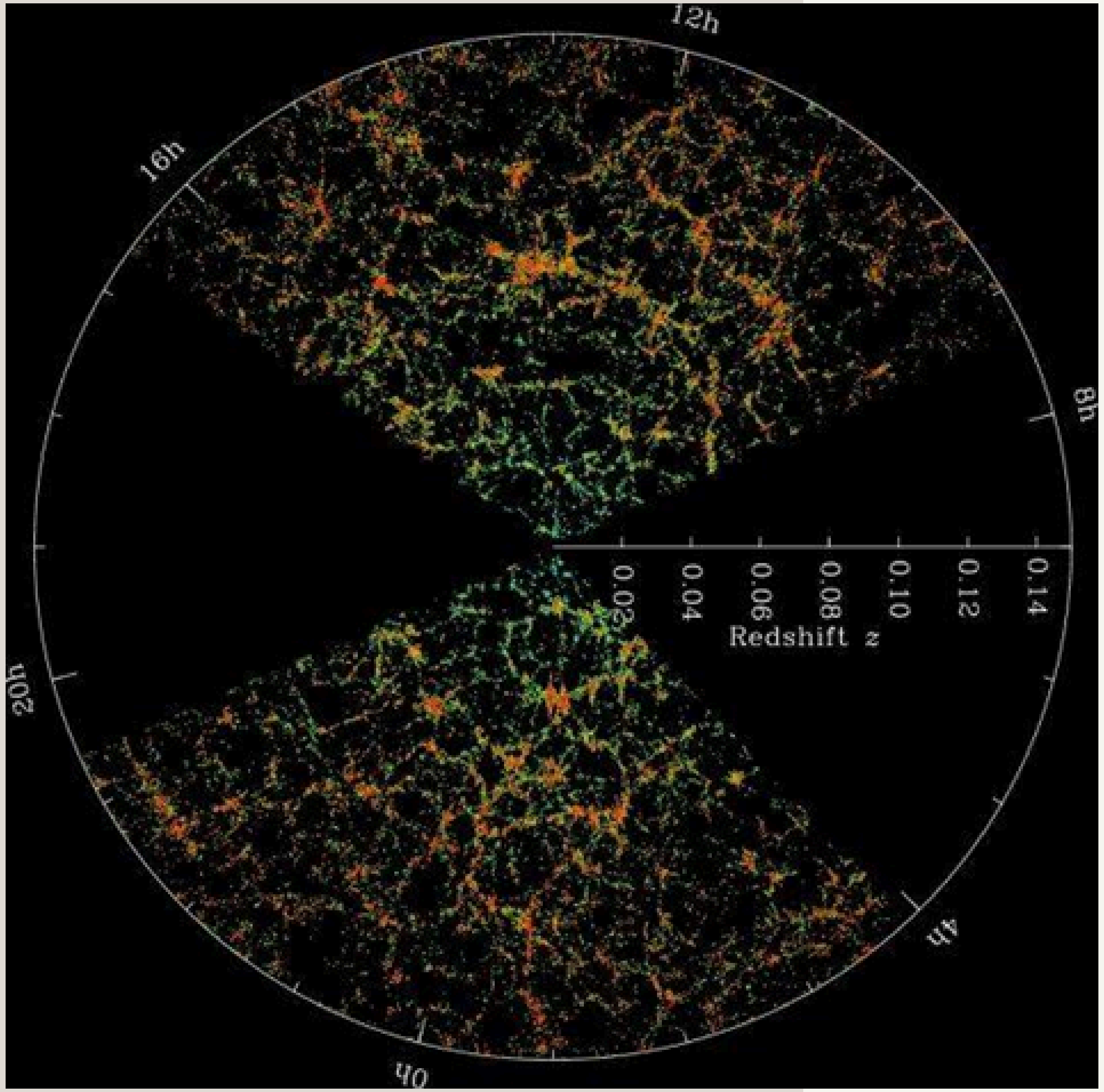

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On the torsion, nonmetricity & gauge invariance in the formulation of the gravitational field equation as an equation of state and in the laws of black hole thermodynamics

