Primordial Black Hole formation from a nonspherical density profile with a misaligned deformation tensor

Chulmoon Yoo(Nagoya Univ.)

CY arXiv: 2403.11147

DYNAMICS OF PRIMORDIAL BLACK HOLE FORMATION II

@NAGOYA, JAPAN

Oct. 7 - 10

INVITED SPEAKERS

Gabriele Franciolini (CERN, Switzerland)
Marcos Flores (LPENS, France)
***John T. Giblin (Kenyon Coll.)**
Juan Carlos Hidalgo (Universidad Nacional Autonoma de Mexico)
Kazunori Kohri (NAOJ, KEK, KIPMU)
Eugene A. Lim (King's Coll. London)
Shi Pi (Chinese Academy of Sciences, China)

*:to be confirmed

***To access the participants and invited speakers list page, you will need the following password:**
dpbhf2

Nagoya?

Chulmoon Yoo

Why you should attend this workshop

◎**Objectives**

To take this opportunity to promote international mutual understanding, build cooperative relationships, and lead to future constructive and cutting-edge international joint research.

◎**It should be a good enough reason, but you may have additional reasons, e.g., …**

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Schedule

Main organizers

Albert Escrivà (Nagoya)

Tomohiro Harada (Rikkyo)

Yuichiro Tada (Nagoya)

Takahiro Terada (KMI, Nagoya),

Shuichiro Yokoyama (KMI, Nagoya)

Chulmoon Yoo (Nagoya)

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Primordial Black Hole

◎Remnants of primordial non-linear inhomogeneity

◎BHs not produced by late time stellar collapse

◎Reliable formation scenario:

Collapse of rarely dense regions generated by quantum fluctuation during inflation
It’s rare, but has a finite probability!!

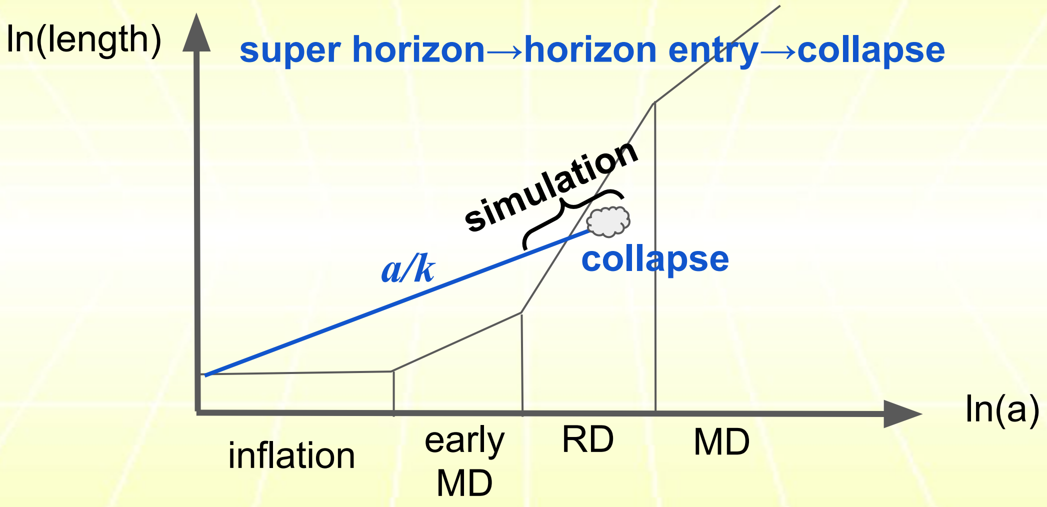
◎If you accept inflation, you should be able to accept the PBH formation

◎PBH is an appealing candidate for dark matter and BH binaries

- ➢ BHs “exist” in our universe
- ➢ BHs behave as dark matter in a cosmological scale
- ➢ Reliable scenario of **PBH** formation

PBH formation

◎Comoving scale of an inhomogeneity ~ $1/k$



◎GR simulation starting from a super-horizon non-linear initial data

◎We assume the linear equation of state p=wρ with w=1/3 and 1/5

Super-horizon scale metric

◎**Leading order metric with gradient expansion**

$$ds^2 = -dt^2 + a^2\Psi^4 f_{ij}dx^i dx^j$$

◎**The next-leading order perturbations**

$$\psi(t,x^k) = \Psi(x^k)(1+\xi(t,x^k)),\ \delta = \frac{\rho-\rho_b}{\rho_b},\ h_{ij} = \tilde{\gamma}_{ij} - f_{ij},\ \chi = \alpha - 1,\ \kappa = \frac{K-K_b}{K_b},\ \tilde{A}_{ij}$$

