Theory of rare $b \to d\,\ell\ell$ decays

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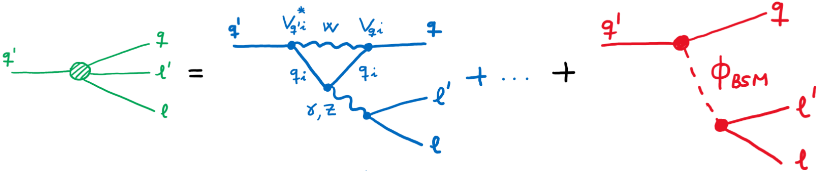
Rare decays probing BSM physics

- **FCNCs are loop and CKM suppressed in the SM.**



- **BSM contributions could be of same size as the SM.**

Bonus if ℓ are attached (rare decays):

- **SM lepton couplings are flavour universal, LU can be tested.**

- **If $\ell \neq \ell'$ (zero in the SM), LFC can be tested as well.**

Excellent place to search for BSM physics!

EFT approach to rare B decays

1. **Symmetries to build all O_i up to desired dimension ($D = 6$):**

$$\mathcal{H}_{\rm eff} \supset \frac{4\,G_F}{\sqrt{2}}\,V_{tq}^* V_{tb} \frac{\alpha_e}{4\pi} \sum_i c_i^{(\prime)}\,O_i^{(\prime)}\,, \quad c_i = C_i^{\rm SM(\prime)} + C_i^{(\prime)}\,,$$

$$\begin{aligned} O_7^{(\prime)} &= \frac{e}{16\pi^2}\, m_b\,(\bar q_{L\,(R)}\,\sigma_{\mu\nu}\,b_{R\,(L)})\,F^{\mu\nu}\,, \\ O_8^{(\prime)} &= \frac{g_s}{16\pi^2}\, m_b\,(\bar q_{L\,(R)}\,\sigma_{\mu\nu}\,T^a b_{R\,(L)})\,G_a^{\mu\nu}\,, \\ O_{9\,(10)}^{(\prime)} &= (\bar q_{L\,(R)}\gamma_\mu b_{L\,(R)})(\bar\ell\,\gamma^\mu(\gamma_5)\,\ell)\,, \ldots \end{aligned}$$

2. **Compute $C_i(\mu_{\rm EW})$ and RGEs to go down $\mu_{\rm low} \approx m_b$.**

$$C_7^{\rm SM}(m_b) \approx -0.3\,,\; C_8^{\rm SM}(m_b) \approx -0.15\,,\; C_9^{\rm SM}(m_b) \approx 4.1\,,\; C_{10}^{\rm SM}(m_b) \approx -4.2\,.$$

3. **$\langle O_i(\mu_{\rm low})\rangle$ from non-perturbative techniques (Lattice, LCSR, ...)**

4. **Include resonances (or better avoid them).**

$b \to s\,\ell\ell$ transitions

- **Over the past decade a set of tensions with SM predictions has emerged in $b \to s\,\ell\ell$ transitions:**

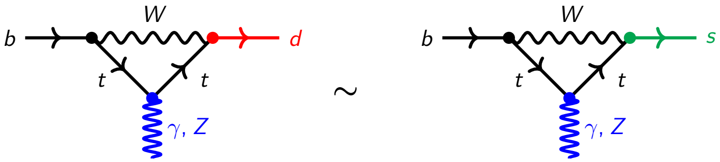
  1. **Branching ratios:** are below the SM values.
  2. **Angular obserbables:** $4\,\sigma$ deviation from the SM in global fits.
  3. **LU ratios:** Experimental LHCb update of R_K revealed consistency with the SM.

- **1 - 2 can be explained consistently together by NP contribution in a single operator:**

$$C_9^{(bs\mu)} \cdot O_9^{(bs\mu)} \approx -1 \cdot (\bar{s}_L \gamma_\mu b_L)(\bar{\mu}\,\gamma^\mu\,\mu)$$

- **3 suggests discrepancies with the SM in $b \to s\,e^+e^-$, specifically reduced BRs and distorted angular distributions.**
- **While this points to NP, further scrutiny is required before firm conclusions can be drawn.**

