Extensions of Maxwell's electrodynamics by axions

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Outline

- One page summary of Maxwell's theory
- Introducing the axion, a $CP = -1$ dynamical field
- Effective Electro-Magneto Dynamics of axions
- Self-sustaining homogeneous axion-electromagnetic configuration in EEMD *
- Electromagnetic Radiation damping in axion stars † ‡
- Outlook

*MPLA **38** (2023) 2350137
†Phys. Rev. D**107** 055017 (2023)
‡Symmetry **14** 1113 (2022)

Maxwell's electrodynamics

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Non-relativistic decomposition

$$F^{0i} = -E_i, \qquad F^{ij} = -\epsilon^{ijk}B_k \qquad \mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -\dot{\mathbf{A}} - \nabla A_0.$$

Dual field strength and Bianchi's identity:

$$F_{d\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}, \qquad \partial_\mu F_d^{\mu\nu} = 0$$

Homogeneous Maxwell-equations follow:

$$\nabla\mathbf{B} = 0, \qquad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}.$$

Electrodynamics is CP-symmetric

$$L_\Theta = \frac{\Theta}{4}F_{\mu\nu}F_d^{\mu\nu} = -\Theta\mathbf{E}\mathbf{B} \qquad \text{FORBIDDEN!}$$

Dynamical equations ($\delta\int d^4xL/\delta A_\nu = 0$):

$$\partial_\mu F^{\mu\nu} = j^\nu, \qquad \nabla\mathbf{E} = \rho, \quad \nabla \times \mathbf{B} - \dot{\mathbf{E}} = \mathbf{j}$$

Why is QCD CP-invariant? (Origin of axion hypothesis)

Non-abelian gluon-dynamics with CP-odd completion

$$L^{QCD}=-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu},\qquad F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^\mu-gf^{abc}A^b_\mu A^c_\nu.$$

$$L^{QCD}_\Theta=\Theta\frac{g^2}{64\pi^2}F^a_{\mu\nu}F^{a\mu\nu}_d,\qquad F^a_{d,\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{a\rho\sigma}.$$

Unnatural smallness: $|\Theta_{exp}|<10^{-10}$.

Peccei-Quinn mechanism

$U_{PQ}(1)$ spontanously broken at scale f_a provides pseudo-Goldstone, pseudoscalar $a(x)$:

$$\Theta\to\Theta_{eff}=\Theta+\xi\frac{\langle a(x)\rangle}{f_a}\approx 0.$$

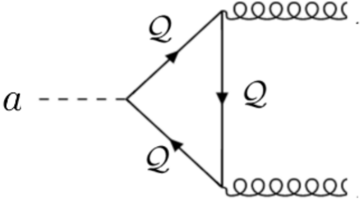
Model: axion-gauge coupling mediated by heavy fermions

Axion-coupling to electromagnetic field:

$$L_{EM+a} = \frac{1}{2}\left[(\partial_\mu a(x))^2 - m_a^2 a(x)^2\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu$$
$$+\frac{1}{4}g_{a\gamma\gamma}a(x)F_{\mu\nu}(x)F_d^{\mu\nu}(x).$$

$F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ $\qquad$ $\partial_\mu F^{\mu\nu} + g_{a\gamma\gamma}\partial_\mu a(x)F_d^{\mu\nu} = j_e^\nu$

$a\gamma\gamma$ coupling might arise from PQ-charged heavy quarks



Axion electrodynamics I.

$$L = \frac{1}{2}\left(\epsilon\mathbf{E}^2(x) - \frac{1}{\mu}\mathbf{B}^2(x)\right) - g_{a\gamma\gamma}a(x)\mathbf{E}(x)\cdot\mathbf{B}(x)$$
$$+\frac{1}{2}\left(\dot{a(x)}^2 - (\nabla a(x))^2 - m_a^2a^2\right) - j_0A_0(x) + \mathbf{j}(x)\mathbf{A}(x),$$

Modified Maxwell equations:

$$\nabla\mathbf{E} + g_{a\gamma\gamma}\mathbf{B}\nabla a(x) = \rho_e,$$
$$-\dot{\mathbf{E}} + \nabla\times\mathbf{B} - g_{a\gamma\gamma}(\dot{a}\mathbf{B} + \nabla a(x)\times\mathbf{E}) = \mathbf{j}_e,$$

Axion equation

$$\ddot{a}(\mathbf{x},t) - \nabla^2 a(\mathbf{x},t) + m_a^2 a(\mathbf{x},t) = g_{a\gamma\gamma}\mathbf{E}\cdot\mathbf{B}.$$

Axion electrodynamics II.

