Constraining 3-form dark energy models

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Introduction

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1. For example, for a wCDM model with w constant and $k = 0$.
- Such an acceleration could be due
  - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
  - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....
- Cosmological tensions H_0 and σ_8

Late-time acceleration of the Universe within GR: dark energy with a constant EoS

Constant equation of state for DE: background-1-

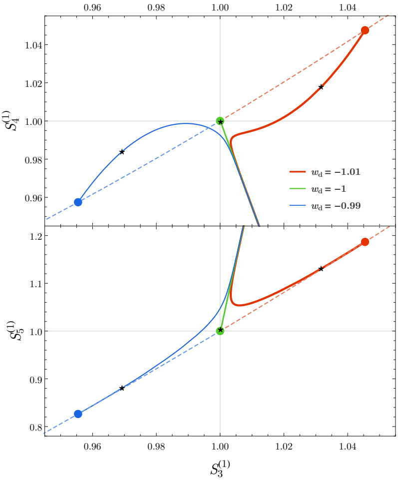
- Cosmic acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi \mathrm{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{de}} + 3p_{\mathrm{de}})$$

- Observation indicates that for $w_{\mathrm{de}} \sim -1$ where $w_{\mathrm{de}} = p_{\mathrm{de}}/\rho_{\mathrm{de}}$.
- Therefore, as soon as DE starts dominating the Universe starts accelerating, i.e. $\ddot{a} > 0$.
- Simplest cases ΛCDM or wCDM.

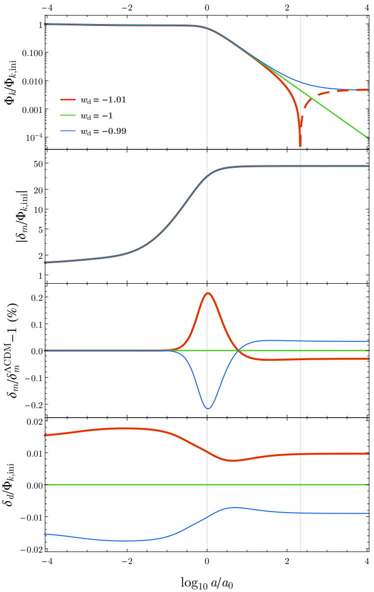
Constant equation of state for DE: background-2-

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
  - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0\,(t - t_0)\right]^n$, where $A_n := a^{(n)}/(a\,H^n)$, $n \in \mathbb{N}$.
  - State finders parameters: $S_3^{(1)} = A_3$, $S_4^{(1)} = A_4 + 3\,(1 - A_2)$, $S_5^{(1)} = A_5 - 2\,(4 - 3A_2)\,(1 - A_2)$
  - $\Omega_\mathrm{m} = 0.309$, $\Omega_\mathrm{d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-2-

- What about $f\sigma_8$ for the three different DE models?

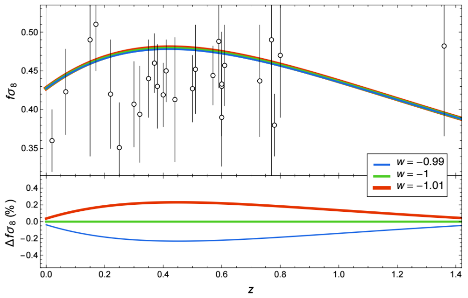


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right)\frac{\delta_{\rm m}\left(z,\ k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\ k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\ k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais [arXiv:1706.01484]

Late-time acceleration through a 3-form field

Can we have something more fundamental to describe DE?