- **Differences between $b \to d\,\ell\ell$ & $b \to s\,\ell\ell$ in the SM:**



(1) CKM matrix elements: V_{td} vs V_{ts}, **(2)** Light quark masses: m_d vs m_s

$$C_i^{(b\to d)} \approx C_i^{(b\to s)}, \text{ (CKMs factorized in } \mathcal{H}_{\text{eff}})$$

$$C_i'^{(b\to d)} \approx \left(\frac{m_d}{m_s}\right) C_i'^{(b\to s)}, \text{ } (O_i' \text{ chiral suppression})$$

- **A violation would signal additional BSM sources of quark flavor violation (beyond (1) and (2)); an agreement would indicate similar effects as the current tensions (maybe NP?).**

Global fit of $b \to d\,\ell\ell$ transitions

What observables do we use?

- **Branching ratios of rare $b \to d\,\mu^+\mu^-, \gamma$ decays:**

  1. $B^+ \to \pi^+\,\mu^+\mu^-$ (3 binned), 1509.00414.
  2. $B_s^0 \to \bar{K}^{*0}\mu^+\mu^-$ (full integrated), 1804.07167.
  3. $B^0 \to \mu^+\mu^-$, 2108.09283.
  4. $\bar{B} \to X_d\gamma$, 1005.4087,1503.01789.

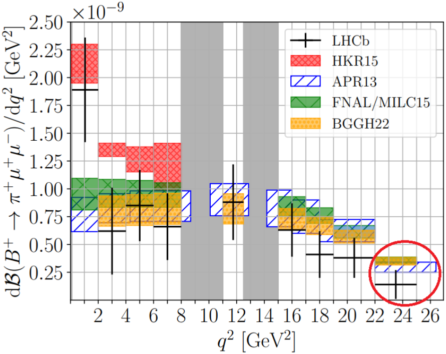
- **In total we use 6 observables, compared with $b \to s\,\ell\ell$ transitions:**

$$\frac{\#\text{ obs. exp. }(b \to d\,\ell\ell)}{\#\text{ obs. exp. }(b \to s\,\ell\ell)} \sim \frac{1}{50} \text{ (ideally 1)}$$

$B^+ \to \pi^+ \mu^+ \mu^-$

2209.04457

k	$[q^2_{\min}, q^2_{\max}]$ [GeV^2]	$\mathcal{B}_k^{(B\pi)}$ SM [10^{-9} GeV^{-2}]	$\mathcal{B}_k^{(B\pi)}$ experiment [10^{-9} GeV^{-2}]
1	[2, 4]	$0.80 \pm 0.12 \pm 0.05 \pm 0.04$ (FFs, CKMs, scale)	$0.62^{+0.39}_{-0.33} \pm 0.02$ (stat., syst.)
2	[4, 6]	$0.81 \pm 0.12 \pm 0.05 \pm 0.05$	$0.85^{+0.32}_{-0.27} \pm 0.02$
3	[6, 8]	$0.82 \pm 0.11 \pm 0.05 \pm 0.07$	$0.66^{+0.30}_{-0.25} \pm 0.02$
4	[11, 12.5]	$0.82 \pm 0.09 \pm 0.05 \pm 0.09$	$0.88^{+0.34}_{-0.29} \pm 0.03$
5	[15, 17]	$0.73 \pm 0.06 \pm 0.04 \pm 0.06$	$0.63^{+0.24}_{-0.19} \pm 0.02$
6	[17, 19]	$0.67 \pm 0.05 \pm 0.04 \pm 0.05$	$0.41^{+0.21}_{-0.17} \pm 0.01$
7	[19, 22]	$0.57 \pm 0.03 \pm 0.03 \pm 0.04$	$0.38^{+0.18}_{-0.15} \pm 0.01$
8	[22, 25]	$0.35 \pm 0.02 \pm 0.02 \pm 0.02$	$0.14^{+0.13}_{-0.09} \pm 0.01$
9	[15, 22]	$0.64 \pm 0.04 \pm 0.04 \pm 0.05$	$0.47^{+0.12}_{-0.10} \pm 0.01$
10	$[4m_\mu^2, (m_{B^+} - m_{\pi^+})^2]$	$17.9 \pm 1.9 \pm 1.1 \pm 1.5^\dagger$ GeV^2	$18.3 \pm 2.4 \pm 0.5$ GeV^2



- Very good agreement (below 1 σ) except for high-q^2 bins with 1.6 σ.
- Low-q^2 bin [0.1, 2] GeV^2, suffers from ρ, ω and ϕ resonances.
- $q^2 \approx 9.5$ GeV^2 & $q^2 \approx 13.5$ GeV^2 suffer from J/ψ and ψ resonances.
- Duality works better for larger bins, we use the largest one for high-q^2.
- Only include the theoretically clean bins: [2, 4], [4, 6], [15, 22] GeV^2.

