EXPLORING THE BIG BANG IN THE LAB WITH FEMTOSCOPY

AND LÉVY-STABLE DISTRIBUTIONS

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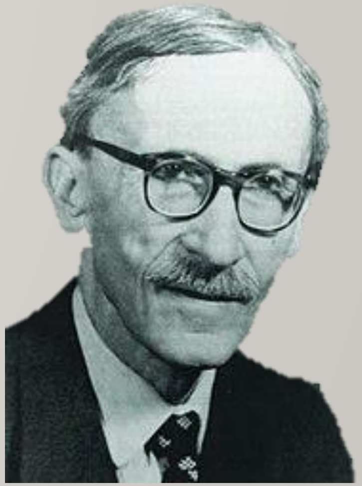
XII BOLYAI-GAUSS-LOBACHEVSKY CONFERENCE (BGL-2024)

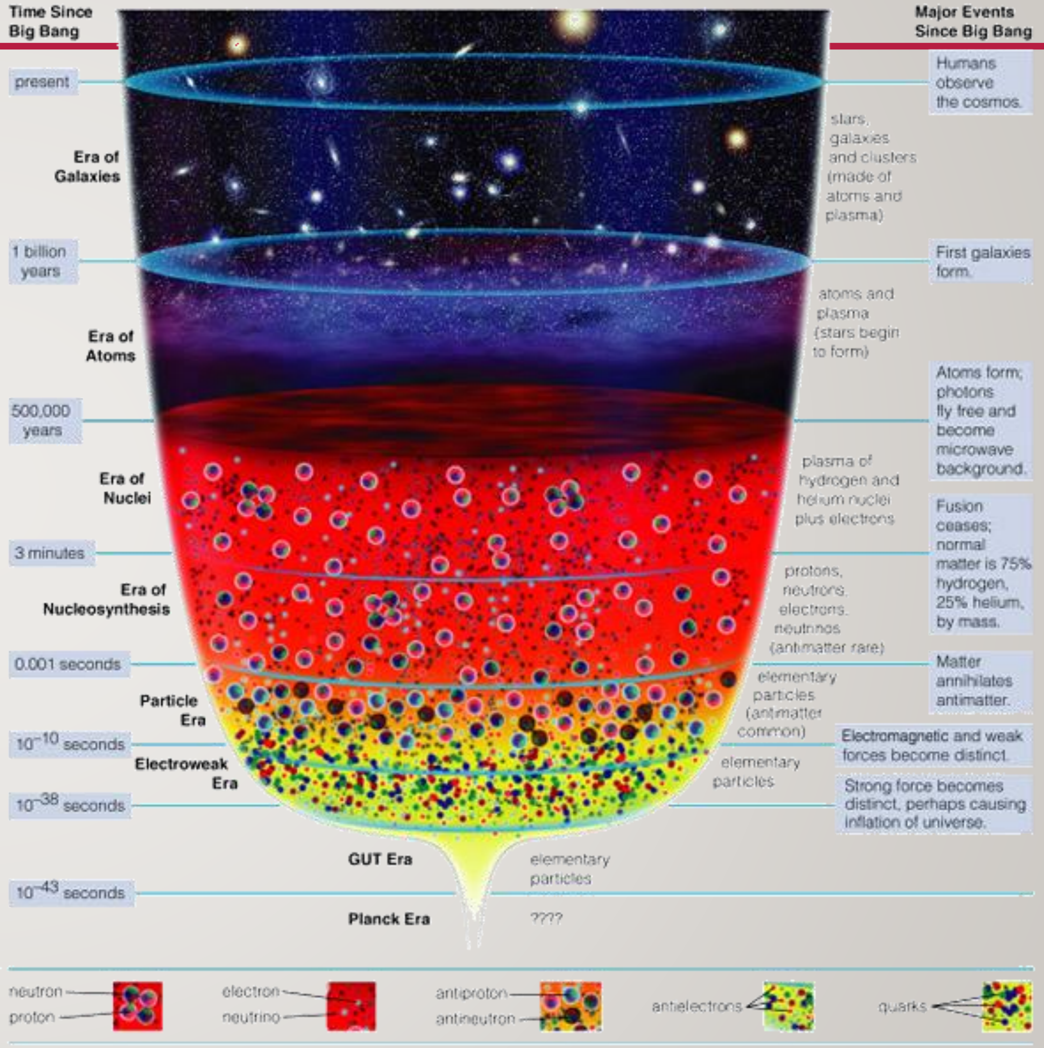


CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

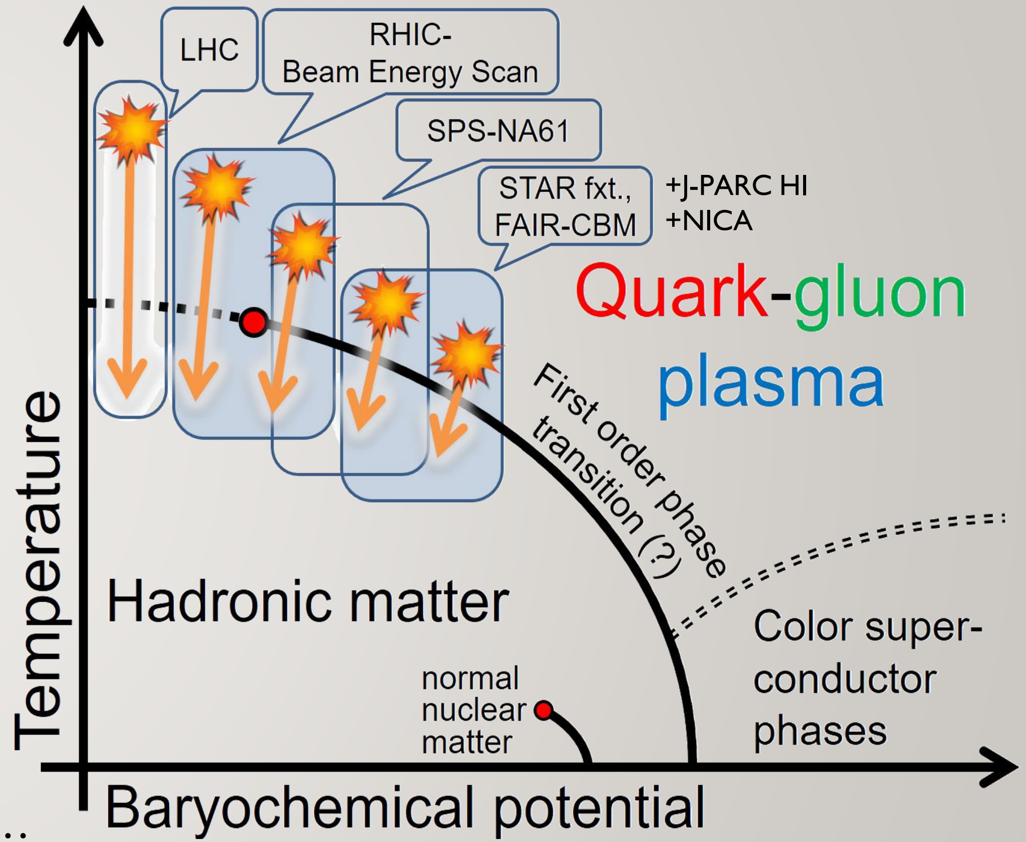
BIG BANG IN THE LAB

- Ages of the Universe:
    - Stars & Galaxies (after few 100M years)
    - Atoms (after ~400k years)
    - Nuclei (after ~3 minutes)
    - Nucleosynthesis (first 3 minutes)
    - Strong interaction era (first microsecond)
    - … ?
- How to investigate?
    - Create little bangs
    - Collisions of heavy ions
    - Record outcoming particles
    - Find clever observables from data



EXPLORING THE PHASE MAP OF QCD

- Properties of nuclear matter? Explore phase map!
- Control parameters:
  - Collision energy, system, geometry
  - Affecting temperature and density
- Phase map: temperature versus matter excess (baryochemical potential μ_B)
  - Crossover at low μ_B and T≅160 MeV
  - Probably 1st order quark-hadron phase transition at high μ_B (NJL, bag model, etc)
  - Critical End Point (CEP) in between, at some μ_B and T_c?
  - High μ_B: nuclear matter, neutron stars, color superconductors…
- Phase transition importance: even in core-collapse supernovae!



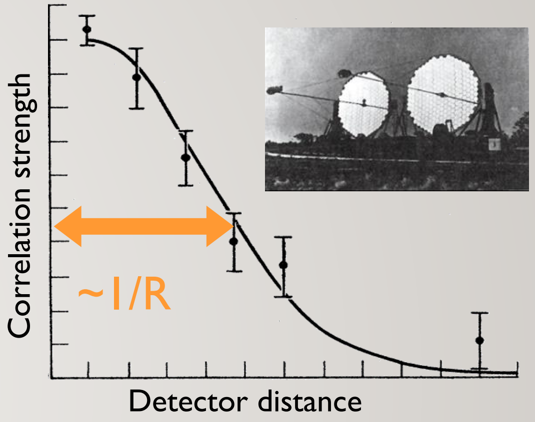
HBT OR FEMTOSCOPY IN HIGH ENERGY PHYSICS

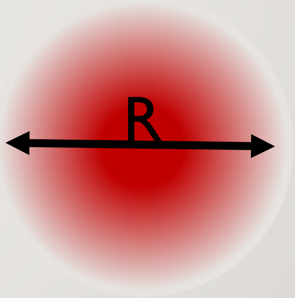
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

    $C(q) \cong 1 + \left| \int S(r) e^{iqr} dr \right|^2$

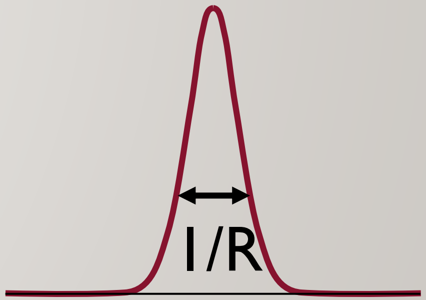
    (under some assumptions)
  - Also with distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy





source function S(r)



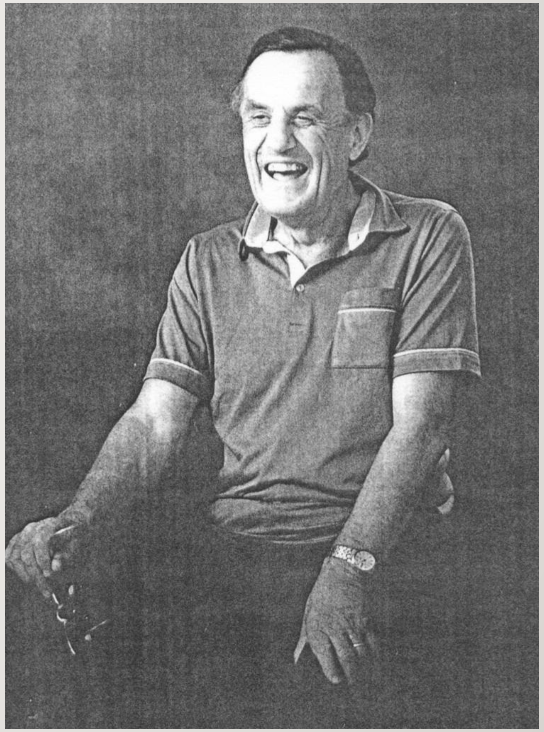
correlation funct. C(q)

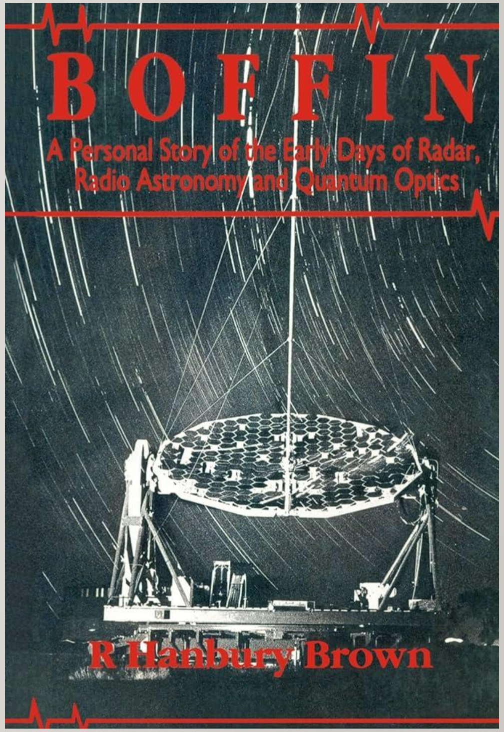
IGNORANCE IS SOMETIMES A BLISS IN SCIENCE

„In fact, to a surprising number of people the idea that the arrival of photons at two separated detectors can ever be correlated was not only heretical but patently absurd, and they told us so in no uncertain terms, in person, by letter, in print, and by publishing the results of laboratory experiments, which claimed to show that *we were wrong*."

„I was a long way from being able to calculate, whether it would be sensitive enough to measure a star.
... my education in physics had stopped far short of the quantum theory. Perhaps just as well, *ignorance is sometimes a bliss in science*."

R. H. Brown, Boffin: a personal story of radar, radio astronomy and quantum optics
(Taylor & Francis, 1991)

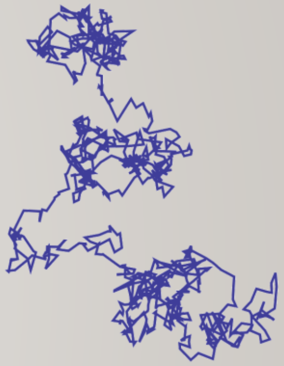


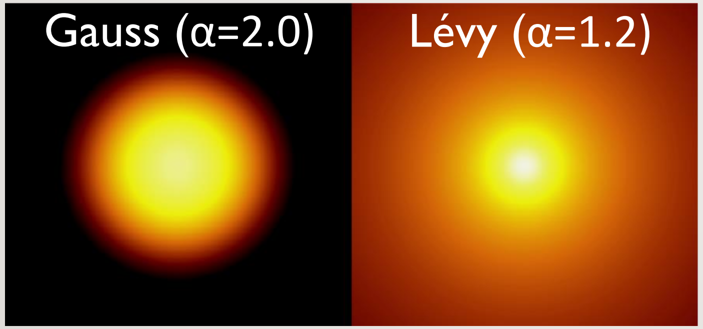


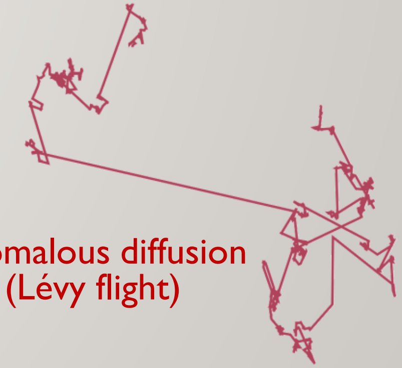
GAUSSIAN & LÉVY DISTRIBUTIONS IN HEAVY-ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$, Gauss if $\alpha = 2$, Cauchy if $\alpha = 1$
  - Suggested by Csörgő, Hegyi and Zajc in Eur.Phys.J. C36 (2004) 67-78
  - First observed in L3 (Novák, Csörgő, Metzger, …) at LEP (e^+e^-) Eur.Phys.J.C 71 (2011) 1648
  - Special cases: α = 2 Gaussian, α = 1 Cauchy

Normal diffusion





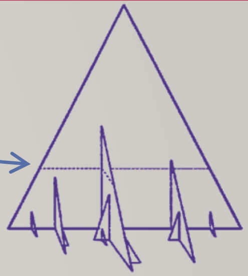


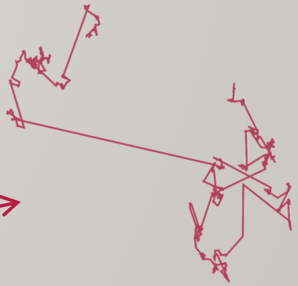
Anomalous diffusion (Lévy flight)

- Shape of the correlation functions with Lévy source: $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$
- A possible reason for Lévy source: anomalous diffusion, many others

WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
    - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;
    Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)
  - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)



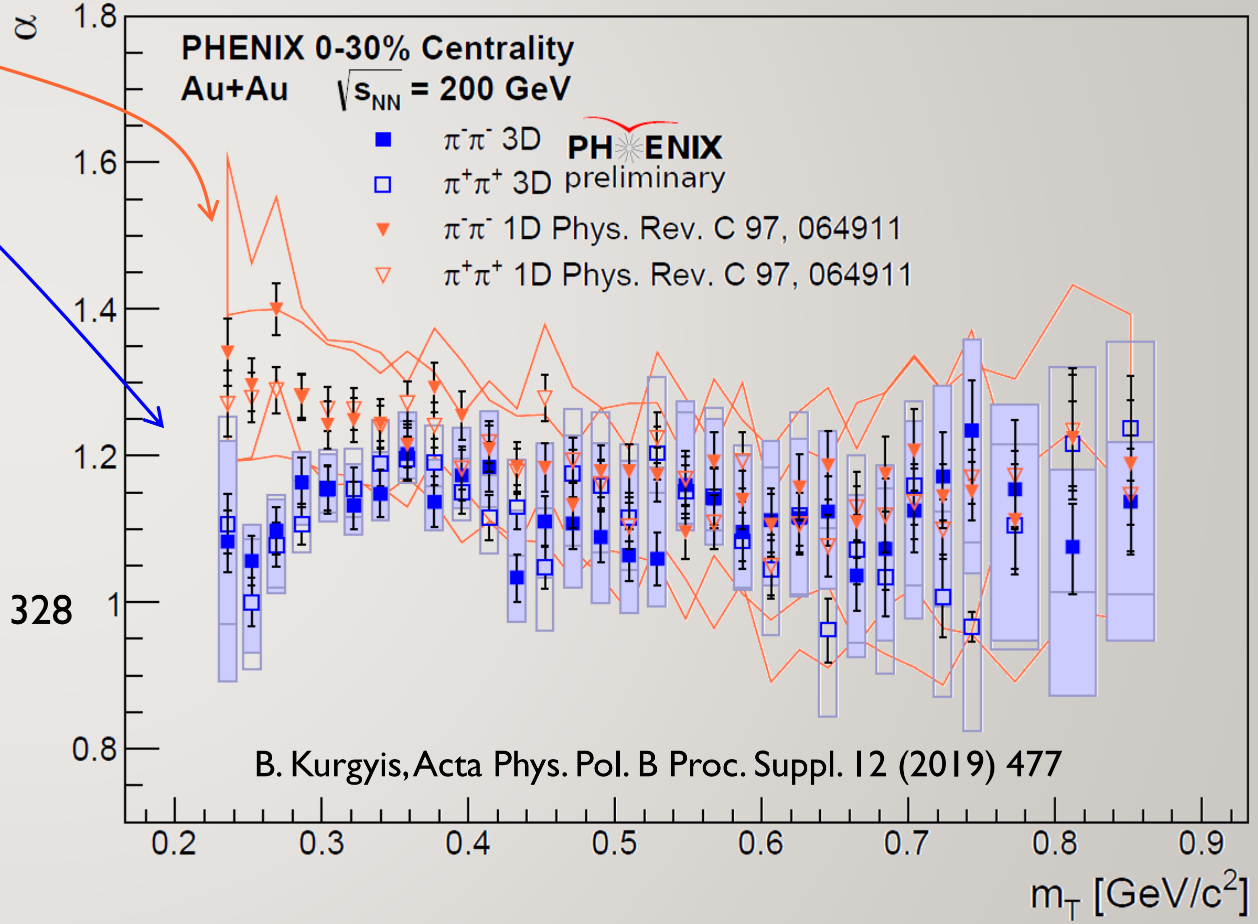


- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search, understanding source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production

LÉVY EXPONENT VERSUS TRANSVERSE MASS, 1D AND 3D

- Lévy exponent α in 3D close to 1D result
- On average still far from 2
- Observable differences at low m_T
  - Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible
    Kurgyis, Kincses, Csanád, Nagy, Universe 9 (2023) 7, 328
  - If spherical in LCMS, radius in PCMS:

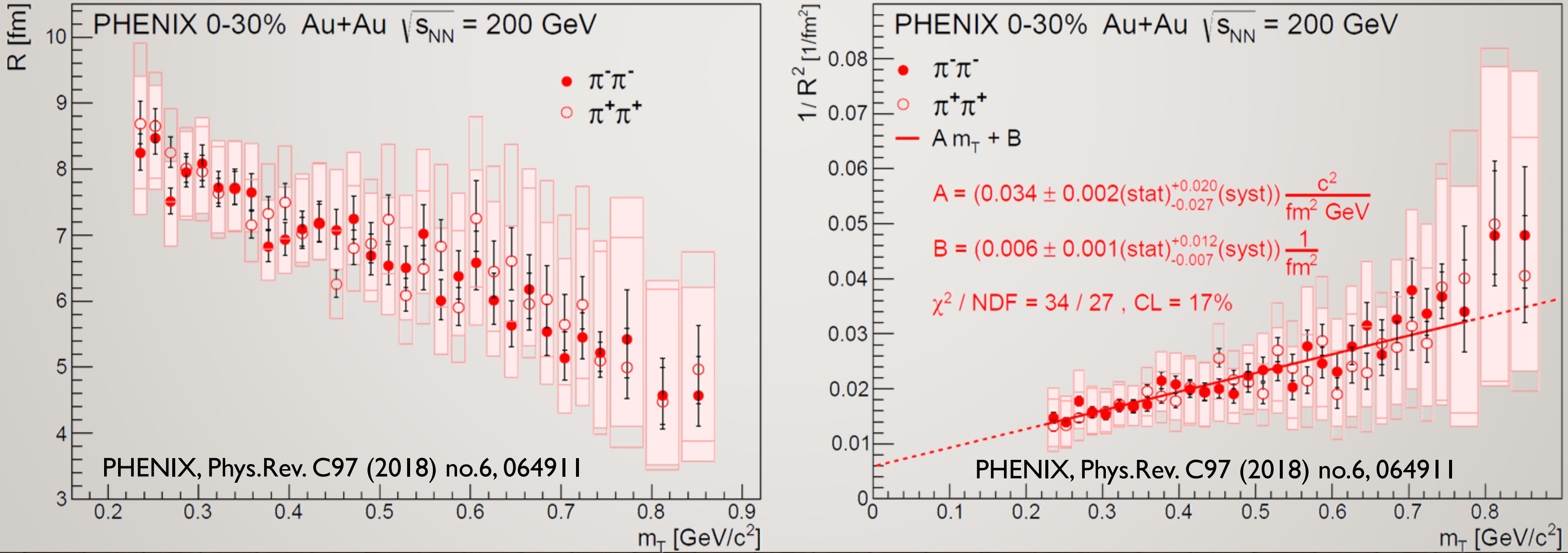
$$R_{PCMS} = \sqrt{\frac{1-2\beta_T^2/3}{1-\beta_T^2}} \cdot R_{LCMS}$$



B. Kurgyis, Acta Phys. Pol. B Proc. Suppl. 12 (2019) 477

LÉVY SCALE PARAMETER R AT RHIC

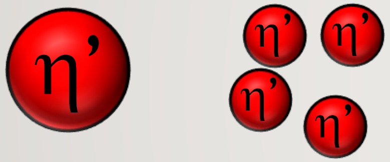
- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$) not invalid; but: predicted for Gaussian case only!



