

# Merge many times

**Davide Gerosa**  
University of Milano-Bicocca

davide.gerosa@unimib.it  
www.davidegerosa.com

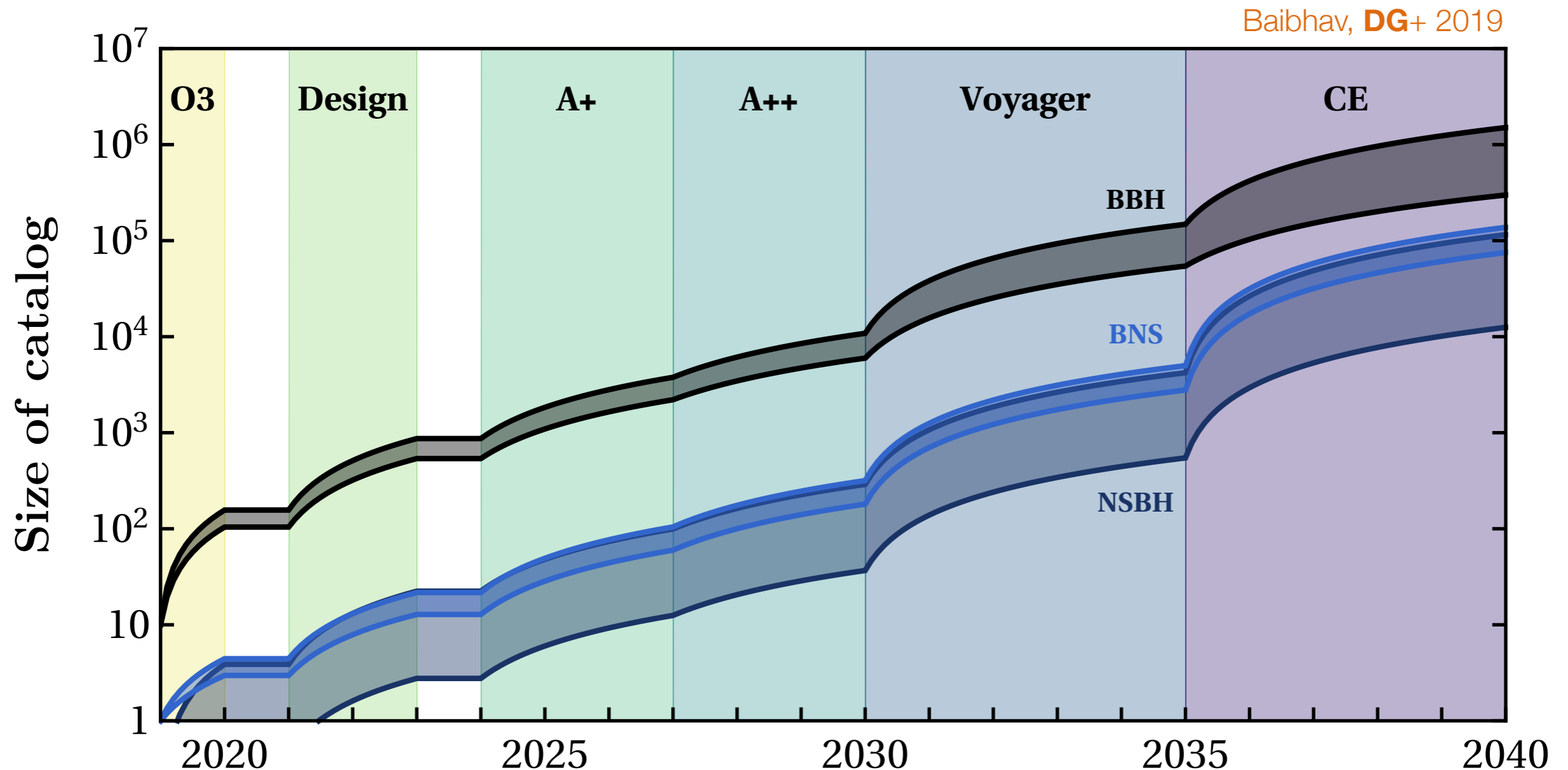
Jul 26th, 2024  
EREP 2024  
Coimbra, Portugal



# The best is yet to come

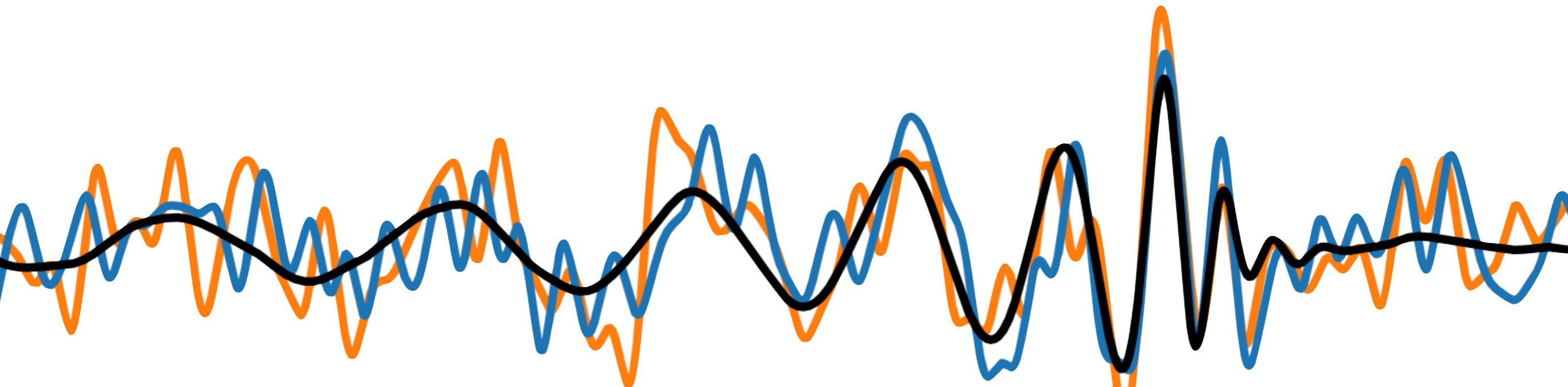
About 100 surprises so far, thousands in ~5 yrs, millions in ~20 years

- Large statistics: details emerging at the **population** level
- Many events means **rare outliers**



# Outline

- **Astro:** formation channels
- **Stats:** Population analyses
- **Relativity:** Hierarchical mergers



# Can black holes really make it?

Power emitted in gravitational waves:

$$\frac{da}{dt} = -\frac{64 G^3 M^3}{5 c^5 a^3} \frac{q}{(1+q)^2}$$

Peters 1964



**GW-driven inspiral timescale**

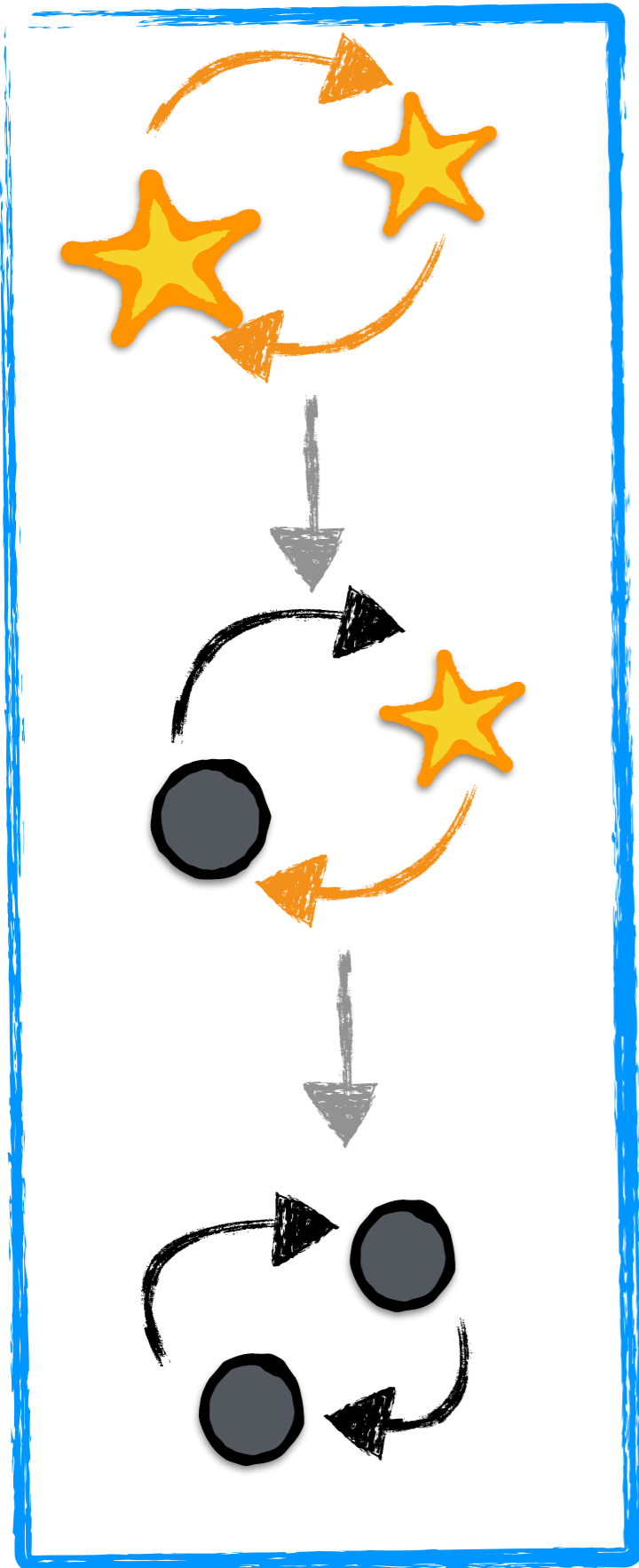
$$t_{\text{GW}} \sim a \frac{dt}{da} \sim a^4$$

**Gravitational waves** are efficient below

$$a_{\text{GW}} = 1.2 \times 10^{11} \left( \frac{t_{\text{GW}}}{1.4 \times 10^{10} \text{yr}} \right)^{1/4} \left( \frac{M}{M_{\odot}} \right)^{3/4} \text{cm} \sim 10 R_{\odot} \quad \text{for stellar-mass BHs}$$

**Relativity** alone cannot explain the LIGO events,  
we need some **astrophysics**

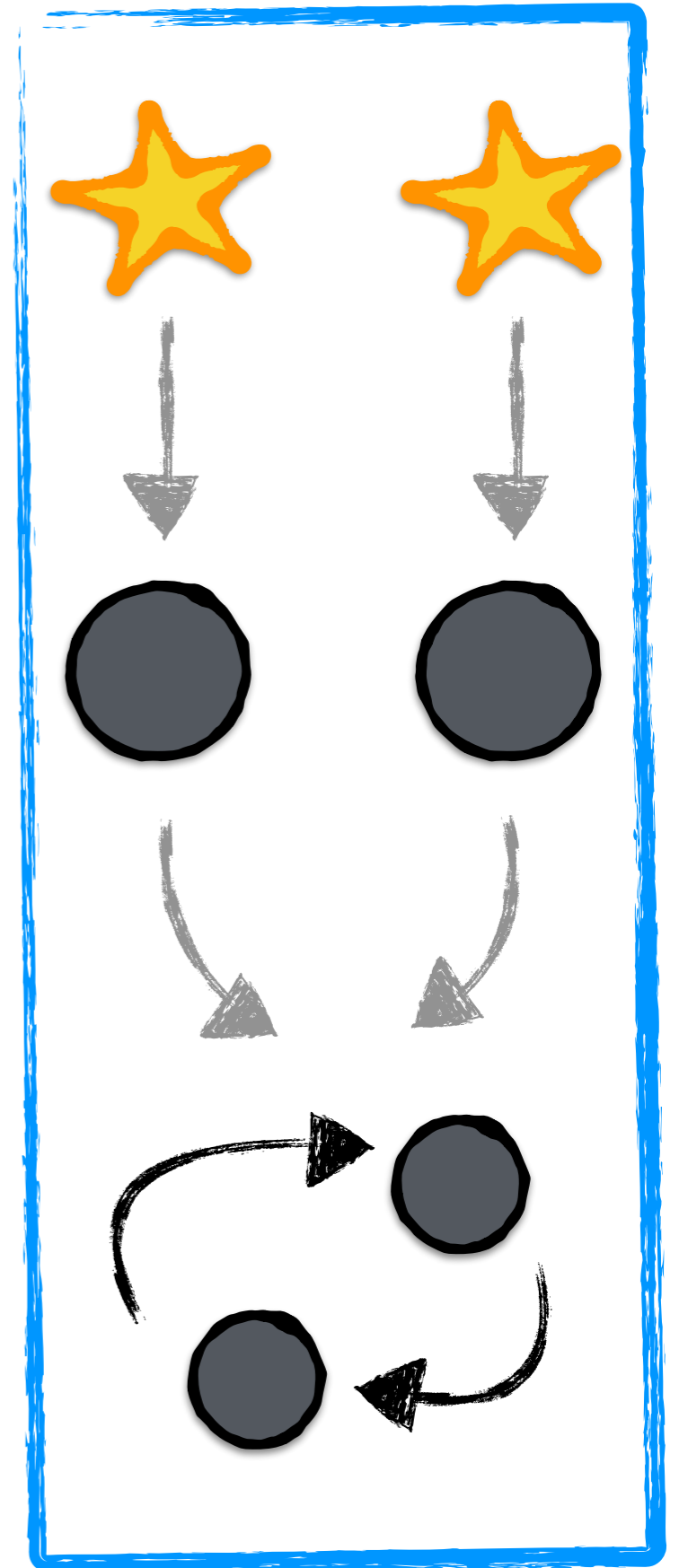
# Have we been together for so long?



**Yes! I've known you  
since you were a star**



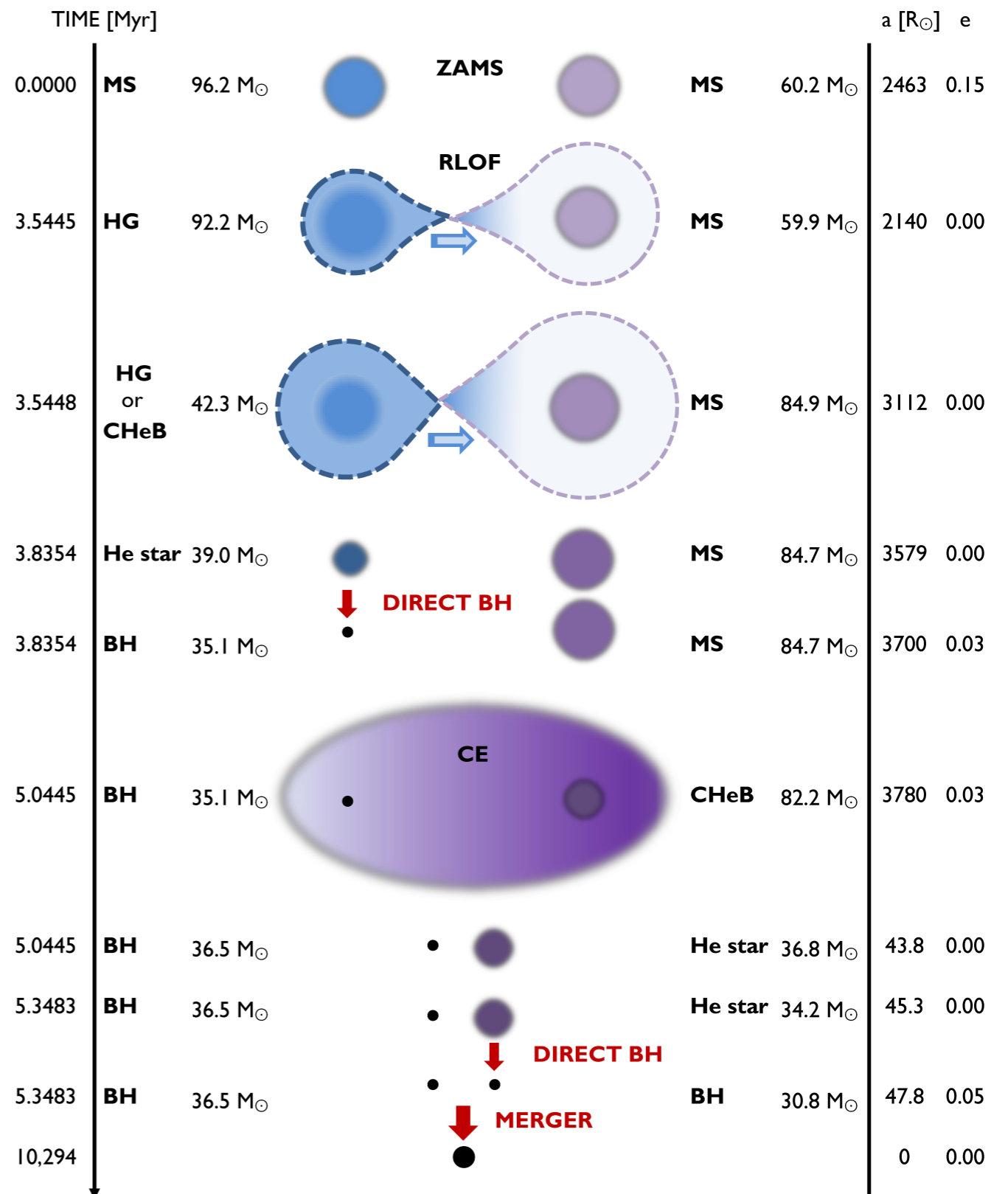
**Don't you remember?  
We just met in cluster**



# Massive stars to BHs: *field* evolution

Belczynski+2016

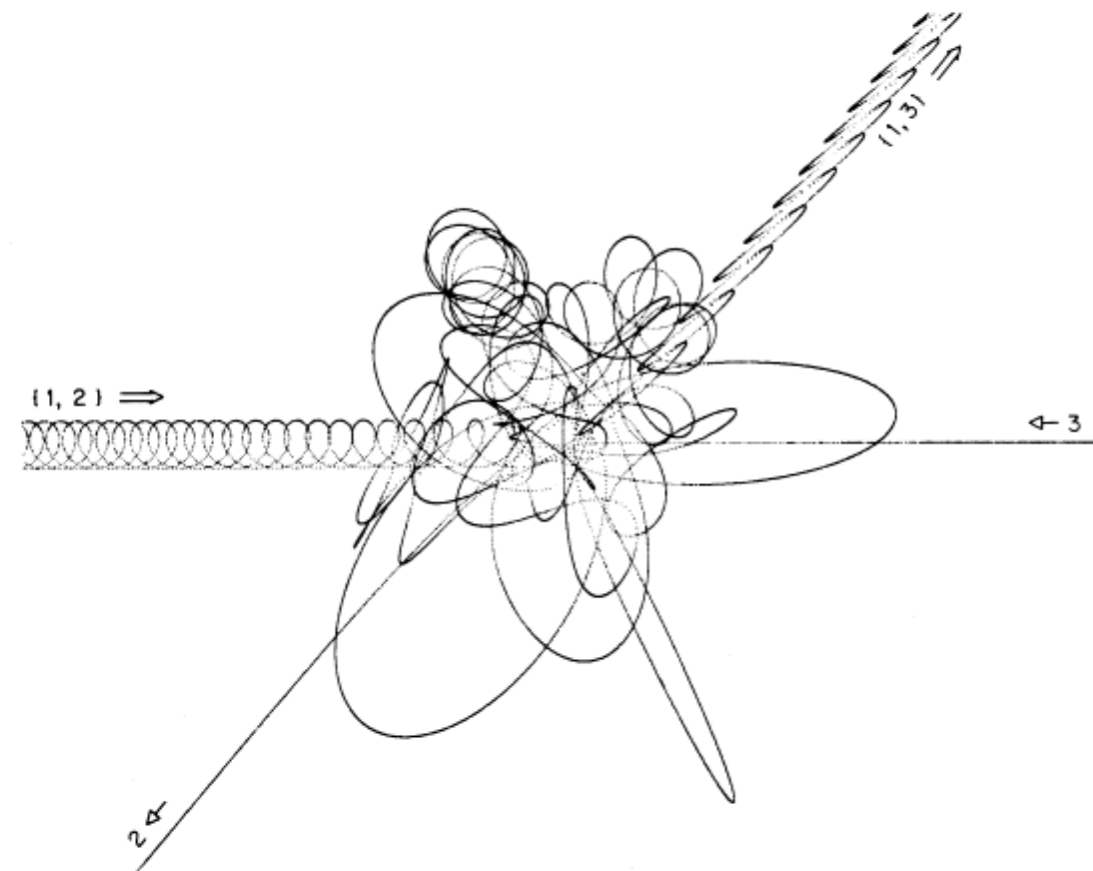
1. **Main-sequence** binary star
2. First evolves to supergiant:  
Roche-lobe overflow, **mass transfer**
3. First goes **supernova** and forms a BH  
*Is it still a binary?*
4. Second evolves to supergiant:  
**common envelope**  
Must be efficient... Critical stage to bring the separation down!  
... but not too much: *is it still a binary?*
5. Second goes **supernova** and forms a BH  
*Is it still a binary?*
6. Inspiral, merger, ringdown and LIGO