$$\xi = -\frac{1}{6w+6} f \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$\chi = -\frac{3w+1}{3w+3} f \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$\delta = f \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$h_{ij} = -\frac{4}{(3w+5)(3w-1)} p_{ij} \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$\tilde{A}_{ij} = \frac{2}{3w+5} p_{ij} H_b \left(\frac{1}{aH_b}\right)^2 + \mathcal{O}(\epsilon^4)$$

$$f(x^k) := -\frac{4}{3}\frac{\bar{\triangle}\Psi}{\Psi^5}$$

$$p_{ij}(x^k) := \frac{1}{\Psi^4}\left[-\frac{2}{\Psi}\left(\bar{\mathcal{D}}_i\bar{\mathcal{D}}_j\Psi - \frac{1}{3}f_{ij}\bar{\triangle}\Psi\right) + \frac{6}{\Psi^2}\left(\bar{\mathcal{D}}_i\Psi\bar{\mathcal{D}}_j\Psi - \frac{1}{3}f_{ij}\bar{\mathcal{D}}^k\Psi\bar{\mathcal{D}}_k\Psi\right)\right]$$

$\Psi(x^k)$ fixes everything

Initial condition

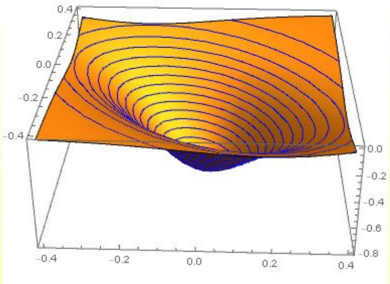
◎Initial curvature perturbation

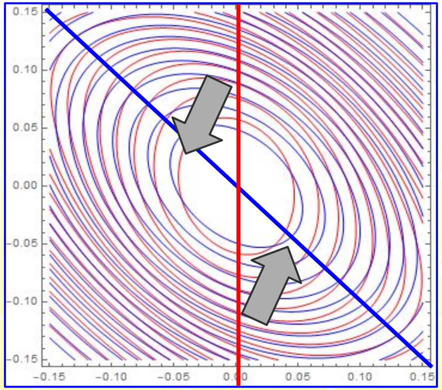
$$\frac{\zeta}{\mu} := -\frac{2}{\mu}\ln\Psi \simeq -1 + \frac{1}{2}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right) + \mathcal{O}(r^4)$$

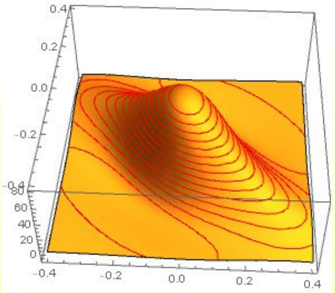
$$\frac{\triangle\zeta}{\mu k^2} \simeq 1 - \frac{1}{2}\left(\kappa_1^2 x^2 + \kappa_2^2 y^2 + \kappa_3^2 z^2\right) + \mathcal{O}(r^4)$$

ζ∼gravitational potential on (x,y) plane

Δζ∼energy density on (x,y) plane







tidal torque ⇒ angular momentum transfer ⇒ spinning PBH

Parameter setting

◎Initial curvature perturbation

$$\frac{\zeta}{\mu} \simeq -1 + \frac{1}{2}\left(k_1^2(x+y)^2/2 + k_2^2(x-y)^2/2 + k_3^2 z^2\right) + \mathcal{O}(r^4)$$

$$k_1^2 = \frac{1}{3}(\xi_1 + 3\xi_2 + \xi_3) \qquad k_2^2 = \frac{1}{3}(\xi_1 - 3\xi_2 + \xi_3) \qquad k_3^2 = \frac{1}{3}(\xi_1 - 2\xi_3)$$

$$\frac{\triangle\zeta}{\mu k^2} \simeq 1 - \frac{1}{2}\left(\kappa_1^2 x^2 + \kappa_2^2 y^2 + \kappa_3^2 z^2\right) + \mathcal{O}(r^4)$$

$$\kappa_1^2 = \frac{1}{3}\left(\tilde{\xi}_1 + 3\tilde{\xi}_2 + \tilde{\xi}_3\right) \qquad \kappa_2^2 = \frac{1}{3}\left(\tilde{\xi}_1 - 3\tilde{\xi}_2 + \tilde{\xi}_3\right) \qquad \kappa_3^2 = \frac{1}{3}\left(\tilde{\xi}_1 - 2\tilde{\xi}_3\right)$$