CORRELATION STRENGTH λ: IN-MEDIUM MASS?

- Connection to chiral restoration
  - Decreased η’ mass → more η’ produced → more decay pions → λ decreases
  - Kinematics: η’ → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η’ mass?

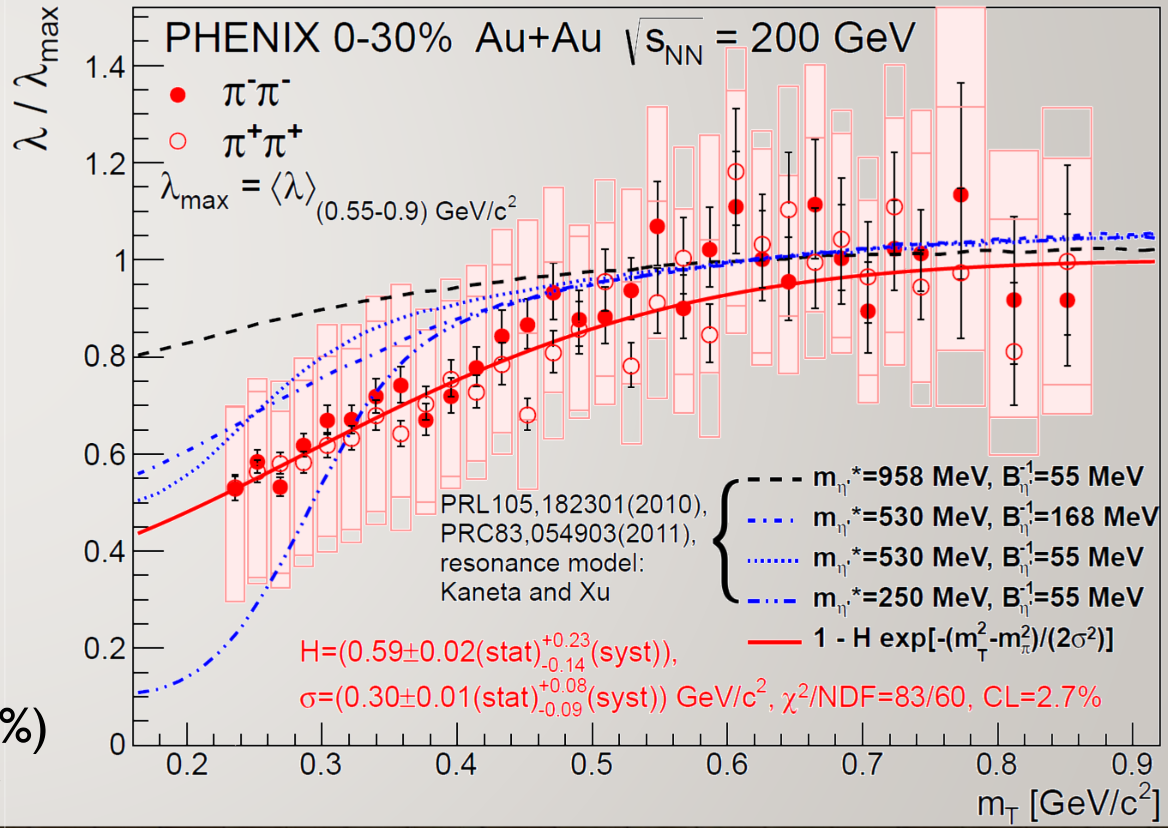
Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$

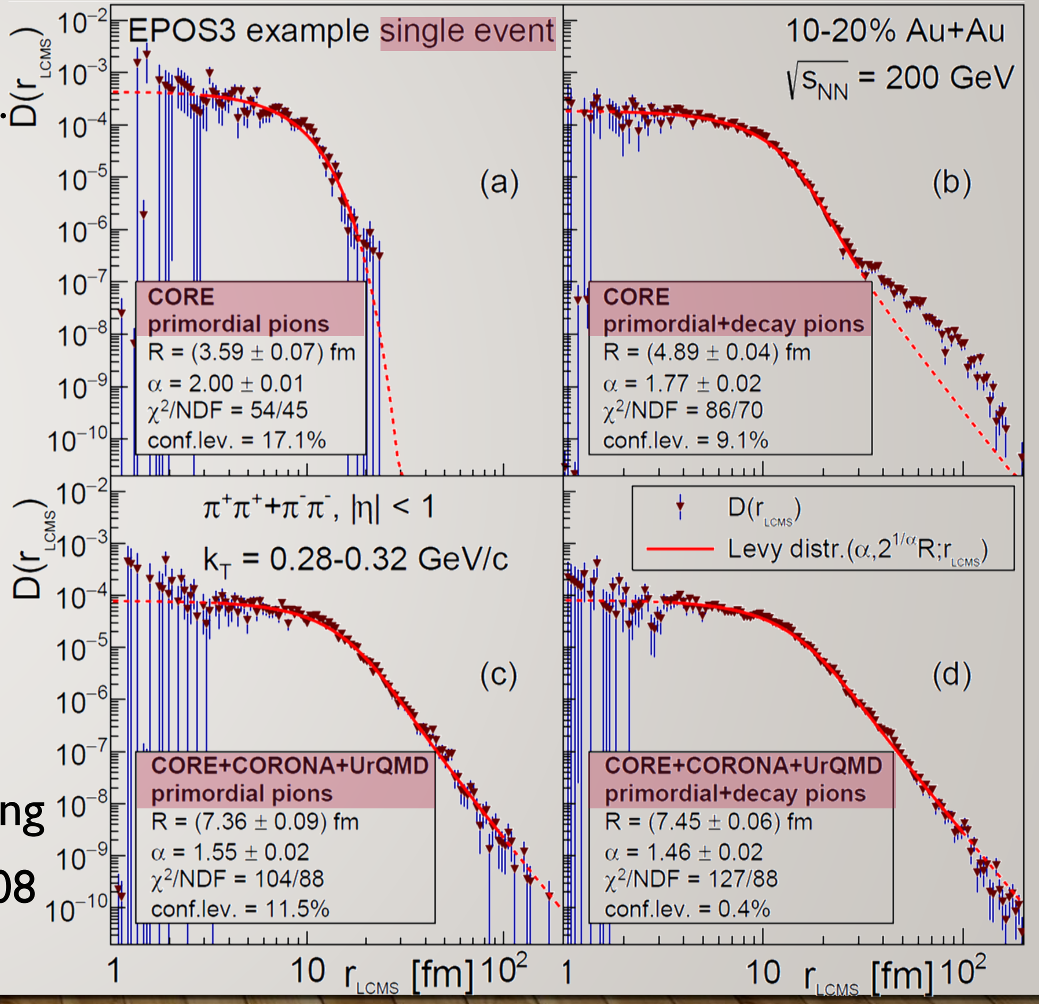
- Results not incompatible with this
- Recall: 3D results similar to 1D
- Would need direct check with photons ($\eta' \to \gamma\gamma$, 2.3%)
- Centrality dependent analysis in collaboration review

PHENIX, Phys.Rev. C97 (2018) no.6, 064911



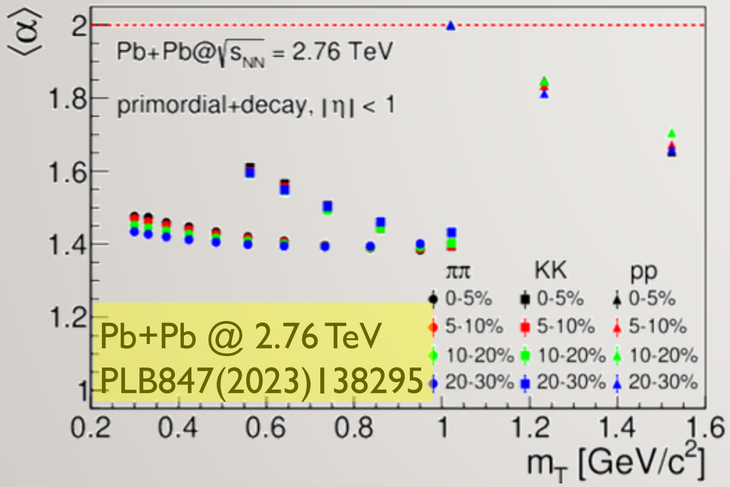
EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

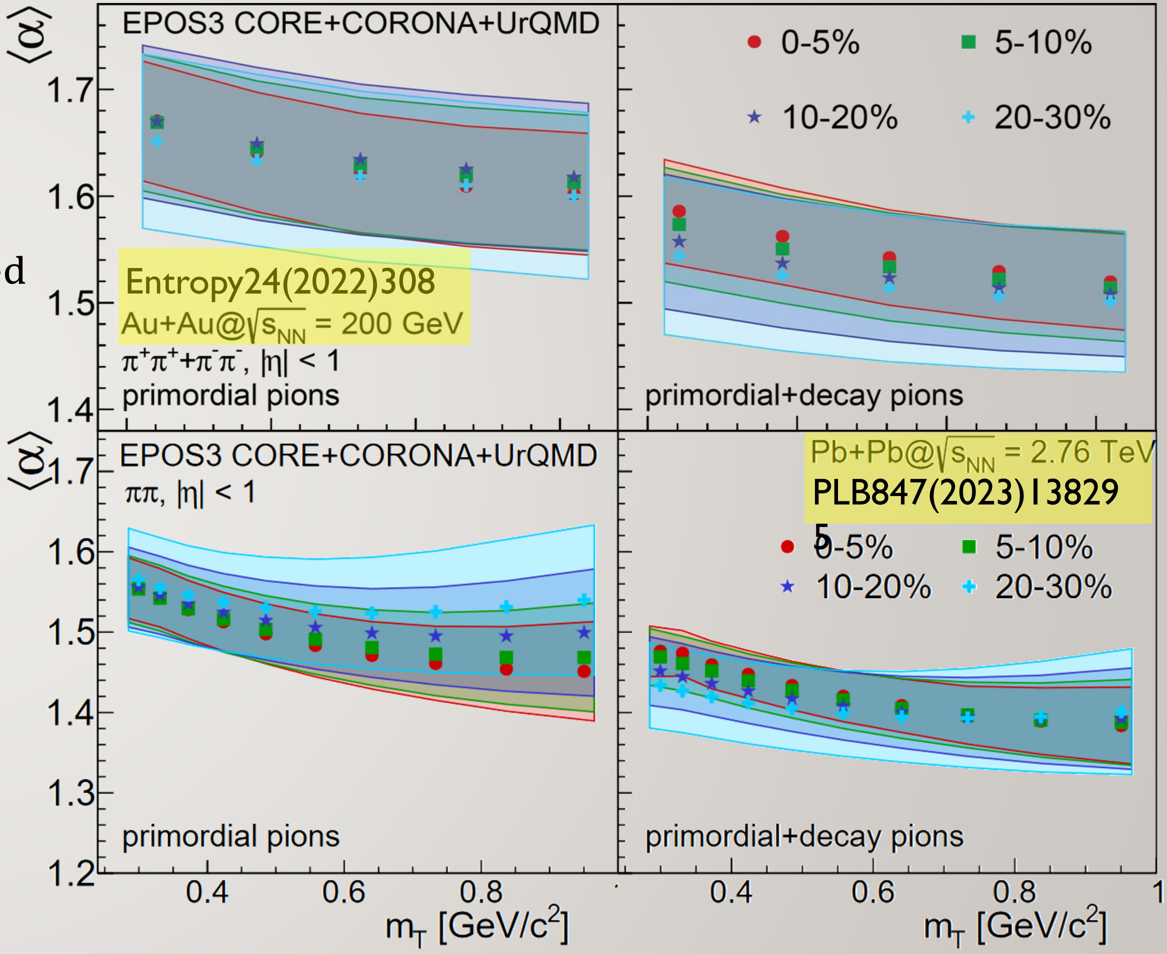
- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona, viscous hydro (vHLLE), cascades, UrQMD
- Pair distribution calculated: $D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$
  - Angle-averaged radial source distribution of like-sign pion pairs
- Investigated cases:
  a) CORE, primordial pions: close to Gaussian
  b) CORE, with decay products: power-law structures
  c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  d) CORE+CORONA+UrQMD, with decay products: Lévy shape
- Lévy shape in single events; source size versus m_T: hydro scaling
  - 200 GeV AuAu: Kincses, Stefaniak, Cs., Entropy 24 (2022) 308
  - 2.76 TeV PbPb: Kórodi, Kincses, Cs., PLB 847 (2023) 138295



AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

- $\langle\alpha\rangle$ versus m_T and centrality: small dependence
  - 200 GeV Au+Au: Entropy 24 (2022) 308
  - 2.76 TeV Pb+Pb: Phys. Lett. B 847 (2023) 138295
- With or without decays at RHIC: $\alpha_{\text{EPOS}} > \alpha_{\text{measured}}$
- Particle type dependence analyzed as well

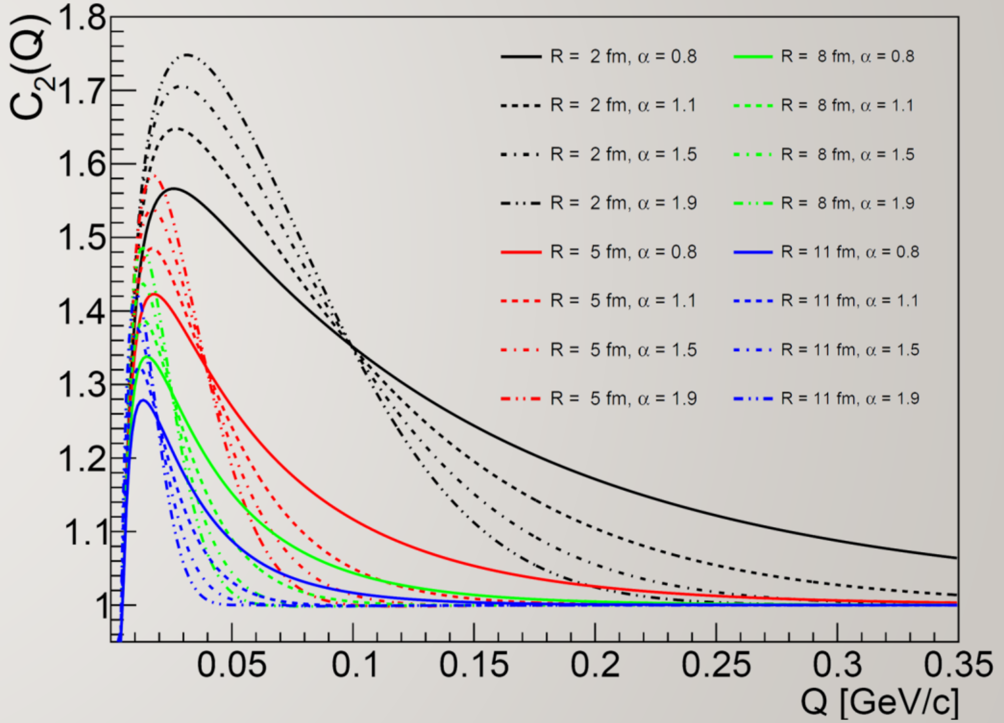




- 3D analysis as well

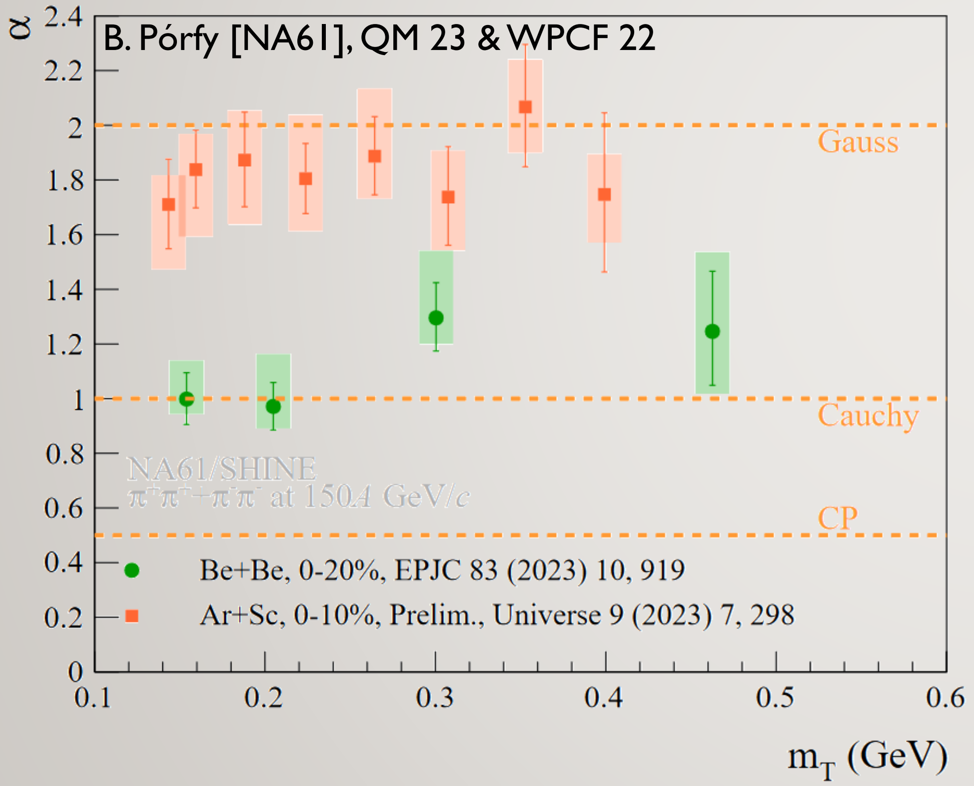
HOW TO CALCULATE THE COULOMB EFFECT

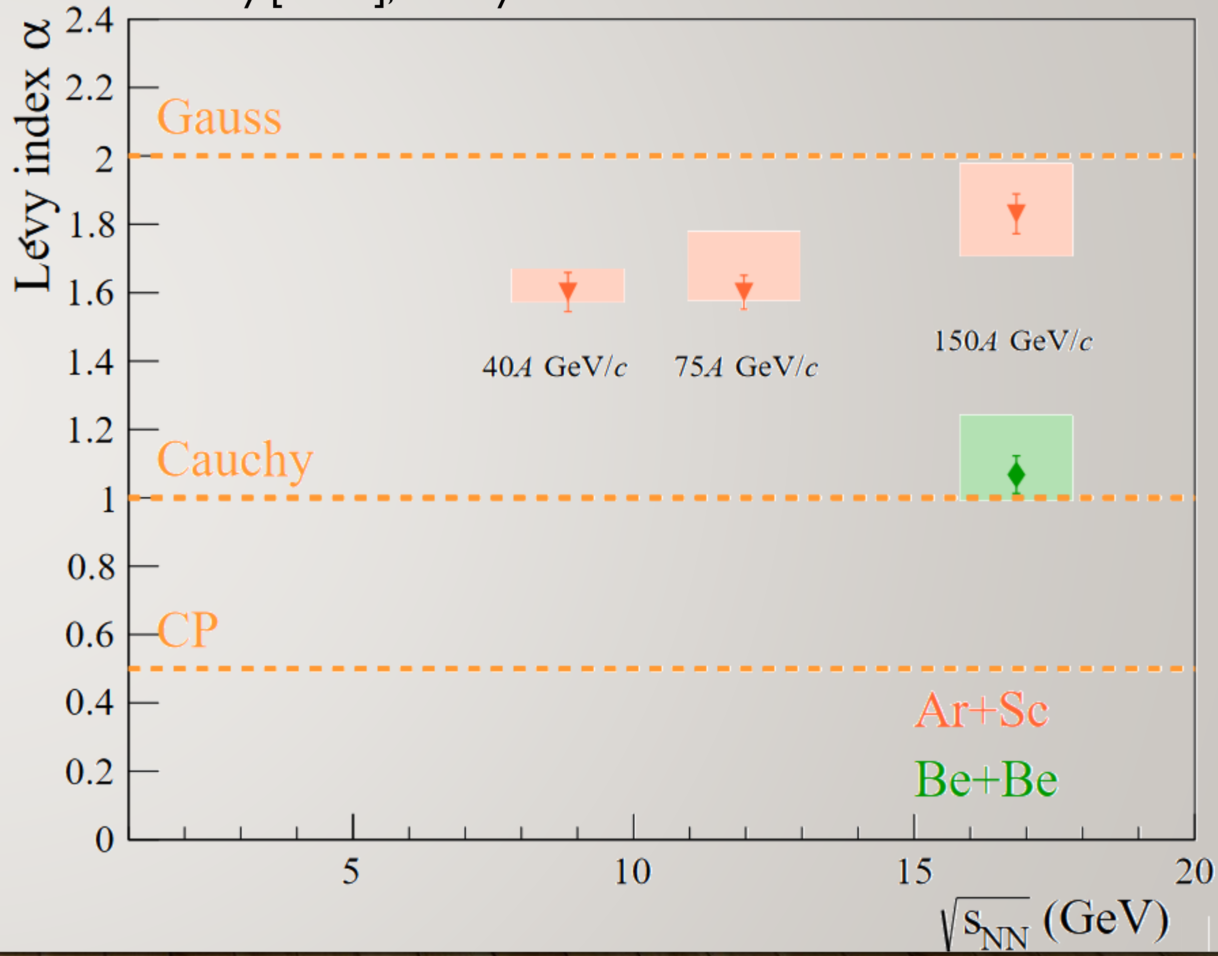
- Calculating correlation functions with the Coulomb effect included: time consuming in the past
- Method used in early analyses: Coulomb correction calculated for fixed radius and shape
  - For example, fixing $R = 5$ fm and $\alpha = 2$
- More consistent method: correlation function with Coulomb FSI precalculated in a tabular form
  - Iterative fitting, see e.g., PHENIX, PRC97 (2018) 6, 064911
- Convenient, but somewhat restricted method: interpolating functional form, in a limited R, α range
  - See Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238, used in arXiv:2306.11574 [CMS], arXiv:2302.04593 [NA61]
- A novel method: see next talk by Márton Nagy
  - Nagy, Purzsa, Csanád, Kincses Eur. Phys. J. C 83, 1015 (2023), code at github.com/csanadm/CoulCorrLevyIntegral



NA61/SHINE RESULTS

- At 150 AGeV: α(Be+Be) < α(Ar+Sc)
- Slight decrease of α for smaller energies in Ar+Sc

