Space-time symmetry breaking and the mystery of dark matter

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Full 4D spacetime diffeomorphism invariance is a staple of physics post General Relativity.

- Physics is invariant under coordinate transformations involving not only time to time coordinates and space to space coordinates, but mixing the two:

$$x^\mu \to x^{\mu\prime} = x^{\mu\prime}(x^\mu)$$

$$x^\mu = (t, \mathbf{x})$$
$$x^{\mu\prime} = (t', \mathbf{x}')$$

- Hence, after GR, a new word entered the dictionary:

"spacetime"

(instead of the proverbial space and time).

This has tremendous philosophical and foundational implications:

- Philosophical implications, obviously: we are ontologically identifying space and time, leading to issues like the “block Universe”, the flow of time as an illusion, etc, etc. (See Smolin and Unger).
- It also has strong technical

implications in Quatum Gravity:

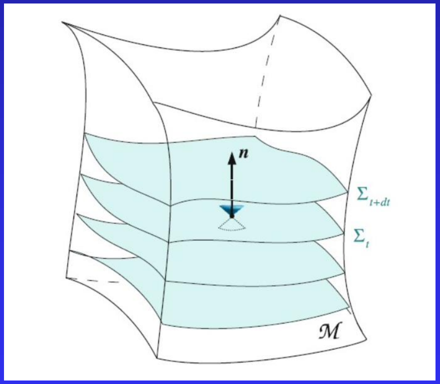
the problem of time

See Isham’s excellent review…

C. J. Isham, NATO Sci. Ser. C **409**, 157-287 (1993) [arXiv:gr-qc/9210011 [gr-qc]].

The purpose of this talk:

- First of all, I won’t be dwelling on quantization.
- Instead I am going to examine the classical implications of breaking this sacred symmetry:
  - Introduce a foliation, i.e. a split 4=3+1:
    (just as one does in the Hamiltonian formalism, ADM, etc. purely as a mathematical device)



  - Make this foliation count in terms of physical effects by breaking the 4D symmetry into a a 3D symmetry plus a global time reparameterization invariance (i.e. the residual symmetries that preserve the foliation).

Giving the punchline away

(before you fall asleep in this hot weather…)

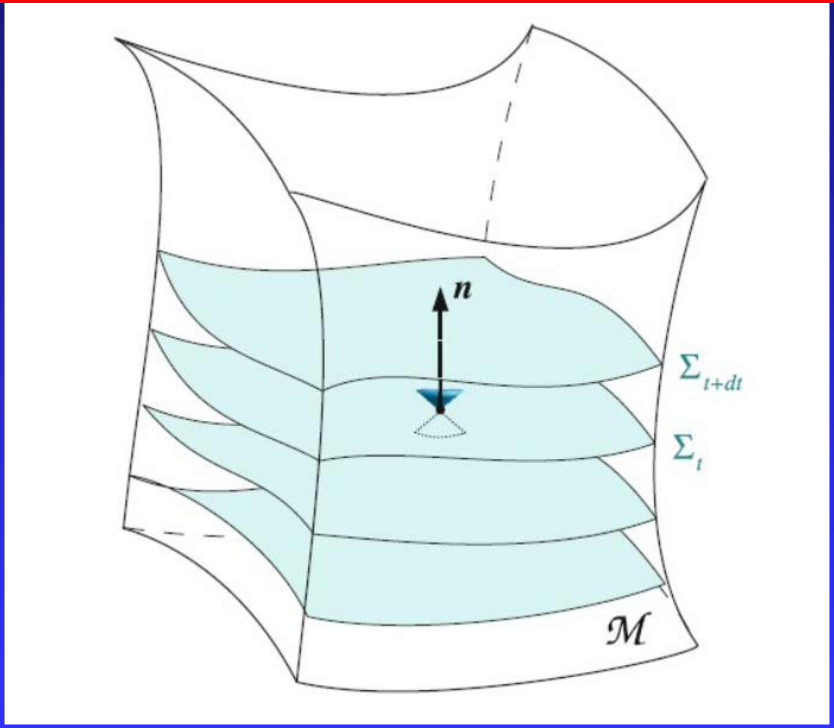
- If you
  - break the symmetry in one regime (e.g., in the early Universe, at high energy, etc)
  - and then restore it (after a phase transition, at low energy, etc),

  you generally are left over with some permanent damage.
- This permanent blemish is equivalent to a theory with full symmetry restoration and a dark matter component.
- (Not a panacea, but it could be an element of the dark matter puzzle.)