◎With reference to the peak theory

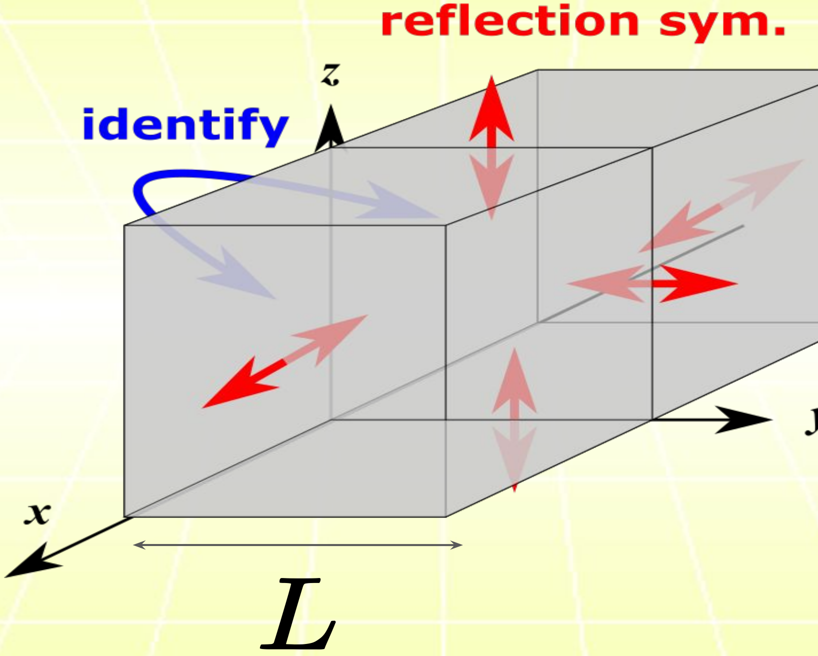
$$\vec{k}^2 = \vec{\kappa}^2 = \xi_1 = \tilde{\xi}_1 = 100/L^2$$

$\xi_3 = \tilde{\xi}_3 = 0$ The most probable values

$\xi_2 = 10/L^2$
$\tilde{\xi}_2 = 15/L^2$

Too large to be statistically expected
⇒Too much idealized for spin generation

Numerical domain



About the numerical code

◎Originally provided by Hirotada Okawa (for E-eqs and real scalar field w/ periodic BC)

◎COSMOS（秋桜）code by C++ [CY, Hirotada Okawa, Ken-ichi Nakao(1306.1389), Hirotada Okawa, Helvi Witek, Vitor Cardoso(1401.1548)]

◎Basically follows the SACRA（桜）code by Fortran

[Tetsuro Yamamoto, Masaru Shibata, Keisuke Taniguchi(arXiv:0806.4007)]

◎Independently developed by CY and Hirotada Okawa

◎In the CY side, it is mainly dedicated to PBH formation as follows

- Inhomogeneous (scale-up) coordinate system has been implemented
  [CY, Taishi Ikeda,Hirotada Okawa(arXiv:1811.00762)]
- Fluid evolution code has been implemented
  [CY, Tomohiro Harada,Hirotada Okawa(arXiv:2004.01042)]
- 1+1 code for spherical systems has been developed based on COSMOS
  [CY, Harada, Hirano, Okawa, Sasaki(arXiv:2112.12335)]
- Recently, a mesh refinement procedure has been implemented

Summary of resolution difference

◎Resolution in the previous simulation 2004.01042 CY, Tomohiro Harada, Hirotada Okawa

- Scale-up reference coordinates x^i related to the Cartesian coord. X^i by

$$X^i = x^i - \frac{S}{1+S}\frac{L}{\pi}\sin\left(\frac{\pi}{L}x^i\right) \text{ with } S = 15$$

- Resolution at the center ($\Delta x = L/100$)

