Beyond Einstein's General Relativity: Hybrid metric-Palatini gravity

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Brief outline of the talk

- Need for new gravitational physics?
- Foundations of gravitation
- Beyond General Relativity (Modified gravity):
  - Extensions of $f(R)$ gravity and some applications
- **Hybrid metric-Palatini gravity**
  - Astrophysical and cosmological applications
  - Graviton propagator, etc
  - Future research, etc

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Introduction

- The perplexing fact of the late-time cosmic acceleration has forced theorists and experimentalists to pose the question:
  - Is General Relativity (GR) the correct relativistic theory of gravitation?
- The fact GR is facing so many challenges:
  - Difficulty in explaining particular observations
  - Incompatibility with other well established theories
  - Lack of uniqueness

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- The fact GR is facing so many challenges:
  - Difficulty in explaining particular observations
  - Incompatibility with other well established theories
  - Lack of uniqueness

Is this indicative of a need for new gravitational physics?

Motivations

- Of course, cosmology is also an ideal testing ground for GR (in particular, late-time cosmic acceleration).
- Promising approach: assume that at large scales GR breaks down, and a more general action describes the gravitational field.
- Generalizations of the Einstein-Hilbert Lagrangian: R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, $\varepsilon^{\alpha\beta\mu\nu}R_{\alpha\beta\gamma\delta}R^{\gamma\delta}_{\mu\nu}$, $C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}$, etc.
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  - possibility of a more realistic representation of the gravitational fields near curvature singularities;
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General Relativity (GR): Hilbert-Einstein action

- GR is a classical theory, therefore no reference to an action is required. Consider the Hilbert-Einstein action:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + L_m(g^{\mu\nu}, \psi) \right] . \tag{1}$$

- But, the Lagrangian formulation is elegant, and has merits:
  - Quantum level: the action acquires a physical meaning, and a more fundamental theory of gravity will provide an effective gravitational action at a suitable limit;
  - Easier to compare alternative gravitational theories through their actions rather than by their field equations;
  - In many cases one has a better grasp of the physics as described through the action, i.e., couplings, kinetic and dynamical terms, etc

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Foundations of Gravitation Theory

- Schiff and Dicke: gravitational experiments do not necessarily test GR, i.e., do not test the validity of specific field equations, experiments test the validity of principles;
- Triggered the development of powerful tools for distinguishing and testing theories, such as the Parametrized Post-Newtonian (PPN) expansion (pioneered by Nordvedt; extended by Nordvedt and Will)
- Indeed, the idea that experiments test principles and not specific theories, implies the need of exploring the conceptual basis of a gravitational theory.

Dicke framework

- **Dicke Framework.** Probably the most unbiased assumptions to start with, in developing a gravitation theory:
  - Spacetime is a 4-dim manifold, with each point in the manifold corresponding to a physical event (note that a metric and affine connection is not necessary at this stage);
  - The equations of gravity and the mathematical entities in them are to be expressed in a form that is independent of the coordinates used, i.e., in a covariant form.
- It is common to think of GR, or any other gravitation theory, as a set of field equations (or an action).
  - However, a complete and coherent axiomatic formulation of GR, or any other gravitation theory, is still lacking.
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Einstein Equivalence Principle (C. Will)

- Einstein Equivalence Principle (EEP): The EEP is at the heart of gravitation theory.
- Thus if the EEP is valid, then gravitation must be a curved spacetime phenomenon, i.e., it must obey the postulates of Metric Theories of Gravity:
  - Spacetime is endowed with a metric (second rank non-degenerate tensor);
  - The world lines of test bodies are geodesics of that metric;
  - In local freely falling frames, Lorentz frames, the non-gravitational laws of physics are those of Special Relativity.

$f(R)$ gravity

- Consider $f(R)$ gravity, for simplicity:

$$S = \int d^4x \sqrt{-g} \left[\frac{f(R)}{2\kappa^2} + L_m(g^{\mu\nu}, \psi) \right] . \tag{2}$$

  Appealing feature: combines mathematical simplicity and a fair amount of generality!