$B_s^0 \to \bar{K}^{*0}\mu^+\mu^-$

$$\mathcal{B}(B_s^0 \to \bar{K}^{*0}\mu^+\mu^-)_{\rm SM} = (46.0 \pm 6.0)\cdot 10^{-9}$$

FFs CKM scale resonances

$$= (46.0 \pm 2.1 \pm 2.8 \pm 3.3 \pm 3.6)\cdot 10^{-9}$$

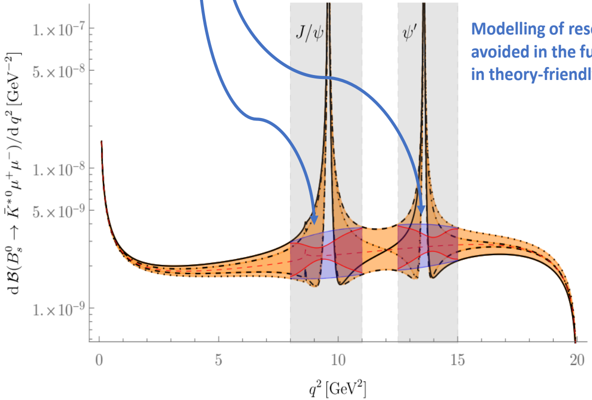
2209.04457

1.4 σ

$>$

$$\mathcal{B}(B_s^0 \to \bar{K}^{*0}\mu^+\mu^-)_{\rm exp} = (29 \pm 11)\cdot 10^{-9}$$

1804.07167



Modelling of resonances can be avoided in the future with data in theory-friendly bins!

$B^0 \to \mu^+\mu^-$ and scalar operators

- In the SM, only the operator O_{10} contributes which yields

$$\mathcal{B}(B^0 \to \mu^+\mu^-)_{\text{SM}} = (1.01 \pm 0.07) \cdot 10^{-10} \ ,$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-)_{\text{exp}} = (1.20 \pm 0.84) \cdot 10^{-10} \ ,$$

in agreement with the experimental value 2108.09283.

- $\mathcal{B}(B^0 \to \mu^+\mu^-)$ is sensitive to $O_{10}^{(\prime)}$, $O_S^{(\prime)}$, and $O_P^{(\prime)}$ operators.

$$\frac{\mathcal{B}(B^0 \to \mu^+\mu^-)}{\mathcal{B}(B^0 \to \mu^+\mu^-)_{\text{SM}}} = |\mathcal{P}|^2 + |\mathcal{S}|^2$$

$$\mathcal{P} = \frac{C_{10}^{\text{SM}} + C_{10-}}{C_{10}^{\text{SM}}} + \frac{m_B^2}{2\,m_\mu}\left(\frac{1}{m_b + m_d}\right)\left(\frac{C_{P-}}{C_{10}^{\text{SM}}}\right)$$
$$\mathcal{S} = \frac{m_B^2}{2\,m_\mu}\sqrt{1 - \frac{4m_\mu^2}{m_B^2}}\left(\frac{1}{m_b + m_d}\right)\left(\frac{C_{S-}}{C_{10}^{\text{SM}}}\right) .$$

- Using the current experimental information

$$-1.8 \lesssim C_{10-} \lesssim 1.7 \quad \text{or} \quad 6.7 \lesssim C_{10-} \lesssim 10.1$$
$$-0.06 \lesssim C_{P-} \lesssim 0.05 \quad \text{or} \quad 0.2 \lesssim C_{P-} \lesssim 0.3 \ ,$$
$$|C_{S-}| \lesssim 0.1 \, ,$$

- $O_S^{(\prime)}$, and $O_P^{(\prime)}$ are more constrained than $O_{10}^{(\prime)}$ (due to m_B/m_μ) not considered in the global fits.

