Work in progress

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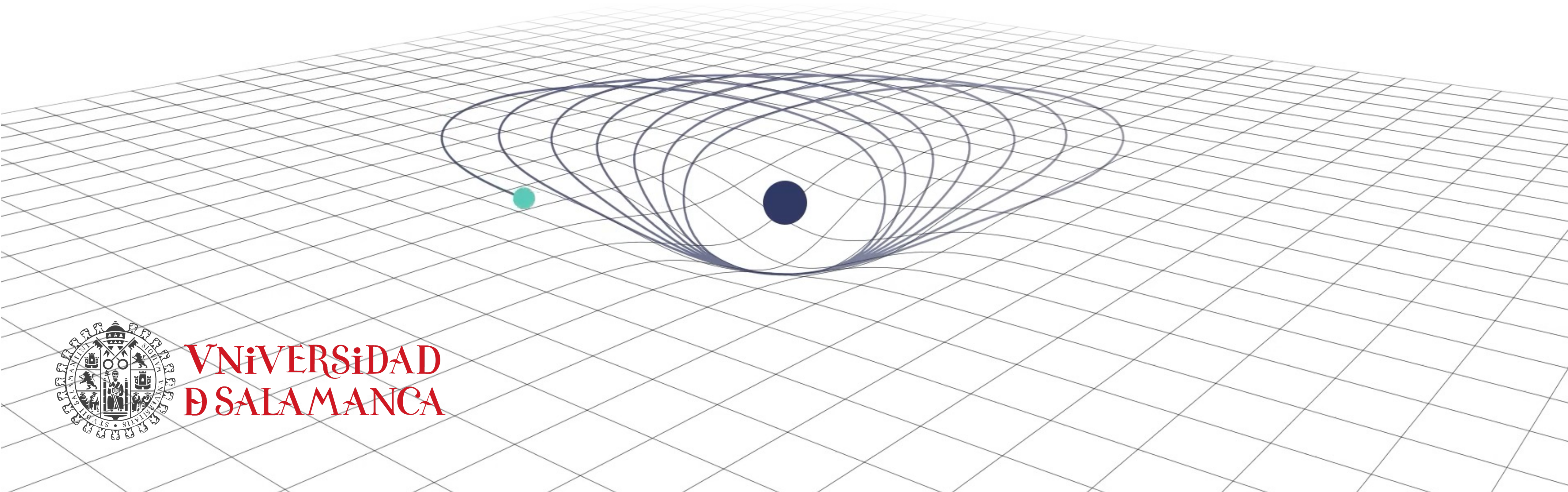
EREP 2024 Coimbra | July 23, 2024

The Galactic Center

as a gravitational laboratory

Riccardo Della Monica

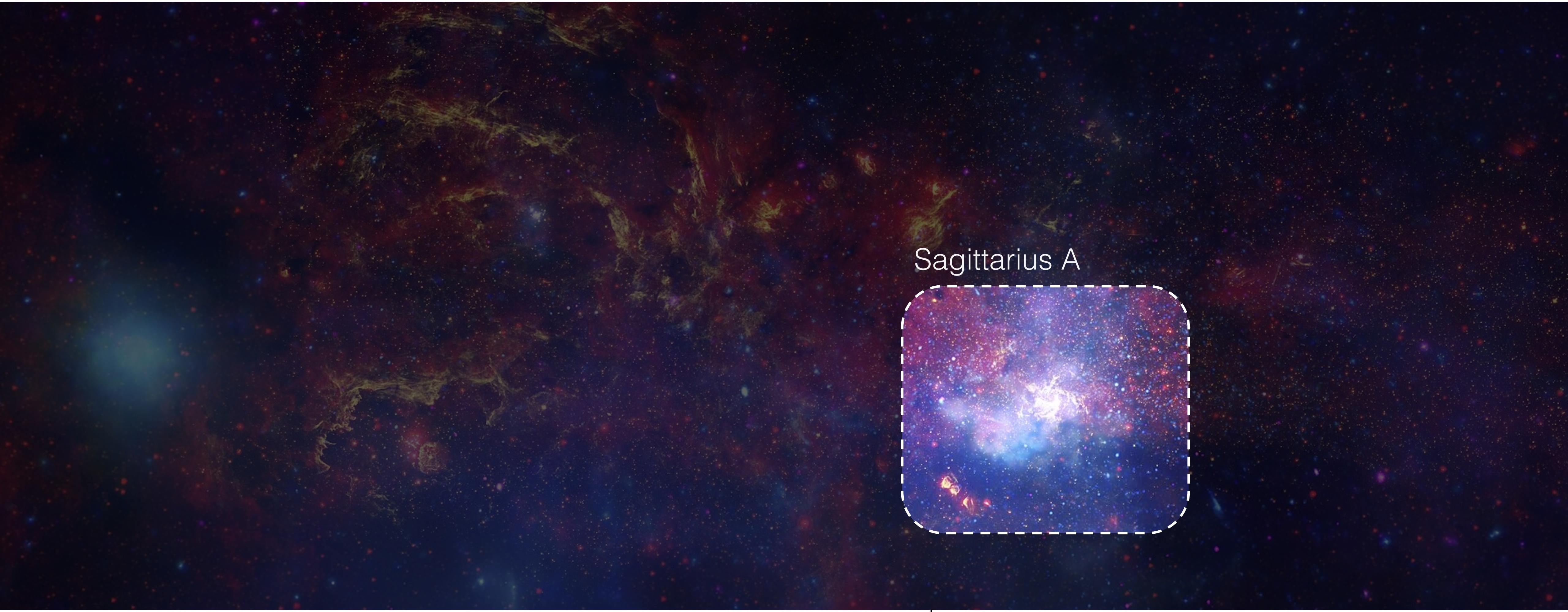


VNiVERSiDAD D SALAMANCA

The Galactic Center

Hubble Space Telescope, the Spitzer Space Telescope, and the Chandra X-ray Observatory (2009)

The Galactic Center



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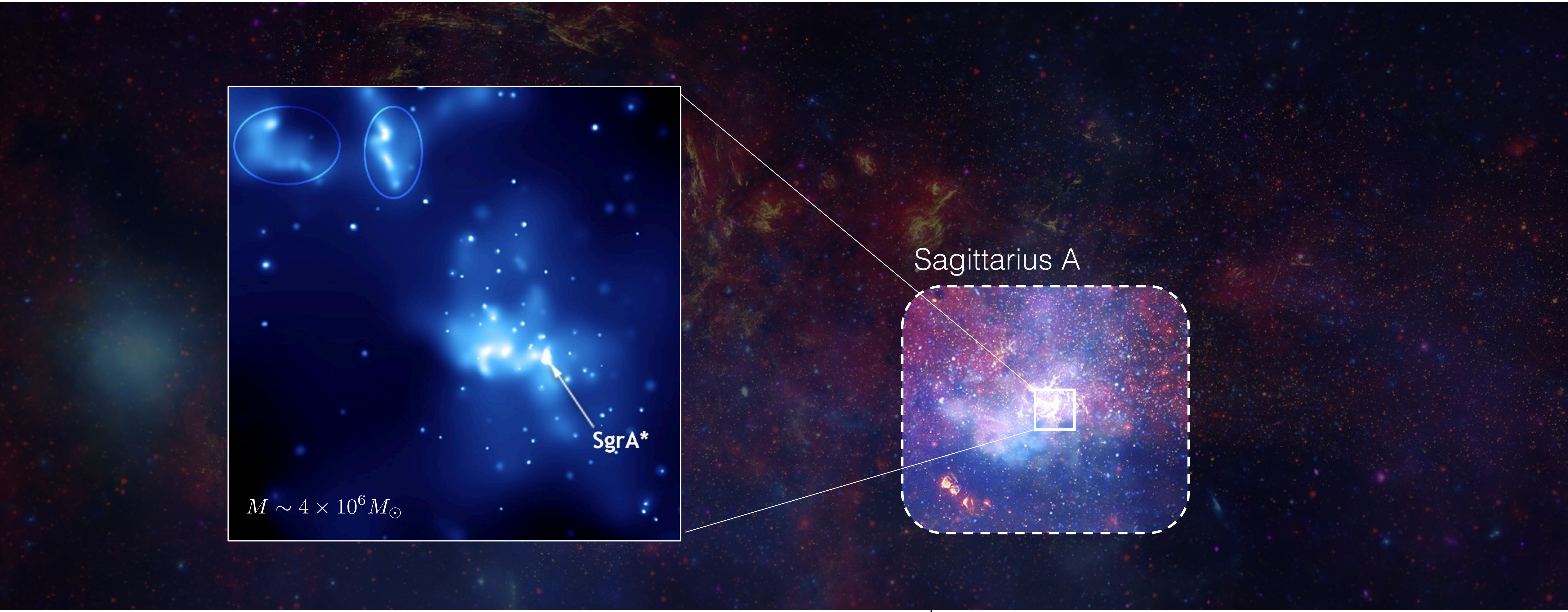
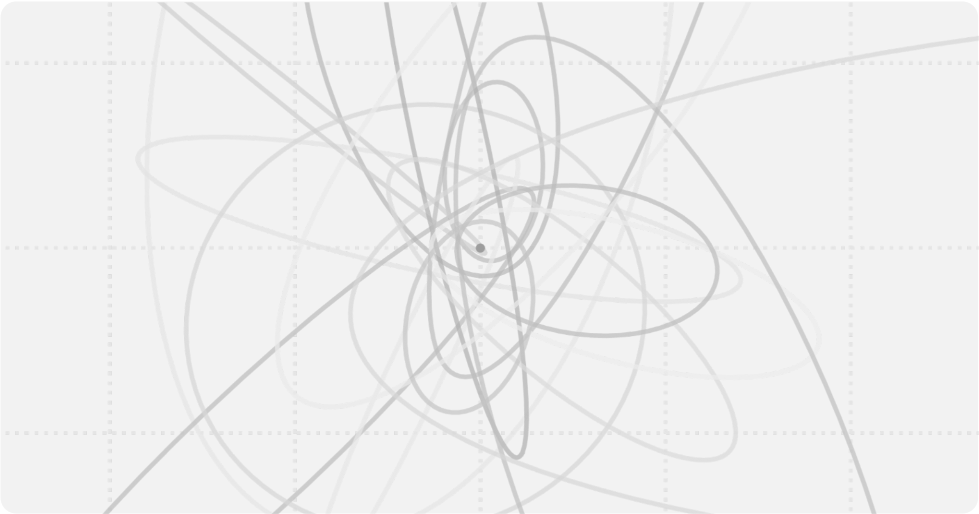


Hubble Space Telescope, the Spitzer Space Telescope, and the Chandra X-ray Observatory (2009)

The Galactic Center represents a **unique laboratory to probe gravity**

S-stars



Black Hole shadow

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Orbital precession

Gravitational redshift

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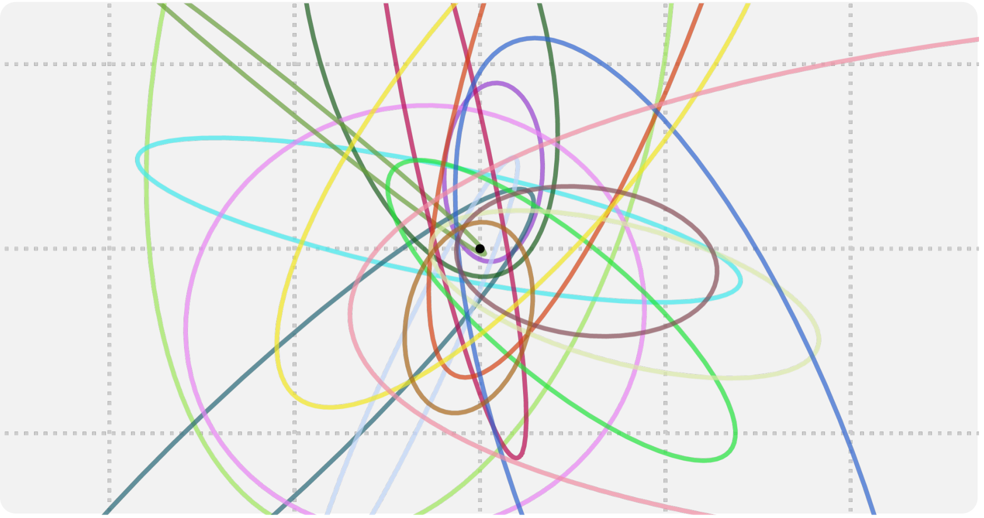

SEE KARIM's TALK LATER

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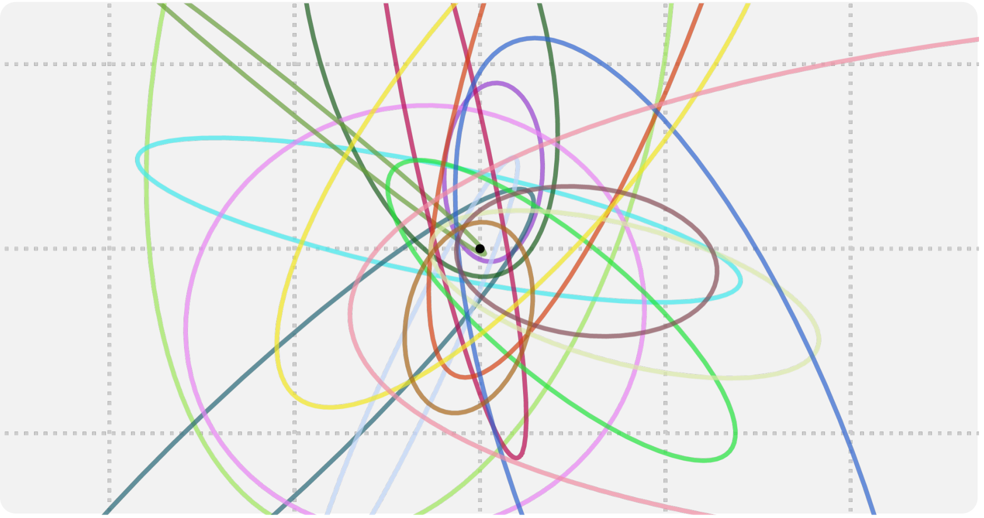

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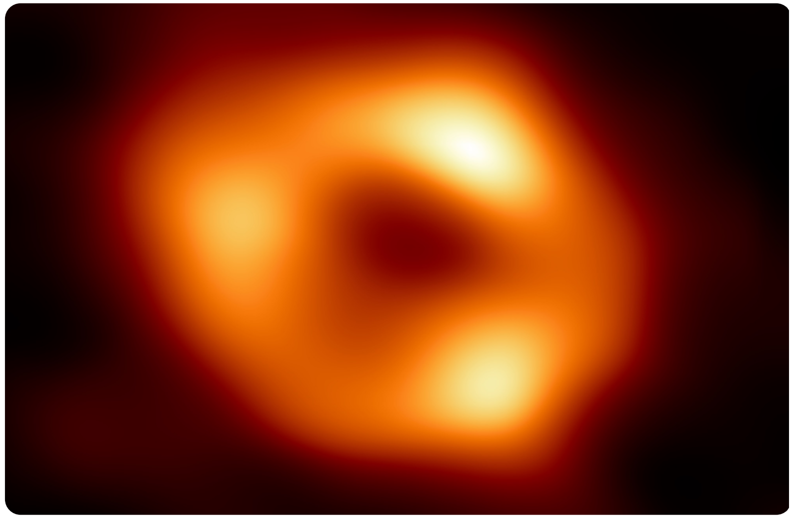

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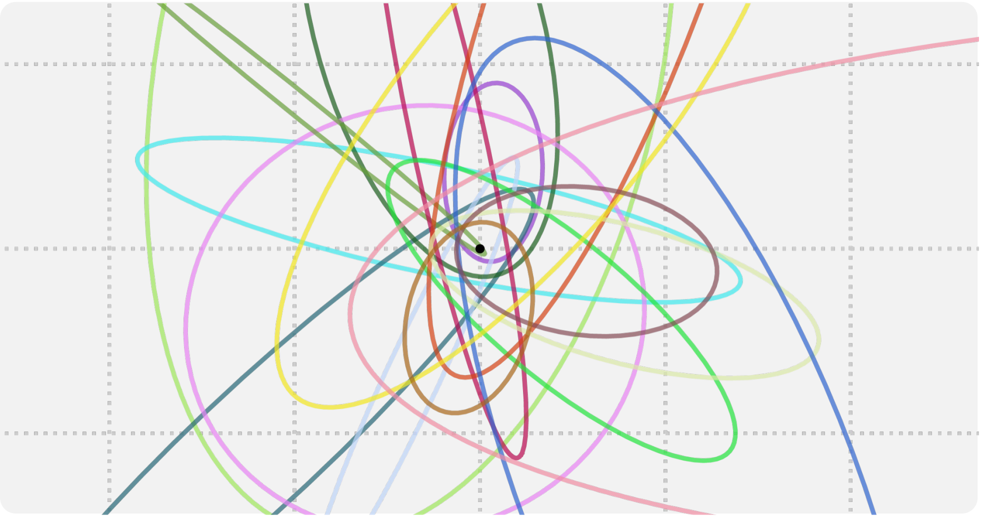

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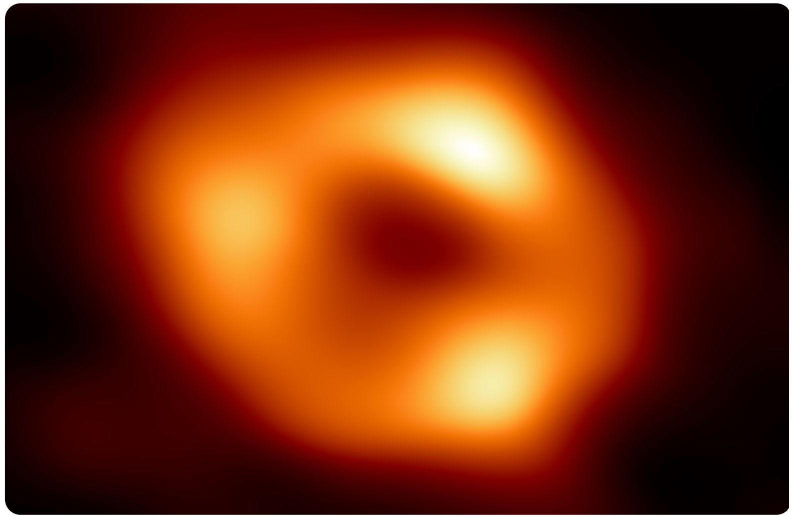

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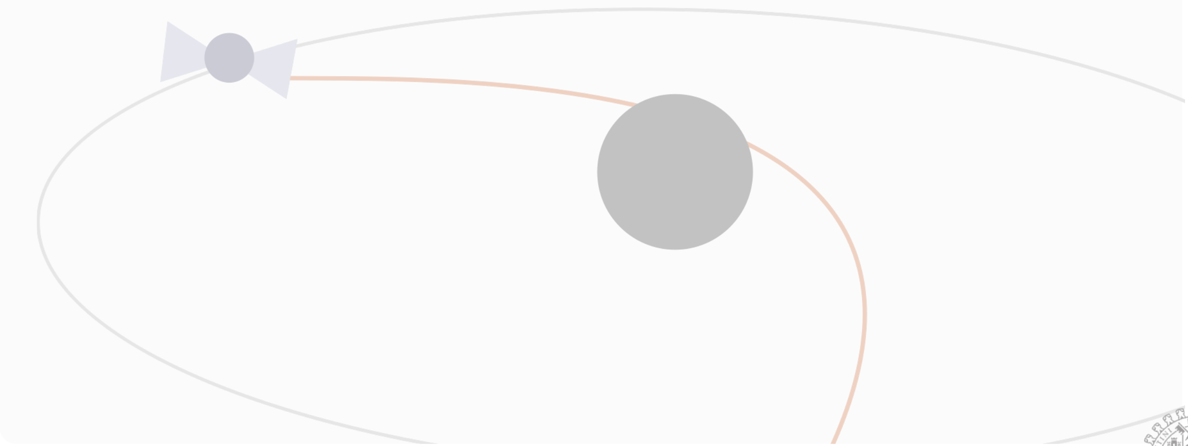
Future prospects

We are moving to an era of **precision astronomy in the Galactic Center**

Closer S-stars

Pulsars



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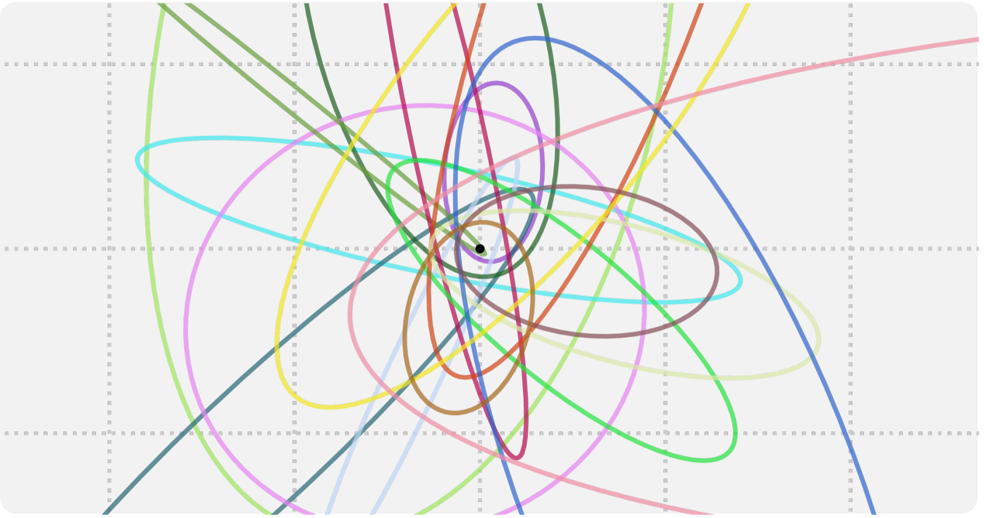

