Physical and cosmological implications of non-minimal geometry-matter couplings

12th Bolyai-Gauss-Lobachevsky Conference (BGL-2024), May 2nd

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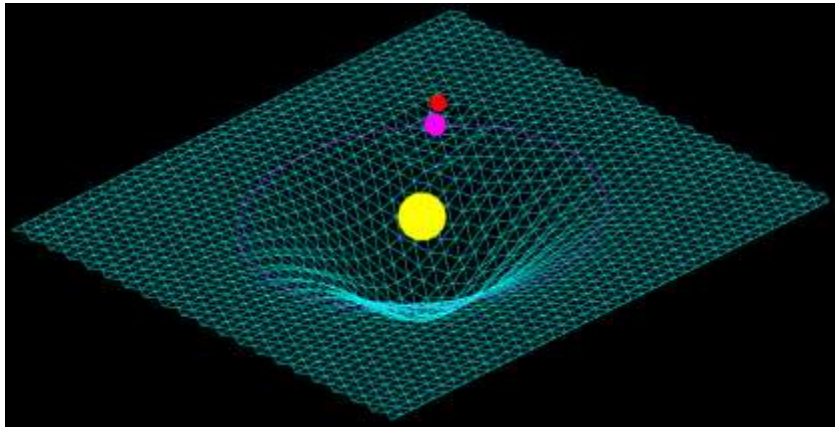
Carl Friedrich Gauss (1777–1855), Johann Bolyai (1802–1860) and Nikolai Lobachevsky (1792–1856)

General Relativity and the ΛCDM Model

1915: General Relativity (GR)

A drastic change in the paradigm of gravitational physics

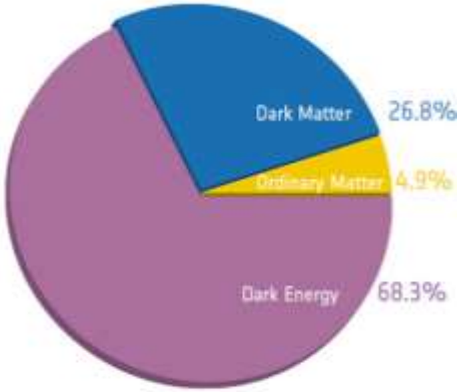
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



ΛCDM Model

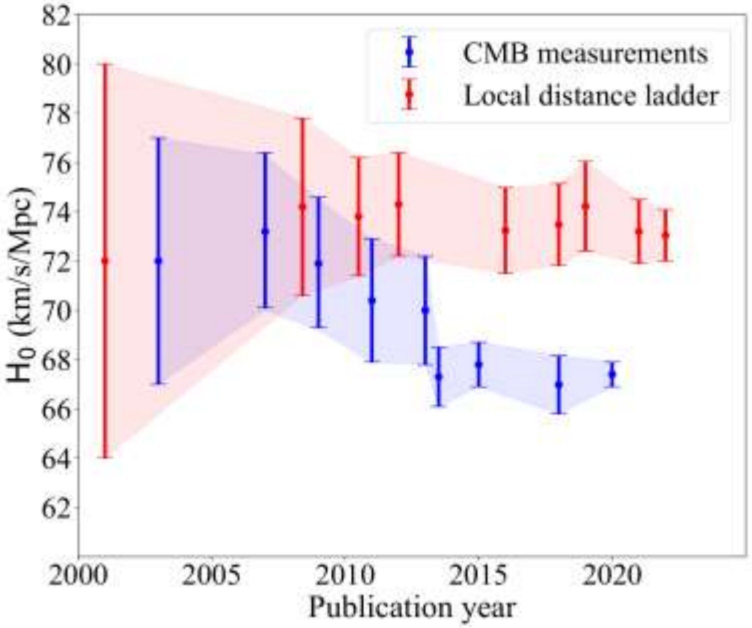
- Geometry (Gravity)
  - **GR**
  - Homogeneity and Isotropy
  - (GR + Homogeneity and Isotropy): Friedmann–Lemaître–Robertson–Walker metric (FRLW) Metric
- Matter Content
  - Cosmological Constant
  - Baryonic and Cold Dark Matter

There are some shortcomings in both GR and ΛCDM Model:



Credits: NRAO

❌ ΛCDM does not explain the nature of dark matter and dark energy

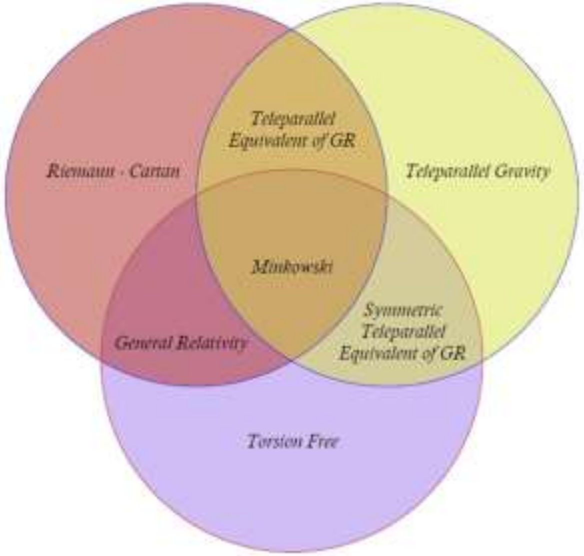


J. P. Hu and F. Y. Wang, Universe 9 (2023) no.2, 94 [arXiv:2302.05709 [astro-ph.CO]]

❌ Hubble Tension - a discrepancy between early-time and late-time measurements of H_0

❌ GR lacks uniqueness – The Geometrical Trinity of Gravity

J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, Universe 5 (2019) no.7, 173 [arXiv:1903.06830 [hep-th]]



Credits: S. Capozziello

Are these problems hints that new physics is needed?

