Charm CP Violation including U-spin breaking effects

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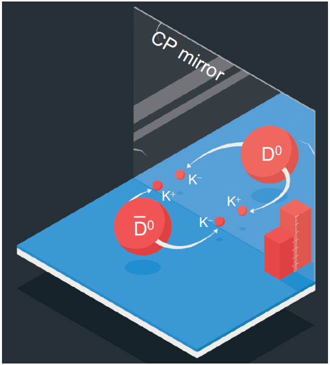
Unique gate to flavor structure of up-type quarks

$$a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) - a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-)$$
$$= (-0.161 \pm 0.028)\% \,.$$

[LHCb 1903.08726, HFLAV 2021]

The problem: Is it SM?



[CERN]

Direct CP Violation is an Interference Effect

$$a_{CP}^{\rm dir}(f) \equiv \frac{|\mathcal{A}(D^0 \to f)|^2 - |\mathcal{A}(\overline{D}^0 \to f)|^2}{|\mathcal{A}(D^0 \to f)|^2 + |\mathcal{A}(\overline{D}^0 \to f)|^2} \approx 2 r_{\rm CKM} r_{\rm QCD} \sin \varphi_{\rm CKM} \sin \delta_{\rm QCD}$$

f = CP-eigenstate.

The decay amplitude:

$$\mathcal{A} = 1 + r_{\rm CKM}\, r_{\rm QCD}\, e^{i(\varphi_{\rm CKM} + \delta_{\rm QCD})}$$

- $r_{\rm CKM}$: real ratio of CKM matrix elements.
- $\varphi_{\rm CKM}$: weak phase.
- $r_{\rm QCD}$: real ratio of hadronic matrix elements.
- $\delta_{\rm QCD}$: strong phase.

Where does the interference come from?

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-, \ldots \xrightarrow{\text{QCD}} \pi^+\pi^-$$

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+\pi^-, \ldots \xrightarrow{\text{QCD}} K^+K^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+K^-$$

Prediction from SM CKM

$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\text{QCD}}\,.$ U-spin: $r_{\text{QCD}} = \mathcal{A}^{\Delta U=0} / \mathcal{A}^{\Delta U=1}\,.$

SU(3): Approximate symmetry for the light quarks u, d, s.

Can we overcome soft QCD in Charm?

Expansion parameters

- In kaon decays we have m/Λ.
- In B decays we have Λ/m.
- In charm…?

Need to revisit toolbox / find new strategies.

The three $\Delta I = 1/2$ rules for $P \to \pi\pi$

- Relevant ratio of strong isospin matrix elements:

$r_{QCD}^{\Delta I=1/2} \equiv A^{\Delta I=1/2}/A^{\Delta I=3/2}$	Kaon	Charm	Beauty
Data	22	2.5	1.5
"No QCD" limit	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
Enhancement	$\mathcal{O}(10)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

[D: Franco Mishima Silvestrini 2012, B: Grinstein Pirtskhalava Stone Uttayarat 2014]

- Rescattering most important in K decays, less important but still significant in D decays, and small in B decays.

Can we tell a loop from a tree?

$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\mathrm{QCD}}\,, \quad r_{\mathrm{QCD}} = \mathcal{A}^{\Delta U=0} / \mathcal{A}^{\Delta U=1}$$

Assuming the SM, the data implies $r_{\mathrm{QCD}}^{\mathrm{Exp}} = \mathcal{O}(1)$.

What is $r_{\mathrm{QCD}}^{\mathrm{SM}} \equiv |P/T|$?

- Light Cone Sum Rules (LCSR): $r_{\mathrm{QCD}}^{\mathrm{SM}} \sim 0.1$.

  [Petrov Khodjamirian 1706.07780, Chala Lenz Rusov Scholtz 1903.10490, Lenz Piscopo Rusov 2312.13245]
- Large non-pert. effects from low energy QCD: $r_{\mathrm{QCD}}^{\mathrm{SM}} = \mathcal{O}(1)$.
  (Like in charm $\Delta I = 1/2$ rule.)

  [Grossman Schacht 1903.10952, Brod Kagan Zupan 1111.5000, Schacht Soni 2110.07619]
- Using $\pi\pi/KK$ rescattering data: Under debate. [Franco Mishima Silvestrini 1203.3131, Bediaga Frederico Magalhaes 2203.04056, Pich Solomonidi Vale Silva 2305.11951]

Determination of P/T in $D \to \pi\pi$: A caveat

“The data implies $|P/T| = \mathcal{O}(1)$”

This statement actually relies on an underlying, commonly made assumption:

- The relative strong phase between P and T is assumed $\mathcal{O}(1)$ (from rescattering).

