Numerical analysis of Gravitational Waves

From eccentric sources

Balázs Kacskovics



Reimann Geometry

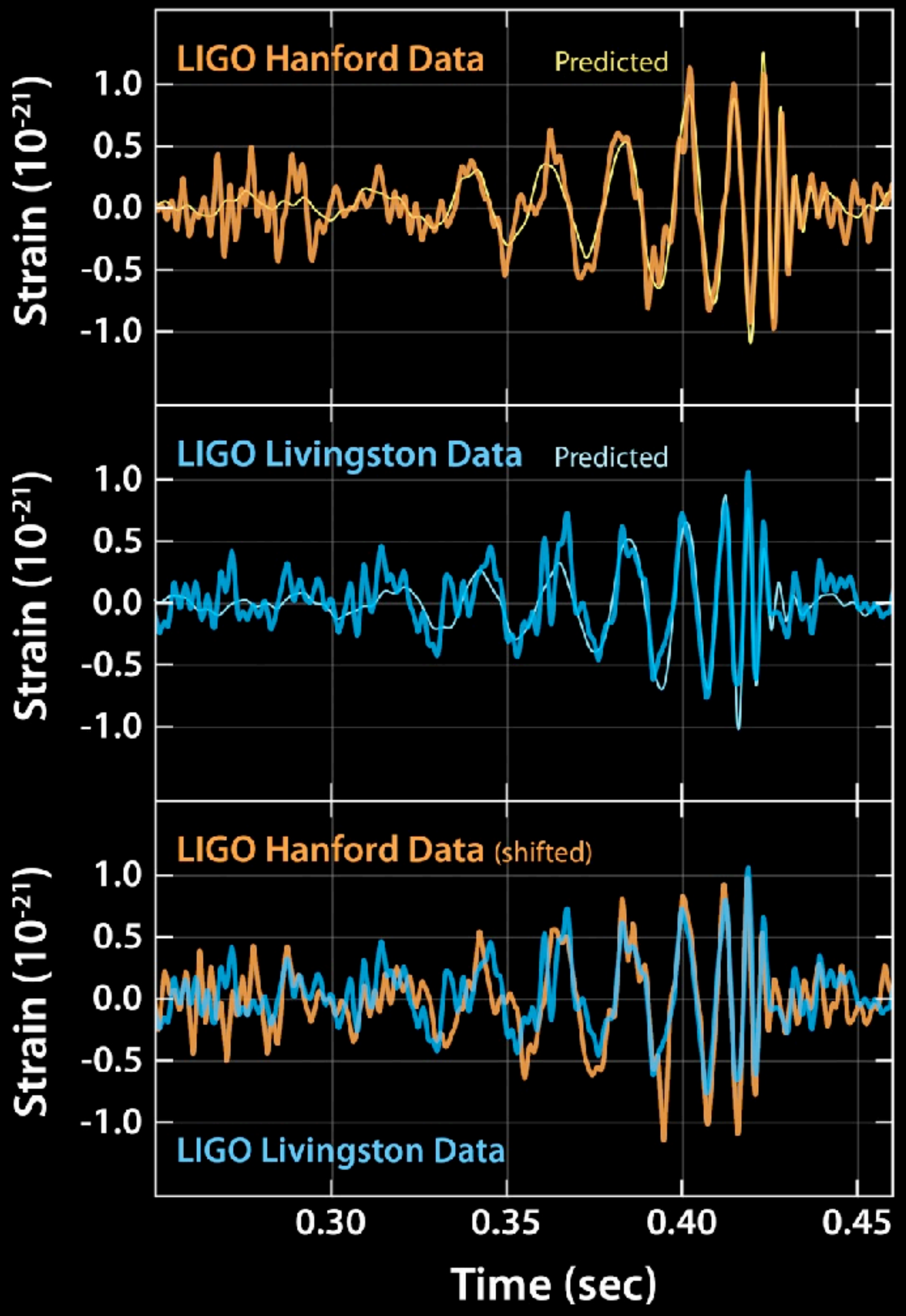
General Relativity

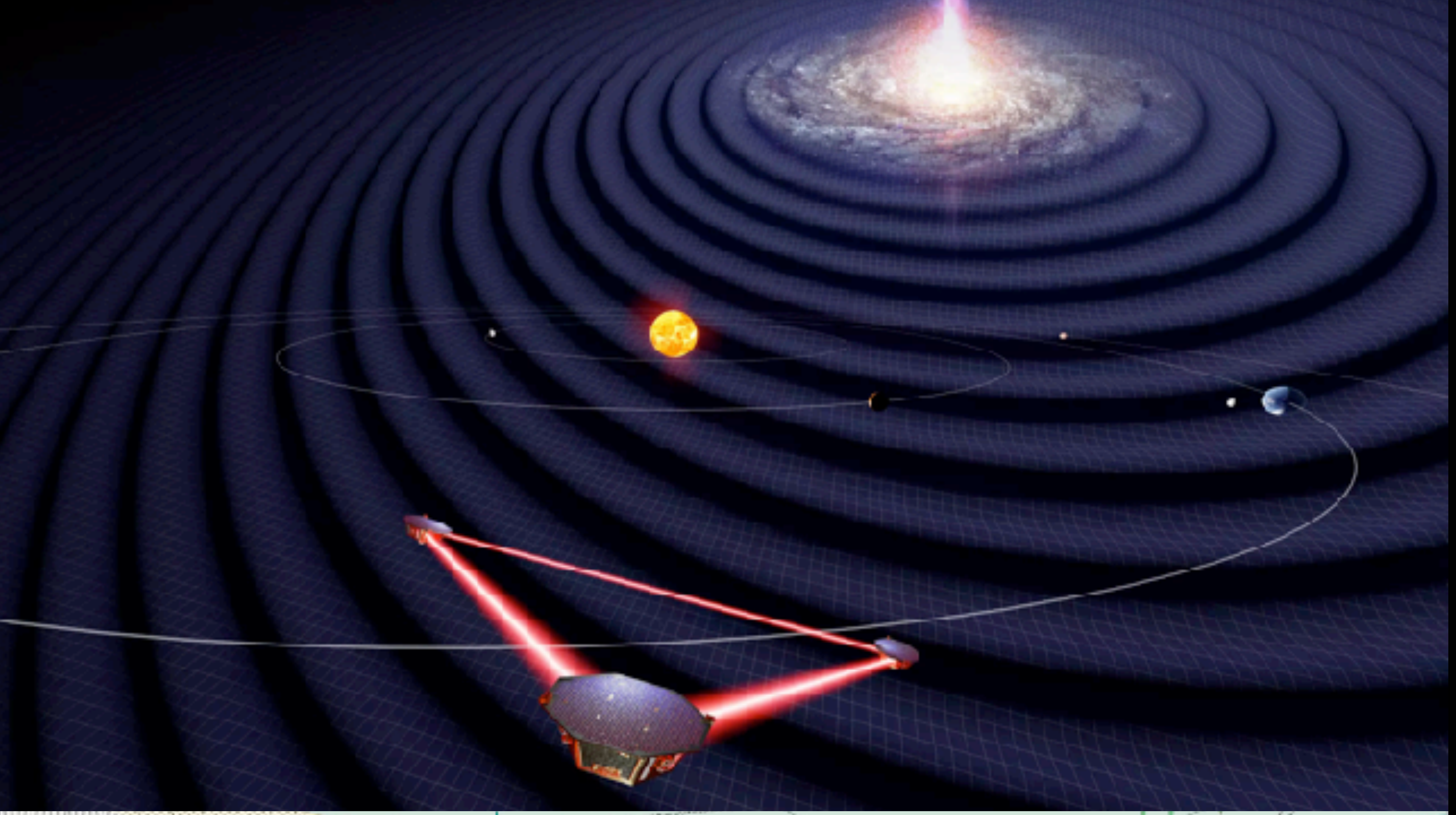
"I am your father's, brother's, nephew's, cousin's, former roommate. - What does that make us? - Absolutely nothing...,,