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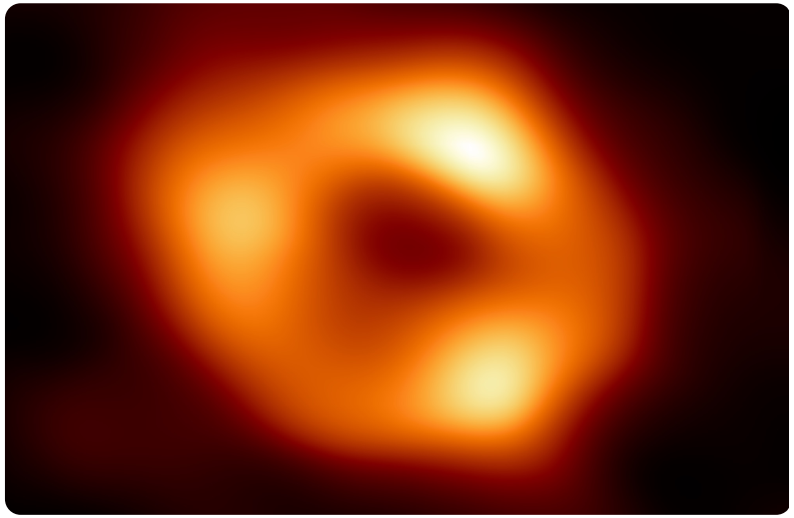

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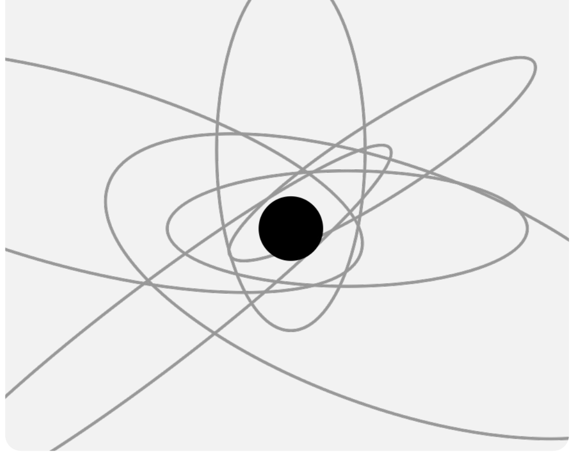

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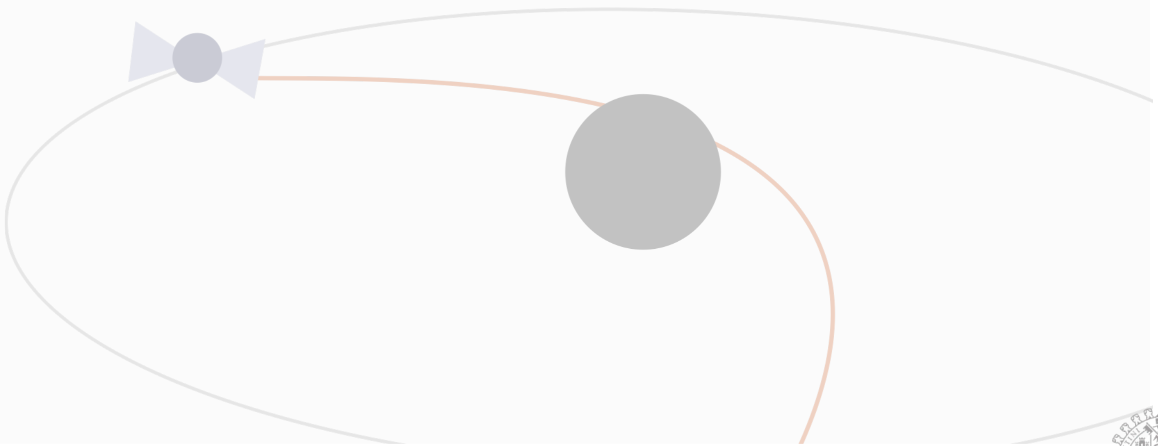

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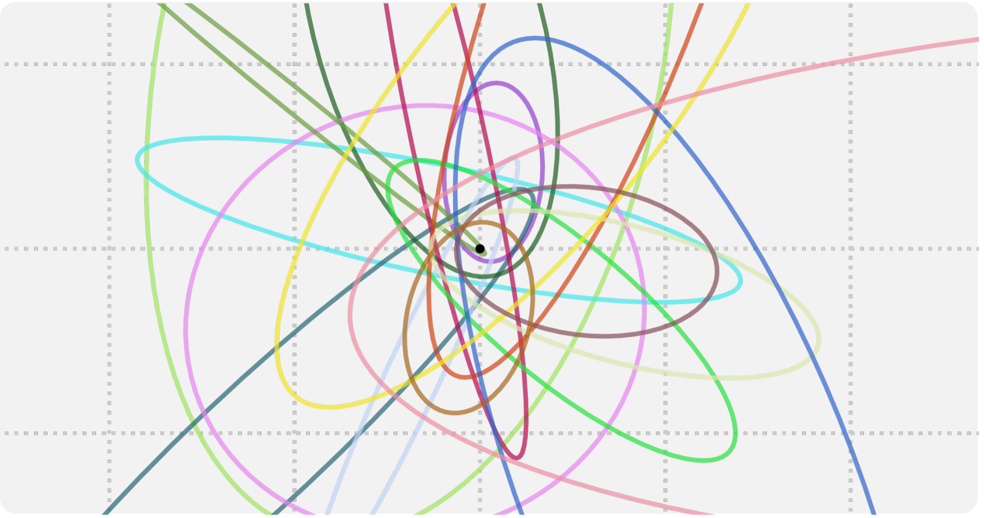

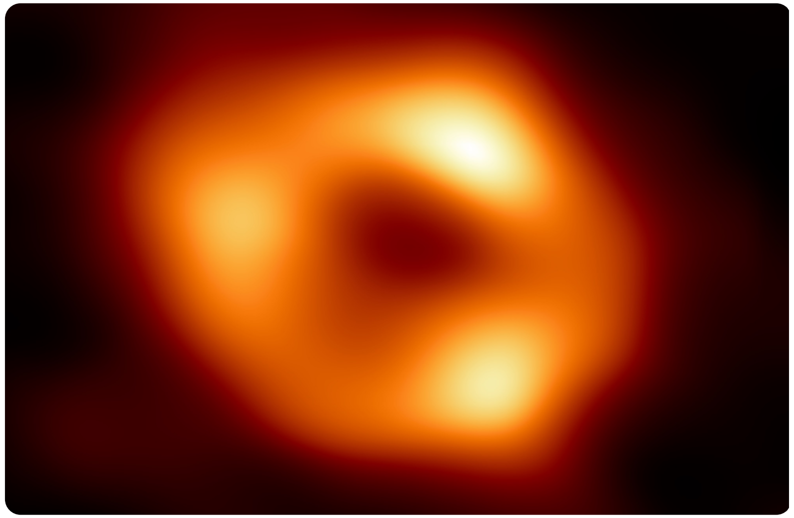

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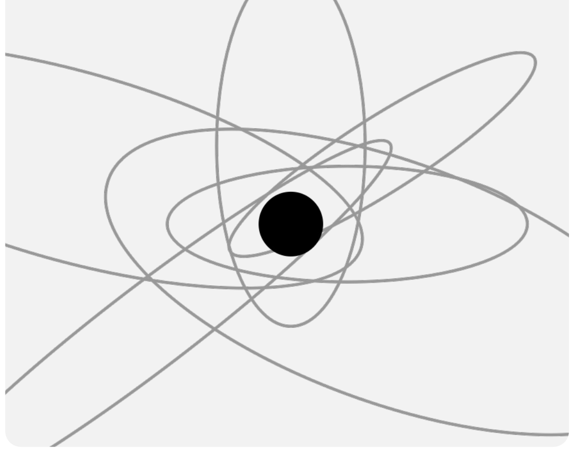

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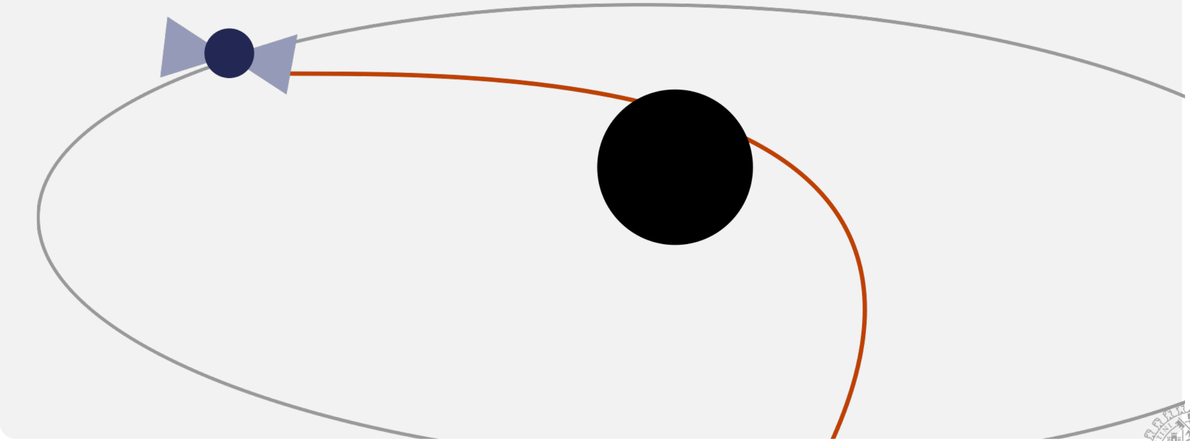

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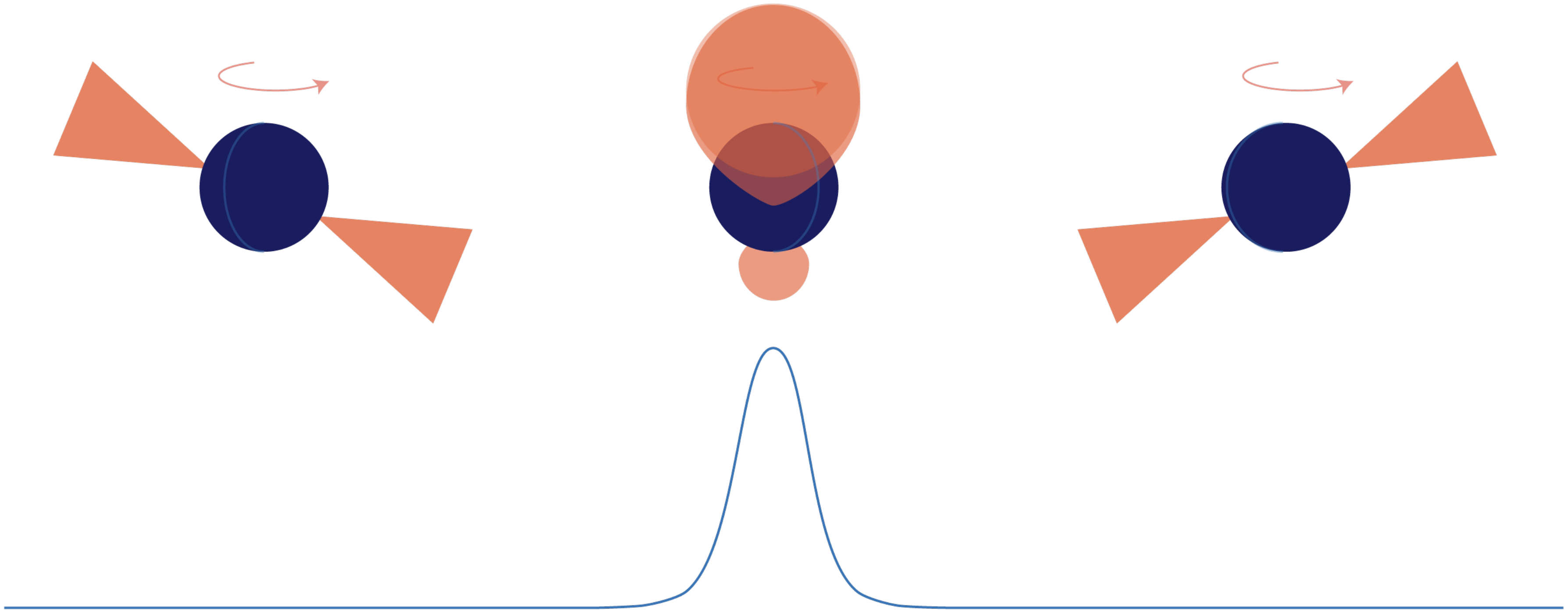

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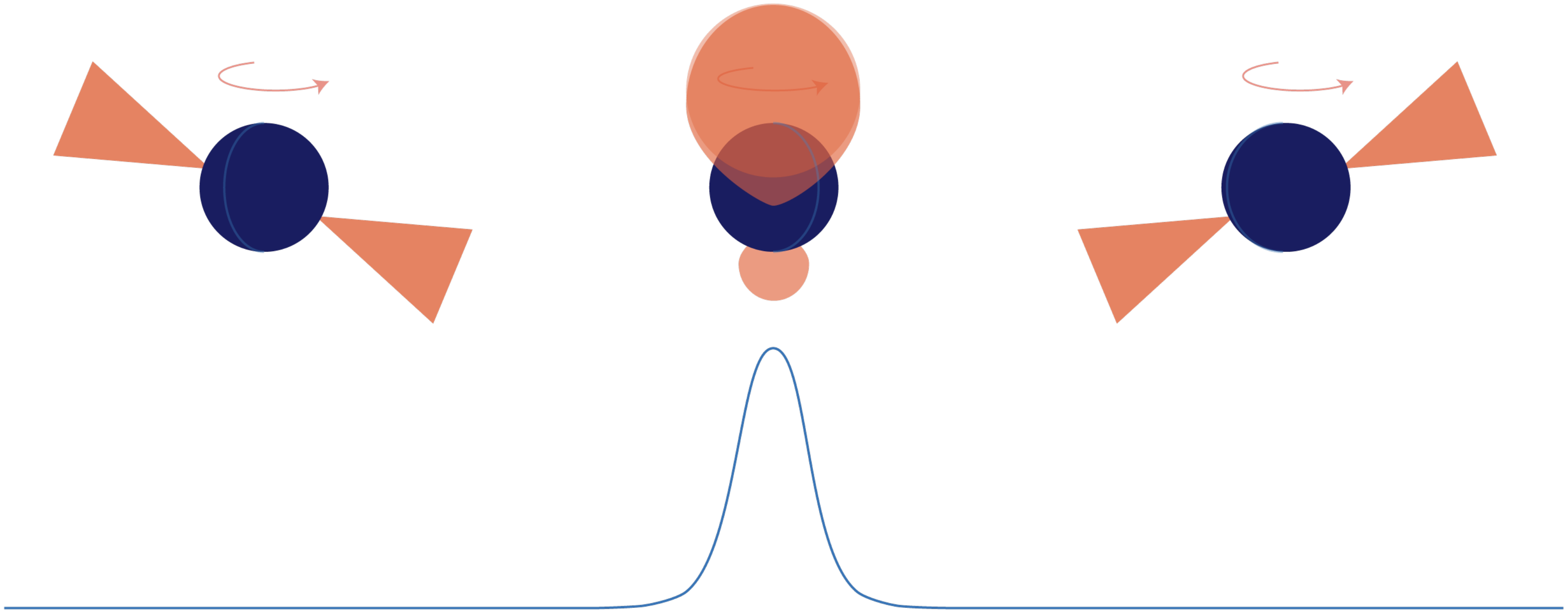
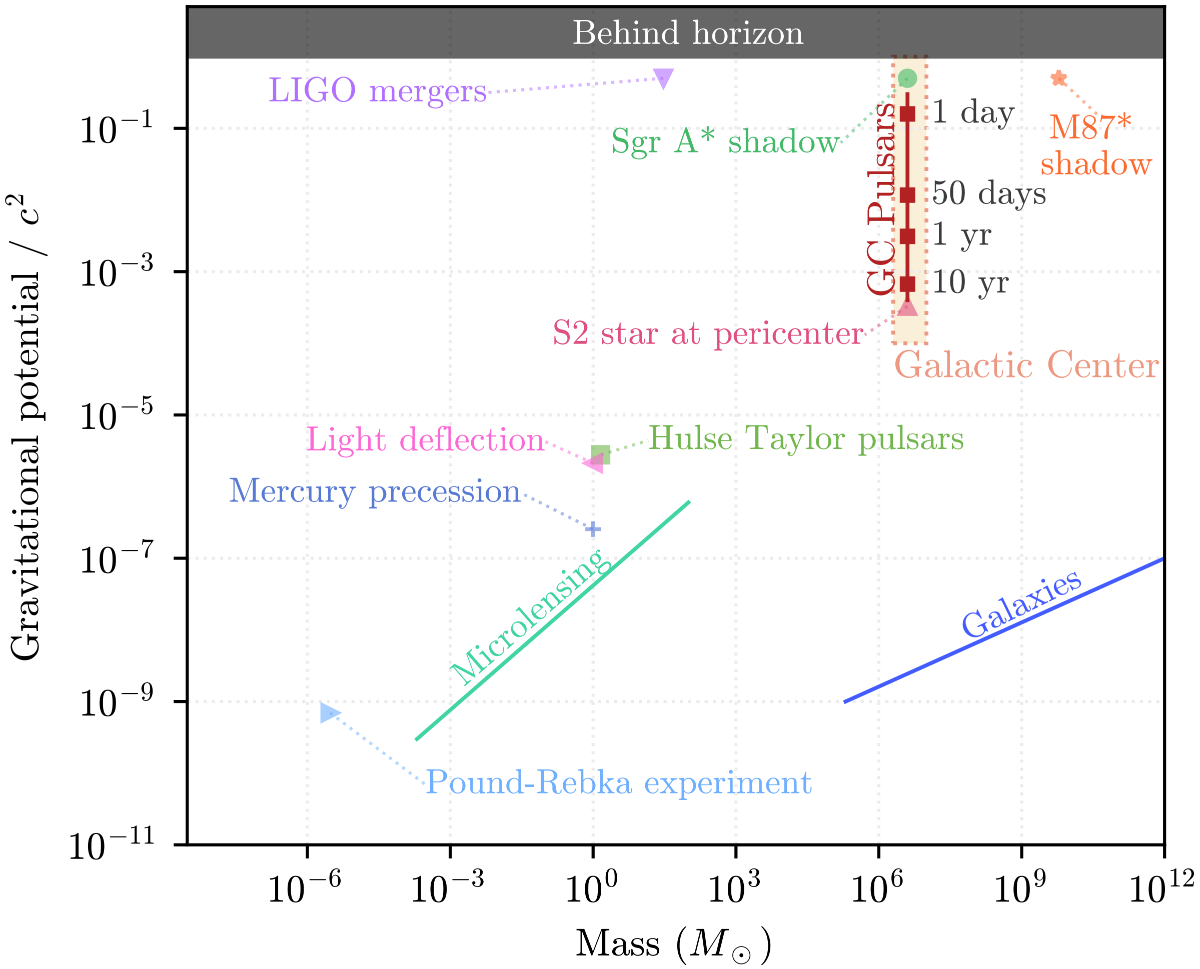