What I am about to say is pretty general

- In fact it only depends on the Dirac Hypersurface Deformation Algebra (a representation of 4D diffs) being broken and then restored.
- But there is no shortage of examples. Some very mainstream some off the beaten track:
  - Horava Lifshitz theory (a renormalizable UV completion of GR).
  - Evolution in the laws of physics
  - Global or Machian interactions.

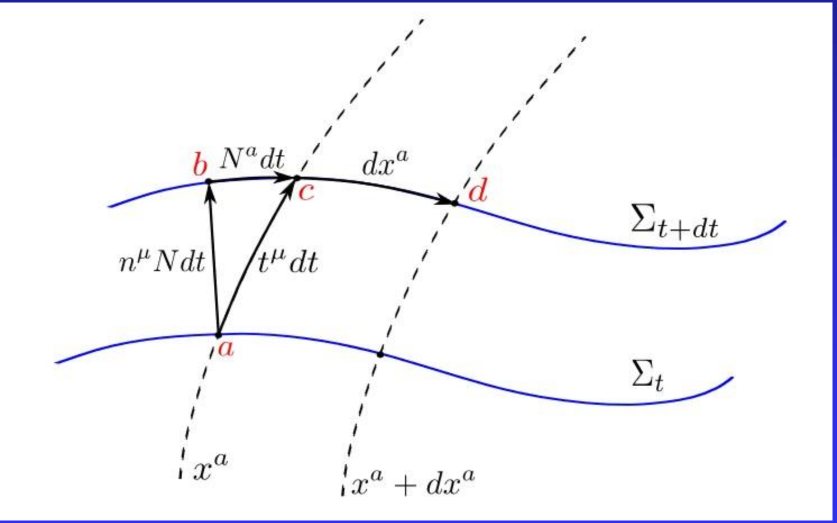
Let's imagine we have a preferred fixed (non-dynamical) foliation



- Obviously from a causal point of view this will feel like a pre-cog situation (cf. Coleman and Lambda; the sequester model, etc)

A quick lecture in ADM formalism

- Start from a space-like foliation (globally hyperbolic)
- Elements:
  - Metric:
    - Lapse, 00, N
    - Shift, 0i, N^i
    - Intrinsic metric, ij, h_{ij}
  - Geometry
    - Intrinsic curvature: Riemann and Co. of h_{ij}
    - Extrinsic curvature (roughly the time derivative of h_{ij})
    - Acceleration (of the normal) $a_\mu = n^\nu \nabla_\nu n_\mu = h_\mu^{\ \nu} \nabla_\nu \ln N$



Decomposing the 4D action into 3+1, if there is 4D diff invariance:

- We find for the metric:
  - Dynamical variables spatial metric (or projector) h_{ij}
  - Lagrange multipliers lapse N, shift N^i,

$$\mathcal{H}_E = N\mathcal{H} + N^i\mathcal{H}_i$$

  - Thus we get 4 constraints:
    - The Hamiltonian constraint: $\mathcal{H} = 0$
    - The Momentum constraint: $\mathcal{H}_i = 0$

The fact that the theory is pure constraint, implies that:

- Its consistency boils down to the constraints closing under the Poisson bracket (i.e. they form an algebra)
- Then, the constraints are preserved by the evolution.
- (This is first class instead of second class constraints.)

Yes, they do close, as enshrined in the Dirac Hypersurface Deformation algebra:

- Algebra (using smoothed constraints)

$$\{H_i(f^i), H_j(g^j)\}_L = H_i([f,g]^i)$$
$$\{H_i(f^i), H(g)\}_L = H(f^i \partial_i g)$$
$$\{H(f), H(g)\}_L = H_i(h^{ij}(f\partial_j g - g\partial_j f))$$

$$H(f) = \int d^3y\, f(y)\mathcal{H}(y)$$
$$H_i(f^i) = \int d^3y\, f^i(y)\mathcal{H}_i(y)$$

- (Small hiccup but that's OK: they are an "algebroid" or "business class")

This can be understood geometrically; the constraints generate 4D diffeomorphisms…