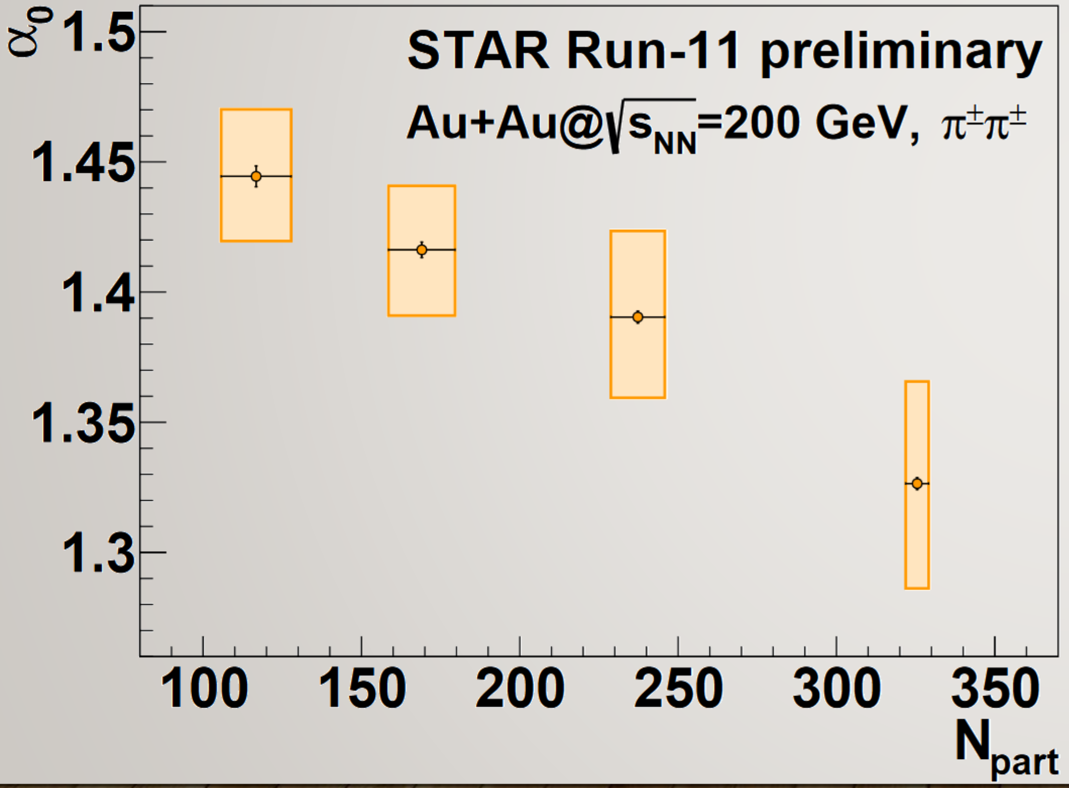




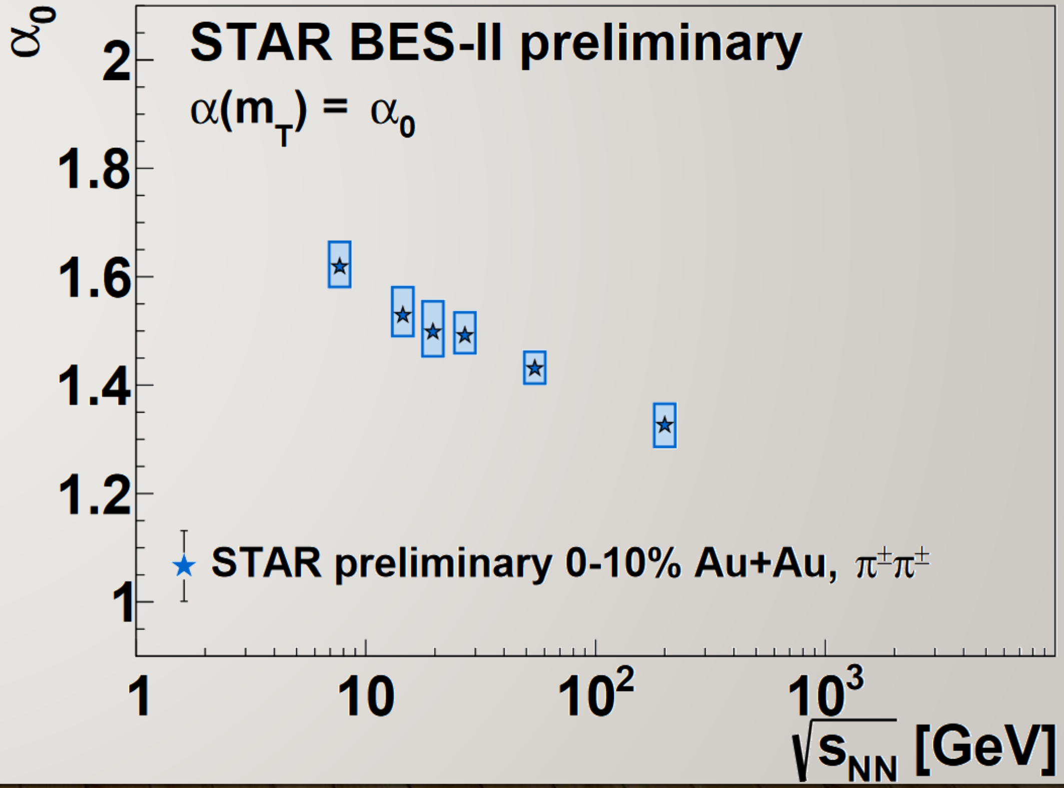
STAR RESULTS

- Lévy distributions observed from $\sqrt{s_{NN}} = 7.7$ GeV to 200 GeV
- Source far from Gaussian, increase of α for peripheral collisions and for smaller collision energies

D. Kincses [STAR], WPCF 2023

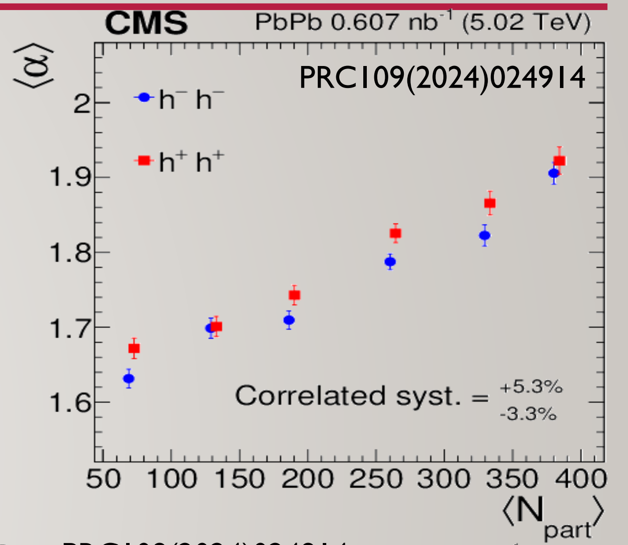


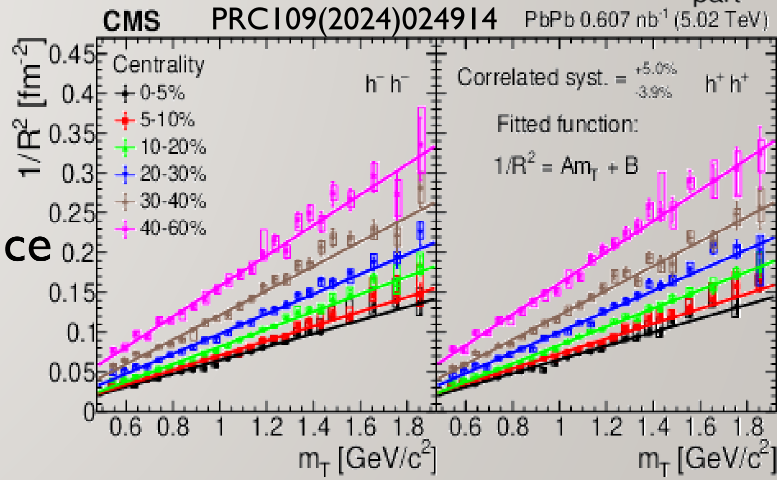
D. Kincses [STAR], Zimányi 2023



CHARGED HADRON ANALYSIS IN 5 TEV PB+PB

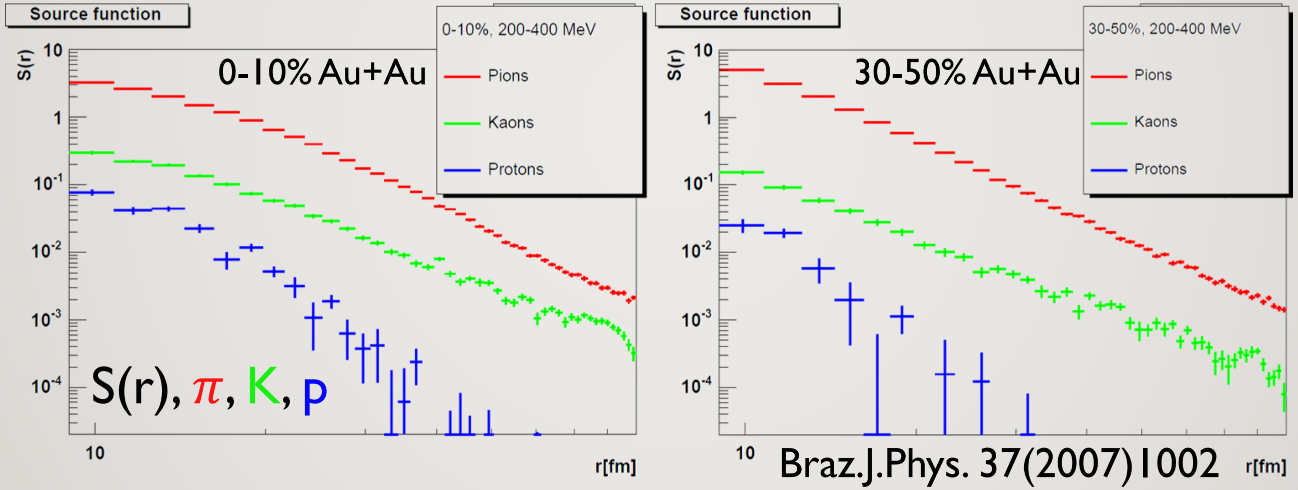
- Lévy index α measured:
  - Far from Cauchy
  - Not exactly Gaussian
  - Closer to Gaussian for large N_{part}, unlike RHIC
- Lévy scale R: hydro scaling confirmed
  - In every centrality class, despite non-Gaussianity
  - Hubble coefficient can be extracted: 0.12-0.18 c/fm, larger than at RHIC
- Correlation strength λ also analyzed
- Low-Q deviation cross-checked with Monte-Carlo: two-track acceptance
- Final CMS result: Phys.Rev.C 109 (2024) 2, 024914
  - Preliminary results in proceedings: B. Kórodi, Universe 9 (2023) 7, 318





THE IMPORTANCE OF A KAON ANALYSIS

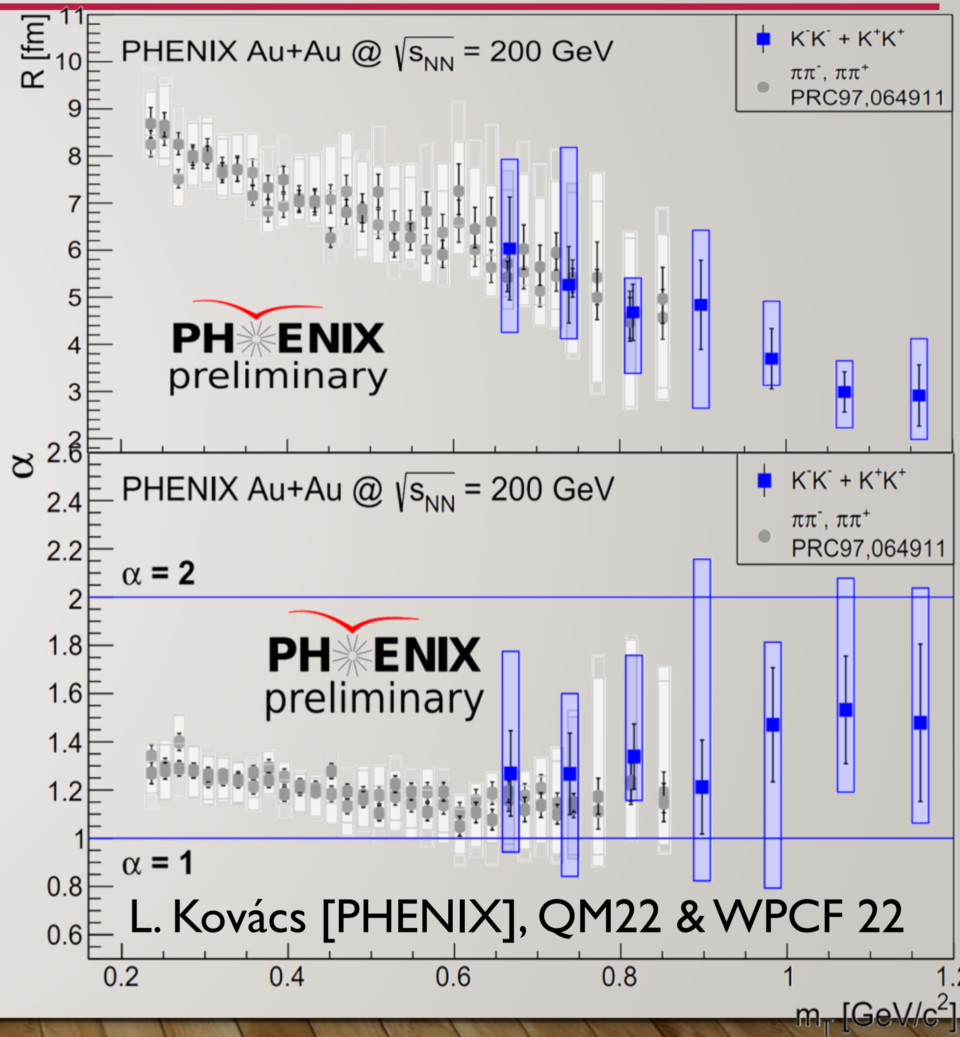
- Kaons: smaller cross-section, larger mean free path
- Mean free path increases more during a time-step → heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$

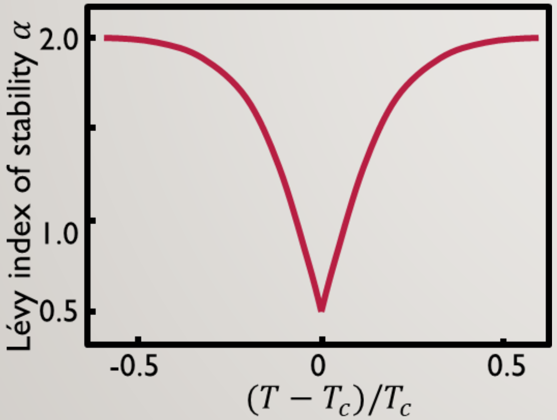
KAONS AT PHENIX AND STAR (SEE TALK BY R. BELMONT)

- Preliminary analysis performed at PHENIX and STAR
- Kaon and pion data seem compatible at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
  - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
- Dominant mechanism creating Lévy source?
  - Not only rescattering?
  - Anomalous hydro at the sQGP stage?
- PHENIX prelim. results: L. Kovács, Universe 9 (2023) 7, 336
- STAR prelim. results: A. Mukherjee, Universe 9 (2023) 7, 300

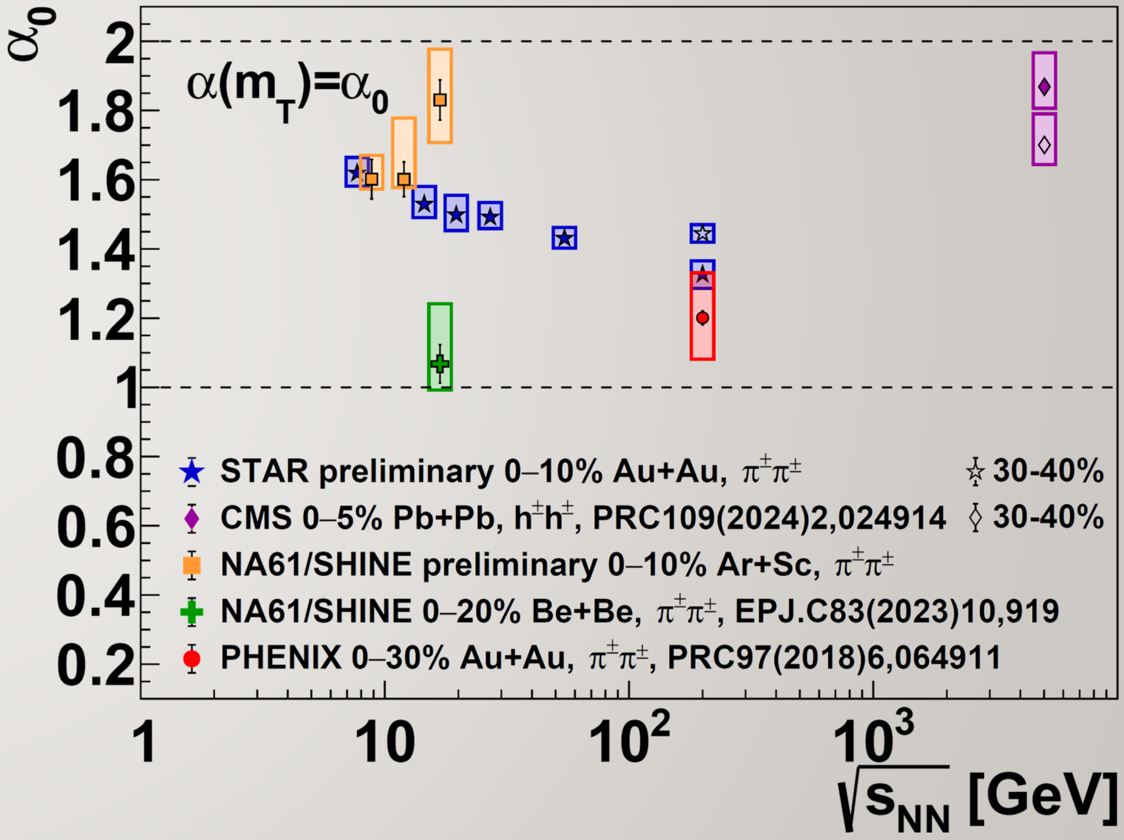


STABILITY PARAMETER α FROM SPS TO LHC

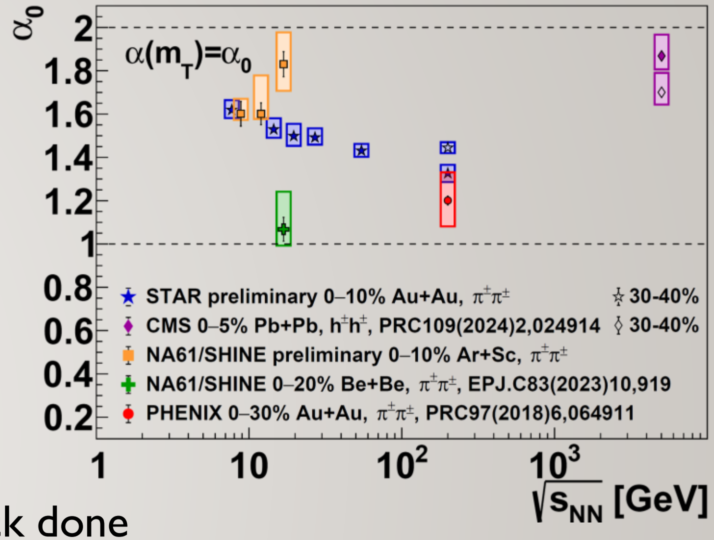
- Different values for small (Be+Be) & medium (Ar+Sc) systems at SPS
  - Also true for Pb+Pb and p+p at LHC? (pp: $\alpha = 1$ assumed)
- Medium and large systems: non-monotonic trend
  - Minimum around top RHIC energy?
- Compare to expectation cartoon based on Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67



Csanád, Kincses, Universe 10 (2024) 54. arXiv:2401.01249

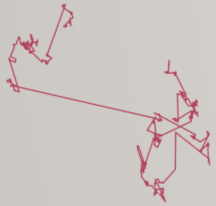


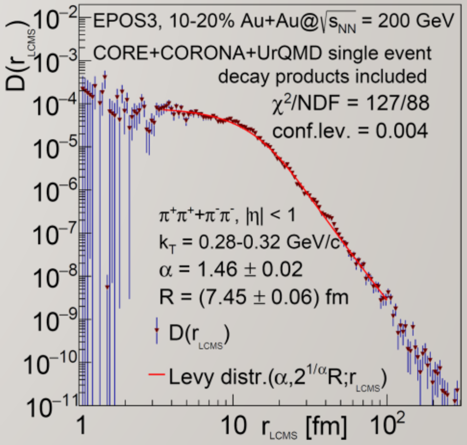
CONCLUSIONS AND OUTLOOK



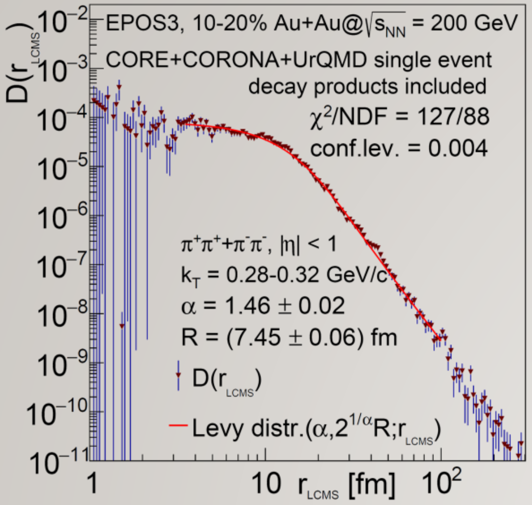
- Lévy sources from SPS to RHIC and LHC
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
  - **Lévy R**: hydro scaling, despite not Gaussian
  - **Lévy λ**: signs of η' in-medium mass modification

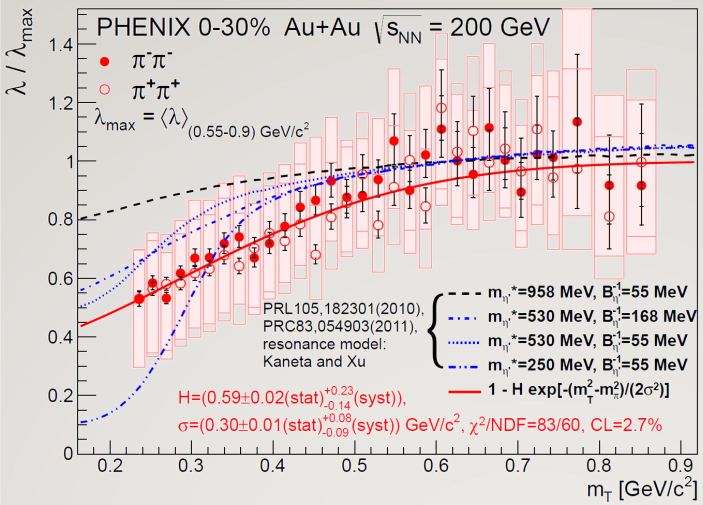
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS
  - Directional averaging → source is (approx.) spherical in LCMS, 3D cross-check done
  - Event averaging → event-by-event simulations show Lévy
  - **Resonance decays** → part of the reason, not enough alone
  - **Hadronic rescattering, Lévy flight** → $\alpha(K) \geq \alpha(\pi)$ puzzling

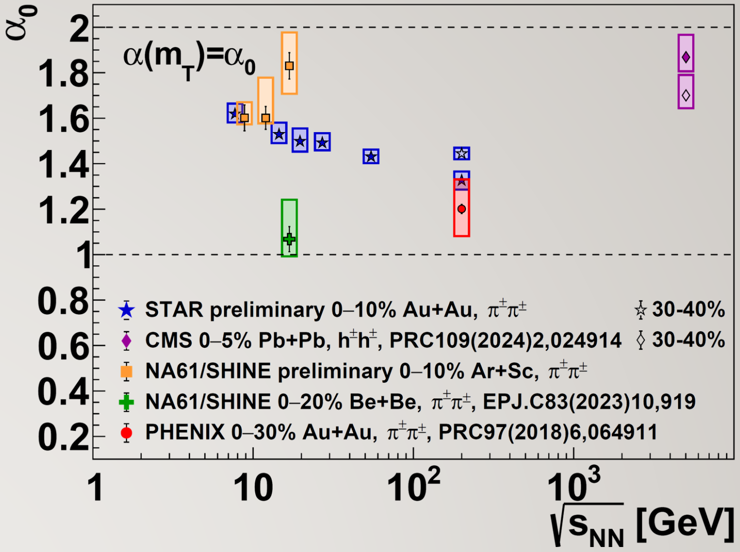




- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Can there be anomalous diffusion in the quark stage?
  - What is the role of finite size and finite time?