- Can we have something more fundamental to describe (phantom) DE models?
  - A possibility come in the form of 3-forms.
  - Inspired in string theory: Copeland, Lahiri,Wands (1995)
  - Massless 3-form as Cosmological Constant (solving CC problem): Turok, Hawking (1998)
  - Inflation or late time acceleration driven by self-interacting 3-forms: Germani and Kehagias (2009), Koivisto, Nunes (2009) and (2010)
  - Non-Gaussianity: Kumar, Mulryne, Nunes, Marto, Moniz (2016)
  - Quantum cosmology with 3-forms: Bouhmadi-López, Brizuela, Garay (2018)
- The answer as we will see in a momment is yes:

Phantom DE models (LSBR): Morais, Bouhmadi-López, Kumar, Marto, Tavakoli (2017),Bouhmadi-López, Marto, Morais and Silva (2018), C.G. Boiza, M.B.-L, H.-W. Chiang and P. Chen work in progress (2024)

What are p-forms?

A p-form is a totally anti-symmetric covariant tensor:

$$\omega_{\mu_1\ldots\mu_p} = \omega_{[\mu_!\ldots\mu_p]}\,.$$

In D-dimensions, the number of degrees of freedom of a massive p-form is

$$\text{degrees of freedom} = \frac{(D-1)!}{(D-1-p)!p!}\,.$$

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
Section based on Morais, B.L, Kumar, Marto and Tavakoli, Phys. Dark Univ. 15, 7 (2017) [arXiv:1608.01679 [gr-qc]]

p-forms in Cosmology

In a 4-dimensional space-time:

- $p = 0$ (scalar field) $\Rightarrow$ 1 degree of freedom
- $p = 1$ (vector field) $\Rightarrow$ 3 degrees of freedom
- $p = 2 \Rightarrow$ 3 degrees of freedom
- $p = 3 \Rightarrow$ 1 degree of freedom

$\Rightarrow$ The scalar field and the 3-form are the only ones compatible with a homogeneous and isotropic universe (in an easy way).

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

The 3-form action

- We will consider the following action for a massive 3-form, $A_{\mu\nu\rho}$, minimally coupled to gravity

$$S^A = \int \mathrm{d}^4\mathbf{x}\sqrt{|\det g_{\mu\nu}|}\left[-\frac{1}{48}F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} - V\left(A^{\mu\nu\rho}A_{\mu\nu\rho}\right)\right].$$

- The strength tensor, a 4-form, is defined through the exterior derivative: $F_{\mu\nu\rho\sigma} \equiv 4\nabla_{[\mu}A_{\nu\rho\sigma]}$
- The equation of motion, obtained from variation of S^A, is

$$\nabla_\sigma F^\sigma{}_{\mu\nu\rho} - 12\frac{\partial V}{\partial\left(A^2\right)}A_{\mu\nu\rho} = 0$$

- $\Rightarrow$ a massless 3-form is equivalent to a cosmological constanst

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
M. Duff and P. Van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)

3-form Cosmology

We consider a homogeneous and isotropic universe described by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j\,.$$

t - cosmic time, $\dot{\{\}} = d\{\}/dt$
a - scale factor
x^i - comoving spatial coordinates (roman indices run from 1 to 3).

Only the purely spatial components of the 3-form are dynamical:

$$A_{0ij} = 0\,, \qquad A_{ijk} = a^3(t)\chi(t)\epsilon_{ijk}\,.$$

T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

3-form Cosmology: background equations

⇒ Friedmann Equation

$$3H^2 = \kappa^2 \rho_\chi = \kappa^2 \left[\frac{1}{2} \left(\dot{\chi} + 3H\chi \right)^2 + V(\chi^2) \right] .$$

⇒ Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_\chi + P_\chi \right) = -\frac{\kappa^2}{2} \chi \frac{\partial V}{\partial \chi} .$$

A 3-form can show phantom-like behavior if $\partial V / \partial \chi^2 < 0$.

⇒ Equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + \frac{\partial V}{\partial \chi} = 0 .$$

3-form Cosmology: evolution of χ-1-

Combining the Raychaudhuri equation and the equation of motion for χ:

$$\ddot{\chi} + 3H\dot{\chi} + \left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi} = 0\,.$$

The static solutions are:

- the critical points of the potential: $\dfrac{\partial V}{\partial \chi} = 0$,
- the limiting points: $\chi = \pm\chi_c$.