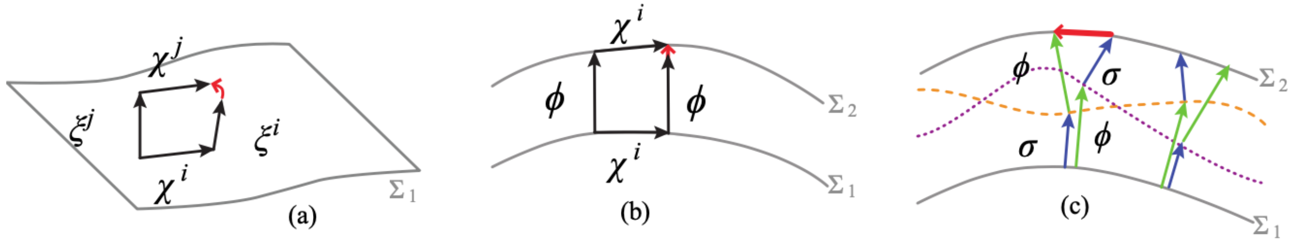


Figure 4: Geometric interpretation of constraints: (a) diffeomorphism constraint, (b) diffeomorphism + Hamiltonian constraints, (c) Hamiltonian constraint [1].

- But otherwise this is just a complicated way to prove that the time derivative of zero is zero.

This is a general property of theories with 4D diff invariance… and by the negative:

- In theories where 4D diffs are degraded to 3D diffs one loses the local Hamiltonian constraint (one is only left with a global Hamiltonian constraint)
  - This happens in Horava-Lifshitz theory.
  - It happens if there is evolution or time dependence in laws of physics.
- But suppose that this is first broken and then restored.

Then, the theory with newly-found 4D diff invariance has a leftover Hamiltonian from a broken phase

- For example:

$$\dot{\mathcal{H}} = \{\mathcal{H}, \mathbf{H}\}_L + \{\mathcal{H}, \mathbf{H}\}_{NL}$$
$$\dot{\mathcal{H}}_i = \{\mathcal{H}_i, \mathbf{H}\}_L + \{\mathcal{H}_i, \mathbf{H}\}_{NL} = 0$$

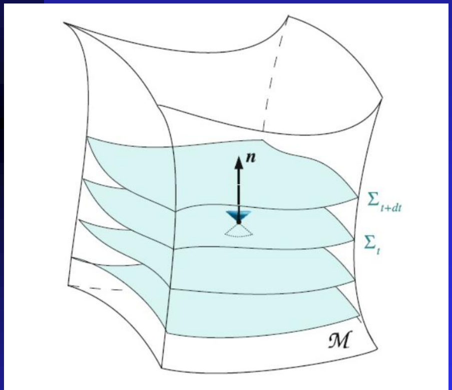
$$\mathcal{H}(x, t_0) \neq 0$$
$$\mathcal{H}_i(x, t_0) = 0$$

- So we have to evolve this according to the Dirac Hypersurface Deformation Algebra

$$\dot{\mathcal{H}} = \{\mathcal{H}, \mathbf{H}\}_L = \partial_i(N^i \mathcal{H}) + \partial_i(\mathcal{H}^i N) + \mathcal{H}^i \partial_i N$$
$$\dot{\mathcal{H}}_i = \{\mathcal{H}_i, \mathbf{H}\}_L = \mathcal{H}\, \partial_i N + D_j(N^j \mathcal{H}_i) + \mathcal{H}_j D_i N^j$$
$$= \mathcal{H}\, \partial_i N + \partial_j(N^j \mathcal{H}_i) + \mathcal{H}_j \partial_i N^j,$$

What follows next depends crucially on a little detail:

- Is the preferred foliation geodesic or not:



$$a_\mu = n^\nu \nabla_\nu n_\mu = h_\mu^\nu \nabla_\nu \ln N$$

- Or equivalently, is the lapse function non-space dependent or not? $N = N(t)$

You can see why…

- If the lapse function has spatial gradients there is no way the momentum constraint cannot be violated, even if it wasn't during the symmetry breaking phase.