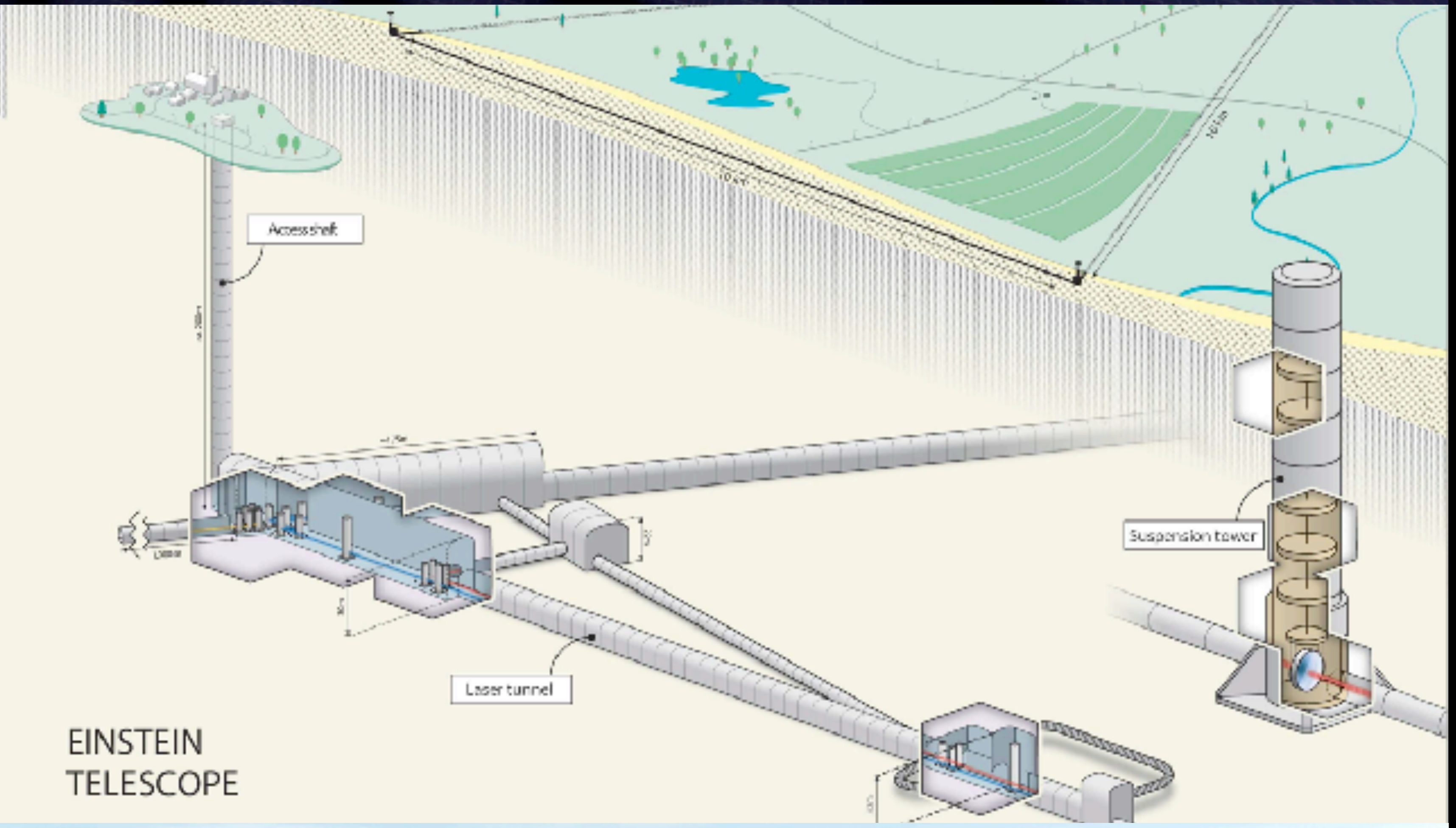
Spaceballs — 1987

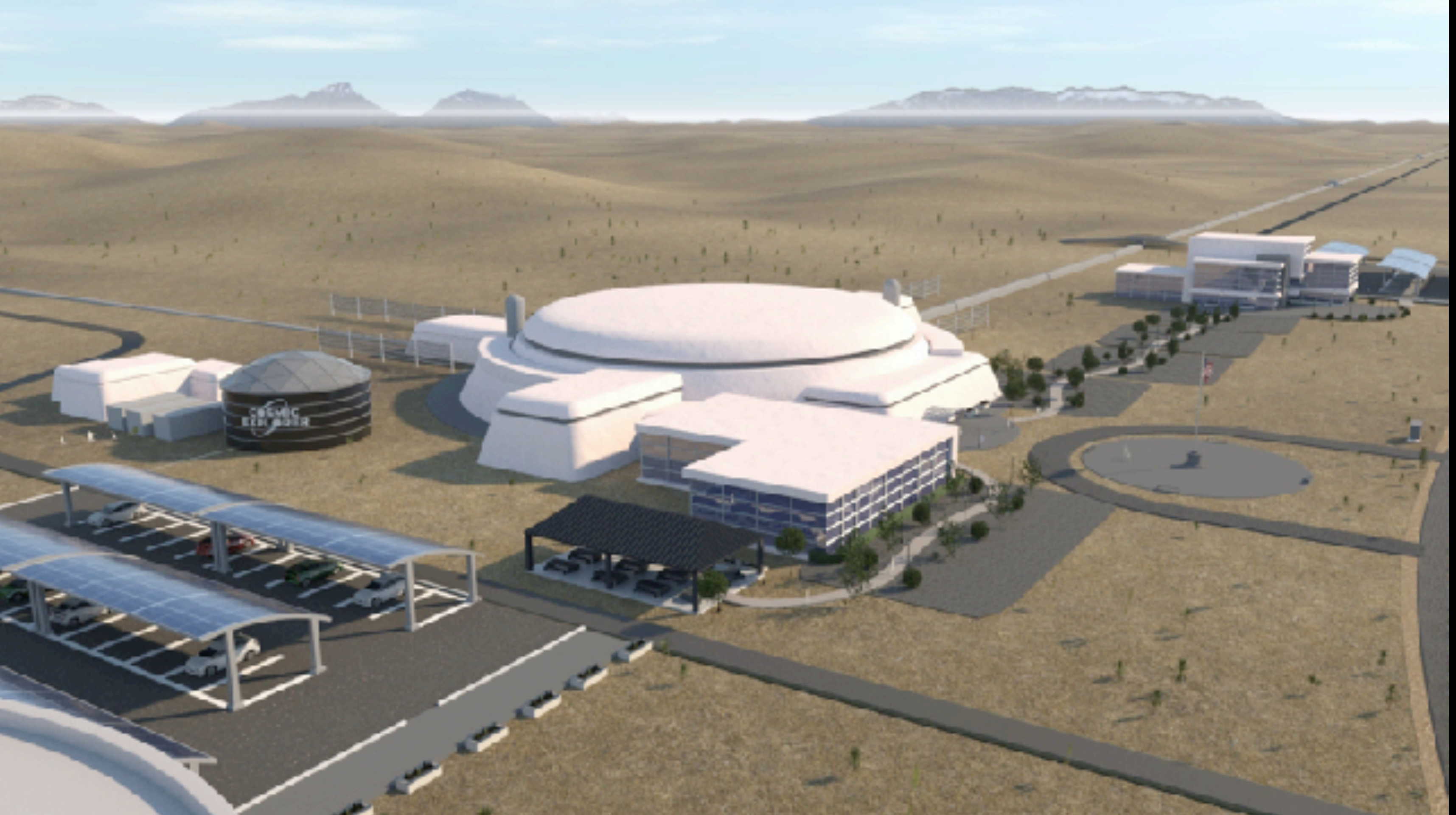
Algorithms Used in GW Search

- In the LAL library (used by the mainstream): Numerical, EOB, Taylor waveforms
- Excellently modeling the currently detectable waveforms
- Problems:
  - ➡ Only short waveforms
  - ➡ Only specific waveforms
  - ➡ No eccentric waveforms
  - ➡ Mostly spin-aligned









New GW Detectors on the Horizon

eLISA, Einstein Telescope, Cosmic Explorer

- Targeting new sources like NS-NS binaries, merging galactic nuclei, supernovae, stochastic background
- Significantly longer observational times $\Longrightarrow$ longer waveforms (up to 6 months — eLISA)
- Research of the inspiral phase
- Eccentricity and spin effects will be important in the orbital evolution of compact binaries
- eLISA got the green light this year

Gravitational Waves

Linearized theory

- Starting from the Einstein equation

$$R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi G}{c^4}T^{ab}$$

- Take a small perturbation of the Einstein eq. around a flat spacetime (gauge symmetry of GR)

$$g_{ab} = \eta_{ab} + h_{ab}, \quad h_{ab} < < 1$$

- The Riemann-tensor expressed in h_{ab} linear order

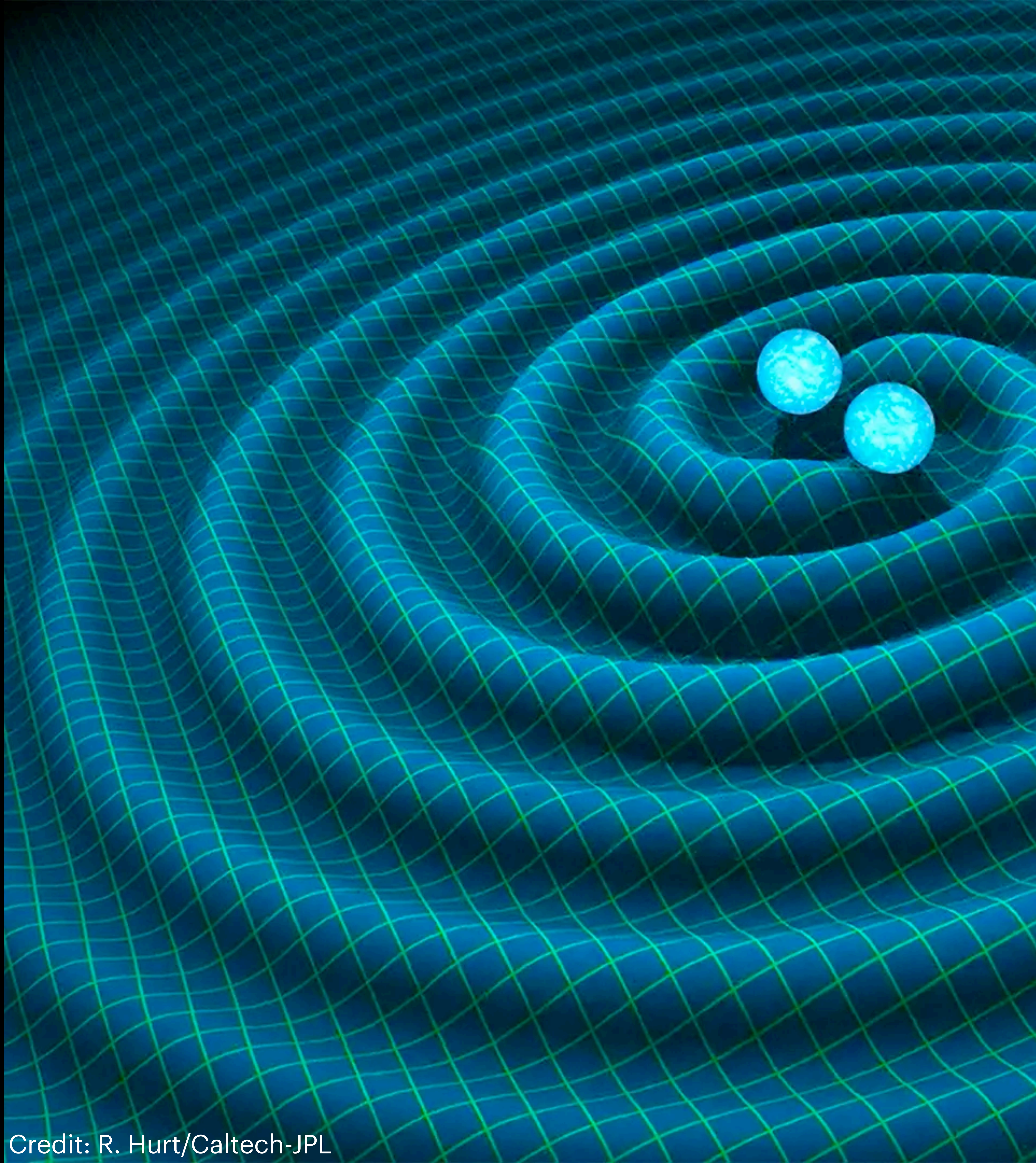
$$R_{abcd} = \frac{1}{2}(\partial_b\partial_c h_{ad} + \partial_a\partial_d h_{bc} - \partial_a\partial_c h_{bd} - \partial_b\partial_d h_{ac})$$