THANK YOU FOR YOUR ATTENTION

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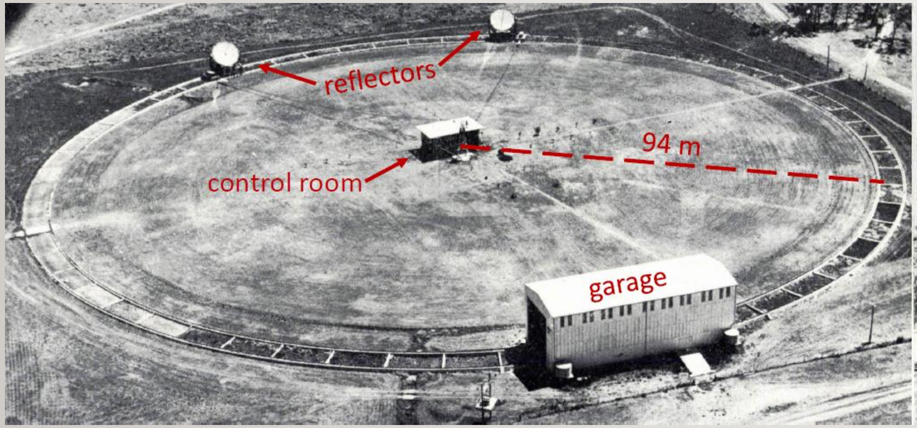
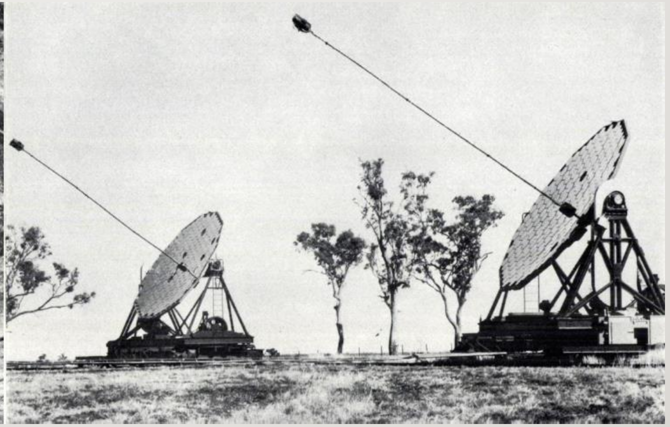
Universe **2024**, *10*(2), 54
https://doi.org/10.3390/universe10020054

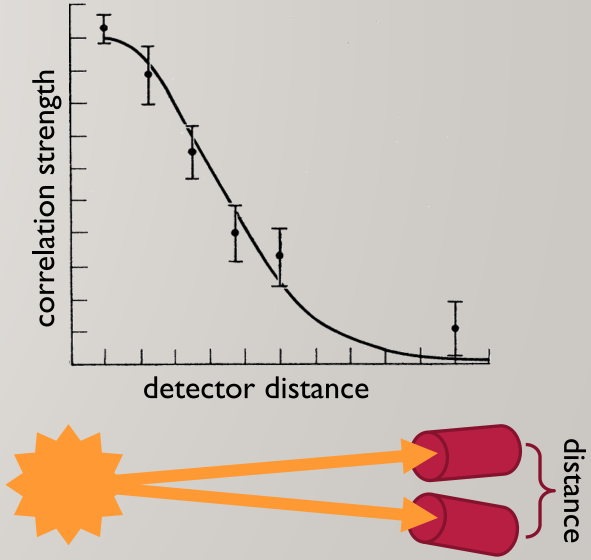
BACKUP

A SURPRISING DISCOVERY: HBT CORRELATIONS

- Radio astronomy: Jansky, 1933, weird 24h oscillation; stars also emit EM radiation in the radio domain!
- R. H. Brown: investigated radio waves from stars
  - Jordell Bank (optical and radio telescopes), tabletop experiment (optics), Narrabri (stellar interferometer)
- R. Q. Twiss helped to understand results mathematically
- Weird correlation in all experiments: joint intensity ”too frequent”, interference?
- „Interference between two different photons never occurs”
  P.A. M. Dirac, Quantum Mechanics (Oxford UP, London, 1958)



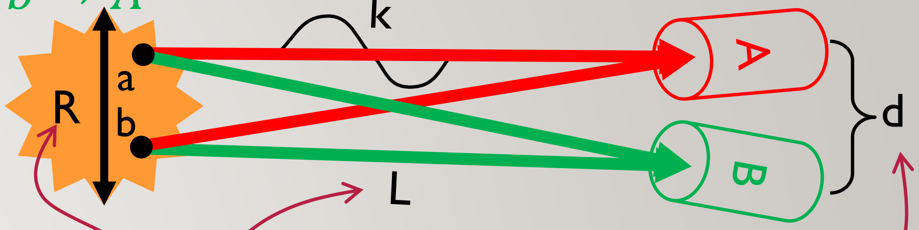


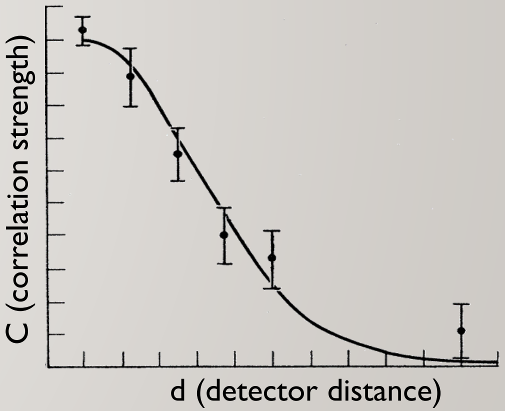


HBT EFFECT: QUANTUM EXPLANATION

- „Symmetrized wavefunction”: $a \to A$ and $b \to B$ or $a \to B$ and $b \to A$
  - Cannot distinguish which photon is happening in which detector
  - Important: photon is the detection event
  - Detection possibilities are symmetrized
- With a plane wave (with k wavenumber) from two point-like sources, the normalized joint probability, called correlation function, is:

$$C(A,B) = \frac{P(A,B)}{P(A)P(B)} = 1 + \cos\left(\frac{kR}{L}d\right)$$

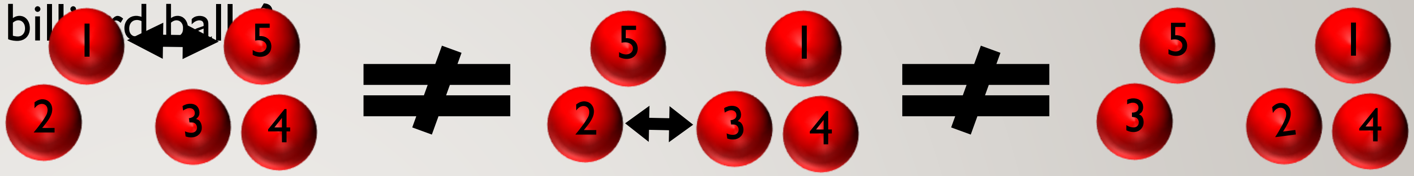




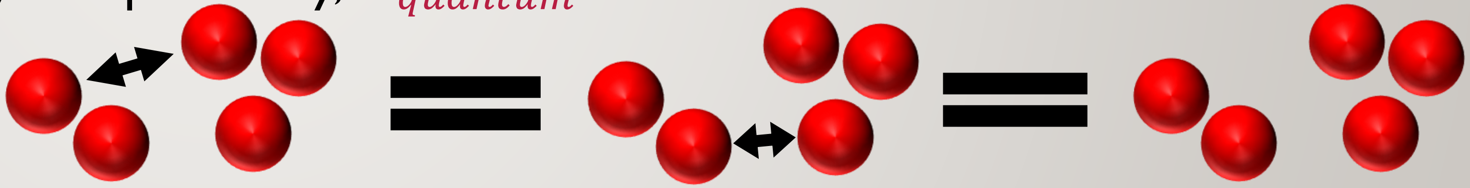
- Correlation width inversely proportional to source size
- Bose-Einstein correlation!
- Similar explanation can be given based on classical waves
  - Deep reason: classical scalar or vector waves behave as bosons, statistically

INDISTINGUISHABILITY OF ELEMENTARY PARTICLES

- How many ways to choose 2 out of 5 billiard balls?



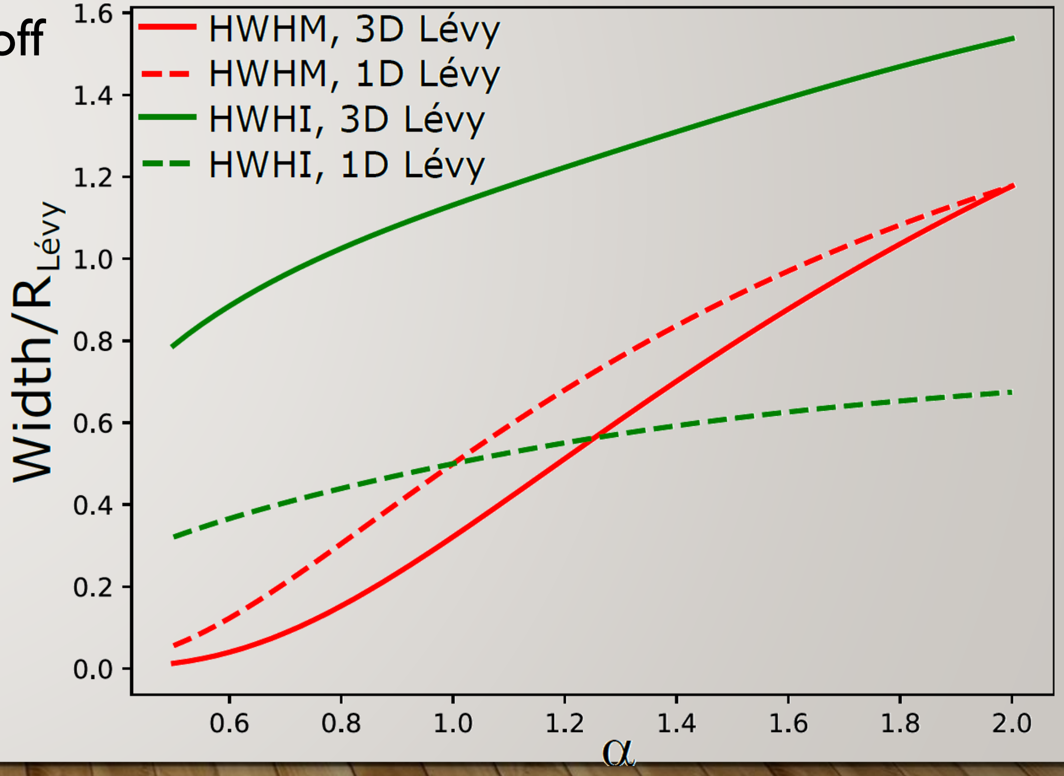
- $W_{classical} = \binom{5}{2} = \frac{5!}{2!3!} = 10$ is the number of possibilities
- What if we have elementary particles instead of balls?
- They are indistinguishable: hence only one possibility, $W_{quantum} = 1$



- Two-particle wave-function must be symmetric!
- Particles 1 and 2 in states A and B, symmetrized wave-function: $\Psi_{12}^{AB} \sim \Psi_1^A \Psi_2^B + \Psi_1^B \Psi_2^A$

WHAT IS THE TRUE SIZE OF THE SOURCE?

- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \to 2^{1/\alpha}R$
- In principle, RMS $= \infty$ if $\alpha < 2$, practice: depends on cutoff
- What do Gaussian HBT radii mean?
- Alternative measures:
  - HWHI: (half) width at half integral
  - HWHM: (half) width at half max
  - Large difference between 1D and 3D relative width
  - Width (normalized by R) nontrivially depends on α
  - If $\alpha = 2$ or $\alpha = 1$ assumed: deviation from true scale

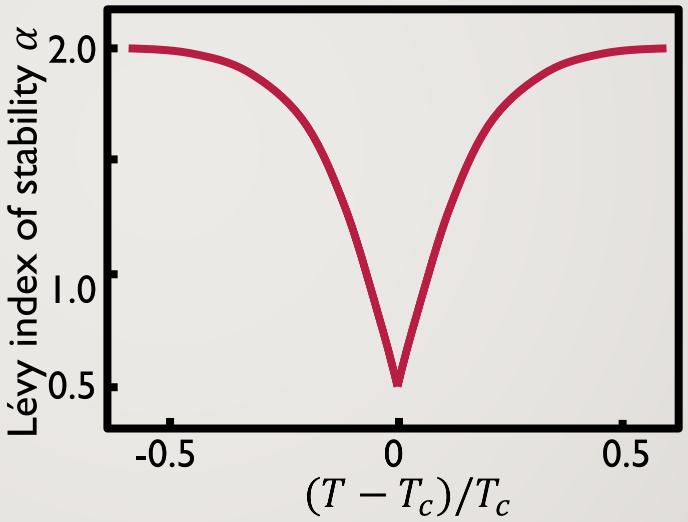


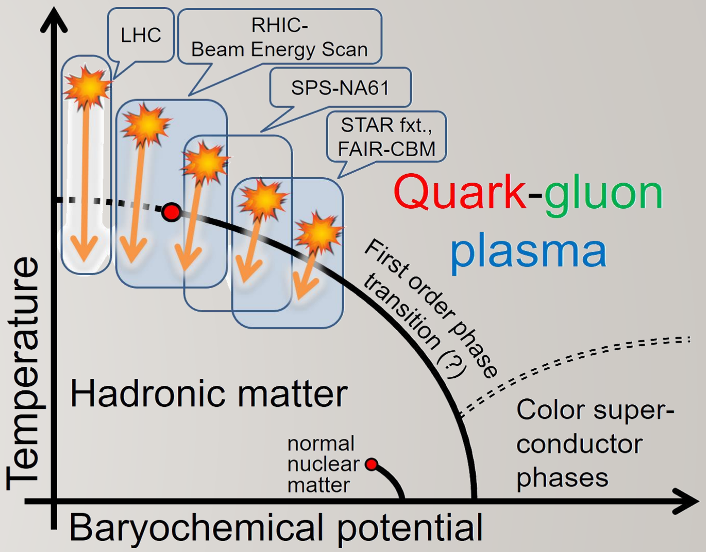
LÉVY INDEX AS A CRITICAL EXPONENT?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?

  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising

  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?

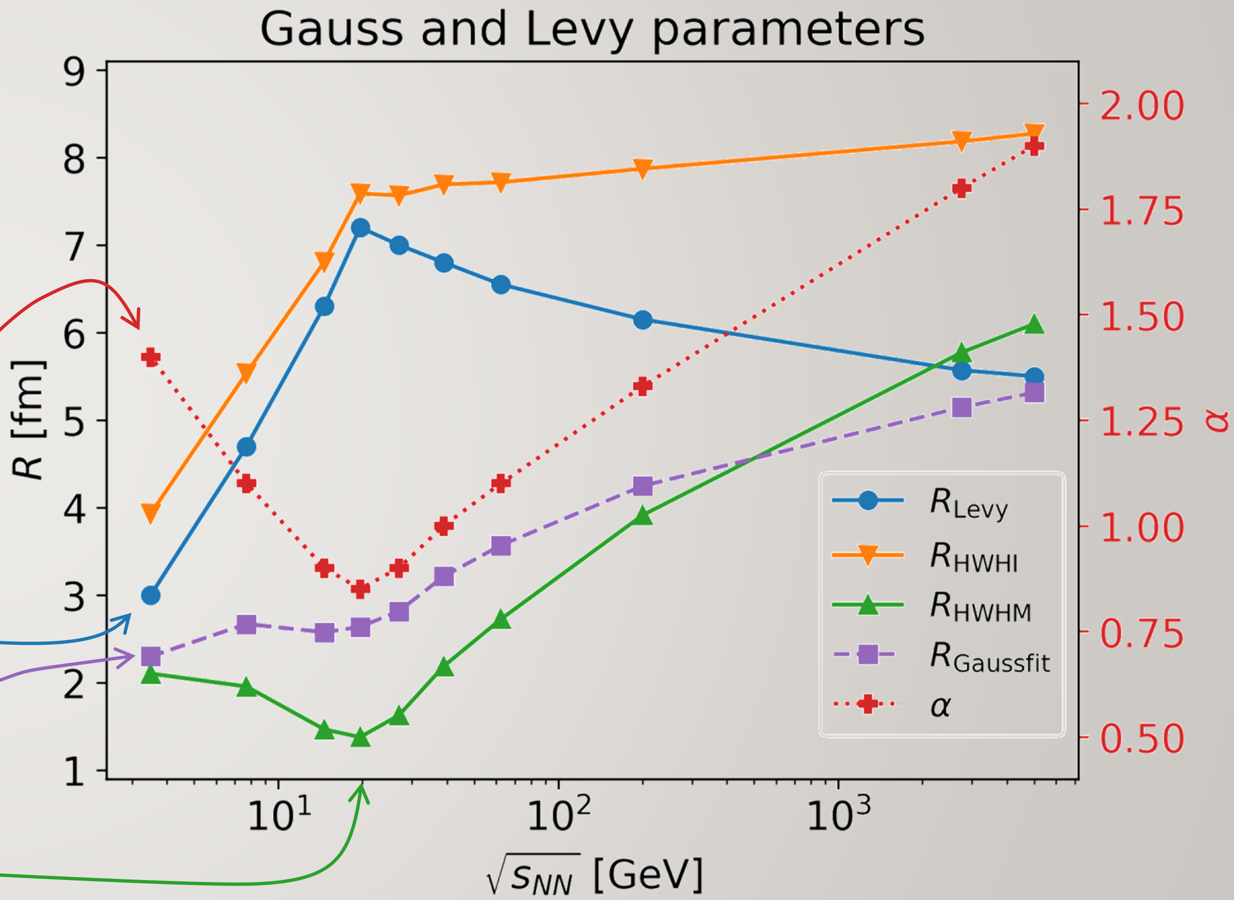




- Finite-size/time & non-equilibrium effects → what does power-law tail mean?
  - Finite-size effects not important? See e.g. Fytas et al, PRE93, 063308 (2016), Ballesteros et al., PLB387 (1996) 125

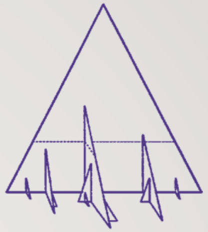
SOURCE SIZE MEASURES AROUND THE CRITICAL POINT?

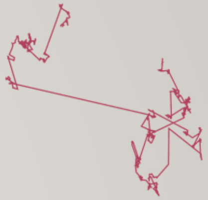
- Main Lévy source parameters: R_{Levy}, α
- Other source size measures:
  - R_{Gaussfit}: $C(Q; R_{\mathrm{Levy}}, \alpha)$ fitted with $\alpha = 2$ fixed
  - R_{HWHM}: half width at half maximum
  - R_{HWHI}: half width at half integral
- **Simulated scenario:**
  - minimum in α vs. s_{NN}
  - maximum in R_{Levy} vs. s_{NN}
- **Observation:**
  - R_{Gaussfit}: approximately monotonic increase
  - Minimum in R_{HWHM}!
  - Trend change in R_{HWHI}!



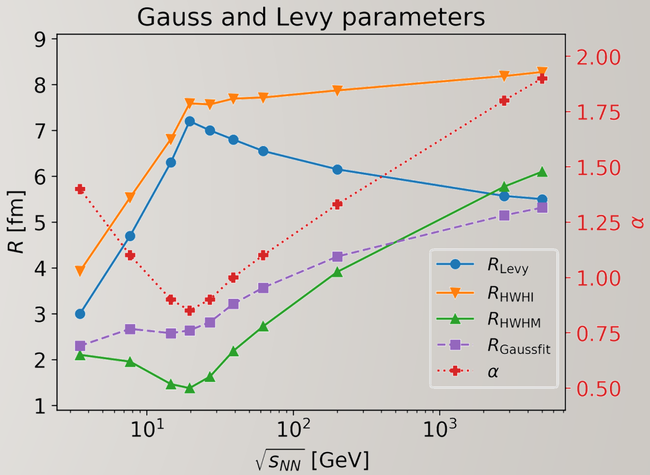
SO WHY SHOULD ANYONE ASSUME LÉVY SOURCES?

- Extra parameter (α) has physical meaning in each of the physical reasons of its appearance
  - Jet fragmentation, critical phenomena, anomalous diffusion





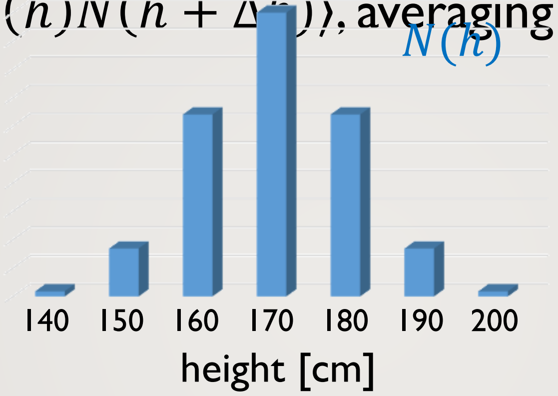
- When measuring source size with Gaussian assumption, actual size and shape information are entangled

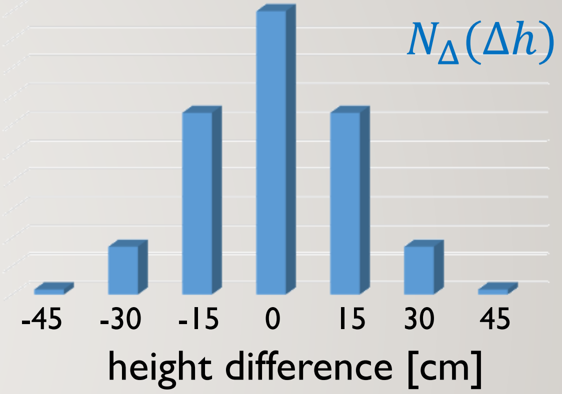


- Why not try it?
  - One more parameter (with its ups and downs, e.g., interparam. corr.)
  - Coulomb more complicated (but by now not too bad, see talk by M. Nagy)
  - Note: radius means something else than Gaussian radius (published often in the past)
- Why assume anything? Can't we just use spherical decomposition and imaging?
  - Not if we want to quantify details of the source, such as size or strength (e.g., to measure R_{out}, R_{side} vs $\sqrt{s_{NN}}$)

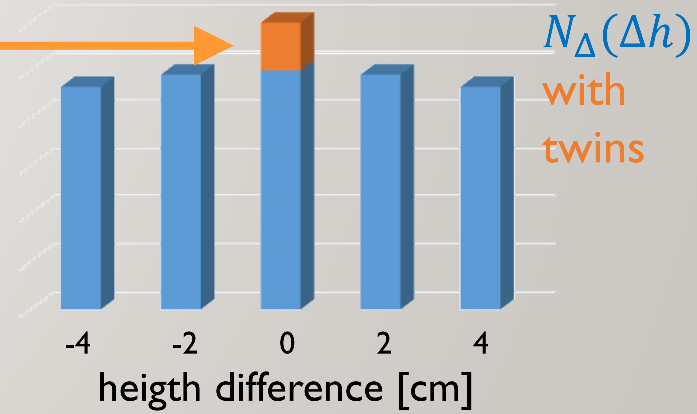
WHY ARE CORRELATIONS USEFUL?