Author: Jhan Nicolás Martínez Lobo
Advisor: Yeinzon Rodríguez García
Co-Advisor: José Fernando Rodríguez

1.RG:Review

The Metric

$$g_{\mu\nu},$$

tensor comparison at different points,

$$\Gamma^{\rho}_{\mu\nu}(g_{\mu\nu}), \quad \Rightarrow R^{\lambda}{}_{\rho\mu\nu},$$

and field equations are given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu},$$

A. Einstein, A. der Wissenschaften, 1915.

1.1.RG:Lovelock

$$G_{\mu\nu} = 0$$

- 4 dimensions,
- minimal gravitational coupling with matter,
- field equations up to second order,
- divergenceless,

D. Lovelock, J. Math. Phys., 1971.

1.2.RG:Gauge

$$h_k{}^\mu, A^{ij}{}_\mu,$$

Global transformations

$$\mathcal{L}_M = \mathcal{L}_M(\phi, \partial_k \phi),$$

$$\Rightarrow$$

Local transformations

$$\tilde{\mathcal{L}}_M = b\mathcal{L}_M(\phi, \nabla_k \phi),$$

$$\nabla_k \phi = h_k{}^\mu \left(\partial_\mu + \frac{1}{2} A^{ij}{}_\mu \Sigma_{ij} \right) \phi,$$

M. Blagojevic, CRC Press, 2001.

1.2.RG:Gauge

Lorentz and traslation field strength are given by:

$$F^{ij}{}_{\mu\nu} \equiv \partial_\mu A^{ij}{}_\nu - \partial_\nu A^{ij}{}_\mu + A^i{}_{s\mu} A^{sj}{}_\nu - A^i{}_{s\nu} A^{sj}{}_\mu,$$

$$F^s{}_{kl} = h_k{}^\mu h_l{}^\nu (\nabla_\mu b^s{}_\nu - \nabla_\nu b^s{}_\mu) \equiv h_k{}^\mu h_l{}^\nu F^s{}_{\mu\nu},$$

which gives the PGT:

$$\widetilde{\mathcal{L}} = b\mathcal{L}_F(F^{ij}{}_{kl}, F^s{}_{kl}) + b\mathcal{L}_M(\phi, \nabla_k \phi),$$

M. Blagojevic, CRC Press, 2001.

1.3.Laws of black hole dynamics

Zeroth law:

$$\kappa^2 \equiv -\frac{1}{2}\xi^{\alpha;\beta}\xi_{\alpha;\beta}, \qquad \kappa_{;\alpha} = 0,$$

First law:

$$(M, J, A) \to (M + \delta M, J + \delta J, A + \delta A),$$

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_H \delta J,$$

J. M. Bardeen, B. Carter, S. W. Hawking, Commun. Math. Phys., 1973.

1.3.Laws of black hole dynamics

second law:

$$T = \frac{\hbar}{2\pi}\kappa, \qquad S = \frac{k_{\mathrm{B}}A}{4\ell_{\mathrm{P}}^2},$$