CP violation as interference of P and T.

$$A = (-V_{cd}^* V_{ud}) \times T - \left(\frac{V_{cb}^* V_{ub}}{2}\right) \times P\,.$$

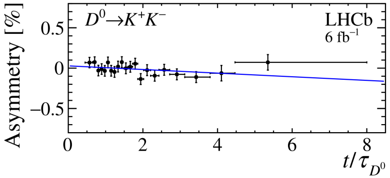
Direct CP asymmetry:

$$a_{CP}^f \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \mathrm{CKM} \times \left|\frac{P}{T}\right| \times \sin\left(\arg\left(\frac{P}{T}\right)\right)$$

How to determine the strong phase from the data

- Strong phases can be obtained from measurements of time-dependent CP violation or quantum-correlated decays. [Xing hep-ph/9606422, Gronau Grossman Rosner hep-ph/0103110, Bevan Inguglia Meadows 1106.5075, Bevan Meadows 1310.0050, Grossman Kagan Zupan 1204.3557, Xing 1903.09566, Grossman Schacht 1903.10952, Schacht 2207.08539]

$$A_{CP}(f,t) \approx a_{CP}^f + \Delta Y^f \frac{t}{\tau_{D^0}}$$



[LHCb 2105.09889]

- No measurements of phases yet, despite big experimental advances.
- ΔY^f have large errors, and sensitivity to phase is subleading.

$$\Delta Y^{K^+K^-} = (-0.3 \pm 1.3 \pm 0.3) \cdot 10^{-4}\,,$$

$$\Delta Y^{\pi^+\pi^-} = (-3.6 \pm 2.4 \pm 0.4) \cdot 10^{-4}\,.$$

Isospin to the rescue

[Gavrilova Grossman Schacht, 2312.10140]

- Assuming SM, isospin allows determination of strong P/T phase from direct CP asymmetries and branching ratios only.
- Also enables extraction of magnitude of P/T without assumptions about phase.
- $D \to \pi\pi$ has same group-theory structure as $B \to \pi\pi$ [Gronau London 1990], however, different approximations are used in the two systems.

Isospin Decomposition

$$A^{+-} = -\lambda_d \left(\sqrt{2}\,(t_{1/2} + t_{3/2}) \right) - \frac{\lambda_b}{2} \left(\sqrt{2}\, p_{1/2} \right) ,$$

$$A^{00} = -\lambda_d \,(2t_{3/2} - t_{1/2}) - \frac{\lambda_b}{2}\,(-p_{1/2}) ,$$

$$A^{+0} = -\lambda_d \,(3\, t_{3/2}) .$$

Assumptions, expected to hold at $\mathcal{O}(1\%)$.

- Isospin limit, neglect isospin breaking effects.
- Neglect electromagnetic corrections and electroweak penguin contributions. Subleading due to smallness of Wilson coefficients.

Consequences:

- No $\Delta I = 5/2$ matrix elements.
- No $\Delta I = 3/2$ contribution to CKM-subleading amplitude.

P/T in terms of operator matrix elements

- Ambiguity to define two interfering amplitudes (using CKM unitarity).
- Therefore, ratio of two interfering amplitudes P/T not clearly defined.
- Unambiguous definition with operator matrix elements:

[Khodjamirian Petrov 1706.07780]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) \right) \equiv \sum_{q=d,s} \lambda_q \mathcal{O}^q \,,$$

$$\langle \mathcal{O}^q \rangle^f \equiv \langle f | \mathcal{O}^q \left| D^0 \right\rangle \,,$$

$$A(D^0 \to f) = \lambda_d \langle \mathcal{O}^d \rangle^f + \lambda_s \langle \mathcal{O}^s \rangle^f = -\lambda_d \left(\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f \right) - \frac{\lambda_b}{2} \left(2 \langle \mathcal{O}^s \rangle^f \right) \,,$$

Unambiguous definition of P/T ratio as:

$$r^f \equiv \left| \frac{2 \langle \mathcal{O}^s \rangle^f}{\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f} \right| \,,$$

Theory Limits $\quad r^f \equiv \left|\frac{2\langle \mathcal{O}^s\rangle^f}{\langle \mathcal{O}^s\rangle^f - \langle \mathcal{O}^d\rangle^f}\right|$

- "Rescattering" ≡ matrix element of operator $\mathcal{O}^s \neq 0$.
- Naive interpretation: $s\bar{s}$ pair rescatters into $d\bar{d}$, hadronization to pions.