There are many paths to choose...

One of them is modifying gravity!

Physical motivations for such modifications of gravity:

- To explain the dark components of the universe;
- To deal with the theoretical issues of GR;
- To provide a more realistic representation of the gravitational fields near curvature singularities;
- To set a viable framework for the quantization of the gravitational interaction.

J. F. Donoghue, Phys. Rev. D 50 (1994), 3874-3888 [arXiv:gr-qc/9405057 [gr-qc]]

Modified Gravity Theories (MGTs)

Lovelock's Theorem (1971):

The Einstein Field Equations are the only equations of motion that come from a (local) gravitational action with 2nd order derivatives of the metric tensor in 4 dimensions.

Consequence: to modifiy the Einstein Field Equations, we can

- Add extra fields to the gravitational sector
- Add more than 2nd order derivatives of the metric
- Include non-locality
- Add extra dimensions

And that is how one does Modified Gravity!

Minimal Coupling Theory

$$S = \int_{\mathcal{M}} \sqrt{-g}\,(\text{Geometry} + \text{Matter})\, d^4x$$

Matter - Lagrangian density $\mathcal{L}_m(g_{\mu\nu}, \Psi, \partial\Psi)$

Flat Minkowski space-time $\eta_{\mu\nu} \longrightarrow$ General space-time $g_{\mu\nu}$

Partial derivative $\partial_\mu \longrightarrow$ Covariant derivative ∇_μ that contains the affine connection coefficients

$\boldsymbol{f(R)}$ gravity

$$S = \frac{c^4}{16\pi G} \int_{\mathcal{M}} \sqrt{-g} \left(f(R) + \frac{16\pi G}{c^4} \mathcal{L}_m(g_{\mu\nu}, \Psi, \partial\Psi) \right) d^4x$$

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NMGM Coupling Theory

$$S = \int_{\mathcal{M}} \sqrt{-g}\,(\text{Geometry} \times \text{Matter})\, d^4x$$

Non-minimal couplings between geometry and matter are terms that **directly** connect geometrical quantities to matter fields

Example: $\boldsymbol{f(R, \mathcal{L}_m)}$ **gravity**

$$S = \int_{\mathcal{M}} \sqrt{-g}\, f(R, \mathcal{L}_{\rm m})\, d^4x$$

T. Harko and F. S. N. Lobo, Eur. Phys. J. C 70 (2010), 373-379 [arXiv:1008.4193 [gr-qc]]

$f(R, T)$

T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D 84 (2011), 024020 [arXiv:1104.2669 [gr-qc]]

$f(R, T, R_{\mu\nu}T^{\mu\nu})$

Z. Haghani, T. Harko, F. S. N. Lobo, H. R. Sepangi and S. Shahidi, Phys. Rev. D 88 (2013) no.4, 044023 [arXiv:1304.5957 [gr-qc]].

Important consequence:

Non-conservation of the (matter) energy-momentum tensor → An extra force appear

$$\nabla^\mu T_{\mu\nu} \neq 0 \qquad\qquad f^\mu \neq 0$$

Violation of the Equivalence Principle (EP)? — An alternative to dark matter?

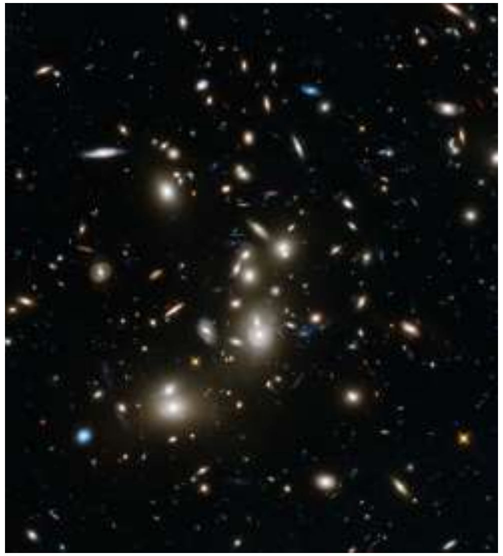
Seems to be a reason to **rule out** these theories, but...