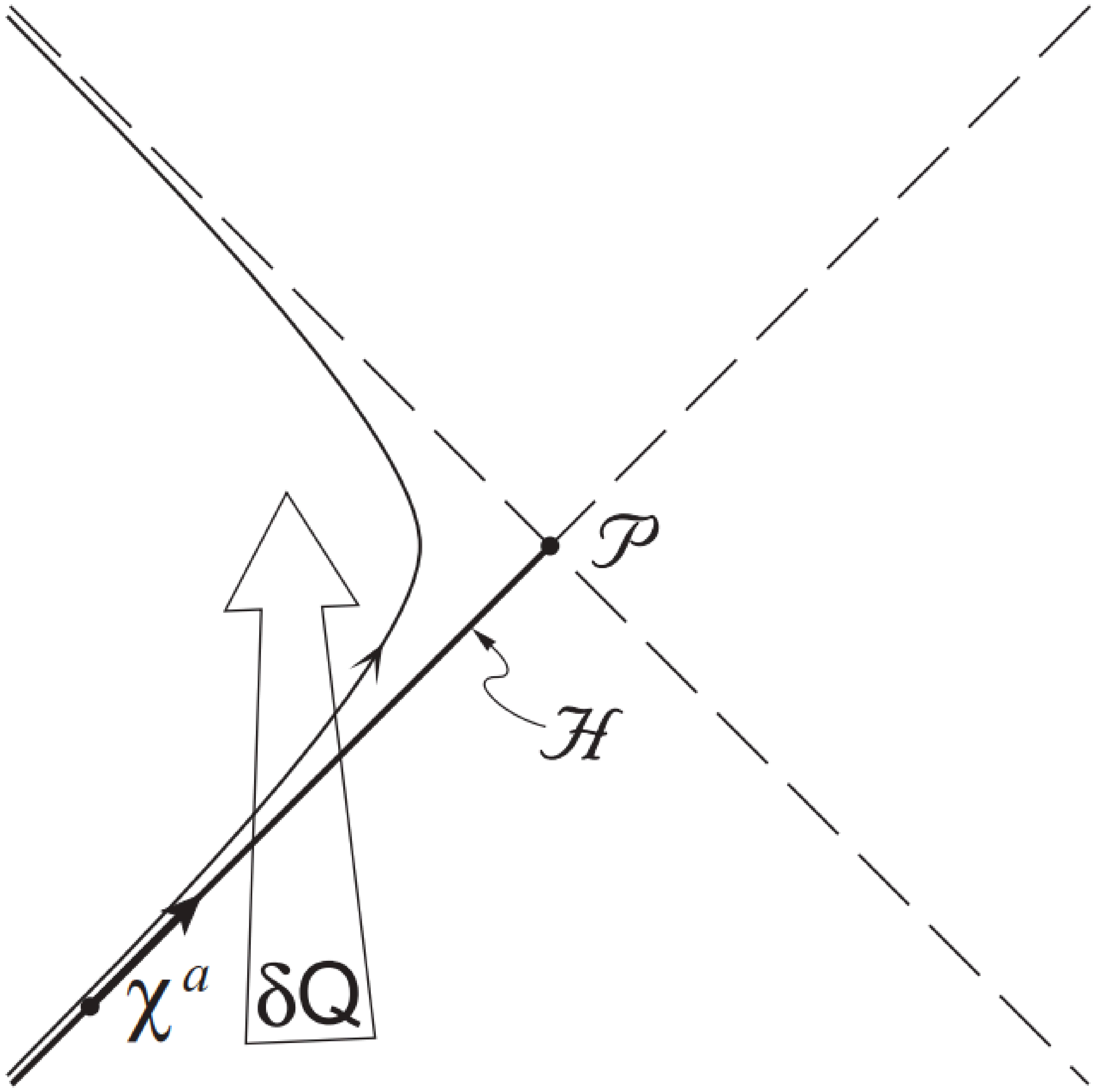
therefore,

$$\Delta S_{Tot} = \Delta S + \Delta S_M \geq 0,$$

S. W. Hawking, Nature, 1974.

J. D. Bekenstein, Phys. Rev. D, 1974.

1.4.RG:Thermodynamics



T. Jacobson, Phys. Rev. Lett., 1995.

1.4.1.Unruh effect

In Minkowski spacetime, accelerated observers measure a thermal bath of temperature:

$$T = \frac{\hbar\kappa}{2\pi},$$

where κ is the acceleration.