- Ricci scalar is a dynamical degree of freedom: $3\Box F + FR - 2f = \kappa T$ (where $F = df/dR$)
- Introduces a new light scalar degree of freedom
  - This produces a late-time cosmic acceleration
  - But, the light scalar strongly violates the Solar System constraints
  - Way out: 'chameleon' mechanism, i.e., the scalar field becomes massive in the Solar System
- Approaches: metric, Palatini, metric-affine formalisms (and the **hybrid metric-Palatini formalism**)

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Action of hybrid metric-Palatini gravity

- Action:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, [R + f(\mathcal{R})] + S_m \ , \tag{3}$$

  - S_m is the matter action, $\kappa^2 \equiv 8\pi G$,
  - R is the Einstein-Hilbert term,
  - $\mathcal{R} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is the Palatini curvature,
  - $\mathcal{R}_{\mu\nu}$ is defined in terms of an independent connection $\hat{\Gamma}^\alpha_{\mu\nu}$:
    $\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}^\alpha_{\mu\nu,\alpha} - \hat{\Gamma}^\alpha_{\mu\alpha,\nu} + \hat{\Gamma}^\alpha_{\alpha\lambda}\hat{\Gamma}^\lambda_{\mu\nu} - \hat{\Gamma}^\alpha_{\mu\lambda}\hat{\Gamma}^\lambda_{\alpha\nu}$.

- (Harko, Koivisto, FL, Olmo, PRD 2012)
  (Capozziello, Harko, Koivisto, FL, Olmo, JCAP 2013)
  (review: Capozziello, Harko, Koivisto, FL, Olmo, 2015)

Gravitational field equations

- Varying the action with respect to the metric, one obtains:

$$G_{\mu\nu} + F(\mathcal{R})\mathcal{R}_{\mu\nu} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \kappa^2 T_{\mu\nu}\,, \tag{4}$$

where the energy-momentum tensor is defined as usual,

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta(g^{\mu\nu})}\,. \tag{5}$$

- Varying the action with respect to $\hat{\Gamma}^{\alpha}_{\mu\nu}$:

$$\hat{\nabla}_{\alpha}\left(\sqrt{-g}F(\mathcal{R})g_{\mu\nu}\right) = 0 \tag{6}$$

  - implies that $\hat{\Gamma}^{\alpha}_{\mu\nu}$ is the Levi-Civita connection of a metric $h_{\mu\nu} = F(\mathcal{R})g_{\mu\nu}$.
  - Thus, $h_{\mu\nu}$ is conformally related to the physical metric $g_{\mu\nu}$, with the conformal factor given by $F(\mathcal{R}) \equiv df(\mathcal{R})/d\mathcal{R}$.

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Scalar-tensor representation

- May be expressed as the following scalar-tensor theory

$$S = \int \frac{d^4x\sqrt{-g}}{2\kappa^2}\left[(1+\phi)R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m. \quad (7)$$

  - Differs from $w = -3/2$ Brans-Dicke theory in the coupling of the scalar to the curvature, which in the $w = -3/2$ theory is ϕR.
  - This simple modification will have important physical consequences.

- The gravitational field equation is given by:

$$(1+\phi)G_{\mu\nu} = \kappa^2 T_{\mu\nu} + \nabla_\mu\nabla_\nu\phi - \Box\phi g_{\mu\nu} - \frac{3}{2\phi}\nabla_\mu\phi\nabla_\nu\phi + \frac{3}{4\phi}\nabla_\lambda\phi\nabla^\lambda\phi g_{\mu\nu} - \frac{1}{2}V g_{\mu\nu}, \quad (8)$$

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Scalar-tensor representation: Einstein frame

- Conformal transformation into the Einstein frame:

$$\hat{g}_{\mu\nu} \equiv (1+\phi)\, g_{\mu\nu}\,, \tag{9}$$

- The Einstein frame Lagrangian becomes:

$$\hat{\mathcal{L}} = \hat{R} + \frac{3}{2\phi}\frac{\hat{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}}{(1+\phi)^2} - \frac{V(\phi)}{(1+\phi)^2}\,. \tag{10}$$

- Put into its canonical form by introducing the rescaled field ψ as

$$\phi = \tan^2\left(\frac{\psi}{2\sqrt{3}}\right)\,. \tag{11}$$

The vacuum theory then becomes a canonical scalar theory with a very specific potential [stemming of course from the original function $f(\mathcal{R})$] in the Einstein frame.

