Schwarzschild-plus-reservoir

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Pelayo V. Calzada[1] Pedro Bargueño[1] Salvador Miret-Artés[2]

[1]Departamento de Física Aplicada, Universidad de Alicante
[2]Instituto de Física Fundamental, Consejo Superior de Investigaciones Científicas

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Motivation: quantum fluctuations

In the absence of a unified framework, some potential alternatives to incorporate **quantum fluctuations** in the **spacetime geometry dynamics** are:

- Stochastic gravity: stochastic noise is added on top of the semiclassical Einstein equations.
- Postquantum Classical Gravity: classical-quantum mechanics framework.

Motivation: open systems approach

We introduce an alternative mechanism, inspired in the **open systems approach** to effectively introduce fluctuations in classical spacetime dynamics:

Weak interactions of the classical spacetime geometry with a **thermal bath** result in modified dynamics that exhibit **stochastic effects**.

Here, we will demonstrate this proposal by constructing a simple toy model for a particular scenario.

Motivation: spacetime foam

Spacetime itself at the smallest scales should manifest **quantum fluctuations of geometry**, conforming a **spacetime foam**.

Applying the open systems approach:

Spacetime foam may be modelled as a **thermal bath** weakly interacting with the classical geometry.

Idea suggested by L. Garay to study spacetime foam perturbations in low-energy fields in a flat spacetime.

L. J. Garay, Spacetime foam as a quantum thermal bath, Phys. Rev. Lett. 80, 2508–2511 (1998).

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Open systems approach: system-plus-reservoir

System-plus-reservoir Hamiltonian formulation

External degrees of freedom on a system are modelled as an **interacting environment**.

Two assumptions are usually made:

(i) the environment is non-relativistic and only weakly perturbed from its equilibrium $\Rightarrow$ Its dynamics can be modelled by a **harmonic reservoir**.

(ii) the coupling to the environment is weak $\Rightarrow$ a **bilinear interaction** is considered.

System-plus-reservoir models

Assuming a system described by a particle in a potential V, a **coordinate coupling** leads to the **Caldeira-Leggett Hamiltonian**:

$$H_{C-L} = \frac{p^2}{2M} + V(q) + \frac{1}{2}\sum_j \left(p_j^2 + \omega_j^2 \left(x_j - \alpha_j q\right)^2\right), \tag{1}$$

which results in the **generalized Langevin equation**:

$$m\ddot{q} + m\int_0^t ds\ \gamma(t-s)\dot{q}(s) + \frac{\partial V}{\partial q} = \xi(t), \tag{2}$$

describing:

- a deterministic damping (determined by the friction kernel $\gamma(t)$),
- a stochastic contribution $\xi(t)$.

Analogous results are obtained considering **momentum coupling** instead.

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Hamiltonian formulation for the Schwarzschild spacetime

Considering a **Schwarzschild observer** for the static patch, **Kuchař's reduced Hamiltonian** may be obtained:

$$\Gamma_S[m, p] = \int (p\dot{m} - m)dt \rightarrow \boxed{H_s = m(t)} \tag{3}$$

where $m(t)$ denotes the Schwarzschild mass.

K. V. Kuchař, Geometrodynamics of Schwarzschild black holes, Phys. Rev. D 50, 3961 (1994).

Schwarzschild-plus-reservoir Hamiltonian

Which bilinear coupling should we consider for a system-plus-reservoir Hamiltonian?

Evolution of p is fixed by Kuchař's reduction procedure $\Rightarrow$ a **momentum coupling** is considered.

The **Schwarzschild-plus-reservoir Hamiltonian** proposed is

$$H = \mathrm{m} + \frac{1}{2}\sum_j \left((p_j - \mu_j \mathrm{p})^2 + \omega_j^2 x_j^2 \right), \tag{4}$$

describing a **harmonic bath** interacting with the **Schwarzschild spacetime** through a **momentum coupling**.

Harmonic bath interpretation

In a **flat spacetime**, L. Garay modelled the **spacetime foam** as a Klein-Gordon scalar field $\Rightarrow$ a set of **harmonic oscillators**.

In a **curved spacetime** harmonic oscillators may still be related to the spacetime foam:

- Spacetime foam can be pictured as a **gas of virtual black holes**.
- Attending to its quasinormal modes spectrum a black hole may be described as an ensemble of **harmonic oscillators**.

  M. Cadoni, M. Oi, & A. P. Sanna, Quasinormal modes and microscopic structure of the schwarzschild black hole, Phys. Rev. D 104 (12) (2021)

Field equations

For the Schwarzschild-plus-reservoir Hamiltonian

$$H = \mathrm{m} + \frac{1}{2}\sum_j \left((p_j - \mu_j \mathrm{p})^2 + \omega_j^2 x_j^2 \right), \tag{5}$$

the obtained equations of motion are

$$\begin{cases} \dot{p}_j = -w_j^2 x_j, & (6) \\ \dot{x}_j = p_j - \mu_j \mathrm{p}, & (7) \end{cases} \qquad \begin{cases} \dot{\mathrm{p}} = -1, & (8) \\ \dot{\mathrm{m}} = \sum_{j=1}^{N} \left(-\mu_j p_j + \mu_j^2 \mathrm{p} \right). & (9) \end{cases}$$

Schwarzschild mass dynamics

Eliminating the reservoir degrees of freedom

$$\mathrm{m}(t) = \widetilde{m} + \frac{2}{\pi}\int_0^\infty d\omega \frac{J(\omega)}{\omega}\cos(wt) + \xi(t), \tag{10}$$

where

- $\widetilde{m}$, represents a new **effective mass**, for $t \to \infty$,
- $J(\omega)$, the spectral density, indicates the coupling intensity at each oscillator frequency,
- $\xi(t)$ represents a gaussian stochastic contribution depending on the temperature T of the bath.

Ohmic spectral density

The Ohmic spectral density with an algebraic cutoff is given by

$$J(\omega) = \frac{\gamma\omega}{1 + (\omega/\omega_c)^2}. \tag{11}$$

which results in

$$\mathrm{m}(t) = \widetilde{m} + \gamma\ \omega_c\ e^{-\omega_c t} + \xi(t), \tag{12}$$

where

- $\widetilde{m} = M - \gamma\ \omega_c$,
- $\xi(t)$ corresponds to gaussian white noise.

Summary

- The **spacetime foam** may be effectively modelled by a **thermal bath**.
- **Bilinear interaction** of this thermal bath with the classical geometry results in a **modified dynamics** that exhibit **stochastic effects**.
- For a **Schwarzschild black hole** this concept has been implemented in a toy model.
- Specifically, for an **Ohmic spectral density** the modified dynamics exhibit a **damping** modulated by a **gaussian white noise**.

Thanks!