Cosmic precision-clocks
Stability of one part in 10^{15}

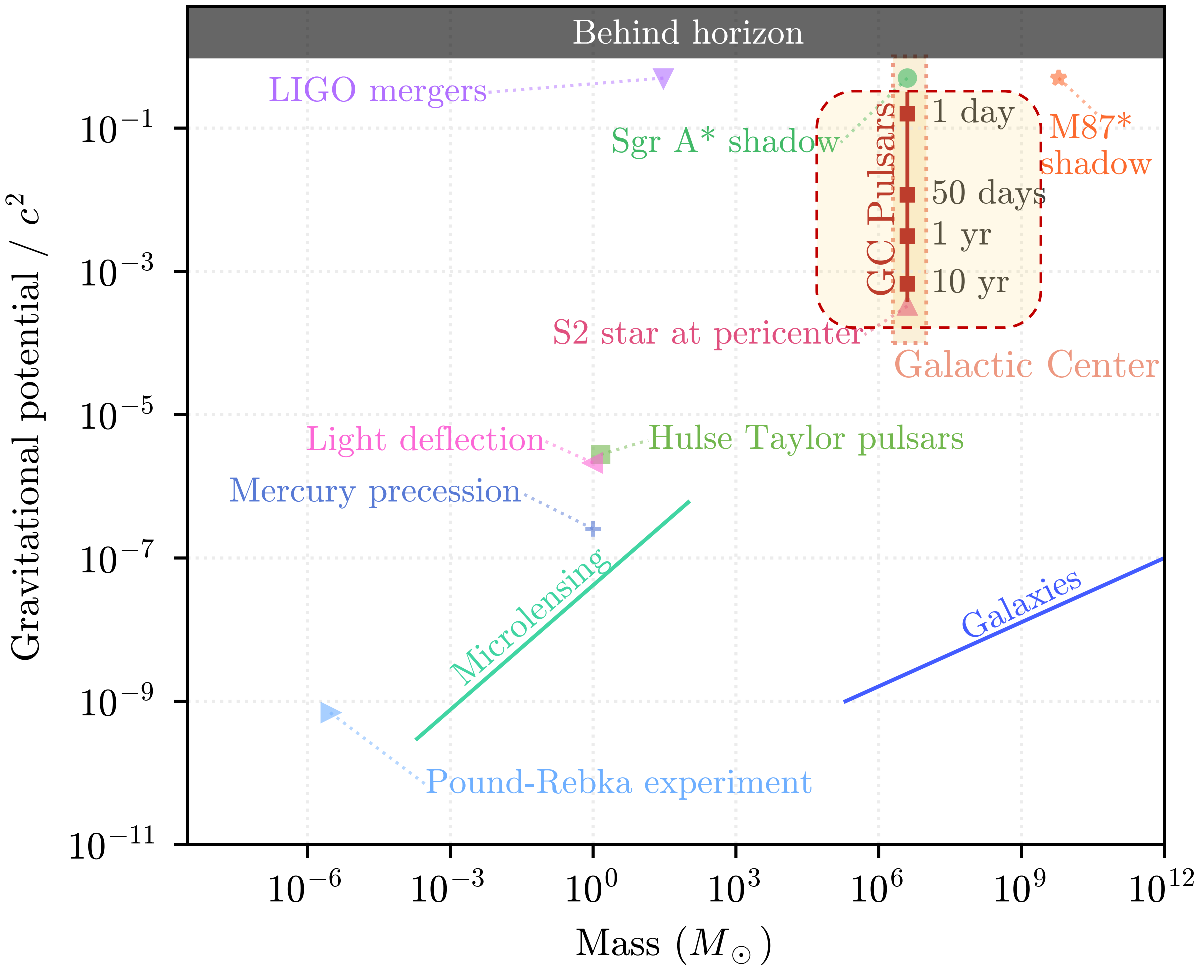
Pulsars



Major scientific goal of future facilities like **Squared Kilometer Array** (SKA) that promise not only to be able to detect them but also to perform **timing analysis.**

$$\sigma_{\rm TOA} \sim 100\,\mu s$$

Pulsars



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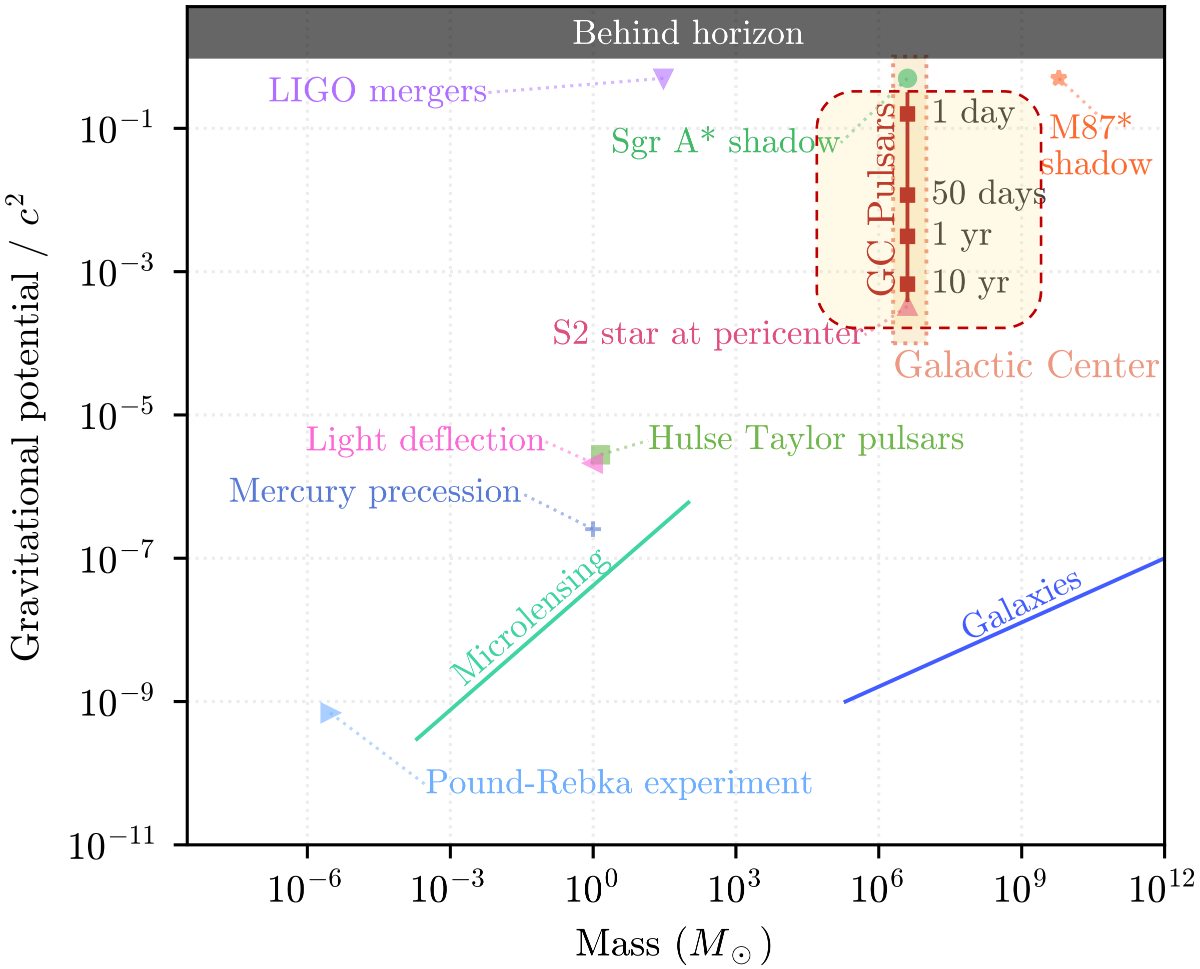

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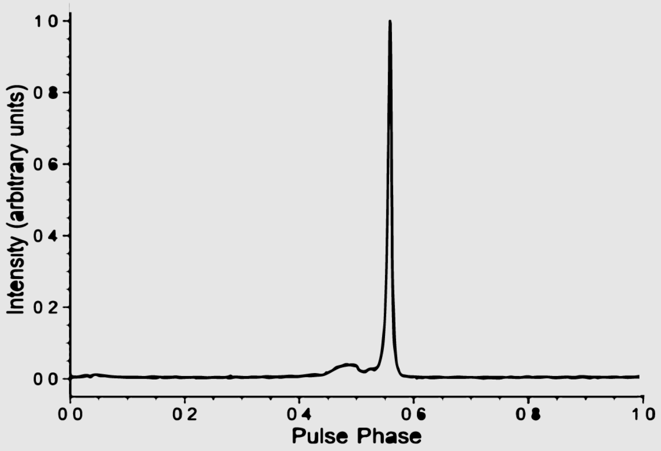
Pulsars data

Times of arrival (TOAs)



Index	**TOA** (MJD)	**Error** (MJD)
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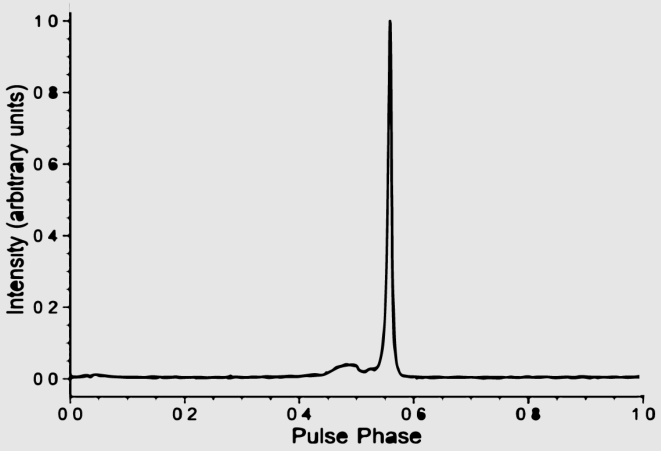

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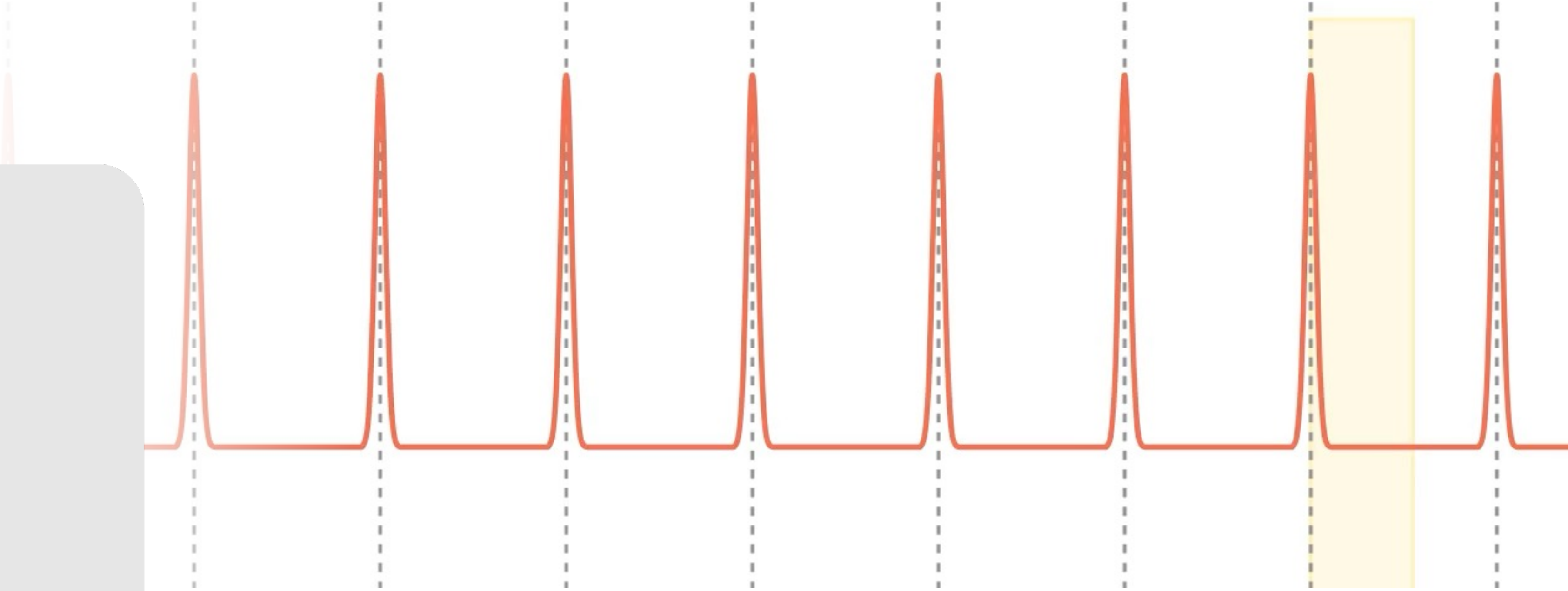

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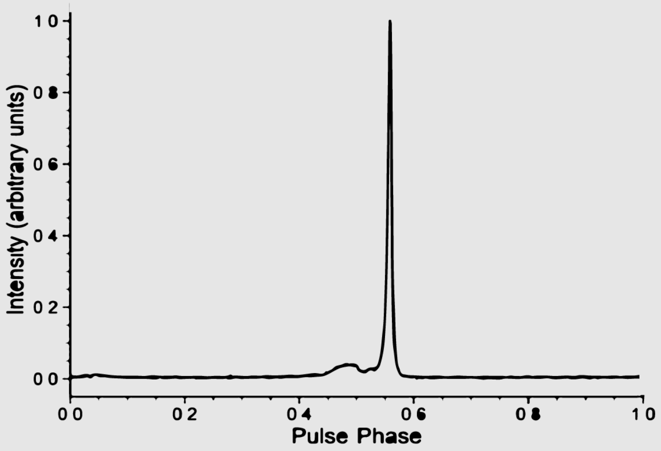

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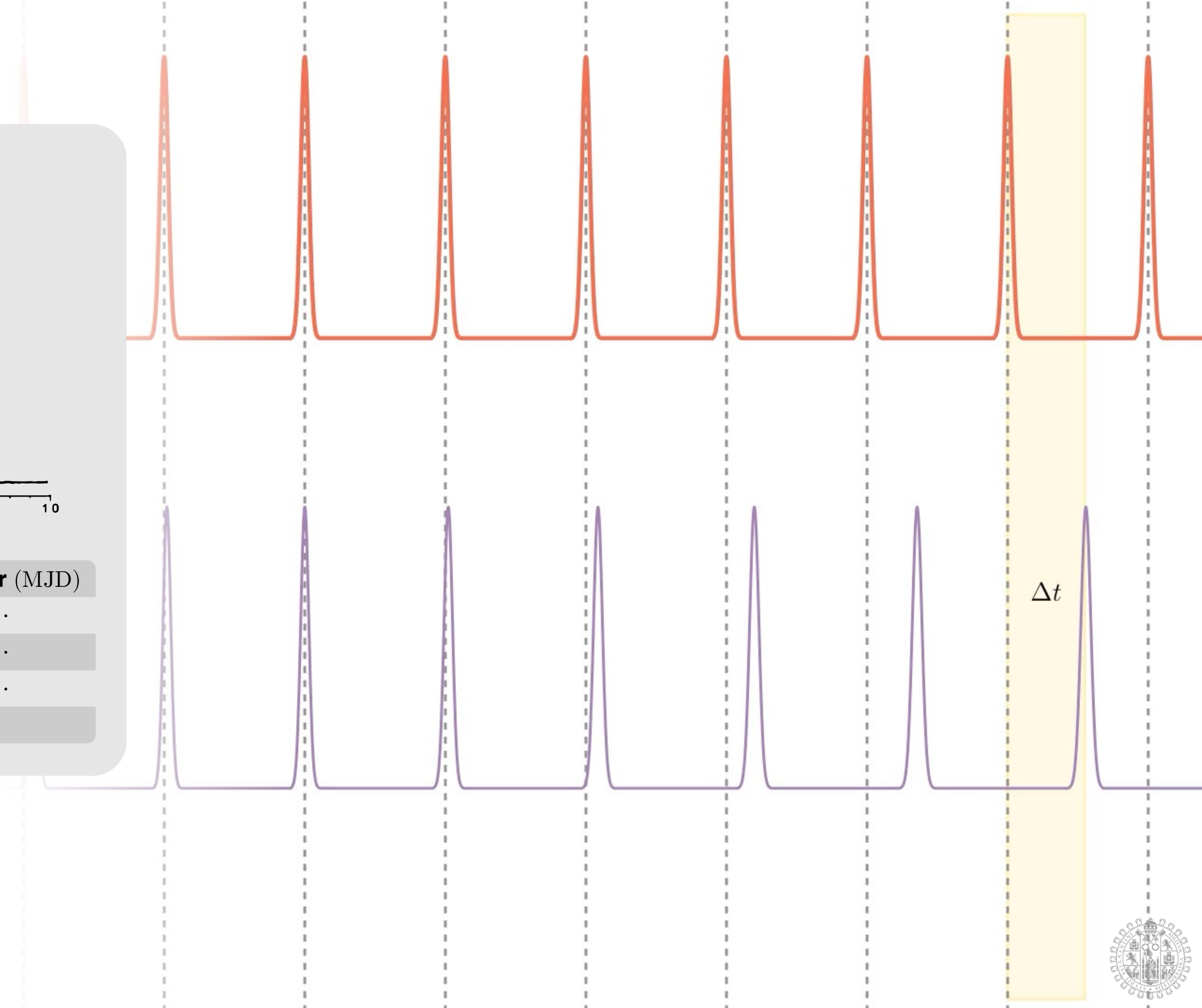

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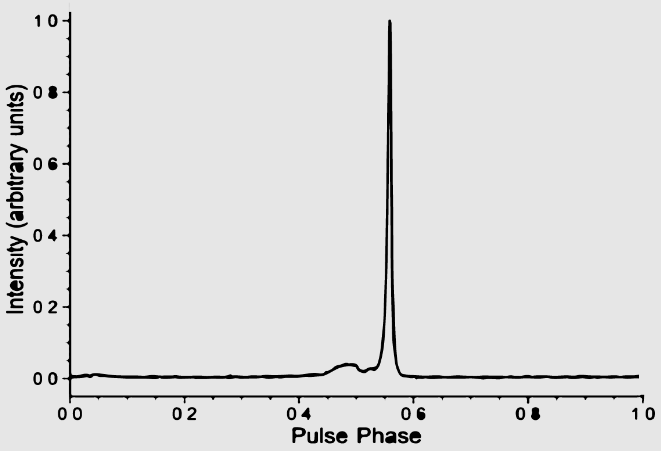

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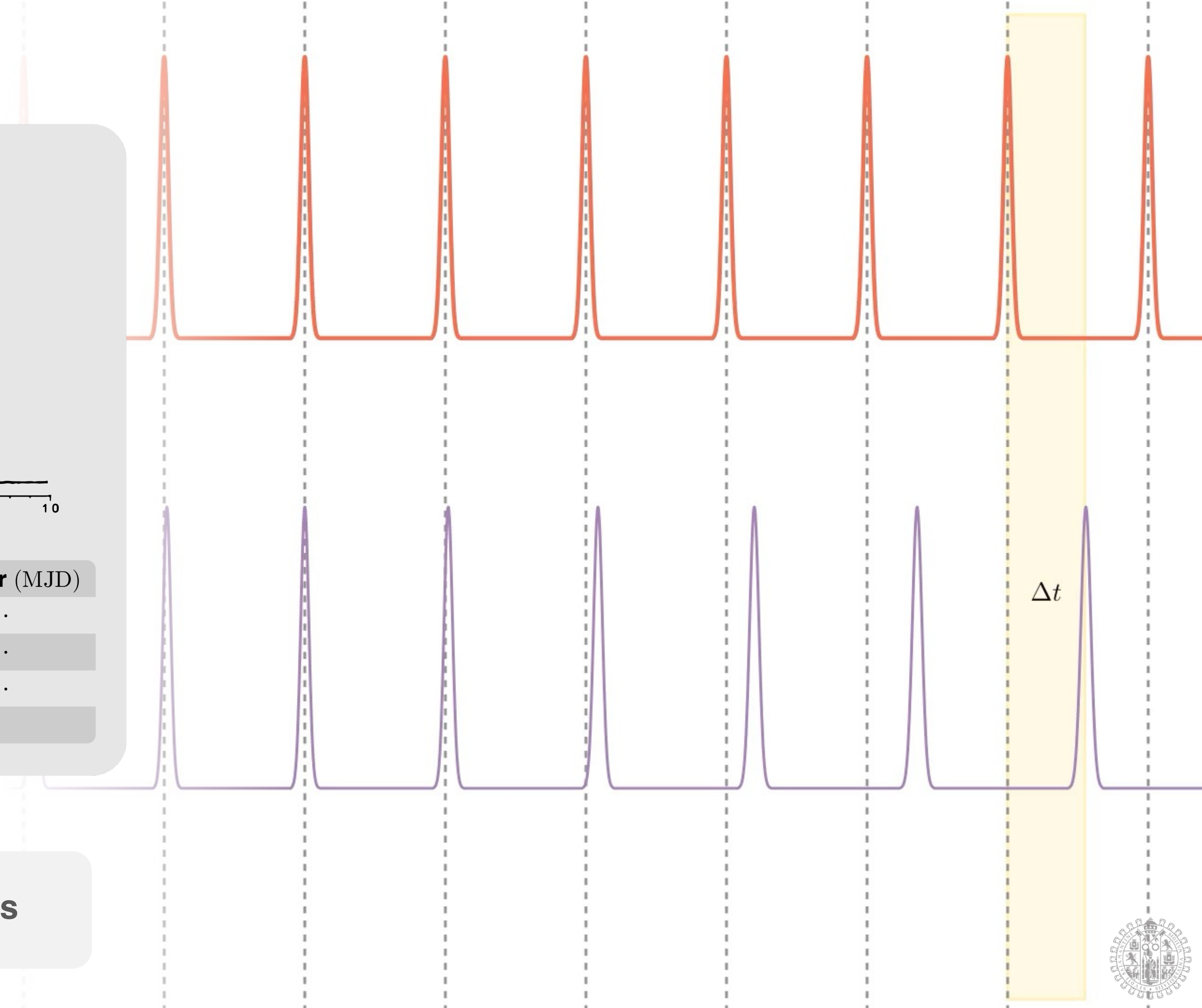
$\Delta t =$ **Classical effects**



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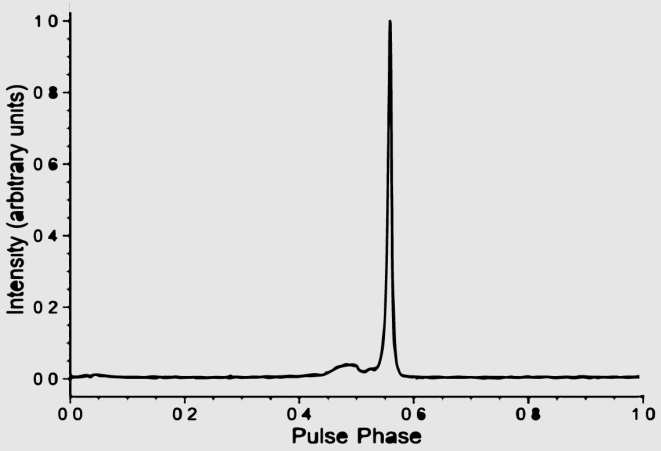