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Weak-field, slow-motion behaviour

- The effective Newton constant $G_{\rm eff}$ and the post-Newtonian parameter (PPN) γ are

$$G_{\rm eff} \equiv \frac{G}{1+\phi_0}\left[1-(\phi_0/3)\,e^{-m_\varphi r}\right] , \quad (12)$$

$$\gamma \equiv \frac{1+(\phi_0/3)\,e^{-m_\varphi r}}{1-(\phi_0/3)\,e^{-m_\varphi r}} . \quad (13)$$

- As is clear from the above expressions, the coupling of the scalar field to the local system depends on the amplitude of the background value ϕ_0.
- If ϕ_0 is small, then $G_{\rm eff} \approx G$ and $\gamma \approx 1$ regardless of the value of the effective mass m_φ^2.

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  - If ϕ_0 is small, then $G_{\rm eff} \approx G$ and $\gamma \approx 1$ regardless of the value of the effective mass m_φ^2.

- This contrasts with the result in the metric version of $f(R)$:

$$\begin{aligned} \varphi &= \frac{2GM}{3r} e^{-m_f r}, && (14) \\ G_{\text{eff}} &\equiv G\left(1 + e^{-m_f r}/3\right)/\phi_0, && (15) \\ \gamma &\equiv \left(1 - \frac{e^{-m_f r}}{3}\right) \Big/ \left(1 + \frac{e^{-m_f r}}{3}\right), && (16) \end{aligned}$$

requires a large mass $m_f^2 \equiv (\phi V_{\phi\phi} - V_\phi)/3$ to make the Yukawa-type corrections negligible in local experiments.

Late-time cosmic speedup

- Consider the FLRW metric ($k=0$): $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$.
- Modified Friedmann equations:

$$3H^2 = \frac{1}{1+\phi}\left[\kappa^2\rho + \frac{V}{2} - 3\dot{\phi}\left(H + \frac{\dot{\phi}}{4\phi}\right)\right], \qquad (17)$$

$$2\dot{H} = \frac{1}{1+\phi}\left[-\kappa^2(\rho+P) + H\dot{\phi} + \frac{3}{2}\frac{\dot{\phi}^2}{\phi} - \ddot{\phi}\right]. \qquad (18)$$

- Scalar field equation:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{2\phi} + \frac{\phi}{3}[2V - (1+\phi)V_\phi] = -\frac{\phi\kappa^2}{3}(\rho - 3P). \qquad (19)$$

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Consistency at Solar System and cosmological scales

- Consider for mathematical simplicity:

$$V(\phi) = V_0 + V_1\phi^2\,. \tag{20}$$

  - Trace of the field eq. automatically implies $R = -\kappa^2 T + 2V_0$.
  - As $T \to 0$ with the cosmic expansion, naturally evolves into a de Sitter phase ($V_0 \sim \Lambda$) for consistency with observations.
  - If V_1 is positive, the de Sitter regime represents the minimum of the potential.
  - The effective mass for local experiments, $m_\varphi^2 = 2(V_0 - 2V_1\phi)/3$, is positive if $\phi < V_0/V_1$.
  - For $V_1 \gg V_0$, the amplitude small enough to pass SS tests.
- The exact de Sitter solution is compatible with dynamics of the scalar field in this model.

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  - The effective mass for local experiments, $m_\varphi^2 = 2(V_0 - 2V_1\phi)/3$, is positive if $\phi < V_0/V_1$.
  - For $V_1 \gg V_0$, the amplitude small enough to pass SS tests.
- The exact de Sitter solution is compatible with dynamics of the scalar field in this model.

Consistency at Solar System and cosmological scales

- Consider for mathematical simplicity:

$$V(\phi) = V_0 + V_1\phi^2 \,. \tag{20}$$

  - Trace of the field eq. automatically implies $R = -\kappa^2 T + 2V_0$.
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Cosmological Perturbations

- To understand the cosmological structure formation, deduce the perturbation equations and analyse them for specific cases.
- This paves the way for a detailed comparison of the predictions with the cosmological data on large scale structure and the cosmic microwave background.
- Consider the Newtonian gauge, which can be parameterized by the two gravitational potentials Φ and Ψ,

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2 \,. \tag{21}$$

- As matter source we consider a perfect fluid, with the background equation of state w and with density perturbation $\delta = \delta\rho_m/\rho_m$, pressure perturbation $\delta p_m = c_s^2\delta\rho_m$ and velocity perturbation v.
- One can show:

$$\Psi + \Phi = -\frac{\varphi}{1+\phi} \,. \tag{22}$$

where we have denoted $\varphi = \delta\phi$.