No-rescattering limit

$$\langle \mathcal{O}^s\rangle_{\text{no rescatt.}} = 0\,, \quad \text{i.e.} \quad r^f_{\text{no rescatt.}} = 0\,.$$

Limit of large rescattering

$$|\langle \mathcal{O}^s\rangle| \sim |\langle \mathcal{O}^d\rangle|\,, \quad \text{i.e.} \quad r^f \gtrsim 1\,, \text{ depending on relative phase.}$$

Strict large N_c limit

[Buras Gerard Bardeen 1401.1385]

$$\left|\frac{t_{1/2}}{t_{3/2}}\right|^{N_c\to\infty} = 2.$$

In $K \to \pi\pi$ this is much larger ($\Delta I = 1/2$ rule).

Closed-form solution [Gavrilova, Grossman, Schacht 2312.10140]

$$\sin\arg(P/T)^{00} = \frac{-\mathrm{sign}\,(a_{CP}^{00})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{+-}}{a_{CP}^{00}}\sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$\sin\arg(P/T)^{+-} = \frac{-\mathrm{sign}\,(a_{CP}^{+-})}{\sqrt{1+\frac{1}{\sin^2\delta_d}\left(\frac{a_{CP}^{00}}{a_{CP}^{+-}}\sqrt{2\frac{\mathcal{P}^{+-}}{\mathcal{B}^{+-}}\frac{\mathcal{B}^{00}}{\mathcal{P}^{00}}}+\cos\delta_d\right)^2}}\,,$$

$$|P/T|^{00} = \frac{1}{|\mathrm{Im}\,(-\lambda_b/\lambda_d)|}\sqrt{(a_{CP}^{00})^2+\frac{(a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}+a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}\cos\delta_d)^2}{2\mathcal{B}^{00}\mathcal{P}^{+-}\sin^2\delta_d}}\,,$$

$$|P/T|^{+-} = \frac{1}{|\mathrm{Im}\,(-\lambda_d/\lambda_d)|}\sqrt{\left(a_{CP}^{+-}\right)^2+\frac{\left(a_{CP}^{00}\sqrt{2\mathcal{B}^{00}\mathcal{P}^{+-}}+a_{CP}^{+-}\sqrt{\mathcal{B}^{+-}\mathcal{P}^{00}}\cos\delta_d\right)^2}{\mathcal{B}^{+-}\mathcal{P}^{00}\sin^2\delta_d}}\,.$$

Knowledge of $D \to \pi^+\pi^-$ translates into $D \to \pi^0\pi^0$

[Gavrilova, Grossman, Schacht 2312.10140]

$$\frac{\sin\delta^{+-}}{\sin\delta^{00}} = \frac{a_{CP}^{+-}}{a_{CP}^{00}} \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}\,,$$

$$\frac{r^{00}}{r^{+-}} = \sqrt{\frac{1}{2}\frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}}\frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}\,.$$

$$r \equiv P/T\,, \qquad \mathcal{P}: \text{ phase-space}\,.$$

- Although we have essentially no information about $\sin\delta^{00}$ we can obtain non-trivial information about r^{00}, due to the correlation to r^{+-} from isospin.

Numerical Results: Systematic Uncertainties

- Our results relate P/T to $\mathcal{B}^f$ and a_{CP}^f.
- In experimental extraction of a_{CP}^f additional assumptions are made.
- For extraction of a_{CP}^{+-} LHCb assumes a universal ΔY.
  We use same ΔY for the extraction of a_{CP}^{00}.
- A universal ΔY is motivated by U-spin [Kagan Silvestrini 2001.07207]
  It follows overall systematic theory uncertainty of $\mathcal{O}(\varepsilon^2) \sim 10\%$.
- In order to reach theory uncertainty of $\mathcal{O}(1\%)$ we need extractions of a_{CP}^f without universality assumption: future data on ΔY^{+-} and ΔY^{00}.
- Our numerics with current data include universality assumption, implying an additional overall theory uncertainty of $\mathcal{O}(10\%)$.