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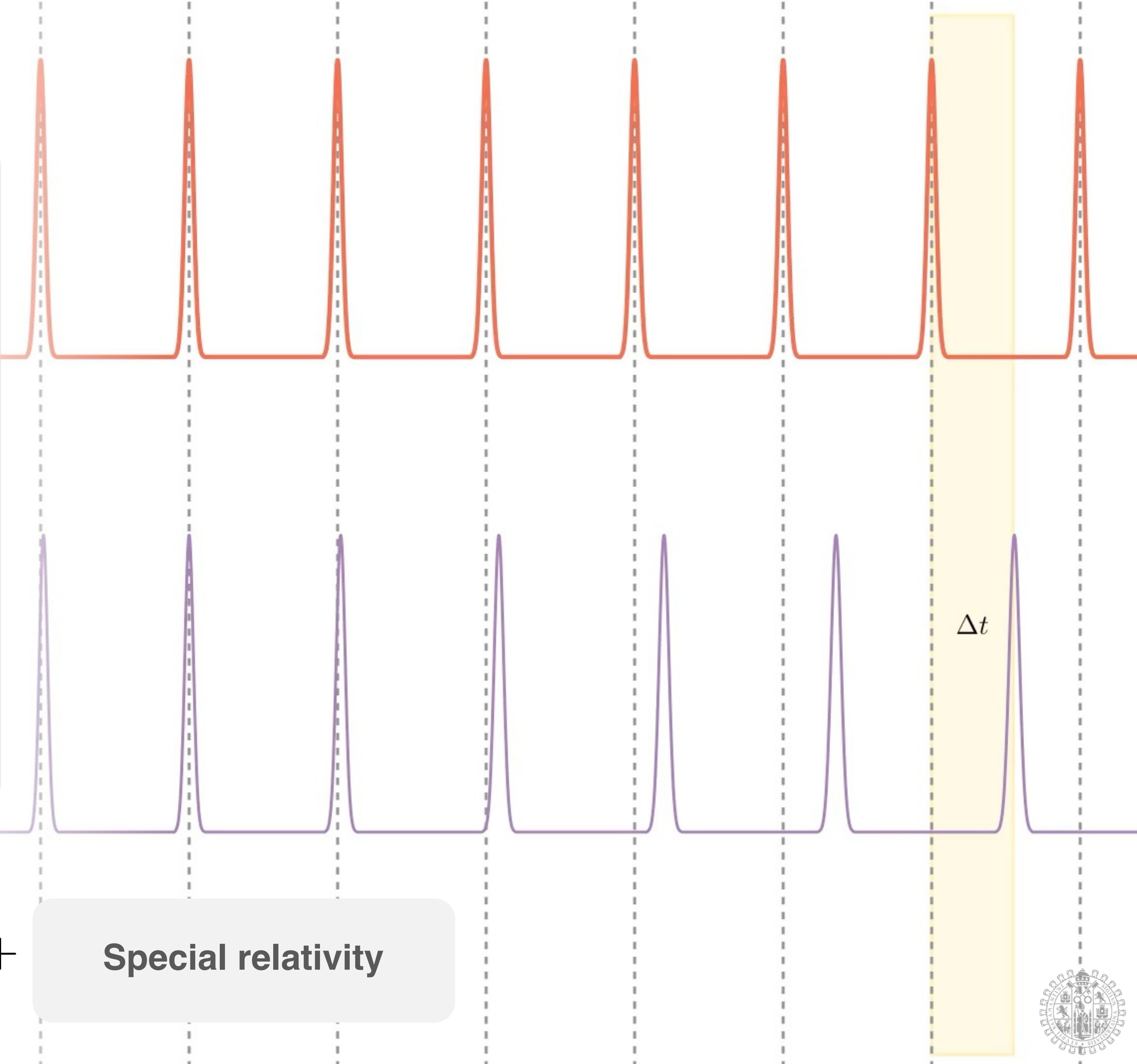

$\Delta t =$ **Classical effects** $+$ **Special relativity**

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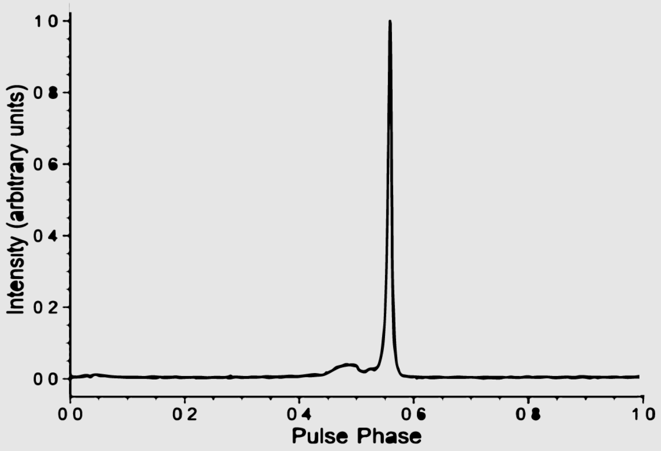

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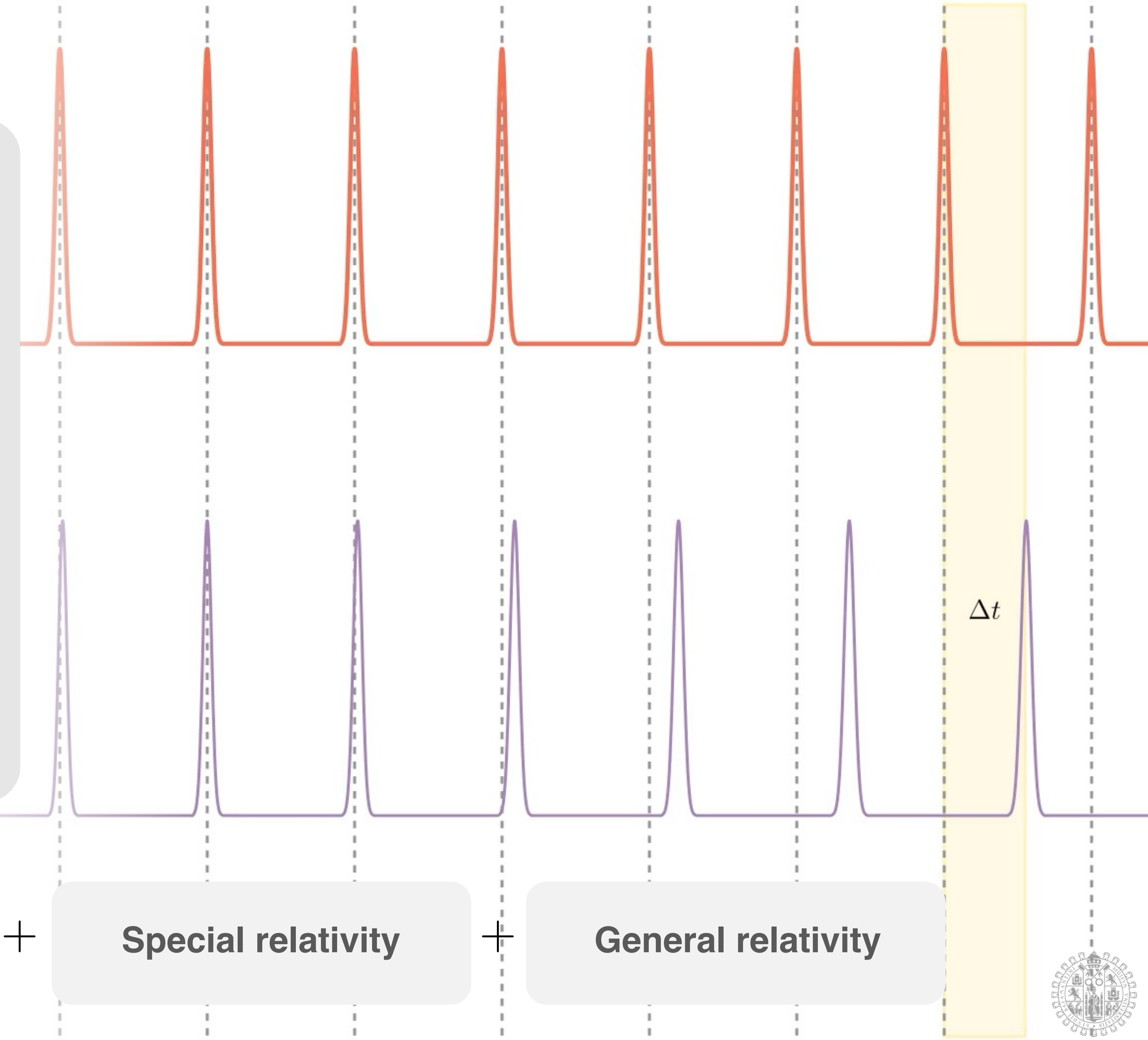

$$\Delta t = \text{Classical effects} + \text{Special relativity} + \text{General relativity}$$

What is **usually** done…

GR effects on the trajectory

Consider a Keplerian orbit with orbital parameters $(a, e, t_P, i, \Omega, \omega)$ and compute the **post-Keplerian** evolution of this parameter from a **post-Newtonian** approximation

$$\dot{\omega} = \frac{6\pi GM}{c^2 a(1-e^2)}, \quad \dots$$

GR effects on the photons

Use **post-Newtonian** approximations for all the different delays, assuming that the total delay is a linear sum of the single effects.

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Hobbs et al. (2006)
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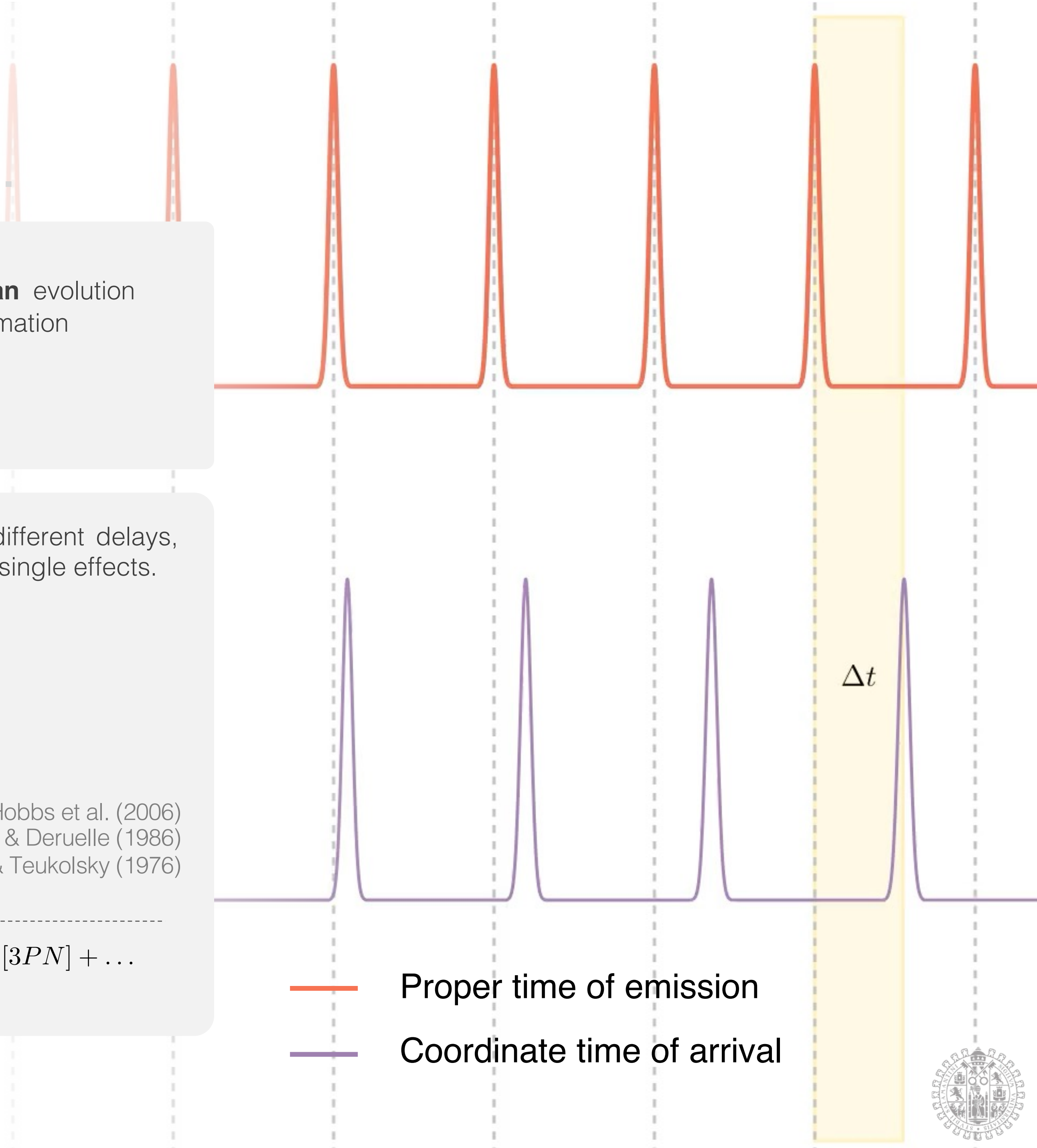

$$\tau_i = \mathrm{TOA}_i - \Delta t_i$$

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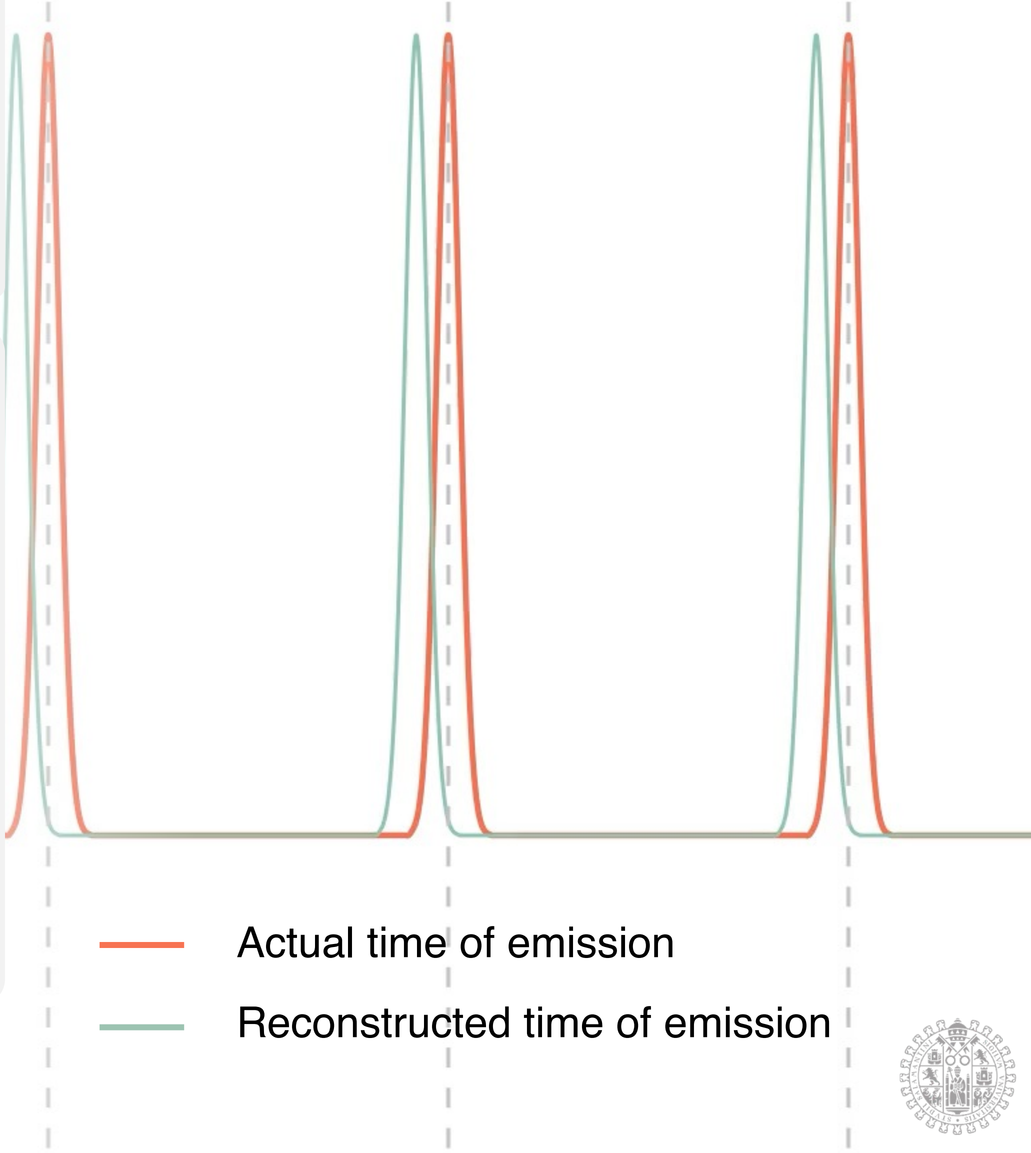

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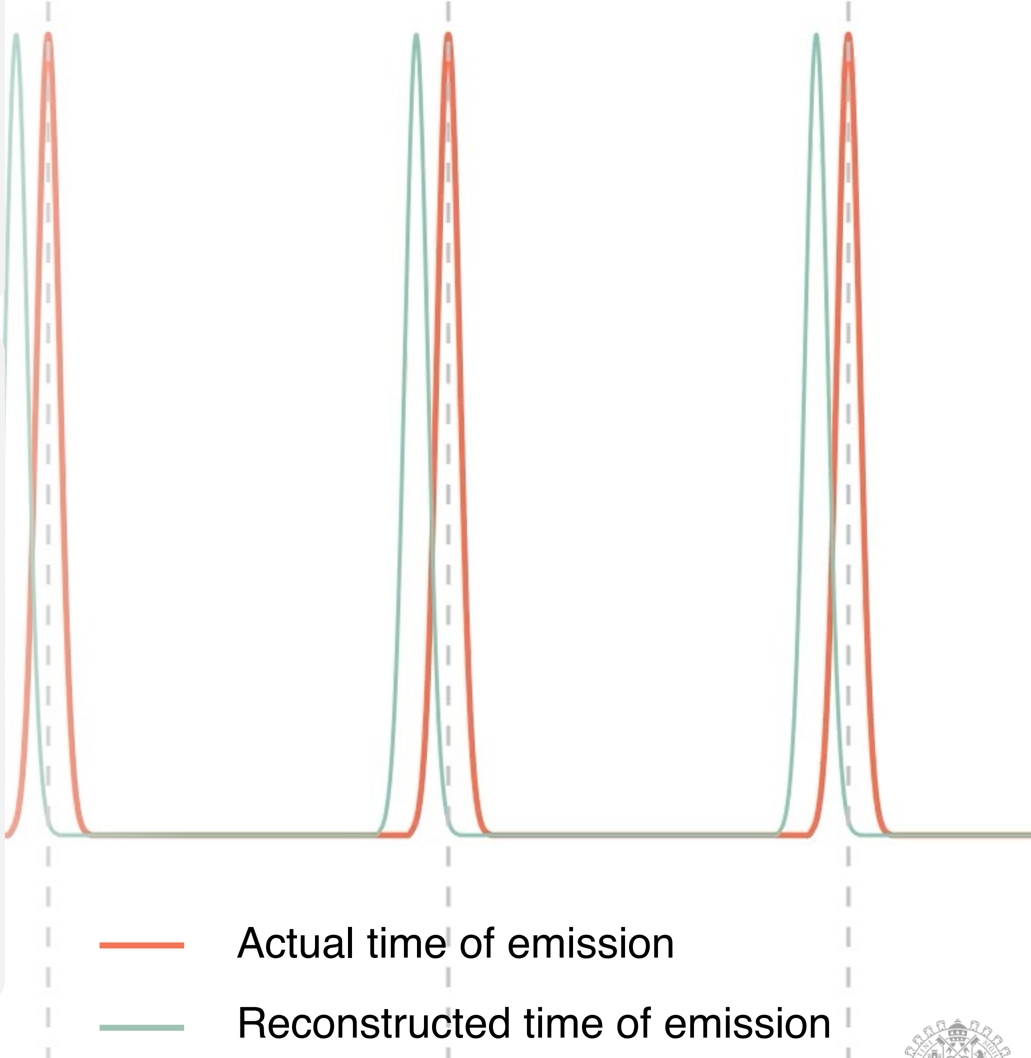

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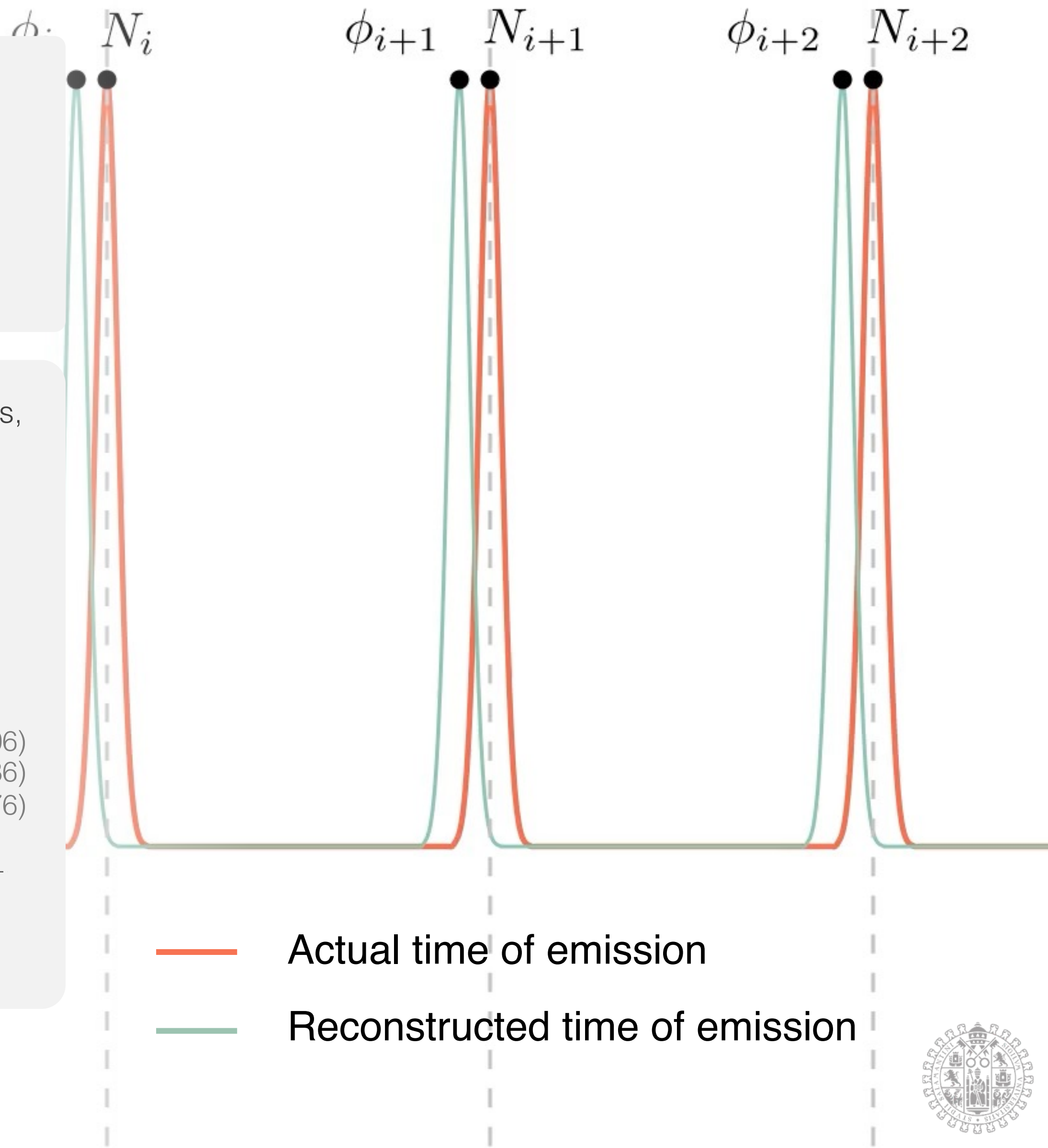