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$$\dot{\delta} + 3H(c_s^2 - w)\delta = -(1+w)\left(3\dot{\Phi} - \frac{k^2}{a}v\right), \tag{23}$$

$$\dot{v} + (1-3c_s^2)Hv = \frac{1}{a}\left(\Psi + \frac{c_s^2}{1+w}\delta\right), \tag{24}$$

respectively.

- Linear part of the Klein-Gordon equation is

$$\ddot{\varphi} + \left(3H + \frac{1}{\phi}\right)\dot{\varphi} + \left(\frac{k^2}{a^2} + \frac{\dot{\phi}^2}{2\phi^2} - \frac{2}{3}V''(\phi)\right)\varphi$$
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Matter dominated cosmology

- Consider the formation of structure in the matter-dominated universe, where $w = c_s^2 = 0$ (assume scales deep inside the Hubble radius: so called quasi-static approximation).
- Combining the continuity and the Euler equation in this approximation, one finally obtains:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\text{eff}}\,\rho_m \delta \,, \tag{26}$$

with

$$G_{\text{eff}} \equiv \frac{1 - \frac{1}{3}\phi}{1+\phi} G \,. \tag{27}$$

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Matter dominated cosmology

Confrontations of specific model predictions with the present large scale structure data and forecasts for the constraints from future experiments (ex. EUCLID mission).

Vacuum fluctuations

- The propagation of the scalar degree of freedom in vacuum is also a crucial consistency check on the theory.
- Set $\rho_m = 0$, and consider the curvature perturbation in the uniform-field gauge ζ. In terms of the Newtonian gauge perturbations this is

$$\zeta = \Phi - \frac{H}{\dot{\phi}}\varphi \,. \tag{28}$$

- We obtain the exact (linear) evolution equation (tedious algebra):

$$\ddot{\zeta} + \left[3H - 2\frac{\ddot{\phi} + 2\dot{H}(1+\phi) - \frac{\dot{\phi}^2}{1+\phi}}{\dot{\phi} + 2H(1+\phi)} + \frac{\phi(1+\phi)}{\dot{\phi}^2} \times \right.$$
$$\left. \times \left(\frac{2\ddot{\phi}\dot{\phi}}{\phi(1+\phi)} + \frac{\dot{\phi}^3(1+\phi)^2\phi}{1-\phi^3(1+\phi)^3}\right)\right]\dot{\zeta} = -\frac{k^2}{a^2}\zeta \,, \tag{29}$$

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Construction of specific models and their observational tests:

- the Einstein-frame formulation might present a convenient starting point for that as it, given the function $f(\mathcal{R})$, presents directly the relevant inflationary potential in terms of the canonic field.

More General Hybrid Metric-Palatini Theories

- The "hybrid" theory space is a priori large. In addition to the metric and its Levi-Civita connection, one also has an additional independent connection as a building block to construct curvature invariants from.
- Thus one can consider various new terms such as

$$\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu}\,,\; R^{\mu\nu}\mathcal{R}_{\mu\nu}\,,\; \mathcal{R}^{\mu\nu\alpha\beta}\mathcal{R}_{\mu\nu\alpha\beta}\,,\; R^{\mu\nu\alpha\beta}\mathcal{R}_{\mu\nu\alpha\beta}\,,\; \mathcal{R}R\,,\; \textit{etc}. \tag{30}$$

- Though an exhaustive analysis of such hybrid theories has not been performed, there is some evidence that the so called hybrid class of theories presented here is a unique class of viable higher order hybrid gravity theories.

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Representative class of more general theories

- Consider a representative class of more general theories:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\, f(R,\mathcal{R},\hat{Q}_H)\,, \qquad \hat{Q}_H = R^{\mu\nu}\mathcal{R}_{\mu\nu} \qquad (31)$$

- Variation with respect to the metric yields the field equation:

$$f_{,R}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + g_{\mu\nu}\Box f_{,R} - \nabla_\mu\nabla_\nu f_{,R} + f_{,\mathcal{R}}\mathcal{R}_{\mu\nu} + 2f_{,\hat{Q}}R^{\lambda}_{\mu}\mathcal{R}_{\nu\lambda}$$