The axion "medium"

$$\mathbf{P} = g_{a\gamma\gamma} a\mathbf{B}, \qquad \mathbf{D} = \mathbf{E} + \mathbf{P}, \qquad \nabla\mathbf{D} = \rho,$$
$$\nabla \times \mathbf{H} - \dot{\mathbf{D}} = \mathbf{j}, \qquad \mathbf{H} = \mathbf{B} - g_{a\gamma\gamma} a\mathbf{E}$$

Or effective charge and current density of electrically neutral axion

$$\rho_{axion} = -\nabla\mathbf{P} = -g_{a\gamma\gamma}\mathbf{B}\nabla a,$$
$$\mathbf{j}_{axion} = \dot{\mathbf{P}} + \nabla \times \mathbf{M} = g_{a\gamma\gamma}(\dot{a}\mathbf{B} + \nabla a \times \mathbf{E})$$

Energy balance

$$W_{charged} = \int d^3x\mathbf{j}\mathbf{E} = \int d^3x\mathbf{E}(\nabla \times \mathbf{B} - \dot{\mathbf{E}} - \mathbf{j}_{axion})$$
$$= -\int d\mathbf{F}(\mathbf{E} \times \mathbf{B}) - \frac{d}{dt}\frac{1}{2}\int d^3x(\mathbf{E}^2 + \mathbf{B}^2) - \int d^3x\mathbf{j}_{axion}\mathbf{E}$$

After rearangement

$$\frac{dE_{EM}}{dt} + \frac{dE_{axion}}{dt} = \int d^3x\mathbf{j}\mathbf{E} - \int d\mathbf{F}(\mathbf{E} \times B)$$

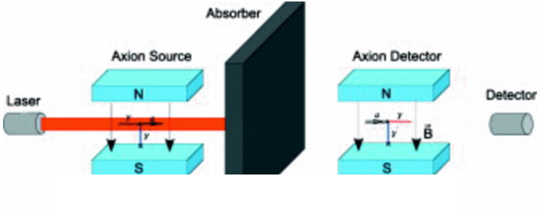
Review: Paul Sikivie, Rev. Mod. Phys.**93** (2020) 15004

Shining through wall

observing axions produced in laboratory

Review: A. Ringwald, arXiv:2404.09036

Original proposition Ringwald, Phys. Lett. B**569** (2003) 51



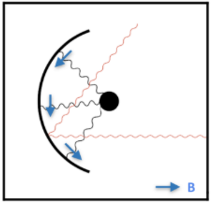
Electromagnetic search for galactic DM axions

Galactic axion-halo:
standing wavelike ($\lambda \sim km$), density $\rho_a \sim 0.45 GeV/cm^3$

$$a(t) = \sqrt{\rho_a}\cos(m_a t)/m_a$$

Tools of observation:
i) *tunable microwave cavity, hitting resonance* ($\omega \approx m_a$) *(Sikivie, 1983)*.
ii) *dish antennas (Horns et al., 2013)*
Static magnetic field $\mathbf{B}_0$ parallel to a metallic surface cooperates with axion standing wave to produce oscillating electric field strength $\mathbf{E}(t)$ by

$$\dot{\mathbf{E}} = -g_{a\gamma\gamma}\mathbf{B}_0\dot{a}(t) = g_{a\gamma\gamma}\mathbf{B}_0\sin(m_a t)\sqrt{\rho_a}$$



Axion Electro-Magneto Dynamics

Axions interacting with dyonic heavy quarks (Q)
(Sokolov and Ringwald, 2022)

Separate magnetic vector potential C^μ is necessary in addition to A^μ:

$$G^{\mu\nu} = \partial^\mu C^\nu - \partial^\nu C^\mu, \qquad G_d^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

$$\partial_\mu G^{\mu\nu} = j_m^\nu, \qquad \partial_\mu G_d^{\mu\nu} = 0 \quad \text{Bianchi-identity},$$

Enforcing unique electromagnetic field strength in field equations

$$F_d^{\mu\nu}|_{\mathbf{B},\mathbf{E}} = G^{\mu\nu}|_{\mathbf{B},\mathbf{E}} \qquad \to G^{0i} = -B_i, \qquad G^{ij} = \epsilon^{ijk} E_k.$$

!! Violates Bianchi-identity on magnetic sources!!
Remedy: Dirac, Zwanziger, ...