- The EP is a heuristic hypothesis introduced by Einstein to construct GR

T. Damour, Class. Quant. Grav. 13 (1996), A33-A42 [arXiv:gr-qc/9606080 [gr-qc]]

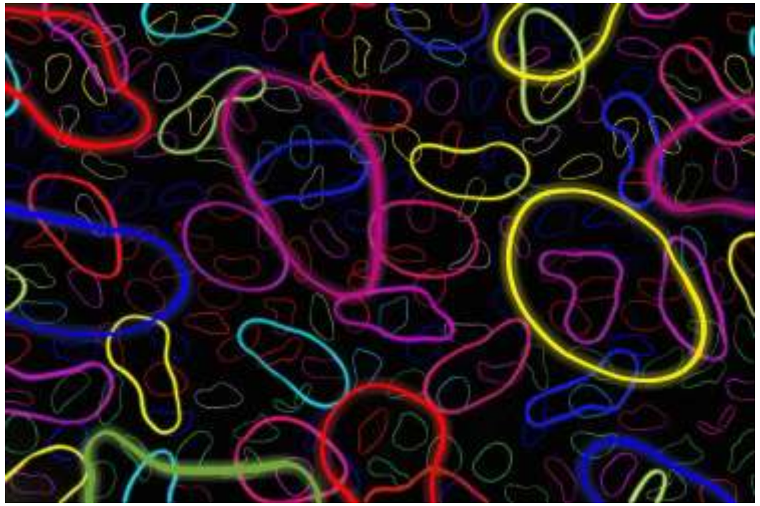
- Dark Energy-Dark Matter Interaction and the Violation of the Equivalence Principle from the Abell Cluster A586

O. Bertolami, F. Gil Pedro and M. Le Delliou, Phys. Lett. B 654 (2007), 165-169 [arXiv:astro-ph/0703462 [astro-ph]]



One of Abell Clusters

String theory predict a scalar particle, the so-called dilaton, that also mediates the gravitational interaction, with a gravitational coupling such that it violates the EP!



Many observable consequences:

- Non-universality of free fall;
- Variation of fundamental “constants”;
- Relative drift of atomic clocks.

Modified Gravity Theories (MGTs): Non-minimal geometry-matter couplings

Some recent papers/results on the literature:

2014 – Thermodynamic interpretation of the generalized gravity models with NMGM couplings

T. Harko, Phys. Rev. D 90 (2014) no.4, 044067 [arXiv:1408.3465 [gr-qc]]

2018 – Investigation of compact stars in $f(R, T_{\mu\nu}T^{\mu\nu})$ gravity

N. Nari and M. Roshan, Phys. Rev. D 98 (2018) no.2, 024031 [arXiv:1802.02399 [gr-qc]]

2020 – Investigation of inflationary scenarios in $f(R,T)$ gravity

S. Bhattacharjee, J. R. L. Santos, P. H. R. S. Moraes and P. K. Sahoo, Eur. Phys. J. Plus 135 (2020) no.7, 576 [arXiv:2006.04336 [gr-qc]]

2021 – Cosmological solutions in scalar-tensor $f(R,T)$ gravity

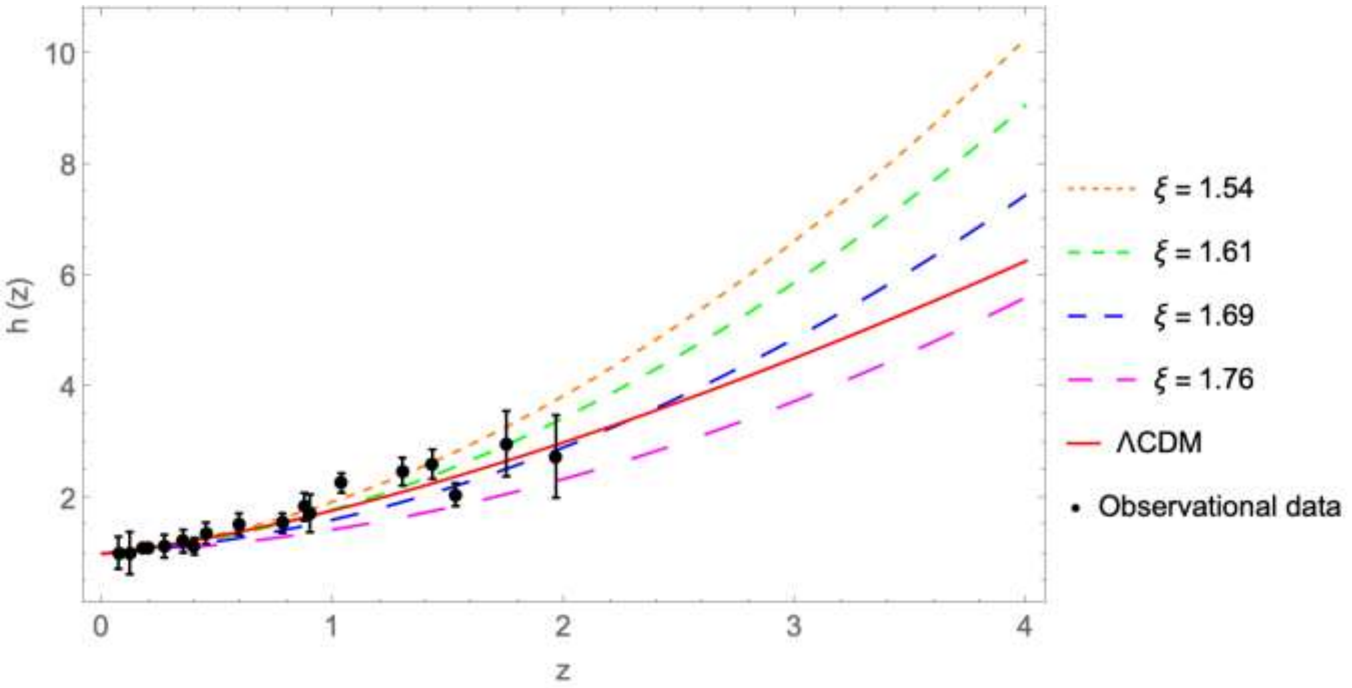
T. B. Gonçalves, J. L. Rosa and F. S. N. Lobo, Phys. Rev. D 105 (2022) no.6, [arXiv:2112.02541 [gr-qc]]