Numerical Results [Gavrilova, Grossman, Schacht 2312.10140]

Direct CP Asymmetries		
a_{CP}^{+0}	$+0.004 \pm 0.008$	[79–82]
a_{CP}^{00}	-0.0002 ± 0.0064	[a][79, 83, 84]
a_{CP}^{+-}	0.00232 ± 0.00061	[2]
Branching Ratios		
$\mathcal{B}(D^0 \to \pi^+\pi^0)$	$(1.247 \pm 0.033) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^+\pi^-)$	$(1.454 \pm 0.024) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \to \pi^0\pi^0)$	$(8.26 \pm 0.25) \cdot 10^{-4}$	[85]
Further Numerical Inputs		
$\mathrm{Im}\,(\lambda_b/(-\lambda_d))$	$(-6.1 \pm 0.3) \cdot 10^{-4}$	[85]

TABLE I. Experimental input data. We use the decay times and masses from Ref. [85]. [a]Our extraction from $A_{CP}(D^0 \to \pi^0\pi^0) = -0.0003 \pm 0.0064$ [79] and $\Delta Y = (-1.0 \pm 1.1 \pm 0.3) \cdot 10^{-4}$ [52].

a_{CP}^{+-}	$(2.32 \pm 0.07) \cdot 10^{-3}$
a_{CP}^{00}	$(-2 \pm 9) \cdot 10^{-4}$

TABLE II. Future data scenario employing the current central values and using prospects for the errors from Table 6.5 of Ref. [86] (300 fb^{-1}) and Table 122 of Ref. [87] (50 ab^{-1}) for $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$, respectively. All other input data is left as specified in Table I.

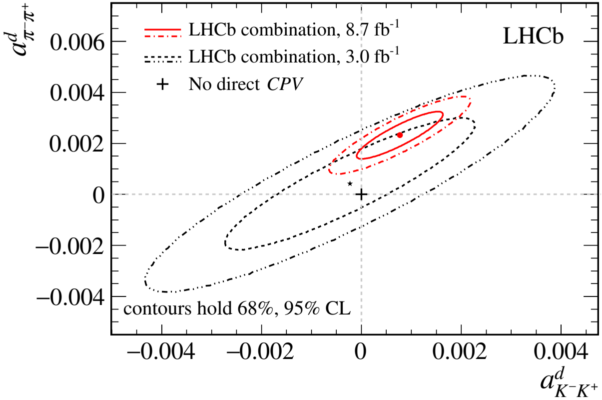
Parameter	Current data	Future data scenario
r_t	3.43 ± 0.06	3.43 ± 0.06
$\cos\delta_t$	0.06 ± 0.02	0.06 ± 0.02
$\cos\delta_d$	-0.68 ± 0.01	-0.68 ± 0.01
$\vert\sin\delta^{00}\vert$	0^{+1}_{-0}	$0.06^{+0.20}_{-0.06}$
$\vert\sin\delta^{+-}\vert$	$0.7^{+0.3}_{-0.5}$	$0.69^{+0.21}_{-0.16}$
r^{00}	$5.2^{+13.3}_{-2.4}$	$5.2^{+1.6}_{-1.2}$
r^{+-}	$5.5^{+14.2}_{-2.7}$	$5.5^{+1.8}_{-1.3}$

TABLE III. Numerical results for current and hypothetical future data. In the future data scenario, the results for r_t, $\cos\delta_t$ and $\cos\delta_d$ are identical to the ones with current data, as these depend only on the branching ratio data which is not modified in the future data scenario compared to current data. Furthermore, in the future data scenario $\sin\delta^{+-} < 0$. The overall additional relative systematic uncertainty of $\mathcal{O}(10\%)$ due to the universality assumption of ΔY for the extraction of the direct CP asymmetries comes on top of the errors shown here, see text for details.

- $|P/T|$ is large. Future data will significantly reduce errors.

Beyond ΔA_{CP}: A U-spin anomaly?

[LHCb, 2209.03179]



- First evidence of direct CPV in single decay, $D^0 \to \pi^+\pi^-$: 3.8σ.
- $a_{CP}^{\text{dir}}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4}$,
  $a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = (23.2 \pm 6.1) \cdot 10^{-4}$.

Violation of a U-spin limit sum rule

- Separate measurement of both CP asymmetries allows for first time test of the U-spin expansion in CKM-suppressed amplitudes.