$$\dot{\mathcal{H}} = \{\mathcal{H}, \mathbf{H}\}_L = \partial_i(N^i\mathcal{H}) + \partial_i(\mathcal{H}^i N) + \mathcal{H}^i\partial_i N$$
$$\dot{\mathcal{H}}_i = \{\mathcal{H}_i, \mathbf{H}\}_L = \mathcal{H}\,\partial_i N + D_j(N^j\mathcal{H}_i) + \mathcal{H}_j D_i N^j$$

In either case: There is a price for full symmetry restoration is a new form of "matter"

- This permanent blemish can be absorbed into a postulated new matter component.

$$\mathcal{H} = -\mathcal{H}_m \neq 0 \implies \bar{\mathcal{H}} = \mathcal{H} + \mathcal{H}_m \approx 0$$
$$\mathcal{H}_i = -\mathcal{H}_i^m \neq 0 \implies \bar{\mathcal{H}}_i = \mathcal{H}_i + \mathcal{H}_i^m \approx 0$$

$$\bar{S}_0 = S_0 + S_m$$

$$N = N(t, x)$$
$$N^i = N^i(t, x)$$

There is a general requirement on this "effective matter" form:

- The new form of matter must follow the evolution dictated by the Dirac algebra:

$$\dot{\mathcal{H}}_m = \partial_i(N^i\mathcal{H}) + \partial_i(\mathcal{H}^i N) + \mathcal{H}^i\partial_i N$$
$$\dot{\mathcal{H}}_i^m = \mathcal{H}_m\,\partial_i N + D_j(N^j\mathcal{H}_i^m) + \mathcal{H}_j^m D_i N^j$$

- … and it just so happens that a generic form of matter does not follow these rules, bur rather:

$$\dot{\mathcal{H}}_m = \{\mathcal{H}_m, \bar{\mathbf{H}}\}_L = \{\mathcal{H}_m, \mathbf{H}_m\}_{q_m,p_m} + \frac{\delta\mathcal{H}_m}{\delta h_{ij}}\dot{h}_{ij}$$
$$\dot{\mathcal{H}}_i^m = \{\mathcal{H}_i^m, \bar{\mathbf{H}}\}_L = \{\mathcal{H}_i^m, \mathbf{H}_m\}_{q_m,p_m} + \frac{\delta\mathcal{H}_i^m}{\delta h_{ij}}\dot{h}_{ij}$$

- More concretely, this is what happens (known from the 1990 QG literature)

$$\{H_i^m(N^i), H_m(N)\} = H_m(N^i\partial_i N) - \{H_i^G(N^i), H_m(N)\}$$

$$\{H_i^m(N^i), H_m(N)\} = H_m(N^i\partial_i N) + \int dx \, \frac{\delta \mathcal{H}_m}{\delta h_{ij}} N(D_i N_j + D_j N_i)$$

$$\dot{\mathcal{H}}_m = [\text{required}] - 2N \frac{\delta \mathcal{H}_m}{\delta h_{ij}} K_{ij}$$

- So, crucially we get the requirement

$$\frac{\delta \mathcal{H}_m}{\delta h_{ij}} = 0$$

The “active” gravity of the new form of matter can be found from its s.e.t.:

- This can be found from the stress energy tensor:

$$T^m_{00} = \frac{-2}{N\sqrt{h}}\frac{\delta S_m}{\delta g^{00}} = \frac{N}{\sqrt{h}}(N\mathcal{H}_m + 2N^i\mathcal{H}^m_i),$$
$$T^m_{0i} = \frac{-2}{N\sqrt{h}}\frac{\delta S_m}{\delta g^{0i}} = \frac{N}{\sqrt{h}}\mathcal{H}^m_i,$$
$$T^m_{ij} = 0,$$

$$\frac{\delta \mathcal{H}_m}{\delta h_{ij}} = 0$$

- Generally the stress-energy tensor can be written "covariantly" as:

$$
\begin{aligned}
T^m_{\mu\nu} &= \rho n_\mu n_\nu - 2\Pi^\alpha n_{(\mu} h_{\nu)\alpha} \\
&= \rho n_\mu n_\nu - (n_\mu \Pi_\nu + n_\nu \Pi_\mu)
\end{aligned}
$$

where:

$$
\rho = n_\mu n_\nu T^{\mu\nu}_m = \frac{\mathcal{H}_m}{\sqrt{h}}
$$

$$
\Pi^\mu = h^{\mu\alpha} n^\nu T^m_{\alpha\nu} = \left(0, \frac{\mathcal{H}^i_m}{\sqrt{h}}\right)
$$

What kind of “matter” is this?