2023 - Traversable wormhole in $f(R, T_{\mu\nu}T^{\mu\nu})$ gravity without exotic matter

J. L. Rosa, N. Ganiyeva and F. S. N. Lobo, Eur. Phys. J. C 83 (2023) no.11, 1040 [arXiv:2309.08768 [gr-qc]]

Time

M. A. S. Pinto, T. Harko and F. S. N. Lobo, Phys. Rev. D 106 (2022) no.4, 044043 [arXiv:2205.12545 [gr-qc]]



- Non-minimal curvature-matter couplings induce particle production - $T = g^{\mu\nu}T_{\mu\nu}$
- We have a cosmology in which the Universe gradually builds up entropy as particles are created;
- The scalar-tensor $f(R,T)$ gravity can explain the late time acceleration without dark energy.

MCMC analysis with Pantheon+ and H(z) datasets

A. Bouali et al., Mon. Not. Roy. Astron. Soc. 526 (2023), 4192-4208 [arXiv:2309.15497 [gr-qc]]

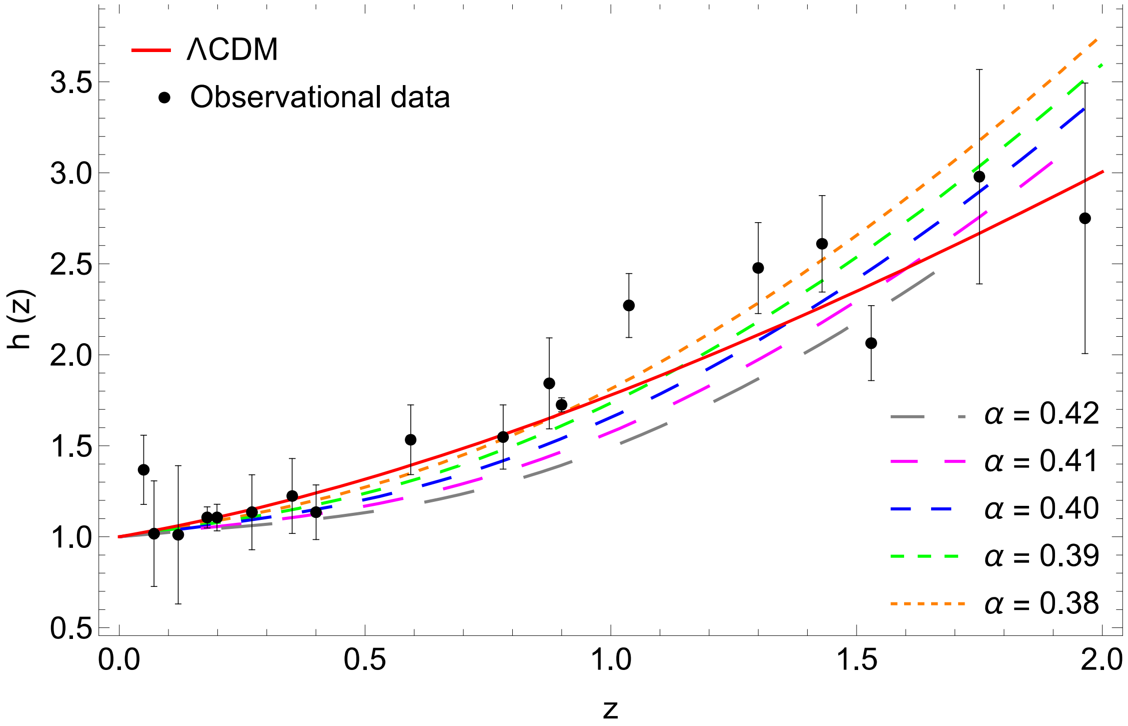
Model	${\chi^2_{tot}}^{min}$	χ^2_{red}	$\mathcal{K}_f$	AIC_c	ΔAIC_c	BIC	ΔBIC
ΛCDM	1656.4528	0.943848	3	1662.4664	0	1678.87	0
$f(R,T)$ Model 1	1647.2268	0.9396	5	1657.2610	−5.20541	1684.59	5.71786
$f(R,T)$ Model 2	1654.4114	0.94376	5	1664.4456	1.97916	1691.77	12.9025

Table 2. Summary of ${\chi^2_{tot}}^{min}$, χ^2_{red}, AIC_c, ΔAIC_c, BIC, and ΔBIC for ΛCDM and $f(R, T)$ models.