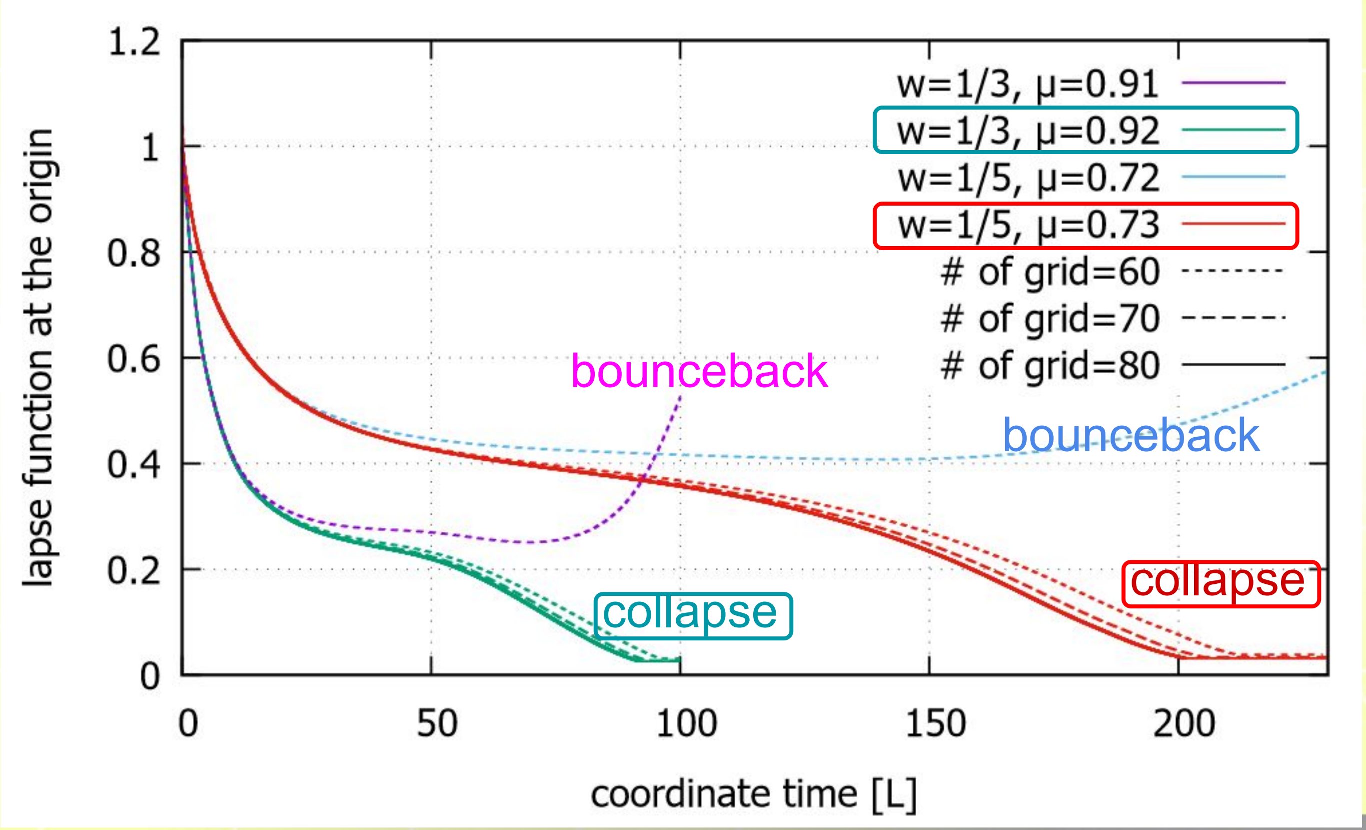
$$\Delta X|_{\text{center}} = \frac{1}{1+S}\Delta x = \frac{1}{16}\frac{L}{100} = \frac{L}{1600}$$

◎New simulation with mesh refinement

- $S = 10, \Delta x = L/80$
- Two additional layers for the mesh refinement

$$\Delta X|_{\text{center}} = \frac{1}{1+S} \times \frac{1}{2^2} \times \Delta x = \frac{1}{44} \times \frac{L}{80} = \frac{L}{3520}$$