Axion Electro-Magneto Dynamics

Two new axion-photon couplings

$$\Delta L_{aEMD} = -\frac{1}{8}a(x)\left(g_{EE}F_{\mu\nu}F_d^{\mu\nu} + g_{MM}G_{\mu\nu}G_d^{\mu\nu} + 2g_{EM}F_{\mu\nu}G_d^{\mu\nu}\right).$$

$$\partial_\mu F^{\mu\nu} + g_{EE}\partial_\mu a(x)F_d^{\mu\nu} + g_{EM}\partial_\mu a(x)G_d^{\mu\nu} = j_e^\nu$$
$$\partial_\mu G^{\mu\nu} + g_{MM}\partial_\mu a(x)G_d^{\mu\nu} + g_{EM}\partial_\mu a(x)F_d^{\mu\nu} = j_m^\nu.$$

In terms of field strengths

$$\nabla\mathbf{E} + g_{EE}\mathbf{B}\nabla a(x) - g_{EM}\mathbf{E}\nabla a(x) = \rho_e,$$
$$\nabla\mathbf{B} - g_{MM}\mathbf{E}\nabla a(x) + g_{EM}\mathbf{B}\nabla a(x) = \rho_m,$$
$$-\dot{\mathbf{E}} + \nabla\times\mathbf{B} - g_{EE}(\dot{a}\mathbf{B} + \nabla a(x)\times\mathbf{E}) + g_{EM}(\dot{a}\mathbf{E} - \nabla a(x)\times\mathbf{B}) = \mathbf{j}_e,$$
$$\dot{\mathbf{B}} + \nabla\times\mathbf{E} - g_{MM}(\dot{a}\mathbf{E} - \nabla a(x)\times\mathbf{B}) + g_{EM}(\dot{a}\mathbf{B} + \nabla a(x)\times\mathbf{E}) = -\mathbf{j}_m.$$

Axion field equation

$$\ddot{a}(\mathbf{x},t) - \nabla^2 a(\mathbf{x},t) + m_a^2 a(\mathbf{x},t) = (g_{EE} - g_{MM})\mathbf{E}\cdot\mathbf{B} - g_{EM}(\mathbf{E}^2 - \mathbf{B}^2).$$

Energy-balance equation (Poynting-theorem)

$$-\int d^3x\,[(\mathbf{j}_e+\mathbf{j}_{axion,e})\cdot\mathbf{E}(t,\mathbf{x})+(\mathbf{j}_m+\mathbf{j}_{axion,m})\cdot\mathbf{B}(t,\mathbf{x})]$$
$$=\int d\mathbf{F}\cdot(\mathbf{E}\times\mathbf{B})+\frac{d}{dt}\int d^3x\frac{1}{2}\left(\mathbf{E}^2(t,\mathbf{x})+\mathbf{B}^2(t,\mathbf{x})\right).$$

With the explicit expressions of axionic currents ($\mathbf{j}_e=\mathbf{j}_m=0$)

$$-\int d^3x\,[\mathbf{j}_{axion,e}\cdot\mathbf{E}(t,\mathbf{x})+\mathbf{j}_{axion,m}\cdot\mathbf{B}(t,\mathbf{x})]$$
$$=-\int d^3x\dot{a}\left[(g_{aEE}-g_{aMM})\mathbf{E}\cdot\mathbf{B}-g_{aEM}(\mathbf{E}^2-\mathbf{B}^2)\right],$$

by the axion equation equals the rate of change of the axion energy

$$-\frac{d}{dt}\int d^3x\frac{1}{2}\left(\dot{a}^2+(\nabla a)^2+m_a^2a^2\right).$$