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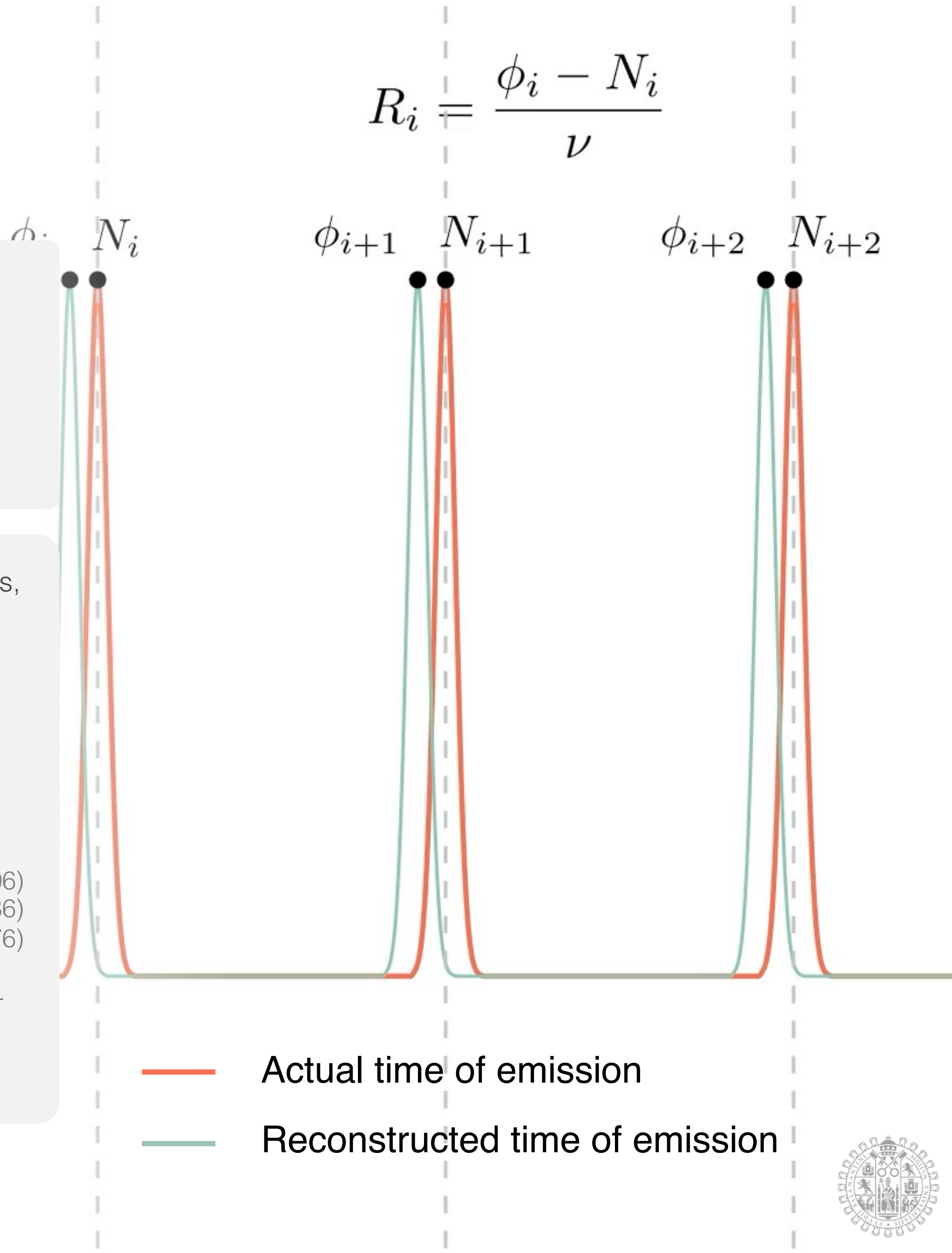

$$\chi^2 = \sum_{i=1}^{N_{\rm data}} \left(\frac{R_i}{\sigma_i}\right)^2$$

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$$\dot{\omega} = \frac{6\pi GM}{c^2 a(1-e^2)}, \quad \dots$$

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Use **post-Newtonian** approximations for all the different delays, assuming that the total delay is a linear sum of the single effects.

$$\Delta t_{\rm Rømer} = \frac{a(1-e^2)\sin i \sin(\omega+\phi)}{c(1+e\cos\phi)}$$

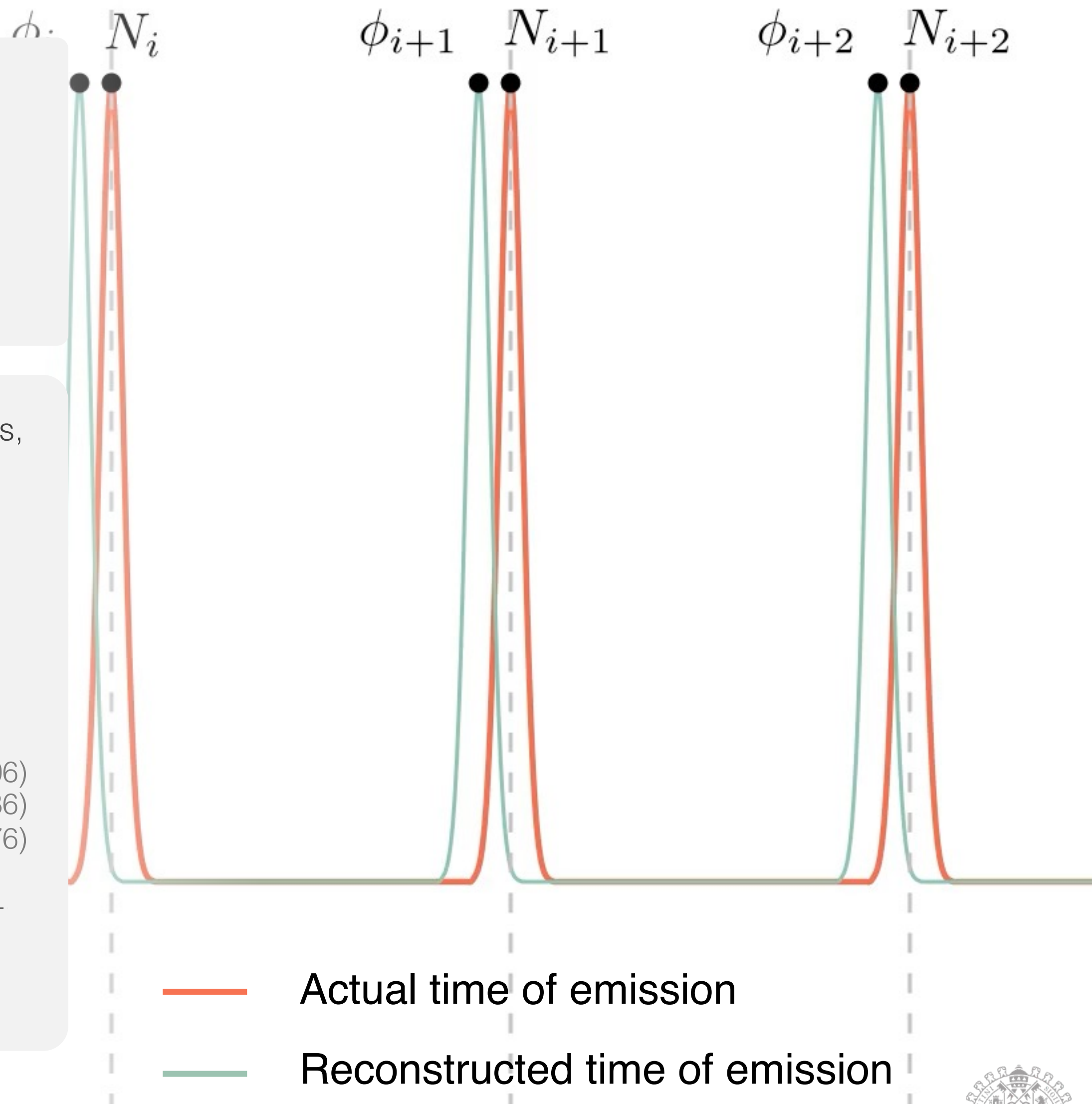
$$\Delta t_{\rm Shapiro} = \frac{2GM}{c^3} \ln\left[\frac{1+e\cos\phi}{1-\sin i\sin(\omega+\phi)}\right]$$

$$\Delta t_{\rm geo} = \frac{2GM}{c^3}\left[\frac{|\vec{r}_\pm - \vec{r}_s|}{R_E}\right]^2$$

$$\Delta t_{\rm Einstein} = \gamma \sin u$$

Hobbs et al. (2006)
Damour & Deruelle (1986)
Blandford & Teukolsky (1976)

$$\Delta t = \Delta t_{\rm Rømer} + \underbrace{\Delta t_{\rm Shapiro} + \Delta t_{\rm geo}}_{[1PN]} + [2PN] + [3PN] + \dots$$



What is **usually** done…

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What is **usually** done… / What **should** be done

GR effects on the **trajectory**

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What should be done

- Integrate the **geodesic equations** for a time-like geodesic describing the motion of a test particle in the BH space-time

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{ds} \frac{dx^\rho}{ds} = 0$$

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Emitter-observer problem

- Find the null geodesic that connects emitter and observer

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- Integrate the geodesic equations for such null geodesic to get the actual photon path in the BH space-time

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Integrate
the pulsar orbit

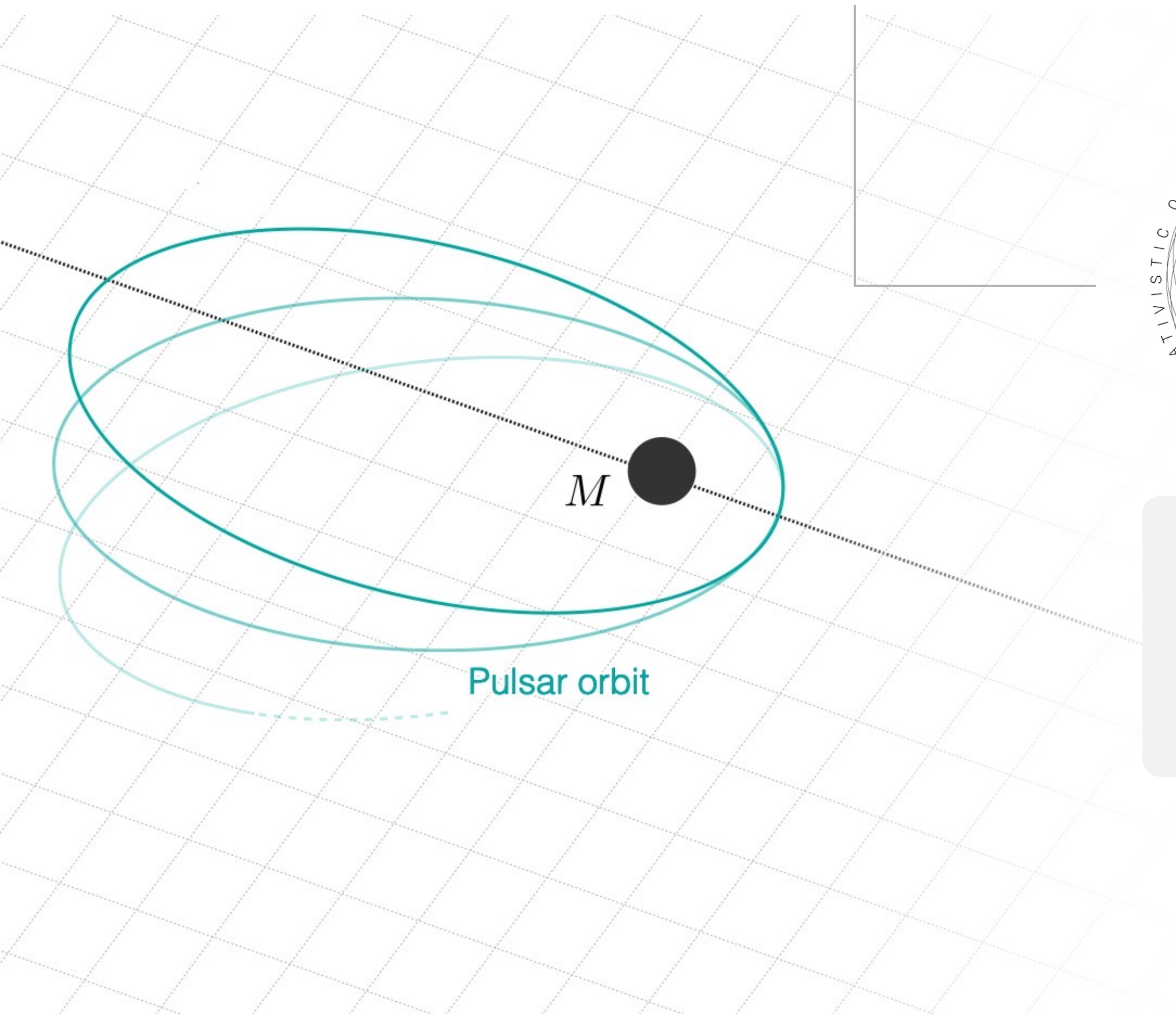
Find the connecting
photon

Compute
the propagation time

Integrate the pulsar orbit

Find the connecting photon

Compute the propagation time



PyGRO

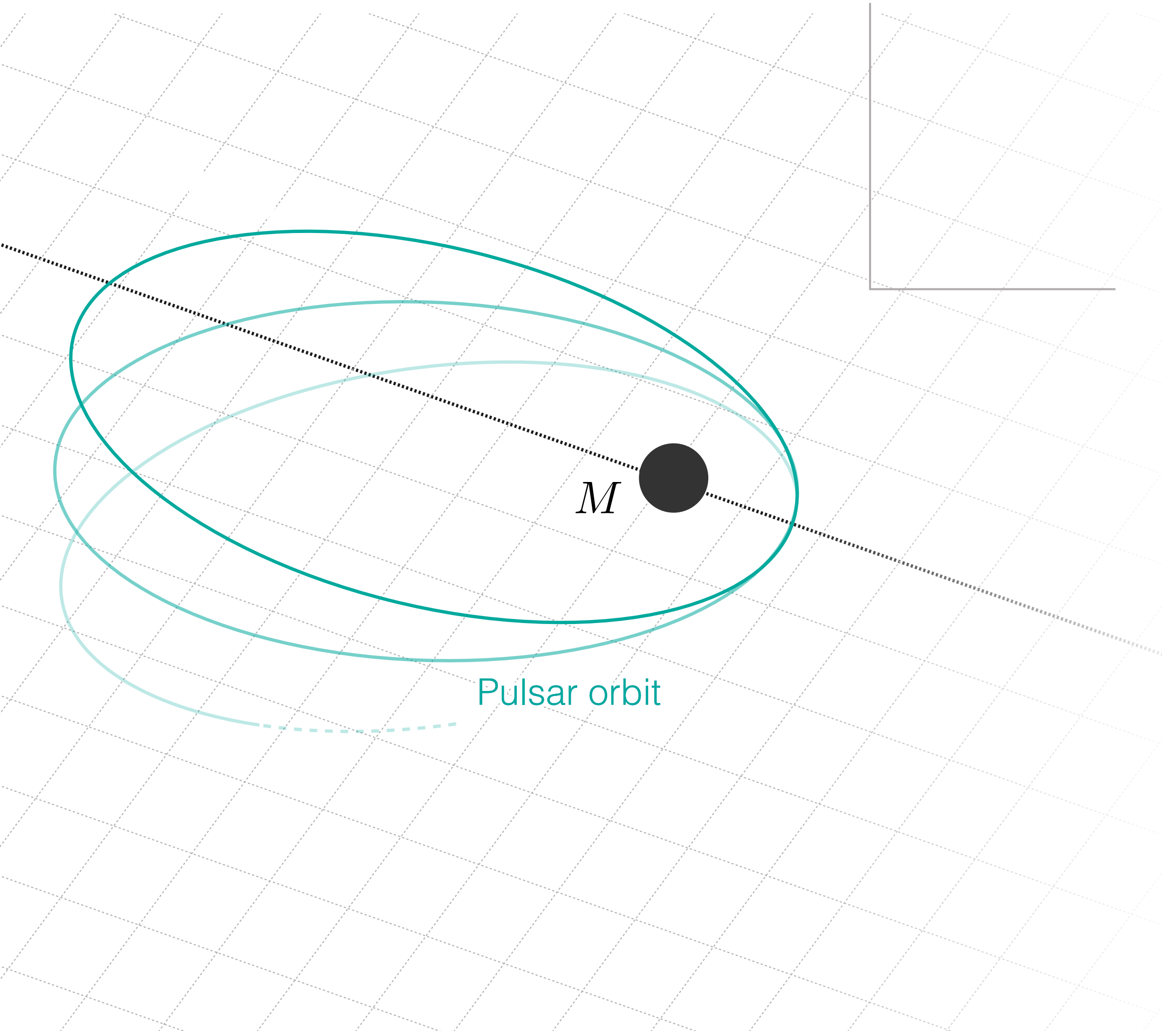
a **Py**thon integrator for **G**eneral **R**elativistic **O**rbits

https://github.com/rdellamonica/pygro

Integrates geodesic equation for both time-like and null geodesics in any given asymptotically-flat spherically symmetric space-time

$$ds^2 = A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

Integrate the pulsar orbit — **Find the connecting photon** — **Compute the propagation time**



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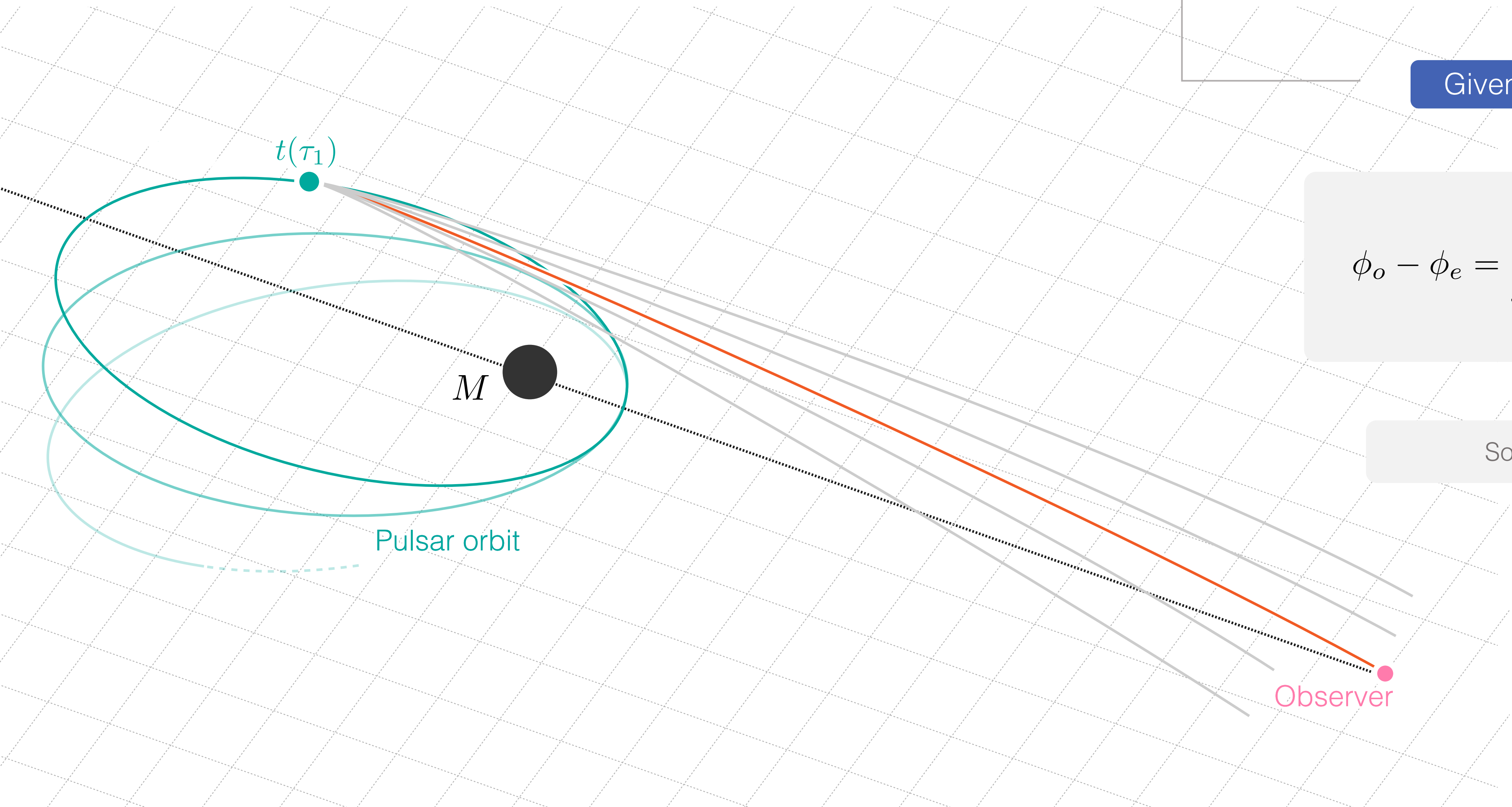
We stick to the Schwarzschild space-time for today's presentation