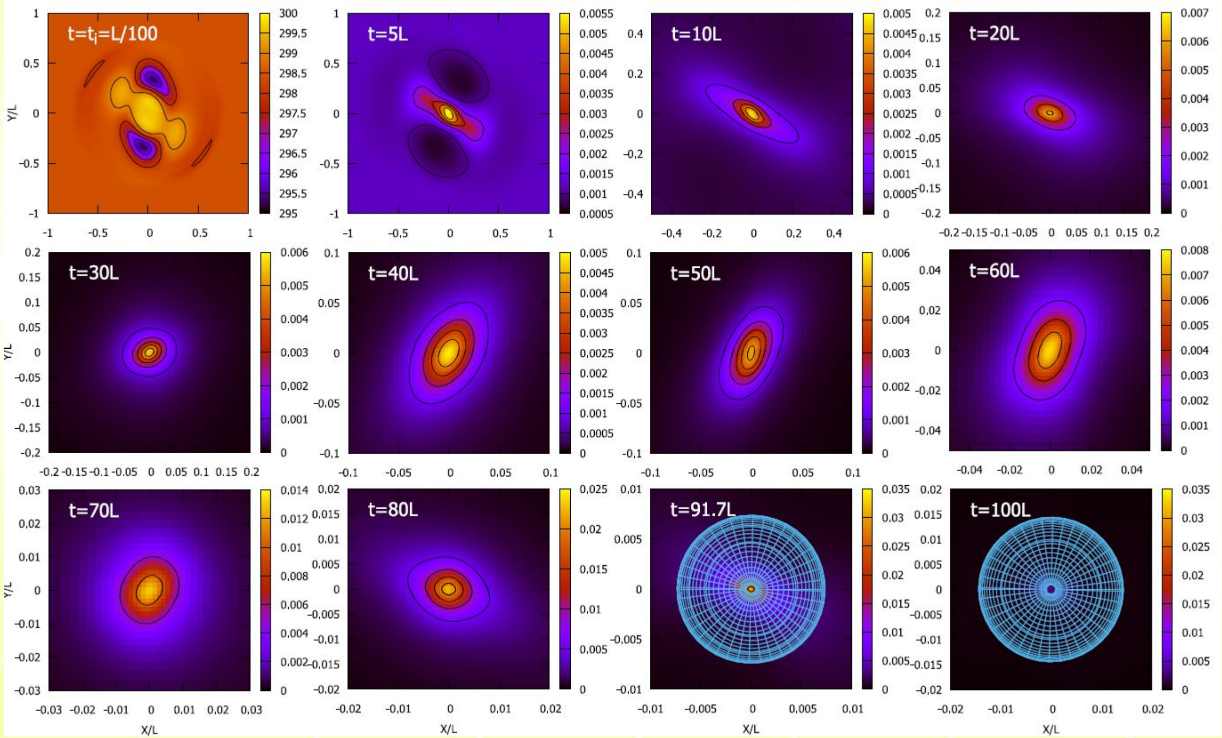
Thresholds



lapse function at the origin
coordinate time [L]
1.2
1
0.8
0.6
0.4
0.2
0
50
100
150
200
w=1/3, μ=0.91
w=1/3, μ=0.92
w=1/5, μ=0.72
w=1/5, μ=0.73
# of grid=60
# of grid=70
# of grid=80
bounceback
bounceback
collapse
collapse