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Graviton propagator

- Considering perturbations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ around Minkowski space $g_{\mu\nu} = \eta_{\mu\nu}$, and inverting the linearised field equations for the physical metric, provides the propagators for the graviton and the additional degrees of freedom that may be present in $h_{\mu\nu}$.
- The propagator $\Pi^{\alpha\beta\gamma\delta}$ is defined by

$$\Pi^{-1\gamma\delta}_{\alpha\beta} h_{\gamma\delta} = \kappa^2 \tau_{\alpha\beta} \,, \tag{33}$$

where $\tau_{\alpha\beta}$ represents the linearised stress energy source.
- In the formalism of the spin-projector operators, the result can be given in Fourier space in terms of two functions a and c as

$$k^2 \Pi_{\alpha\beta\gamma\delta} = \frac{\mathcal{P}^2_{\alpha\beta\gamma\delta}}{a(-k^2)} - \frac{\mathcal{P}^0_{\alpha\beta\gamma\delta}}{a(-k^2) - 3c(-k^2)} \,, \tag{34}$$

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  where $\mathcal{P}^2_{\alpha\beta\gamma\delta}$ picks up the spin-2 and $\mathcal{P}^0_{\alpha\beta\gamma\delta}$ the scalar modes of the fluctuations.

- The functions a and c can be determined immediately given the form of $f(R, \mathcal{R}, \hat{Q}_H)$. They depend upon the combinations

$$A = \frac{6f^{(0)}_{\mathcal{R}\mathcal{R}} + f^{(0)}_{,\hat{Q}}}{2f^{(0)}_{,\mathcal{R}}}, \quad \text{and} \quad B = \frac{f^{(0)}_{,\hat{Q}}}{f^{(0)}_{\mathcal{R}}}, \tag{35}$$

in the following way:

$$a(\Box) = f^{(0)}_{,R} + f^{(0)}_{,\mathcal{R}} - f^{(0)}_{,\hat{Q}} \frac{B}{4}\Box^2, \tag{36}$$

$$\begin{aligned} c(\Box) = {} & f^{(0)}_{,R} + f^{(0)}_{,\mathcal{R}} - 2\left(f^{(0)}_{,RR} + 4f^{(0)}_{,R\mathcal{R}} + f^{(0)}_{,\hat{Q}}\right)\Box \\ & + \left[f^{(0)}_{,R\mathcal{R}}(6A + B) + f^{(0)}_{,\hat{Q}}\left(2A + \frac{B}{4}\right)\right]\Box^2 . \end{aligned} \tag{37}$$

Metric $f(R)$ models

- In the pure metric $f(R)$ case, $f^{(0)}_{,R\mathcal{R}} = A = 0$ and we have

$$\Pi^{\alpha\beta\gamma\delta}_{f(R)} = \Pi^{\alpha\beta\gamma\delta}_{GR} + \frac{1}{2\left(k^2 + (3f^{(0)}_{,RR})^{-1}\right)}\mathcal{P}^{0\alpha\beta\gamma\delta} \,. \tag{38}$$

  - Thus we have an extra scalar degree of freedom, as we expect since the $f(R)$ models are known to be equivalent to Brans-Dicke theories with a vanishing parameter $\omega_{BD} = 0$.
  - The mass of the "scalaron" is $m^2 = (3f^{(0)}_{,RR})^{-1}$, and as long as $f''(R) > 0$ the theory is stable, otherwise a tachyonic mass spoils the stability around Minkowski space.

Palatini $f(\mathcal{R})$ models

- The Palatini-type $f(\mathcal{R})$ models are equivalent to Brans-Dicke theories with the parameter $\omega_{BD} = -3/2$.
    - This particular value corresponds to vanishing kinetic term of the field, which is thus nondynamical.
    - Therefore we expect that no additional scalar degree of freedom should appear.
- For a proper normalisation we may assume that $f^{(0)}_{,\mathcal{R}} = 1$, and we have that $f^{(0)}_{,RR} = f^{(0)}_{,R\mathcal{R}} = f^{(0)}_{,\hat{Q}} = 0$. Hence,

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Hybrid metric-Palatini models

- In Ricci-flat spacetimes the hybrid metric-Palatini theories share the properties of Palatini-$f(\mathcal{R})$ theories, which in vacuum reduce to GR with a possible cosmological constant.
- Therefore it is not a surprise that we find no new propagating degrees of freedom in Minkowski vacuum,

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Hybrid Ricci-squared $f(\mathcal{R},\hat{Q})$ theories ($\hat{Q}_H = R^{\mu\nu}\mathcal{R}_{\mu\nu}$)