- Perhaps not surprisingly, this is not quite like normal matter (remember it is the degrees of freedom of gravity and normal matter that are driving the evolution of their non-vanishing total Hamiltonian and Momentum)
- Some quirks are inevitable…
- They depend crucially on whether Σ_t is geodesic or not

What kind of matter is this? (Part I: geodesic preferred frame)

- If the momentum constraint is never violated this is just a dust fluid (Cold Dark Matter?)

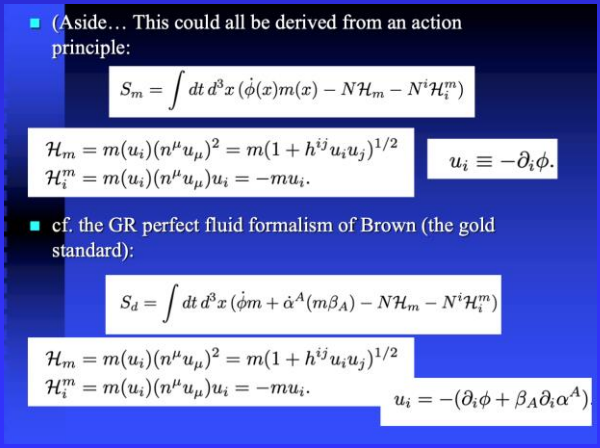
$$T^m_{\mu\nu} = \rho_m(x) n_\mu n_\nu$$

- …with 4-velocity aligned with the preferred frame Σ_t

So there is nothing weird about that, right?

- Wrong! See the small print:

------- small print:



- (Aside… This could all be derived from an action principle:

$$S_m = \int dt\, d^3x\, (\dot{\phi}(x) m(x) - N\mathcal{H}_m - N^i \mathcal{H}^m_i)$$

$$\mathcal{H}_m = m(u_i)(n^\mu u_\mu)^2 = m(1 + h^{ij} u_i u_j)^{1/2}$$
$$\mathcal{H}^m_i = m(u_i)(n^\mu u_\mu) u_i = -m u_i.$$

$$u_i \equiv -\partial_i \phi.$$

- cf. the GR perfect fluid formalism of Brown (the gold standard):

$$S_d = \int dt\, d^3x\, (\dot{\phi} m + \dot{\alpha}^A (m\beta_A) - N\mathcal{H}_m - N^i \mathcal{H}^m_i)$$

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$$u_i = -(\partial_i \phi + \beta_A \partial_i \alpha^A)$$

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J. D. Brown, "Action functionals for relativistic perfect fluids," *Class. Quant. Grav.* **10** (1993), 1579–1606, gr-qc/9304026.

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$$u_i = -(\partial_i\phi + \beta_A \partial_i \alpha^A)$$

Here α^A and β_A $(A = 1, 2, 3)$ are 6 space-time scalars containing the hydrodynamical degrees of freedom.

Hence the equivalence with a perfect fluid is not complete

- Equivalent to a perfect fluid with 4 out of its 5 degrees of freedom frozen:
  - The entropy is frozen (equation of state of dust)
  - The hydrodynamical degrees of freedom are frozen (there is a preferred frame that is never erased).
- This is equivalent to imposing 4 first class constraints on the fluid (one fixing the dust equation of state, 3 identifying the 4-velocity and the normal of the preferred frame).

What kind of matter is this? (Part II: non-geodesic preferred frame)

- Things get significantly weirder if the preferred frame is non-geodesic. Recall:

$$
\begin{aligned}
T^m_{\mu\nu} &= \rho n_\mu n_\nu - 2\Pi^\alpha n_{(\mu} h_{\nu)\alpha} \\
&= \rho n_\mu n_\nu - (n_\mu \Pi_\nu + n_\nu \Pi_\mu)
\end{aligned}
$$

$$
\begin{aligned}
T^m_{00} &= \frac{-2}{N\sqrt{h}}\frac{\delta S_m}{\delta g^{00}} = \frac{N}{\sqrt{h}}(N\mathcal{H}_m + 2N^i\mathcal{H}^m_i), \\
T^m_{0i} &= \frac{-2}{N\sqrt{h}}\frac{\delta S_m}{\delta g^{0i}} = \frac{N}{\sqrt{h}}\mathcal{H}^m_i, \\
T^m_{ij} &= 0,
\end{aligned}
$$

- Hence we have a matter form which in one frame has energy density and momentum, but no pressure or any other spatial stresses.