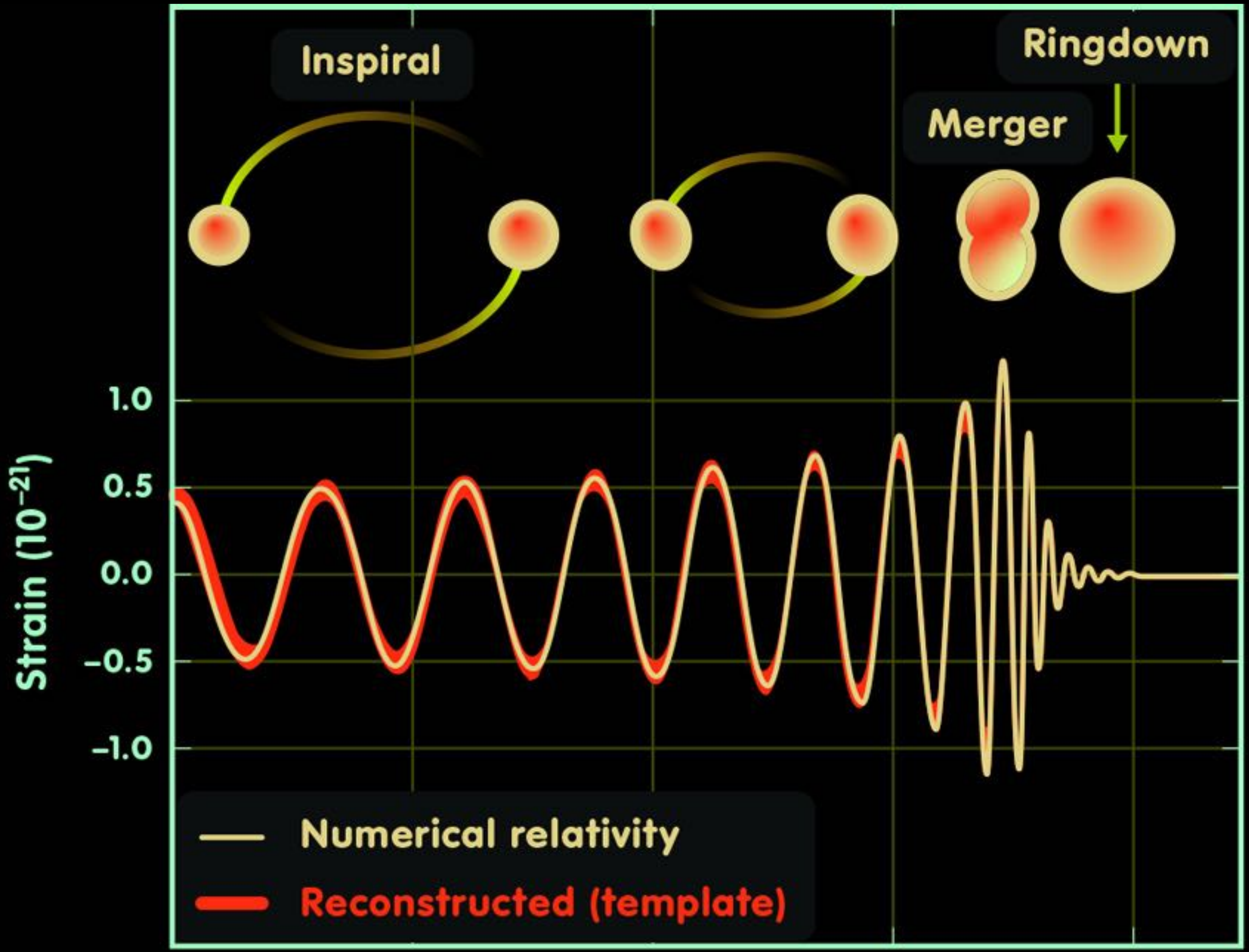
- The linearized Einstein equation

$$\Box \bar{h}_{ab} + \eta_{ab}\partial^c\partial^d\bar{h}_{cd} - \partial^c\partial_b\bar{h}_{ac} - \partial^c\partial_a\bar{h}_{bc} = -\frac{16\pi G}{c^4}T_{ab}$$

- Using the gauge freedom of GR and choosing the De Donder gauge, $\partial^b\bar{h}_{ab} = 0$

$$\Box \bar{h}_{ab} = -\frac{16\pi G}{c^4}T_{ab}$$

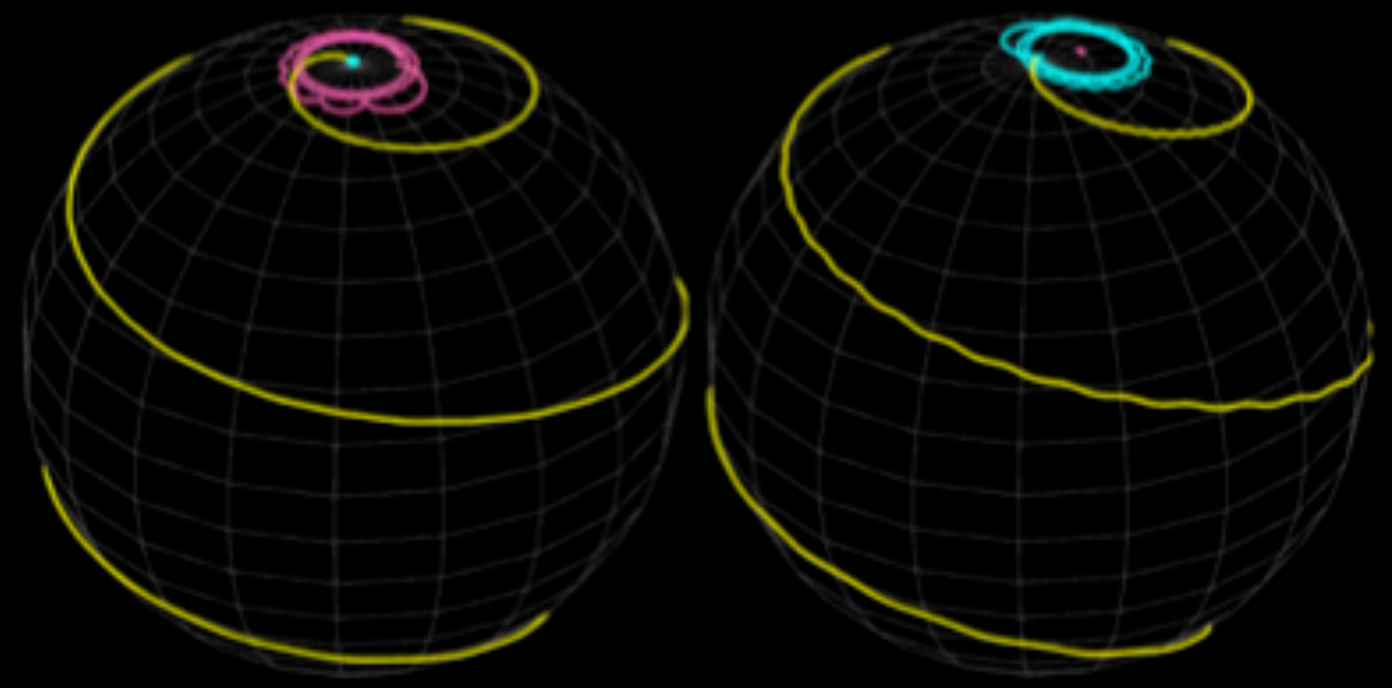




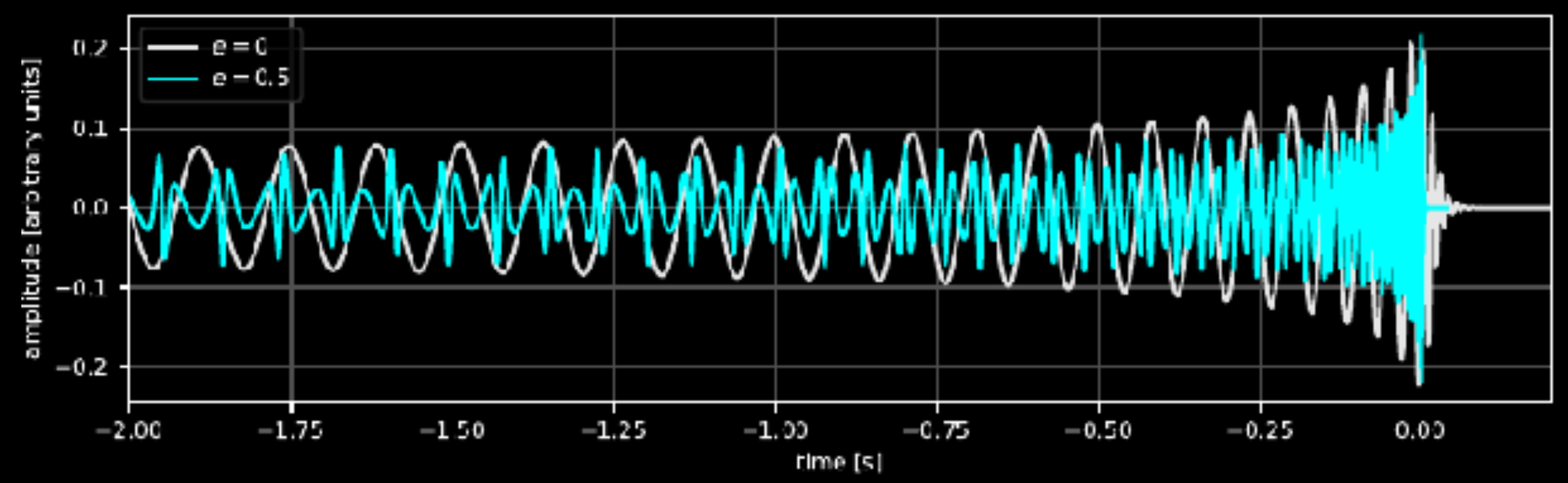
LIGO / Redesign: Daniela Leitner

Motivation

PRL 114, 141101 (2015)



ApJ 883, 149 (2019)