Once inside the interval $[-\chi_c,\, \chi_c]$, the field χ evolves towards a local minimum of V. However…

3-form Cosmology: evolution of χ-2-

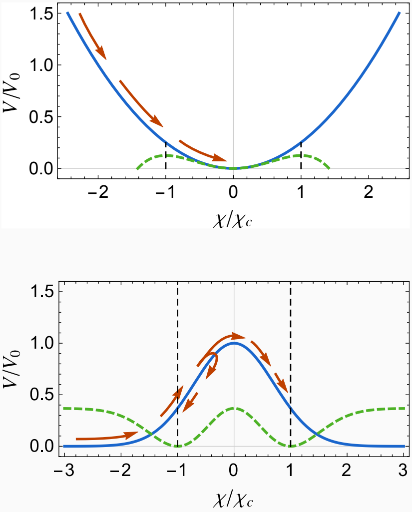
- Independently of the shape of a regular potential, in absence of DM interaction, the 3-form decays rapidly towards the interval $[-\chi_c, \chi_c]$ Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

- In an expanding Universe, once inside the interval $[-\chi_c, \chi_c]$, the 3-form will end up in one of the minima of the potential (notice $V_{\rm eff} \neq V$).

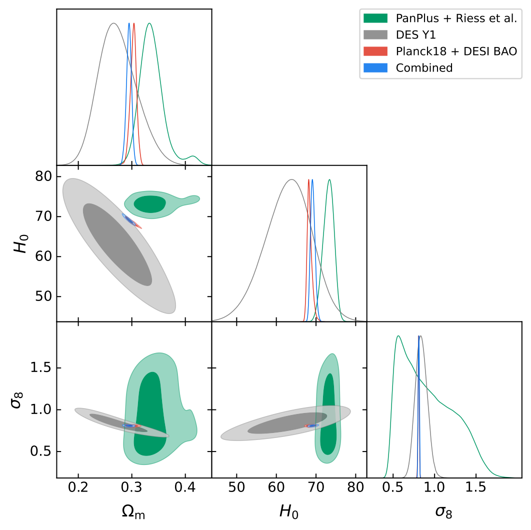
- If the 3-form stops at the limits of this interval:

$$\chi = \pm\chi_c \quad \text{and} \quad \dot{\chi} = 0$$

- $\longrightarrow$ Universe heads towards a LSBR event $\left(\chi_c = \sqrt{2/3\kappa^2}\right)$