In other words a completely new form of matter… take a moment to savour how odd this is:

- Note how, for example for dust, one always has pressure if there is momentum:

$$T^d_{\mu\nu} = \rho_d \gamma^2 n_\mu n_\nu + \rho_d \gamma (n_\mu w_\nu + n_\nu w_\mu) + \rho_d w_\mu w_\nu$$

$$u^\mu = \gamma n^\mu + w^\mu$$

- It is true that locally the s.e.t can be diagonalized to find a rest frame description: a fluid with anisotropic stresses (but that is a contrived description)

One thing is normal: The "passive" gravity is trivial

- The evolution dictated by the Dirac algebra amounts to conservation of stress-energy-momentum:

$$n_\mu \nabla_\nu T_m^{\mu\nu} = 0 \iff \dot{\mathcal{H}}_m = \partial_i(N^i \mathcal{H}_m) + \partial_i(\mathcal{H}^i_m N) + \mathcal{H}^i_m \partial_i N,$$
$$h_{\alpha\mu} \nabla_\nu T_m^{\mu\nu} = 0 \iff \dot{\mathcal{H}}^m_i = \mathcal{H}_m \,\partial_i N + \partial_j(N^j \mathcal{H}^m_i) + \mathcal{H}^m_j \partial_i N^j,$$

- For the case of dust this amounts to geodesic motion even for the ``effective dust"

What I am about to say is pretty general

- In fact it only deper rsurface Deformation Algeb f 4D diffs) being broken and then res

- But there is no shor ne very mainstream some off the beaten track:
  - Horava Lifshitz theory (a renormalizable UV completion of GR).
  - Evolution in the laws of physics
  - Global or Machian interactions.

Horava-Lifshitz theory is a good example

- Take spacetime apart, and mess with it so that it cannot be reassembled, but must remain space and time.
- For example, make lambda different from 1 in the UV then bring it back to 1 in the IR in the ADM action:

$$S_{HL} = \frac{M_{Pl}^2}{2} \int dt\, d^3x \sqrt{h} N \left[(K_{ij}K^{ij} - \lambda K^2) - \mathcal{V}[h_{ij}] \right]$$

- It is well known that one then loses the local Hamiltonian constraint:

S. Mukohyama, Phys. Rev. D **80**, 064005 (2009) doi:10.1103/PhysRevD.80.064005 [arXiv:0905.3563 [hep-

- As an aside, if the IR is not fully restored, you have violations of energy conservation:

$$-n_\mu \nabla_\nu T_m^{\mu\nu} = c_g^2 M_{Pl}^2 (\lambda - 1) D^2 K$$

(P.Bassani, JM., S.Mukohyama; in preparation)

- … so it is possible to end up with a truly strange form of dark matter. Even the "passive gravity" may be different.

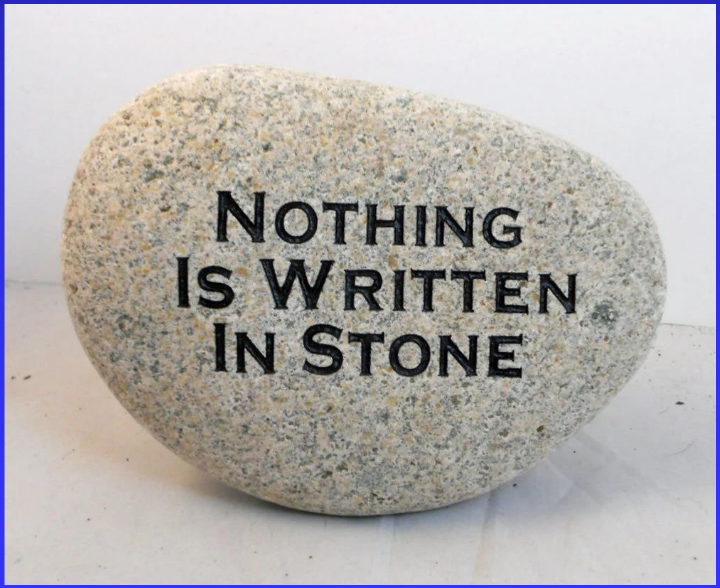
What I am about to  general

- In fact it only depends on the Deformation Algebra (a repr...) being broken and then restored.