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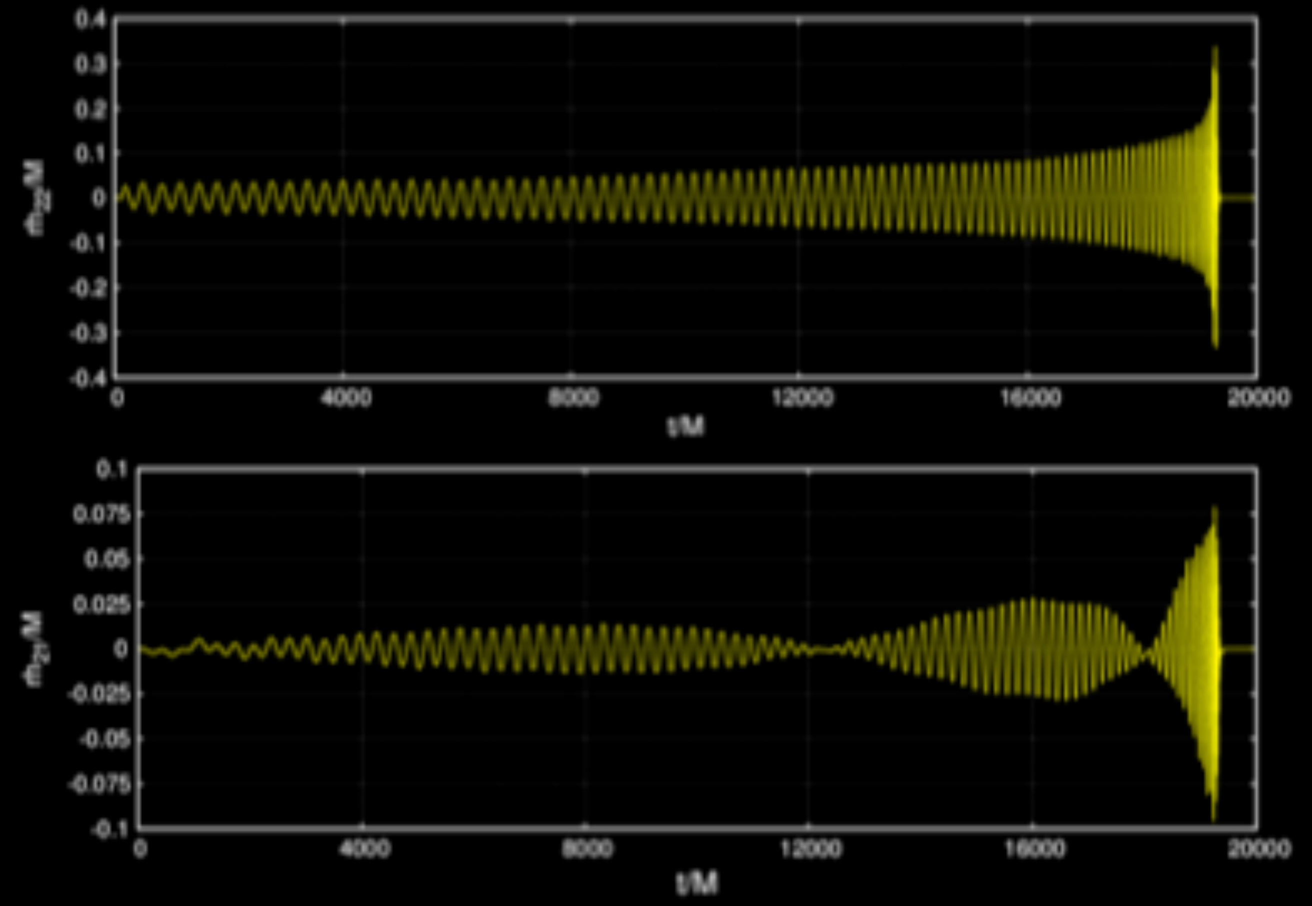
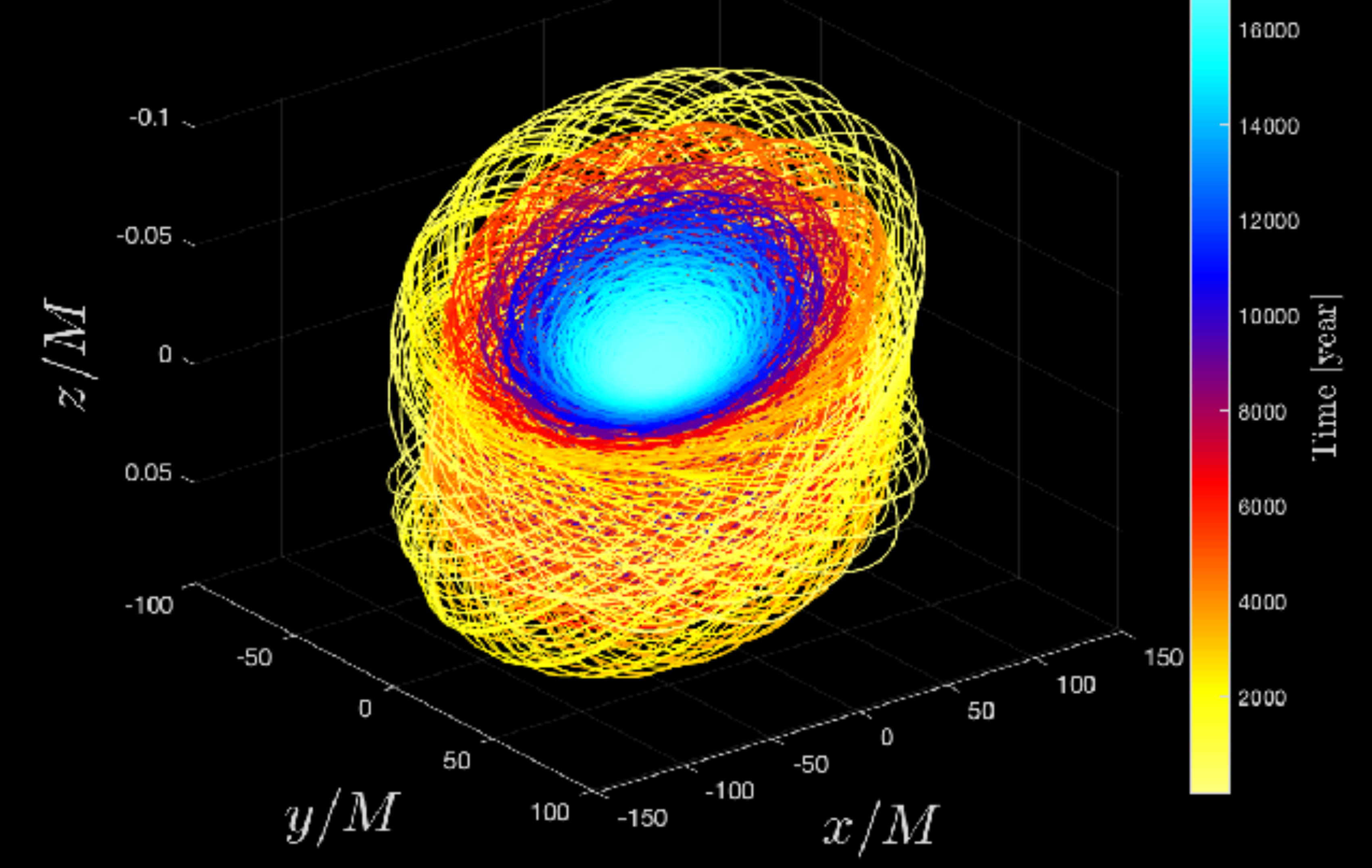
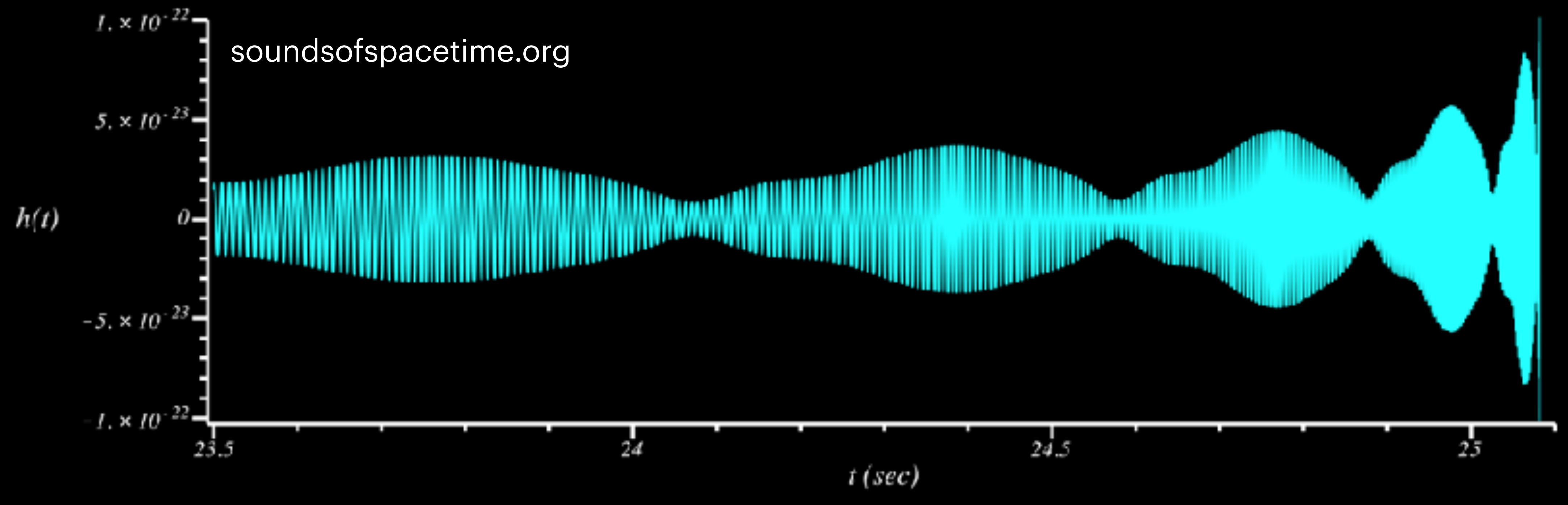


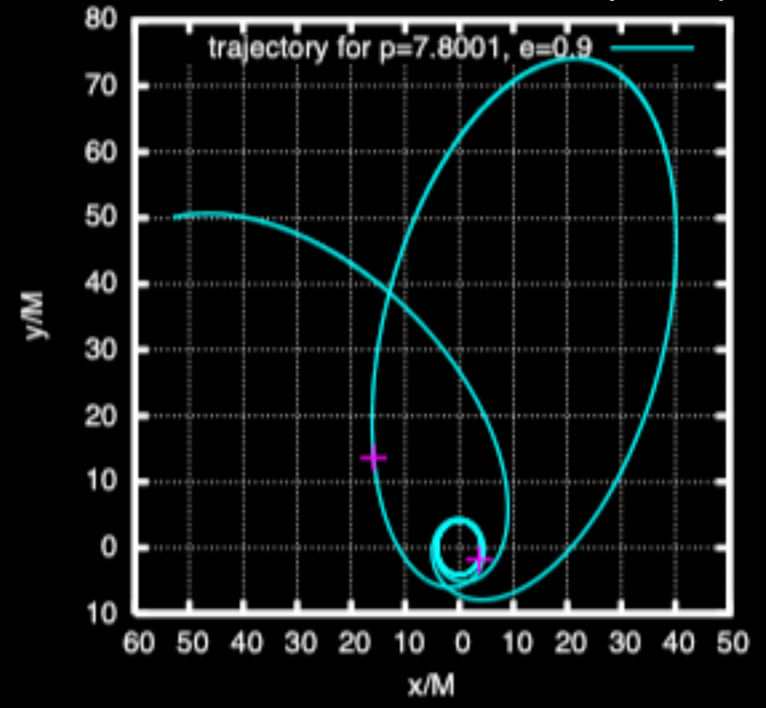
Class. Quantum Grav. 39, 095007 (2022)



