Black Holes in LIV Gravity

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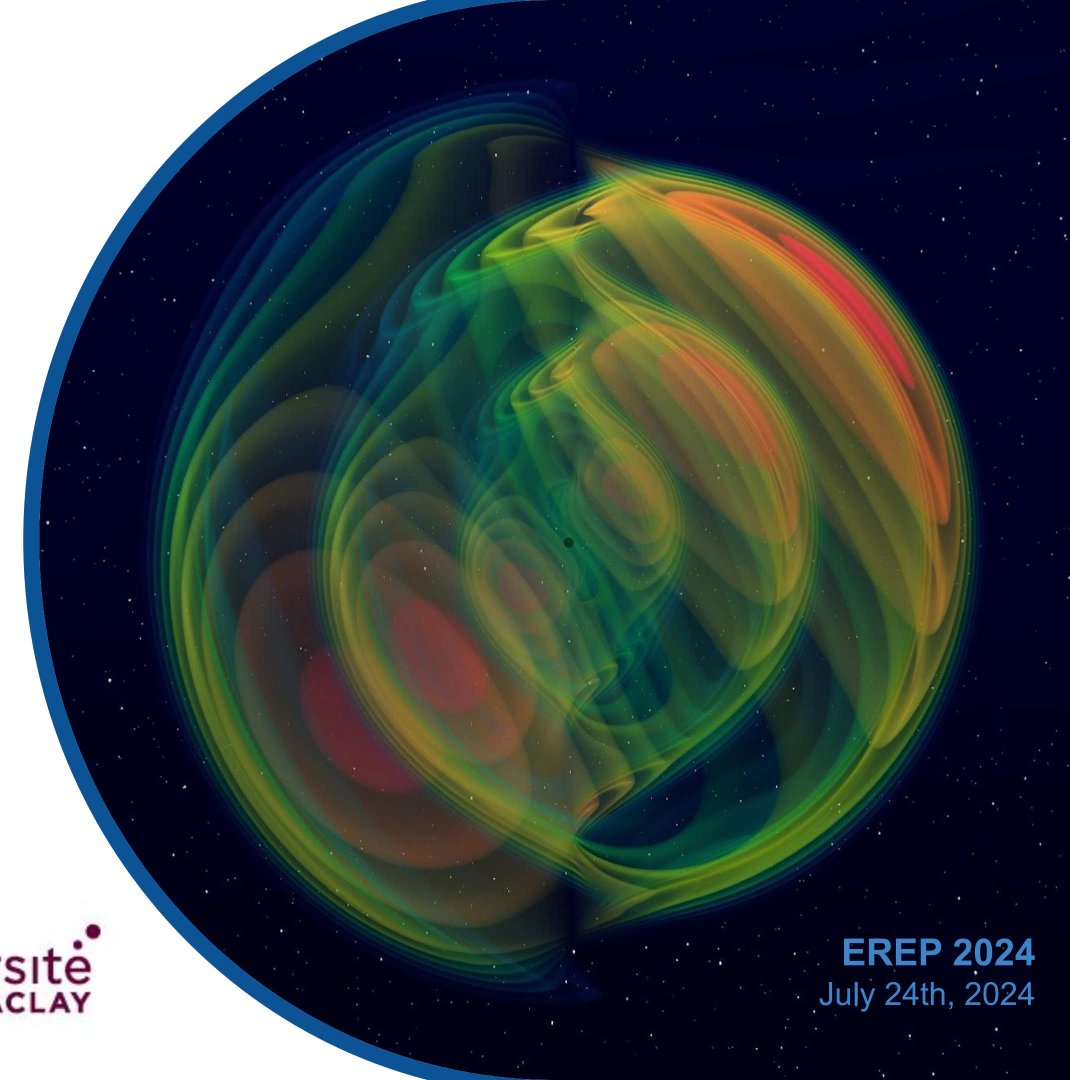

EREP 2024
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Several QG scenarios point to

Lorentz
Invariance
Violation

framework for studying LIV in gravity

LIV Gravity $\subset$ modified gravity

superluminal propagation

non-linear dispersion relations:

$$\omega^2(\vec{k}) = |\vec{k}|^2 + \frac{|\vec{k}|^4}{\Lambda^2} + \ldots$$

[unbounded propagation speeds]

THE FIRST FEW TIMES EINSTEIN IMAGINED FLYING ALONGSIDE A BEAM OF LIGHT, HE DIDN'T HAVE ANY PARTICULAR INSIGHTS.