- We can arrange the result for the propagator in the form

$$\Pi^{\alpha\beta\gamma\delta}_{f(\mathcal{R},\hat{Q})} = \frac{3f^{(0)}_{,\hat{Q}}\left(1+\frac{3}{4}f^{(0)}_{,\hat{Q}}k^2\right)\ \mathcal{P}^{0\alpha\beta\gamma\delta}}{2\left(1-\frac{1}{4}\left(f^{(0)}_{,\hat{Q}}\right)^2 k^4\right)\left(1+3f^{(0)}_{,\hat{Q}}k^2+2\left(f^{(0)}_{,\hat{Q}}\right)^2 k^4\right)} + \frac{\Pi^{\alpha\beta\gamma\delta}_{GR}}{\left(1-\frac{1}{4}\left(f^{(0)}_{,\hat{Q}}\right)^2 k^4\right)}\,. \qquad (41)$$

- We have a modulated graviton propagator which adds two extra poles. In addition, there appears a scalar propagator that has five poles.
    - This theory is seriously haunted by ghosts and thus not physical.
    - It is easy to convince oneself that this occurs very generically once one builds the action from higher order hybrid curvature invariant.

Dark matter in hybrid metric-Palatini gravity

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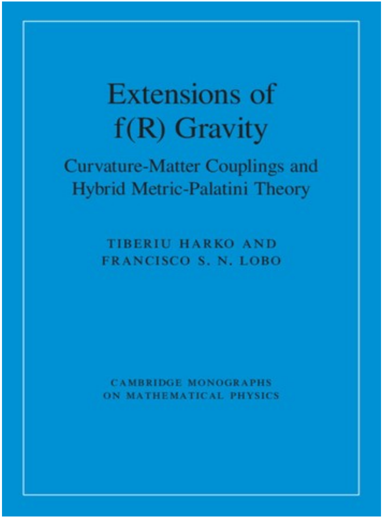
Based on book: Cambridge University Press (2018)



Recent work

1. T. Harko, F. S. N. Lobo and H. M. R. Silva, "Cosmic stringlike objects in hybrid metric-Palatini gravity," Phys. Rev. D **101** (2020), 124050 [arXiv:2003.09751 [gr-qc]].
2. B. Danila, T. Harko, F. S. N. Lobo and M. K. Mak, "Spherically symmetric static vacuum solutions in hybrid metric-Palatini gravity," Phys. Rev. D **99** (2019), 064028 [arXiv:1811.02742 [gr-qc]].
3. B. Danila, T. Harko, F. S. N. Lobo and M. K. Mak, "Hybrid metric-Palatini stars," Phys. Rev. D **95** (2017), 044031 [arXiv:1608.02783 [gr-qc]].

Generalized hybrid metric-Palatini gravity

1. H. M. R. da Silva, T. Harko, F. S. N. Lobo and J. L. Rosa,
"Cosmic strings in generalized hybrid metric-Palatini gravity,"
Phys. Rev. D **104** (2021), 124056 [arXiv:2104.12126 [gr-qc]].
2. J. L. Rosa, F. S. N. Lobo and G. J. Olmo,
"Weak-field regime of the generalized hybrid metric-Palatini gravity,"
Phys. Rev. D **104** (2021), 124030 [arXiv:2104.10890 [gr-qc]].
3. J. L. Rosa, F. S. N. Lobo and D. Rubiera-Garcia,
"Sudden singularities in generalized hybrid metric-Palatini cosmologies,"
JCAP **07** (2021), 009 [arXiv:2103.02580 [gr-qc]].
4. J. L. Rosa, S. Carloni and J. P. S. Lemos,
"Cosmological phase space of generalized hybrid metric-Palatini theories of gravity,"
Phys. Rev. D **101** (2020), 104056 [arXiv:1908.07778 [gr-qc]].

Summary and Conclusion

1. We presented a hybrid metric-Palatini framework for theories of gravity, and tested the new theories it entails using a number of theoretical consistency checks and observational constraints.
2. One may evade altogether the chameleon mechanism.
3. One excludes theories inhabited by ghost-like, superluminally propagating and otherwise pathological degrees of freedom.
4. In a monistic view of Physics, one would expect Nature to make somehow a choice between the two distinct possibilities offered by metric and Palatini formalisms.
    1. We have shown that a theory consistent with observations and combining elements of these two approaches is possible.
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THANK YOU FOR YOUR TIME AND ATTENTION!