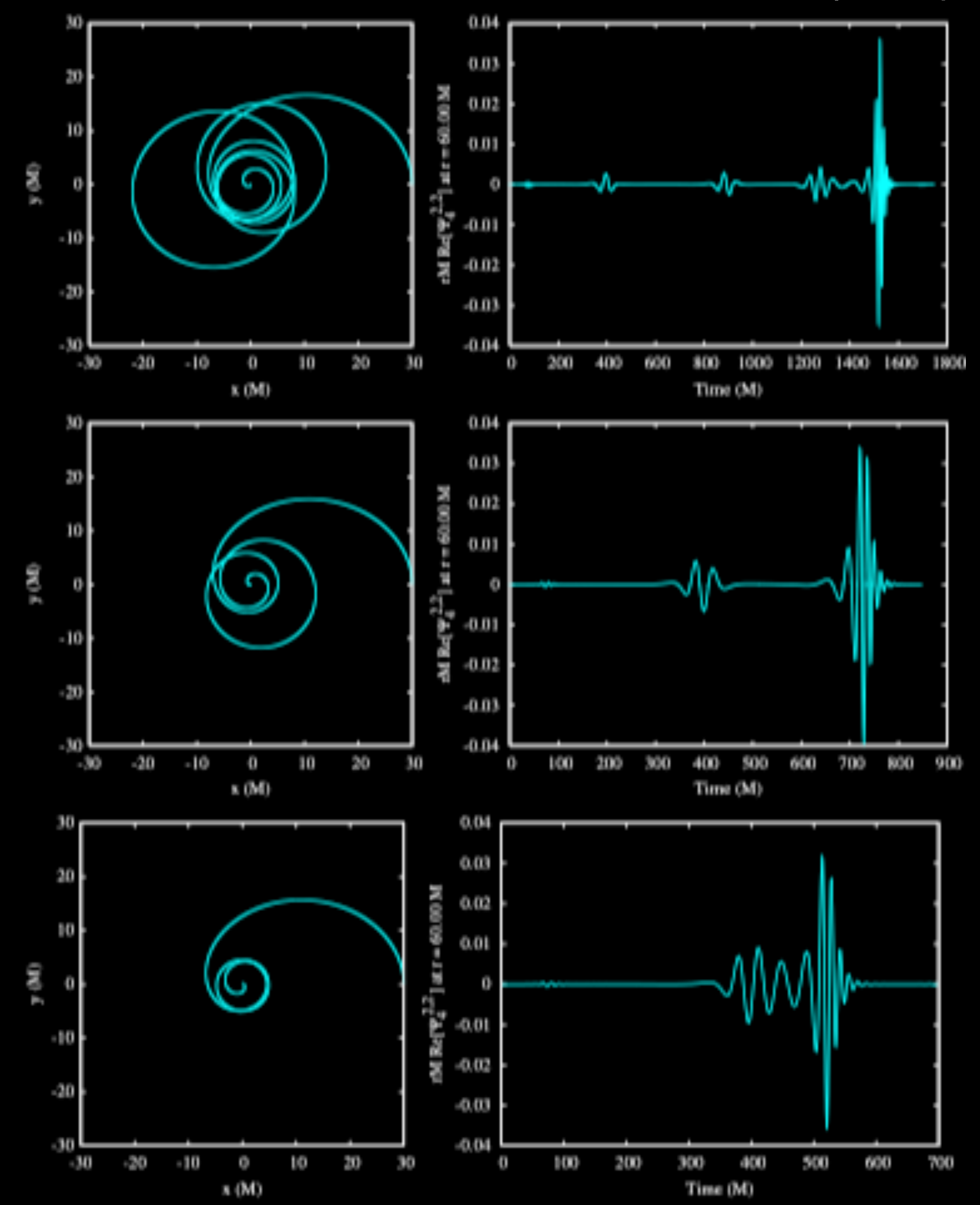
soundsofspacetime.org



PRD 75, 124011 (2007)

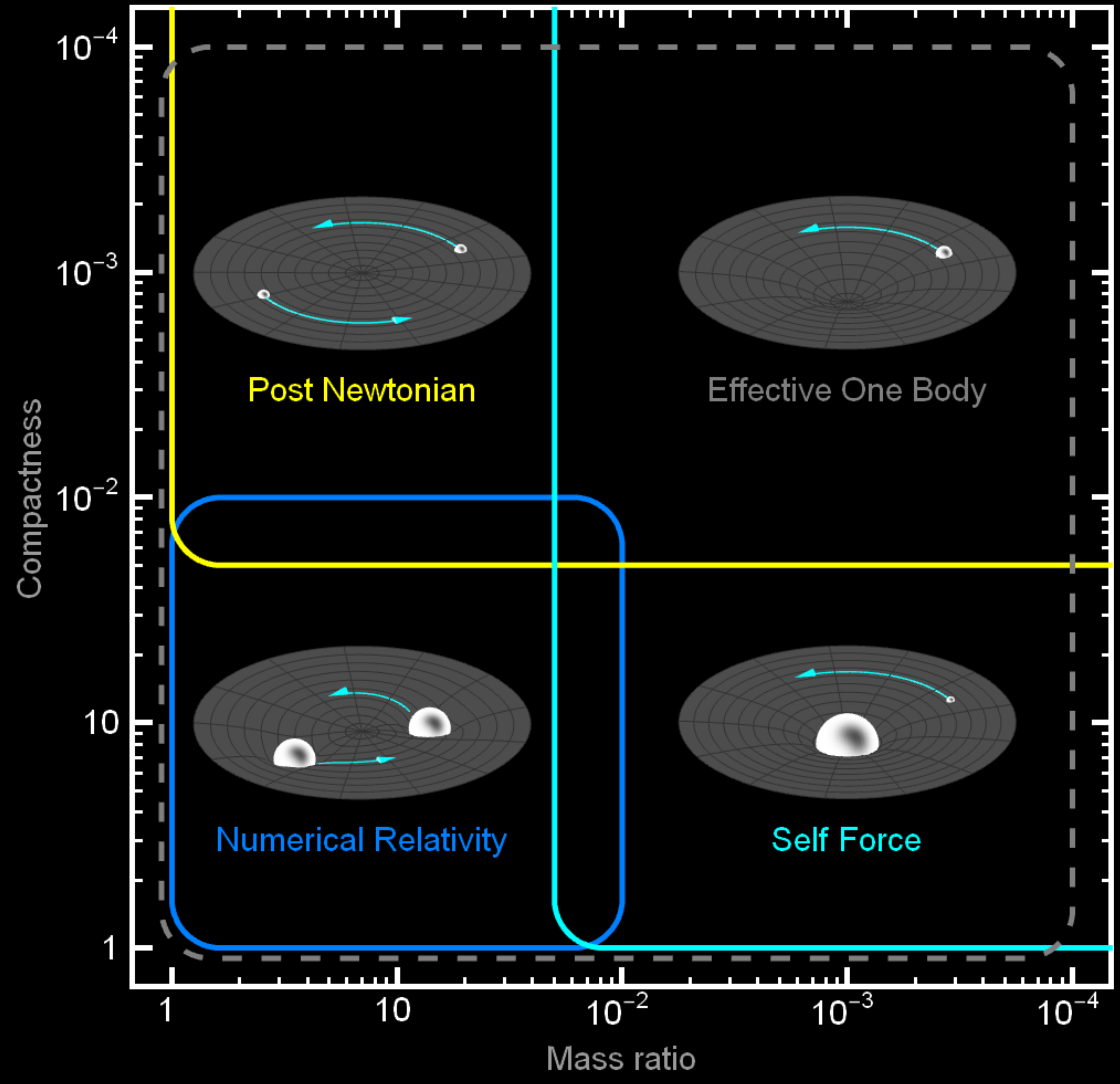


PRL 103, 131101 (2009)



Two-body problem of General Relativity

Maarten van de Meent - Own work



Post-Newtonian Expansion

- Built upon two assumptions:
    1. gravity inside the source is weak like in the post-Minkowsikian expansion
    2. the motion of the components of the source is slow
- The equation of motion

$$\mathbf{a} = \mathbf{a}_{\mathrm{N}} + \mathbf{a}_{\mathrm{PN}} + \mathbf{a}_{\mathrm{2PN}} + \mathbf{a}_{\mathrm{3PN}} + \mathbf{a}_{\mathrm{4PN}} + \underset{\mathrm{SO}}{\mathbf{a}}{}_{\mathrm{1.5PN}} + \underset{\mathrm{SS}}{\mathbf{a}}{}_{\mathrm{2PN}} + \underset{\mathrm{BT}}{\mathbf{a}}{}^{\mathrm{RR}}_{\mathrm{2.5PN}} + \underset{\mathrm{SO}}{\mathbf{a}}{}_{\mathrm{2.5PN}}$$

$$+ \underset{\mathrm{SO}}{\mathbf{a}}{}_{\mathrm{3.5PN}} + \underset{\mathrm{BT}}{\mathbf{a}}{}^{\mathrm{RR}}_{\mathrm{3.5PN}} + \underset{\mathrm{SS}}{\mathbf{a}}{}^{\mathrm{RR}}_{\mathrm{3.5PN}} + \underset{\mathrm{SO}}{\mathbf{a}}{}^{\mathrm{RR}}_{\mathrm{3.5PN}}$$