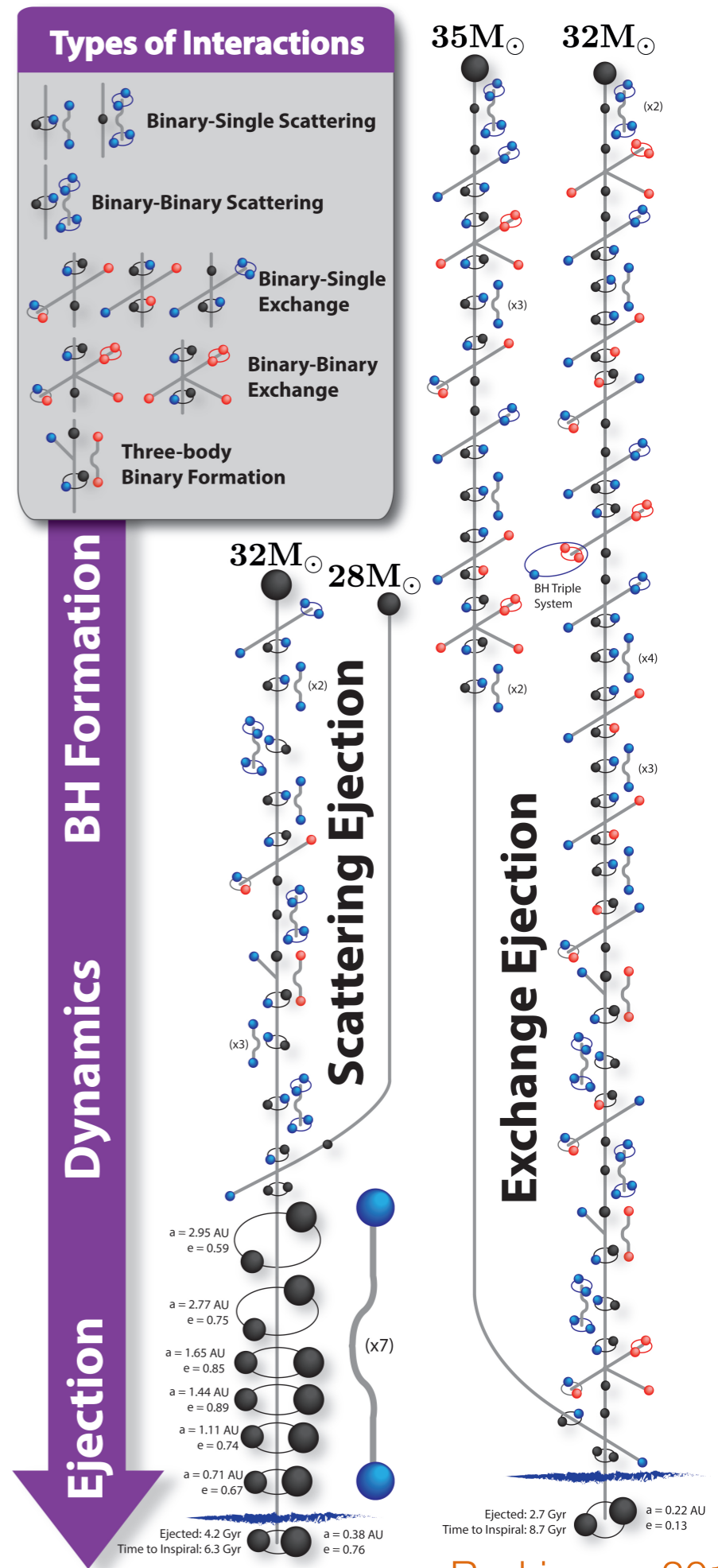
# Dynamical assembling: *cluster evolution*

1. Dense stellar clusters, many three body interactions
2. Dynamical friction: heavy objects sink towards the center
3. Soft binaries become softer, hard binaries become harder

**Key point:** stellar evolution is separate! They meet, swap, meet again, etc...



Hut Bachall 1983



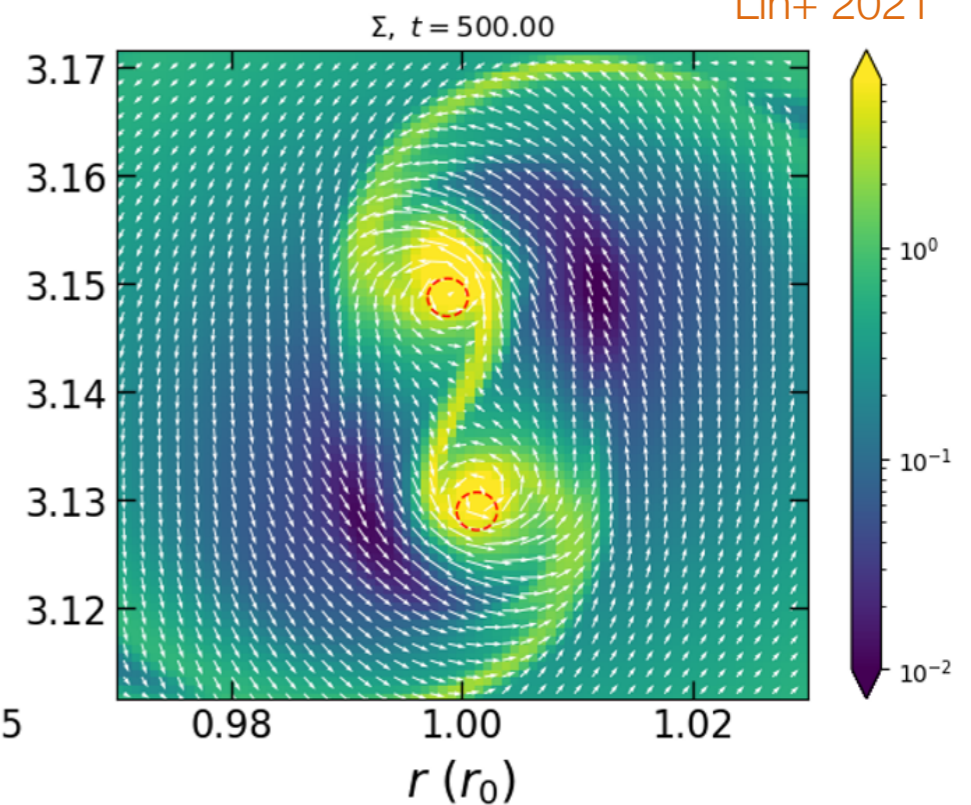
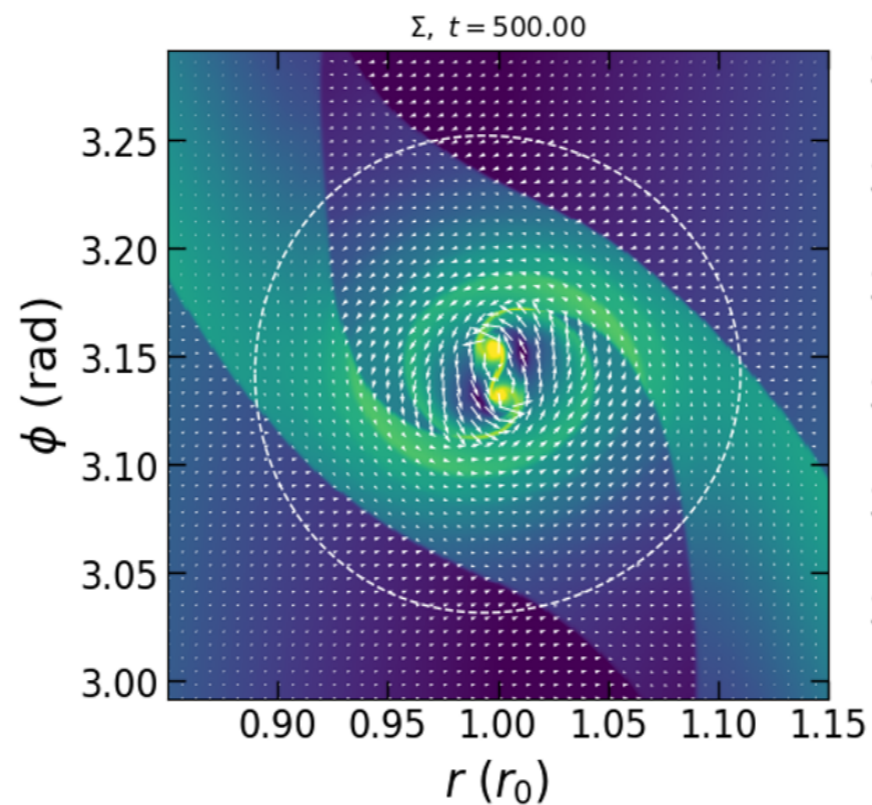
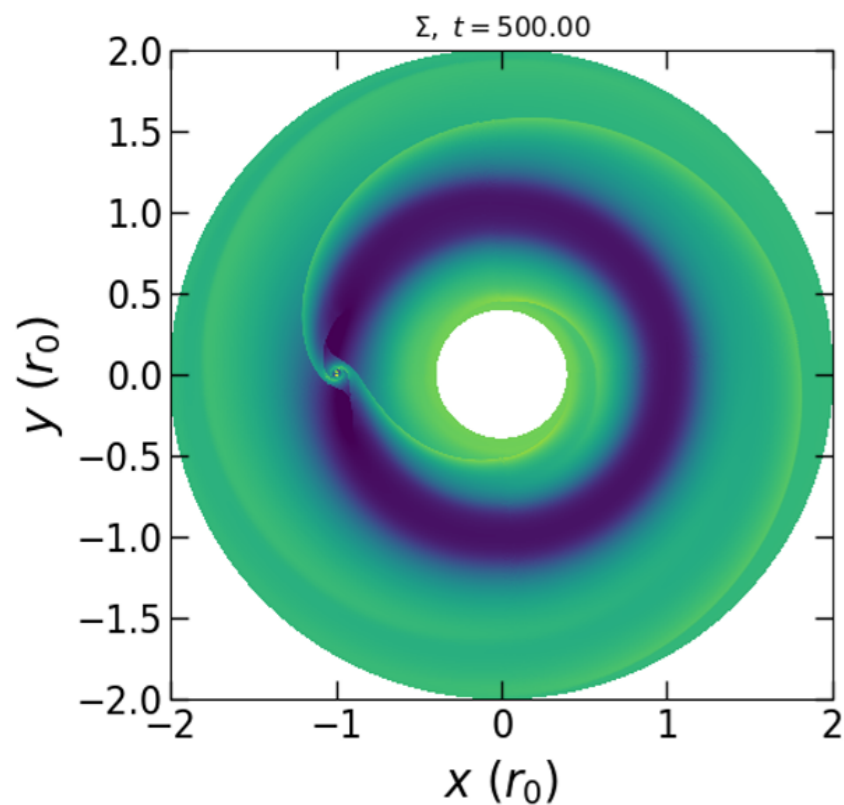
Rodriguez+2016

# AGN disks: “planetary” migration

- 1.** Gaseous disks surrounding supermassive BHs in active galactic nuclei
- 2.** A binary of stellar-mass BHs ends up there (formed there? captured there?)
- 3.** Induced wakes, gravitational torques, migration
- 4.** Maybe migration traps?

Secunda+ 2019

Lin+ 2021





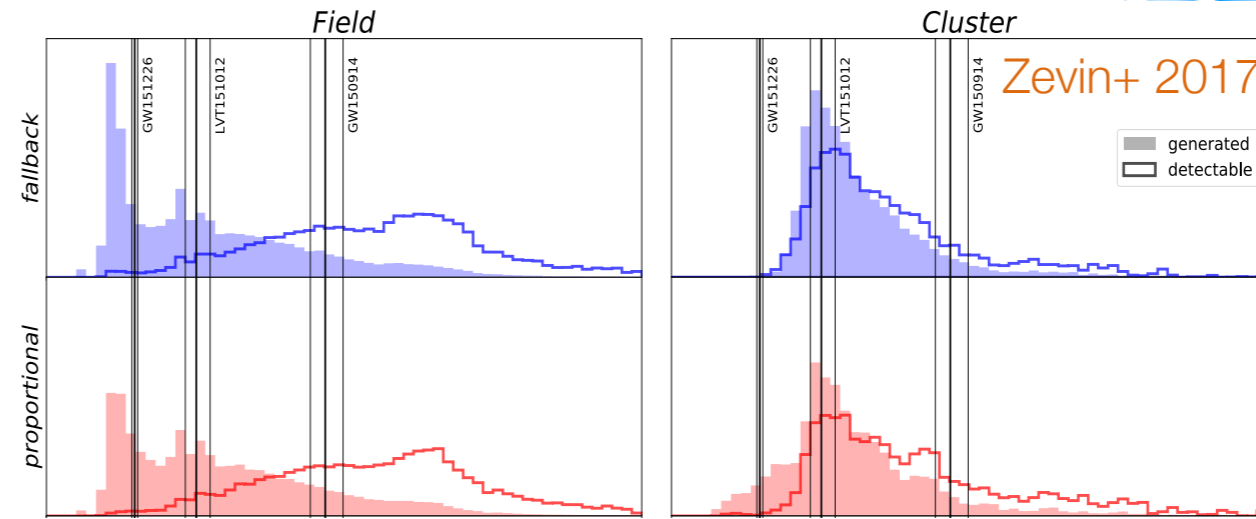
# Can we tell them apart?

What we thought in 2017-2018...

## Masses and rates

~**100 events** needed to distinguish these populations with masses and rates?

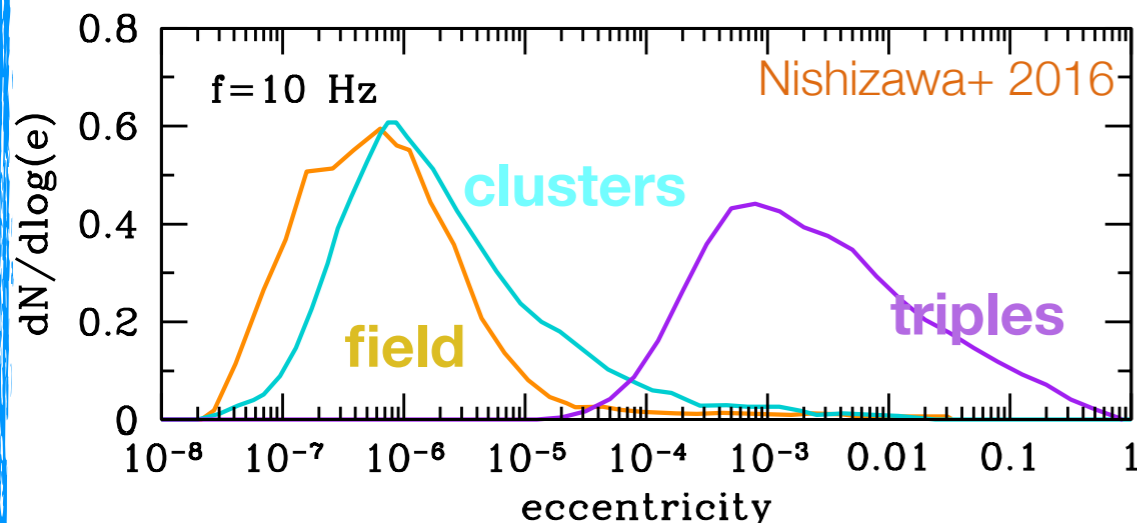
Stevenson+ 2015, Zevin+ 2017



## Eccentricities

**Promising!** Especially for specific scenarios like triples Antonini Perets 2012

**Go to LISA...** Brievik+ 2016, Nishizawa+ 2016

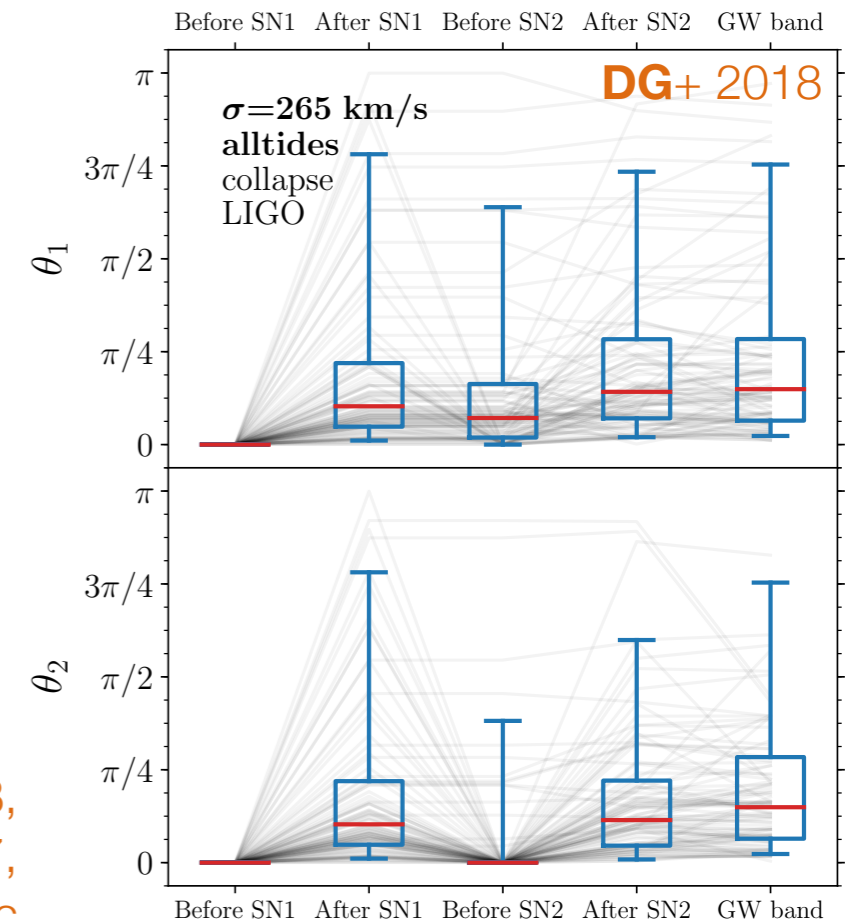


## Spins have secrets!

Binary star interactions imprint correlations on the spin directions:

- SN kicks
- Tides
- Mass transfer

DG+ 2013, 2018, Stevenson+2017, Talbot Thrane 2017, etc



# In reality...

**Almost 10 years of GW astronomy and we have not solved the formation channel problem (yet?)**

- Many unknown processes (astrophysics is dirty)
- Intrinsic degeneracies
- Selection effects
- The stats is hard
- Theoretical astrophysicists are “creative” (predictions change)



# Populations, the Bayes way

$\theta$  **Single-event parameters:** masses, spins, redshifts

$\lambda$  **Population parameters:** spectral index of mass distribution, cutoffs

Inhomogeneous Poisson process:

Loredo 2004, Mandel+ 2019,  
Thrane, Talbot 2019, Vitale, **DG+** 2022,

$$p(\lambda|d) \propto \pi(\lambda) \sigma^{-N}(\lambda) \prod_{i=1}^N \int p_{\text{pop}}(\theta|\lambda) \mathcal{L}(d_i|\theta) d\theta$$

Population prior

Population posterior

N events...