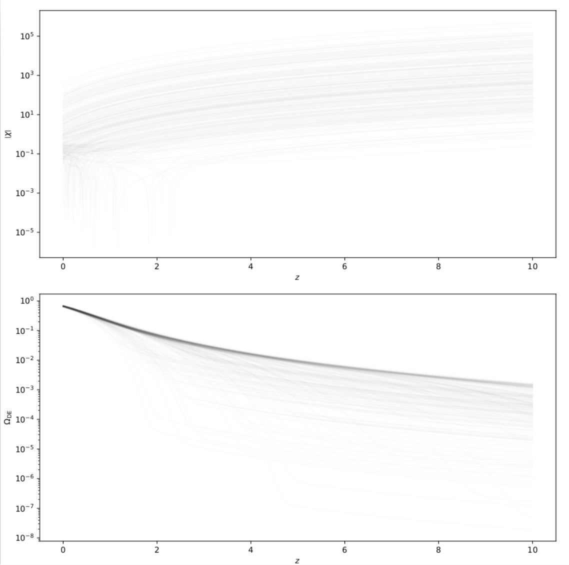


Fitting the model-1-

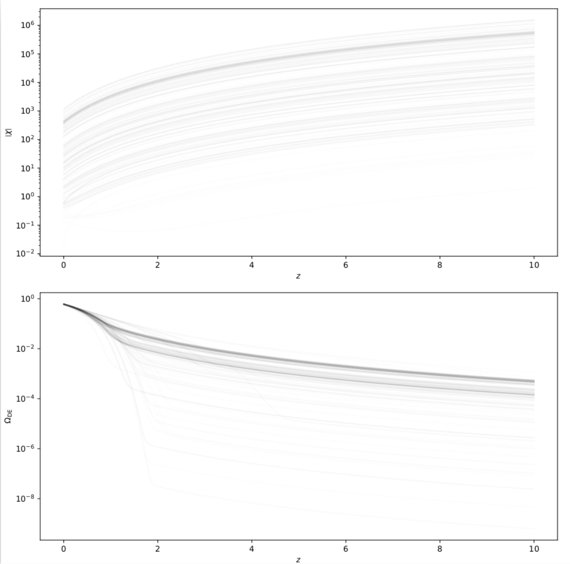


C.G. Boiza, M.B.-L, H.-W. Chiang, H.-M. Huang and P. Chen work in progress (2024)

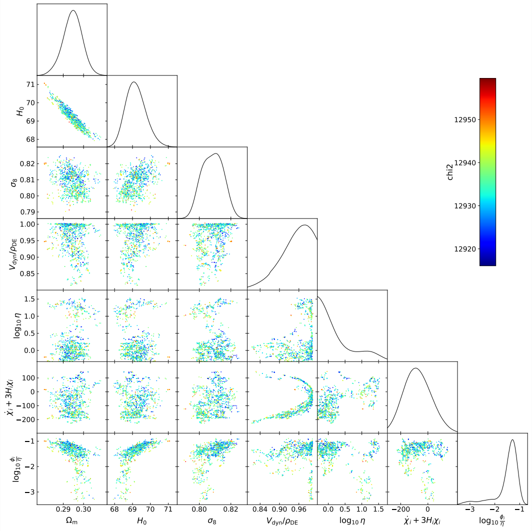
Constraining the evolution of 3-forms from SNIa (smaller Ω_m)



Constraining the evolution of 3-forms from SNIa (larger Ω_m)

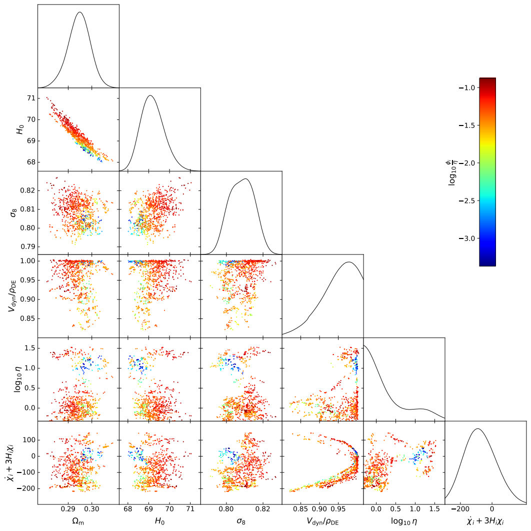


Fitting the model-2-



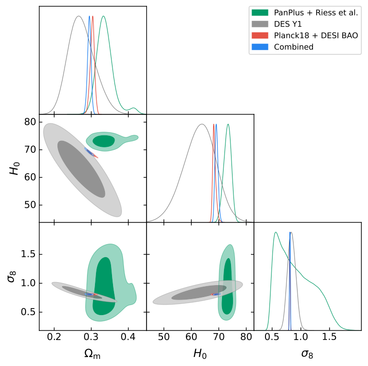
C.G. Boiza, M.B.-L, H.-W. Chiang, H.-M. Huang and P. Chen work in progress (2024)

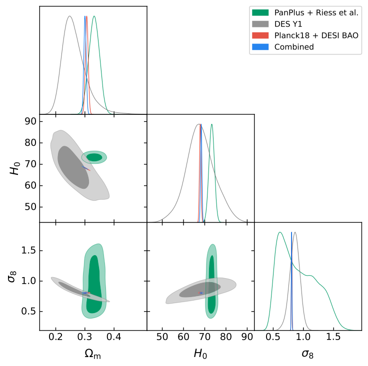
Fitting the model-3-



C.G. Boiza, M.B.-L, H.-W. Chiang, H.-M. Huang and P. Chen work in progress (2024)