- But there is no shortage of examples. Some very mainstream some off the beaten track:
  - Horava Lifshitz theory (a renormalizable UV completion of GR).
  - Evolution in the laws of physics
  - Global or Machian interactions.

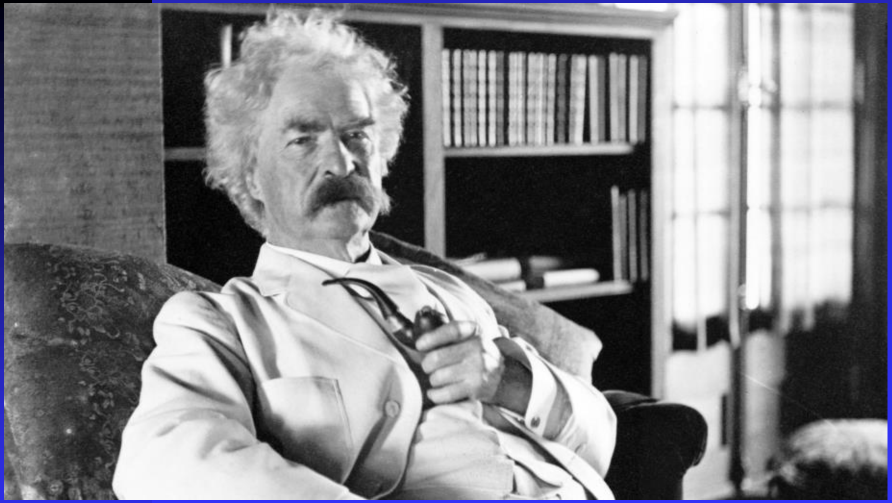
Could the laws of physics change in time?

- One might ask: what's the big deal? Our human "laws" do change after all (look at the Brexit mess), so why not the laws of the Universe?



Time is what stops everything from happening all at the same time

Mark Twain,
John Archibald Wheeler



Time can be seen as the conjugate of the constants of nature, if a foliation is defined.

- The example of unimodular gravity:
  - Make the cosmological “constant” a constant of motion, rather than a fixed parameter
  - Its canonical dual is a measure of time (given by 4-volume to the past, as defined by the foliation).

The covariant reformulation of unimodular gravity:

- Add to the standard action a new term

$$S_0 \to S = S_0 - \int d^4x \Lambda \partial_\mu T^\mu_U = S_0 + \int d^4x (\partial_\mu \Lambda) T^\mu_U$$

- The EOMs are the on-shell constancy of Lambda and a time formula:

$$\frac{\delta S}{\delta T^\mu_U} = 0 \implies \partial_\mu \Lambda = -\frac{\delta S_0}{\delta T^\mu_U} = 0,$$

$$\frac{\delta S}{\delta \Lambda} = 0 \implies \partial_\mu T^\mu_U = \frac{\delta S_0}{\delta \Lambda} = -\frac{\sqrt{-g}}{8\pi G_N}.$$

- This can be done with any “constant”, alpha, and classically there is nothing new…

Time evolution in the laws of physics

- But if instead there is time dependence it all changes…If the Hamiltonian depends on such times, the conjugate "constants" are no longer constant.

$$\begin{aligned}\dot{\alpha} &= \{\alpha, \mathbf{H}\} = \frac{\partial \mathcal{H}_T}{\partial T_\alpha} \\ \dot{T}_\alpha &= \{T_\alpha, \mathbf{H}\} = -\frac{\partial \mathcal{H}_T}{\partial \alpha}\end{aligned}$$

- And the Hamiltonian stops being a constant of motion (reflecting the diff degradation from 4D to 3D):

$$\mathcal{H}(t,x) = \int_C \left(\frac{\partial \mathcal{H}}{\partial \alpha}\dot{\alpha} + \frac{\partial \mathcal{H}}{\partial T_\alpha}\dot{T}_\alpha \right)$$