- Frequency of a given height in a population/sample: $N(h)$
- Height densities: $N_\Delta(\Delta h) = \langle N(h)N(h+\Delta h)\rangle$, averaging on h
- Simplest case:
  both $N(h)$ and $N_\Delta(\Delta h)$
  Gaussian





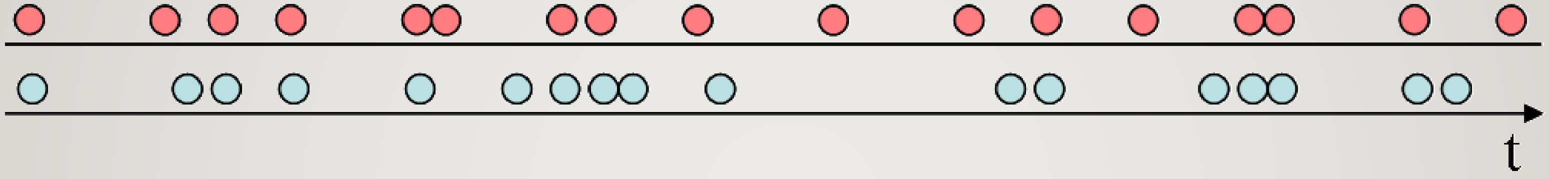
- Except if lot of identical twins:
  unexpected number of same height pairs –
  increase at zero!



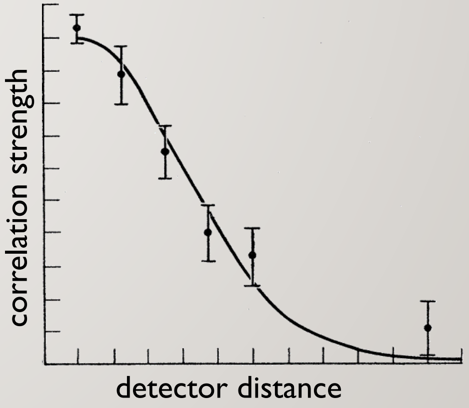
- Then $N_\Delta(0) > \langle N(h)N(h)\rangle$
- Correlation: tells the number of identical twins!

THE HBT CORRELATION

- Observation by R. H. Brown: decreasing detector distance increases correlations in detector signals
- Joint intensity ”too frequent”: $I(A,B) > I(A)I(B)$



- Reason for correlations? Interference?
- „Interference between two different photons never occurs”
  P.A. M. Dirac, Quantum Mechanics (Oxford UP, London, 1958)
- Why does the correlation reduce with distance?

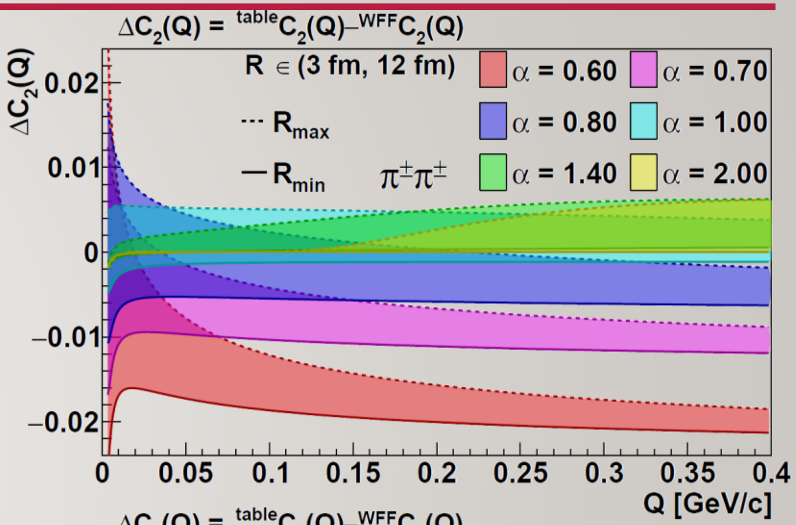


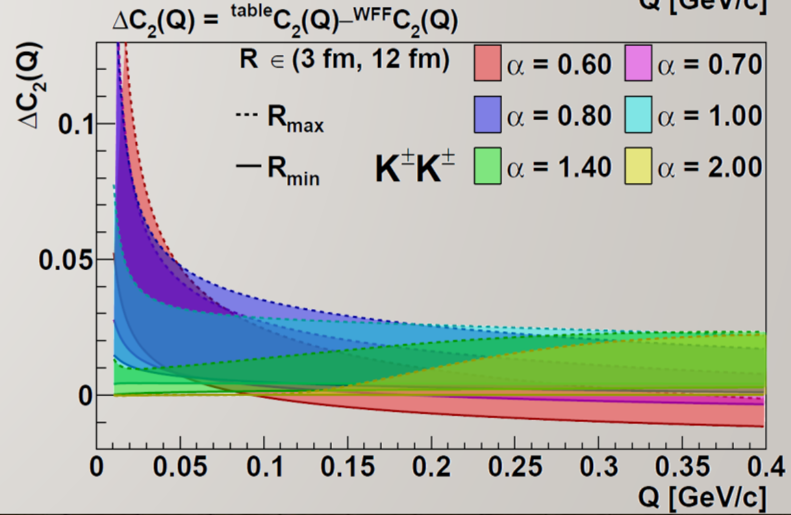
A NOVEL METHOD FOR LÉVY SHAPES WITH COULOMB FSI

- New mathematical development:

  Coulomb integral $C_2(Q) = \int d^3r \left|\psi_Q(r)\right|^2 D(r)$ can be performed
  - $D(r)$ is expressible as a Fourier transform: $D(r) = \int d^3q e^{iqr} f(q)$, for example $D(r)$ Lévy: $f(q) = e^{-|qR|^\alpha}$
  - Integrals $\int d^3r$ and $\int d^3q$ unfortunately cannot be exchanged
  - Calculation can still be performed via Lebesgue and Fubini theorems
- Result: $C_2(Q) = |\mathcal{N}|^2 \left(1 + f(Q) + \frac{\eta}{\pi}[A_{1s}[f](Q) + A_{2s}[f](Q)]\right)$,

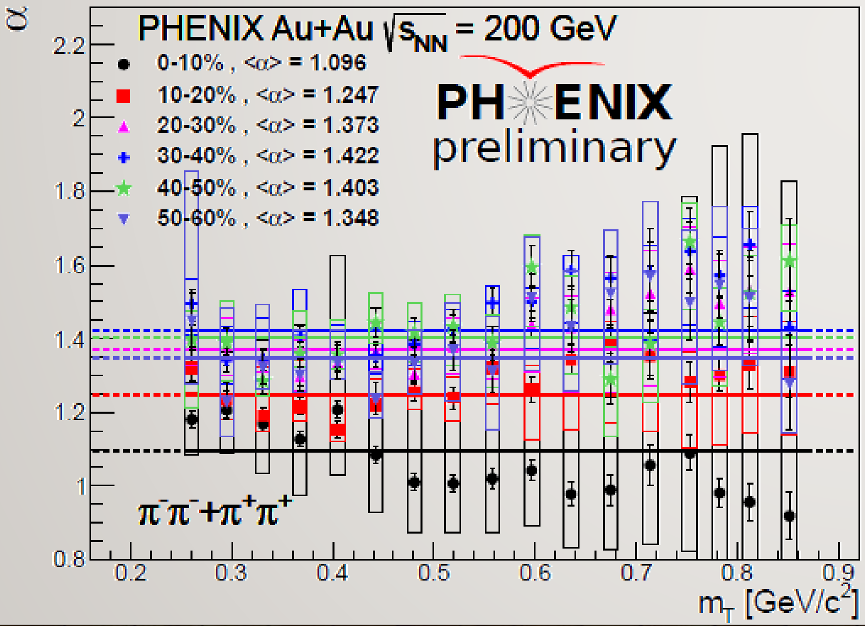
  where $|\mathcal{N}|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ (Gamow), $\eta = \frac{mc^2\alpha}{\hbar c Q}$, A_{ns} functionals
  - Few percent difference to numerical (tabularized) values used earlier
  - Details in Nagy, Purzsa, Csanád, Kincses Eur. Phys. J. C 83, 1015 (2023), code at github.com/csanadm/CoulCorrLevyIntegral

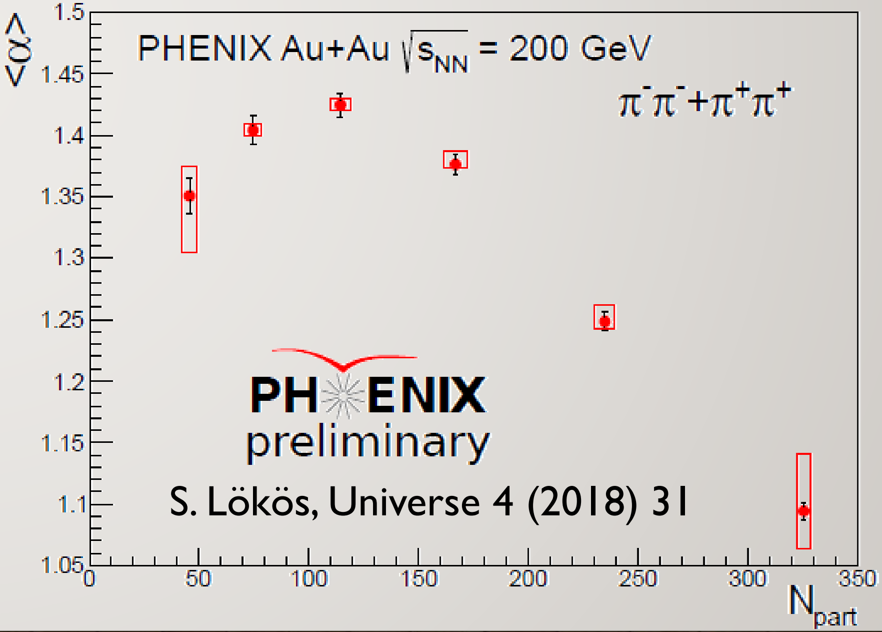




ANALYZING THE CENTRALITY DEPENDENCE

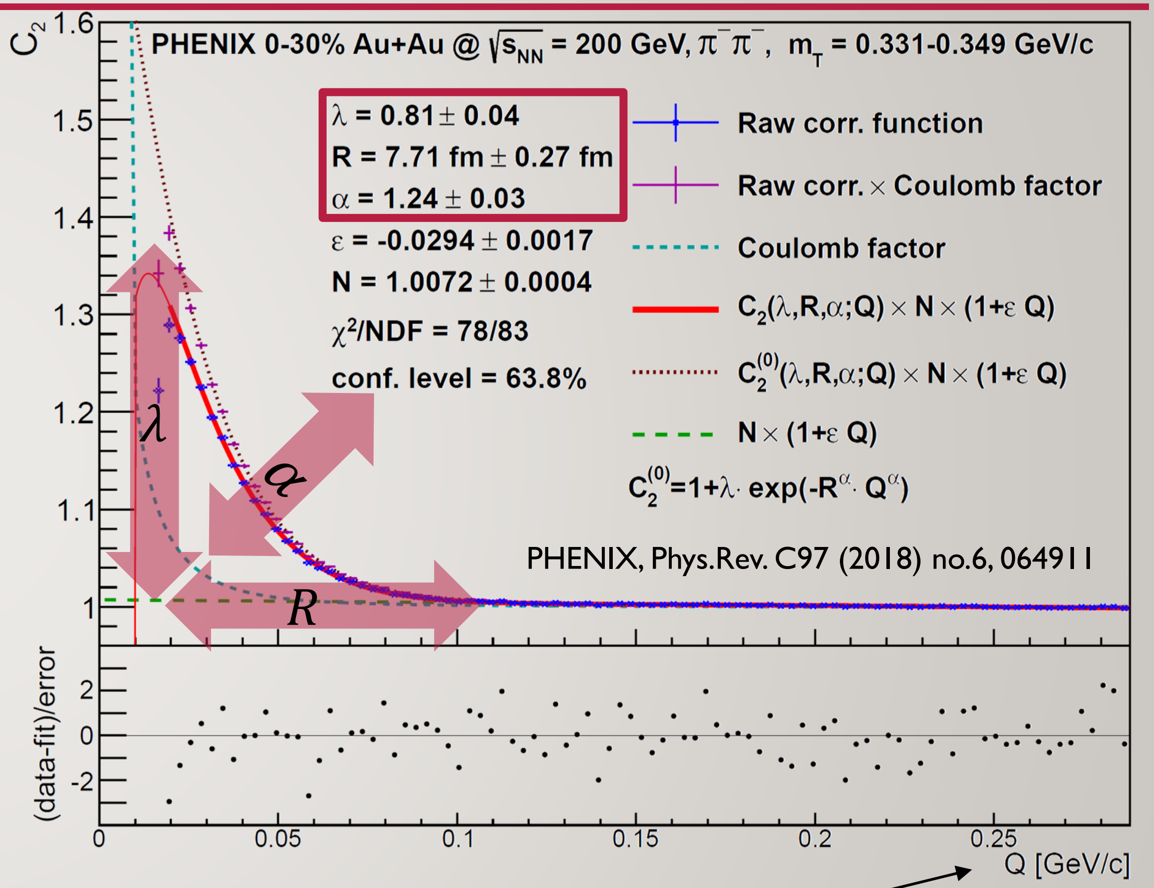
- α vs m_T: Slightly non-monotonic, averaging still possible
- $\langle\alpha\rangle$ vs N_{part}: Slightly non-monotonic, strong decreasing for large N_{part}
- No clear interpretation or understanding of this trend, need theory comparison
- Final data and publication in under collaboration review in PHENIX





EXAMPLE $C_2(Q_{LCMS})$ CORRELATION FUNCTION

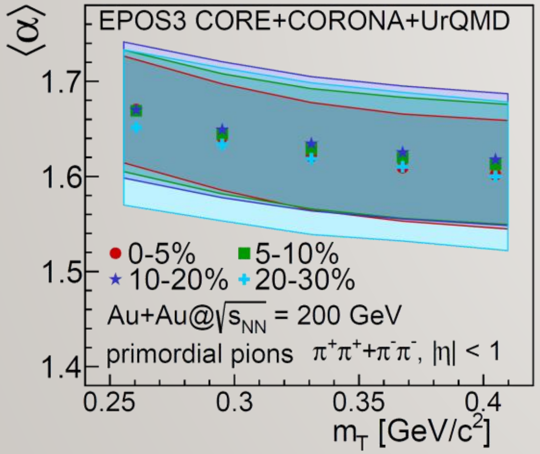
- Correlation function: spherical in LCMS
  - 1D measurement possible
  - Done in several m_T bins
- Fit with calculation based on Lévy distribution
- Only converging fits with good confidence level should be accepted
- Physical parameters:
  R, λ, α measured versus pair m_T

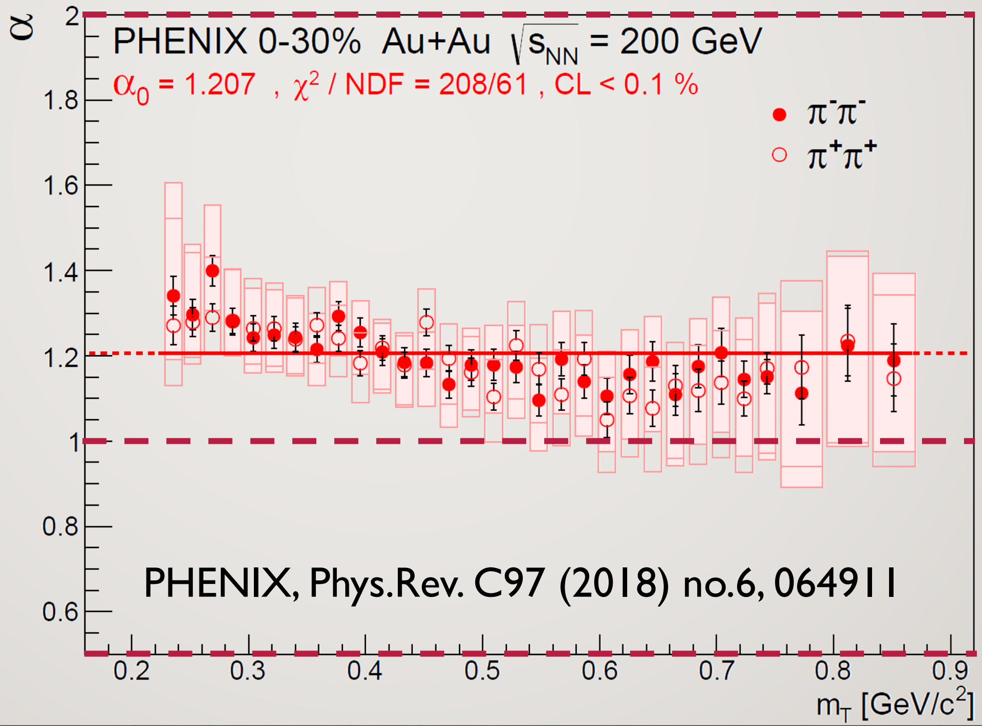


Q: 1D momentum difference in Longitudinally CoMoving System (LCMS)

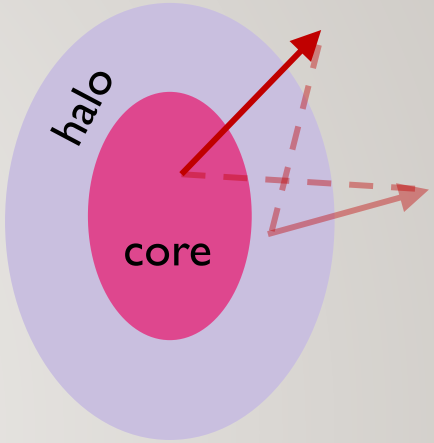
LÉVY EXPONENT α IN 200 GEV AU+AU AT RHIC

- Measured value far from Gaussian (α = 2), inconsistent with expo. (α = 1)
- Far from random field 3D Ising value at CEP (α = 0.5)
- Approximately constant (at least within systematic uncertainties)
- What do models and calculations say?
- EPOS evt-by-evt analysis:



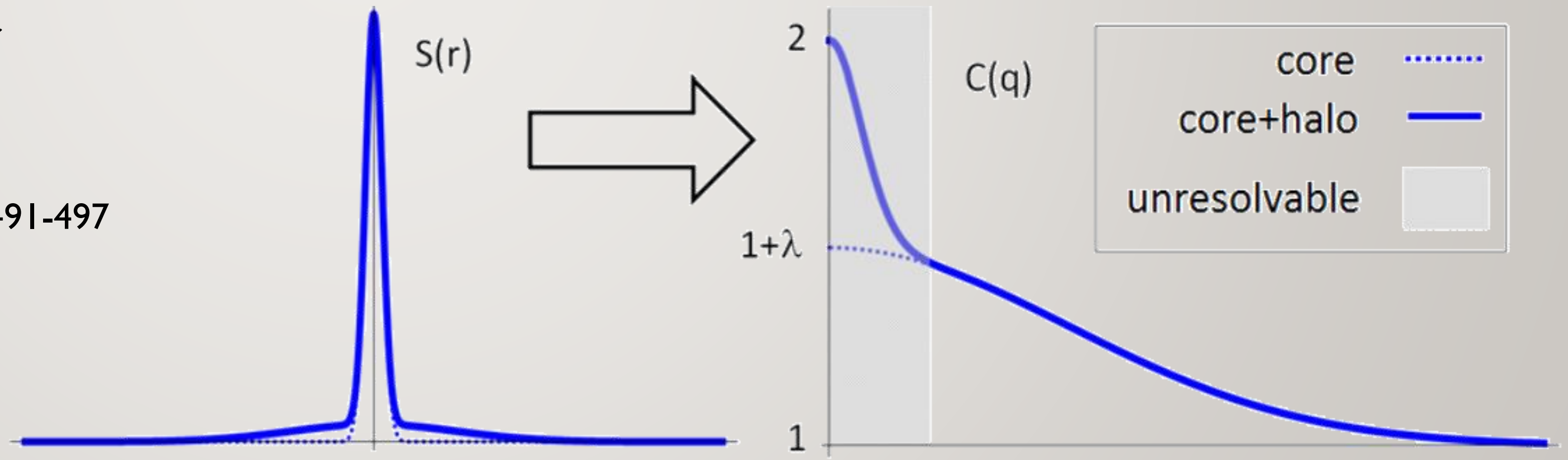


CORRELATION STRENGTH λ: CORE/HALO



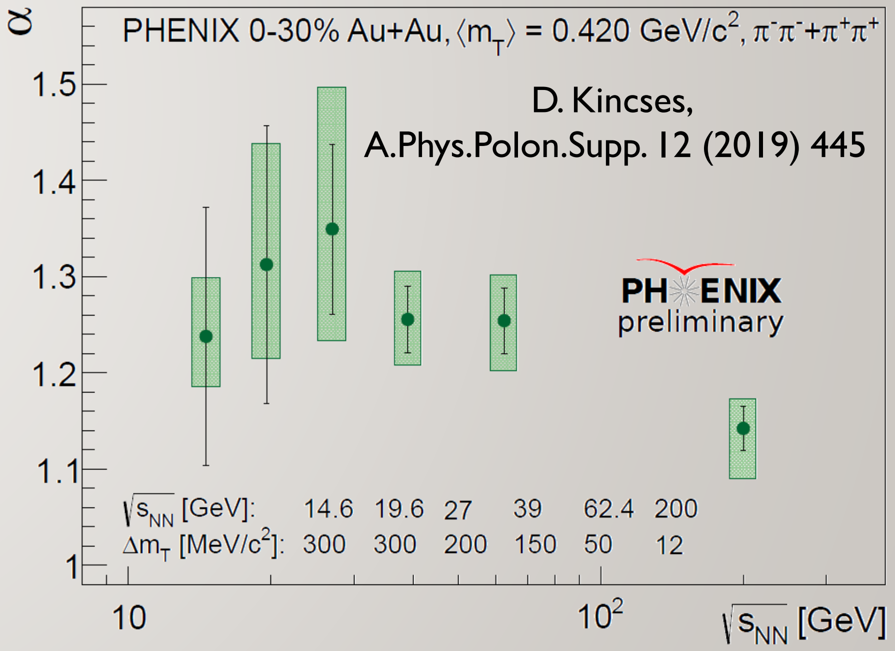
- Two-component core+halo source
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances (≳ 10 fm/c, ω,η,η',K_0^S,...)
  - Unresolvable experimentally
  - Define $f_C = N_{\rm core}/N_{\rm total}$
- True $q \to 0$ limit: $C(0) = 2$
- Apparently $C(q \to 0) \to 1 + \lambda$
- $\lambda(m_{\rm T}) = f_C^2(m_{\rm T})$