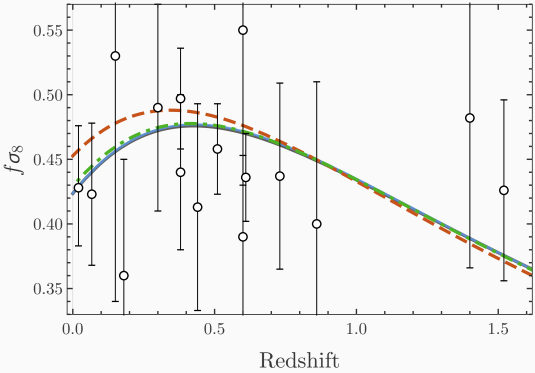
Comparing the model to LCDM





C.G. Boiza, M.B.-L, H.-W. Chiang, H.-M. Huang and P. Chen work in progress (2024)

Behaviour of fσ_8



Further consideration with 3-forms from a gravitational point of view

Let me add that 3-forms can be quite interesting for further reasons as:

- They allow naturally for regular BHs (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC 2021)
- They naturally support wormholes without changing the sign of the kinetic energy (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021).

Conclusions

Conclusions

- We started reviewing the late-time acceleration of the universe through a *wcdm* phenomenological approach component
- We have then described DE through a more fundamental field encoded in a 3-form which mimics a phantom-like behaviour.
- We have discussed if such a DE model could help to release the H_0 and/or σ_8 tensions
- It can be shown that Bhs free of singularities and also Whs can be supported by 3-forms (slides afterwards).

Thank you for your attention !!! Gratissima!!!

Former study and our main goal

- Black holes (exterior) and naked singularity has been analysed in: Barros, Danila, Harko, Lobo (2020)
- We are rather interested in getting regular BHs; i.e. keeping the essence and avoiding the singularity

BHs interior and KS space-time

- Inside the event horizon, the space-time of a static and spherically symmetric black hole can be described by the Kantowski-Sachs anisotropic metric
- The geometry is caracterised by 2 scale factors $a(t)$ and $b(t)$
- The matter sector will be described by the 3-form (incoded in χ) and its potential (incoded in $V(\chi)$)
- We assume that close to the event horizon we recover Schwarzschild solution.
- We assume a functional form for $b(t)$

Regular BHs supported by 3-forms: geometry and matter content

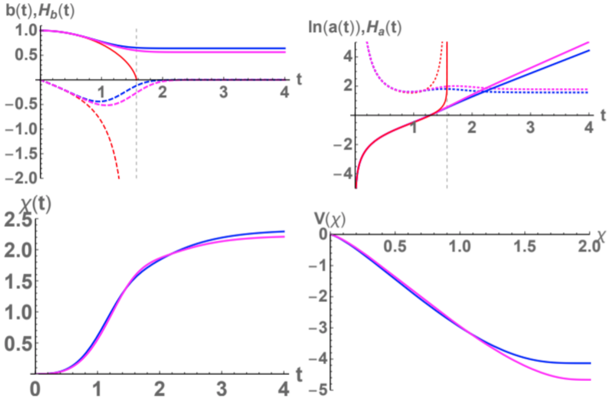
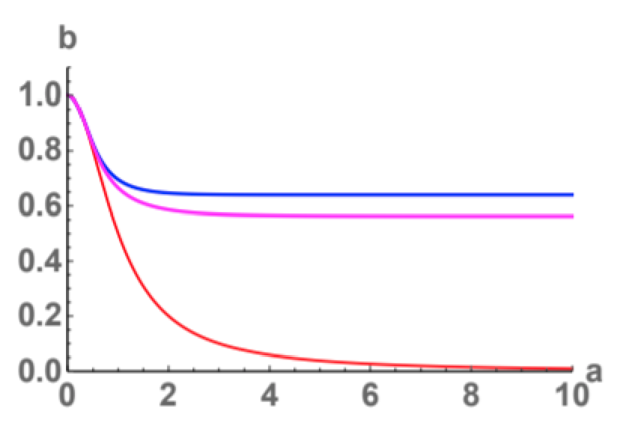