Population model

Single-event likelihood

Selection effects:  $\sigma(\lambda) = \int p_{\text{pop}}(\theta|\lambda) p_{\text{det}}(\theta) d\theta$

Detection probability

# What model for the Universe?

Option 1: **Simple, parametrized functional forms**

LIGO/Virgo and many others

↳ Encode some physical intuitions (but hard coded)

Option 2: **Non-parametric models** Rinaldi DelPozzo 2021, Edelman+ 2022,  
Callister Farr 2023

↳ Purely data-driven, but physical interpretation becomes tricky

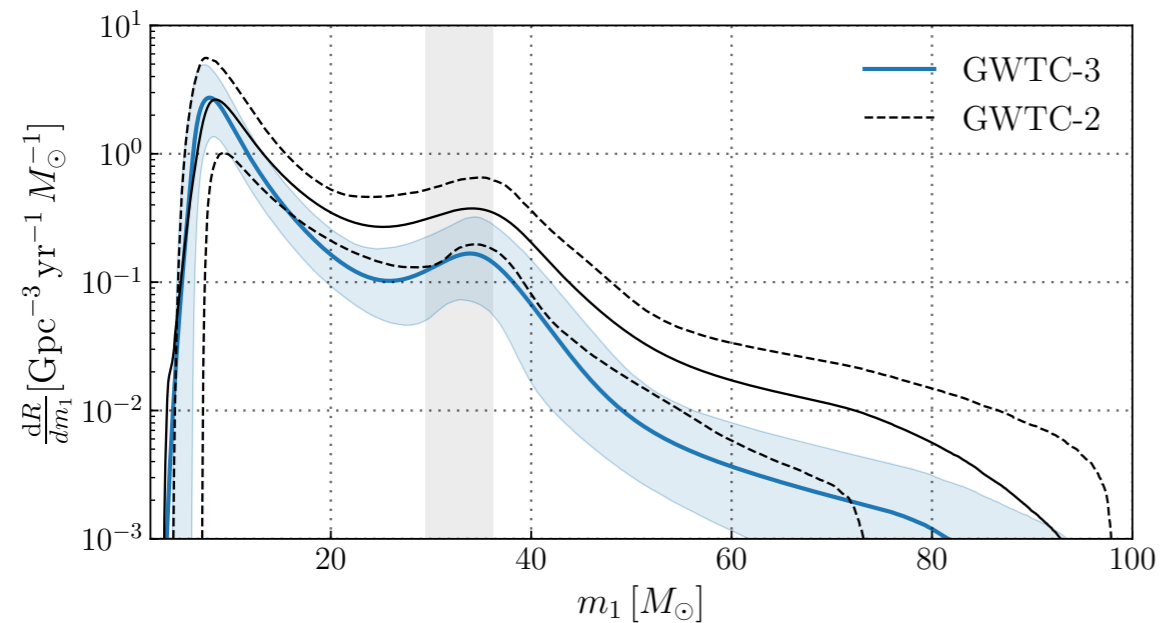
Option 3: **Direct fits to astro sims** Taylor **DG** 2018, Wong+ 2020, Mould **DG**+ 2022

↳ Astrophysics! But very model dependent!  
Need some deep learning to make it work

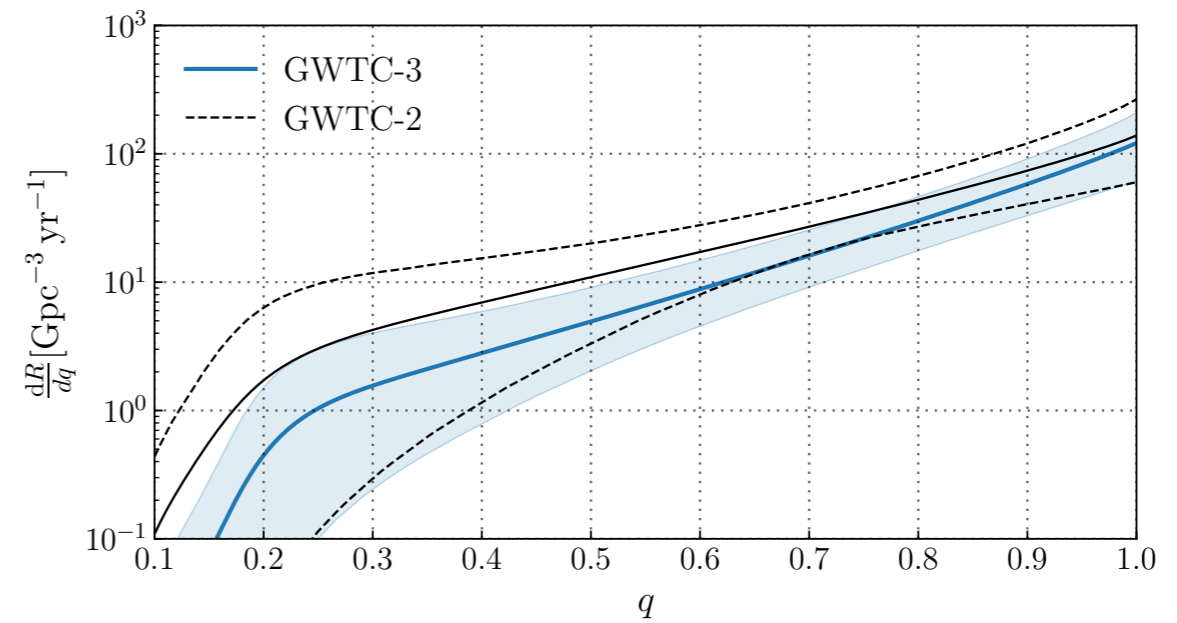
# The population of merging BHs

LIGO/Virgo 2021

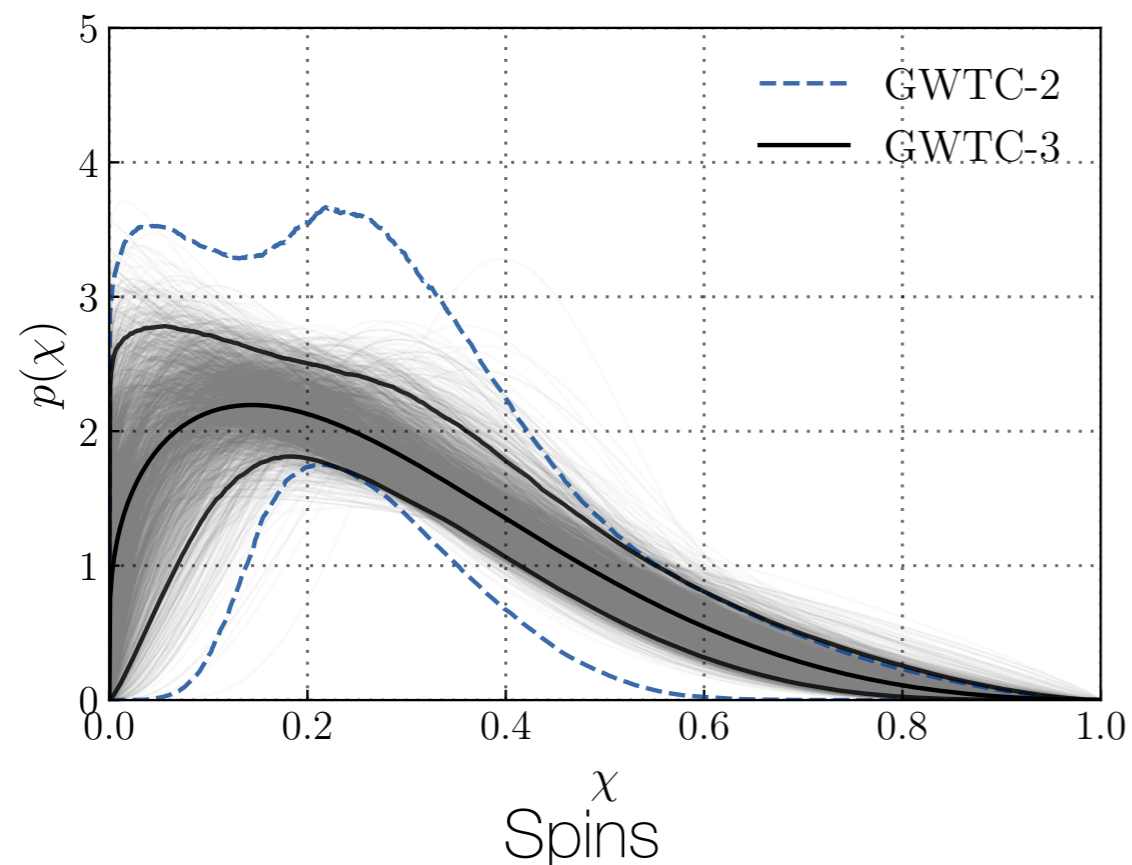
Using all the events up to GWTC3:



Primary mass



Mass ratio



Spins

- Good handle on the marginal distributions...
- Still largely relying on simple parametrized models
- But some very interesting steps into the wild, “non parametric” inference

# Are BHs correlated?

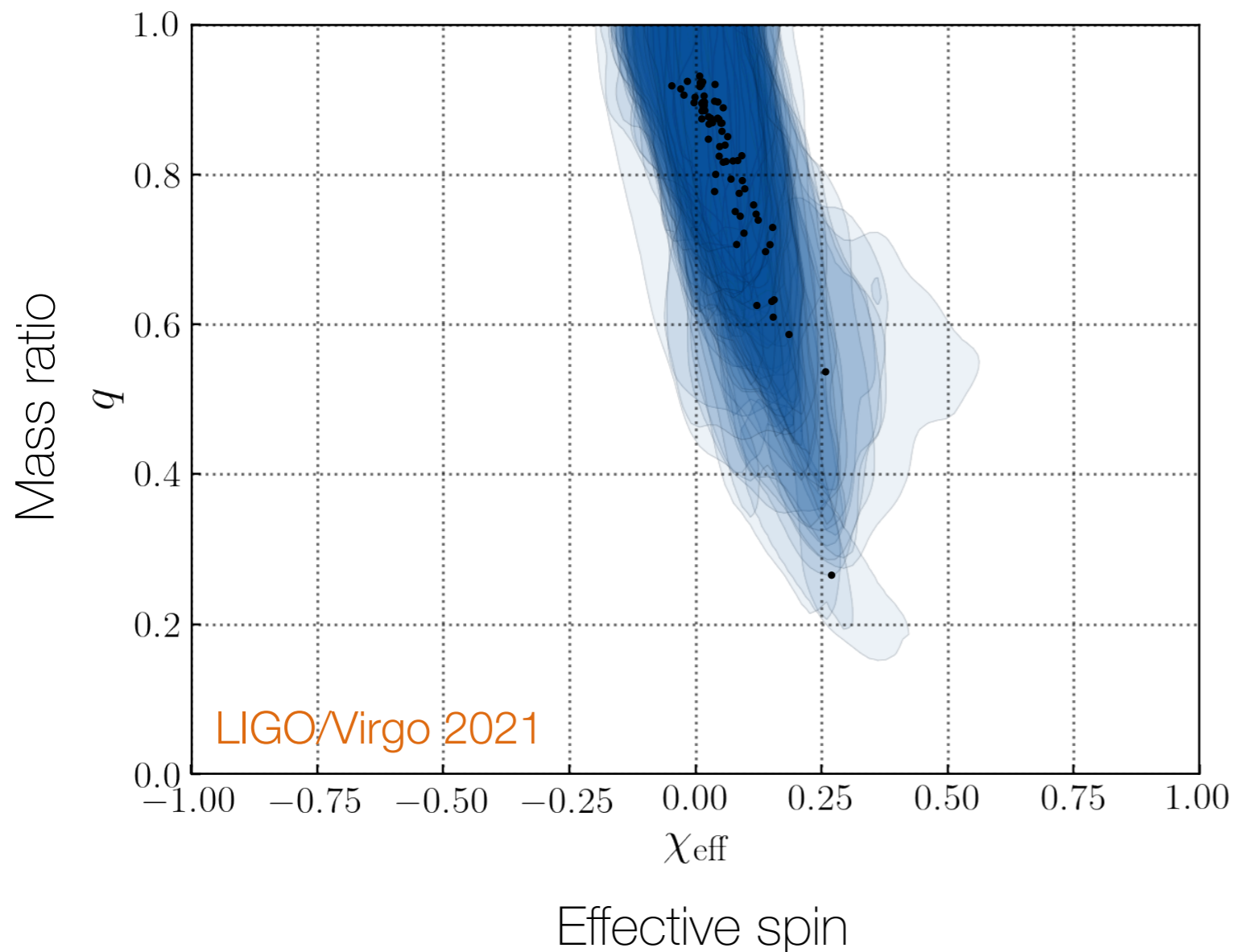
- Next: correlations! E.g. does the mass spectrum evolve with redshift?

Hints, not all models [Rinaldi+ 2023](#)

- But masses and spins are definitely anticorrelated!

$$q = \frac{m_2}{m_1} \leq 1$$

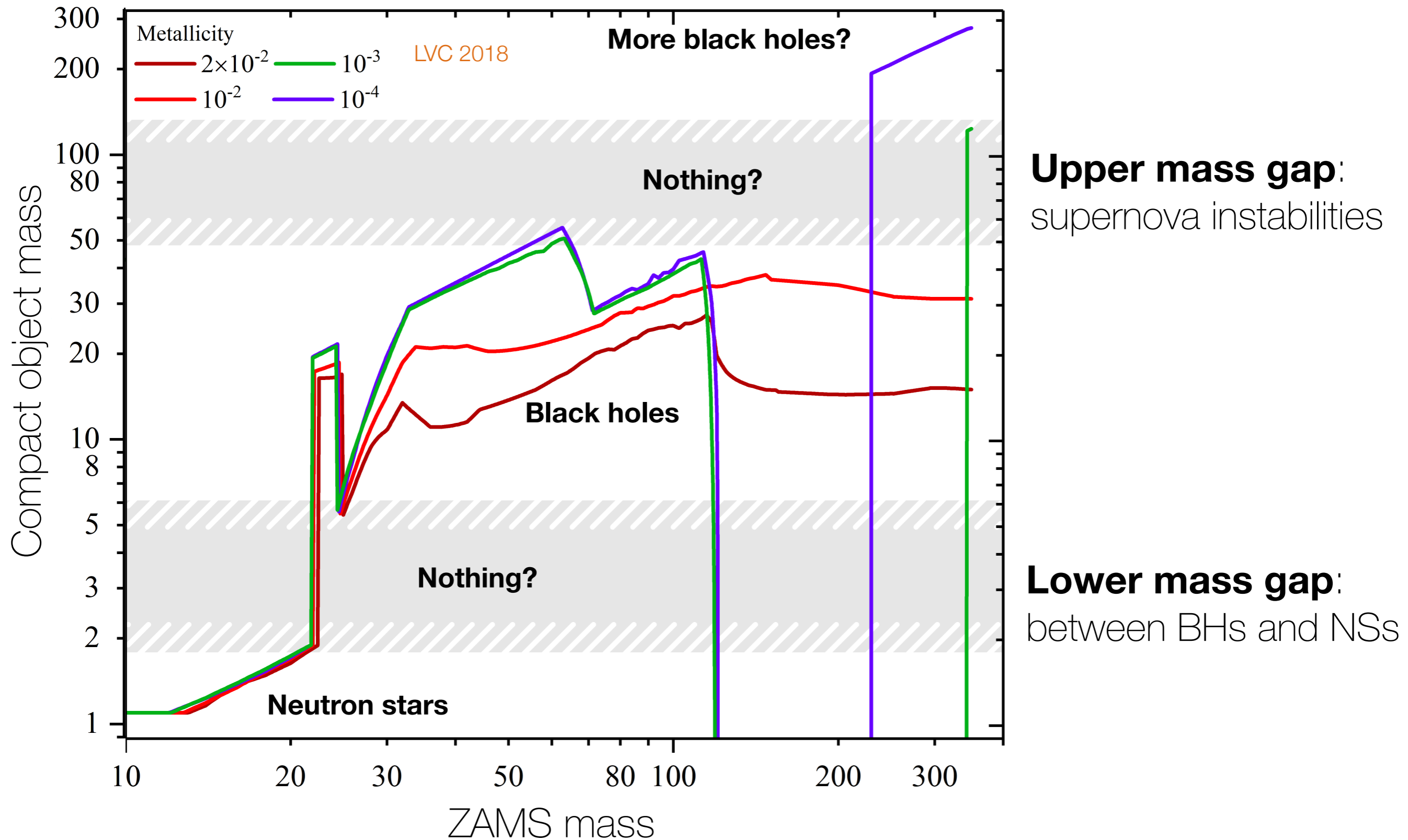
$$\chi_{\text{eff}} = \frac{\chi_1 \cos \theta_1 + q \chi_2 \cos \theta_2}{1 + q} \in [-1, 1]$$



- First discovered  
[Callister+ 2021](#)
- Confirmed with more data  
[LIGO/Virgo 2021](#)
- Confirmed with more stats  
[Adamcewicz+ 2022, 2023](#)

*"I tried very hard to kill it but couldn't"* Tom Callister (July 2023)

# A solid prediction? The gaps



# Pair-instability supernovae

As the mass of the core increases:

1. Electron-positron production
2. Radiation pressure drops
3. Core contracts
4. Temperature goes up
5. Explosive oxygen burning
6. Entire star is gone (PISN)
7. Repeated pulsations (PPISN)

Heger Woosley 2002, Belczynski+ 2016,  
Woosley+ 2017, Spera Mapelli 2017,  
Marchant+ 2018, Stevenson+ 2019

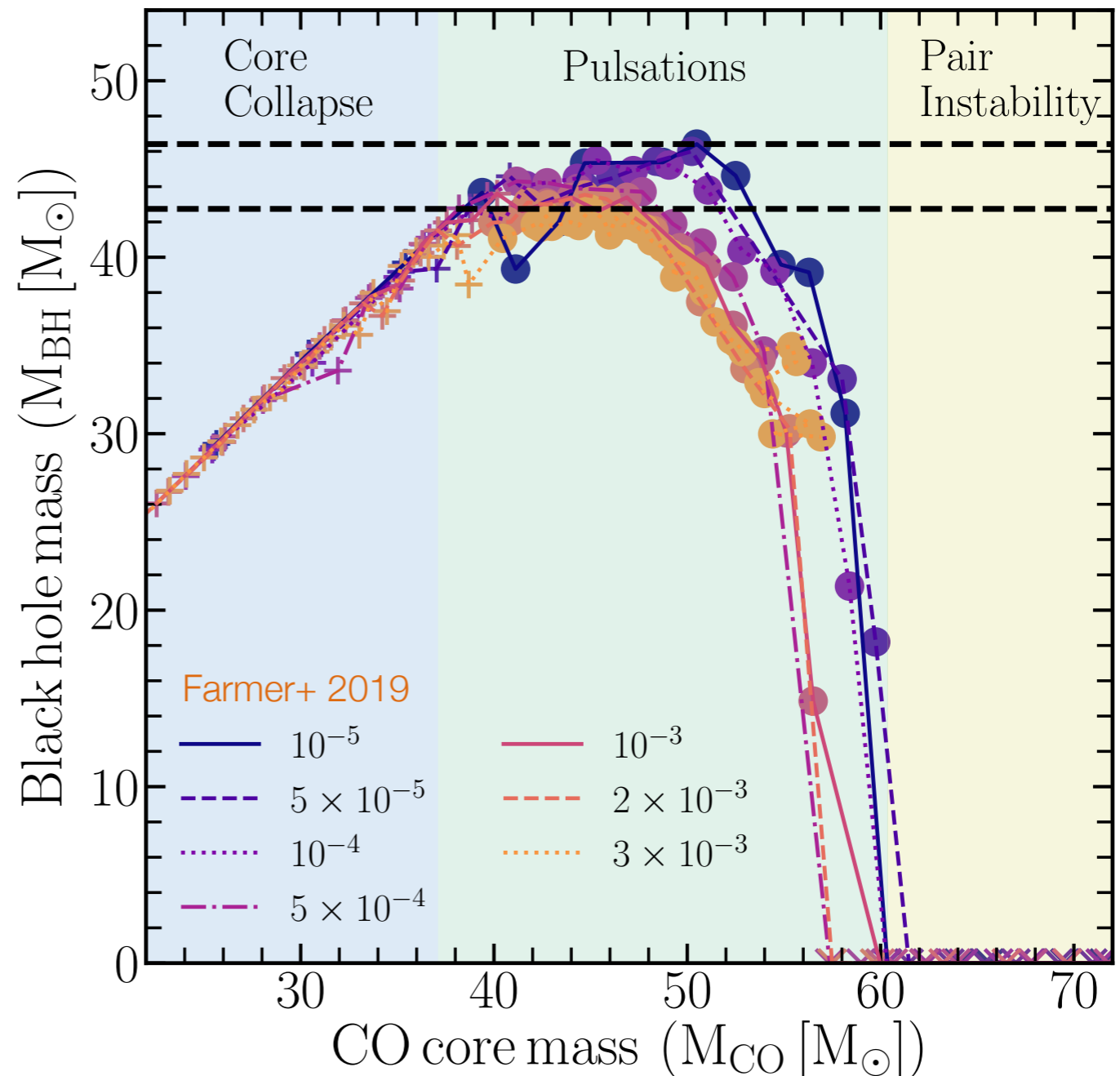
BH forbidden for  
 $M \gtrsim 50M_{\odot}$

This limit is very solid...