Bolz et al, Phys.Rev. D47 (1993) 3860-3870;
Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497



COLLISION ENERGY DEPENDENCE

- $\langle\alpha\rangle$ approximately monotonic versus $\sqrt{s_{NN}}$
  - No clear interpretation or understanding of this trend
  - With widely varying m_T interval (due to statistics) → may influence outcome
  - Important w.r.t. shape averaging interpretation of $\alpha \neq 2$
- Lévy exponent α still far from conjectured CEP limit of 0.5
  - Very much dependent on m_T bin width
  - Working on final results…

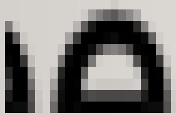


HOLE IN $\lambda(m_T)$: ALL MEASUREMENTS AT RHIC

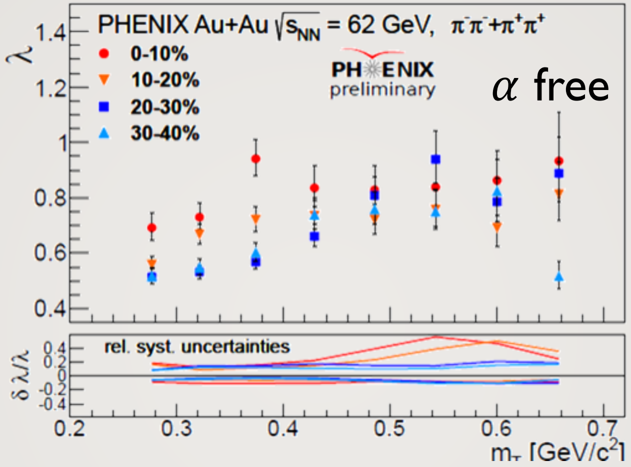
- Hole apparent for $\sqrt{s_{NN}} \geq 39$ GeV, ~independently of centrality

S. Lökös, Universe 4 (2018) 31

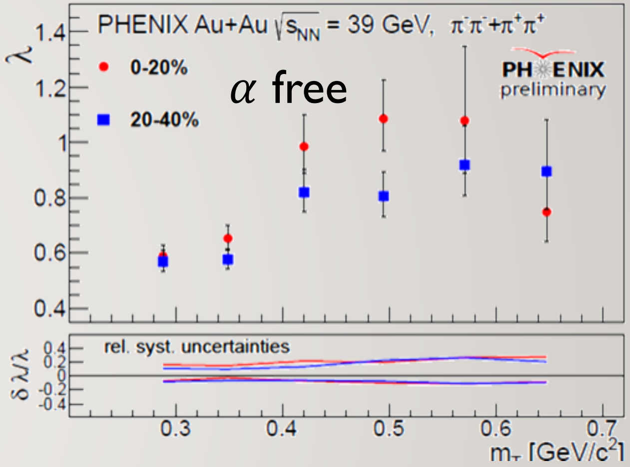
α fixed



D. Kincses, Universe 4 (2018) 11



D. Kincses, Universe 4 (2018) 11



- Due to reduced η' mass?
- Sign for chiral restoration?
- To be cross-checked with photons, dileptons, etc.
- Working on finalized PHENIX results

COHERENCE WITH THREE-PION LÉVY HBT

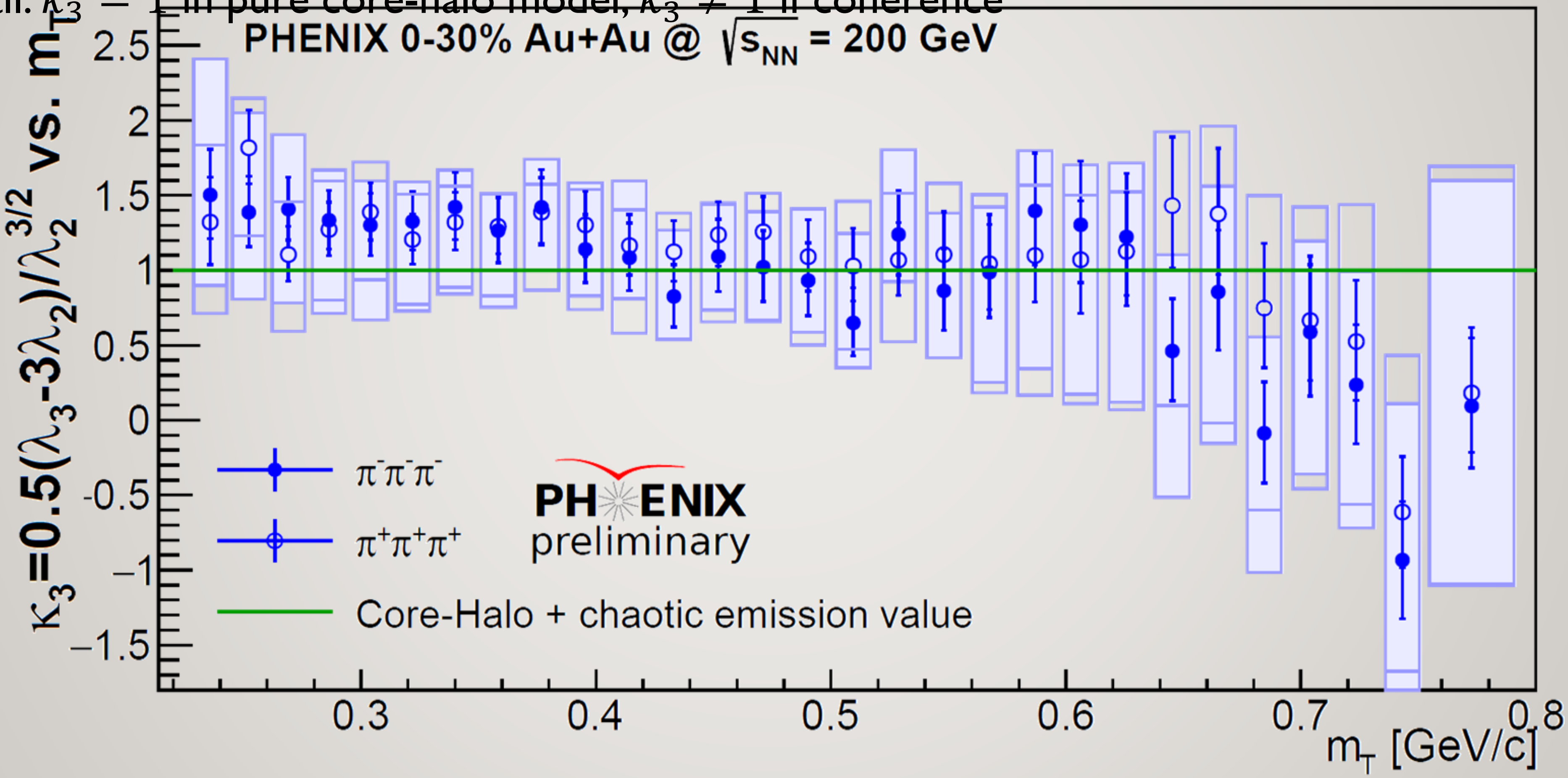
- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$
$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
  - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

TEST OF CORE-HALO MODEL / COHERENCE

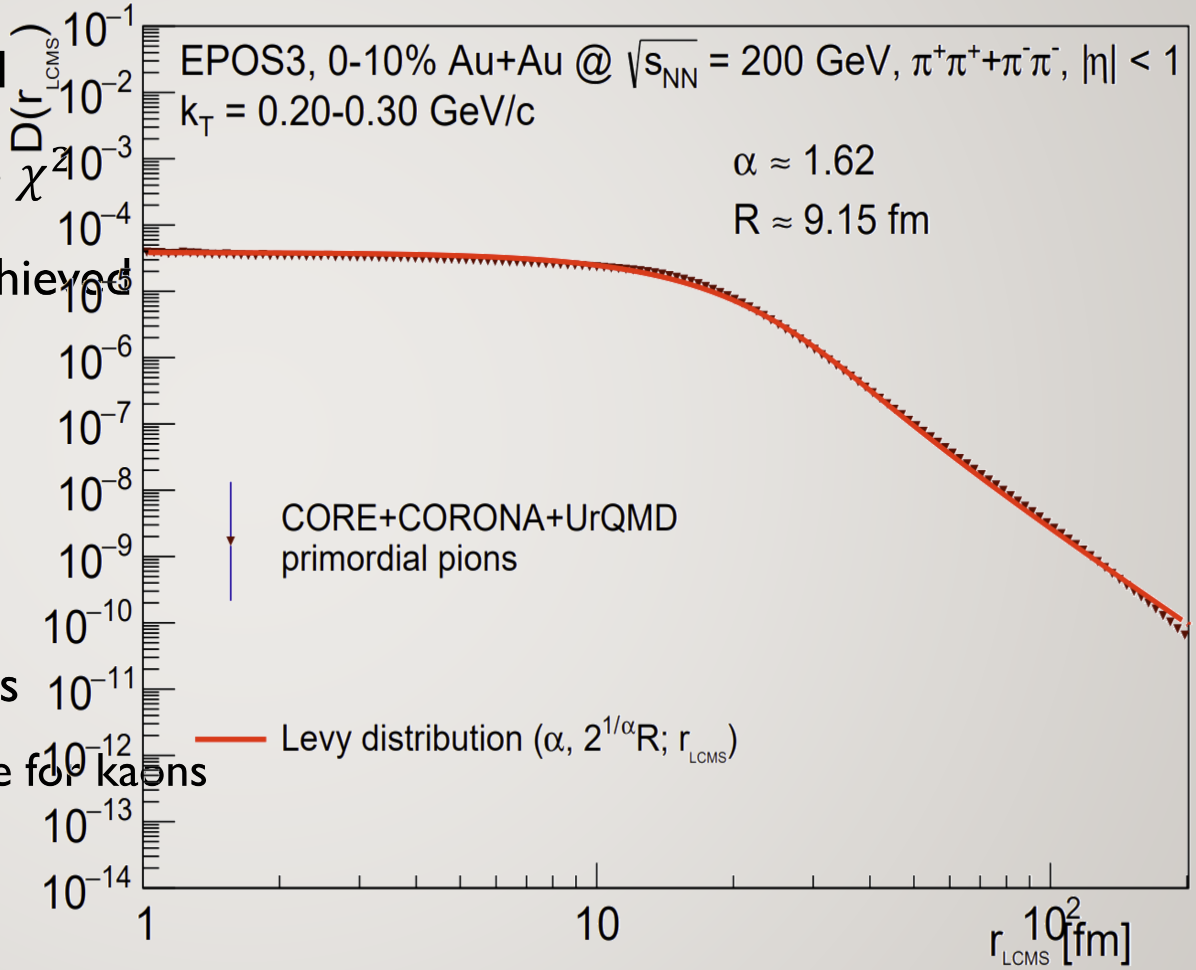
- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



WPCF 19, B. Kurgyis, Phys. Part. Nuclei 51 (2020) 263-266

ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
  - Event averaged:
    $\alpha \approx 1.62, R \approx 9.15$ fm
  - Event-by-event:
    $\alpha \approx 1.66, R \approx 8.96$ fm
- More reasonable approach for kaons
  - No event-by-event analysis possible for kaons



SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, …):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2$$

$$\cong 1 + \left|\int S(r,K) e^{iqr} dr\right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r,K) = \int S\left(\rho+\frac{r}{2}, K\right) S\left(\rho-\frac{r}{2}, K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K) e^{iqr} dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]
Kincses, Nagy, Csanad Phys. Rev. C102, 064912 (2020) [arXiv:1912.01381]

INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$:

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K) e^{iqr} dr$$

- If there is interaction:

$$\Psi_2^{(0)}(r) \to \Psi_2^{(\text{int})}(r_1, r_2)$$

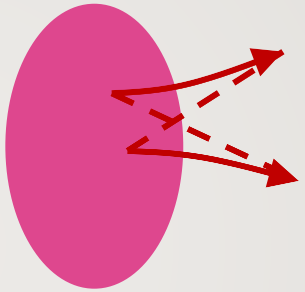
- For Coulomb:

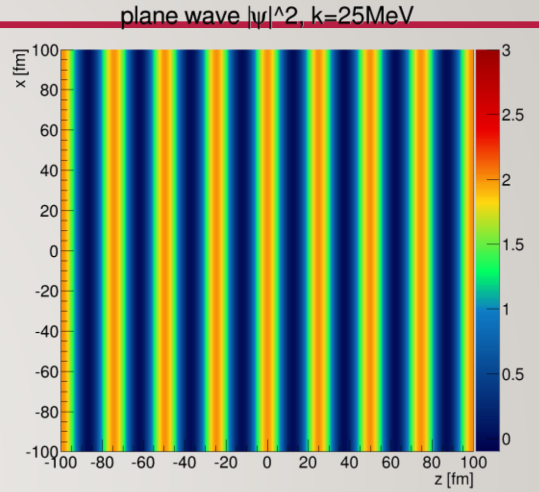
$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot (\text{complicated hypergeometric expression})$$

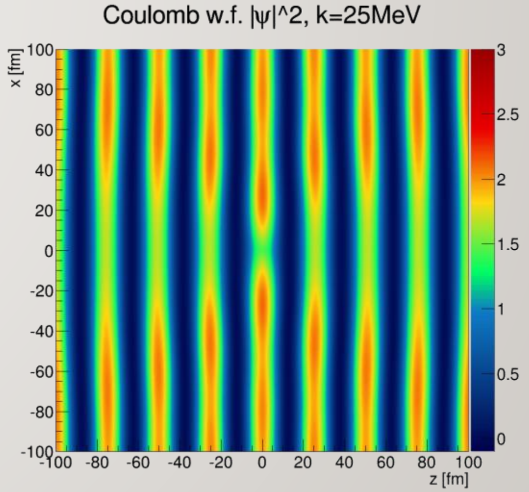
- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\text{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- Complication: need for integrating power-law tails
- In this analyis: assuming spherical source
- Parametrization possible Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238





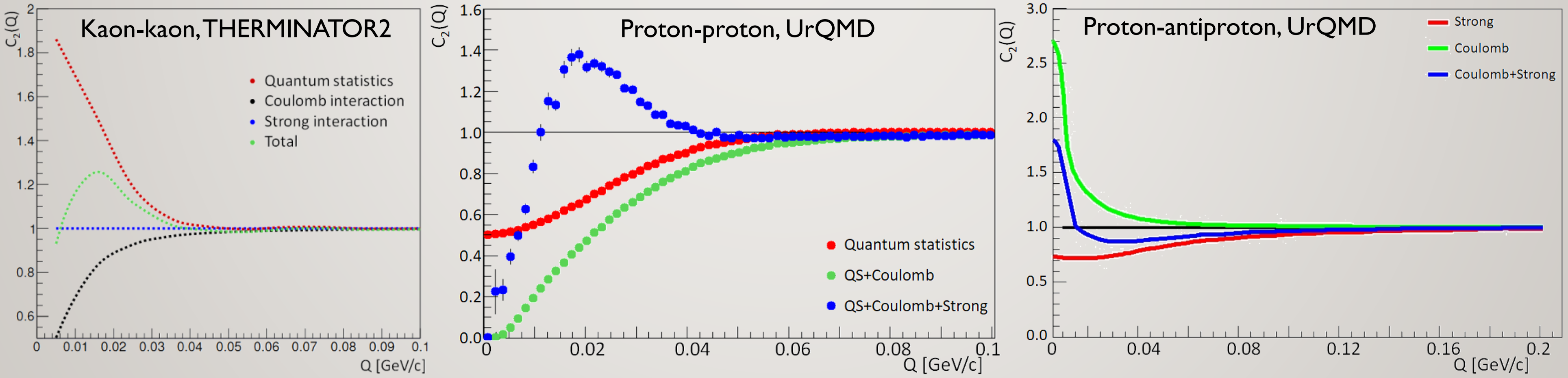


ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr$$

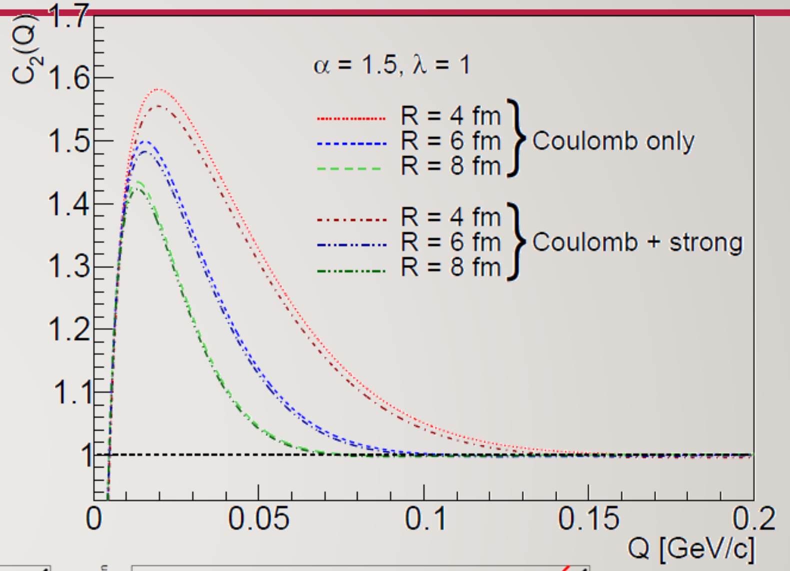
- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function

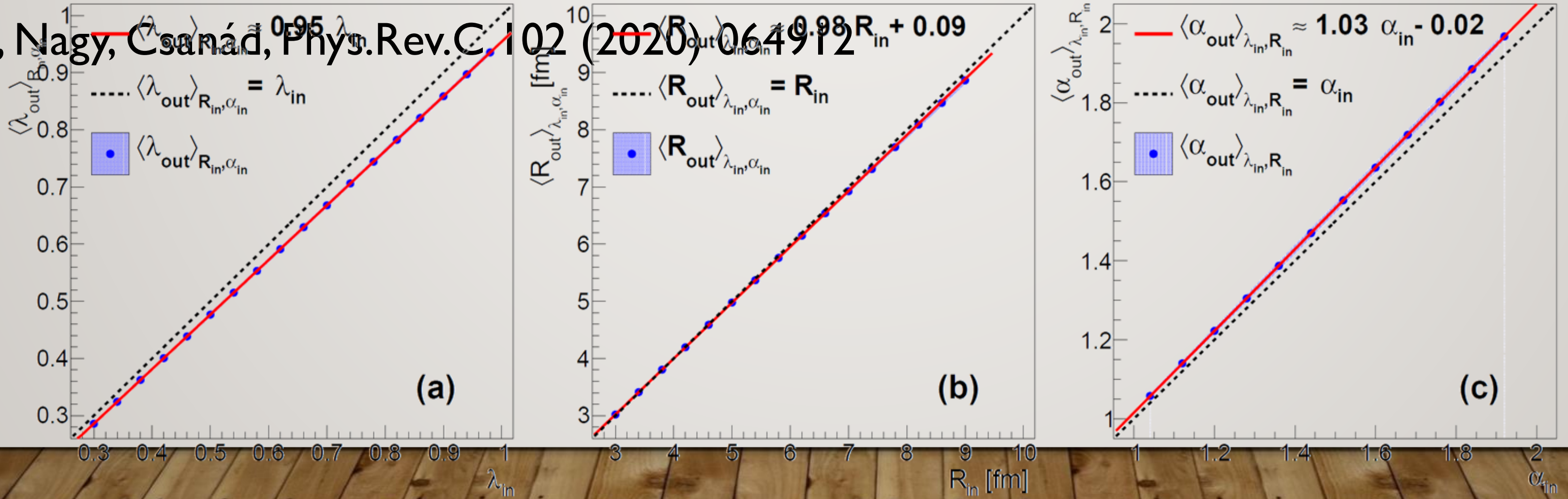


From e.g. H. Zbroszczyk's talk at Zimányi School 2019

STRONG INTERACTION FOR PION PAIRS

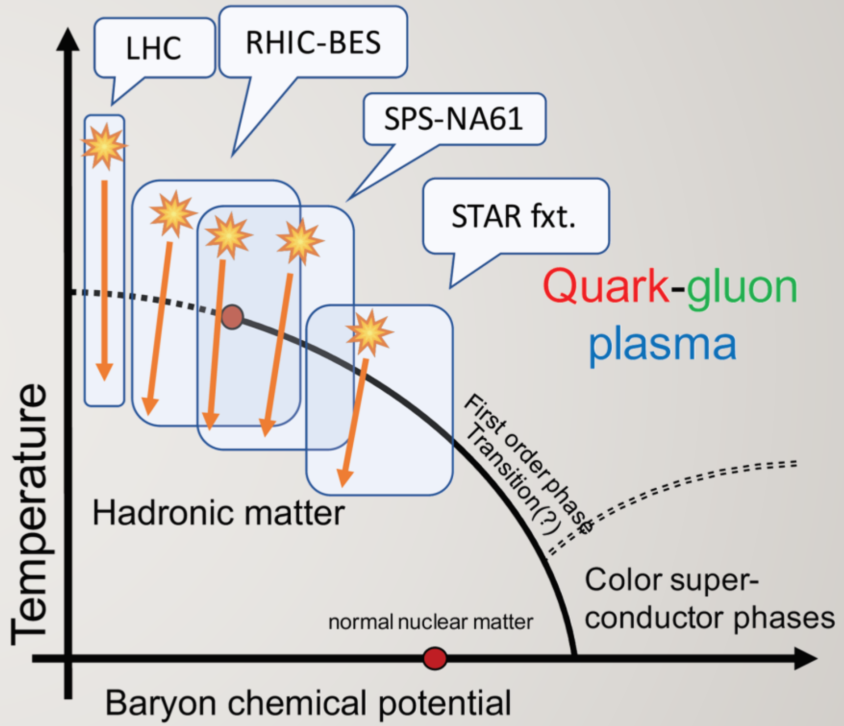
- Additional potential appearing
- Possible handling: strong phase shift,
  Modify s-wave component in wave func.
  R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α
  Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 064912