$\bar{B} \to X_d\gamma$

- The SM prediction for the CP-averaged $\bar{B} \to X_d\gamma$ branching ratio

$$\mathcal{B}(\bar{B} \to X_d\gamma)_{\text{SM}} = (16.8 \pm 1.7) \cdot 10^{-6} \ ,$$
$$\mathcal{B}(\bar{B} \to X_d\gamma)_{\text{exp}} = (14.1 \pm 5.7) \cdot 10^{-6} \ ,$$

in very good agreement.

- $\mathcal{B}(\bar{B} \to X_d\gamma)$ is sensitive to $O_7^{(\prime)}$ and $O_8^{(\prime)}$ operators.

$$\mathcal{B}(\bar{B} \to X_d\gamma) = \sum_{i=1}^{9} a_i^{(\bar{B}X_d)} w_i^{(\bar{B}X_d)}$$

$$w_i^{(\bar{B}X_d)} = \{1, C_7, C_8, C_7^2, C_8^2, (C_7')^2, (C_8')^2, C_7 \cdot C_8, C_7' \cdot C_8'\}$$

In units of 10^{-5}

$a_1^{(\bar{B}X_d)}$	$a_2^{(\bar{B}X_d)}$	$a_3^{(\bar{B}X_d)}$
1.68	−6.17	−0.28
$a_4^{(\bar{B}X_d)} = a_6^{(\bar{B}X_d)}$	$a_5^{(\bar{B}X_d)} = a_7^{(\bar{B}X_d)}$	$a_8^{(\bar{B}X_d)} = a_9^{(\bar{B}X_d)}$
7.66	0.28	0.53

Fit approach

We work within a frequentist framework based on the approximation of Gaussian likelihood

$$\mathcal{L}(\theta) = e^{-\chi^2(\theta)/2}$$

Wilson coefficients

$$\chi^2(\theta) = -2\ln\mathcal{L}(\theta) = \sum_{i,j}^{n_{\rm obs}} \Delta_i(\theta)\, V_{ij}^{-1}(\theta)\, \Delta_j(\theta)\,,$$

Central values: $\Delta_i(\theta) = \Delta_i^{(\rm th)}(\theta) - \Delta_i^{(\rm exp)}\,,$

Covariance matrix: $V_{ij}(\theta) = V_{ij}^{(\rm th)}(\theta) + V_{ij}^{(\rm exp)}\,.$

Usually WCs=0, here the experimental is less stringent so it is important to include these effects.

6 observables

$$\vec{\Delta} = \{\mathcal{B}_1^{(B\pi)}, \mathcal{B}_2^{(B\pi)}, \mathcal{B}_9^{(B\pi)}, \mathcal{B}(B_s^0 \to \bar{K}^{*0}\mu^+\mu^-), \mathcal{B}(B^0 \to \mu^+\mu^-), \mathcal{B}(\bar{B} \to X_d\gamma)\}\,.$$

Minimization of chi-square: Maximum likelihood method $\partial\chi^2/\partial\theta_i|_{\hat{\theta}} = 0 \longrightarrow \hat{\theta}$ Best-fit points

Confidence regions: $\Delta\chi^2(\theta) \leq \eta(l, n_{\rm par})$ where $\Delta\chi^2(\theta) = \chi^2(\theta) - \chi^2_{\rm min}$

Value where the chi-square cumulative distribution function reaches the probability associated with l sigmas

$\eta(l,1) = l^2,\ \eta(l,2) = (2.30, 6.18, \ldots),$ etc.

In practice: * MIGRAD from the Python package iminuit to conduct the numerical minimization.

* Confidence intervals are computed using MINOS algorithm from iminuit.