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.2.Energy flux

$$\delta Q = \int_{\mathcal{H}} T_{\mu\nu}\chi^{\mu}\mathrm{d}\Sigma^{\nu},$$

$$\chi^{\mu} = -\kappa\lambda k^{\mu}, \qquad d\Sigma^{\mu} = k^{\mu} d\lambda d\mathcal{A},$$

therefore,

$$\delta Q = -\kappa \int_{\mathcal{H}} \lambda T_{\mu\nu} k^{\mu} k^{\nu} d\lambda d\mathcal{A},$$

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.3.Horizon entropy

$$dS = \eta \delta \mathcal{A},$$

$$\delta \mathcal{A} = \int_{\mathcal{H}} \theta \mathrm{d}\lambda \mathrm{d}\mathcal{A},$$

using the Raychaudhuri equation,

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{\mu\nu}k^\mu k^\nu,$$

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.4. Area variation

$$\theta = -\lambda R_{\mu\nu} k^{\mu} k^{\nu},$$

then,

$$\delta \mathcal{A} = -\int_{\mathcal{H}} \lambda R_{\mu\nu} k^{\mu} k^{\nu} d\lambda d\mathcal{A},$$

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.5. Clausius relation

$$\delta Q = TdS = \frac{\hbar\kappa}{2\pi}\eta\delta\mathcal{A},$$

$$\kappa\int_{\mathcal{H}}\lambda T_{\mu\nu}k^{\mu}k^{\nu}d\lambda d\mathcal{A} = \frac{\hbar\kappa}{2\pi}\eta\int_{\mathcal{H}}\lambda R_{\mu\nu}k^{\mu}k^{\nu}d\lambda d\mathcal{A},$$

but, for an arbitrary horizon:

$$T_{\mu\nu}k^{\mu}k^{\nu} = \frac{\hbar\eta}{2\pi}R_{\mu\nu}k^{\mu}k^{\nu}, \qquad \Rightarrow \qquad \frac{2\pi}{\hbar\eta}T_{\mu\nu} = R_{\mu\nu} + fg_{\mu\nu},$$

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.6.Field equation

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \Rightarrow \qquad f = -R/2 + \Lambda,$$

finally,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{2\pi}{\hbar\eta} T_{\mu\nu},$$

where,

$$\eta = \frac{1}{4\hbar G}, \qquad \Rightarrow \qquad \eta^{-1/2} = 2 l_p,$$

T. Jacobson, Phys. Rev. Lett., 1995.

1.4.7.¿Field equation or state equation?

- Viewed in this way, RG is an equation of state.
- It may be no more appropriate to canonically quantize the Einstein equation than it would be to quantize the wave equation for sound in air
- The cosmological constant remains as enigmatic as ever.
- High amplitude and frequency perturbations of the gravitational field could no longer be described by GR due to the lack of local equilibrium conditions.
- Entropy production in the free expansion of a gas is not described by Clausius' Relation.
- Deformations of ideal crystal can be described by a metric like in GR, but local deffects, require structures like torsion and non metricity to be described.

T. Jacobson, Phys. Rev. D, 1995.
F. Falk, J. Elast., 1981.
R. Kupferman et. al., J. Geom. Mech., 2015.
R. Kupferman et. al., Isr. J. Math., 2017.

2.RG: reasons to modify it

- Dark matter: $25.89\% \pm 0.57\%.$

N. Aghanim et. al., Astron. Astrophys.,2020.

- Dark energy: $69.11\% \pm 0.62\%.$

- Hubble tension: $H_0 = (67.4 \pm 0.5)\text{km/s/Mpc},$

$$H_0 = (73.52 \pm 1.62)\text{km/s/Mpc}.$$

A. G. Riess et al., Astrophys. J., 2018.

2.RG: reasons to modify it

- Singularities.

R. Penrose, Phys. Rev. Lett., 1965.

- Renormalizability and unitarity.