Farmer+ 2019, Renzo+ 2020

...until it isn't

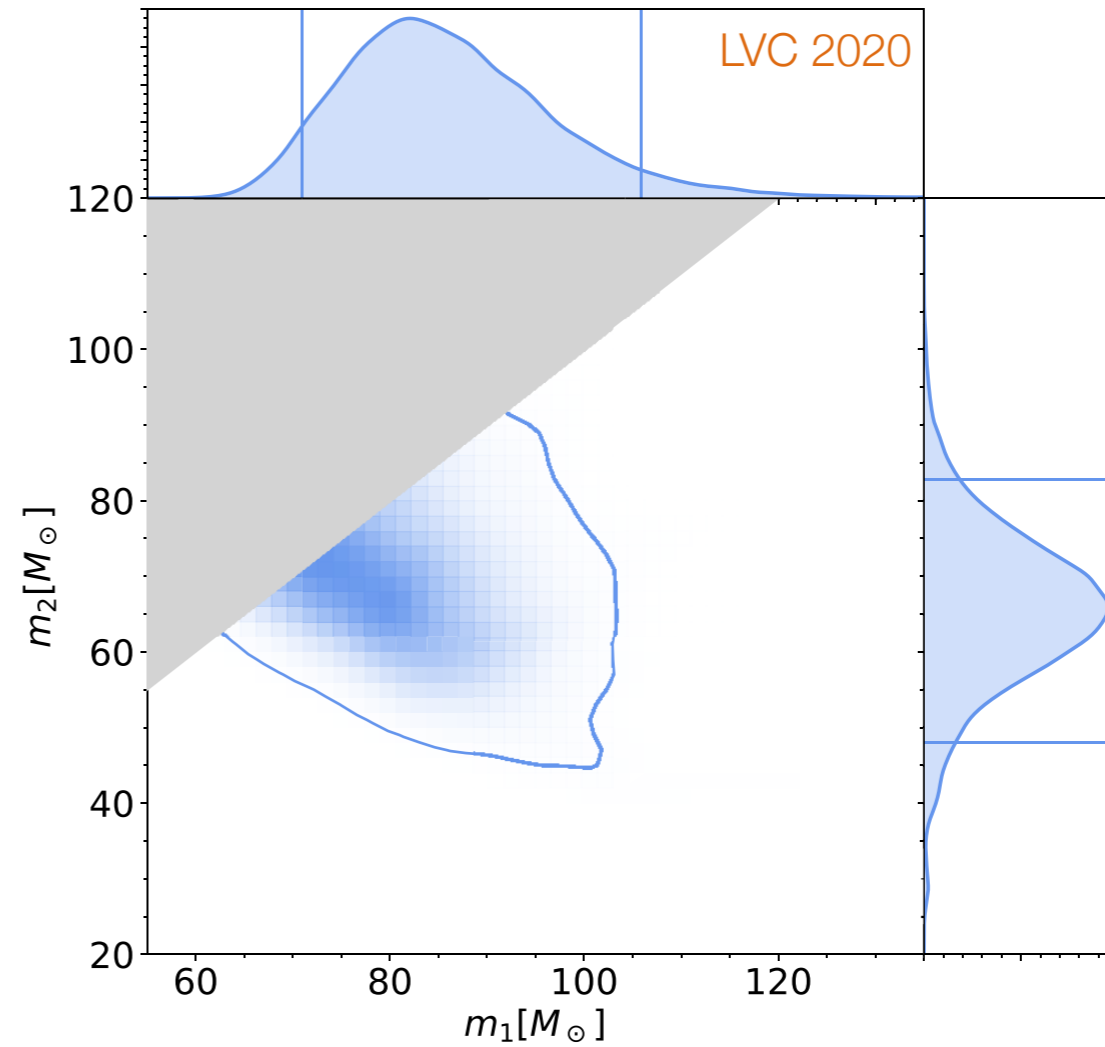
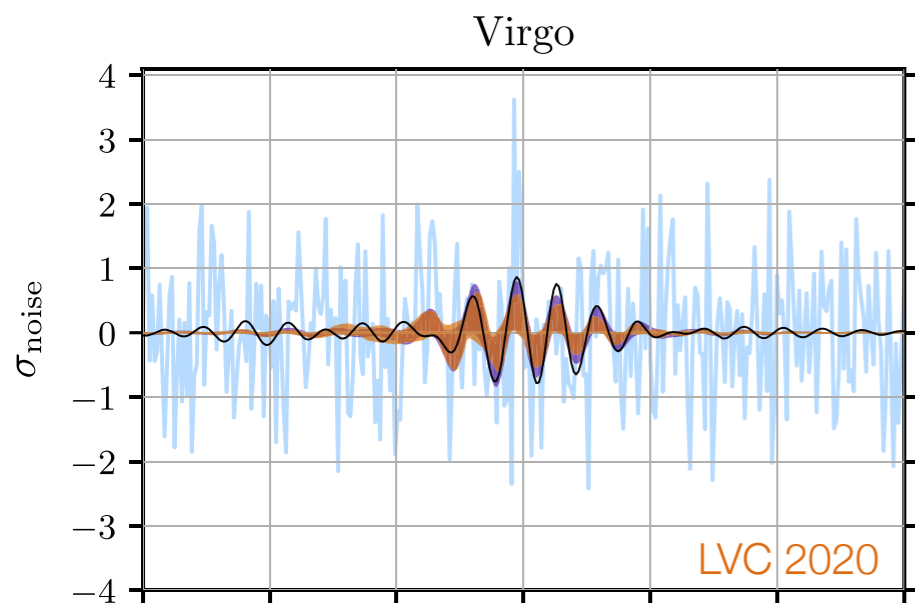
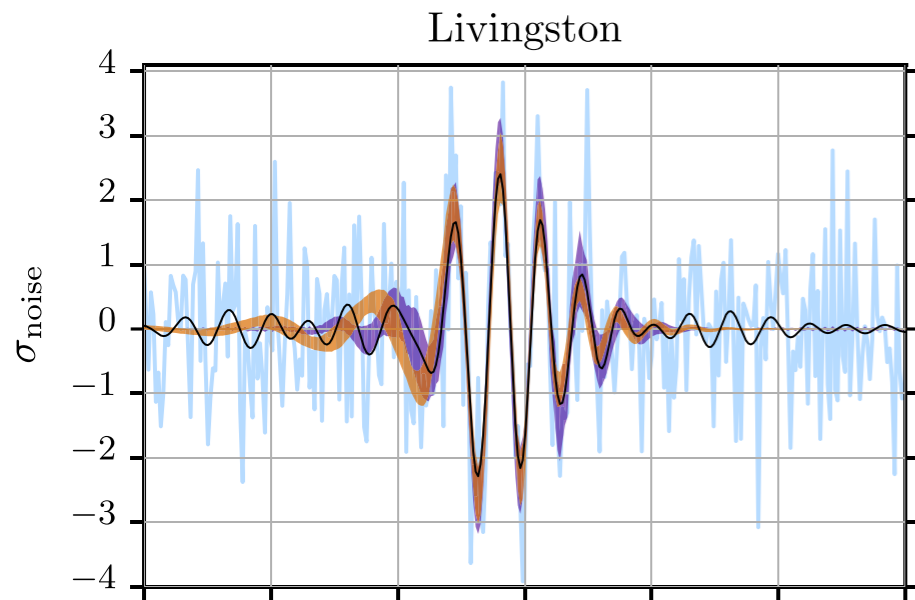
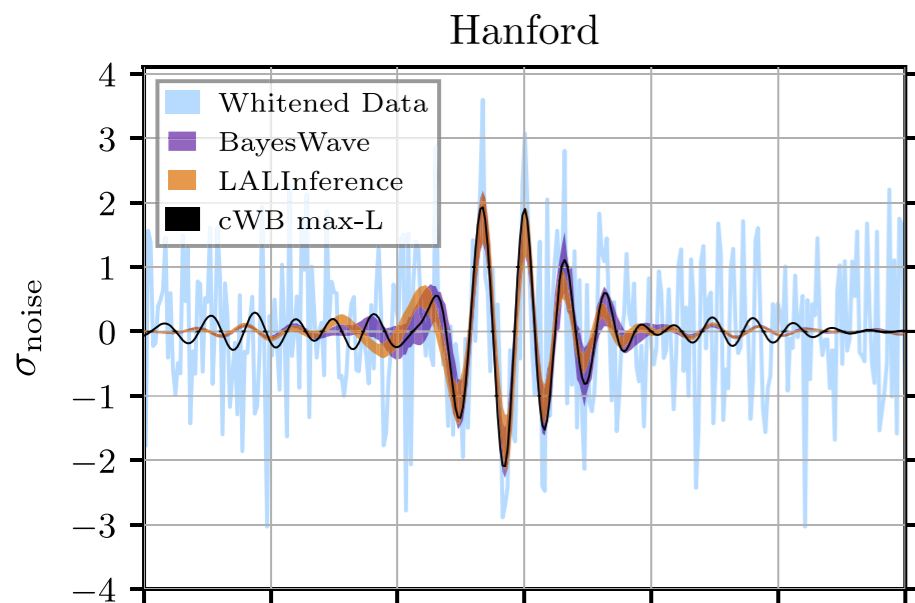
Belczynski+ 2019, 2020, Farmer+ 2020,  
Costa+ 2021, Farag 2022



**Can we bypass stars and  
use black holes?**

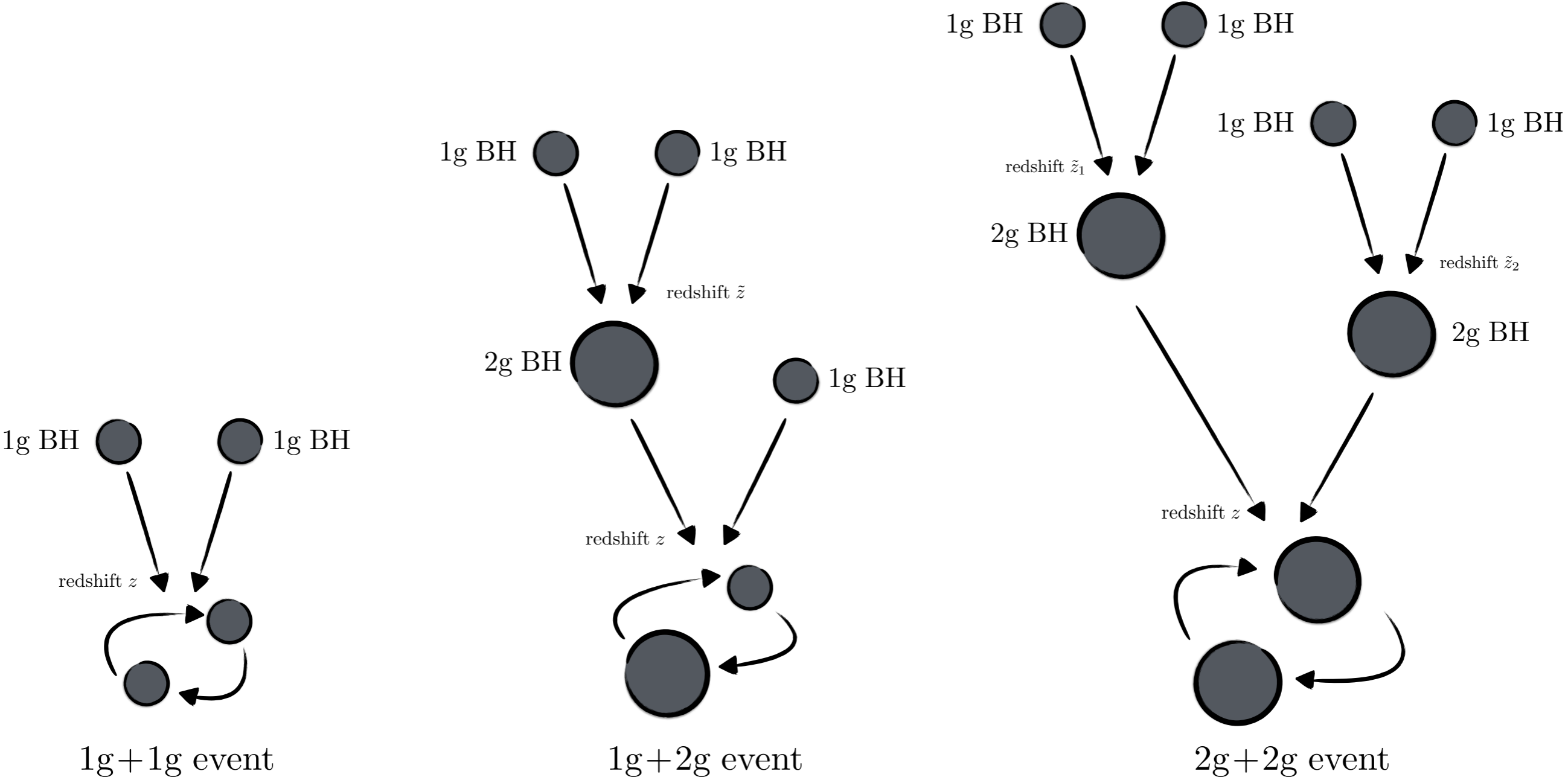


# GW190521: The impossible BH



An extremely confident detection of black holes with  $\sim 65 M_\odot$  and  $\sim 85 M_\odot$ .

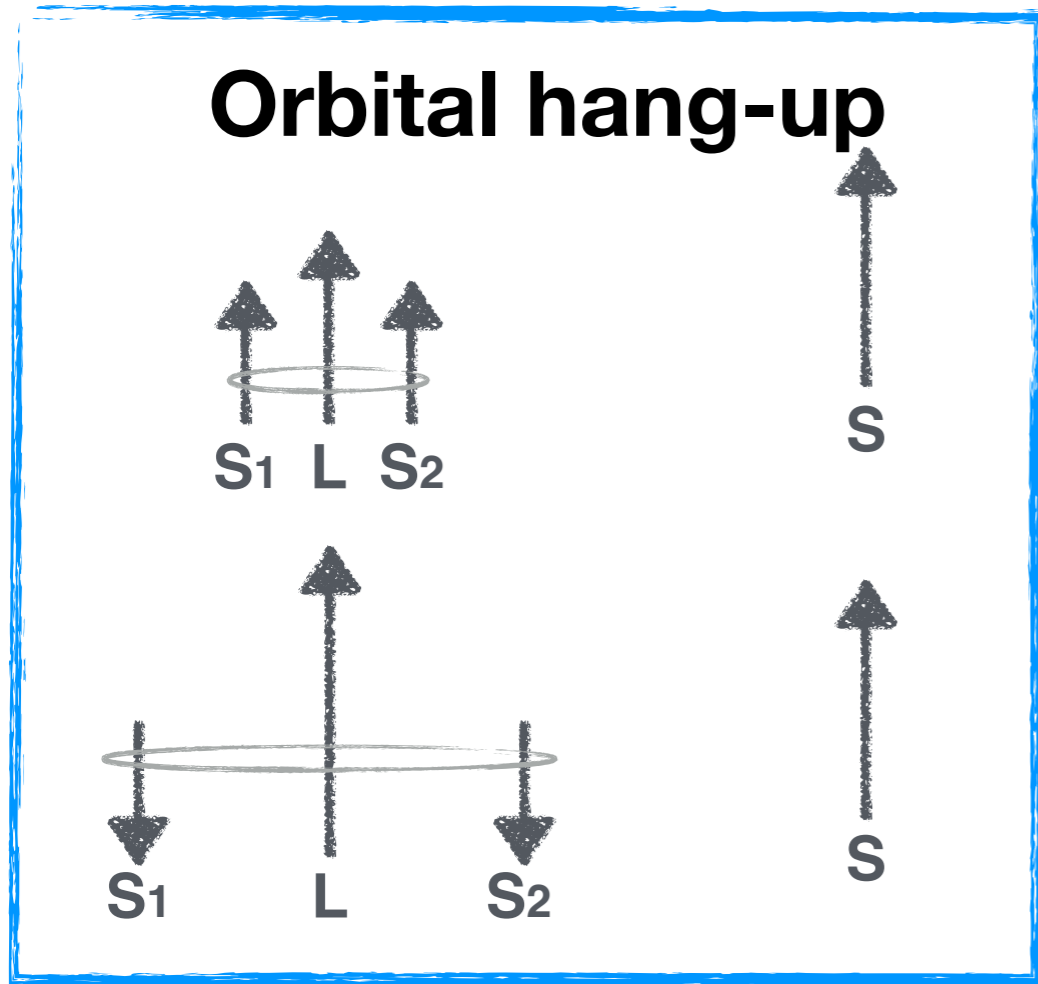
# Black hole generations



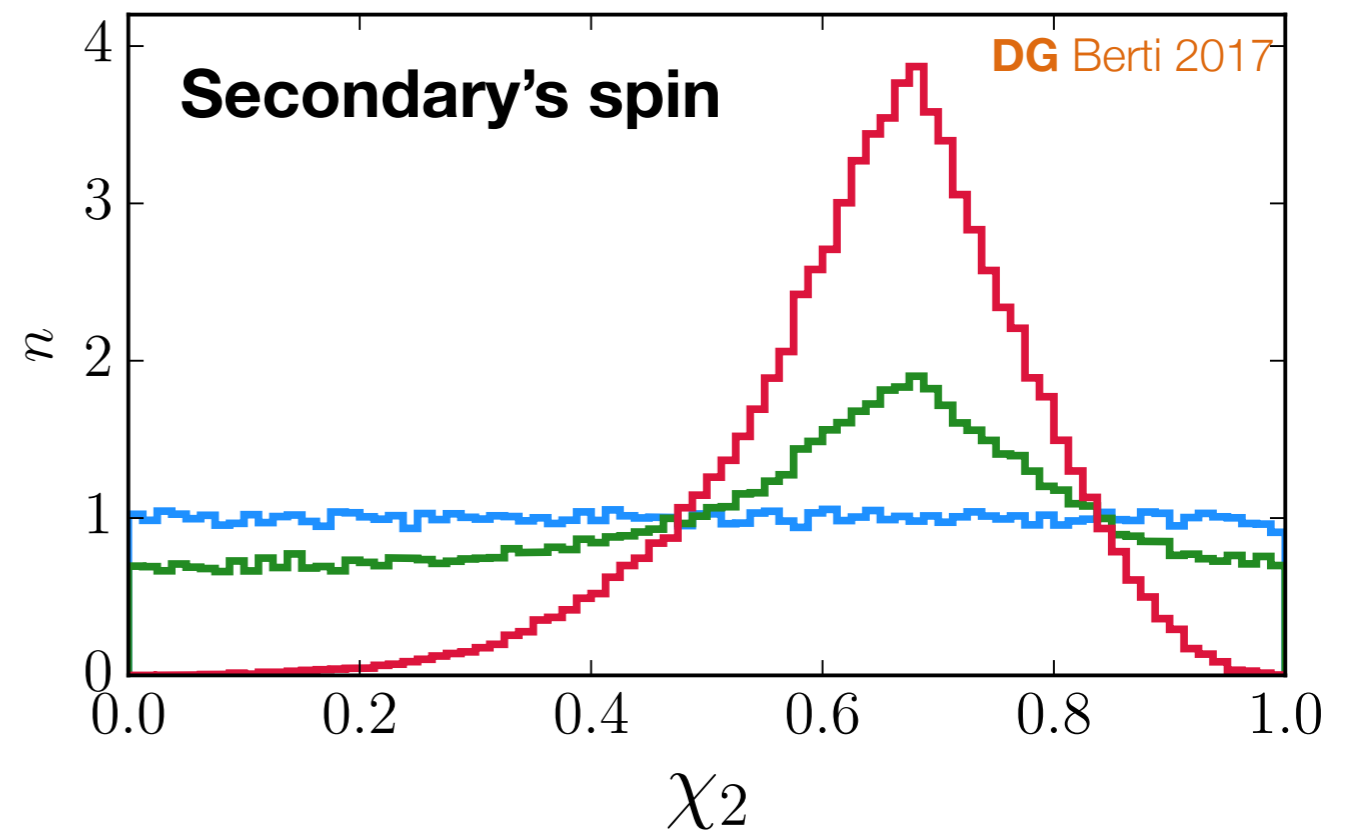
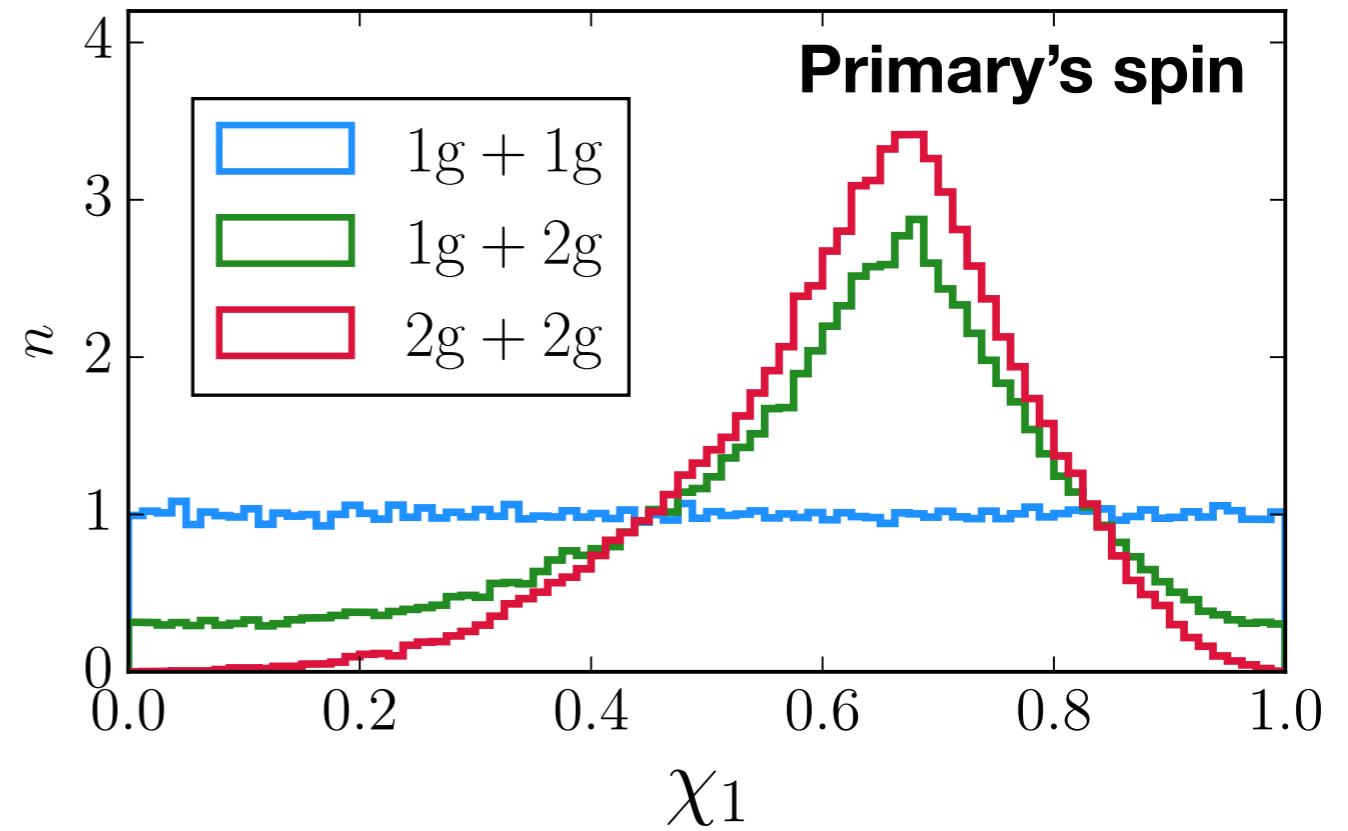
**Orthogonal**, but complementary, direction to the usual field vs. cluster debate