- Indeed this results from an interesting Hamiltonian structure, separating local and global variables:

$$
\begin{aligned}
S &= \int dt\, V_\infty \dot{\alpha} T_\alpha + S_0 \\
&= \int dt \left[V_\infty \dot{\alpha} T_\alpha + \int_{\Sigma_t} d^3x \left(\dot{q}(x) p(x) - \mathcal{H}_E \right) \right]
\end{aligned}
$$

- The Poisson bracket breaks into local vs global components:

$$
\begin{aligned}
\{q(x), p(y)\} &= \delta(x, y) \\
\{\alpha, T_\alpha\} &= \frac{1}{V_\infty}.
\end{aligned}
$$

$$
\{f, g\}_{NL} = \frac{1}{V_\infty} \sum_{(\alpha, T_\alpha)} \frac{\partial f}{\partial \alpha} \frac{\partial g}{\partial T_\alpha} - \frac{\partial f}{\partial T_\alpha} \frac{\partial g}{\partial \alpha}
$$

$$
\{f, g\} = \{f, g\}_L + \{f, g\}_{NL}
$$

- And the Hamiltonian and Momentum evolution take the form I introduced earlier:

$$
\begin{aligned}
\dot{\mathcal{H}} &= \{\mathcal{H}, \mathbf{H}\}_L + \{\mathcal{H}, \mathbf{H}\}_{NL} \\
\dot{\mathcal{H}}_i &= \{\mathcal{H}_i, \mathbf{H}\}_L + \{\mathcal{H}_i, \mathbf{H}\}_{NL}
\end{aligned}
$$

More generally, this is an example of global interactions

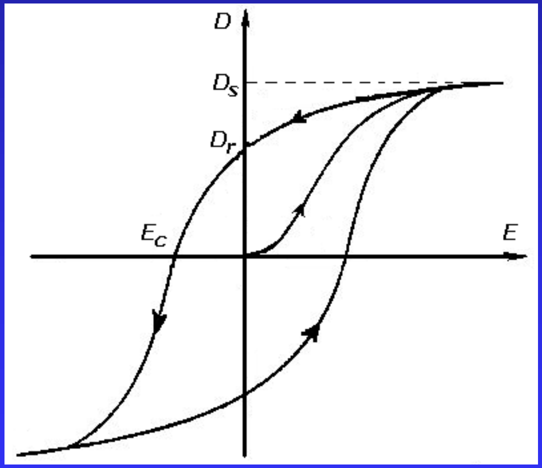
- Not just a non-local dependence on local variables (i.e. through derivatives higher than second).
- Hence it bypasses many of the pathologies of global interactions

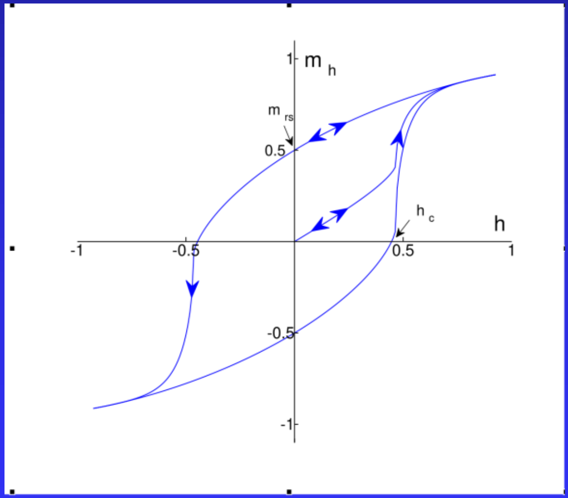
Conclusion:

- Here’s a reverse example of topological defects (spontaneous symmetry breaking, with leftovers from a phase with full symmetry).
- Here we restore a symmetry, but the broken phase leaves a legacy.
- The legacy could play a role in the dark matter conundrum. WATCH THIS SPACE…

Conclusion: A hysteresis effect in cosmology!

- The physics here and now depends on the full past history.
- This happens for example with magnetic materials in physics:





- But it is an ubiquitous effect in everything else… ecology, economy, human life in general, etc, etc, etc.

The arithmetic of the number of degrees of freedom

- Dirac taught us that the number of degrees of freedom of a theory is the phase space dimension divided by 2 minus the number of first class constraints.

- This is where the scalar graviton goes: not a graviton at all but a constrained form of cold dark matter.