- $f(R,T)$ Model 1 is observationally **more supported** than Lambda CDM model

Previous works: Gravitationally matter creation in scalar-tensor $f(R, T_{\mu\nu}T^{\mu\nu})$ gravity

R. A. C. Cipriano, T. Harko, F. S. N. Lobo, **M. A. S. Pinto** and J. L. Rosa, Phys. Dark Univ. 44 (2024), 101463 [arXiv:2310.15018 [gr-qc]]



- Same conclusions of the first work – consistency!

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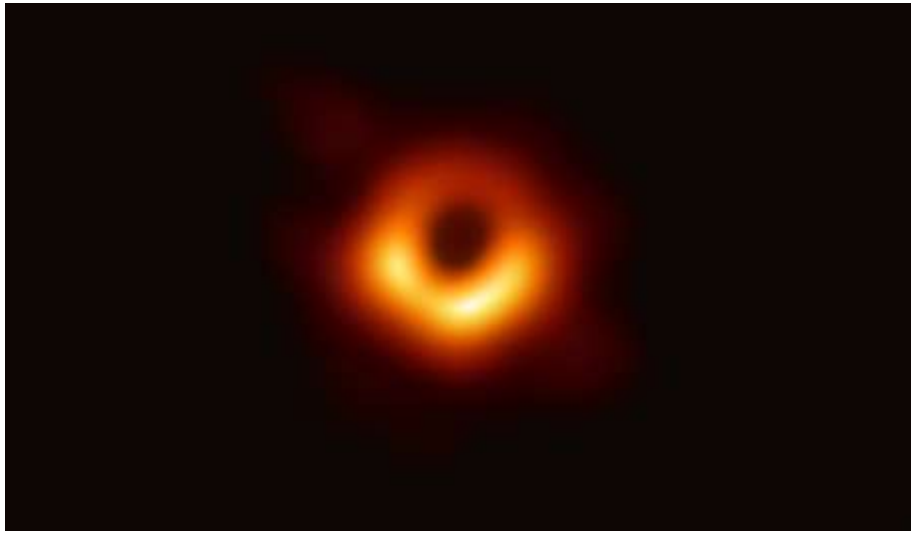

Credits: Event Horizon Telescope Collaboration

- It looks like black holes exist...
- But they display some problems (singularities, closed timelike curves, etc...)
- A quantum theory of gravity aims to solve these issues, but we still do not have such a theory in our hands...
- What can we do? Regular black hole phenomenology!

Non-linear electrodynamics	+	$f(R,T)$ gravity	+	(2+1) dimensions
Generates regular black hole geometry		A possible link to quantum gravity		$\mathrm{AdS}_3/\mathrm{CFT}_2$

Regular black hole solutions in (2+1)-dimensional $f(R,T)$ gravity coupled to nonlinear electrodynamics

(in collaboration with Roberto V. Maluf)

In 2+1 dimensions, the action of $f(R,T)$ gravity is given by: $$S=\int d^3x\sqrt{-g}\left[\frac{1}{2\kappa^2}f(R,T)+L(F)\right]$$

$$\kappa^2=8\pi\tilde{G} \qquad T=g^{\mu\nu}T_{\mu\nu}$$

$$F=F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L(F)\right)}{\delta g^{\mu\nu}}=L(F)g_{\mu\nu}-4L_F F_\mu{}^\alpha F_{\nu\alpha}$$

$$L_F=\frac{dL}{dF}$$

Modified Field Equations: $$f_R R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}f(R,T)+\left(g_{\mu\nu}\nabla^\alpha\nabla_\alpha-\nabla_\mu\nabla_\nu\right)f_R=\kappa^2T_{\mu\nu}-f_T\left(T_{\mu\nu}+\Theta_{\mu\nu}\right)$$

$$\Theta_{\mu\nu}=g^{\rho\sigma}\frac{\delta T_{\rho\sigma}}{\delta g^{\mu\nu}}=-L(F)g_{\mu\nu}+2\left(L_F-4L_{FF}F\right)F_\mu{}^\alpha F_{\nu\alpha}$$

Gauge Field Equations: $$\nabla^\alpha\left[L_F F_{\alpha\beta}-\frac{1}{2\kappa^2}f_T\left(L_F+4L_{FF}F\right)F_{\alpha\beta}\right]=0$$

Assumptions:

$$f(R,T) = R - 2\Lambda + \lambda T \qquad ds^2 = -b(r)dt^2 + \frac{1}{b(r)}dr^2 + r^2 d\theta^2 \qquad F_{\mu\nu} = E(r)\left(\delta^t_\mu \delta^r_\nu - \delta^r_\mu \delta^t_\nu\right)$$

$$\Lambda = -1/\ell^2 \qquad F = F_{\mu\nu}F^{\mu\nu} = -2E^2(r)$$

Resulting Equations:

$$b(r) = -M - \Lambda r^2 + 2\kappa^2 \int r\left[L\left(F(r)\right) + 4E^2(r)L_F\right] dr + \lambda \int r\left[3L\left(F(r)\right) + 4E^2(r)\left(L_F + 8E^2(r)L_{FF}\right)\right] dr$$

The constant of integration is chosen so that we recover the BTZ solution

$$\frac{L'}{4E'} + \frac{\lambda}{4\kappa^2}\left(\frac{L'}{2E'} - \frac{EL''}{E'^2} + \frac{EL'E''}{E'^3}\right) = \frac{q}{r}\left(1 - \frac{\lambda}{2\kappa^2}\right) \qquad L_F = \frac{dL}{dr}\frac{dr}{dF} = L'\frac{dr}{dF}$$