# Spins: the magic number

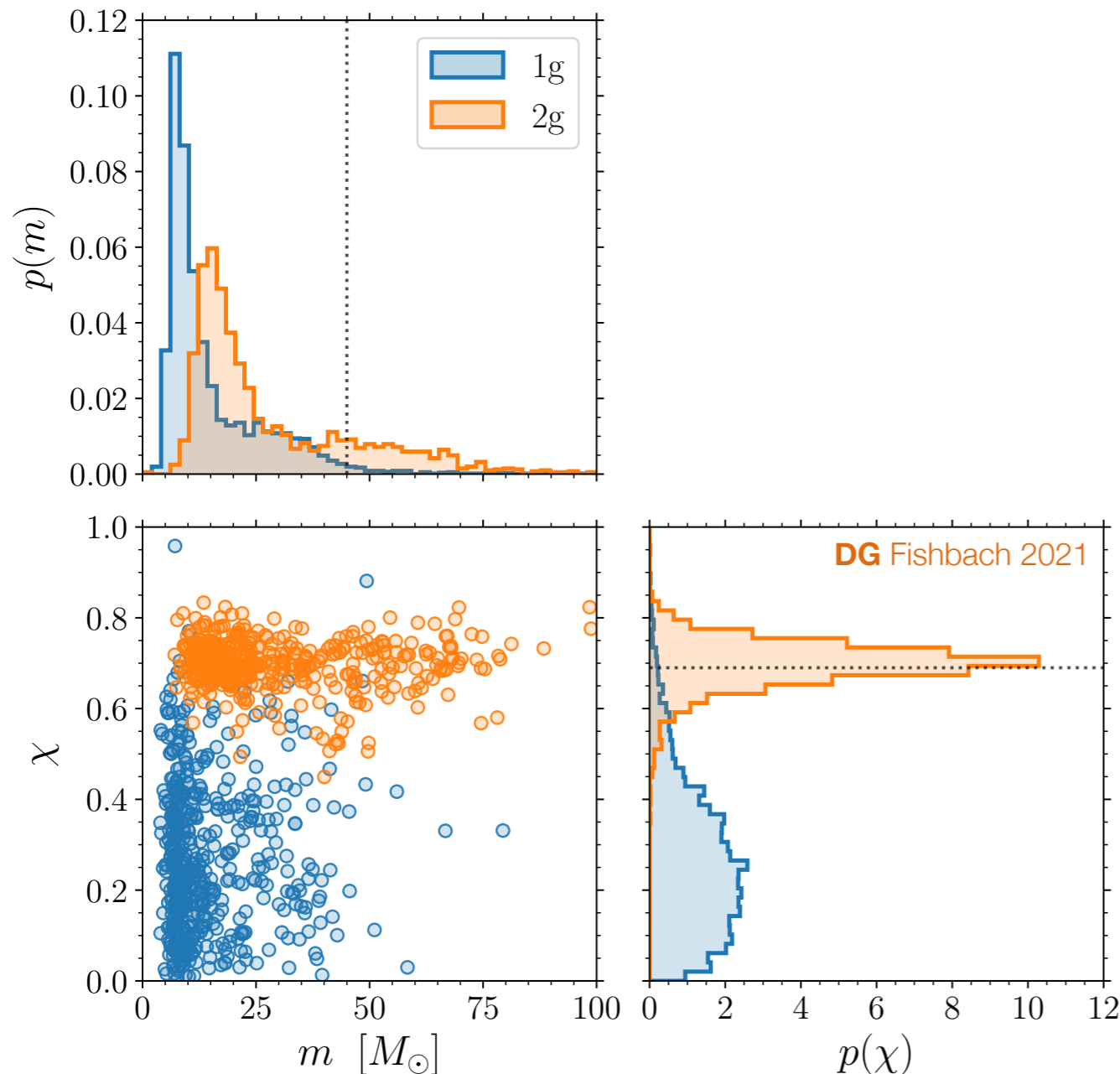
DG Berti 2017, Fishbach+ 2017, Berti Volonteri 2008



Peculiar spin distribution  
peaked at **0.7**



# An explosion of new predictions

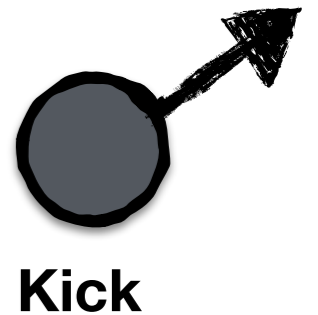
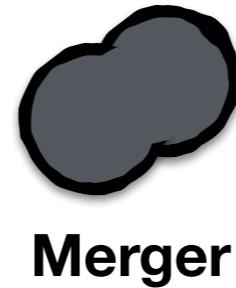


- Masses in the pair-instability mass gap  
Heger+ 2003, Woosley+ 2007
- Peculiar spin distribution peaked at 0.7  
DG Berti 2017, Fishbach+ 2017
- GW kicks require large escape speed  
DG Berti 2019
- Very frequent in AGNs  
Yang+ 2019, Tagawa+ 2020
- Promising for GW190412  
DG Vitale Berti 2020, Rogriguez+ 2020
- Leading explanation for GW190521  
LIGO/Virgo 2020
- Several events in the LIGO catalog?  
Kimball+ 2021, Mould DG Taylor 2022
- Don't overdo it!  
Zevin Holz 2022
- Perhaps reproducing correlations?  
Santini, DG+ 2023

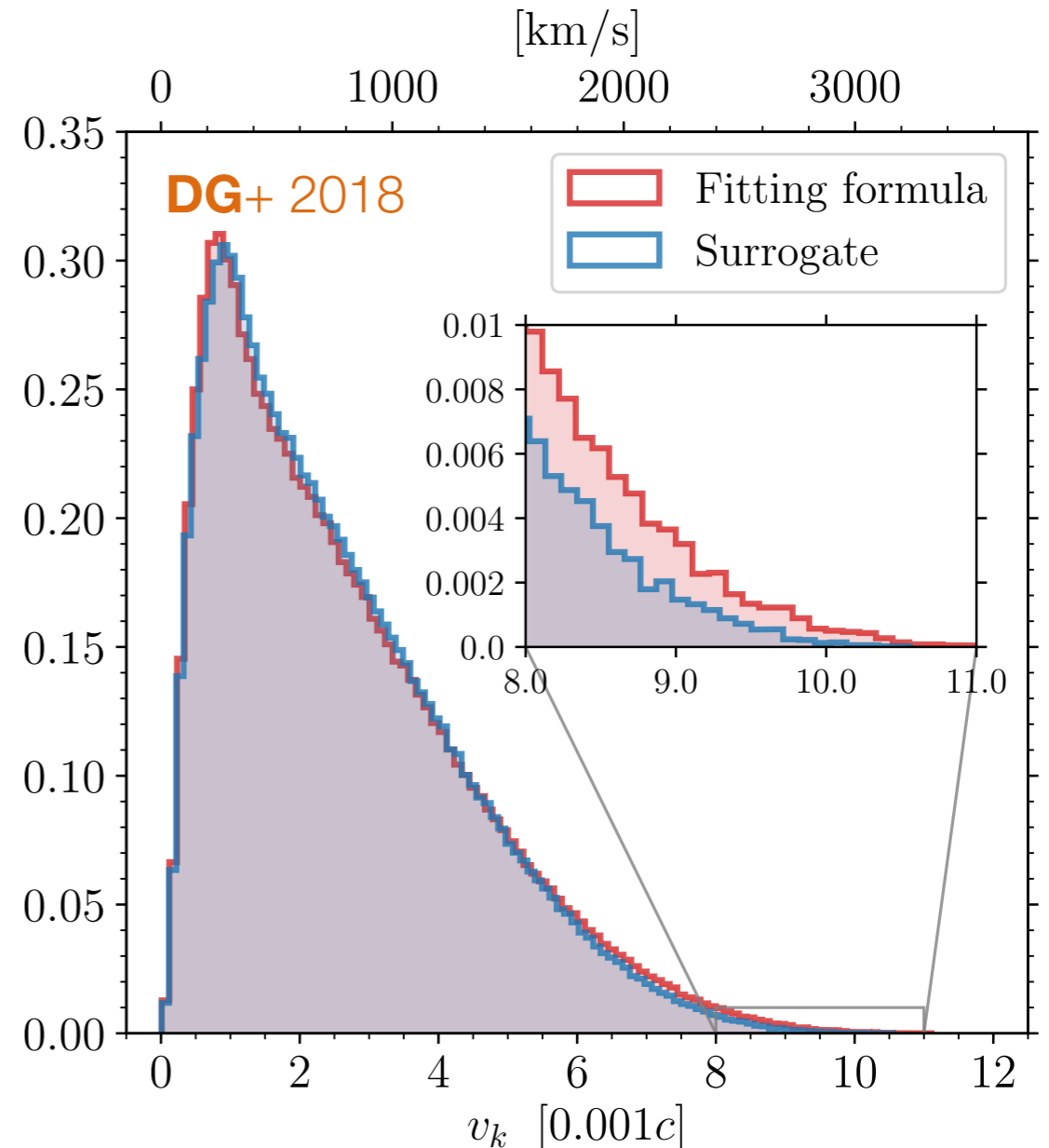
Actually enough for a dedicated review DG Fishbach 2021

# Black-hole recoils

We should say **IMRK**  
instead of **IMR**...



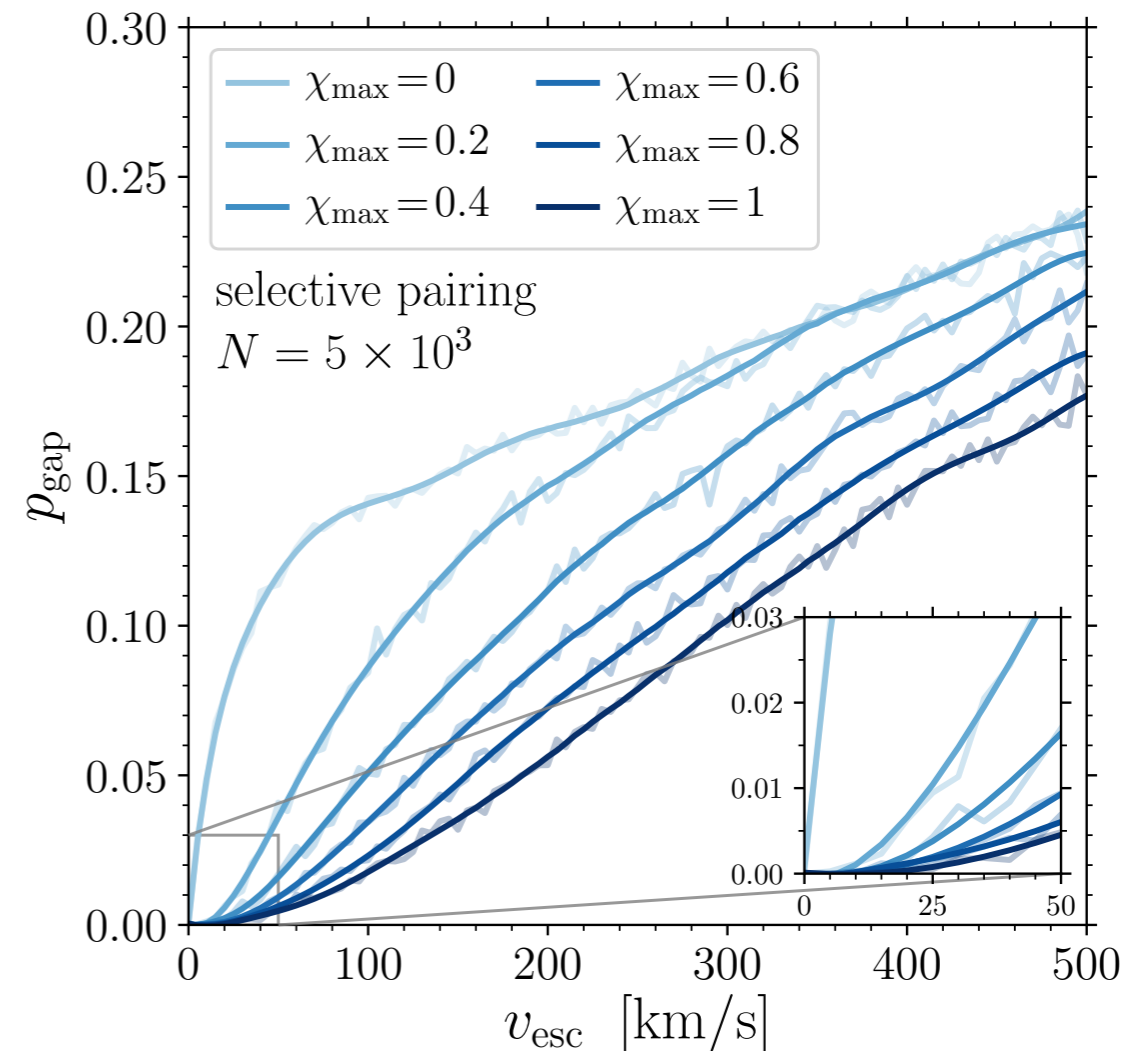
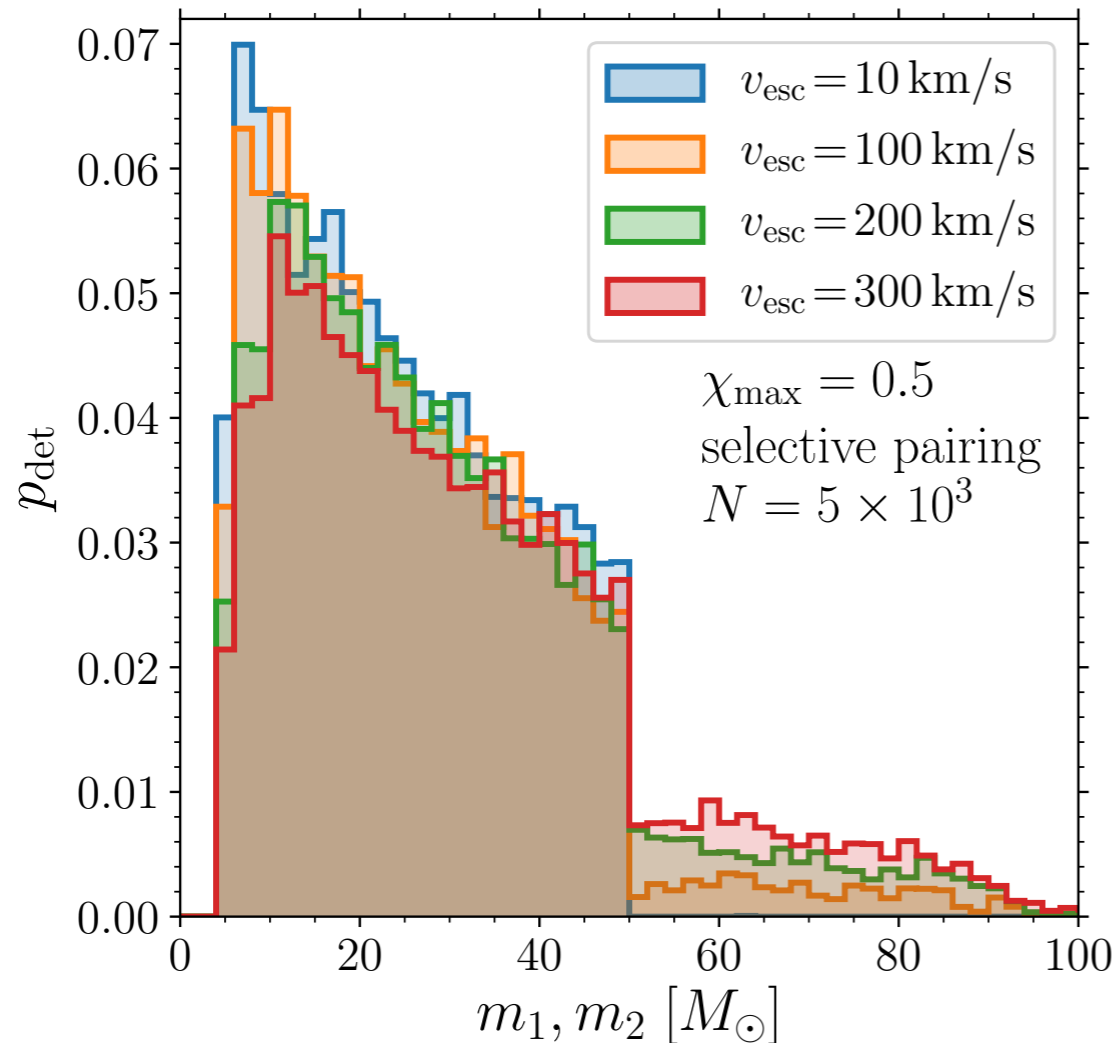
- Anisotropic emission of GWs causes the final BH to recoil
- Flux of linear momentum accumulates at the very end  
**DG+ 2018**
- Typical recoils are of  $O(100)$  km/s
- Go into the 1000s km/s if lucky  
**Campanelli+ 2007, Gonzalez+ 2007**
- Only environment with large escape speeds can retain remnants!  
**DG, Berti 2019**
- Perhaps avoidable with spin fine-tuning  
**Rodriguez+ 2019**