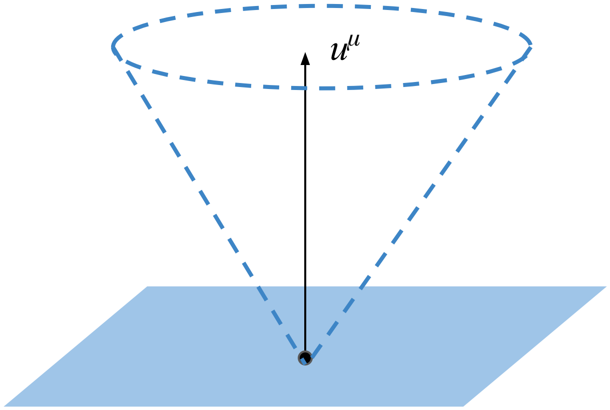
[xkcd]

A road to LIV

Introduce vector field

$u_\mu(x^\alpha)$ everywhere timelike,
gives preferred time direction

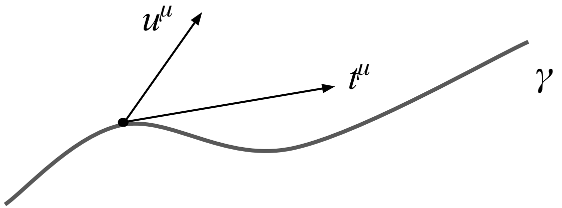
has to be dynamical to ensure
background independence



use it to define
causality

curve with tangent t^μ is:

$$\begin{cases} u_\mu t^\mu < 0 & \text{causal, future directed} \\ u_\mu t^\mu = 0 & \text{acausal} \\ u_\mu t^\mu > 0 & \text{causal, past directed} \end{cases}$$



An example: Einstein–æther theory

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\Big\{R - \Big[\tfrac{1}{3}c_\vartheta \vartheta^2 + c_\sigma \sigma^2 + c_\omega \omega^2 - c_\mathfrak{a} \mathfrak{a}^2\Big]\Big\}$$

$$p_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \quad \text{(projector)}$$

$$\begin{cases} \mathfrak{a}_\mu = u^\alpha \nabla_\alpha u_\mu & \text{(acceleration)} \\ \vartheta = \nabla_\mu u^\mu & \text{(expansion)} \end{cases} \qquad \begin{cases} \sigma_{\mu\nu} = \nabla_{(\mu} u_{\nu)} + u_{(\mu}\mathfrak{a}_{\nu)} - \frac{\vartheta}{3} p_{\mu\nu} & \text{(shear)} \\ \omega_{\mu\nu} = \nabla_{[\mu} u_{\nu]} + u_{[\mu}\mathfrak{a}_{\nu]} & \text{(twist)} \end{cases}$$

+ constraint

$$\mathcal{L} + \zeta(u^\mu u_\mu + 1)$$

unit-norm constraint

+ higher orders

$$\nabla_\mu = -u_\mu \left(u^\alpha \nabla_\alpha\right) + p_\mu^{\ \alpha} \nabla_\alpha$$

‘temporal – spatial’ splitting

Particular case:

$$u_\mu = \frac{\nabla_\mu T}{\sqrt{-\nabla_\alpha T \nabla^\alpha T}} \quad \Leftrightarrow \quad \omega_{\mu\nu} = 0$$