U-spin limit sum rule: Broken at 2.7σ [LHCb, 2209.03179]

$$\Sigma a_{CP}^{dir} \equiv a_{CP}^{dir}(D^0 \to K^+K^-) + a_{CP}^{dir}(D^0 \to \pi^+\pi^-) \overset{\text{U-spin}}{=} 0$$

Improved U-spin limit sum rule: Broken at 2.1σ [Schacht, 2207.08539]

$$-\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \frac{a_{CP}^{\text{dir}}(D^0 \to K^+K^-)}{a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)} = -0.93^{+0.62}_{-0.41} \neq +1.$$

- U-spin breaking is expected: Only approximate symmetry.
- Amount goes beyond generic expectations of $\sim 30\%$ at 1.9σ.

Model-Independent Predictions

- Large U-spin breaking indicates large $\Delta U = 1$ operator(s).
- It follows $\mathcal{O}(1)$ breaking of U-spin limit sum rule:

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = -\frac{a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-)}{a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-)} \quad \text{broken at } \mathcal{O}(1)\,,$$

- Connected to wider class of decays via SU(3)-flavor symmetry.

Expect $$\frac{\Gamma(D^+ \to \overline{K}^0 K^+)}{\Gamma(D_s^+ \to K^0\pi^+)} = -\frac{a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0\pi^+)}{a_{CP}^{\mathrm{dir}}(D^+ \to \overline{K}^0 K^+)}$$ also broken at $\mathcal{O}(1)$.

- Improved versions of these sum rules: [Müller Nierste Schacht 1506.04121]

$$a_{CP}^{dir}(D^0 \to K^+K^-)\,,\quad a_{CP}^{dir}(D^0 \to \pi^+\pi^-)\,,\quad a_{CP}^{dir}(D^0 \to \pi^0\pi^0)\,,\quad \text{and}$$
$$a_{CP}^{dir}(D^+ \to K_S K^+)\,,\quad a_{CP}^{dir}(D_s^+ \to K_S\pi^+)\,, a_{CP}^{dir}(D_s^+ \to K^+\pi^0)\,.$$

These should also be broken at $\mathcal{O}(1)$.

Explanations beyond the SM: “$\Delta U = 1$ models”

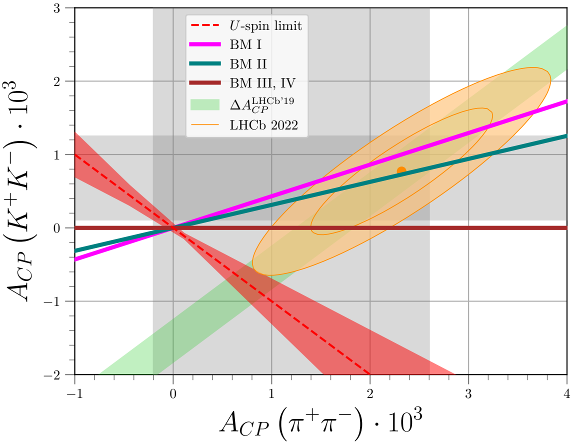
- NP models with $\Delta U = 1$ operators can explain breaking of U-spin limit sum rules. [Hiller Jung Schacht 1211.3734]
- Additional operators with flavor content $\bar{s}c\bar{u}s$ and/or $\bar{d}c\bar{u}d$ with non-universal coefficients.
- Example: Z' models where fermion charges depend on generation.

[Bause Gisbert Golz Hiller 2004.01206],
[Bause Gisbert Hiller Höhne Litim Steudtner 2210.16330]

$$\begin{aligned}
\mathcal{L}_{Z'} \supset \Big(& g_L^{uc}\bar{u}_L\gamma^\mu c_L Z'_\mu + g_R^{uc}\bar{u}_R\gamma^\mu c_R Z'_\mu + \text{h.c.}\Big) \\
&+ g_L^d \bar{d}_L\gamma^\mu d_L Z'_\mu + g_R^d \bar{d}_R\gamma^\mu d_R Z'_\mu \\
&+ g_L^s \bar{s}_L\gamma^\mu s_L Z'_\mu + g_R^s \bar{s}_R\gamma^\mu s_R Z'_\mu \\
&+ g_L^{ll} \bar{l}_L\gamma^\mu l_L Z'_\mu + g_R^{ll} \bar{l}_R\gamma^\mu l_R Z'_\mu
\end{aligned}$$

Z' model predictions

[Bause Gisbert Hiller Höhne Litim Steudtner 2210.16330]



- Viable models with leptophobic Z' below $\mathcal{O}(20\ \text{GeV})$.
- Pattern of CP violation in $D \to \pi\pi$, including $a_{CP}^{dir}(D^+ \to \pi^+\pi^0) \neq 0$.