# The role of the escape speed

An escape speed of **~50 km/s** is necessary to populate the mass gap

DG Berti 2019

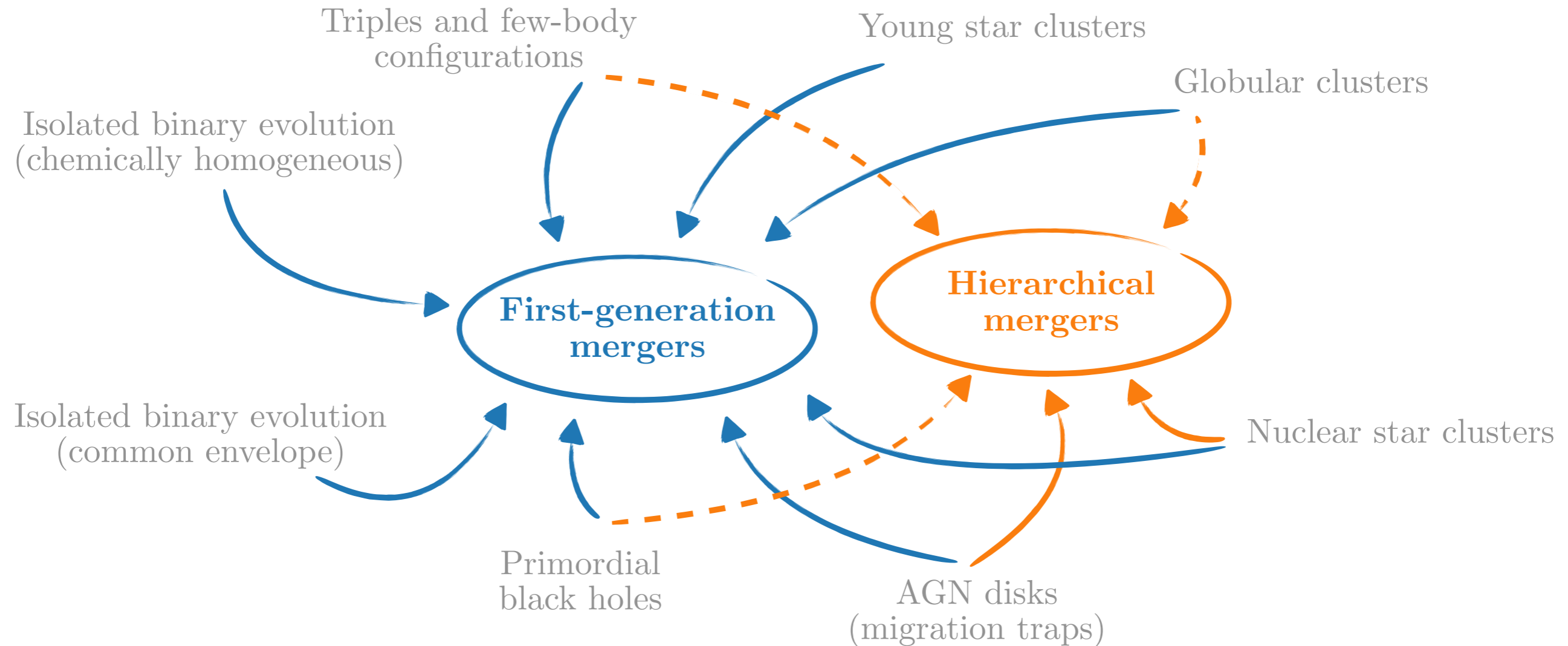


**~50 km/s is more than most globular clusters.**

- Nuclear star cluster [Antonini+ 2016](#)
- Triples [Antonini+ 2017](#), [Bin+ 2019](#)
- Disc-assisted migration  
[Stone+ 2017](#), [Bartos+ 2017](#)

# Where do hierarchical black-hole mergers come from?

DG Fishbach 2021



# The gaps are precious

Baibhav, **DG+** 2020

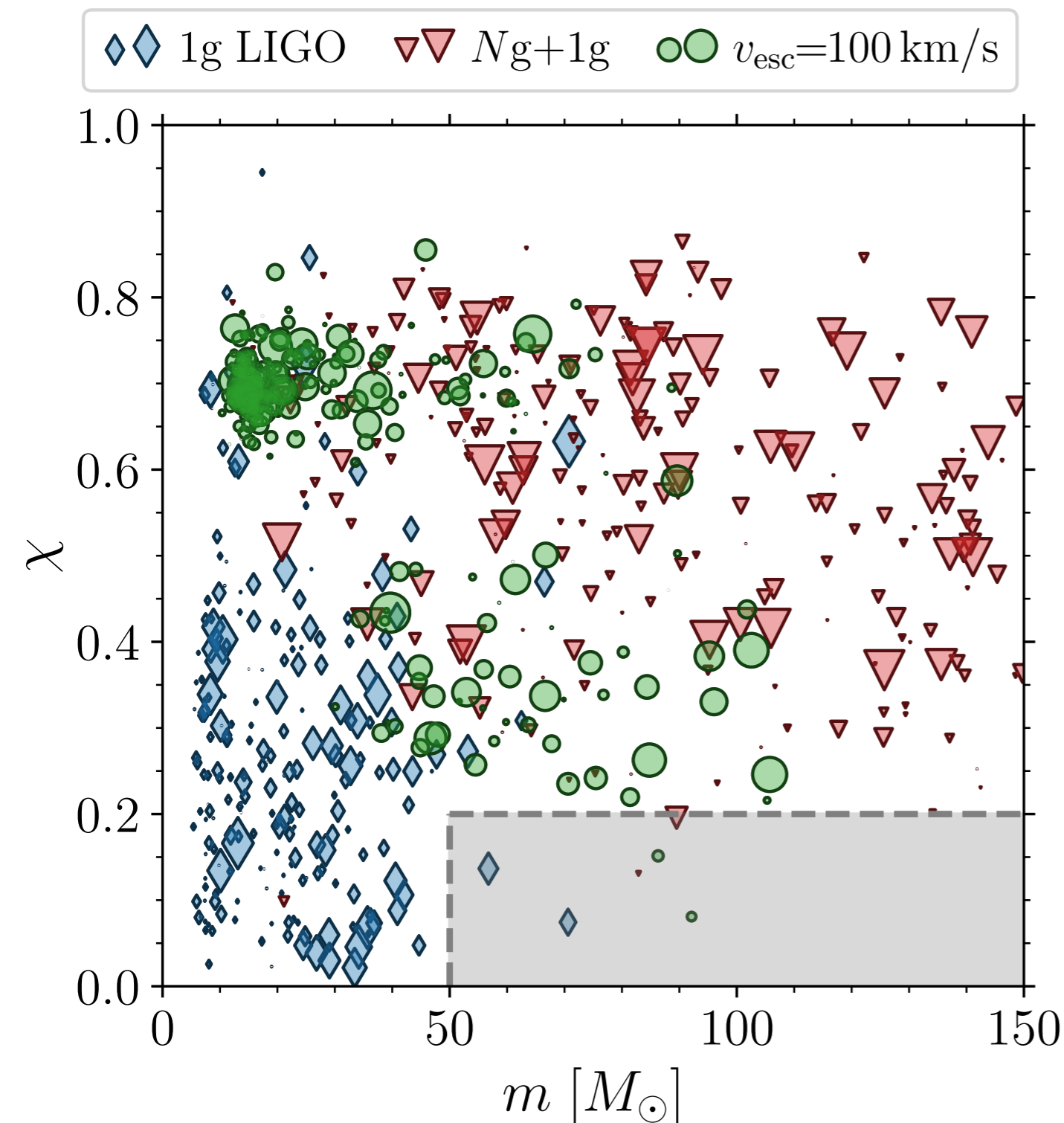
- Two channels “field” and “cluster”:  $N = N_{\text{field}} + N_{\text{cluster}}$
- Some are in the gap:  $N = N_{\text{no gap}} + N_{\text{gap}}$
- The gap is exclusive:  $N_{\text{field,gap}} = 0$        $N_{\text{cluster,gap}} = N_{\text{gap}}$
- A predicted efficiency:  $\lambda \equiv \frac{N_{\text{cluster,gap}}}{N_{\text{cluster}}}$
- Individual contributions:

$$N_{\text{cluster}} = \frac{N_{\text{gap}}}{\lambda} \qquad N_{\text{field}} = N - \frac{N_{\text{gap}}}{\lambda}$$



# High mass but low spin?

DG, Giacobbo, Vecchio 2021



## Hierarchical mergers cannot do it

(even if you try hard)

If a future event is there...  
we need something else!

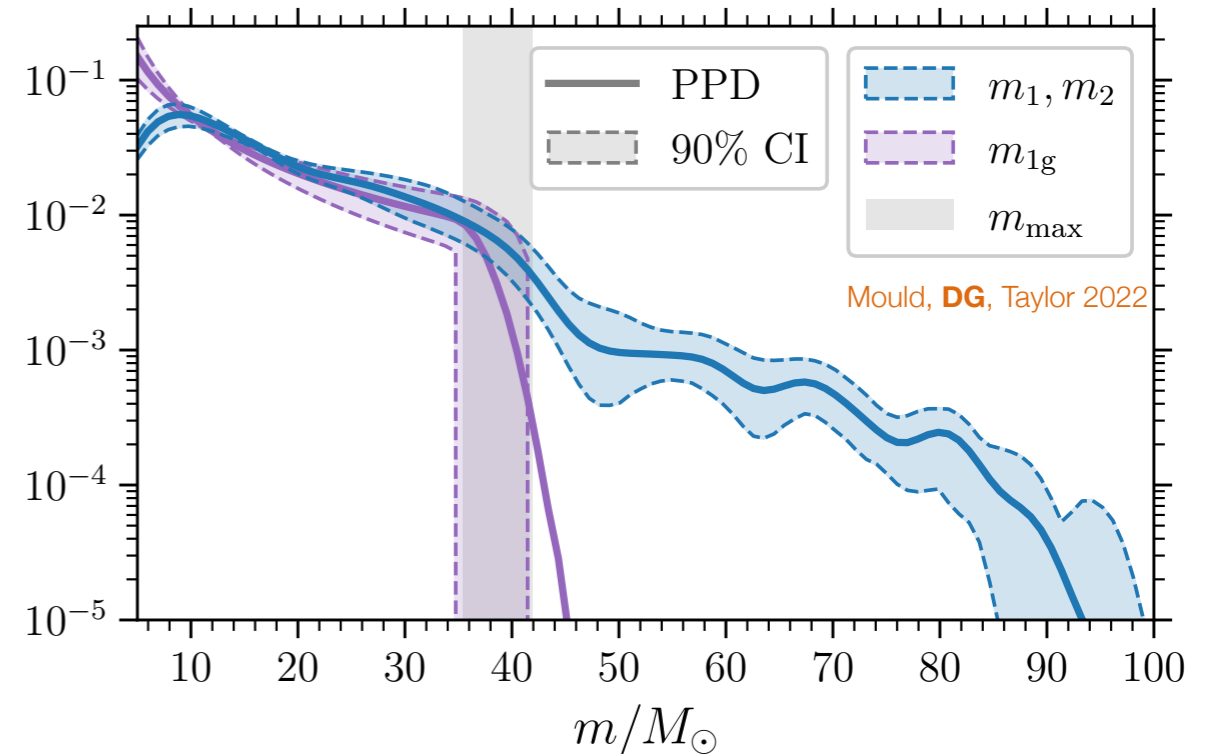
- Lowered CO reaction rate  
e.g. Farmer+ 2020, Costa+ 2021, Farag+ 2022
- Weaker stellar winds  
e.g. Leung+ 2019, Belczynski+ 2020
- Rotation  
e.g. Marchant Moriya 2020, Woosley Heger 2021
- Stellar collisions  
e.g. Di Carlo+ 2019, Renzo+ 2020
- Accretion  
e.g. van Son et al. 2020, Natarajan 2021
- Pop III stars  
e.g. Farrell et al. 2020, Kinugawa et al. 2021

# Here comes deep learning

Mould, **DG**, Taylor 2022

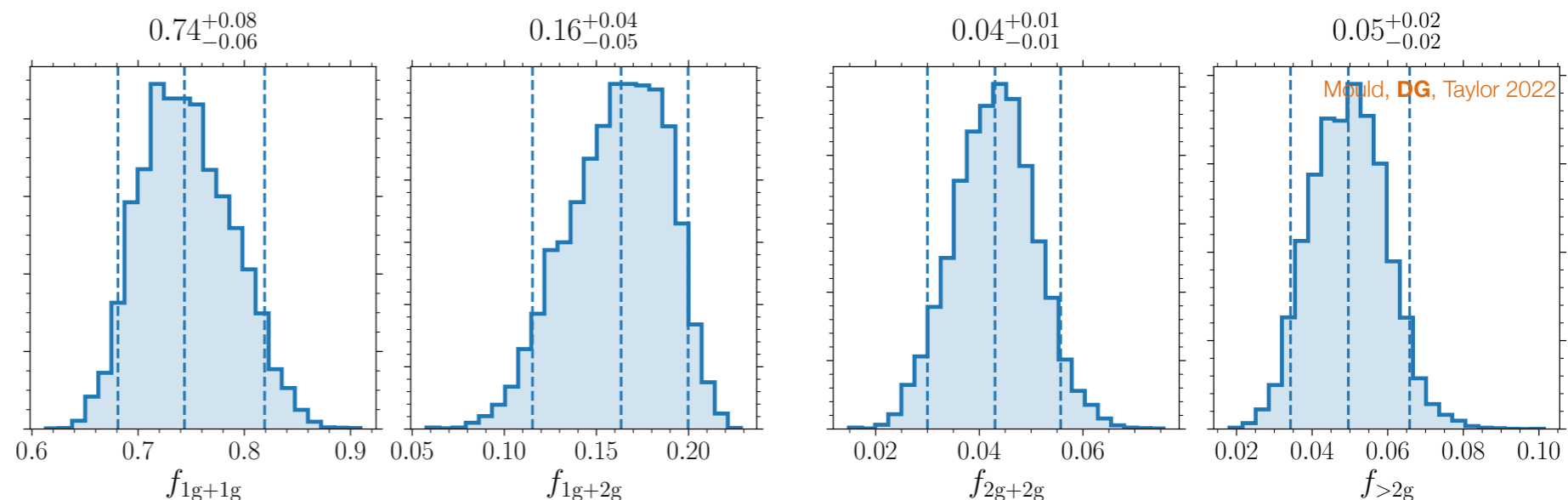
Consistently include hierarchical mergers when fitting the data?  
Awesome but the population is not analytic anymore.

- Cluster-inspired training simulations
- FFT-based KDEs
- Neural network **~70k parameters**
- Selection-effect modeling
- Hierarchical Bayesian analysis with nested sampling

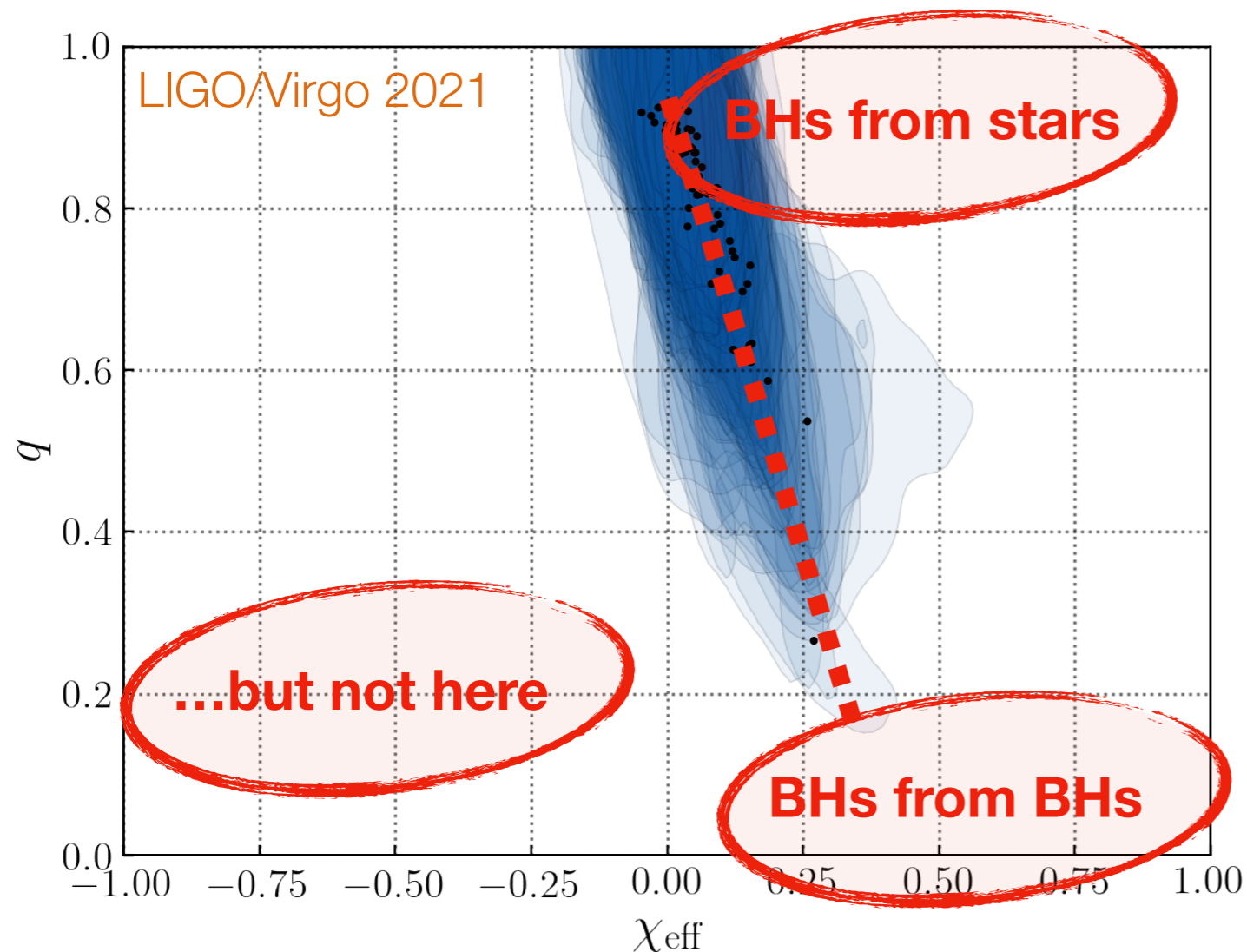


- **We can tell the generations apart**
- Additional structure in the gap due to higher generations
- Similar results from a cluster-tuned model

Kimball+ 2021, 2022



# What's breaking the symmetry?



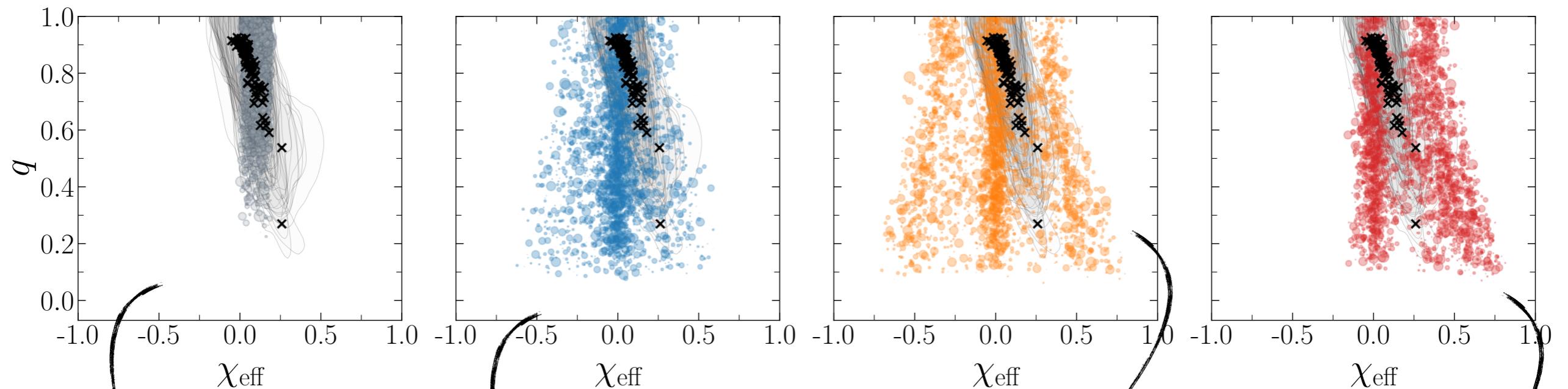
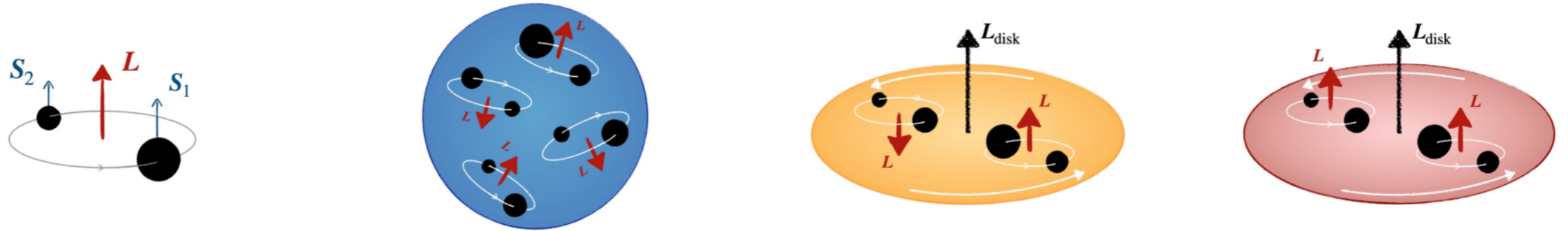
- Hierarchical mergers make small mass ratios...
- Hierarchical mergers make big spins...
- But how on Earth only **positive** effective spins?
- Why not **negative** as well?
- There must be a preferential direction

We argue... Santini **DG+** 2023

**The symmetry of the environment is the secret!**

# Here comes the disk!

Santini **DG+** 2023



**Field binaries**

**Clusters**  
(spherical symmetry)

**Disks?**  
(co/counter-alignment)

**Disks!**  
(prograde only)

A mix of grey and red gives a decent fit!