Hypersurface **O**rthogonality

low energy limit of non-projectable Hořava gravity

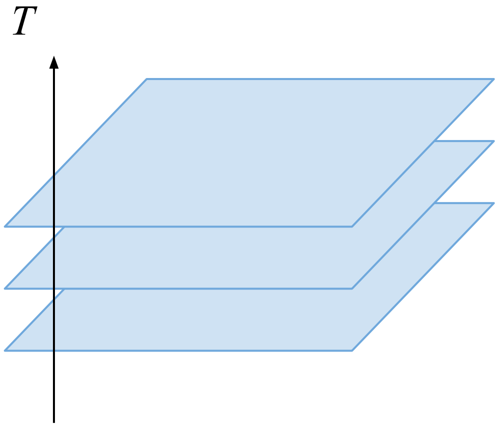
[renormalisable QFT of gravity]

complete to full Hořava w/

✗ $u_\mu \left(u^\alpha \nabla_\alpha \right)$, $p_\mu^\alpha \nabla_\alpha$ ✓

twist vanishes (Frobenius)

foliation via hypersurfaces of simultaneity

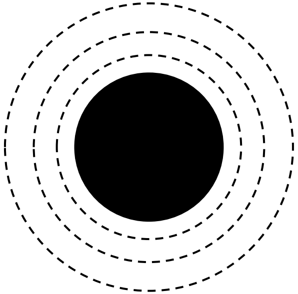


Black Holes (?)

different
propagation speeds

↓

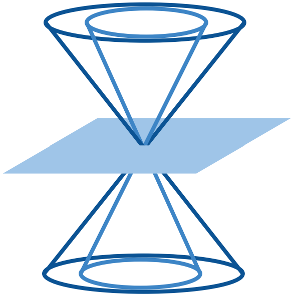
multiple
nested horizons



non-linear $\omega(k)$'s

↓

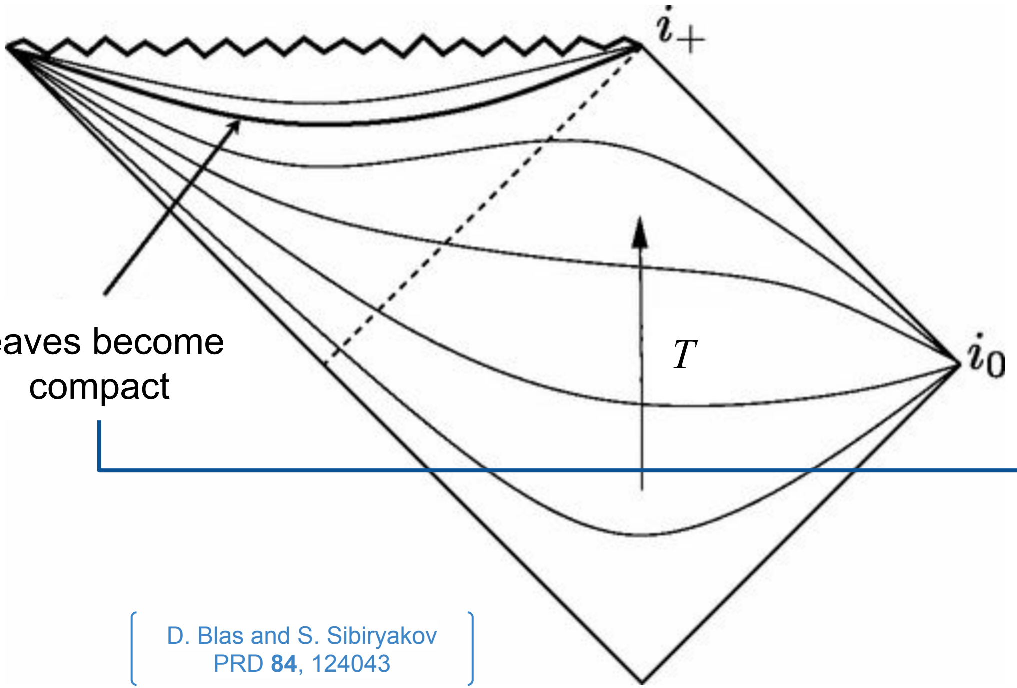
metric horizons
are permeable



Black Holes (!)