FIG. 1: The results of the regular black hole model given by Eq. (19) for $x = 4$ (blue) and $x = 3$ (magenta), respectively. The red curves are the results of the Schwarzschild spacetime. Top-left: $b(t)$ (solid) and $H_b(t)$ (dashed). Top-right: $\ln a(t)$ (solid) and $H_a(t)$ (dotted). The 3-form field $\chi(t)$ and the potential $V(\chi)$ are shown in the bottom-left and bottom-right panels, respectively. The vertical dashed line ($t = \pi/2$) stands for the singularity in the Schwarzschild black hole. Note that we have rescaled $t/r_s \to t$. In addition, the gravitational constant is set to $\kappa = 1$ in these plots. Likewise for the plots below.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: a closer look at the geometry



Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The curvature(s)

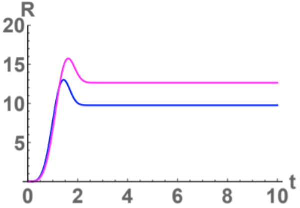


FIG. 3: The Ricci scalar R and the Kretschmann scalar K of the regular black hole model (the magenta and blue curves correspond to $x = 3$ and $x = 4$ respectively), compared to the Schwarzschild case in red. The Ricci scalar is of course identically zero in the Schwarzschild case, hence not shown in the plot.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The null energy condition

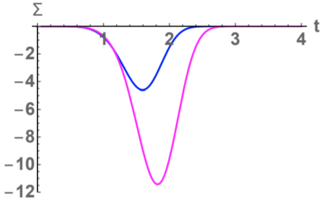


FIG. 5: The null energy condition is violated in the regular black hole model ($\Sigma := T_{\mu\nu}k^{\mu}k^{\nu} < 0$). The blue and the magenta curves correspond to $x = 4$ and $x = 3$, respectively.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The Penrose diagram

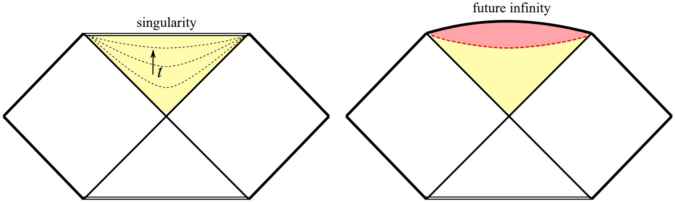


FIG. 4: Left: The causal structure of a maximally extended Schwarzschild black hole, where our coordinate covers inside the event horizon (yellow colored region), where the time coordinate varies from $t = 0$ (horizon) to $t = \pi/2$ (singularity). Right: The effects of the 3-form field is to modify the solution near the putative singularity. The areal radius approaches a constant and the singularity is replaced by the topology $\mathrm{dS}_2 \times \mathrm{S}^2$ (red colored region). Therefore, one can interpret that the internal structure will evolve to a spacelike future infinity rather than a spacelike singularity.

$$ds^2 = -dt^2 + \cosh^2\frac{t}{a}dy^2 + b^2 d\Omega_2^2,$$

- Instead of a singularity a Nariai space-time is reached asymptotically
- Geodesic completeness: One takes infinite proper time to reach the minimum value of $b(t)$; i.e. $b_m \neq 0$
- Notice that we are reaching an effective positive cosmological constant (de Sitter) for a negative potential (!!!!)

Wormholes supported by 3-forms A: Model building

- We assume a quartic potential

$$V(\mathbf{A}^2) = \mu^2 \mathbf{A}^2 + \lambda \mathbf{A}^4$$

.