One-dimensional fits

scenario	fit parameter	best fit	1σ	2σ	$\chi^2_{H_i,\,\mathrm{min}}$	Pull_{H_i}	p-value (%)
H_1	C_7	0.01	[-0.07, 0.11]	[-0.15, 0.25]	3.74	0.15	58
H_2	C_8	0.04	[-0.88, 1.44]	[-1.51, 2.27]	3.76	0.04	58
H_3	C_9	-1.37	[-2.97, -0.47]	[-7.65, 0.26]	1.12	1.63	95
H_4	C_{10}	0.96	[0.3, 1.75]	[-0.29, 2.92]	1.51	1.50	91
H_5	C_7'	-0.02	[-0.18, 0.16]	[-0.31, 0.3]	3.75	0.11	58
H_6	C_8'	-0.04	[-1.16, 1.13]	[-1.86, 1.85]	3.76	0.03	58
H_7	C_9'	-0.21	[-0.91, 0.47]	[-1.63, 1.15]	3.67	0.32	59
H_8	C_{10}'	0.22	[-0.37, 0.8]	[-0.98, 1.38]	3.63	0.37	60
H_9	$C_9 = +C_{10}$	0.19	[-0.57, 1.02]	[-1.24, 1.79]	3.71	0.24	59
H_{10}	$C_9 = -C_{10}$	-0.53	[-0.89, -0.19]	[-1.29, 0.14]	1.27	1.58	93
H_{11}	$C_9' = +C_{10}'$	0.10	[-0.68, 0.86]	[-1.41, 1.53]	3.75	0.13	58
H_{12}	$C_9' = -C_{10}'$	-0.13	[-0.46, 0.22]	[-0.8, 0.57]	3.63	0.37	60
H_{13}	$C_9 = -C_9'$	-1.74	[-3.26, -0.27]	[-4.04, 0.44]	1.96	1.34	85
H_{14}	$C_9 = +C_9'$	-0.55	[-1.29, -0.07]	[-4.13, 0.34]	2.42	1.16	78
H_{15}	$C_9 = -C_{10} = -C_9' = -C_{10}'$	-0.58	[-1.06, -0.2]	[-4.04, 0.12]	1.17	1.61	94
H_{16}	$C_9 = -C_{10} = +C_9' = -C_{10}'$	-0.24	[-0.46, -0.04]	[-0.7, 0.16]	2.35	1.19	79

What do we learn from the one-dimensional fits?

- The most favored scenario is H_3 NP in C_9 (pull=1.63, p-value=95%), followed by H_4 with NP in C_{10} (pull=1.50, p-value=91%).
- Scenarios relating 2 WCs (H_9,...,H_{14}):
  1. H_{10} with LH quarks and LH leptons, $C_9 = -C_{10}$, is preferred by data (pull=1.58, p-value=93%). For comparison, we explore benchmark H_9, LH quarks and RH leptons $C_9 = C_{10}$, results are close to the SM.
  2. We work correlations in $C'_{9,10}$ and find p-values closer to the SM one.
  3. We consider $C_9 = \pm C'_9$, where we find similar results (pull≈1.3, p-value≈80%).

⋆ **Consistency with the SM, but data shows a preference to include NP via C_9, similar as in global fits to $b \to s\,\mu^+\mu^-$ data.**

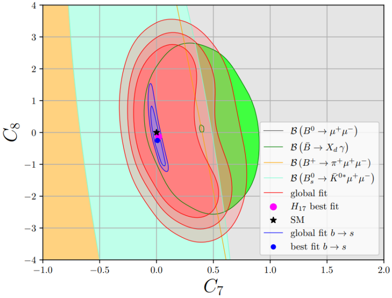
⋆ **Future data is welcome to confirm or refute this preference!**

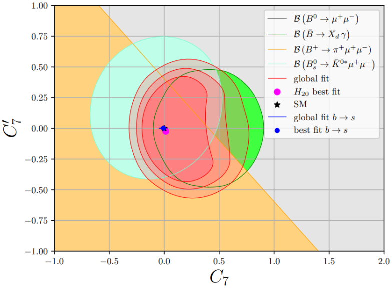
Let's further entertain with two-dimensional fits!