A spherically symmetric surprise!

[sph. sym. ⇒ h.o.]



leaves become compact

D. Blas and S. Sibiryakov
PRD **84**, 124043

universal horizon
(UH)

traps all future-directed causal curves

similar to GR horizons:

- quasi-local characterisation

$$u_\mu \, \chi^\mu \big|_{\mathrm{UH}} = 0$$

- mechanics
- Hawking radiation

How general is this? **?**

Introducing rotation

In h.o. case, UHs exist
(though no example apart from spherical symmetry)

Bhattacharyya, Colombo, Sotiriou, Class. Quantum Grav. 33, 235003 (2016) [1509.01558]

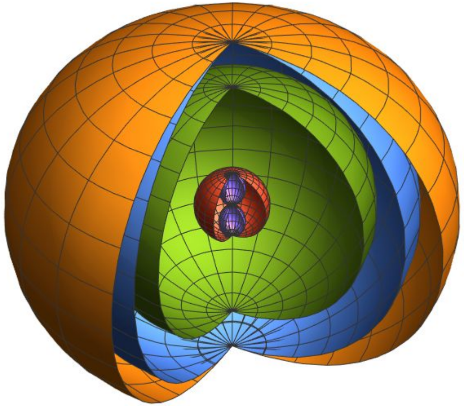
What about non-h.o. case?

We found:
analytical solution
in corner of parameter space

Franzin, Liberati, **JM**
PRD **109**, 084028
[2312.06891]