- The 3-form **A** gives rise to the d.o.f. χ
- Our geometry is described by a static, spherically symmetric metric:

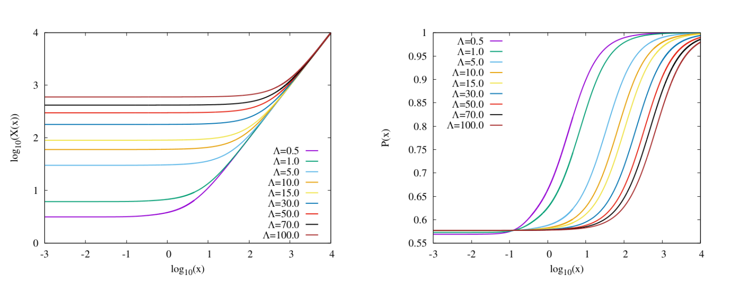
$$ds^2 = -P(r)^2 dt^2 + dr^2 + R(r)^2 d\Omega_2^2$$

- It is useful to introduce a dimensionless representation to numerically solve the equations from the throat outward:

$$r = \frac{x}{\mu}, \quad R = \frac{X}{\mu}, \quad \lambda = \kappa \mu^2 \Lambda, \quad \chi = \frac{\Psi}{\sqrt{\kappa}}.$$

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

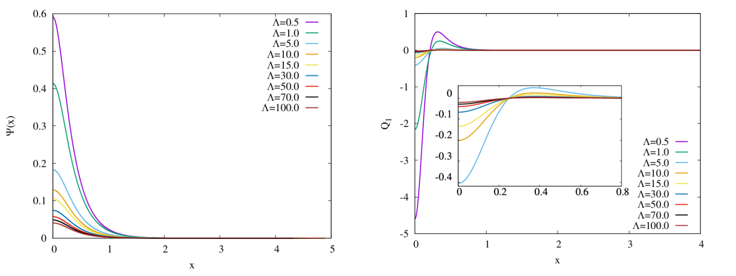
3-form wormholes: The geometry



At the throat: $(x = 0)$, the 2-sphere radius X is minimum and satisfies the flaring-out condition

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

3-form wormholes: The matter content



3-form field evolution + the violation of NEC is concentrated at the throat.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

3-form wormholes: A geometrical approach

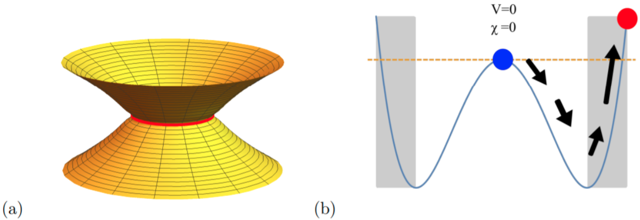
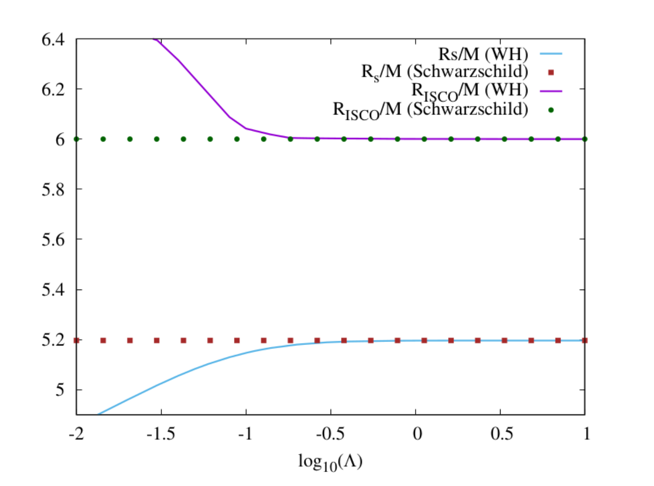


Figure 2. (a) The embedding diagram of the wormhole spacetime supported by the 3-form field. The red curve indicates the wormhole throat. (b) This schematic plot shows the evolution of the 3-form field on the potential in the wormhole spacetime. The shaded regions represent the regions where the NEC is violated.

3-form field evolution

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

3-form wormhole as a Black hole mimicker



3-form wormhole as a Black hole mimicker

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