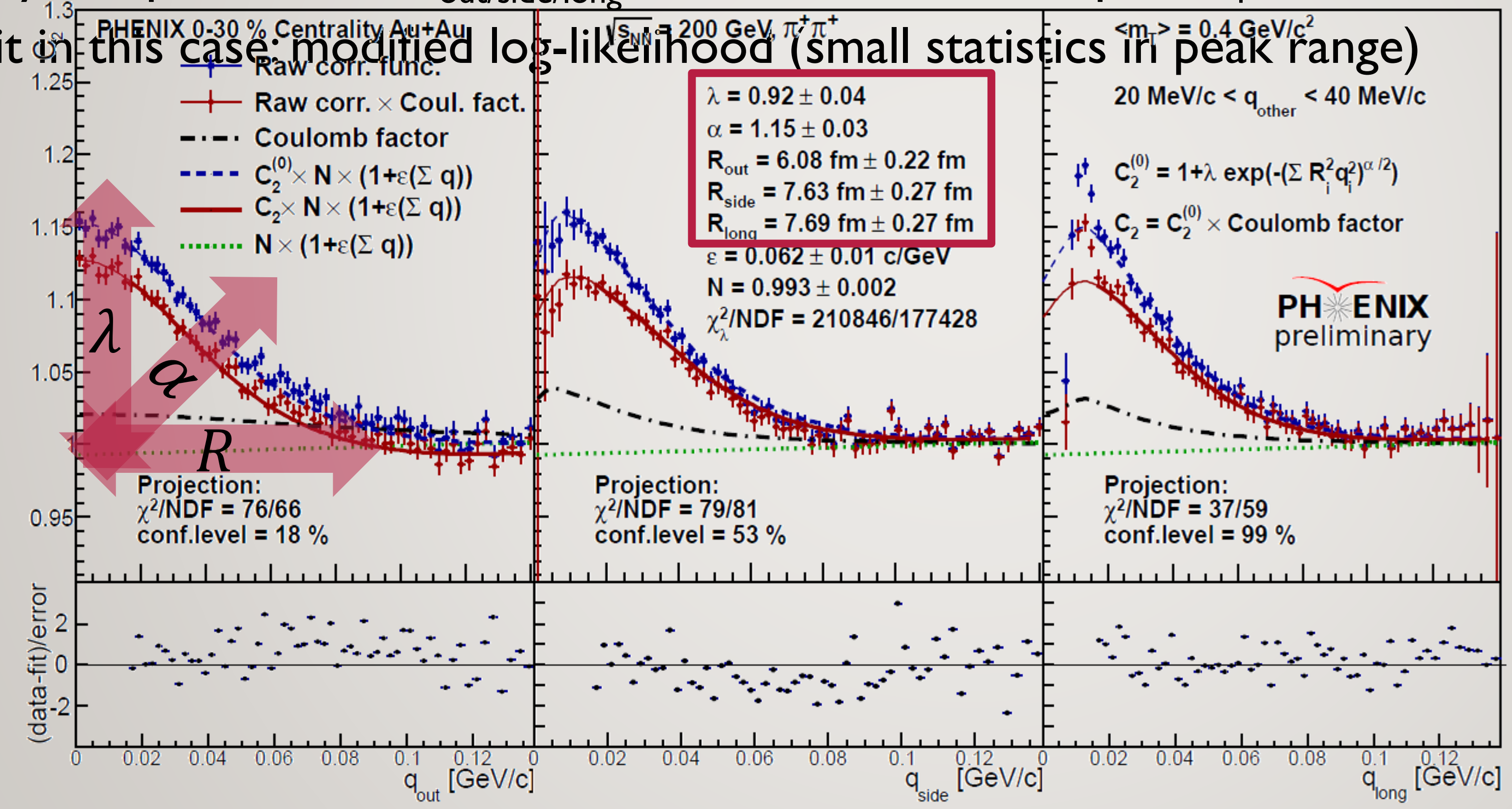
HBT MEASUREMENTS AND THE PHASE DIAGRAM

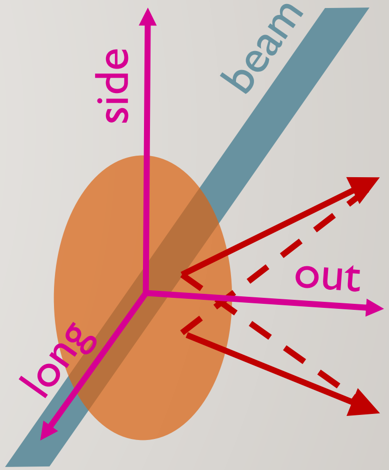
- LHC: measurement at CMS
  - 2-5 ATeV energy, p+p & Pb+Pb
- RHIC: measurement at PHENIX+STAR
  - 10-200 AGeV energy, Au+Au
- SPS: measurement at NA61
  - 17 AGeV energy, Be+Be
- Phase diagram can be investigated



A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

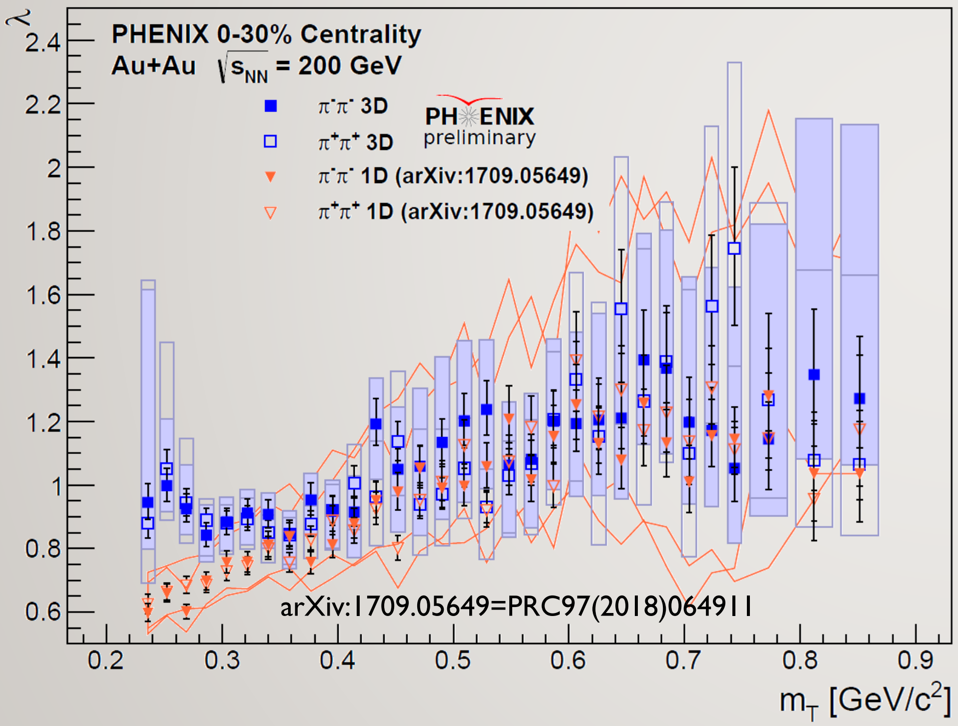
- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out/side/long}$ λ, α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)

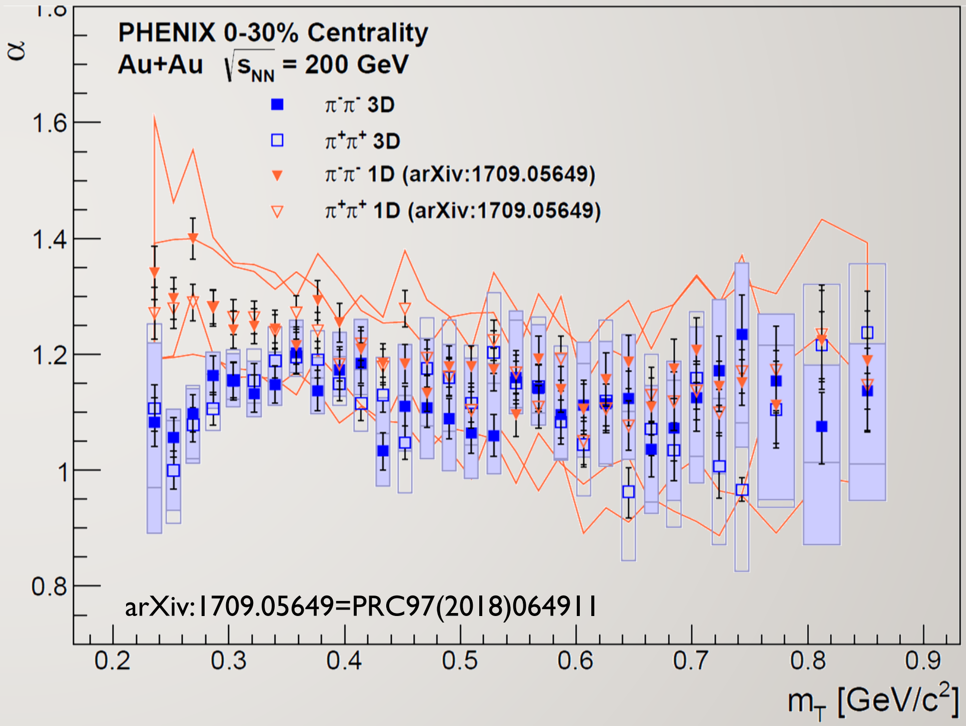




3D VERSUS 1D: STRENGTH λ AND SHAPE α

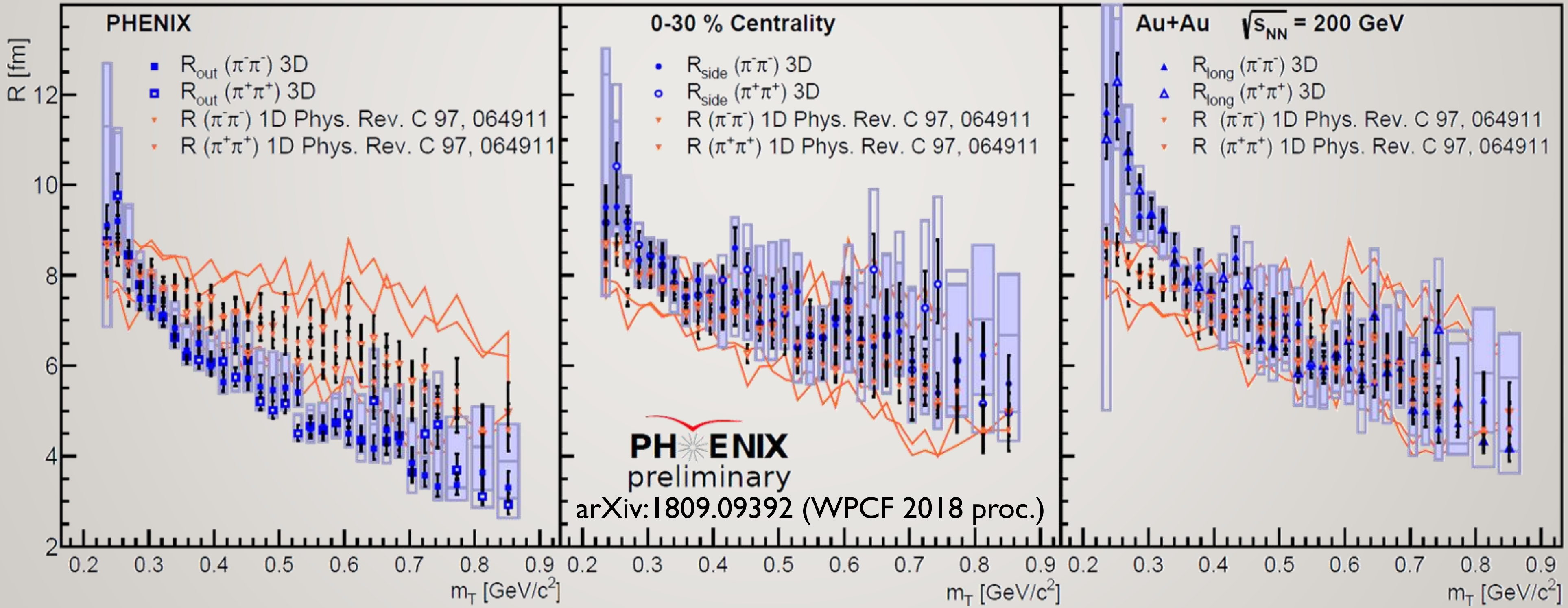
- Compatible with 1D (Q_{LCMS}) measurement of PRC97(2018)064911
- Small discrepancy at small mT: due to large Rlong at small mT?





LÉVY SCALES IN 3D

- Compatibility with 1D Lévy analysis
- Similar decreasing trend as Gaussian HBT radii, but hydro prediction based on Gaussian source
- Asymmetric source for small m_T, validity of Coulomb-approximation?



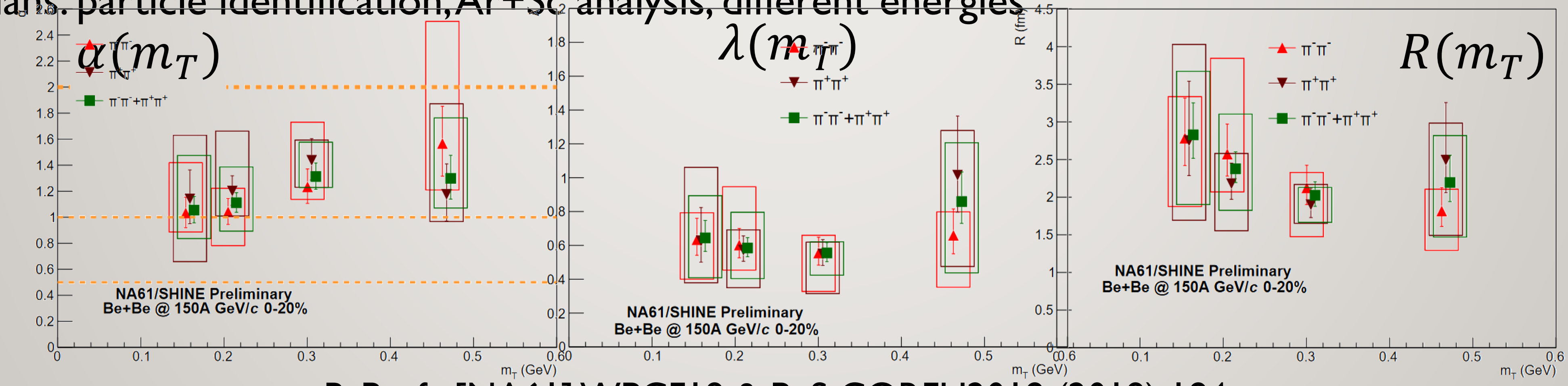
arXiv:1809.09392 (WPCF 2018 proc.)

OPEN QUESTIONS

- Collision energy and centrality dependence of Lévy parameters?
  - Non-monotonicity in $\alpha\left(\sqrt{s_{NN}}\right)$ or $\alpha(\text{centrality})$?
  - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η'?
- Reason for the appearance of Lévy distributions for pions?
  - What is the Lévy exponent for kaons?
  - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
  - Does m_T scaling hold for Lévy scale R?
- Correlation strength versus core-halo picture: are there other effects?
  - Three-particle correlations may show if coherence or other effects play a role
  - Other effects may also play a role (finite meson sizes, random field phase shift, etc)

RESULTS AT NA61/SHINE

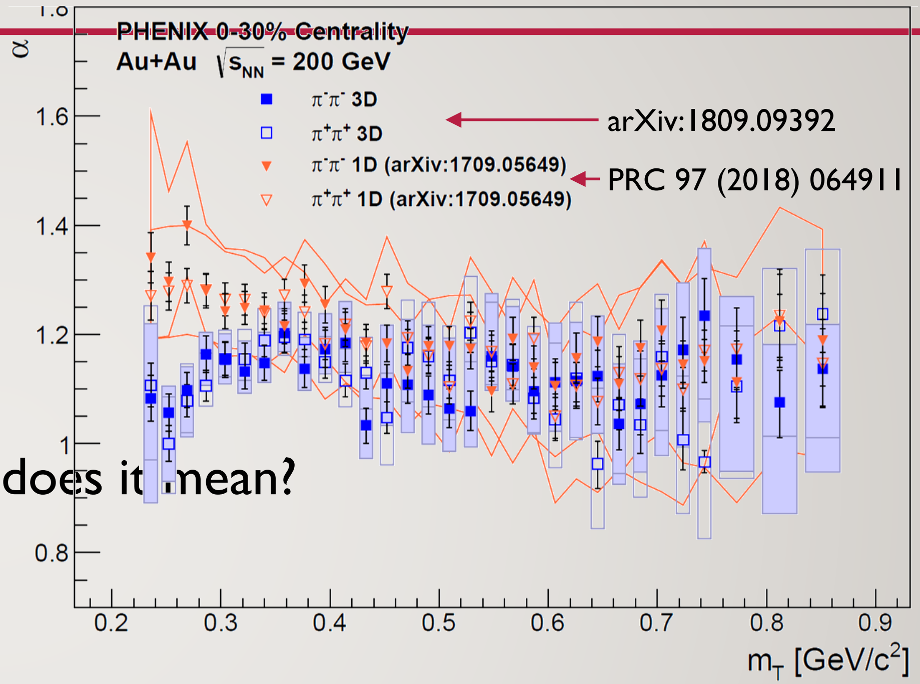
- Be+Be collisions at 150 AGeV beam momentum (17.3 AGeV in c.m.s.)
- Lévy fits describe correlation functions
  - Shape parameter α: far from Gaussian and CEP conjecture
  - Strength parameter λ: nearly constant as previous SPS results, unlike RHIC
  - Spatial scale R: weakly decreasing trend → hydro
- Plans: particle identification, Ar+Sc analysis, different energies

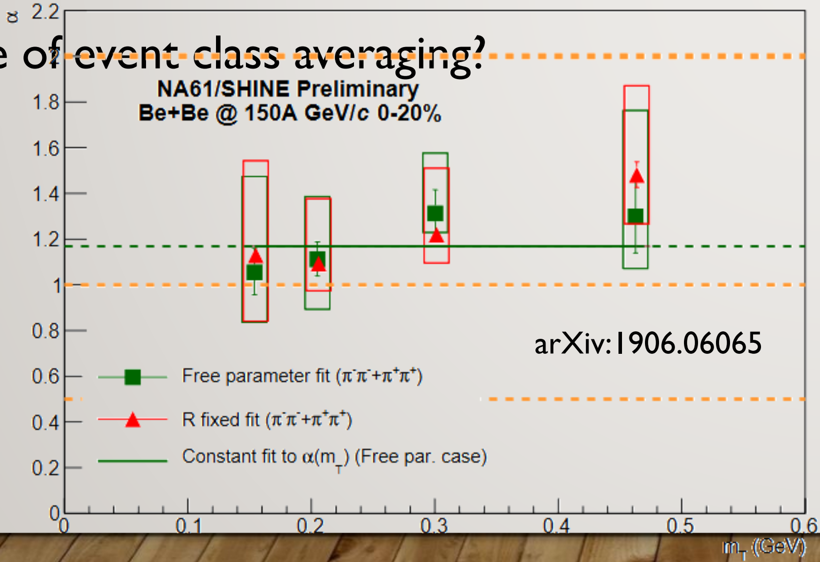


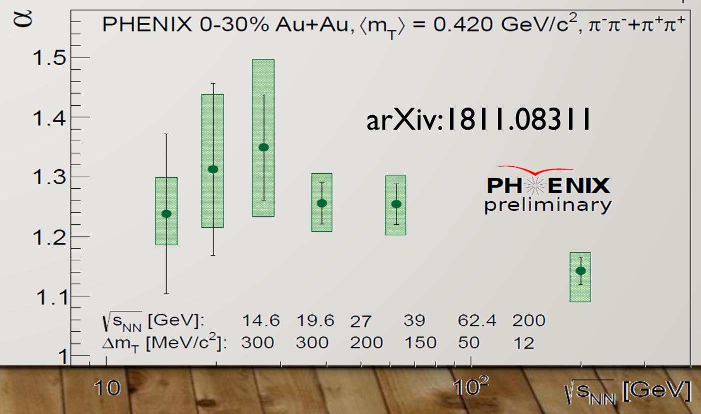
B. Porfy [NA61] WPCF19 & PoS CORFU2018 (2019) 184

LÉVY HBT MEASUREMENTS

- Many experimental results
  - PHENIX Au+Au: $\alpha \approx 1 - 1.5$
  - STAR Au+Au: ongoing
  - NA61 Be+Be: $\alpha \approx 1 - 1.5$
  - CMS Pb+Pb: $\alpha = 1$ fixed
- Where does this Lévy shape come from? What does it mean?
  - Role of event class averaging?