stealth solution:
$g_{\mu\nu}$ = Kerr, $u_\mu = \ldots$

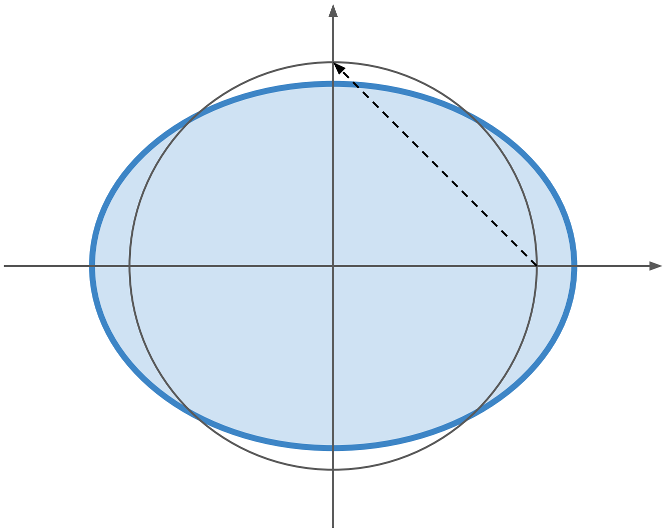
æther has twist

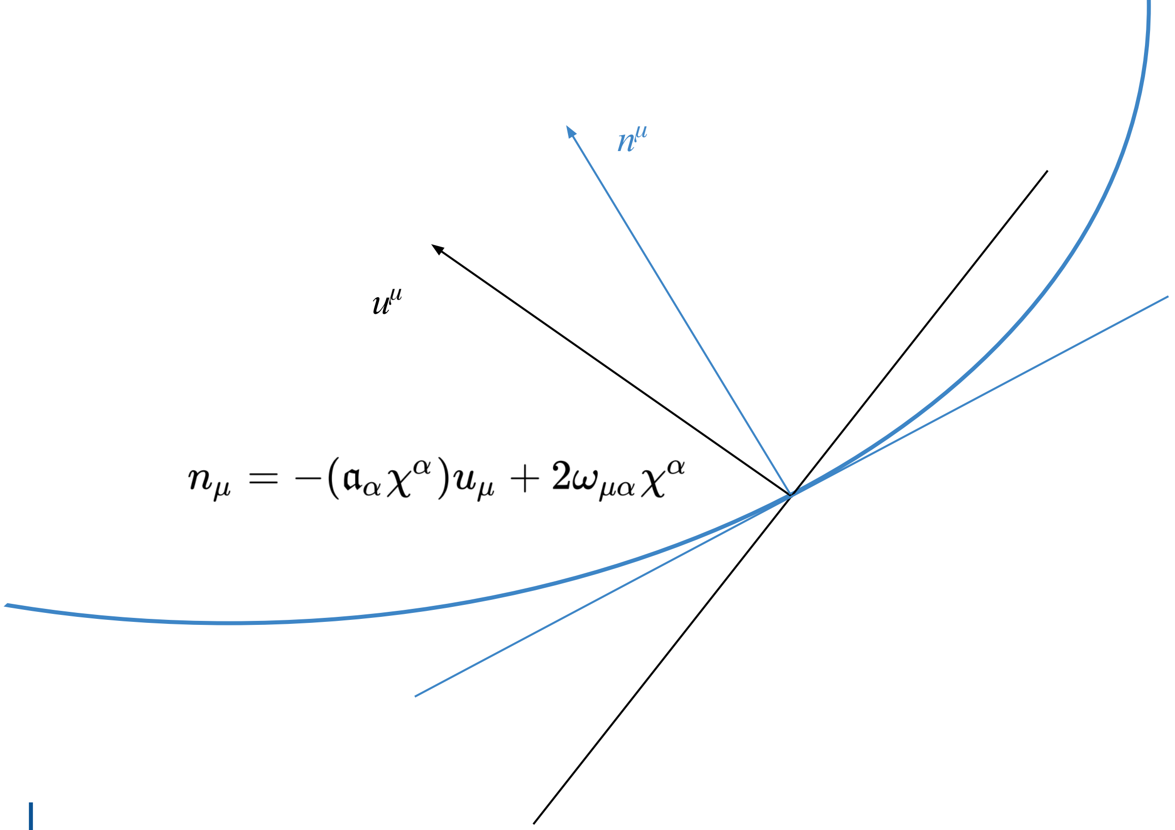


has 'candidate' UH
(we called it quasi-UH)

Quasi UH

QUH not a
causal boundary





$$n_\mu = -(\mathfrak{a}_\alpha \chi^\alpha) u_\mu + 2\omega_{\mu\alpha}\chi^\alpha$$

QUH not orthogonal to u_μ

it can be escaped following a
causal curve

Quasi UH

Still…

1 Better understanding of UHs in non-h.o. setting:

they can exist, but ‘fragile’

[Nathan W. Pyle]

2 QUHs might still be interesting phenomenologically

escaping is ‘difficult’:
need high (group) velocity in a particular direction

momentum always directed ‘inwards’, but not tangent to trajectories

(great example of differences between phase/group/front velocity)

work in progress… (Del Porro, Liberati, **JM**
???)

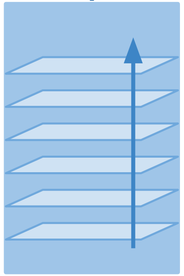
Upshot

Modified DRs change notion of causality

BHs not black
in general

sometimes, BHs are 'saved'

universal horizon



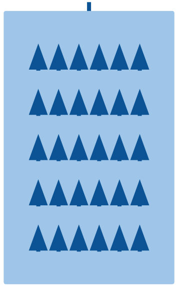
preferred foliation:

yes

preferred threading

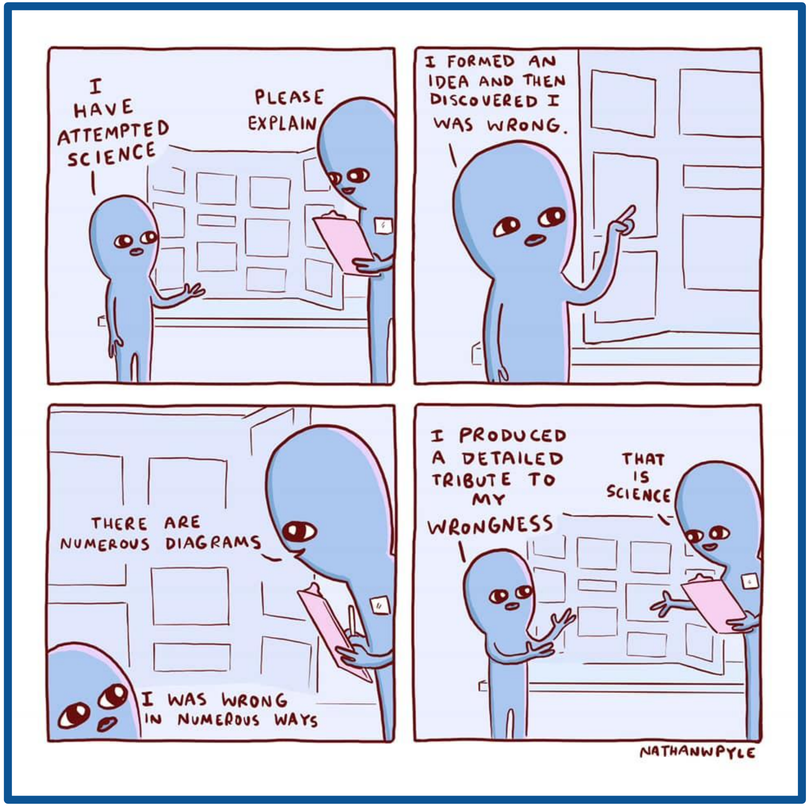
yes-ish

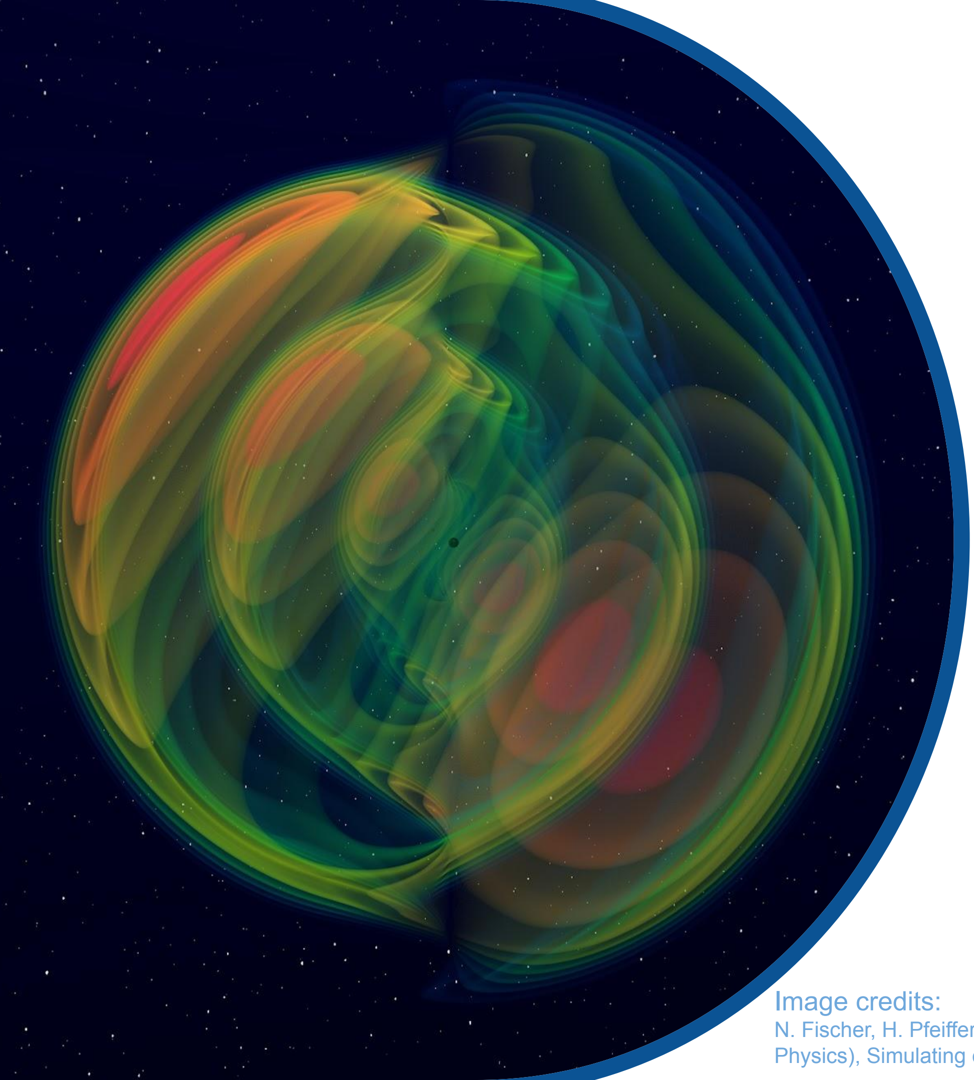
twist must vanish on UH



(stability?)

(phenomenology?)





Thanks!

Get in touch

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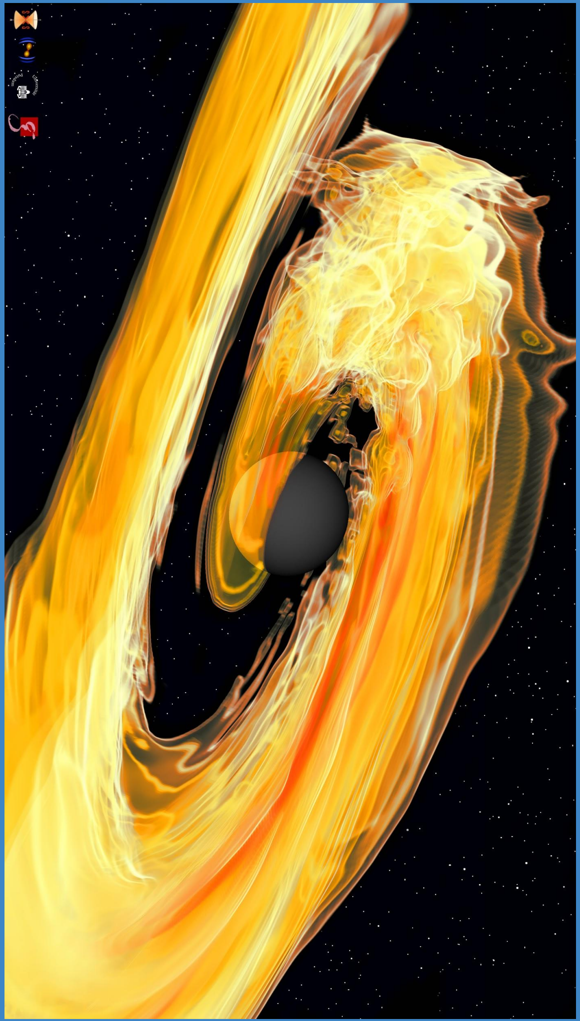
Image credits:
N. Fischer, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics), Simulating eXtreme Spacetimes (SXS) Collaboration

Backup Slides



Stealth solution

$$c_\sigma = c_\omega = c_\mathfrak{a} = 0$$

'minimal æ-theory'

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left\{R - \left[\frac{1}{3}c_\vartheta \vartheta^2 + c_\sigma \sigma^2 + c_\omega \omega^2 - c_\mathfrak{a} \mathfrak{a}^2\right]\right\}$$

choose Kerr
(Ricci flat)

solve

$$\begin{cases} \nabla_\mu u^\mu = 0 \\ u_\mu u^\mu = -1 \end{cases}$$

underdetermined problem

solution contains some arbitrariness

N.B.