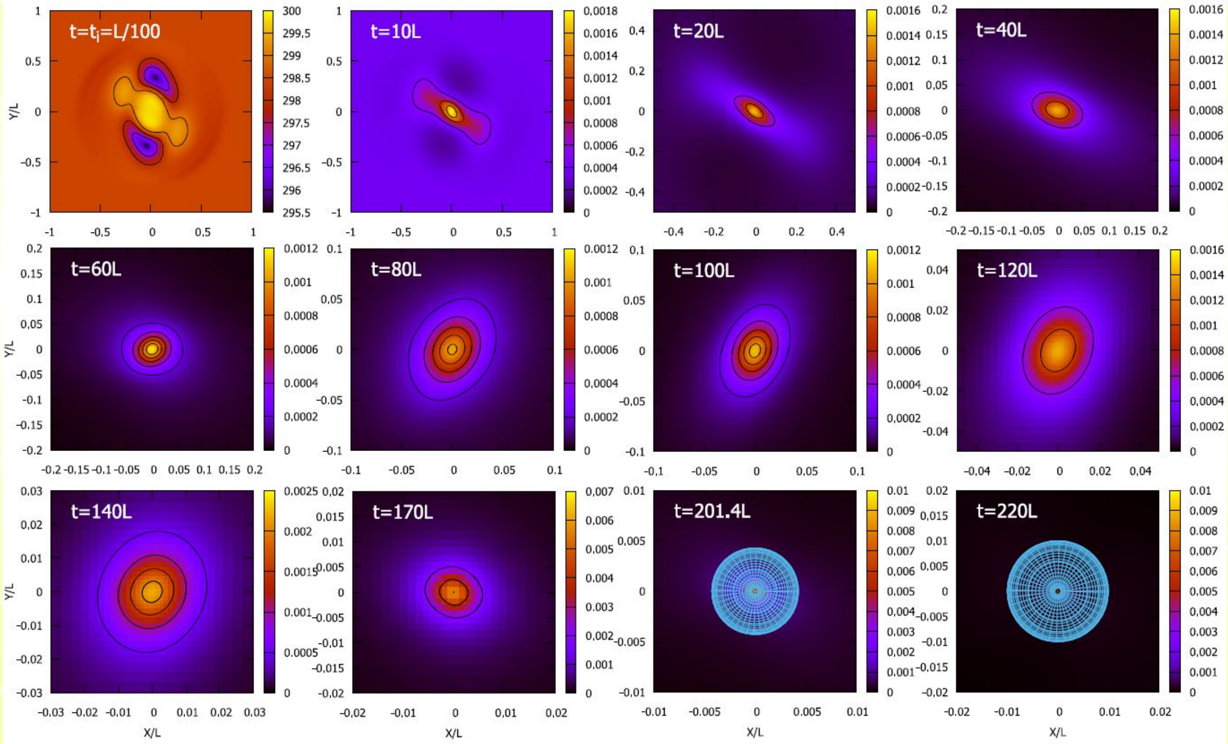
Effective spin parameter

◎w=p/ρ=1/3, amplitude μ=0.92 case



Softer equation of state

◎p/ρ=w=1/5, amplitude μ=0.73 case



Effective spin parameter

◎Kerr black hole case

$$\text{area}: \ A_{\rm Kerr} = 8\pi\left(M^2 + \sqrt{M^4 - a^2 M^2}\right)$$

$$\text{equatorial circumference}: \ d_{\rm Kerr} = 4\pi M$$

$$\Rightarrow s^2_{\rm Kerr} := \frac{a^2}{M^2} = \frac{4\pi A_{\rm Kerr}\left(d^2_{\rm Kerr} - \pi A_{\rm Kerr}\right)}{d^4_{\rm Kerr}}$$