Integrate the pulsar orbit

Find the connecting photon

Compute the propagation time



Given an emission point

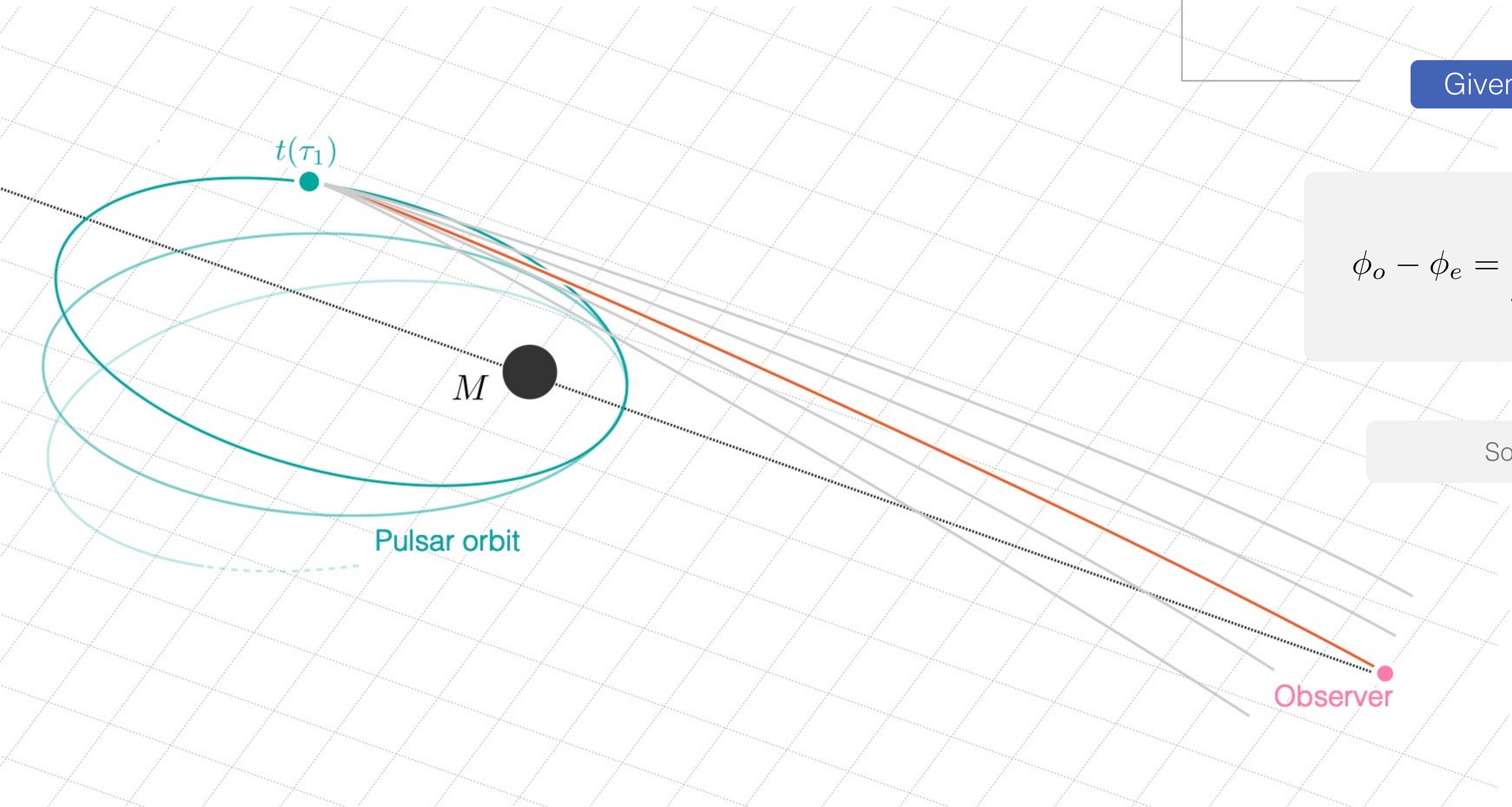
$$\phi_o - \phi_e = \int_{r_{e,\gamma}}^{r_o} \frac{b}{r} \sqrt{\frac{A(r)B(r)}{r^2 - b^2 A(r)}} dr$$

Solve numerically for b

Integrate the pulsar orbit

Find the connecting photon

Compute the propagation time



Given an emission point

$$\phi_o - \phi_e = \int_{r_{e,\gamma}}^{r_o} \frac{b}{r} \sqrt{\frac{A(r)B(r)}{r^2 - b^2 A(r)}}\, dr$$

Solve numerically for b

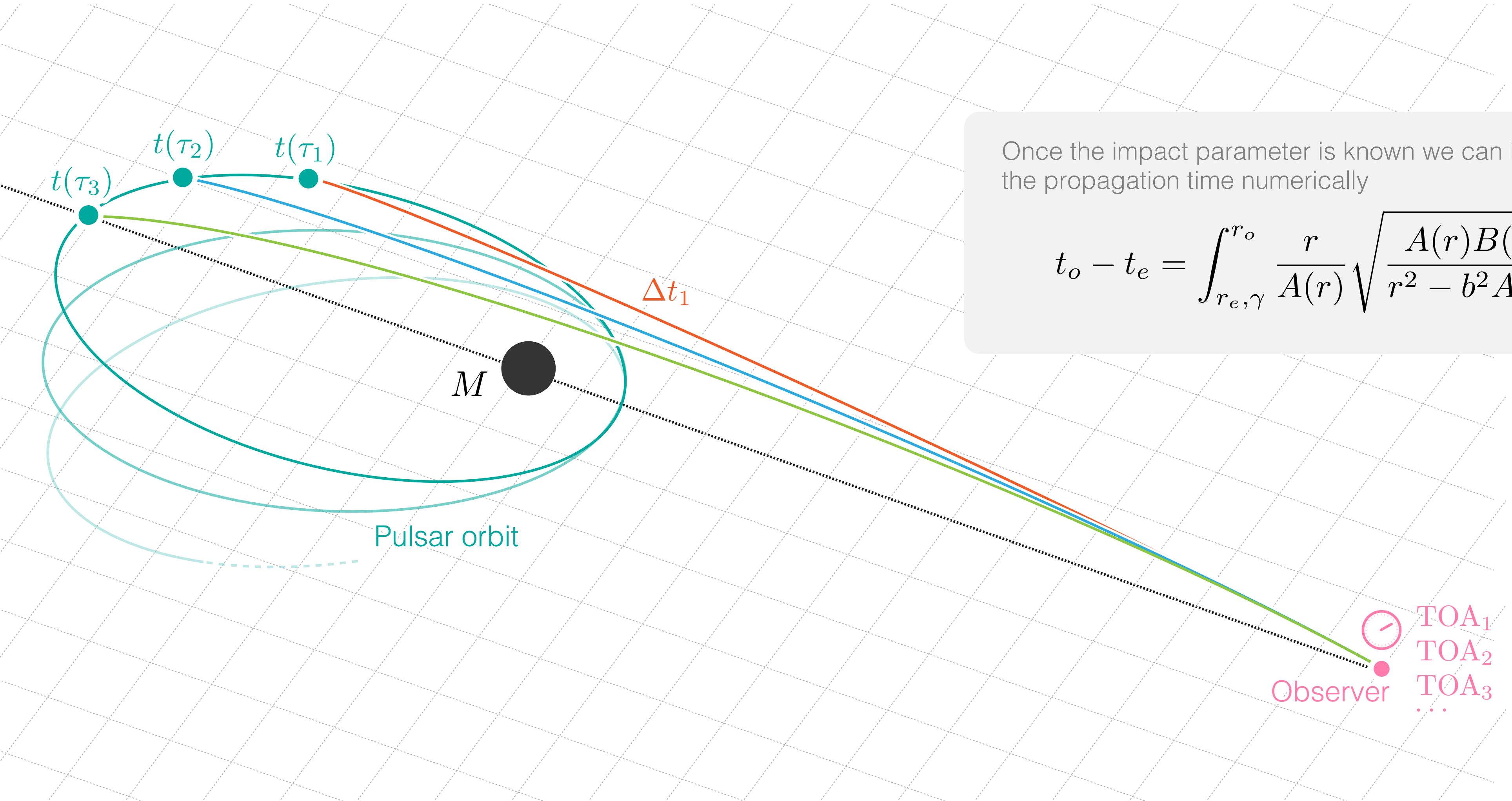
Integrate the pulsar orbit

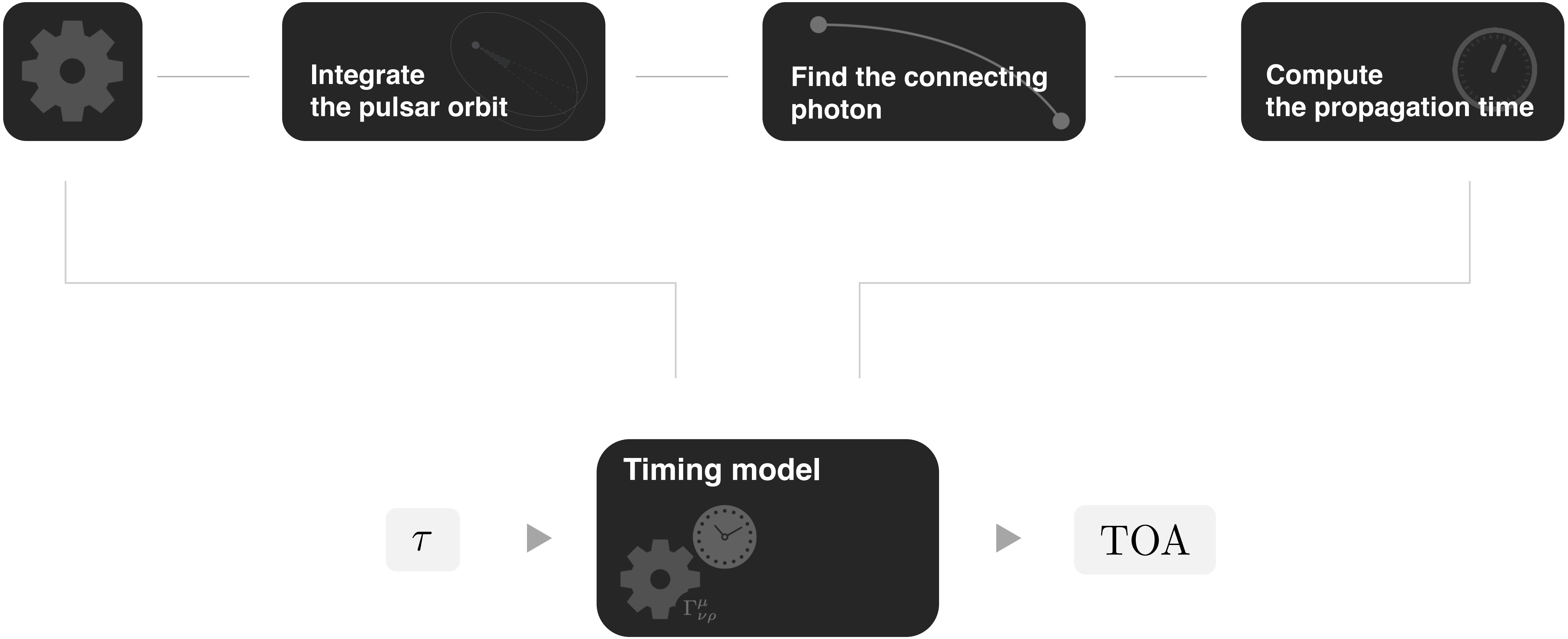
Find the connecting photon

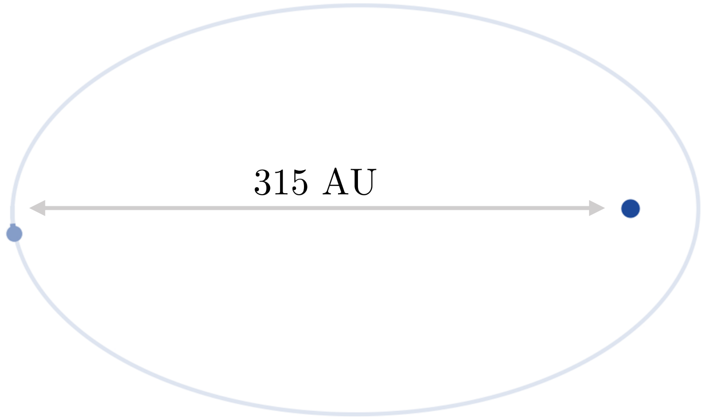
Compute the propagation time

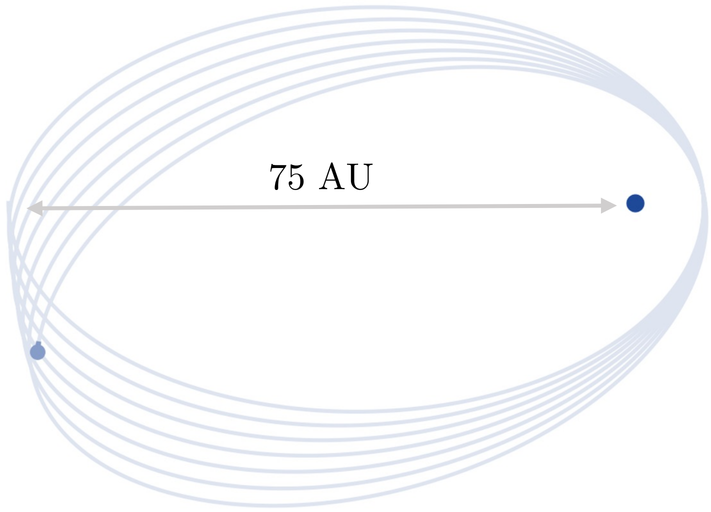
Once the impact parameter is known we can integrate the propagation time numerically

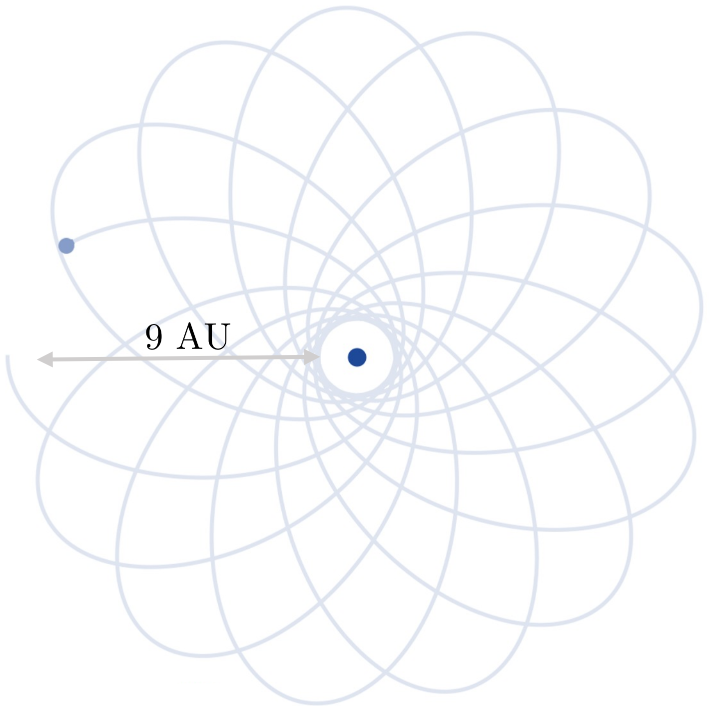
$$t_o - t_e = \int_{r_{e,\gamma}}^{r_o} \frac{r}{A(r)} \sqrt{\frac{A(r)B(r)}{r^2 - b^2 A(r)}} dr$$




Integrate
the pulsar orbit
Find the connecting
photon
Compute
the propagation time
Timing model
τ
$\Gamma^{\mu}_{\nu\rho}$
TOA







Toy 1

$a = 4385 r_g = 175.4$ AU

$e = 0.800$

$T \sim 1$yr

Toy 2

$a = 1095 r_g = 43.8$ AU

$e = 0.800$

$T \sim 50$ days

Toy 3

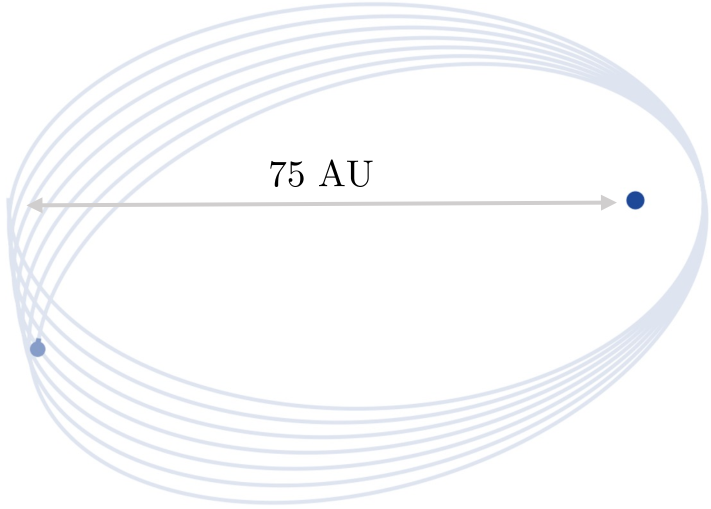
$a = 125 r_g = 5$ AU

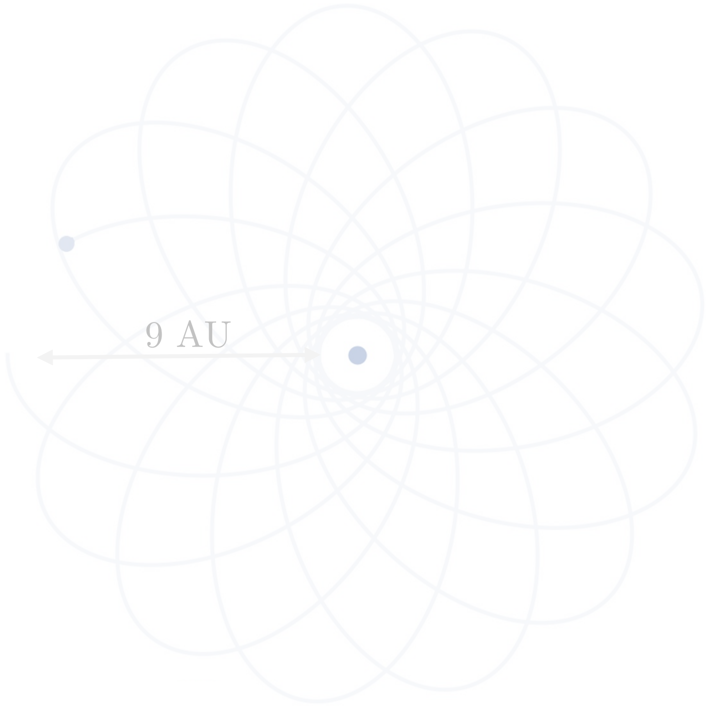
$e = 0.786$

$T \sim 2$ days

Increasingly extreme orbital features

7 Pulsar toy models





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Increasingly extreme orbital features

Toy 2

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↓

M
t_p
a
e
i
ω
P
$\dot{P}$
τ_P
ϕ_0

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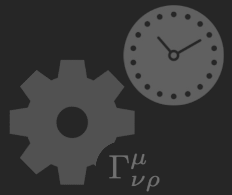
M
t_p
a
e
i
ω
P
$\dot{P}$
τ_P
ϕ_0

Timing model



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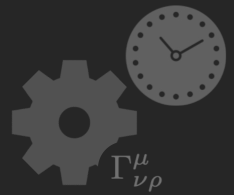
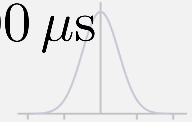