scen.	fit parameters	best fit	1σ	2σ	$\chi^2_{H_i,\,\mathrm{min}}$	Pull_{H_i}	p-v. (%)
H_{17}	(C_7, C_8)	(0.02, -0.13)	([-0.08, 0.21], [-1.57, 1.46])	([-0.16, 0.43], [-2.37, 2.41])	3.74	0.02	44
H_{18}	(C_7, C_9)	(0.05, -1.45)	([-0.04, 0.19], [-3.0, -0.54])	([-0.13, 0.94], [-9.65, 0.22])	0.86	1.19	93
H_{19}	(C_7, C_{10})	(0.04, 1.02)	([-0.05, 0.17], [0.34, 1.87])	([-0.13, 0.73], [-0.26, 3.32])	1.33	1.05	85
H_{20}	(C_7, C_7')	(0.01, -0.02)	([-0.07, 0.12], [-0.21, 0.17])	([-0.15, 0.28], [-0.36, 0.34])	3.73	0.02	44
H_{21}	(C_7, C_9')	(0.02, -0.23)	([-0.07, 0.12], [-0.92, 0.46])	([-0.15, 0.26], [-1.64, 1.15])	3.63	0.08	45
H_{22}	(C_7, C_{10}')	(0.02, 0.23)	([-0.07, 0.12], [-0.36, 0.81])	([-0.15, 0.26], [-0.97, 1.4])	3.60	0.10	46
H_{23}	(C_9, C_{10})	(-1.67, 8.55)	([-7.43, 0.65], [6.48, 9.37])	([-9.13, 1.86], [-1.42, 9.85])	1.00	1.15	91
H_{24}	(C_7', C_9)	(0.05, -1.4)	([-0.17, 0.23], [-2.95, -0.49])	([-0.35, 0.36], [-7.64, 0.26])	1.08	1.12	89
H_{25}	(C_9, C_9')	(-2.22, 1.18)	([-6.55, -0.63], [-2.99, 2.89])	([-7.58, 0.23], [-3.92, 3.81])	0.76	1.22	94
H_{26}	(C_9, C_{10}')	(-1.79, -0.35)	([-6.59, -0.57], [-1.19, 0.36])	([-7.61, 0.27], [-1.8, 1.05])	0.88	1.18	92
H_{27}	(C_7', C_{10})	(0.04, 0.99)	([-0.16, 0.22], [0.31, 1.84])	([-0.3, 0.35], [-0.29, 3.25])	1.48	1.00	83
H_{28}	(C_9', C_{10})	(0.21, 7.34)	([-0.58, 0.99], [6.29, 8.09])	([-1.39, 1.79], [-0.3, 8.72])	1.35	1.04	85
H_{29}	(C_{10}, C_{10}')	(7.45, -0.01)	([6.53, 8.13], [-0.79, 0.97])	([-0.30, 8.73], [-4.54, 4.49])	1.42	1.02	84
H_{30}	(C_7', C_9')	(0.02, -0.26)	([-0.18, 0.21], [-1.07, 0.57])	([-0.32, 0.34], [-1.88, 1.36])	3.66	0.07	45
H_{31}	(C_7', C_{10}')	(0.0, 0.22)	([-0.17, 0.18], [-0.41, 0.84])	([-0.31, 0.32], [-1.04, 1.46])	3.63	0.08	45
H_{32}	(C_9', C_{10}')	(-0.08, 0.17)	([-1.07, 0.83], [-0.65, 0.99])	([-2.04, 1.63], [-1.39, 1.74])	3.62	0.09	45
H_{33}	$(C_9 = -C_9', C_{10} = +C_{10}')$	(-1.73, 0.44)	([-3.34, -0.19], [0.04, 0.95])	([-4.1, 0.51], [-0.34, 4.52])	0.77	1.22	94
H_{34}	$(C_9 = -C_9', C_{10} = -C_{10}')$	(-1.73, 0.01)	([-3.65, 0.15], [-0.45, 0.91])	([-4.6, 1.05], [-0.84, 5.06])	1.96	0.83	74
H_{35}	$(C_9 = +C_9', C_{10} = +C_{10}')$	(0.6, 2.18)	([0.26, 0.89], [-0.58, 4.77])	([-4.95, 1.19], [-0.92, 5.1])	2.15	0.76	70
H_{36}	$(C_9 = -C_{10}, C_9' = +C_{10}')$	(-0.58, 0.57)	([-3.11, -0.2], [-1.37, 3.38])	([-8.03, 0.13], [-3.14, 4.05])	1.17	1.10	88
H_{37}	$(C_9 = -C_{10}, C_9' = -C_{10}')$	(-0.6, 0.15)	([-1.07, -0.21], [-0.27, 0.65])	([-1.86, 0.15], [-0.66, 1.47])	1.15	1.10	88