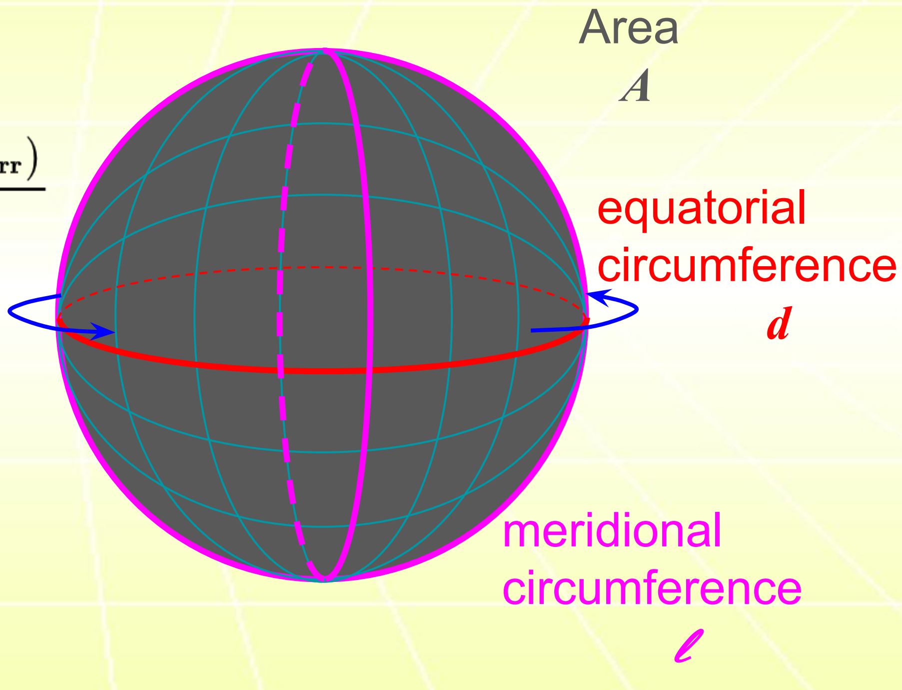
◎Effective dimensionless spin

$$s^2 := \frac{4\pi A\left(d^2 - \pi A\right)}{d^4}$$

◎Asphericity parameters

$$\text{meridional circumference}: \ \ell_{x=0}, \ell_{y=0}$$

$$\alpha := \frac{\ell_{x=0}}{d} \qquad \beta := \frac{\ell_{y=0}}{d}$$



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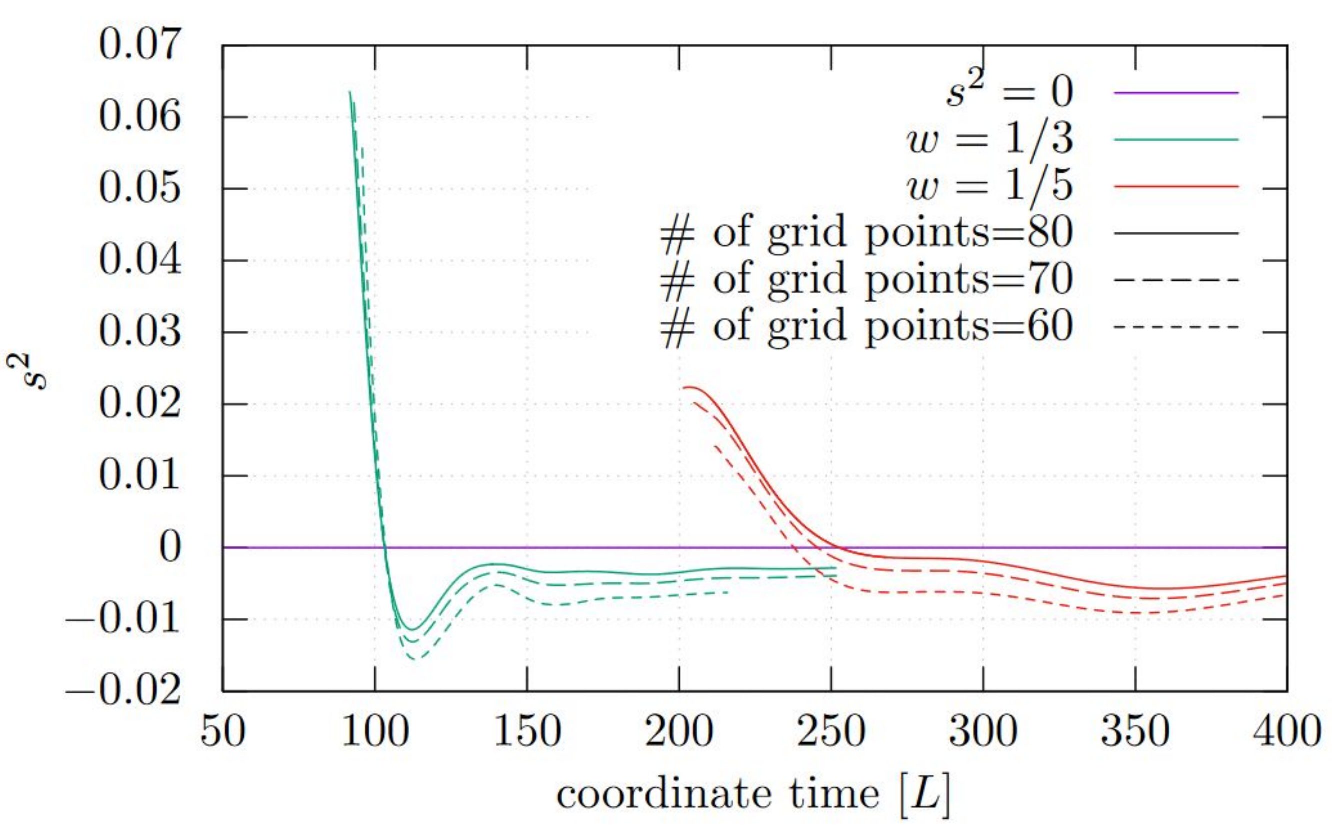