Gaussian noise

$\sim 100\,\mu\text{s}$



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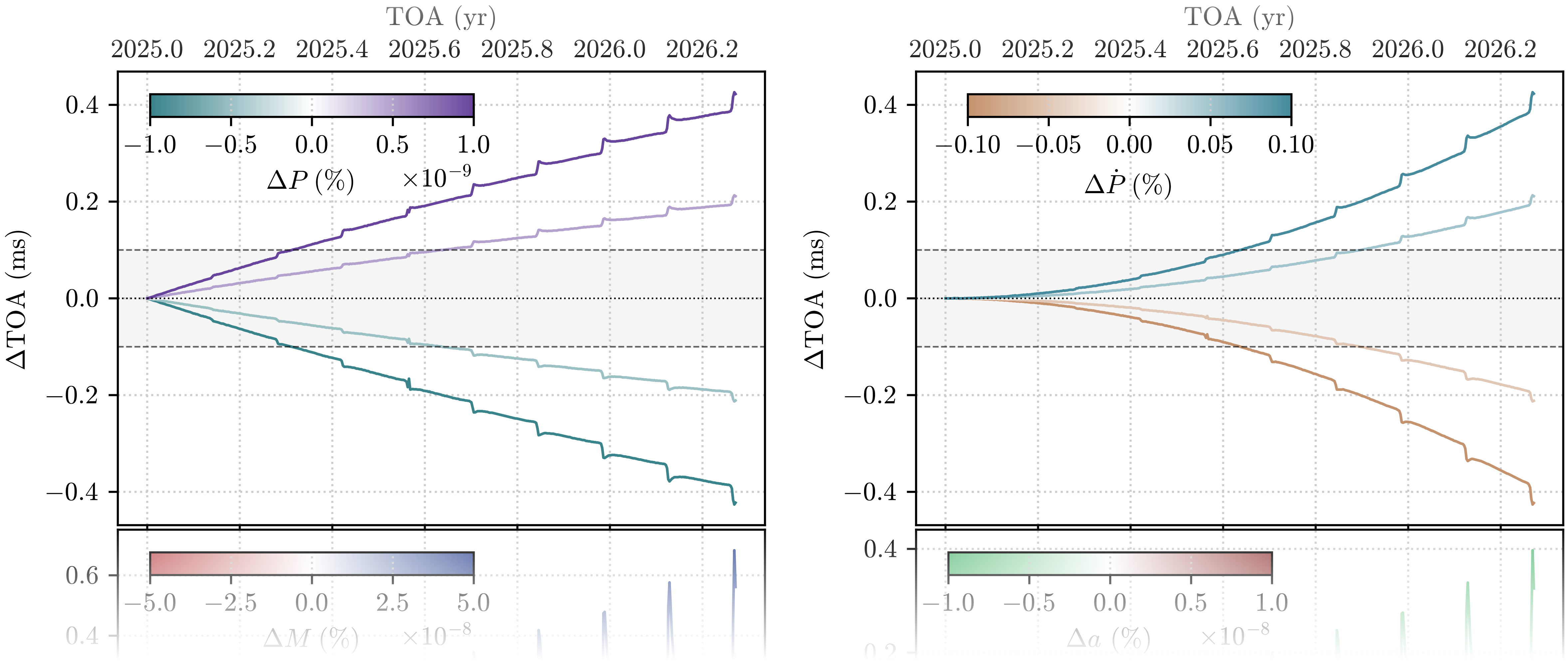
M, t_p, a, e, i, ω, P, $\dot{P}$, τ_P, ϕ_0

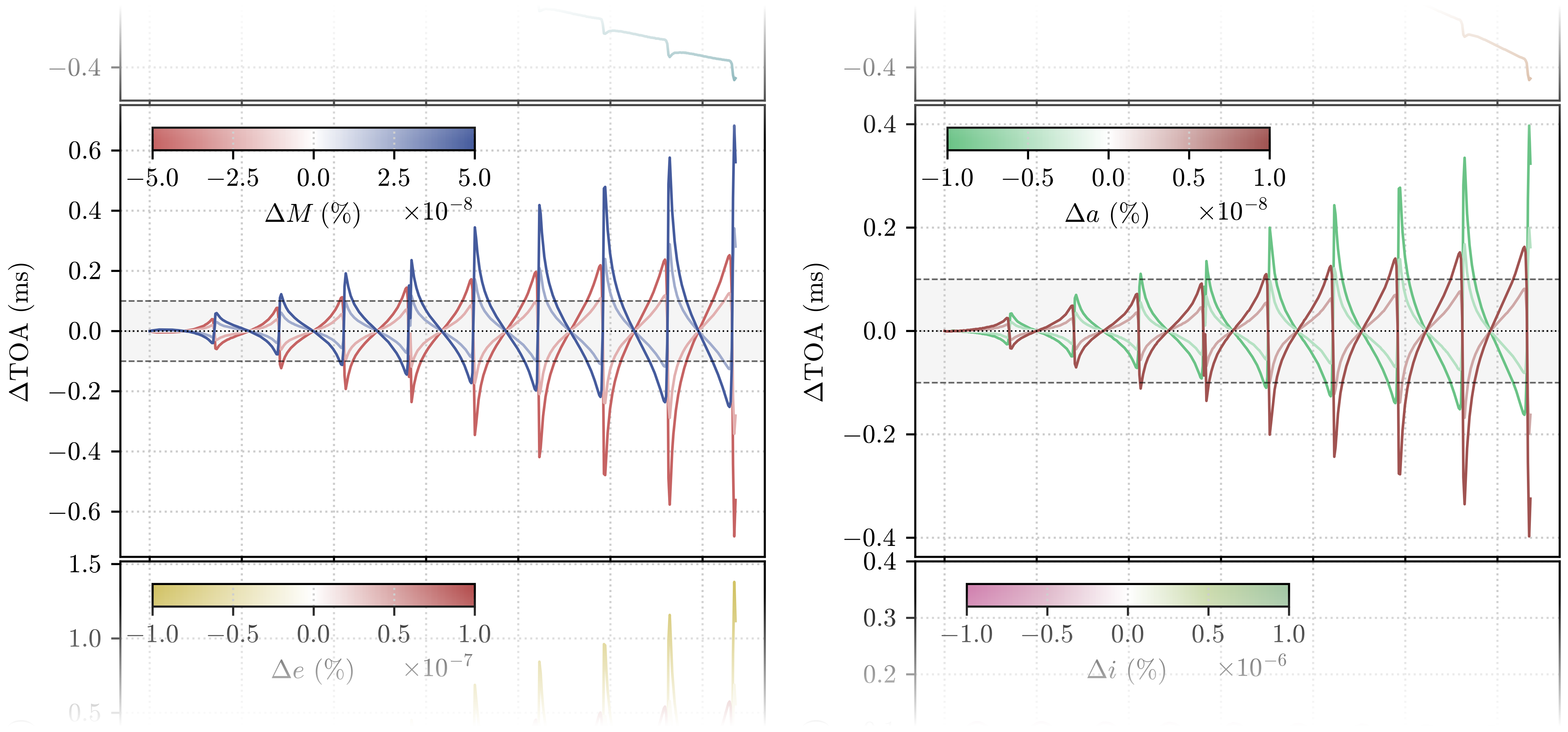
Timing model ($\Gamma^{\mu}_{\nu\rho}$)

Gaussian noise

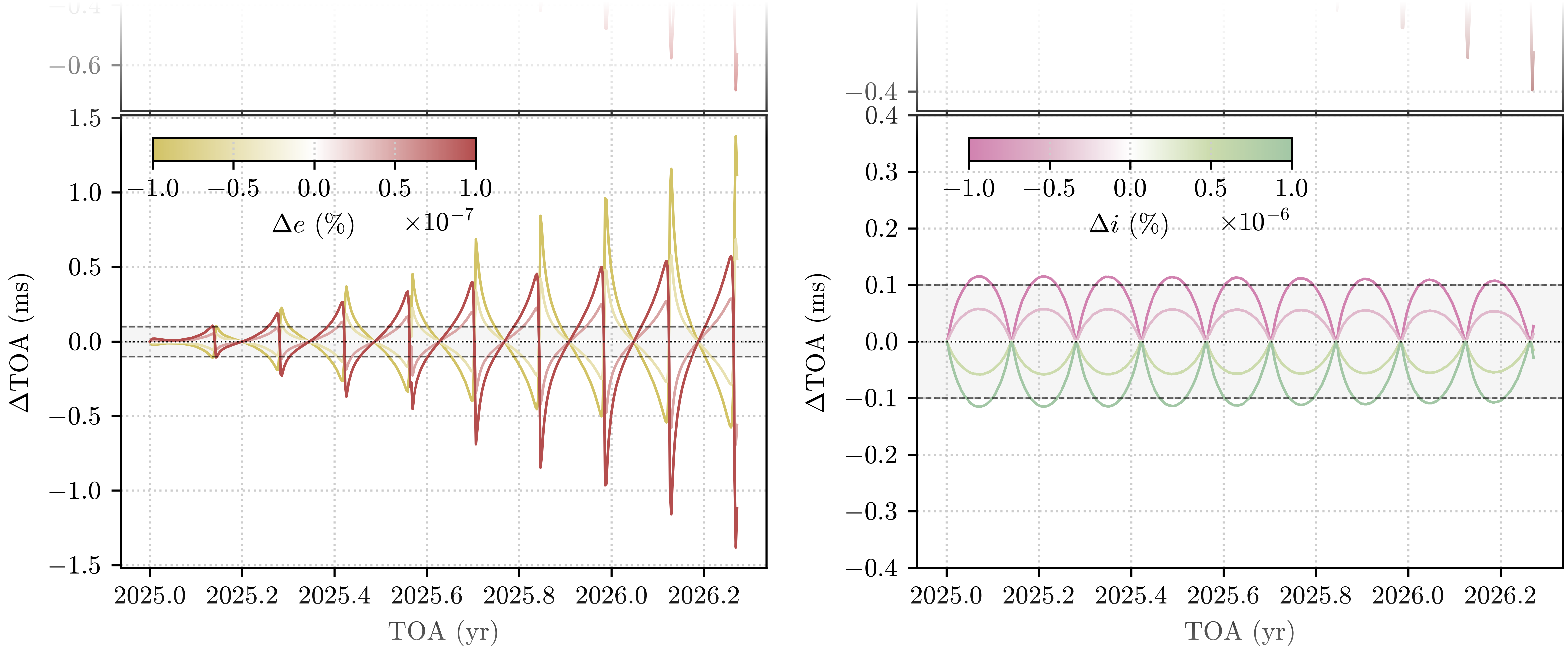
$\sim 100\,\mu\mathrm{s}$

Mock catalogue of TOAs


TOA (yr)
2025.0 2025.2 2025.4 2025.6 2025.8 2026.0 2026.2
ΔTOA (ms)
0.4 0.2 0.0 −0.2 −0.4
−1.0 −0.5 0.0 0.5 1.0
ΔP (%) ×10^−9
−0.10 −0.05 0.00 0.05 0.10
ΔṖ (%)
0.6 0.4
−5.0 −2.5 0.0 2.5 5.0
ΔM (%) ×10^−8
−1.0 −0.5 0.0 0.5 1.0
Δa (%) ×10^−8

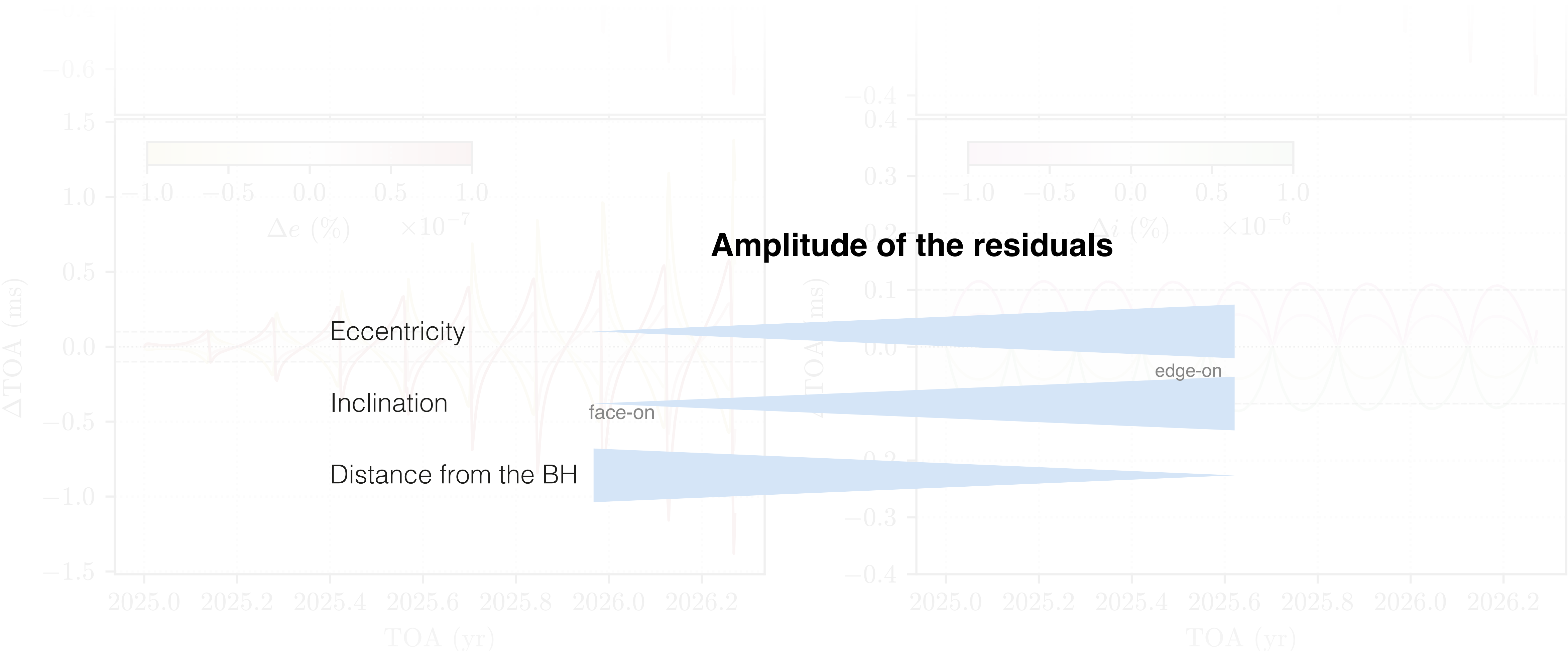


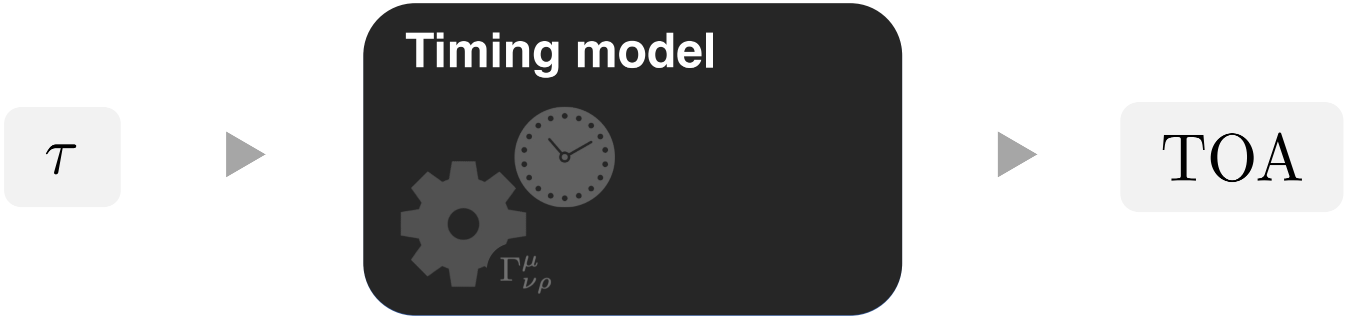
−0.4
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
1.5
1.0
0.5
ΔTOA (ms)
−5.0 −2.5 0.0 2.5 5.0
ΔM (%) ×10−8
−1.0 −0.5 0.0 0.5 1.0
Δe (%) ×10−7
0.4
0.2
0.0
−0.2
−0.4
0.3
0.2
Δa (%) ×10−8
Δi (%) ×10−6

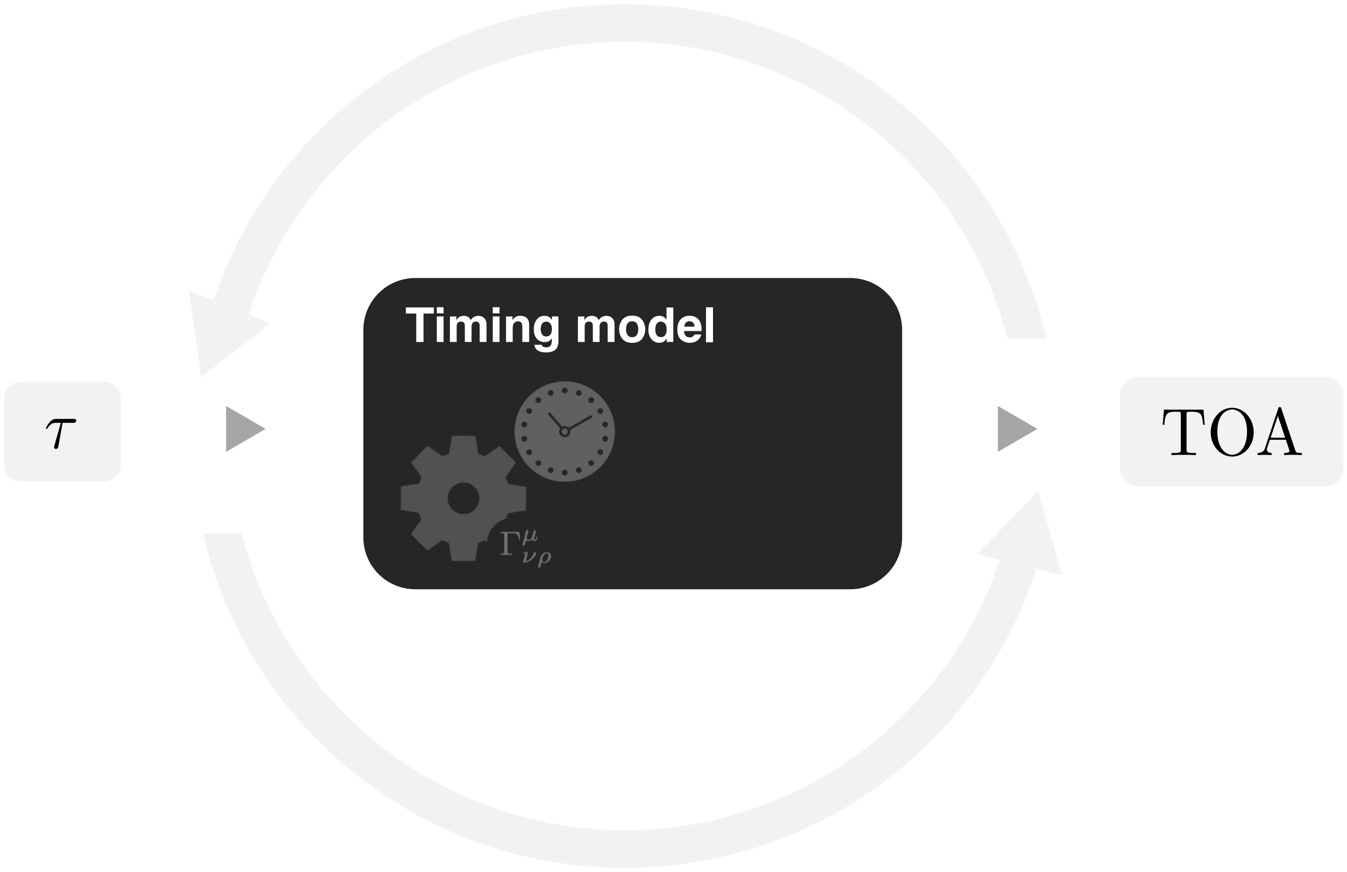


−0.6
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0 −0.5 0.0 0.5 1.0
Δe (%) ×10^−7
ΔTOA (ms)
2025.0 2025.2 2025.4 2025.6 2025.8 2026.0 2026.2
TOA (yr)
−0.4
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−1.0 −0.5 0.0 0.5 1.0
Δi (%) ×10^−6
ΔTOA (ms)
2025.0 2025.2 2025.4 2025.6 2025.8 2026.0 2026.2
TOA (yr)