# Why I think repeated mergers are exciting

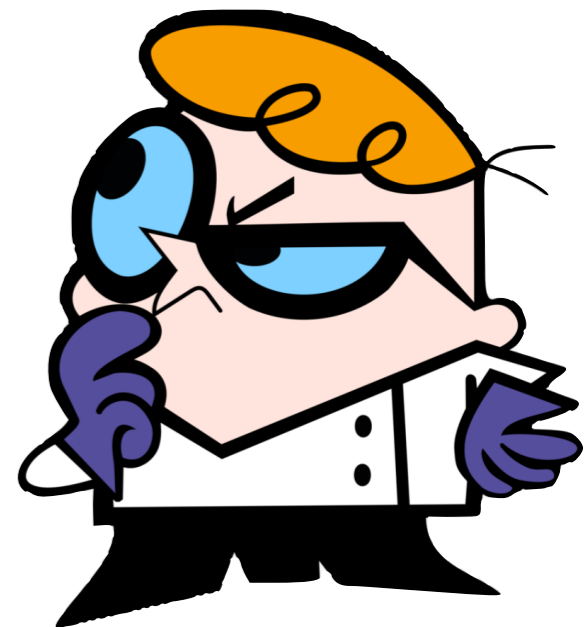
Remember that

**gravitational waves = relativity + astrophysics**

Relativity is clean, astrophysics is dirty...

... but relativity is “vacuum”, astrophysics is full of stuff to discover

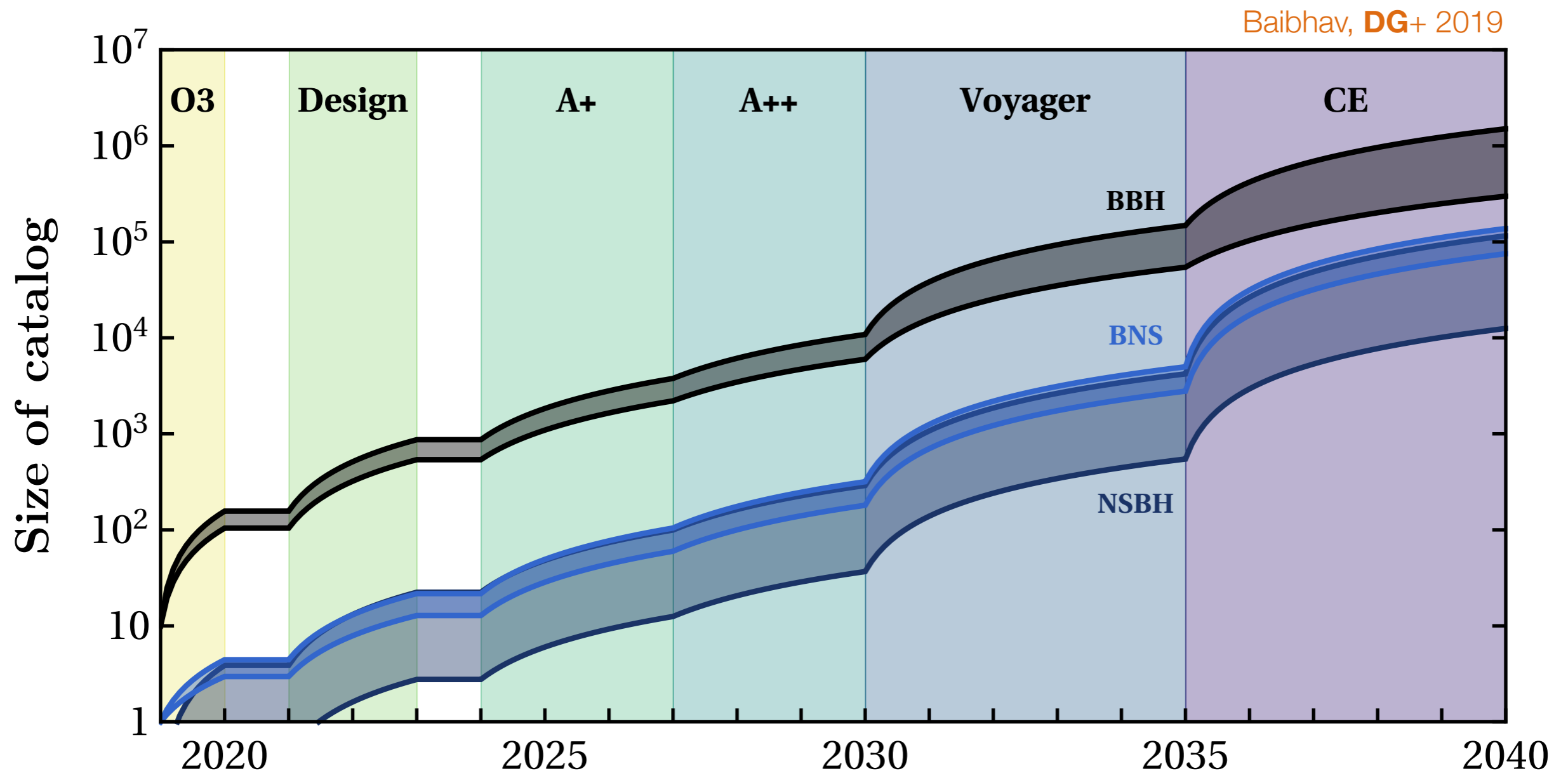
Hierarchical mergers largely rely on **relativity** while providing key insights on the underlying **astrophysics**



# The best is yet to come

About 100 surprises so far, thousands in ~5 yrs, millions in ~20 years

- Large statistics: details emerging at the **population** level
- Many events means **rare outliers**



# Merge many times

**Davide Gerosa**  
University of Milano-Bicocca

davide.gerosa@unimib.it  
www.davidegerosa.com

Jul 26th, 2024  
EREP 2024  
Coimbra, Portugal







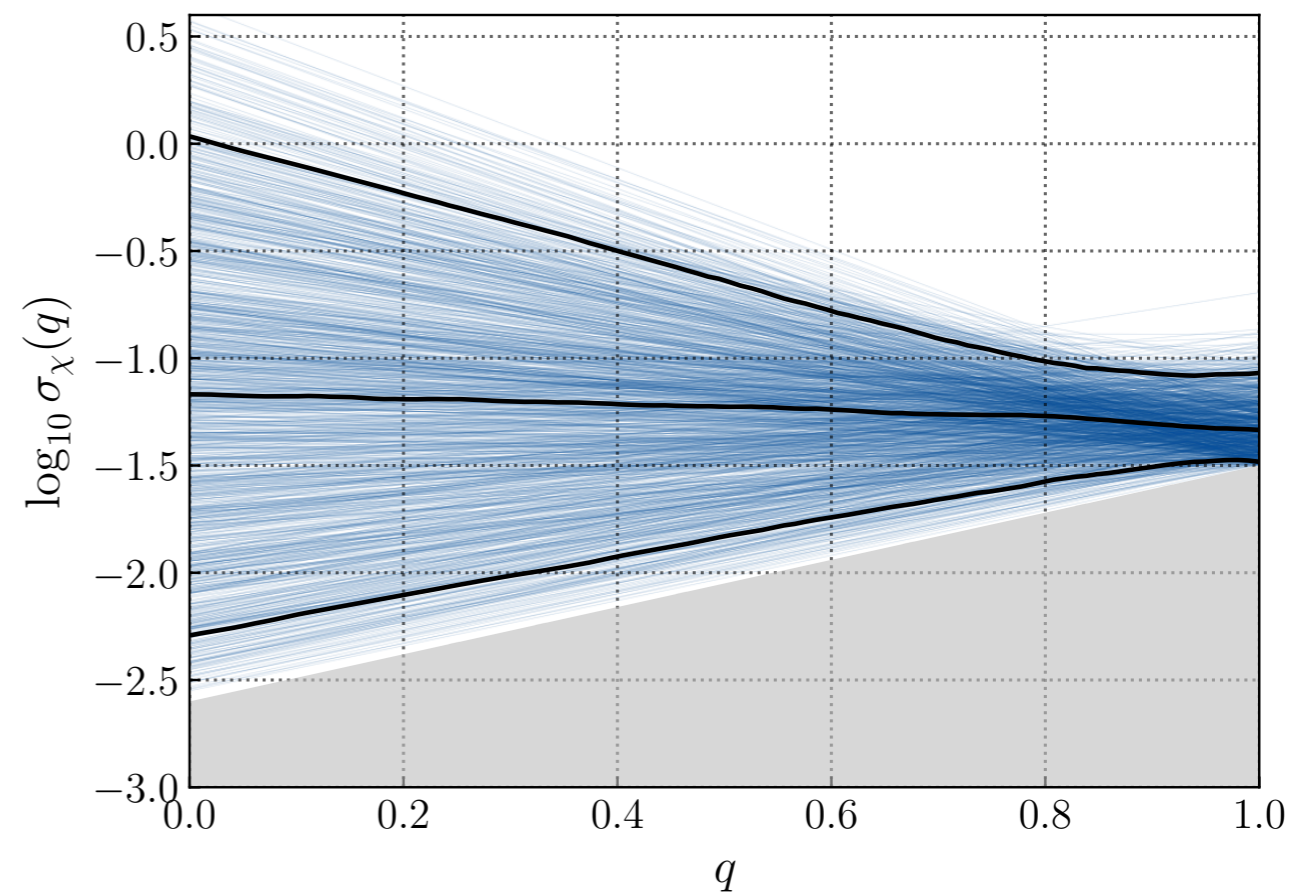
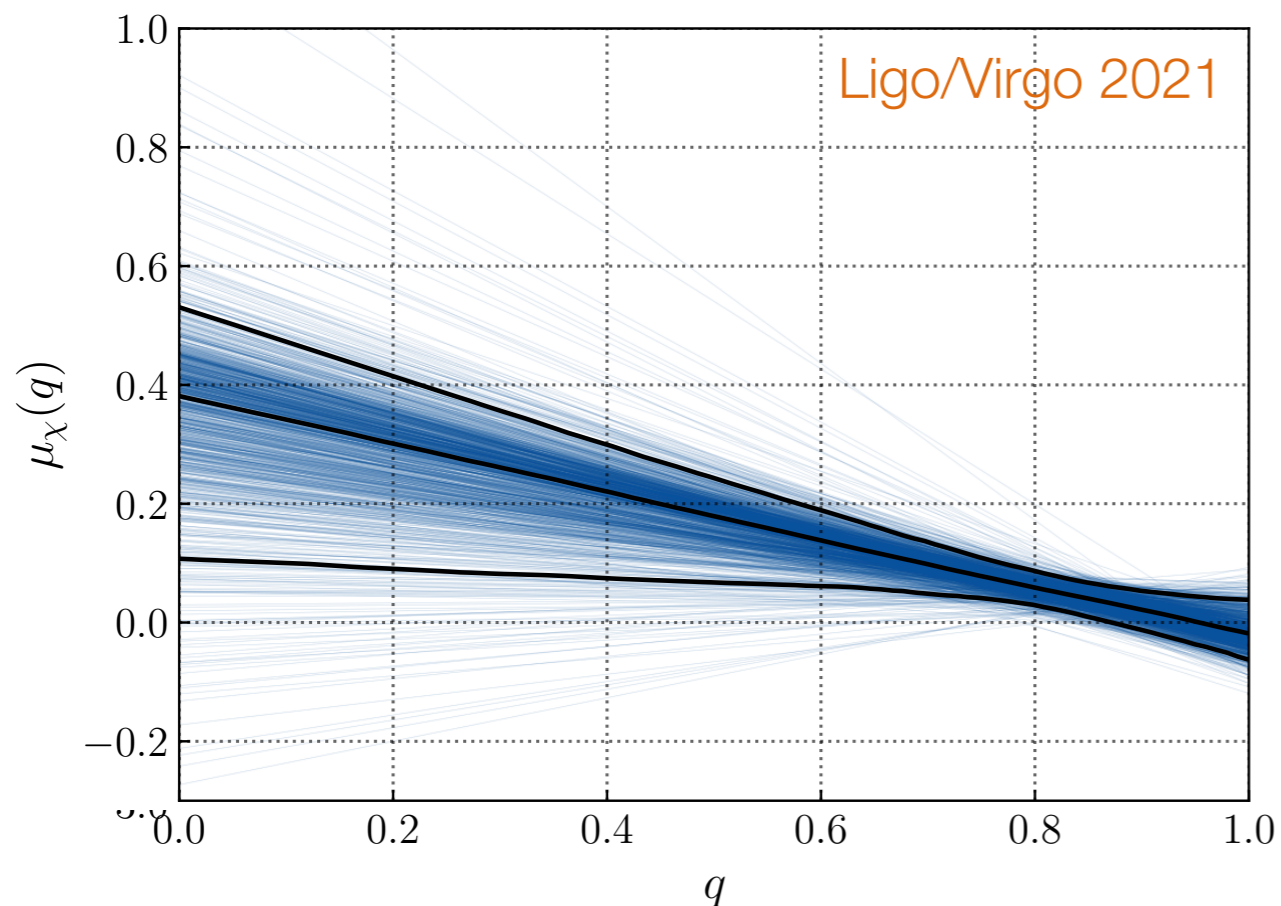
# A targeted population model

A linear population model for the first two moments of the conditional distribution  
(Could this be a restrictive assumption?) **Future work**

$$p(\chi_{\text{eff}}|q) = \frac{1}{\sqrt{2\pi\sigma_{\chi}^2(q)}} \exp\left\{-\frac{[\chi_{\text{eff}} - \mu_{\chi}(q)]^2}{2\sigma_{\chi}^2(q)}\right\}$$

$$\mu_{\chi}(q) = \mu_0 - \mu_1(1 - q),$$

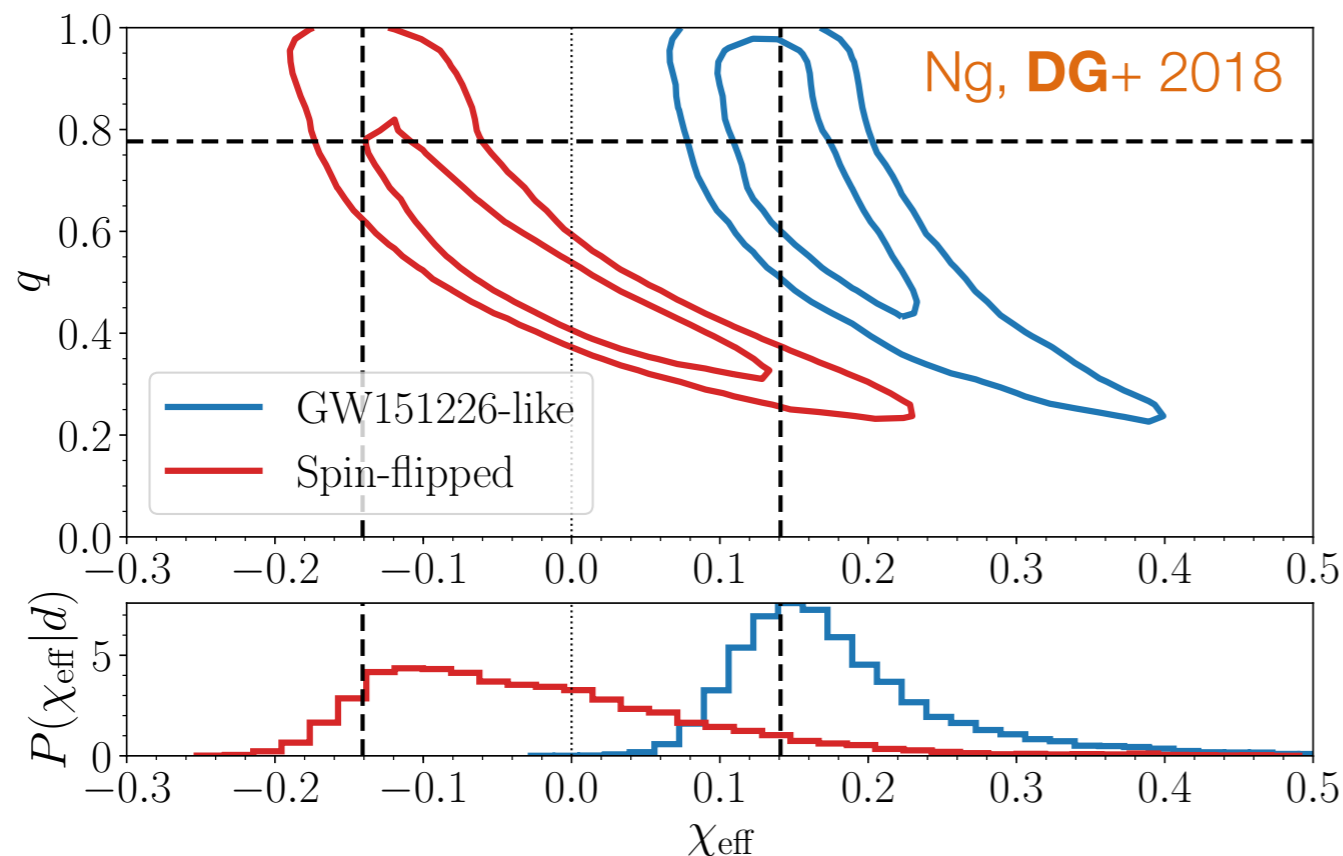
$$\log_{10} \sigma_{\chi}(q) = \log_{10} \sigma_0 - \log_{10} \sigma_1(1 - q). \quad \text{Callister+ 2021}$$



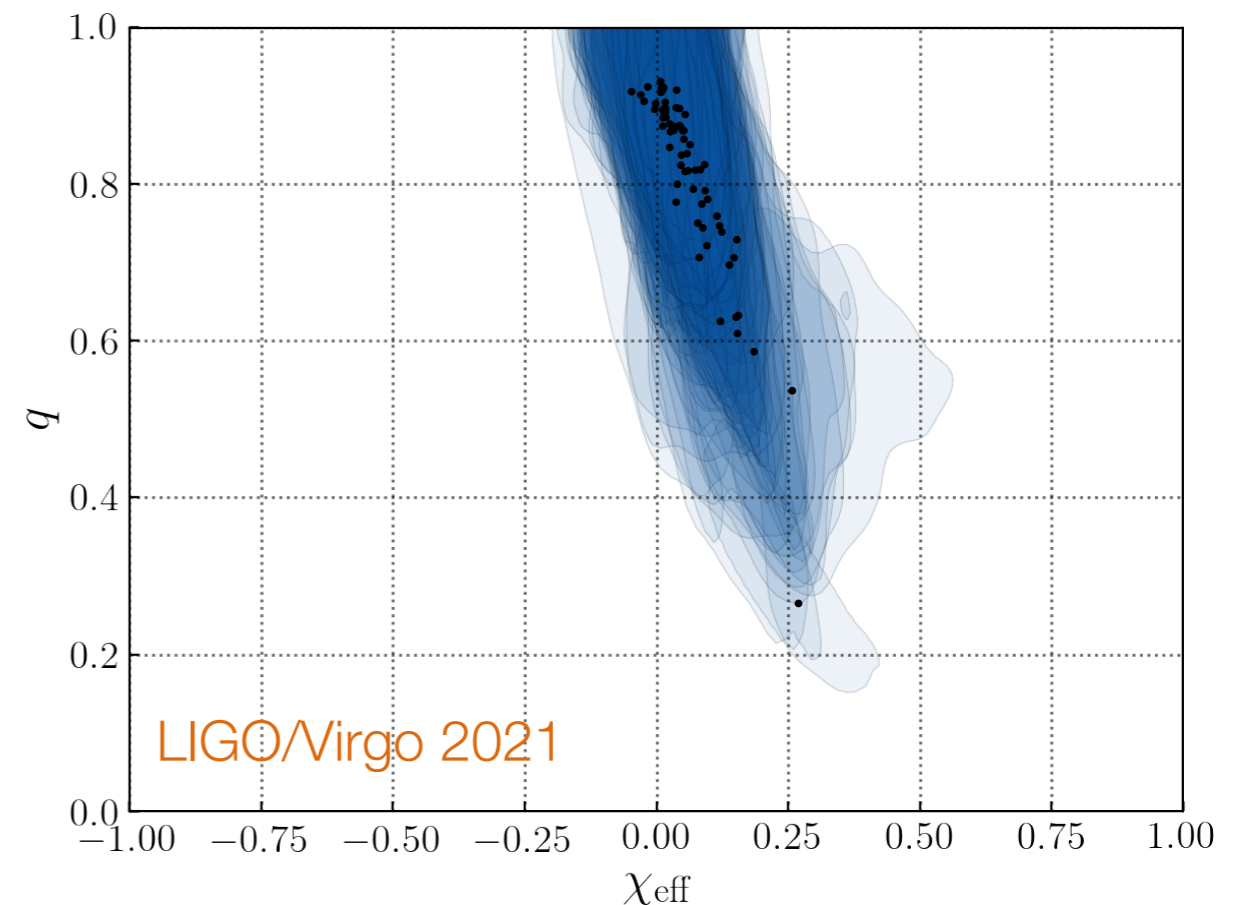
# Careful, it's a population correlation!