R. M. Wald, University of Chicago Press, 2010.
K. S. Stelle, Phys. Rev. D, 1977.

— 3.State of the art

	Lovelock	Gauge	Thermodynamics
$g_{\mu\nu}$	R D. Lovelock, J. Math. Phys., 1971.	$R + \mathcal{O}\left(R^2_{\mu\nu\alpha\beta}\right)$ F. W. Hehl, Nato science series B, 1979.	R T. Jacobson, Phys. Rev. Lett., 1995.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu},$$

— 3.State of the art

	Lovelock	Gauge	Thermodynamics
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$g_{\mu\nu}, T^{\mu}{}_{\alpha\beta}$	R A. Mardones et. al., Class Quant. Grav., 1991.	$R+\mathcal{O}\left(R^2_{\mu\nu\alpha\beta}, T^2_{\alpha\mu\nu}\right)$ F. W. Hehl, Nato science series B, 1979.	R T. De Lorenzo et. al., Phys. Rev. D., 2018.

$$\bar{G}_{\mu\nu} = 8\pi G\left(T^M_{\mu\nu} + \left(\bar{\nabla}_\sigma + T_\sigma\right)\left(\tau^\sigma{}_{\mu\nu} - \tau_{\mu\nu}{}^\sigma - \tau_{\nu\mu}{}^\sigma\right)\right),$$

$$S^\sigma{}_{\mu\nu} = 16\pi G \tau^\sigma{}_{\mu\nu},$$

N. J. Poplawski, Astron. Rev., 2013.
lM. Gasperini, Phys. Rev. Lett., 1986.
A. Kasem et. al., Int. J. Mod. Phys. A, 2021.
Y. Bonder, Int. J. Mod. Phys. D, 2016.
B. A. Costa et. al., Phys. Lett. B, 2024.
F. Cabral et. al., Class Quant. Grav., 2021.

4.MAG

$$\mathbf{R}^{1,3} \rtimes \mathrm{O}(1,3)\,, \quad \Rightarrow \quad \mathbf{R}^{1,3} \rtimes \mathrm{GL}(1,3)\,,$$