Other 'minimal' choices possible,
this is the simplest one that is non trivial

Metric

Kerr in Boyer–Lindquist coordinates

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)\mathrm{d}t^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2 - \frac{4Mra\sin^2\theta}{\Sigma}\mathrm{d}t\mathrm{d}\varphi + \frac{A\sin^2\theta}{\Sigma}\mathrm{d}\varphi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2Mr + a^2$$
$$A = (r^2 + a^2)^2 - \Delta a^2\sin^2\theta$$

Æther

Lie-dragged along Killing vectors

$$u_\mu(x^\alpha) = u_\mu(r,\theta)$$

One simple solution is

$$u_\mu = \left\{ \mp\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Delta}, 0, 0 \right\}_\mu$$

$$u^\mu = \left\{ \pm\frac{A}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}}, -\frac{M^2\Theta}{\Sigma}, 0, \pm\frac{2Mar}{\Delta\Sigma}\sqrt{\frac{\Sigma\Delta + M^4\Theta^2}{A}} \right\}^\mu$$

$u_\theta = 0$

$u_\varphi = 0$

'ZAMO' solution
more or less required

$$\omega_{\mu\nu} \neq 0,\ \sigma_{\mu\nu} \neq 0,\ \mathfrak{a}_\mu \neq 0$$

$\Theta(\theta)$ free function of angle

Fixing $\boldsymbol{\Theta}$

Many possibilities…

One interesting option:

define

$$r_{\mathrm{QUH}}(\theta): \quad \partial_r(\Delta\Sigma)|_{r=r_{\mathrm{QUH}}} = 0$$

then

$$M^4\Theta^2 = -\Delta\Sigma|_{r=r_{\mathrm{QUH}}}$$

so that

$$(u_\mu \chi^\mu)|_{r=r_{\mathrm{QUH}}} = 0$$

(also, pick the ± so that it changes sign)

r_{QUH} is a perfect candidate for a UH

(‘Q’ stands for ‘quasi’)

Probing the QUH

Use toy model of matter with non-linear DRs

test scalar, with modified KG eq.

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{\Lambda^2}\left(p^{\mu\nu}\nabla_\mu\nabla_\nu\right)^2\phi = 0$$

WKB approximation

$$\phi\left(x^\alpha\right) = A\left(x^\alpha\right)e^{iS\left(x^\alpha\right)}$$
$$\nabla_\mu A, \nabla_\mu\nabla_\nu S, \cdots \ll \partial_\mu S =: k_\mu$$

Three (?) equations

dispersion relation (DR)

$$k_\mu k^\mu + \frac{1}{\Lambda^2}\left(p^{\mu\nu}k_\mu k_\nu\right)^2 = 0$$

conservation of Killing energy

$$k_\mu \chi^\mu = -\Omega$$

conservation of Killing angular momentum

$$k_\mu \psi^\mu = m$$

Disformal transformations

The field redifinition

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - D\, u_\mu u_\nu$$
$$\tilde{u}_\mu = \frac{u_\mu}{\sqrt{1+D}}$$

is an internal map of æ-theory
(and T-theory)

$$\tilde{c}_\vartheta = (1+D)c_\vartheta - 2D$$
$$\tilde{c}_\sigma = 1 + (1+D)(c_\sigma - 1)$$
$$\tilde{c}_\omega = 1 + \frac{c_\omega - 1}{1+D}$$
$$\tilde{c}_\mathfrak{a} = c_\mathfrak{a}$$

Disforming
stealth Kerr,

we get new solutions
(of ‘different’ æ-theories)

resulting metric

- depends on D (3-param family)
- is non-stealth
- is non-circular
- its (metric) horizons not Killing

…