- $q$  and  $\chi_{\text{eff}}$  are known to correlate at the single-event level! Ng, **DG+** 2018
- Both enter the waveform phase at 1.5PN Cutler Flanagan 1994
- But this is something else!
- Potential caveat: waveform systematics leaking into the population fit?
- Combination with selection effects? Future work

Single-event correlation: **waveforms**



Population correlation: **astrophysics?**



# Let's parametrize this idea...

Mixing fraction  $f_{\text{disk}} \in [0, 1]$

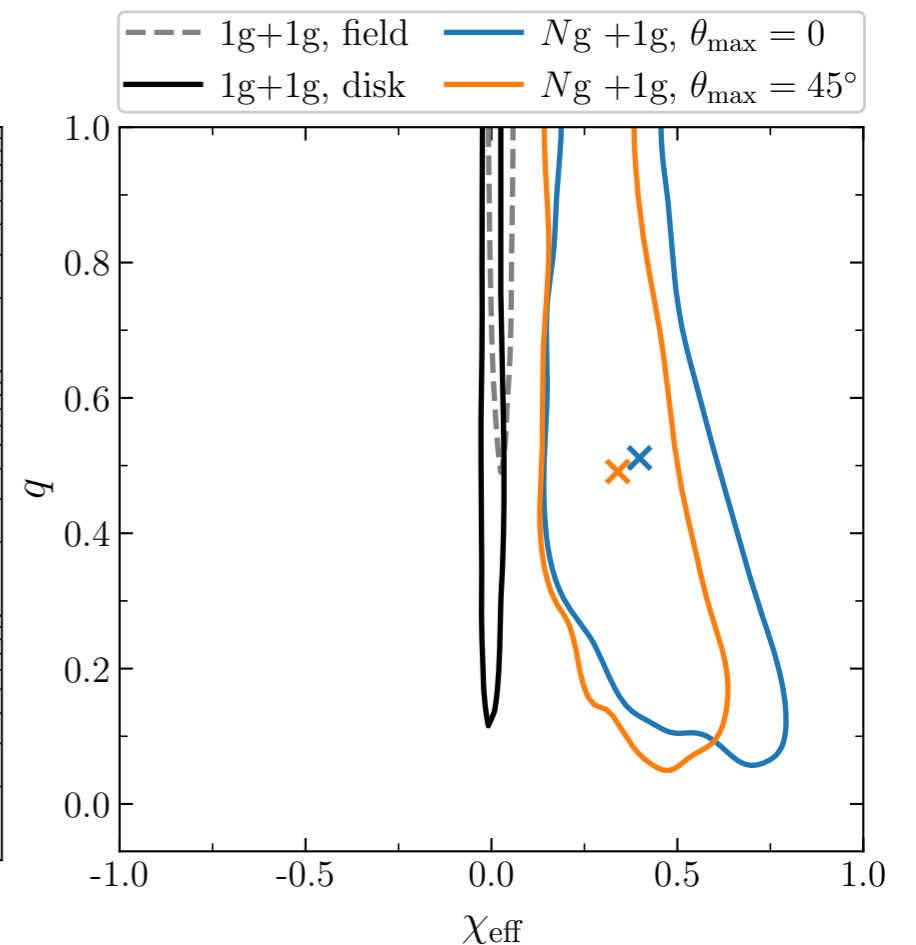
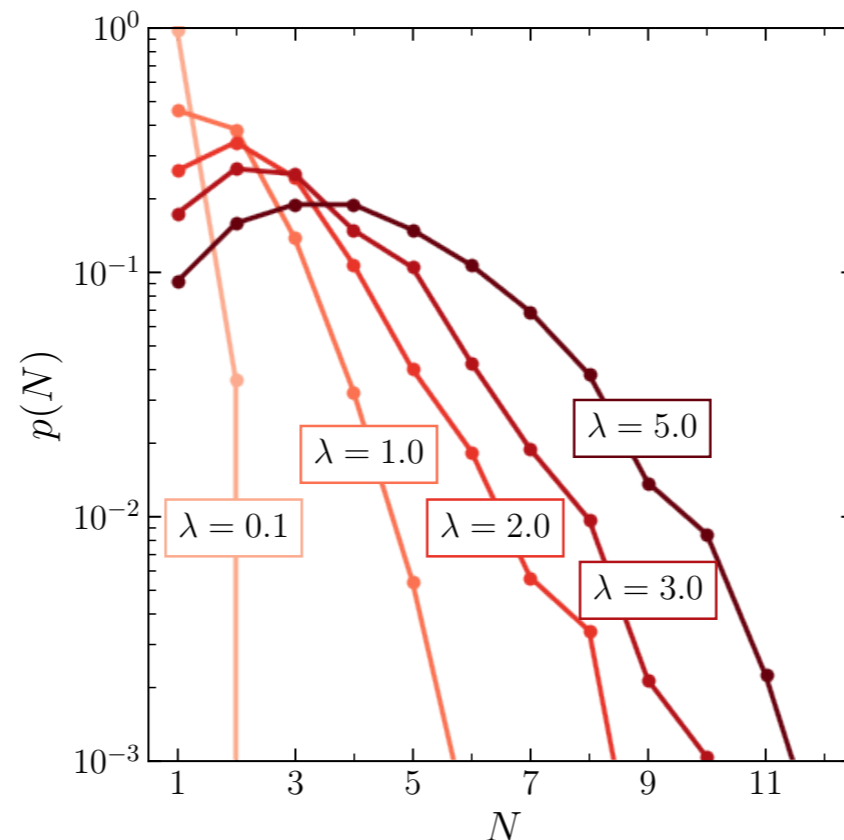
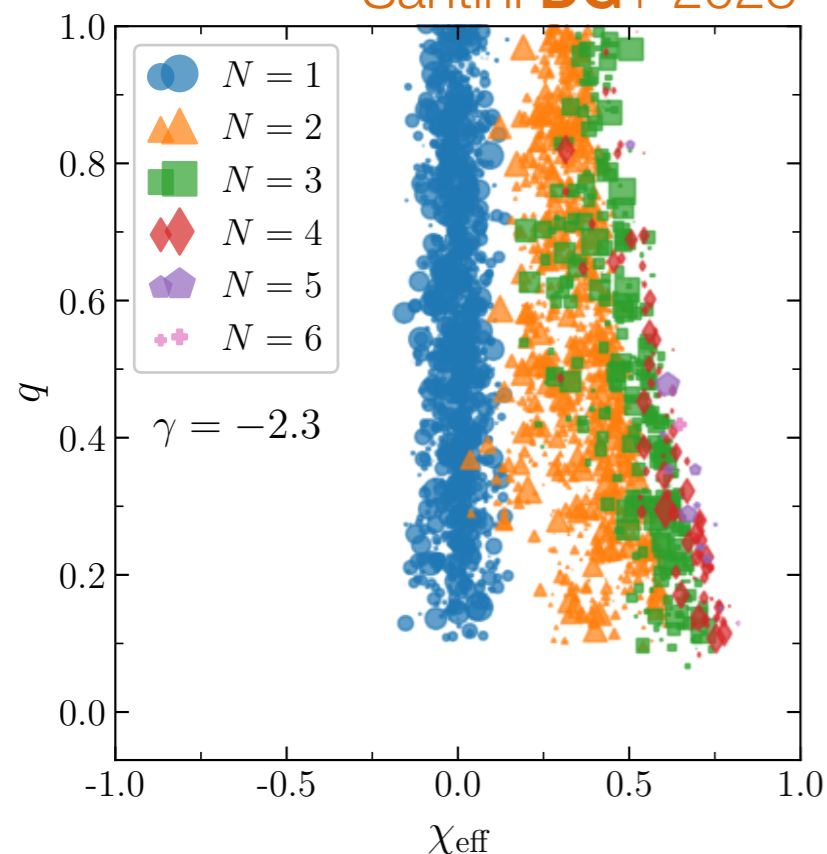
Field binaries  $p(m_1) \propto m_1^\alpha$   $p(m_2|m_1) \propto m_2^\beta$

Disk binaries  $p(m) \propto m^\gamma$  Spins  $\chi \in [0, \chi_{\text{max}}]$

Ng+1g merger chains  $N \sim \text{Poisson}(\lambda)$

Yang+ 2019

Santini **DG+** 2023



# Let's parametrize this idea...

Mixing fraction  $f_{\text{disk}} \in [0, 1]$

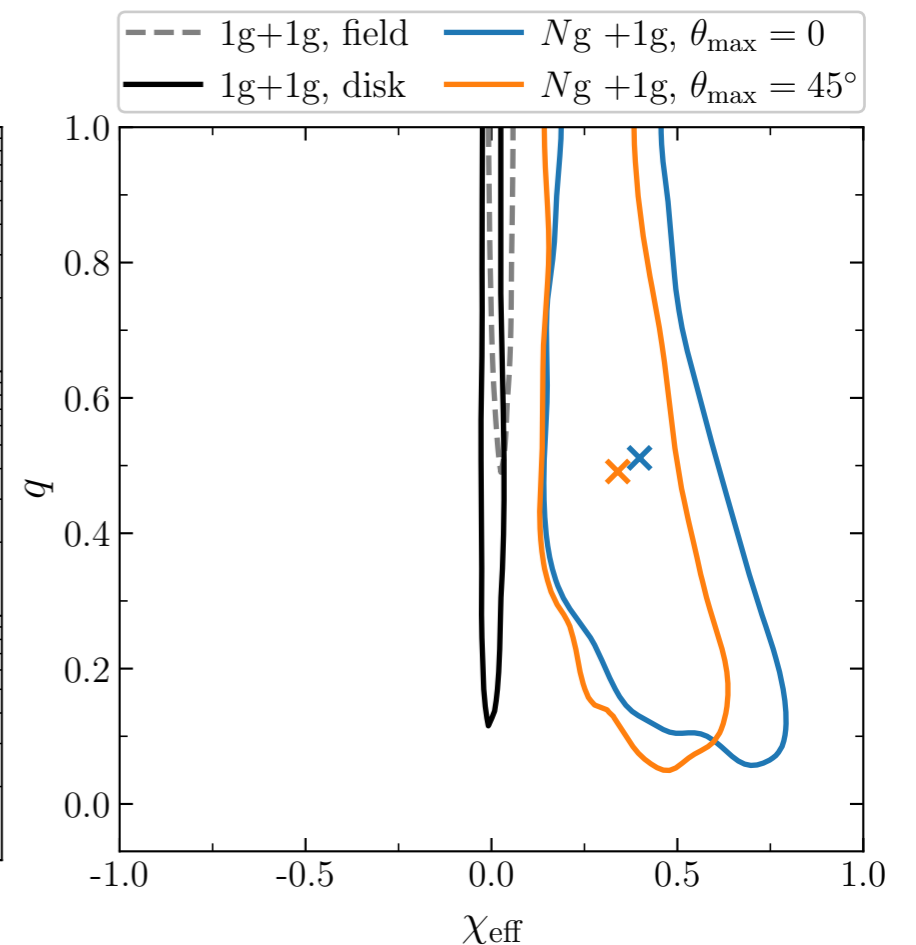
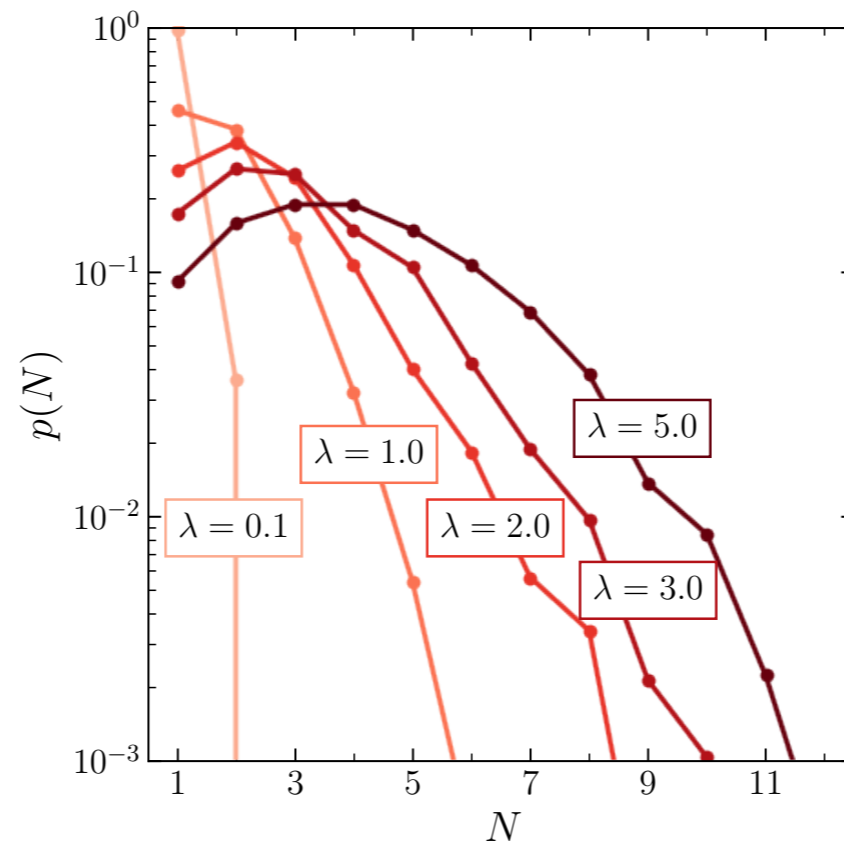
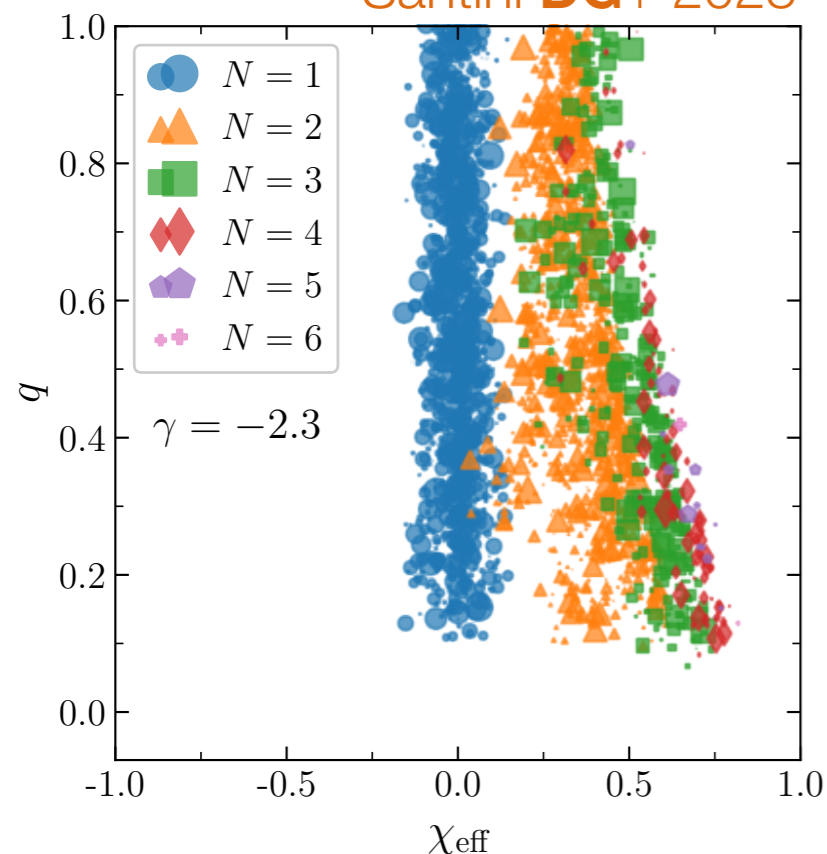
Field binaries  $p(m_1) \propto m_1^\alpha$   $p(m_2|m_1) \propto m_2^\beta$

Disk binaries  $p(m) \propto m^\gamma$  Spins  $\chi \in [0, \chi_{\text{max}}]$

Ng+1g merger chains  $N \sim \text{Poisson}(\lambda)$

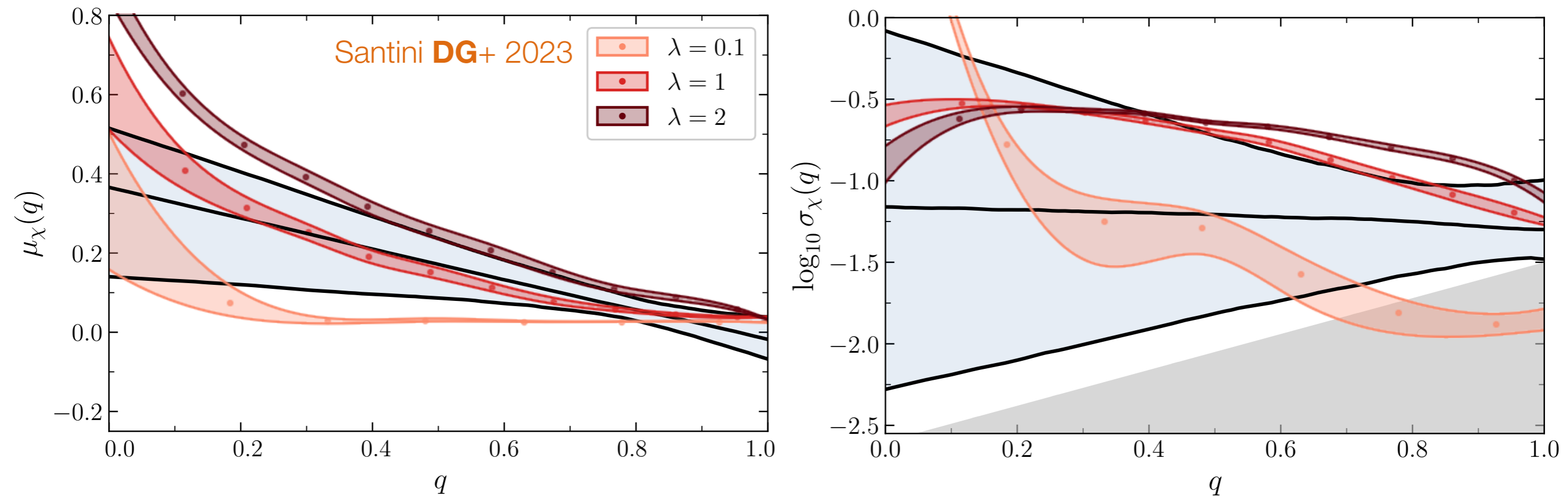
Yang+ 2019

Santini **DG+** 2023



# Looks like it works

Fiducial model:  $\alpha = -3.5$   $\beta = 1.1$   $\gamma = -2.3$   $f_{\text{disk}} = 0.2$   $\chi_{\text{max}} = 0.05$



- Error bars via statistical bootstrapping
- Grid exploration with a simple least-square optimization
- **Careful:** this is not a full Bayesian fit. The model is just too simple.
- The focus here is targeting the key trend.
- Only other astro attempt also uses disks McKernan+ 2021, Vaccaro+ 2023