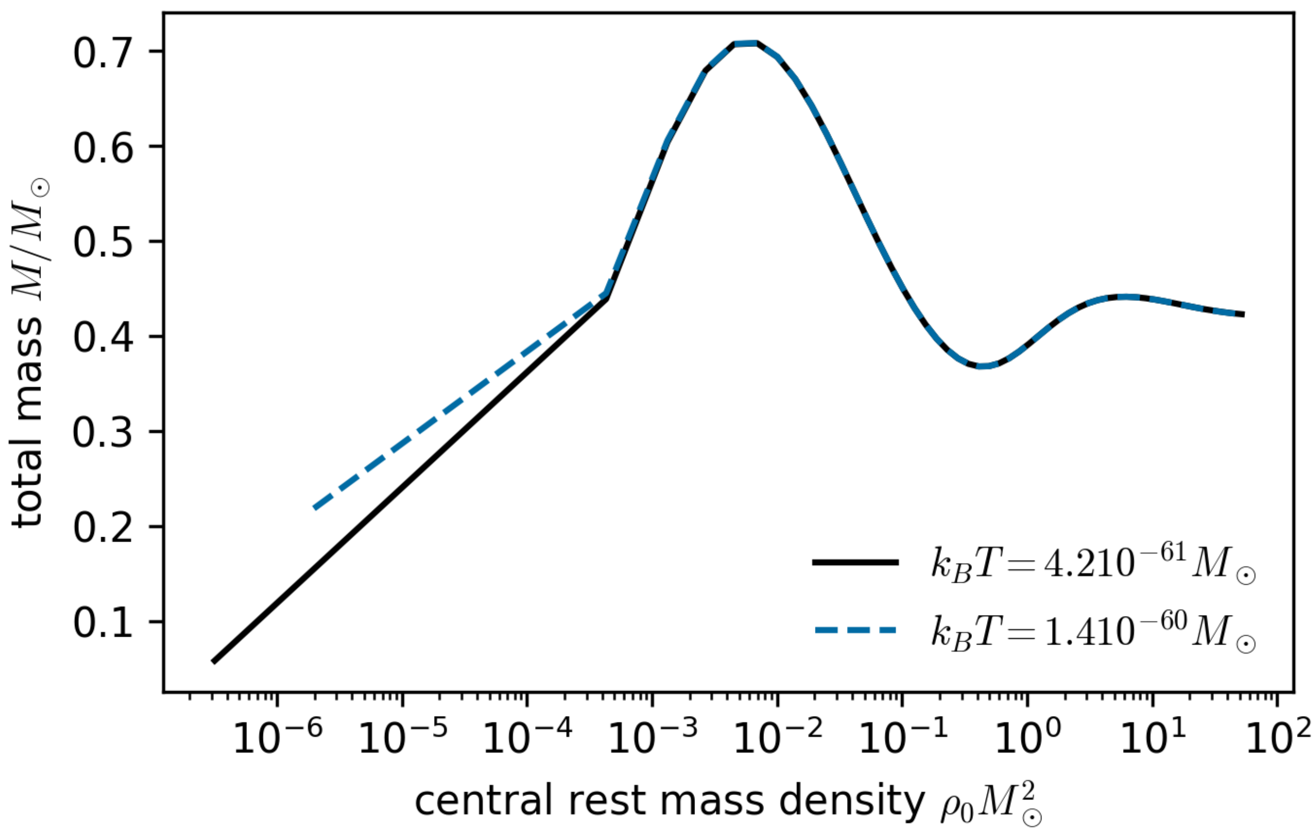


Stellar Profile

- Rest mass density



Total Mass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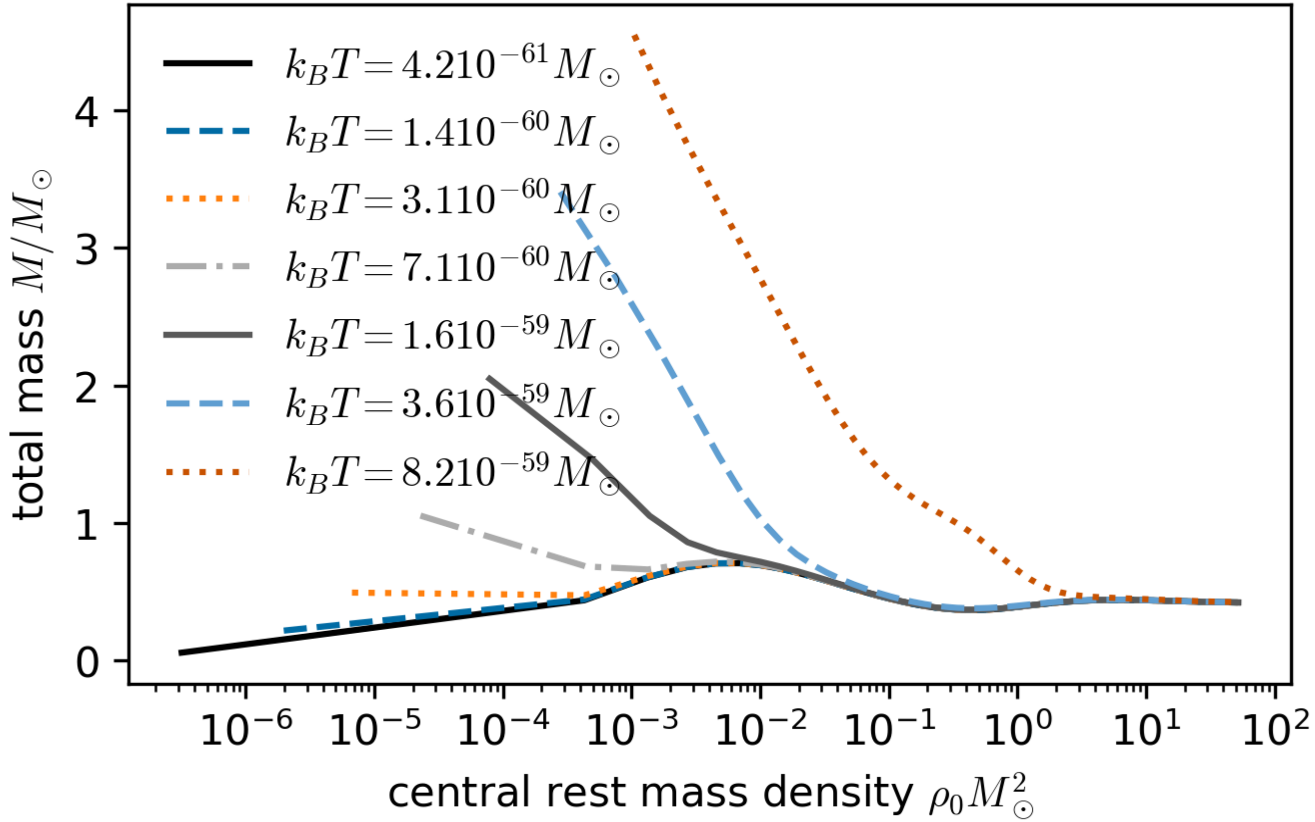
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
central rest mass density $\rho_0 M_\odot^2$

Total Mass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10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
central rest mass density $\rho_0 M_\odot^2$