But is U-spin actually a good symmetry?

Spectroscopy: Eightfold way.

[Gell-Mann, Ne'eman 1961]

- SU(3)$_F$ limit agrees with baryon octet mass splitting to $\sim 10\%$

[Greiner Müller 1989]

Does it work for rates, too?

- Estimate for breaking on amplitude level: $f_K/f_\pi - 1 \sim 0.2$.
- Two often-cited examples of seemingly $\mathcal{O}(1)$ U-spin breaking:

$$\left.\frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)}\right|_{\text{exp}} \sim 3\,, \qquad \left.\frac{\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+K^-)}\right|_{\text{exp}} \sim 0.03\,.$$

- Strict SU(3)$_F$ limit (including phase space):

$$\frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} = 1\,, \qquad \frac{\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+K^-)} = 0\,,$$

Yes. (But we keep testing it at every opportunity.)

[detailed review in Schacht 2207.08539]

A closer look

- Amplitude-level $SU(3)_F$ breaking of $\varepsilon \sim 30\%$ suffices in order to explain the data. [Savage 1991]

$$\frac{(1+\varepsilon)^2}{(1-\varepsilon)^2} \sim 3\,.$$

- Amplitude-level $SU(3)_F$ -breaking in $D^0 \to K_S K_S$:

$$\varepsilon' \sim \sqrt{\frac{\mathcal{B}(D^0 \to K^0 \overline{K}^0)}{\mathcal{B}(D^0 \to K^+ K^-)}} = \sqrt{\frac{2\mathcal{B}(D^0 \to K_S K_S)}{\mathcal{B}(D^0 \to K^+ K^-)}} \sim 0.26\,,$$

- Observations agree with global fits.

[Hiller Jung Schacht 1211.3734, Müller Nierste Schacht 1503.06759]

The picture holds at higher order, too.

[Brod Grossman Kagan Zupan 1203.6659]

Ratio of branching ratios:

$$R_{DPP} \equiv \frac{|\mathcal{A}(D^0 \to K^+K^-)/(V_{cs}V_{us})| + |\mathcal{A}(D^0 \to \pi^+\pi^-)/(V_{cd}V_{ud})|}{|\mathcal{A}(D^0 \to K^+\pi^-)/(V_{cd}V_{us})| + |\mathcal{A}(D^0 \to K^-\pi^+)/(V_{cs}V_{ud})|} - 1$$

U-spin prediction

$$R_{DPP}^{\text{th}} = \mathcal{O}(\varepsilon^2)\,.$$

Data

$$R_{DPP}^{\text{exp}} = 0.046 \pm 0.008\,,$$

- If U-spin breaking were $\mathcal{O}(1)$, we would have $R_{DPP}^{\text{exp}} = \mathcal{O}(1)$.
- Instead, perfectly consistent with $\mathcal{O}(\varepsilon^2)$.

The problem of going to higher order in $SU(3)_F$

- SU(3) is very useful, but $\mathcal{O}(30\%)$ breaking from corrections.
- Going to higher order: complicated.

$$(\mathbf{15}) \otimes (\mathbf{8}) = (\mathbf{42}) \oplus (\mathbf{24}) \oplus (\mathbf{15_1}) \oplus (\mathbf{15_2}) \oplus (\mathbf{15'}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$

$$(\bar{\mathbf{6}}) \otimes (\mathbf{8}) = (\mathbf{24}) \oplus (\mathbf{15}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})$$