The constant of integration is chosen in such a way that $E = q/r$ when $L(F) = -F$ (Maxwell case)

Ongoing work: Regular black holes in (2+1)-dimensional $f(R,T)$ gravity

We propose a generalized form for the electric field that recovers Maxwell in the asymptotic limit.

$$E(r) = \frac{q r^{\alpha}}{\left(r^{\beta} + a^{\beta}\right)^{\frac{\alpha+1}{\beta}}}$$

Simplest case: $a = \alpha = 0 \quad \beta = 1$

$$E(r) = \frac{q}{r} \implies L(r) = \frac{2q^2}{r^2} - \frac{2c_1\lambda}{(2\kappa^2 + 3\lambda)} \frac{1}{r^{\frac{3}{2}+\frac{\kappa^2}{\lambda}}} + c_2 \overset{c_1 = c_2 = 0}{\implies} L(r) = -F = 2E^2(r) = \frac{2q^2}{r^2}$$

$$b(r) = -M - \Lambda r^2 - 2\left(2\kappa^2 - \lambda\right) q^2 \ln r \overset{\lambda = 2\kappa^2}{\implies} b(r) = -M - \Lambda r^2$$

Charged BTZ Solution

BTZ Solution

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- **GR** ($\lambda = 0$): $\frac{L'}{4E'} = \frac{q}{r}$ | $b(r) = -M - \Lambda r^2 + 2\kappa^2 \int r \left[L(F(r)) + 4E^2(r) L_F \right] dr$

Let us obtain the general solutions assuming

$$E(r) = \frac{q r^\alpha}{\left(r^\beta + a^\beta\right)^{\frac{\alpha+1}{\beta}}}$$

$$L(r) = -\frac{2q^2}{a^2} + \frac{4q^2 r^{\alpha-1}}{(\alpha+\beta-1)\left(r^\beta + a^\beta\right)^{\frac{\alpha+1}{\beta}}} \left[\frac{\alpha(\alpha+\beta-1)}{\alpha-1} \, {}_2F_1\left(1, -\frac{2}{\beta}; \frac{\alpha+\beta-1}{\beta}; -\left(\frac{r}{a}\right)^\beta\right) - \left(\frac{r}{a}\right)^\beta {}_2F_1\left(1, \frac{\beta-2}{\beta}; \frac{\alpha-1}{\beta}+2; -\left(\frac{r}{a}\right)^\beta\right) \right]$$

$\alpha > 1, \beta > 0$ To guarantee the convergence of the hypergeometric function

$$b(r) = -M - \left(\Lambda + \frac{2\kappa^2 q^2}{a^2}\right) r^2 - 4\kappa^2 q^2 \int dr \frac{r^\alpha}{\left(r^\beta + a^\beta\right)^{\frac{\alpha+1}{\beta}}} \left[1 - \left(\frac{\alpha+1}{\alpha-1}\right) {}_2F_1\left(1, -\frac{2}{\beta}; \frac{\alpha+\beta-1}{\beta}; -\left(\frac{r}{a}\right)^\beta\right) \right]$$

- **GR** ($\lambda = 0$):

Case $\alpha = \beta = 2$

$$E(r) = \frac{qr^2}{(r^2+a^2)^{3/2}} \quad \Longrightarrow \quad L(r) = \frac{2q^2\left[2r^3 + 4ra^2 - (r^2+a^2)^{3/2}\right]}{a^2(r^2+a^2)^{3/2}}$$

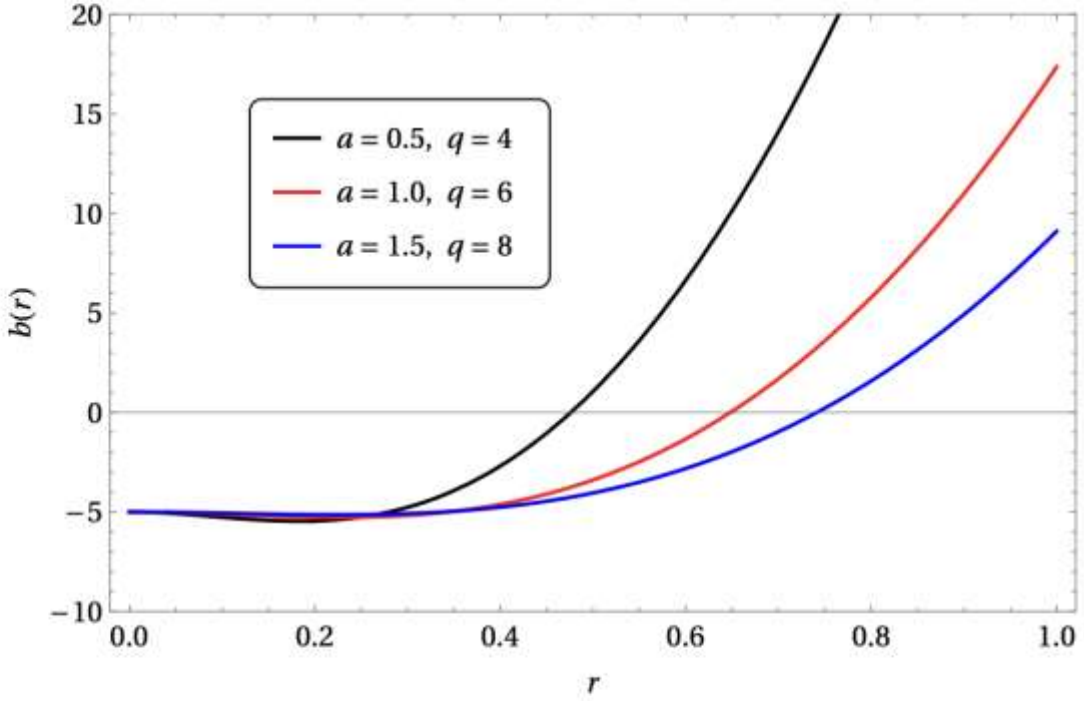
$$b(r) = -M - \left(\Lambda + \frac{2\kappa^2 q^2}{a^2}\right) r^2 + 4\kappa^2 q^2 \left[\frac{r}{a^2}\sqrt{r^2+a^2} - \tanh^{-1}\left(\frac{r}{\sqrt{r^2+a^2}}\right)\right]$$