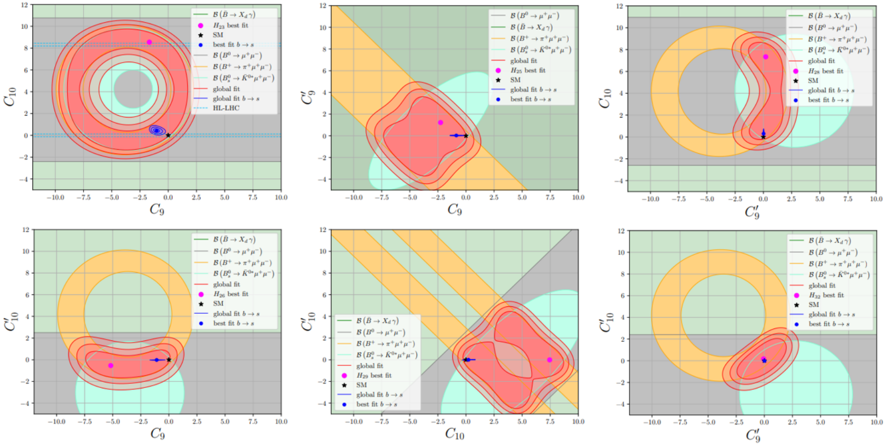
⋆ **Similar pattern as 1D fits, if C_9 present, p-values are large, $\sim$ 90%!**

2D contours of dipole coefficients

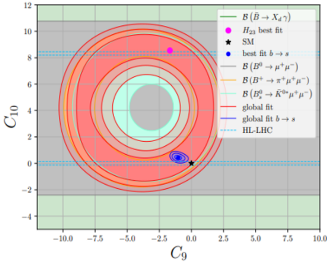


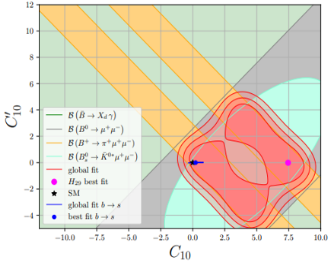


- ⋆ **Excellent complementarity between different observables!**
- ⋆ **Improved limits on $C_7^{(\prime)}$ compared to previous works.** 1106.5499
- ⋆ **Data is consistent with the hypothesis of minimal quark flavor violation.** 2109.01675 & 2209.04457


$\mathcal{B}(\bar{B}\to X_d\gamma)$
H_{23} best fit
SM
best fit $b\to s$
$\mathcal{B}(B^0\to\mu^+\mu^-)$
$\mathcal{B}(B^+\to\pi^+\mu^+\mu^-)$
$\mathcal{B}(B_s^0\to\bar{K}^{0*}\mu^+\mu^-)$
global fit
global fit $b\to s$
HL-LHC
C_9
C_{10}
H_{25} best fit
C_9'
H_{28} best fit
C_{10}'
H_{26} best fit
H_{29} best fit
H_{32} best fit

Summary of 2D contours of $C_{9,10}^{(\prime)}$





- Complementarity between the observables is not currently as good as for dipole coefficients, leading to weaker limits on C_9 and C_{10} .
- The branching ratios of $B^+ \to \pi^+\mu^+\mu^-$ and $B_s^0 \to \bar{K}^{0*}\mu^+\mu^-$ cooperate to reduce the thickness of the annulus (red area) but do not lift the degeneracy between C_9 and C_{10} .
- The branching ratio of $B^0 \to \mu^+\mu^-$ can help due to its dependence on $C_{10}^{(\prime)}$, however, the present precision is insufficient.
- Note that due to the flat likelihood along the ring (red area) the best-fit point (magenta) is only shown for completeness but has little statistical preference over other points in this flat direction.
- All 2D contours make visible discrete ambiguities, for instance the two yellow bands in C_{10}, C'_{10} .
- To remove all these ambiguities additional complementary observables are necessary.
- Data is consistent with the hypothesis of minimal quark flavor violation.

Conclusions & Outlook

$\star$ Model-independent analysis of rare radiative and semileptonic $|\Delta b| = |\Delta d| = 1$ process.

$\star$ Data consistent with the SM, but leave sizable room for NP.

$\star$ Same pattern of $b \to s\, \mu^+ \mu^-$ branching ratios suppressed with respect to the SM, although within larger uncertainties.

$\star$ Improving the fit is not just higher statistics, but also of adding observables sensitive to different combinations of WCs. (i.e. forward-backward asymmetry $A_{\text{FB}}^{\ell} \propto C_9\, C_{10}$, etc)

Thank you!