- The radiation field equation

$$h_{ij} = \frac{2G\mu}{c^4 D}[Q_{ij} + P^{0.5}Q_{ij} + PQ_{ij} + P^{1.5}Q_{ij} + P^2Q_{ij} + PQ_{ij}^{SO}$$

$$+P^{1.5}Q_{ij}^{SO} + P^2Q_{ij}^{SO} + PQ_{ij}^{SS} + P^{1.5}Q_{ij}^{tail}]$$

Effective One-Body Approach

- reduce the conservative dynamics of the general relativistic two-body problem
- Mathisson–Papapetrou–Dixon equation is taken on a deformed Kerr black hole
- Hamiltonian of the Mathisson–Papapetrou–Dixon equations:

$$H_{\text{eff}} = M\eta\left(\beta^i p_i + \alpha\sqrt{1 + \gamma^{ij}p_i p_j + Q_4(p)} + H_{\text{S}}\right) + H_{SC}$$

$$H = M\sqrt{1 + 2\eta\left(\frac{H_{\text{eff}}}{M\eta} - 1\right)}$$

- In the EOBNR framework, the quasicircular part of the radiation field is divided into two:
  - the inspiral-plunge
  - post-merger phase

$$h_{lm}^{(C)} = h_{lm}^{(N,\epsilon)}\hat{S}_{\text{eff}}^{(\epsilon)}T_{lm}e^{i\delta_{lm}}(\rho_{lm})^l N_{lm}$$

$$h_{lm}^{(N,\epsilon)} = \frac{M\eta}{D}n_{lm}^{(\epsilon)}c_{l+\epsilon}V_{\Phi}^{l}Y^{l-\epsilon,-m}\left(\frac{\pi}{2},\Phi\right)$$

- For the eccentric part, in the radiation field terms up to the second post-Newtonian order are considered

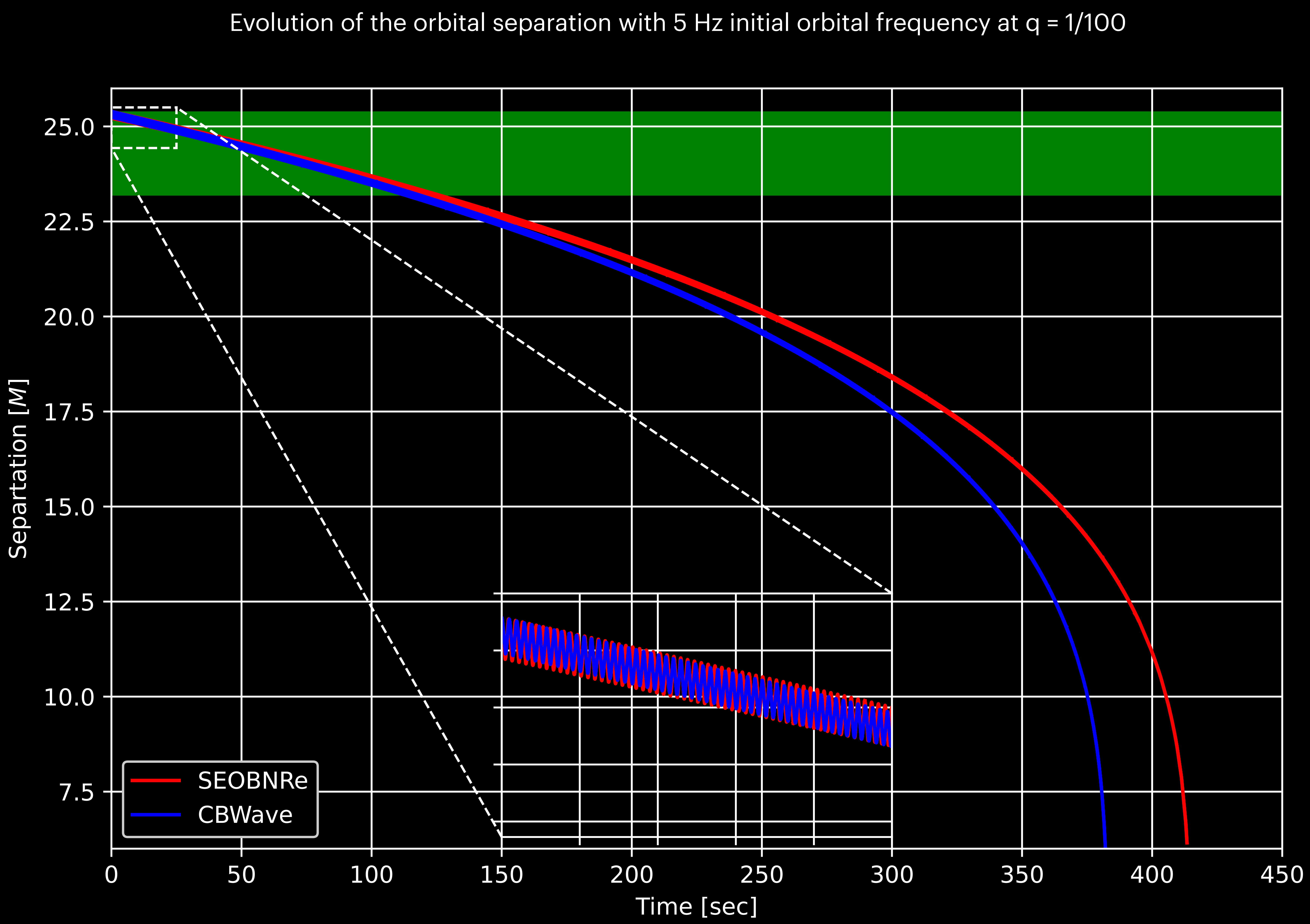
Numerical Results

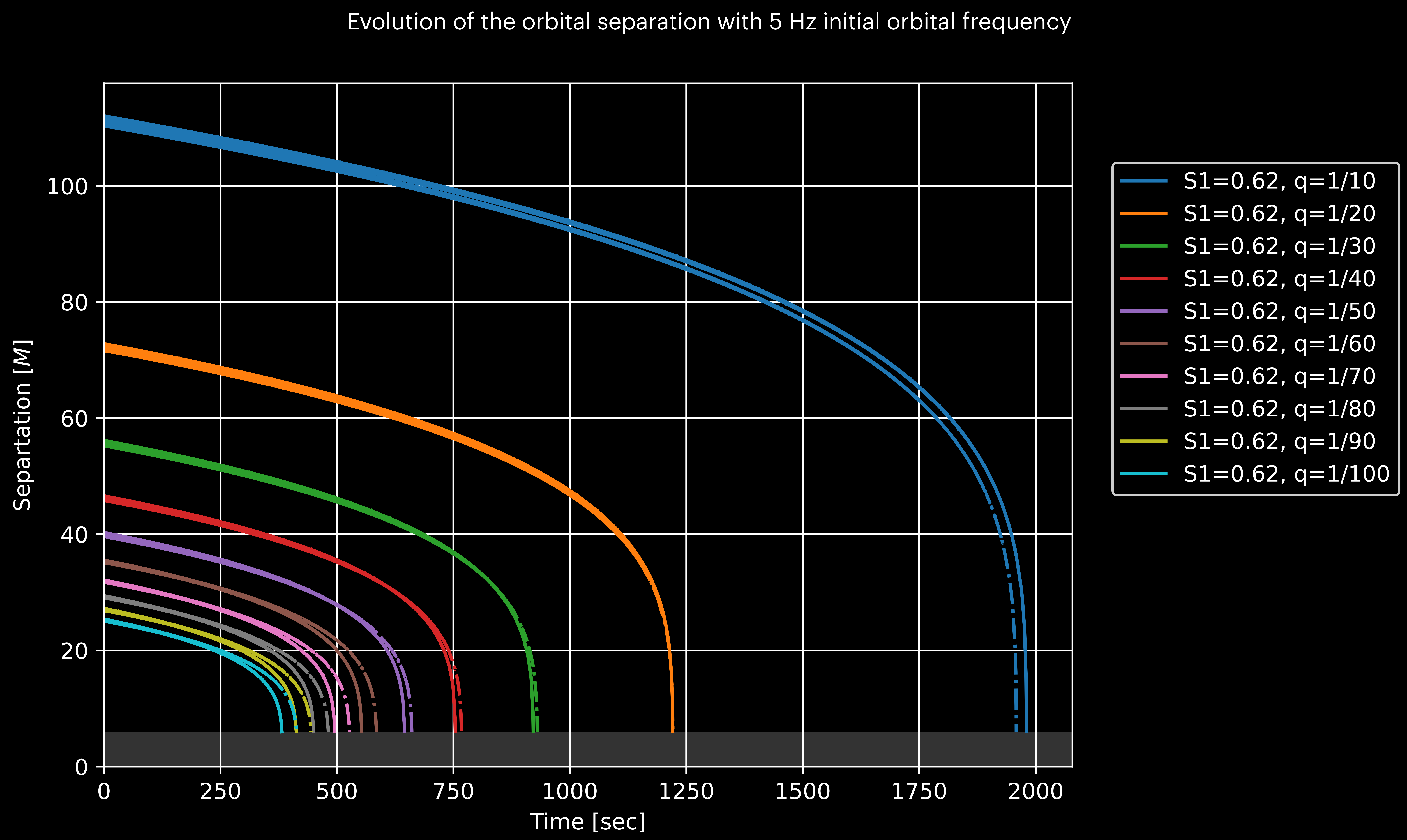
- 2 codes were used; one based on the PN, **CBwaves**; and one based on EOB, **SEOBNRE**
- both codes use a 4th-order Runge—Kutta integrator
- on an identical initial parameter space

Initial Parameters

m_1 [$M_\odot$]	10 … 100
m_2 [$M_\odot$]	10 … 100
R [M_{tot}]	30
R_{min} [M_{tot}]	6
e_0	0.003
dt [sec]	1/4096

- **SEOBNRE** uses the initial orbital frequency: $f_{\text{init}} = \dfrac{c^3}{\pi G(m_1 + m_2)M_\odot\sqrt{\mathbf{r}_0^3}}$


Evolution of the orbital separation with 5 Hz initial orbital frequency at q = 1/100
Separation [M]
Time [sec]
25.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
0
50
100
150
200
250
300
350
400
450
SEOBNRe
CBWave



Evolution of the orbital separation with 5 Hz initial orbital frequency
Separation [M]
Time [sec]
0
20
40
60
80
100
0
250
500
750
1000
1250
1500
1750
2000
S1=0.62, q=1/10
S1=0.62, q=1/20
S1=0.62, q=1/30
S1=0.62, q=1/40
S1=0.62, q=1/50
S1=0.62, q=1/60
S1=0.62, q=1/70
S1=0.62, q=1/80
S1=0.62, q=1/90
S1=0.62, q=1/100

Mismatch/Unfaithfulness

- To calculate the mismatch, one first has to calculate the Overlap:

$$\mathcal{O} = \frac{\langle h_1, h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}}$$

where

$$\langle h_1, h_2 \rangle = 4\Re \int_{f_{\max}}^{f_{\min}} \frac{\tilde{h}_1 \tilde{h}_2^*}{S_n(f)} \mathrm{d}f$$

- The mismatch (or unfaithfulness) is the marginalized overlap over some quantities