Decay d	$B_1^{3_1}$	$B_1^{3_2}$	$B_8^{3_1}$	$B_8^{3_2}$	$B_8^{\bar{6}_1}$	$B_8^{\bar{6}_2}$	$B_8^{15_1}$	...
$D^0 \to K^+K^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{10}$	$-\frac{1}{10\sqrt{2}}$	$-\frac{7}{10\sqrt{122}}$	...
$D^0 \to \pi^+\pi^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$-\frac{1}{10}$	$\frac{1}{10\sqrt{2}}$	$-\frac{11}{10\sqrt{122}}$	...
$D^0 \to \bar{K}^0K^0$	$-\frac{1}{4\sqrt{10}}$	$-\frac{1}{8}$	$\frac{1}{5\sqrt{2}}$	$\frac{1}{2\sqrt{5}}$	0	0	$-\frac{9}{5\sqrt{122}}$	...
$D^0 \to \pi^0\pi^0$	$-\frac{1}{8\sqrt{5}}$	$-\frac{1}{8\sqrt{2}}$	$-\frac{1}{20}$	$-\frac{1}{4\sqrt{10}}$	$\frac{1}{10\sqrt{2}}$	$-\frac{1}{20}$	$\frac{11}{20\sqrt{61}}$	...
...	...	...	...	...	...	...	...	...

Solving the Problem of Higher Order U-spin

[Gavrilova Grossman Schacht, 2205.12975]

Theorems enabling calculations to arbitrary order.

- We are able to determine a priori up to which order sum rules exist.
- We do not need explicit Clebsches. Big complexity reduction.
- Hope: Opens the door for precision in hadronic multi-body decays.

What next? Generalization to $SU(3)_F$, implications for observables.

Systematics of U-spin breaking

- U-spin breaking from mass difference of strange and down quarks:

$$\varepsilon = \frac{m_s - m_d}{\Lambda_{\mathrm{QCD}}} \sim 0.3\,.$$

- Parametrized by triplet-operator H_ε:

$$\mathcal{H}_{\mathrm{eff}} = \sum_{m,b} f_{u,m} \left(H^u_m \otimes H_\varepsilon^{\otimes b} \right)\,, \qquad H_\varepsilon^{\otimes b} \equiv \underbrace{H_\varepsilon \otimes \cdots \otimes H_\varepsilon}_{b}\,.$$

- Any system can be constructed from tensor products of doublets.
- Moving irreps (“crossing sym.”) does not affect structure of sum rules.
- Without loss of generality, consider doublet-only system with

$$0 \to \left(\frac{1}{2}\right)^{\otimes n}$$ and singlet Hamiltonian.

Properties of U-spin pairs

[Gavrilova Grossman Schacht, 2205.12975]

- Amplitude:

$$A_j = \underbrace{(-,-,+,-,+,\dots,+)}_{n} = \sum_\alpha C_{j\alpha} X_\alpha \,.$$

- U-spin conjugated amplitude (complete interchange $s \leftrightarrow d$):

$$\overline{A}_j = \underbrace{(+,+,-,+,-,\dots,-)}_{n} = (-1)^p \sum_\alpha (-1)^b C_{j\alpha} X_\alpha \,.$$

- Notation: Abbreviate m-quantum number: $\pm 1/2 \mapsto \pm$.
  X_α: Reduced matrix element. $C_{j\alpha}$: Clebsches.
- Define (anti-)symmetric combinations of U-spin pairs:

$$a_j \equiv \underbrace{A_j - (-1)^p \overline{A}_j}_{\text{odd in } b}\,, \qquad s_j \equiv \underbrace{A_j + (-1)^p \overline{A}_j}_{\text{even in } b}\,.$$

Results: Sum Rules at any order of U-spin breaking

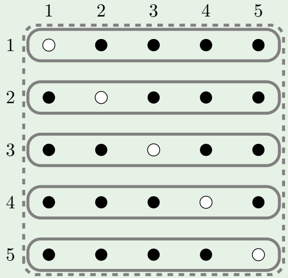
[Gavrilova Grossman Schacht, 2205.12975]

All sum rules at any order b can be written as:

$$\Sigma_j a_j = 0\,, \qquad \Sigma_j s_j = 0\,.$$

Example: $n = 6$ doublets. Lattice dimension $d = n/2 - 1 = 2$.

- Each node ⇔ U-spin pair.
- Each node (points):
  a-type sum rule valid to $b = 0$.
- Sums of nodes in lines:
  s-type sum rules valid to $b = 1$.
- Sum of all nodes in plane:
  a-type sum rule valid up to $b = 2$.



What next? Generalization to $SU(3)_F$, implications for observables.

Conclusions

- This is just the beginning of the exploration of charm CPV.
- Charm is a unique gate to flavor structure of up-type quarks.
- Can we tell a loop from a tree?

- No matter what, we will learn sth new: QCD or New Physics.