Amplitude of the residuals




τ
Timing model
$\Gamma^{\mu}_{\nu\rho}$
TOA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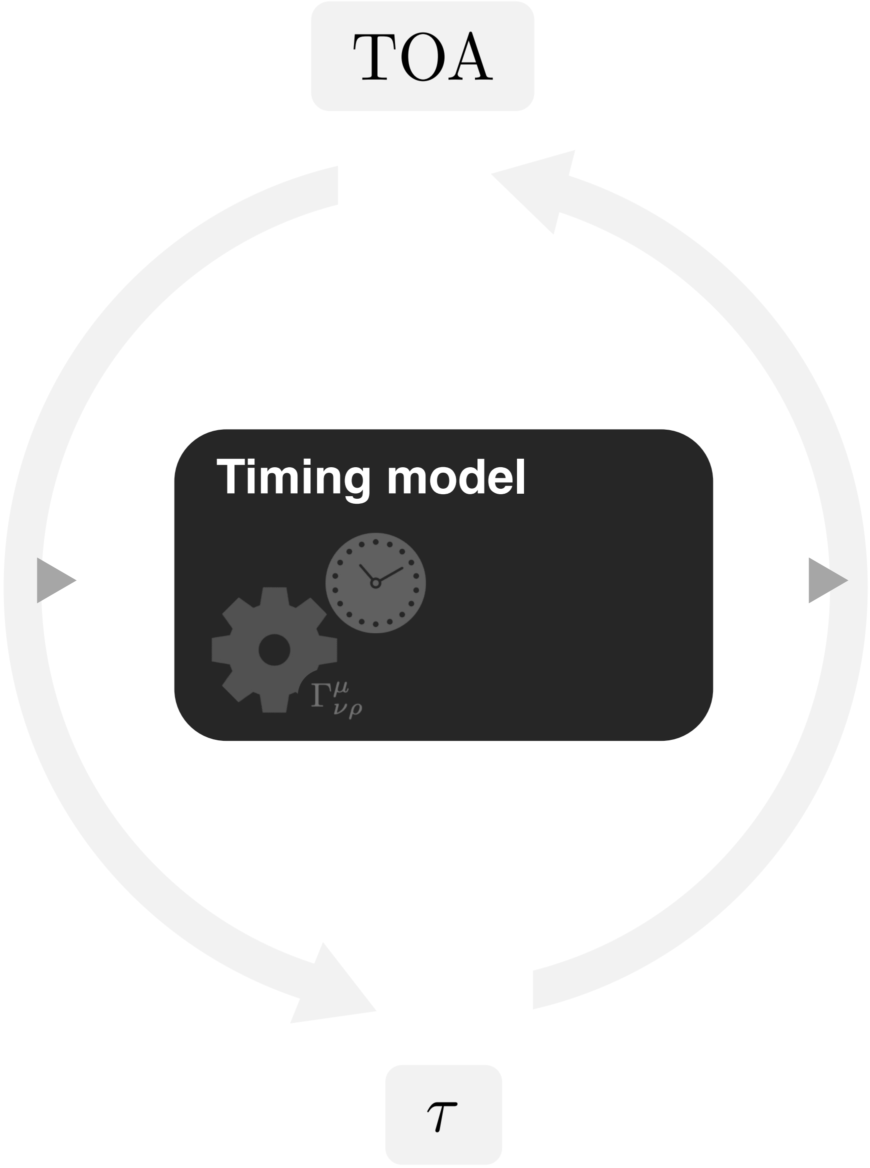
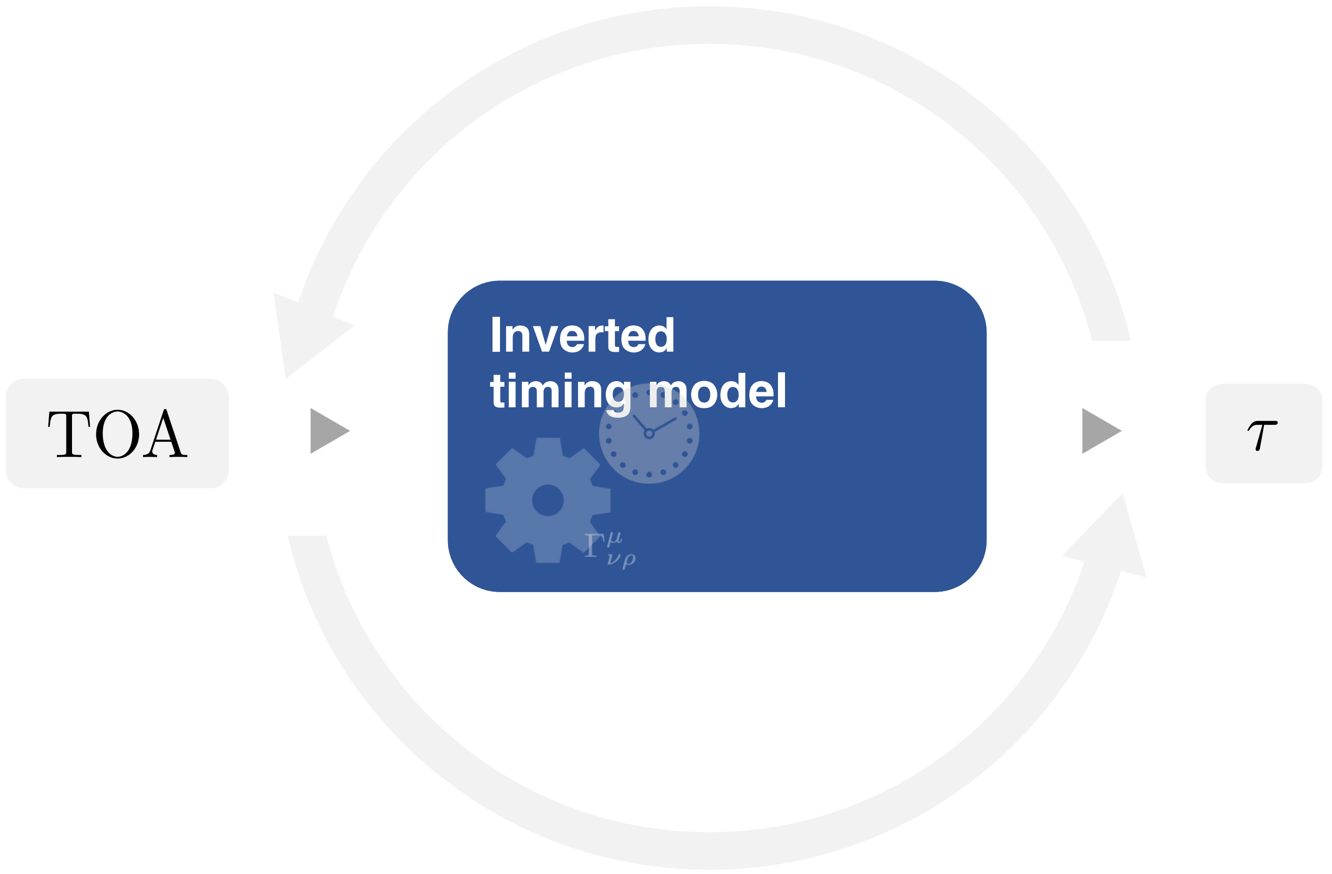

TOA
Timing model
$\Gamma^{\mu}_{\nu\rho}$
τ


TOA
Inverted
timing model
$\Gamma^{\mu}_{\nu\rho}$
τ

TOA ▶ **Inverted timing model** ($\Gamma^{\mu}_{\nu\rho}$) ▶ τ

TOA ▶ **Inverted timing model** ▶ τ ▶ $R_i = \dfrac{\phi(\tau_i) - N_i}{\nu}$

$\Gamma^{\mu}_{\nu\rho}$

TOA ▶ **Inverted timing model** ($\Gamma^{\mu}_{\nu\rho}$) ▶ τ ▶ $R_i = \dfrac{\phi(\tau_i) - N_i}{\nu}$

Spline interpolation

Samples of the relation $\mathrm{TOA} = \mathrm{TOA}(\tau)$

$$N_{\mathrm{data}} \leq N_{\mathrm{dense}} \leq N_{\mathrm{pulses}}$$

$M,\ t_p,\ a,\ e,\ i,\ \omega,\ P,\ \dot{P},\ \tau_P,\ \phi_0$

TOA

Inverted timing model

$\Gamma^{\mu}_{\nu\rho}$

τ

$$R_i = \frac{\phi(\tau_i) - N_i}{\nu}$$

Likelihood

$$\log \mathcal{L} = -\frac{1}{2} \sum_{i=1}^{N_{\rm data}} \left(\frac{R_i}{\sigma_i} \right)^2$$

Spline interpolation

Samples of the relation $\mathrm{TOA} = \mathrm{TOA}(\tau)$

$$N_{\rm data} \leq N_{\rm dense} \leq N_{\rm pulses}$$

MCMC analysis

$M,\ a,\ e,\ P,\ \dot{P}$

Subset of parameters

TOA

Inverted timing model

τ

$$R_i = \frac{\phi(\tau_i) - N_i}{\nu}$$

Likelihood

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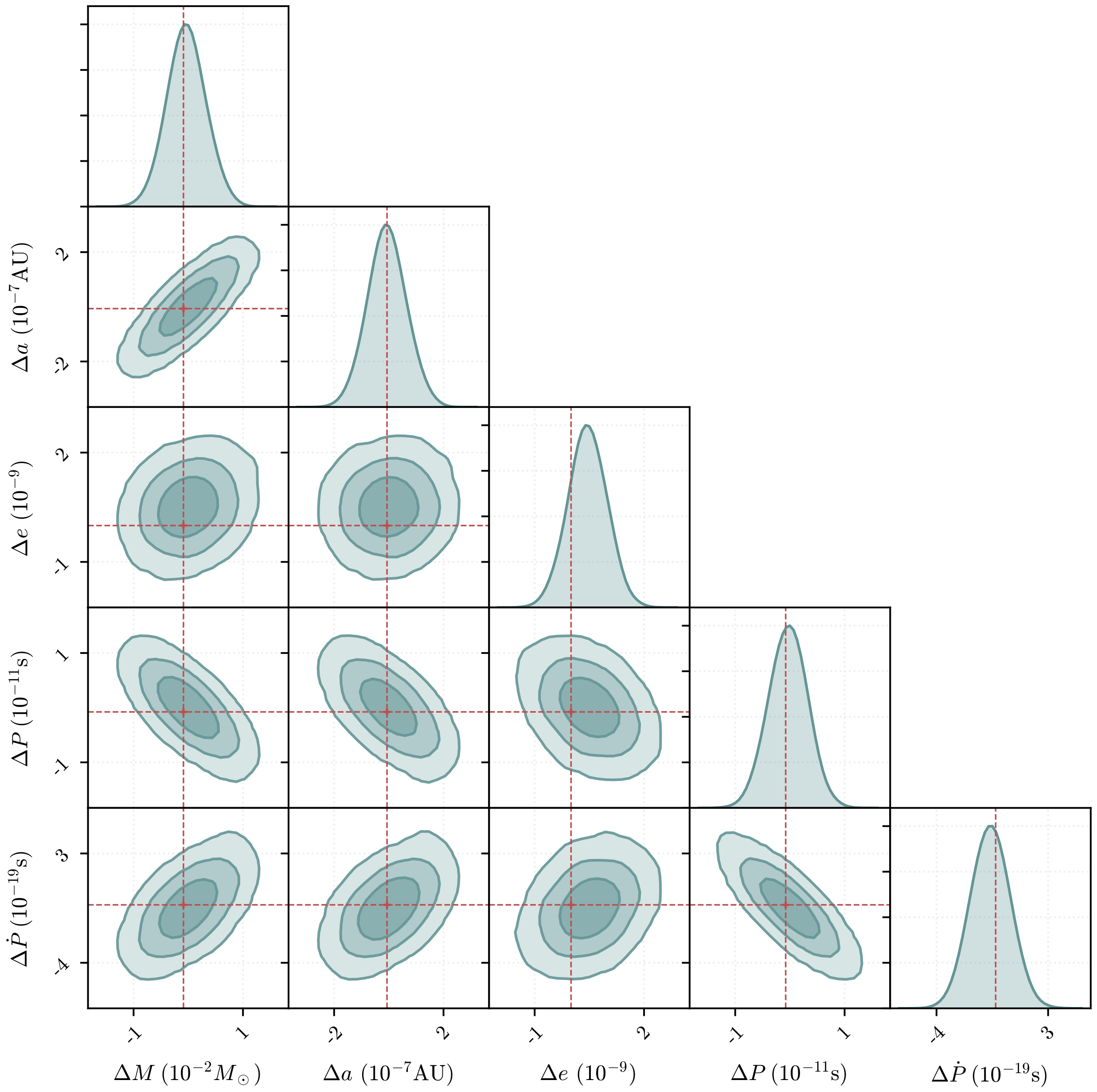
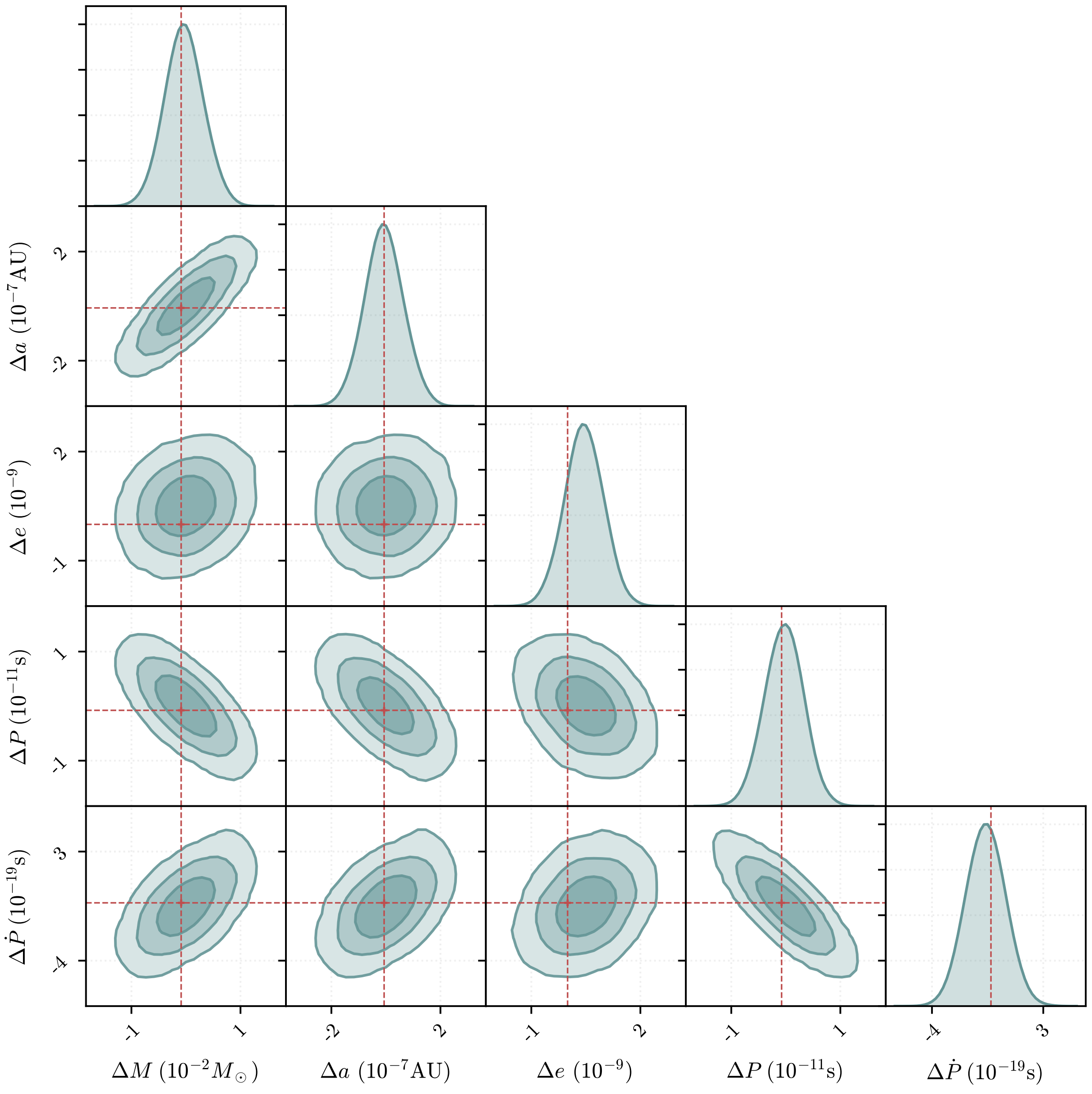
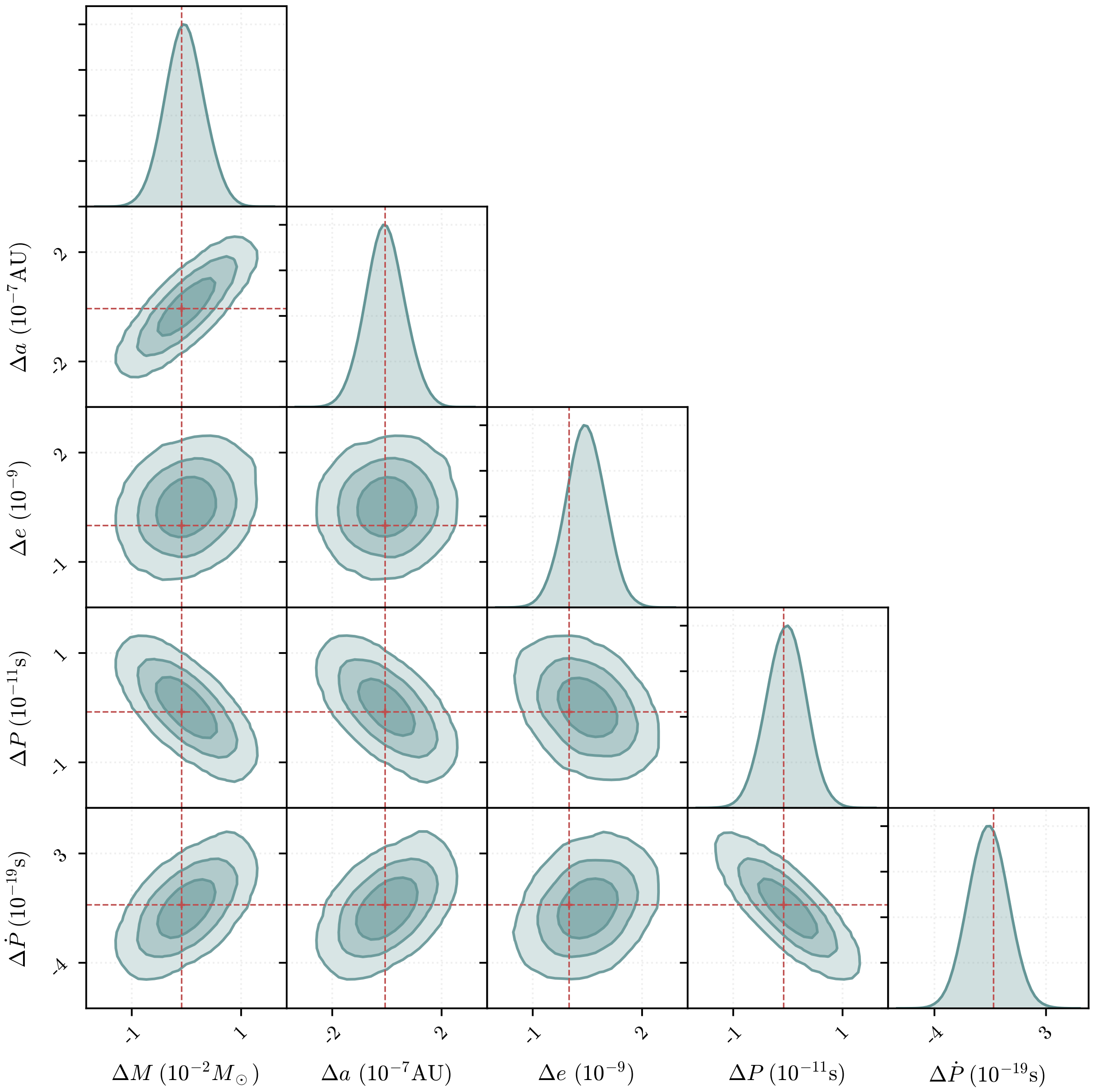
MCMC analysis


$\Delta a\ (10^{-7}\mathrm{AU})$
$\Delta e\ (10^{-9})$
$\Delta P\ (10^{-11}\mathrm{s})$
$\Delta \dot{P}\ (10^{-19}\mathrm{s})$
$\Delta M\ (10^{-2}M_{\odot})$
$\Delta a\ (10^{-7}\mathrm{AU})$
$\Delta e\ (10^{-9})$
$\Delta P\ (10^{-11}\mathrm{s})$
$\Delta \dot{P}\ (10^{-19}\mathrm{s})$

Results of the analysis



Parameter (unit)	**Posterior**	**Precision** (%)
M ($10^6 M_\odot$)	4.2610000011(41)	9×10^{-8}
a (AU)	175.400000006(55)	3×10^{-8}
e	0.80000000045(46)	5×10^{-8}
P (s)	2.0000000000009(41)	2×10^{-10}
$\dot{P}$ (10^{-15} s)	0.99998(12)	1×10^{-2}



Parameter (unit)	**Posterior**	**Precision** (%)
$M\ (10^6 M_\odot)$	4.2610000011(41)	9×10^{-8}
a (AU)	175.400000006(55)	3×10^{-8}
e	0.80000000045(46)	5×10^{-8}
P (s)	2.0000000000009(41)	2×10^{-10}
$\dot{P}\ (10^{-15}$ s)	0.99998(12)	1×10^{-2}

Conclusions

Pulsars at Galactic Center

Timing analysis of Galactic Center pulsars on relativistic orbits ($T < 10$ yr) with accuracy of 100 microseconds as promised by next observational facilities enables unprecedented constraints

*Nature and physical properties of Sgr A**

Underlying theory of gravity

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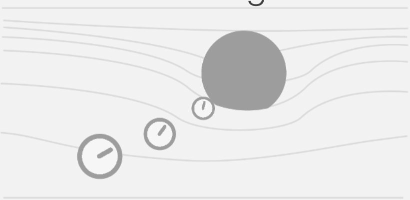
All **relativistic effects** are self-consistently included in the integrated observables



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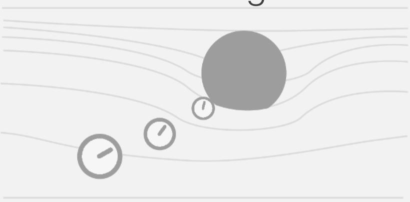
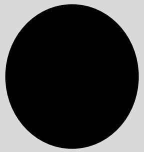


Limitation

- We have limited ourselves to spherically symmetric models

With future observations the spin of Sgr A cannot be neglected*

No Hair Theorem



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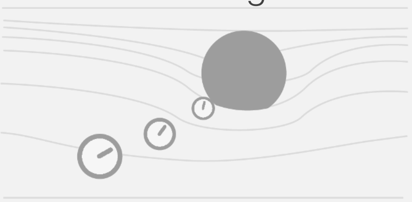

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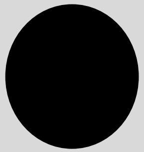

Future prospects

- Extend the methodology to assign initial conditions both for time-like and null particles to axisymmetric spacetime

- Publish, document and maintain PyGRO as an open-source Python package for the benefit of the community

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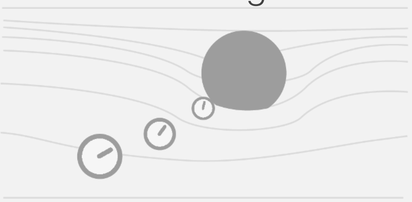

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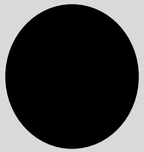

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Thank you
for your attention

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