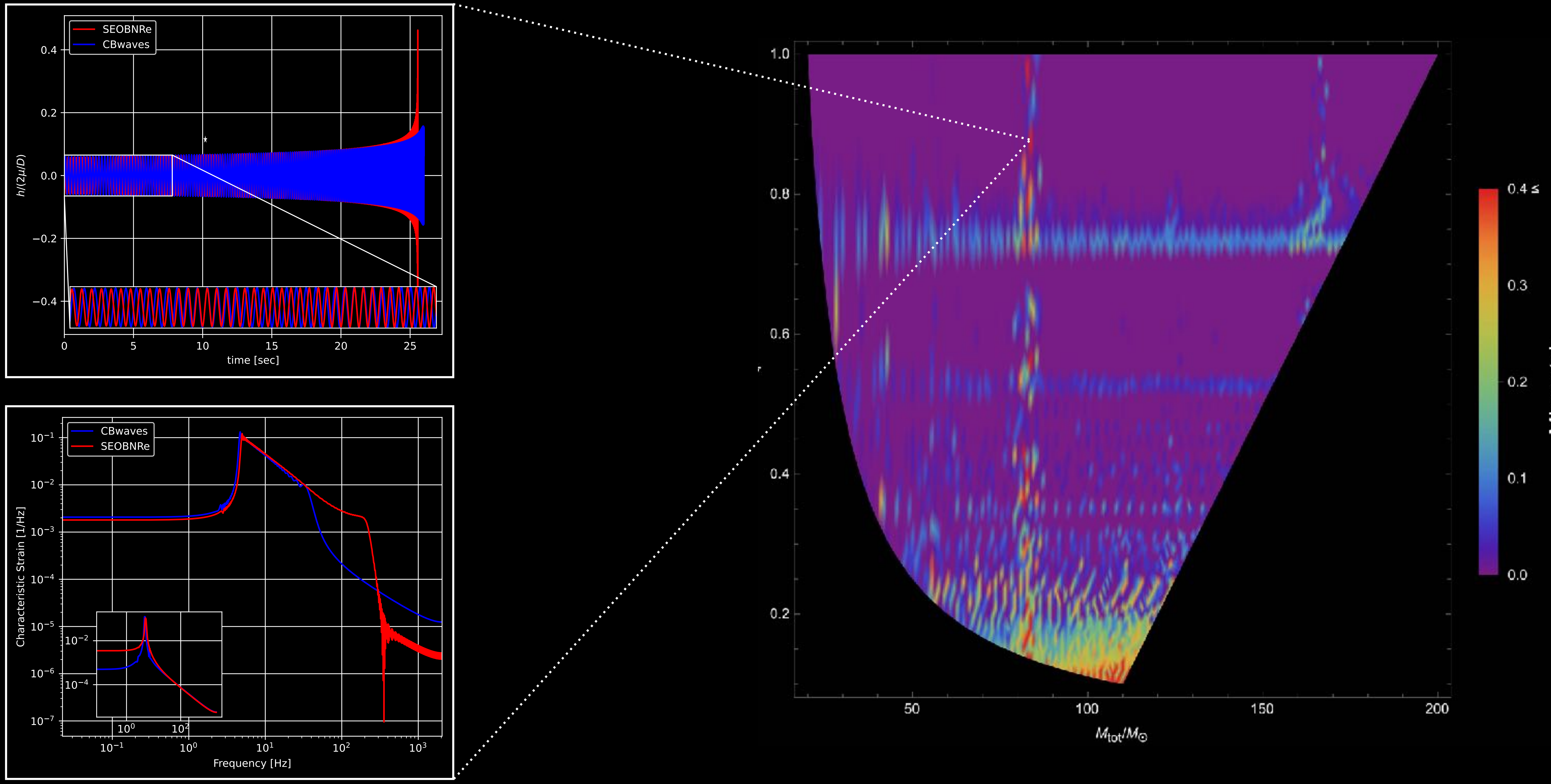
$$\mathcal{M} = \max_{t,\phi,\psi} \mathcal{O}(h_1, h_2)$$

where the $\max$ was taken over timeshifts, polarization angles, and phase

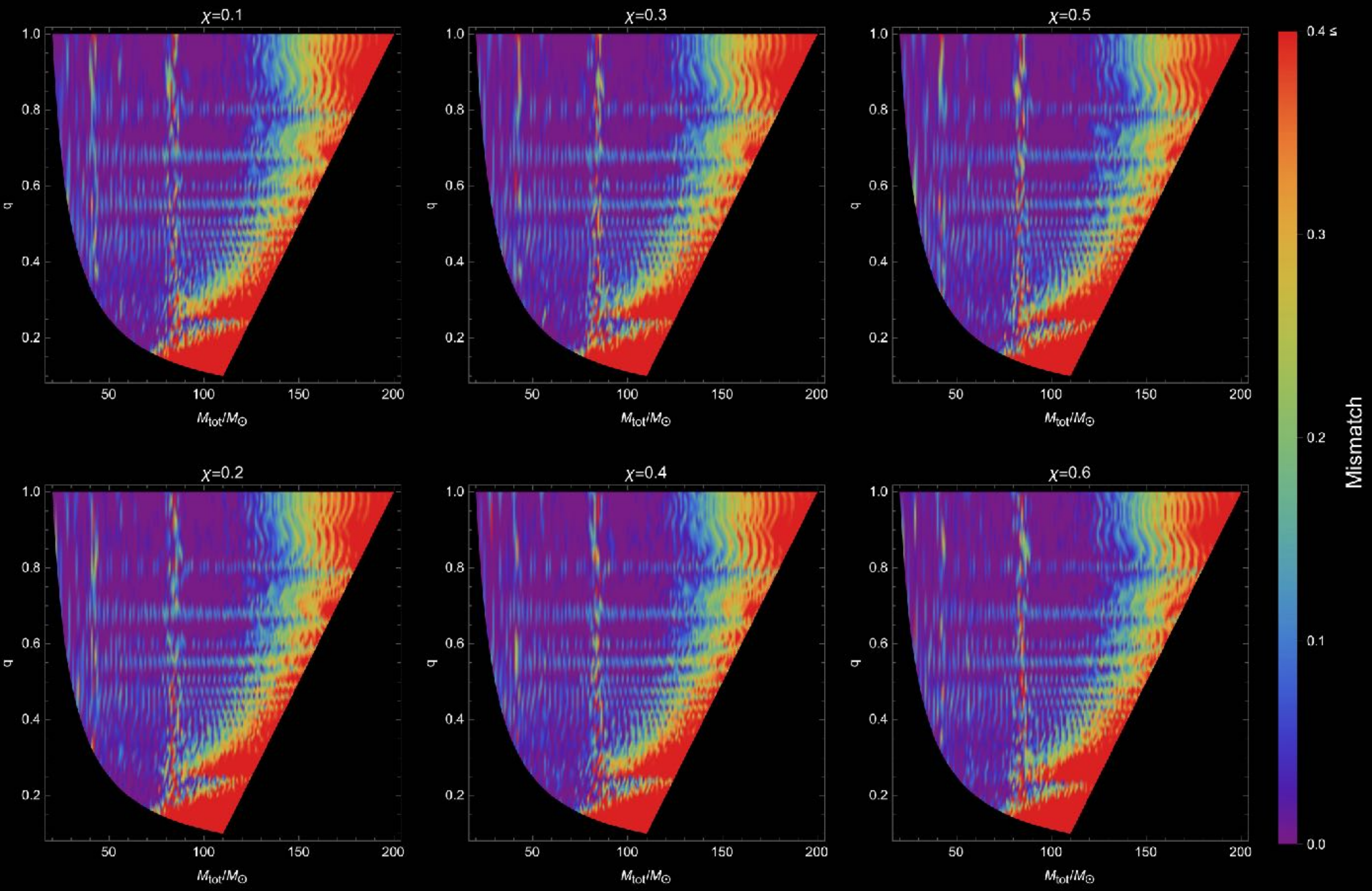
- The kuibit was used.

Mismatch map for the not-spinning configurations

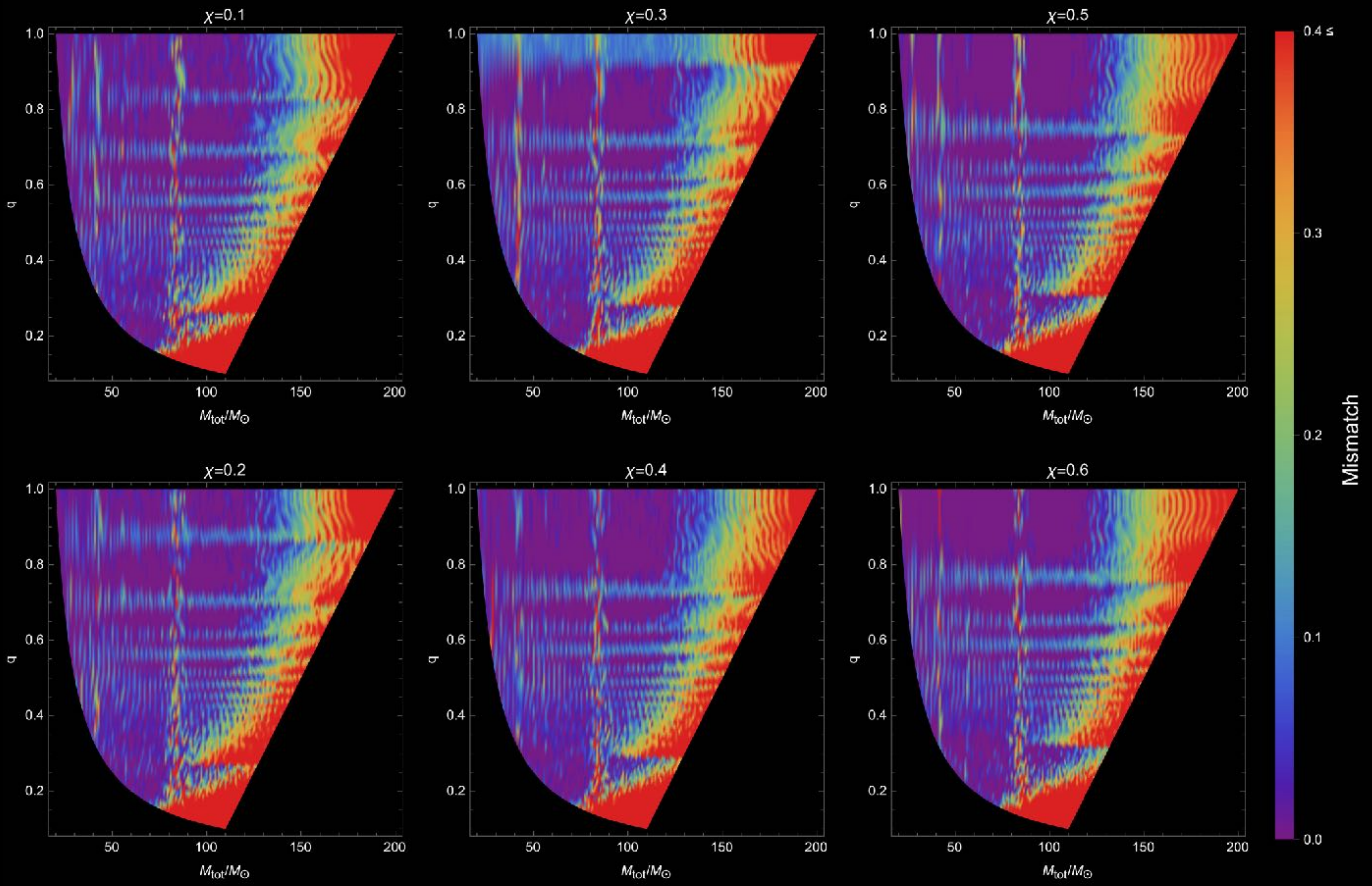
$m_1 = 44.5455\ M_\odot$, $m_2 = 39.0909\ M_\odot$ and $q = 0.8775499209$