$r \to \infty$ $\qquad$ $L(r) \to 2\frac{q^2}{a^2}$ $\qquad$ Does not recover Maxwell ✗

$$b(r) = -M - \left(\Lambda + \frac{2\kappa^2 q^2}{a^2}\right) r^2 + 4\kappa^2 q^2 \left[\frac{r}{a^2}\sqrt{r^2+a^2} - \tanh^{-1}\left(\frac{r}{\sqrt{r^2+a^2}}\right)\right]$$

- **GR** ($\lambda = 0$):

  Case $\alpha = \beta = 2$



1 Event Horizon

Figure 1: Metric coefficient for the regular black holes given from Eq. (34), as a function of the radial coordinate, for some values of a and q, with $M = 5.0$ and $\Lambda = -20$ in Planck units.

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- **GR** $(\lambda = 0)$:

Case $\alpha = \beta = 2$

$$R = 6\Lambda + \frac{4\kappa^2 q^2 \left[3\left(a^2+r^2\right)^{3/2} - 8a^2 r - 6r^3\right]}{a^2\left(r^2+a^2\right)^{3/2}}$$

$$K = 4\left\{3\Lambda^2 + \frac{4\Lambda\kappa^2 q^2 \left[3\left(r^2+a^2\right)^{3/2} - 8a^2 r - 6r^3\right]}{a^2\left(r^2+a^2\right)^{3/2}} + \frac{4\kappa^4 q^4 \left[41a^2 r^4 + 33a^4 r^2 + 3a^6 + 15r^6 - 4r\left(3r^2+4a^2\right)\left(r^2+a^2\right)^{3/2}\right]}{a^4\left(r^2+a^2\right)^3}\right\}$$

Both Ricci and Kretschmann scalars do not diverge! $\Longrightarrow$ This solution is regular!

How can the Lagrangian have a Maxwell-like behavior?

$$\lim_{r\to+\infty} L(r) = \frac{2q^2}{a^2}\left(\frac{\Gamma\left(\frac{\beta+2}{\beta}\right)\Gamma\left(\frac{\alpha-1}{\beta}\right)}{\Gamma\left(\frac{\alpha+1}{\beta}\right)} - 1\right) \begin{cases} \text{constant} & \alpha \neq \beta+1 \quad \alpha>1, \beta>0 \\ 0 & \alpha = \beta+1 \quad \text{(Maxwell)} \end{cases}$$

$$E(r) = \frac{q r^{\beta+1}}{\left(r^\beta + a^\beta\right)^{\frac{\beta+2}{\beta}}}, \qquad L(r) = \frac{2q^2\left(r^\beta - a^\beta\right)}{\left(r^\beta + a^\beta\right)^{\frac{\beta+2}{\beta}}}$$

$$b(r) = -M - \Lambda r^2 - 2\kappa^2 q^2 \frac{r^2}{a^2}\, {}_2F_1\left(\frac{2}{\beta}, \frac{2}{\beta}; \frac{\beta+2}{\beta}; -\left(\frac{r}{a}\right)^\beta\right)$$