Self-sustaining homogeneous axion-electromagnetic field configuration

$\mathbf{E}(t), \mathbf{B}(t), a(t)$ initiated by external uniform magnetic field $\mathbf{B}_0$

$$\dot{\mathbf{E}}(t) = -g_{EE}\dot{a}_0(t)(\mathbf{B}_0 + \mathbf{B}(t)) + g_{EM}\dot{a}_0(t)\mathbf{E}(t)$$
$$\dot{\mathbf{B}}(t) = g_{MM}\dot{a}_0(t)\mathbf{E}(t) - g_{EM}\dot{a}_0(t)(\mathbf{B}_0 + \mathbf{B}(t)).$$

Ansätze

$$\mathbf{E} = f_E(a_0(t))\mathbf{B}_0, \qquad \mathbf{B} = f_B(a_0(t))\mathbf{B}_0$$

Initial conditions

$$f_E(a_0(t_0) = a_i) = 0, \qquad f_B(a_0(t_0) = a_i) = 0.$$

Solution with arbitrary $a_0(t)$ $\quad [\lambda^2 = g_{EE}g_{MM} - g_{EM}^2 > 0]$

$$f_E(a_0) = -\frac{g_{EE}}{\lambda}\sin[\lambda(a_0(t) - a_i)],$$
$$f_B(a_0) = \cos[\lambda(a_0(t) - a_i)] - 1 - \frac{g_{EM}}{\lambda}\sin[\lambda(a_0(t) - a_i)].$$

Back-reaction on axion motion

Axion equation emerging after the "elimination" of electromagnetic fields

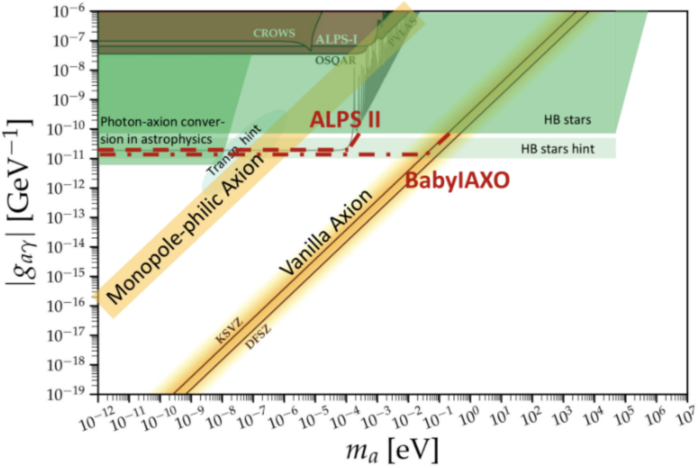
$$\ddot{a}_0(t) + m_a^2 a_0(t) =$$
$$= [(g_{EE} - g_{MM}) f_E(a_0)(f_B(a_0) + 1) - g_{EM}(f_E(a_0)^2 - (f_B(a_0) + 1)^2)] B_0^2$$

completes the stationary $a_0(t), \mathbf{E}(a_0), \mathbf{B}(a_0)$ configuration induced by $\mathbf{B}_0$.

Small amplitude motion $|\lambda(a_0(t) - a_i)| << 1$ results in a mass-shift of axions:

$$\ddot{a}_0(t) + M_a^2 a_0(t) = 0, \qquad M_a^2 = m_a^2 + [(g_{EE} - g_{MM}) g_{EE} + 2g_{EM}^2] B_0^2.$$

Status of search for AxionLike Particles (ALP)



Charge quantisation argues for coupling hierarchy

$g_{MM} >> g_{EM} >> g_{EE}$

Summary

- Electrically neutral axion in extended Maxwell's theory carry effective electric charge and current densities
- Electro-magneto dynamics allows construction of larger set of axion-photon coupling
- In electro-magneto dynamics
  self-sustaining homogeneous axion+electromagnetic configurations appear,
  (possibly enhancing dish antenna signal)

Some interesting phenomena

- Gravitationally self-bound axion stars emit electromagnetic radiation at slow rate
- Rotation of polarization plane of electromagnetic radiation propagating through inhomogeneous axion medium
- Echo of electromagnetic radiation from axion cloud via induced axion decay into two photons
- Parametric electromagnetic instabilities of axion stars