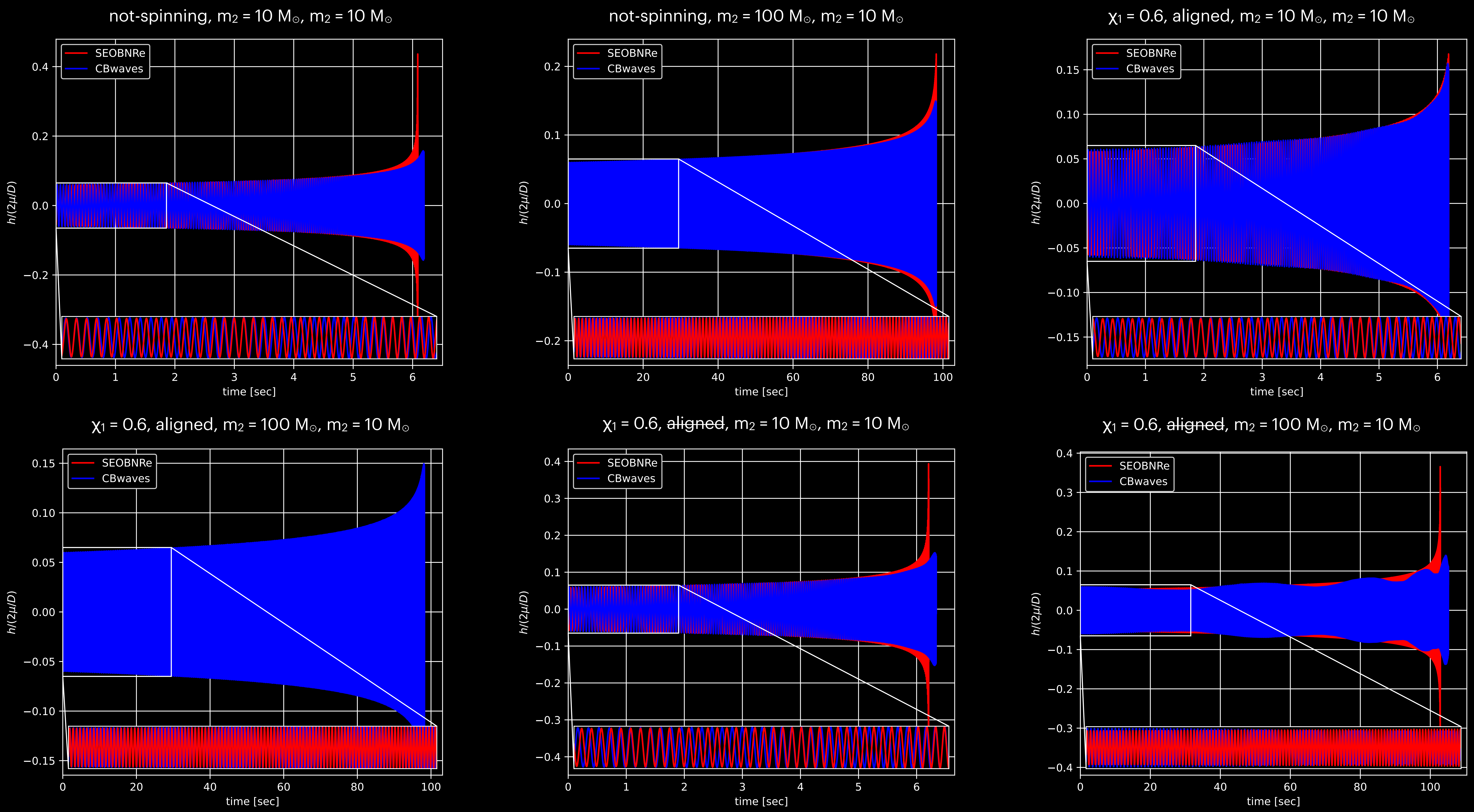


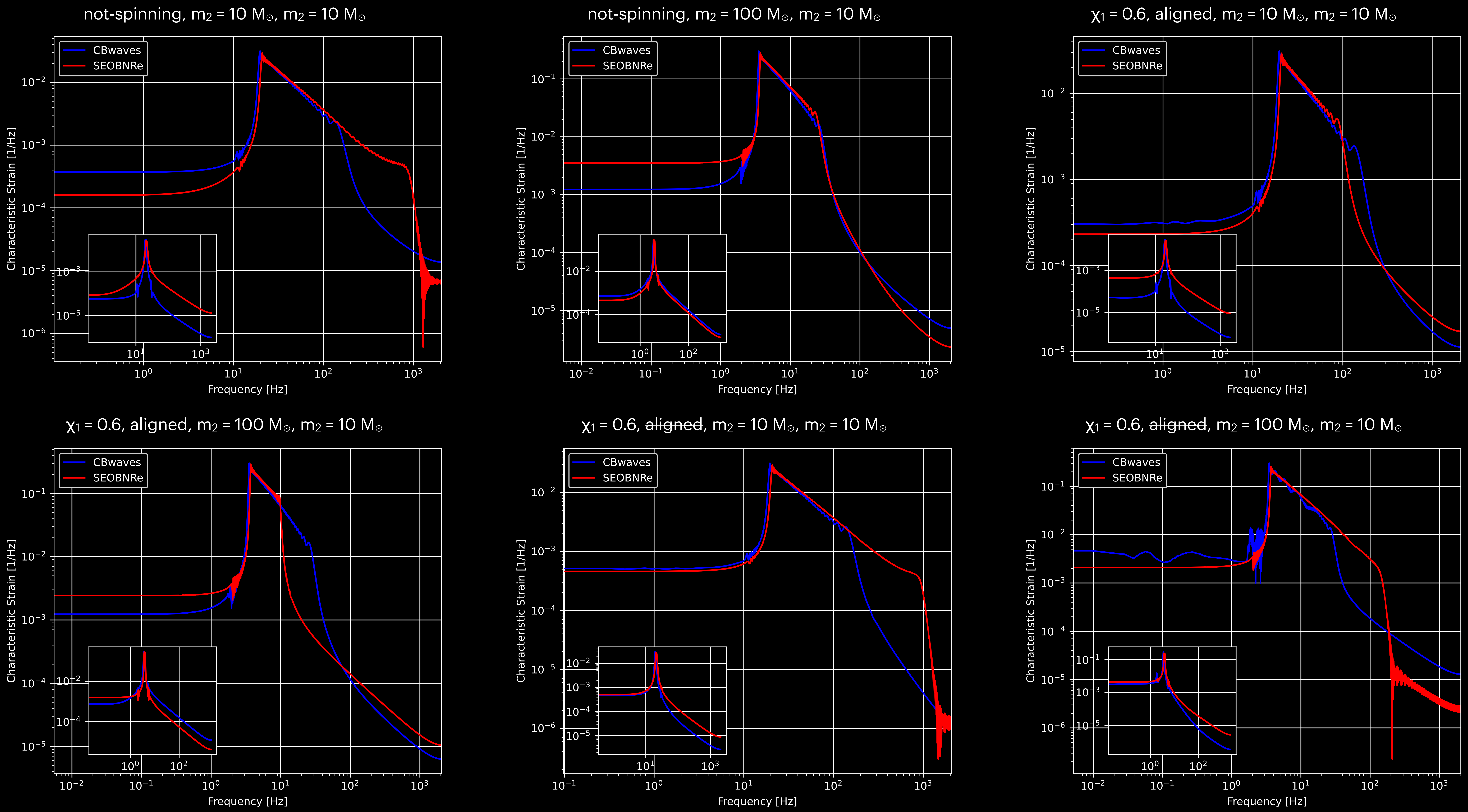
Mismatch map for the spin-aligned configurations



Mismatch map for the non-aligned spin configurations




not-spinning, m2 = 10 M⊙, m2 = 10 M⊙
not-spinning, m2 = 100 M⊙, m2 = 10 M⊙
χ1 = 0.6, aligned, m2 = 10 M⊙, m2 = 10 M⊙
χ1 = 0.6, aligned, m2 = 100 M⊙, m2 = 10 M⊙
χ1 = 0.6, ~~aligned~~, m2 = 10 M⊙, m2 = 10 M⊙
χ1 = 0.6, ~~aligned~~, m2 = 100 M⊙, m2 = 10 M⊙
SEOBNRe
CBwaves
h/(2μ/D)
time [sec]



not-spinning, m2 = 10 M⊙, m2 = 10 M⊙
not-spinning, m2 = 100 M⊙, m2 = 10 M⊙
χ1 = 0.6, aligned, m2 = 10 M⊙, m2 = 10 M⊙
χ1 = 0.6, aligned, m2 = 100 M⊙, m2 = 10 M⊙
χ1 = 0.6, ~~aligned~~, m2 = 10 M⊙, m2 = 10 M⊙
χ1 = 0.6, ~~aligned~~, m2 = 100 M⊙, m2 = 10 M⊙
CBwaves
SEOBNRe
Characteristic Strain [1/Hz]
Frequency [Hz]

“Publish or perish!”

Eugene Garfield — The Academic Man: A Study in the Sociology of a Profession (1942)

Acknowledgment

- Deeply grateful to WSCLab. for computational resources provided
- This work was made in collaboration with Dániel Barta
- Thanks for the insight and pieces of advice given by László Á. Gergely

In memoriam of Mátyás Zs. Vasúth

Thanks for your attention!