Ongoing work: Regular black holes in (2+1)-dimensional $f(R,T)$ gravity

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β	Electric field $E(r)$	Lagrangian $L(r)$	Metric function $b(r)$	References
1	$\frac{qr^2}{(r+a)^3}$	$\frac{2q^2(r-a)}{(r+a)^3}$	$-M-\Lambda r^2+\frac{4\kappa^2q^2r}{r+a}-4\kappa^2q^2\ln\left(\frac{r+a}{a}\right)$	Case III [2]
2	$\frac{qr^3}{(r^2+a^2)^2}$	$\frac{2q^2\left(r^2-a^2\right)}{(r^2+a^2)^2}$	$-M-\Lambda r^2-2\kappa^2q^2\ln\left(\frac{r^2}{a^2}+1\right)$	Case II [1, 2]
4	$\frac{qr^5}{(r^4+a^4)^{3/2}}$	$\frac{2q^2\left(r^4-a^4\right)}{(r^4+a^4)^{3/2}}$	$-M-\Lambda r^2-2\kappa^2q^2\sinh^{-1}\left(\frac{r^2}{a^2}\right)$	Case IV [2]
$\frac{1}{2}$	$\frac{qr^{3/2}}{\left(\sqrt{r}+\sqrt{a}\right)^5}$	$\frac{2q^2\left(\sqrt{r}-\sqrt{a}\right)}{\left(\sqrt{r}+\sqrt{a}\right)^5}$	$-M-\Lambda r^2+\frac{4\kappa^2q^2\sqrt{r}\left(15\sqrt{ar}+6a+11r\right)}{3\left(\sqrt{r}+\sqrt{a}\right)^3}-8\kappa^2q^2\ln\left(\sqrt{\frac{r}{a}}+1\right)$	Maluf et al.
$\frac{2}{n-1}, \forall n\in\mathbb{N}, n>1$	$\frac{qr^{\frac{n+1}{n-1}}}{\left(r^{\frac{2}{n-1}}+a^{\frac{2}{n-1}}\right)^n}$	$\frac{2q^2\left(r^{\frac{2}{n-1}}-a^{\frac{2}{n-1}}\right)}{\left(a^{\frac{2}{n-1}}+r^{\frac{2}{n-1}}\right)^n}$	Logarithmic term $\sim \ln\left[\left(\frac{r}{a}\right)^{\frac{2}{n-1}}+1\right]$	Maluf et al.

Table I: Incomplete List of some NLED models.

[1] M. Cataldo and A. Garcia, Phys. Rev. D 61, 084003 (2000) [arXiv:hep-th/0004177 [hep-th]]

[2] Y. He and M. S. Ma, Phys. Lett. B 774, 229-234 (2017) [arXiv:1709.09473 [gr-qc]]

Ongoing work: Regular black holes in (2+1)-dimensional $f(R,T)$ gravity

- **$f(R,T)$ gravity** ($\lambda \neq 0$): $b(r) = -M - \Lambda r^2 + 2\kappa^2 \int r\left[L\left(F(r)\right) + 4E^2(r)L_F\right]dr + \lambda \int r\left[3L\left(F(r)\right) + 4E^2(r)\left(L_F + 8E^2(r)L_{FF}\right)\right]dr$

$$\frac{L'}{4E'} + \frac{\lambda}{4\kappa^2}\left(\frac{L'}{2E'} - \frac{EL''}{E'^2} + \frac{EL'E''}{E'^3}\right) = \frac{q}{r}\left(1 - \frac{\lambda}{2\kappa^2}\right)$$

Case $\beta = 2$

$$E(r) = \frac{qr^3}{\left(r^2 + a^2\right)^2}$$

$$L(r) = c_1 + \frac{2q^2(r^2 - a^2)}{\left(r^2 + a^2\right)^2} + \frac{2c_2\lambda r^{\frac{9}{2} + \frac{3\kappa^2}{\lambda}}\left(r^2 + a^2\right)^{-3 - \frac{2\kappa^2}{\lambda}}}{2\kappa^2 + 3\lambda} + \frac{32\lambda q^2}{6\kappa^2 + 5\lambda}\int \frac{r\left(r^2 - 3a^2\right)}{\left(r^2 + a^2\right)^3}\, {}_2F_1\left(-\frac{3\kappa^2}{2\lambda} - \frac{5}{4}, -\frac{2\kappa^2}{\lambda}; -\frac{(6\kappa^2 + \lambda)}{4\lambda}; -\frac{r^2}{a^2}\right) dr$$

$$b(r) = -M - \Lambda r^2 + \frac{4\lambda q^2 a^2}{r^2 + a^2} - \left(2\kappa^2 - \lambda\right) q^2 \ln\left(\frac{r^2}{a^2} + 1\right) + \int^r \left[\frac{64\lambda^2 q^2 r''^3}{\left(6\kappa^2 + 5\lambda\right)\left(r''^2 + a^2\right)^2}\, {}_2F_1\left(1, \frac{\kappa^2}{2\lambda} - \frac{1}{4}; -\frac{(6\kappa^2 + \lambda)}{4\lambda}; -\frac{r''^2}{a^2}\right) + \left(2\kappa^2 + 3\lambda\right) r'' \int^{r''} \frac{32\lambda q^2 r'\left(r'^2 - 3a^2\right)}{\left(6\kappa^2 + 5\lambda\right)\left(r'^2 + a^2\right)^3}\, {}_2F_1\left(1, \frac{\kappa^2}{2\lambda} - \frac{1}{4}; -\frac{(6\kappa^2 + \lambda)}{4\lambda}; -\frac{r'^2}{a^2}\right) dr'\right] dr''$$

Summary and Conclusions

- There is a lot of work being published regarding non-minimal geometry-matter couplings;
- They can provide interesting physical phenomena, such as gravitationally induced particle production, late-time acceleration without dark energy and an extra force alternative to dark matter;
- In the regular black holes case, the non-minimal coupling introduces interesting effects, such as the modification of the electric charge, which does not happen in GR;
- There is currently a lot of discussion about whether these theories are relevant to cosmology or not.

Köszönöm! Questions?