the non metricity tensor is defined as

$$Q_{\mu\alpha\beta} \equiv -\nabla_\mu g_{\alpha\beta}\,,$$

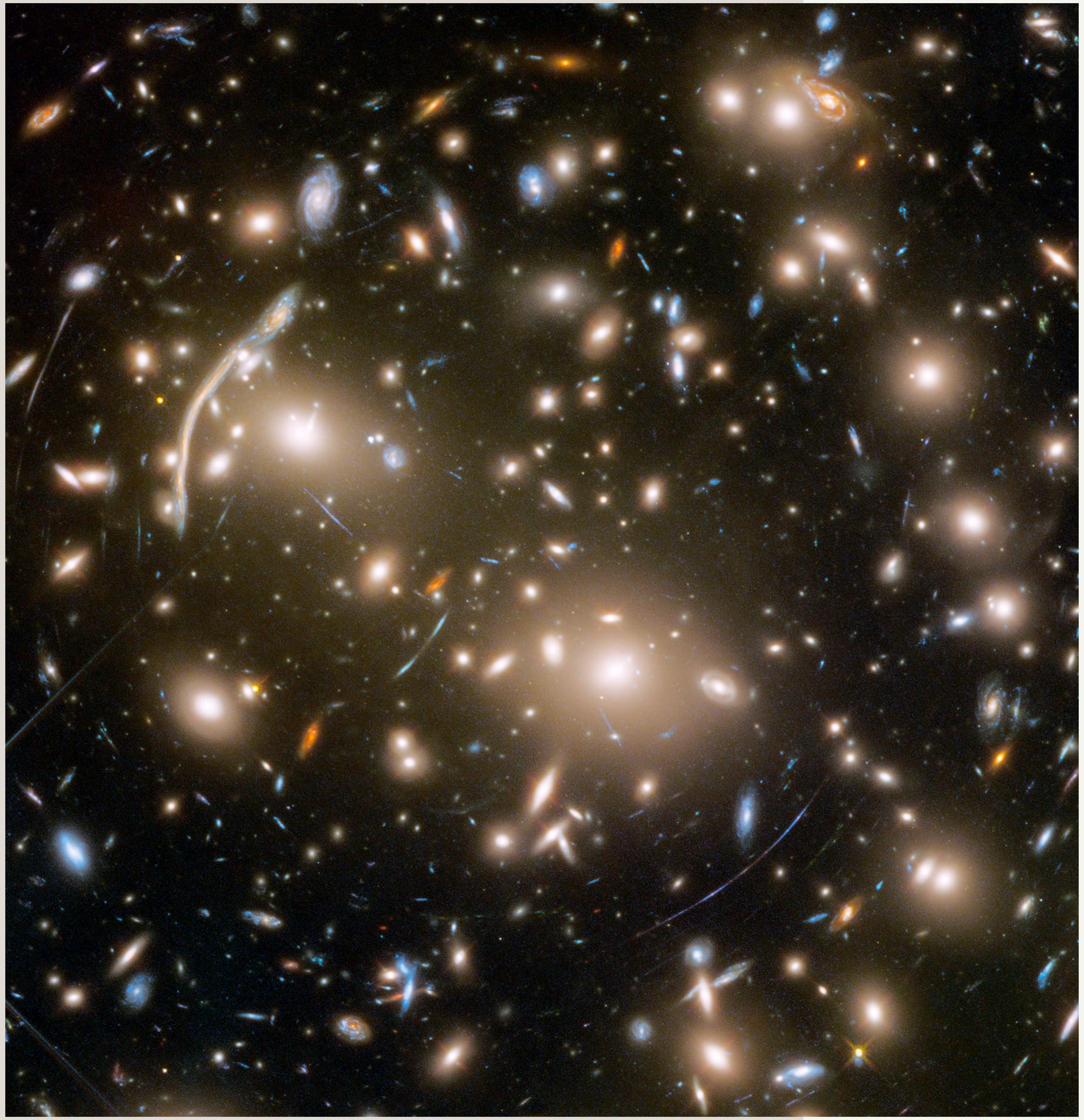
and modifies the inner product under parallel transport as

$$\partial_\rho(g_{\mu\nu}V^\mu W^\nu) = \nabla_\rho(g_{\mu\nu}V^\mu W^\nu) = -Q_{\rho\mu\nu}V^\mu W^\nu,$$

F. W. Hehl et. al., Phys. Rept., 1995.

— 4.MAG

	Lovelock	Gauge	Thermodynamics
$g_{\mu\nu}$	R D. Lovelock, J. Math. Phys., 1971.	$R+\mathcal{O}\left(R^2_{\mu\nu\alpha\beta}\right)$ F. W. Hehl, Nato science series B, 1979.	R T. Jacobson, Phys. Rev. Lett., 1995.
$g_{\mu\nu}, T^{\mu}{}_{\alpha\beta}$	R A. Mardones et. al., Class Quant. Grav., 1991.	$R+\mathcal{O}\left(R^2_{\mu\nu\alpha\beta}, T^2_{\alpha\mu\nu}\right)$ F. W. Hehl, Nato science series B, 1979.	R T. De Lorenzo et. al., Phys. Rev. D., 2018.
$g_{\mu\nu}, T^{\mu}{}_{\alpha\beta}, Q^{\mu}{}_{\alpha\beta}$	$R+\mathcal{O}\left(Q^2_{\alpha\mu\nu}\right)$ A. Jiménez-Cano, Theor. Phys. A., 2022.	$R+\mathcal{O}\left(R^2_{\mu\nu\alpha\beta}, T^2_{\alpha\mu\nu}, Q^2_{\alpha\mu\nu}\right)$ F. W. Hehl et. al., Phys. Rept., 1995.	¿?



Thanks a lot!