Free axions embedded in $\mathbf{B}_0$-induced medium with gravitational interaction

$$U_{eff} = \frac{1}{2}\left(\dot{a}^2 + (\nabla a)^2 + M_a^2 a^2\right) + U_{grav}$$

$$U_{grav} = -\frac{G_N}{2}\int d^3x \int d^3y \frac{\rho_{axion}(\mathbf{x})\rho_{axion}(\mathbf{y})}{|x-y|}.$$

Approximate lowest energy configuration searched in the form $\psi(\mathbf{x},t) = e^{i\mu_g t}\phi(\mathbf{x})$. μ_g: specific gravitational binding energy

$$a(\mathbf{x},t) = \frac{1}{\sqrt{2M_a}}\left(e^{-iM_a t}\psi(\mathbf{x},t) + e^{iM_a t}\psi^*(\mathbf{x},t)\right),$$

$$\int d^3x|\psi(\mathbf{x},t)|^2 = N_{axion}, \quad \rho_{axion} = (M_a - \mu_g)|\psi(\mathbf{x},t)|^2$$

Variational estimate: $\mu_g/M_a \approx 10^2(G_N M_a^2 N)^2 << 1$ for $N \sim 10^{61}$.

$$a(\mathbf{x},t) = \sqrt{\frac{2}{M_a}}wF(\xi)\cos(M_a t), \qquad w \sim \sqrt{N}, \qquad \xi = \frac{|\mathbf{x}|}{R}$$

EEMD electromagnetic radiation from an axion star

$$\Box A^\mu = j^\mu_{axion,e}[\phi, \mathbf{B_0}], \qquad \Box C^\mu = j^\mu_{axion,m}[\phi, \mathbf{B}_0],$$
$$\mathbf{e} = -\dot{\mathbf{A}} - \nabla A^0, \qquad \mathbf{b} = -\dot{\mathbf{C}} - \nabla C^0$$

$$\int d^3x \left(\mathbf{j}_{axion,e} \cdot \mathbf{e}(t,\mathbf{x}) + \mathbf{j}_{axion,m} \cdot \mathbf{b}(t,\mathbf{x})\right)$$
$$= -\int d^3x \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|}\Bigg[\mathbf{j}_{axion,e}(\mathbf{x},t)\cdot\frac{\partial}{\partial t}\mathbf{j}_{axion,e}(\mathbf{x}',t-|\mathbf{x}-\mathbf{x}'|)$$
$$+\frac{\partial}{\partial t}\rho_{axion,e}(\mathbf{x},t)\rho_{axion,e}(\mathbf{x}',t-|\mathbf{x}-\mathbf{x}'|)\Bigg]$$
$$-\int d^3x \int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|}\Bigg[\mathbf{j}_{axion,m}(\mathbf{x},t)\cdot\frac{\partial}{\partial t}\mathbf{j}_{axion,m}(\mathbf{x}',t-|\mathbf{x}-\mathbf{x}'|)$$
$$+\frac{\partial}{\partial t}\rho_{axion,m}(\mathbf{x},t)\rho_{axion,m}(\mathbf{x}',t-|\mathbf{x}-\mathbf{x}'|)\Bigg].$$

EEMD electromagnetic radiation from an axion star

$$\overline{\frac{dE_{ax}}{dt}}^{T} = -\frac{NB_0^2 X^3}{C_2}\Big[(g_{EE}^2 + g_{EM}^2) I_{mag}^2 - \frac{1}{3X^2}(g_{EE}^2 - g_{EM}^2 - 2g_{MM}^2) I_{el}^2\Big],$$

where $X = M_a R$ gives the size of the star in units of M_a^{-1}.

$$I_{mag} = 4\pi \int d\xi \xi^2 \frac{\sin(X\xi)}{X\xi} F(\xi),$$

$$I_{el} = 4\pi \int d\xi \xi^2 \left(\frac{\sin(X\xi)}{(X\xi)^2} - \frac{\cos(X\xi)}{X\xi}\right) F'(\xi).$$

Reinterpretation of $\overline{\frac{dE_{ax}}{dt}}^{T}$ as $M_a \overline{\frac{dN(t)}{dt}}^{T}$ yields $N(t) \sim t^{-1/5}$.