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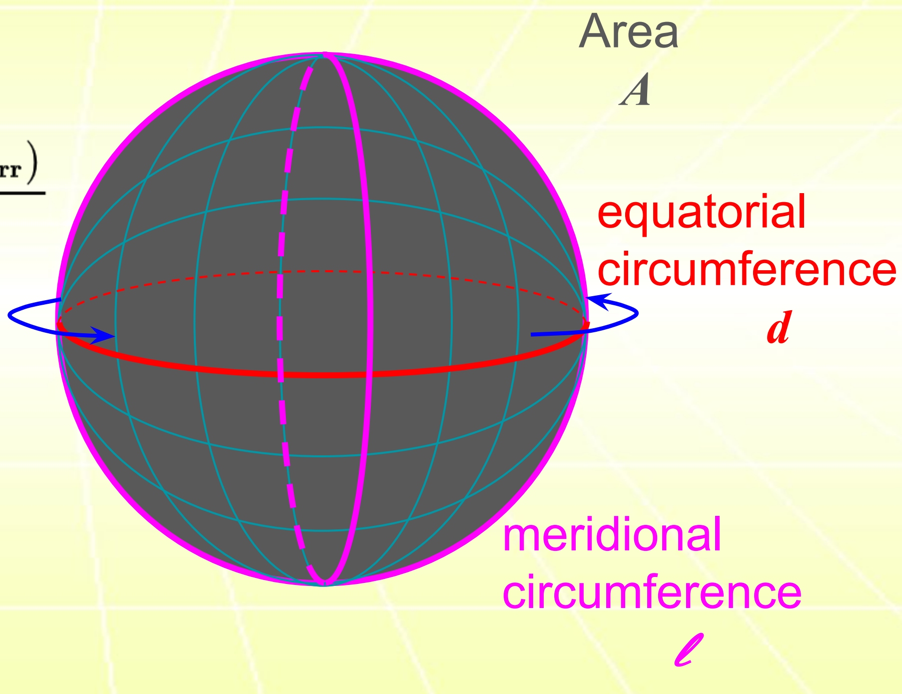
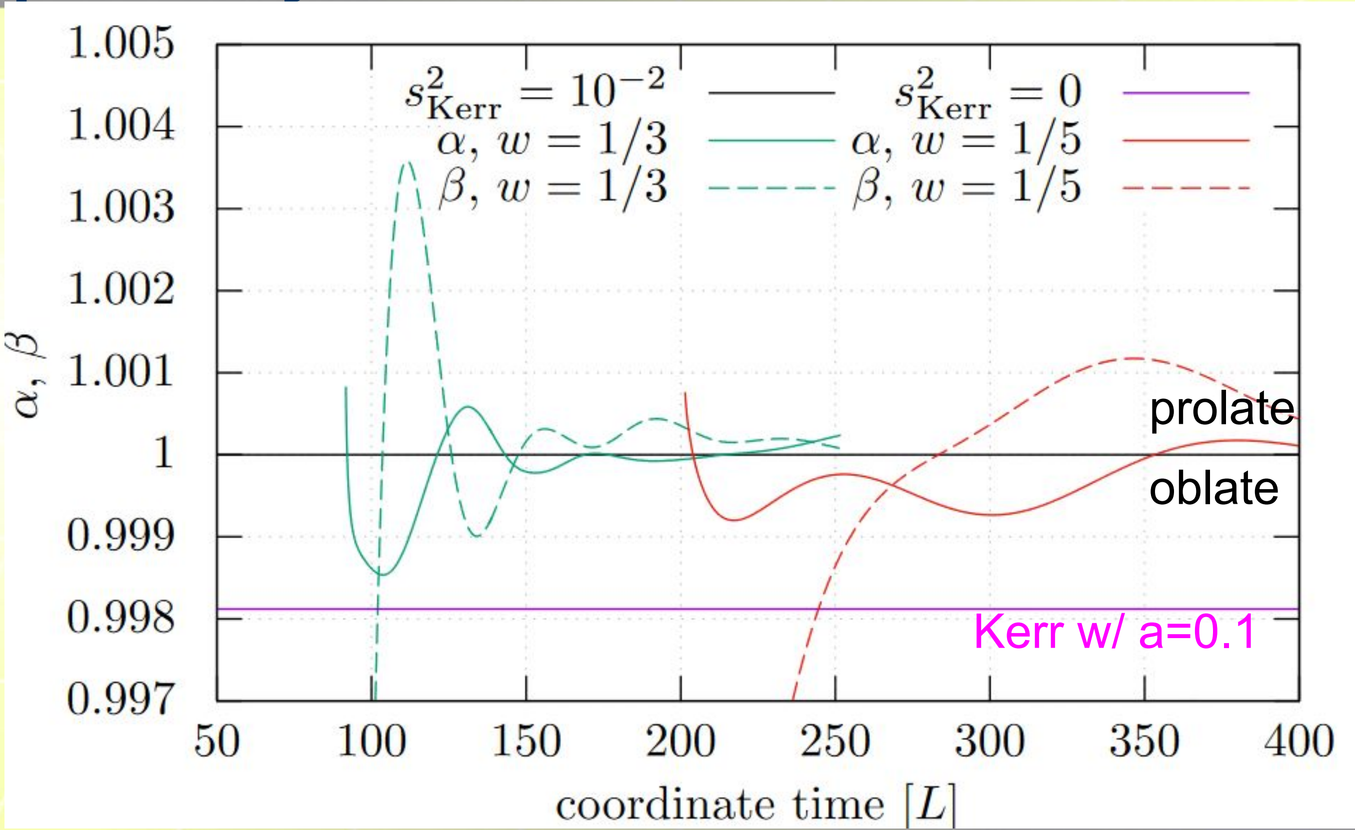


Non-sphericity



Spin of PBH is very small

for the equation of states $p=w\rho$ with $w>1/5$

◎Caveats

- Results only for a specific initial profile
- The case in which the amplitude is very close to the critical value
- etc.

Thank you for your attention

Chulmoon Yoo