THE EPOS MODEL

- **E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons ladders, **O**ff-shell remnants, and **S**plitting of parton ladders
  - K. Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
- Based on Monte-Carlo simulation
- Theoretical framework: parton-based Gribov-Regge theory (PBGRT)
- Three main parts of the model:
  - Core-Corona division (based on dE/dx of string segments)
  - Hydrodynamical evolution (vHLLE 3D+1 viscous hydro)
  - Hadronic cascades (UrQMD afterburner)
- Effects/components to be turned on or off (on top of Core):
  - Corona
  - Rescattering
  - Decays

TWO-PARTICLE SPATIAL CORRELATIONS

- Object to be investigated: two-particle source

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

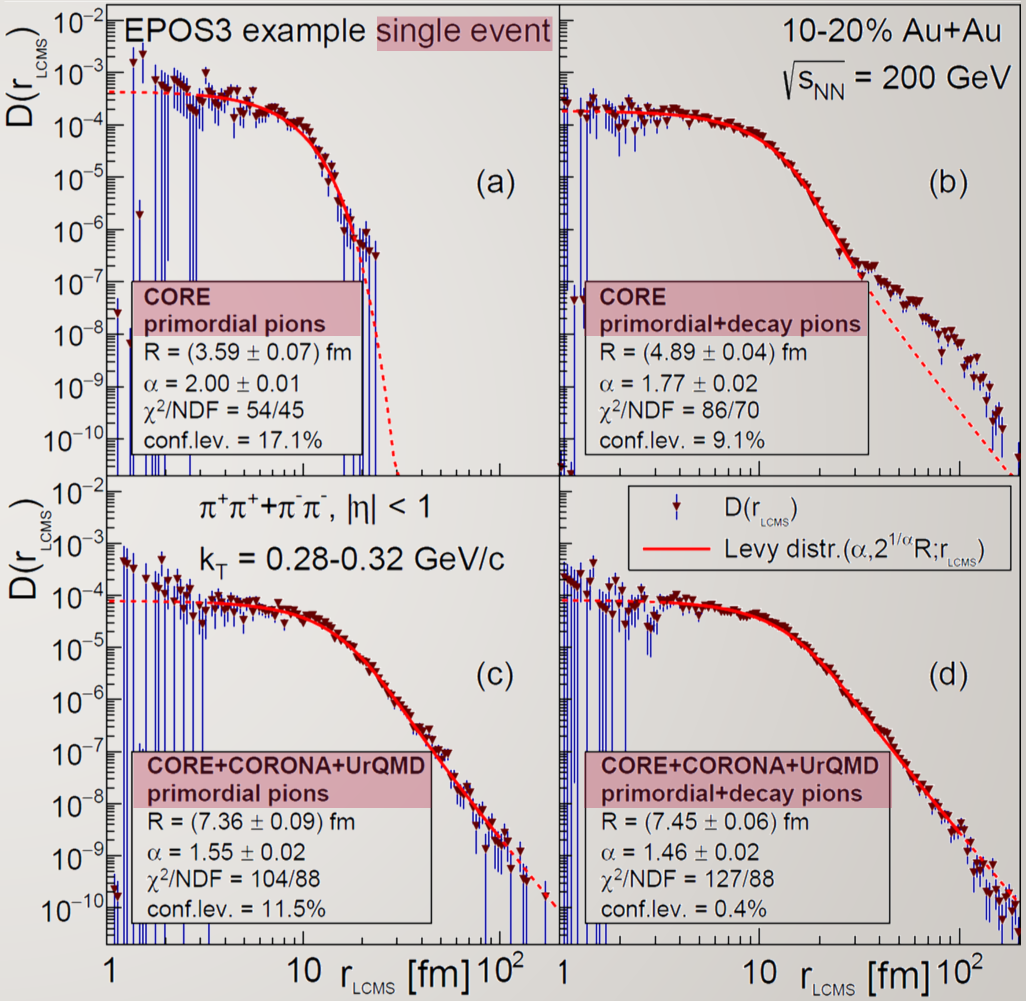
- Experimental results measure power-law tails, Lévy shapes
  - Measure momentum-space correlations, reconstruct $D(r)$ or fit its parameters
- Why do these Lévy shapes appear?
  - What physics does contribute to it? Rescattering, decays?
  - What role does event averaging have in it?
    Cimerman, Plumberg, Tomasik, Phys.Part.Nucl. 51 (2020) 282, PoS ICHEP2020 538
  - What do specific α values mean?
- Event generator models (like EPOS) – direct access to pair-source!
  - Phenomenological investigations of $D(r)$ possible
  - Effects can be turned off or on, investigated separately

VARIOUS PARTICLE SETS COMPARED

- CORE, primordial pions
  - Gaussian source
- CORE + decays
  - power-law structures
- CORE+CORONA+UrQMD
  - Lévy-shape
- CORE+CORONA+UrQMD+decays
  - Lévy-shape
- Important: Lévy appears in all single events!
- Source size versus mT: hydro scaling apparent

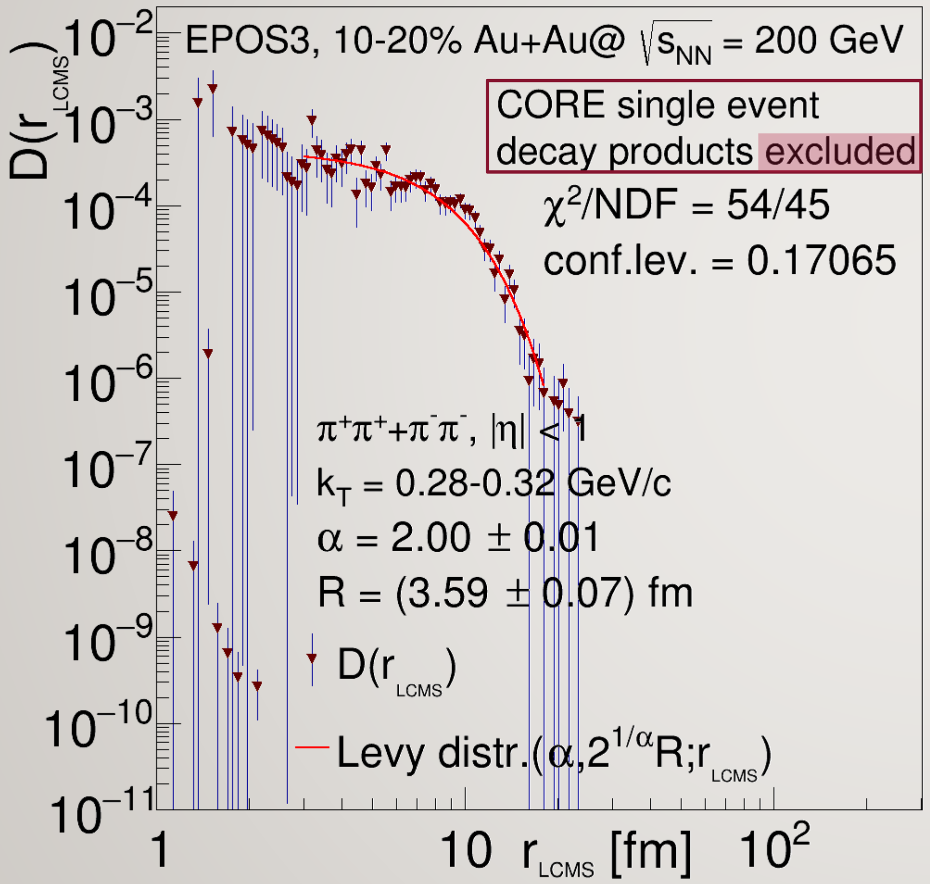
200 GeV AuAu analysis: Kincses et al., Entropy 24 (2022) 308

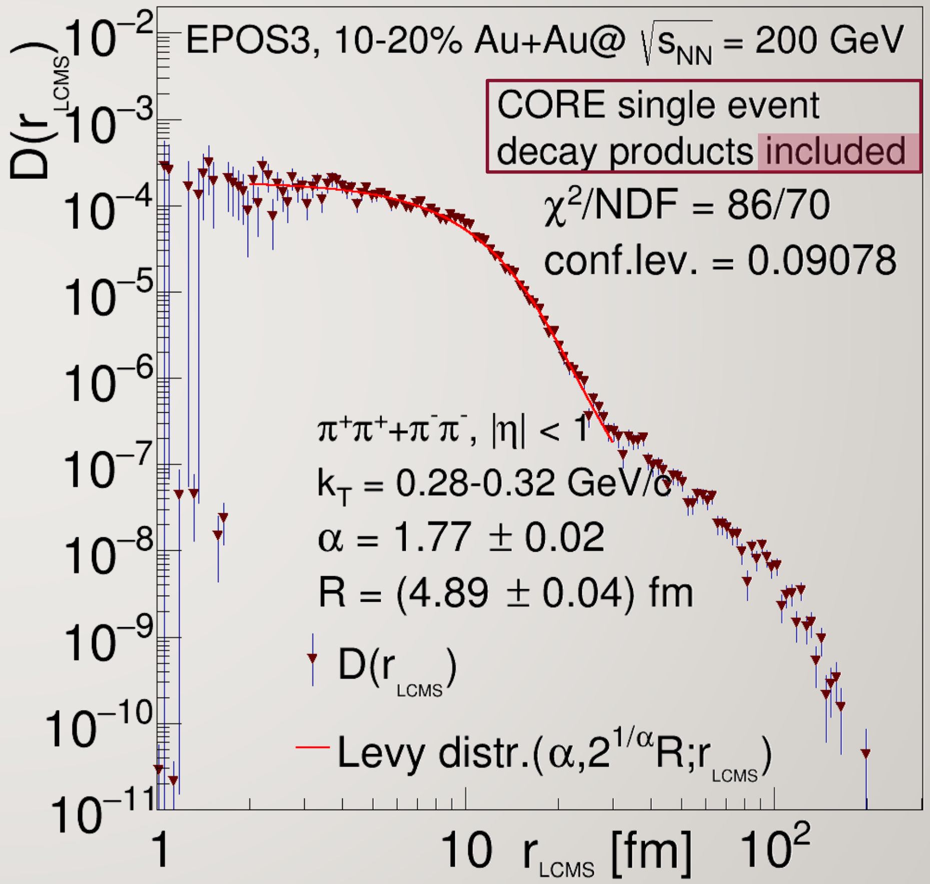
2.76 TeV PbPb analysis: Kincses et al., arXiv:2212.02980



EXAMPLE SINGLE EVENT, CORE ONLY

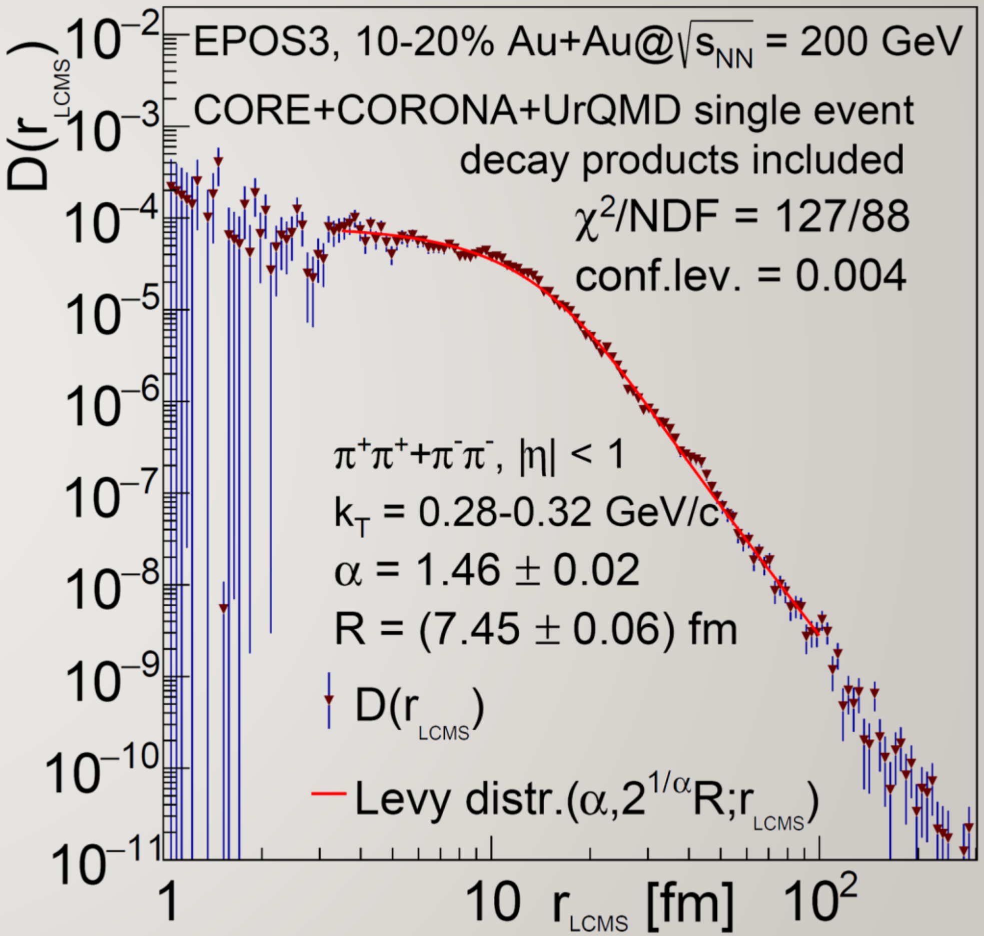
- Gaussian shape without decays, additional structure with decays





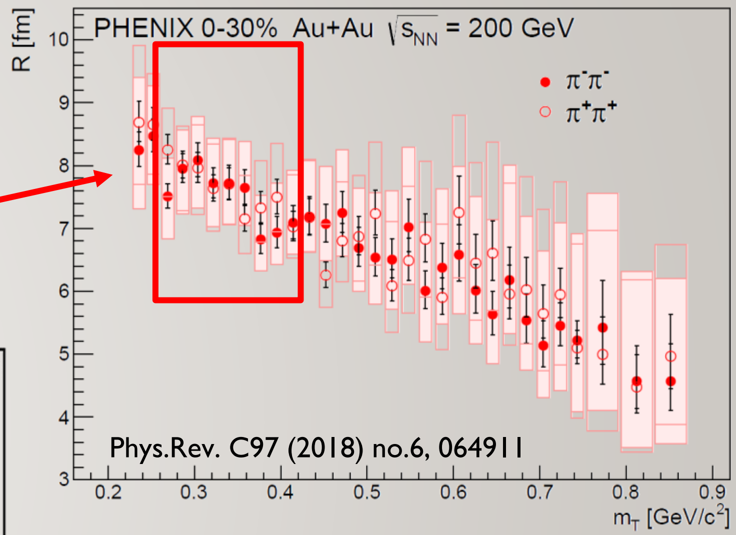
EXAMPLE EVENT, CORE+CORONA+URQMD

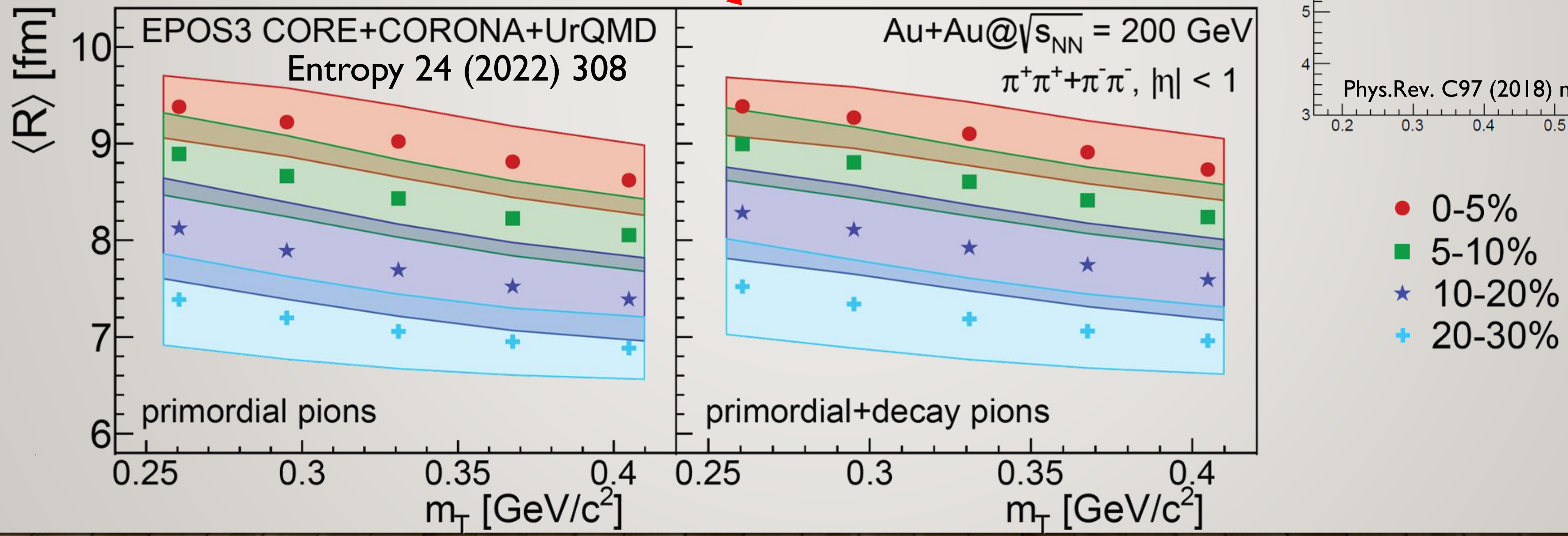
- Investigating $D(r)$ event-by-event
- Lévy-fits provide good description (2-100 fm range)
- Repeat such fits for thousands of events
- Extract α, R distribution



AVERAGE LÉVY SCALE R VS TRANSVERSE MASS

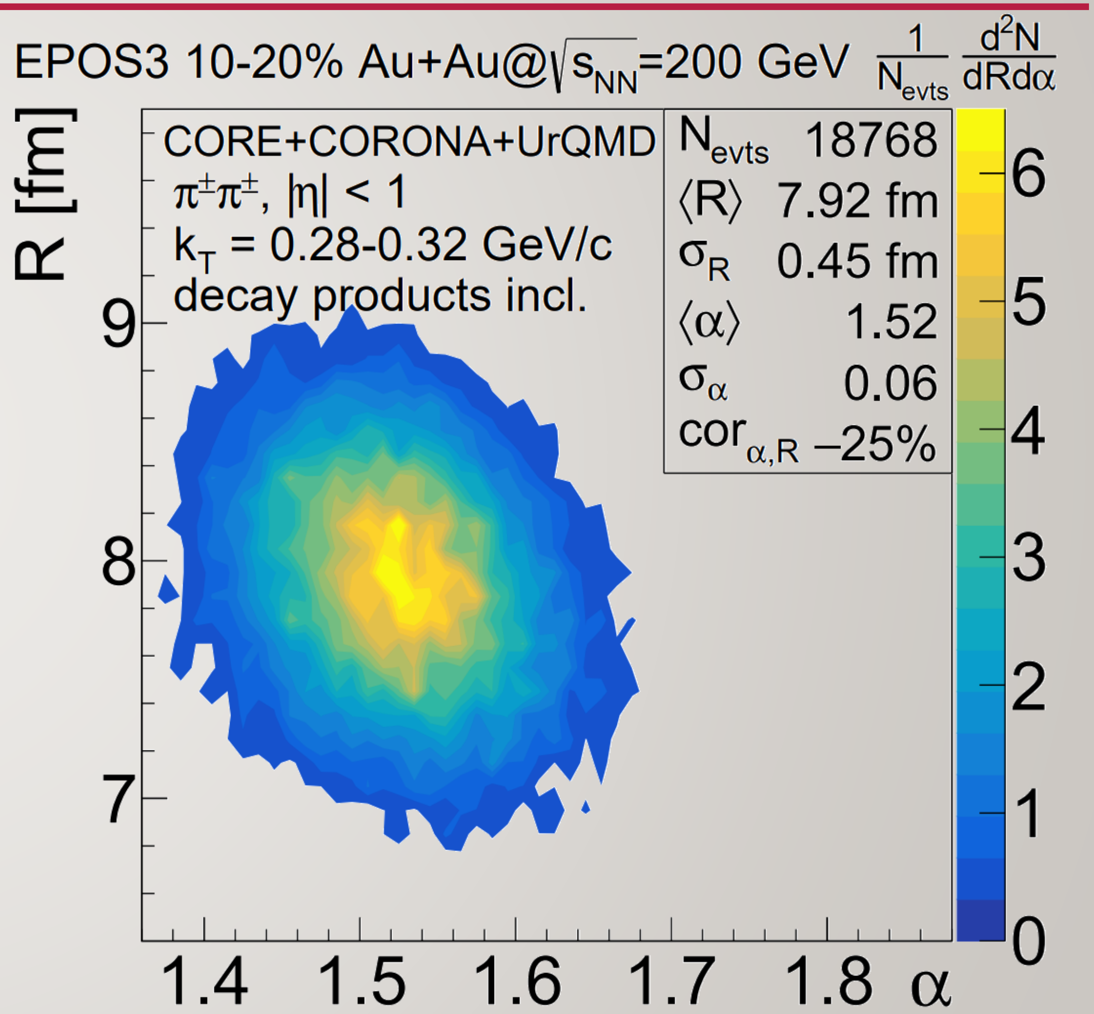
- $\langle R \rangle$ as a function of m_T and centrality
  - Clear dependence on both
  - Distribution width displayed as uncertainty band
- Trends, magnitudes like data
- With decays: slightly higher $\langle R \rangle$ values





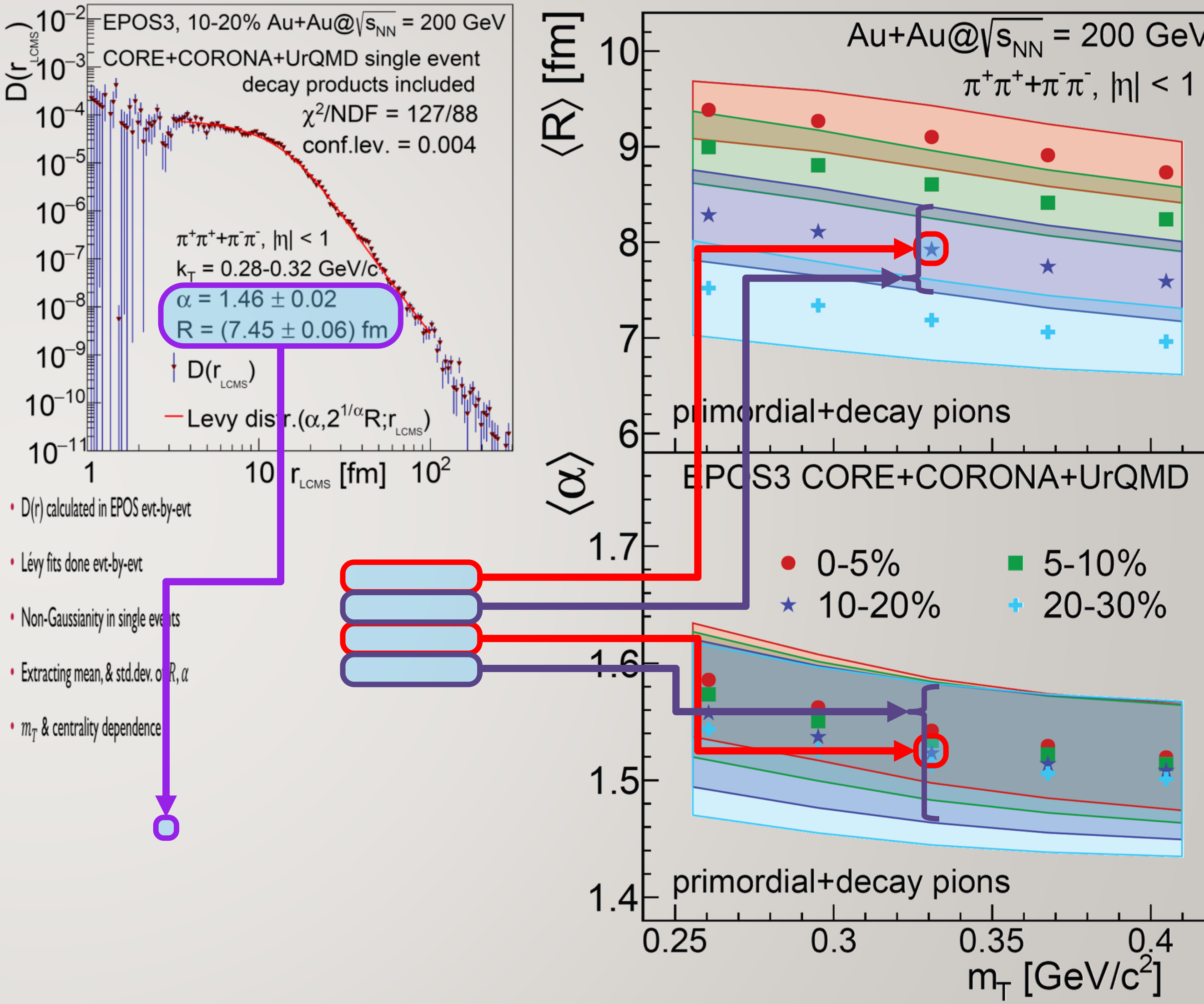
DISTRIBUTION OF α, R PARAMETERS

- Normal distribution of α, R for given centrality & k_T
- Extract mean and std.dev,
- Investigate centrality & k_T dependence
- kT dependence investigated around the peak of the pair-kT distr. to have adequate stat.

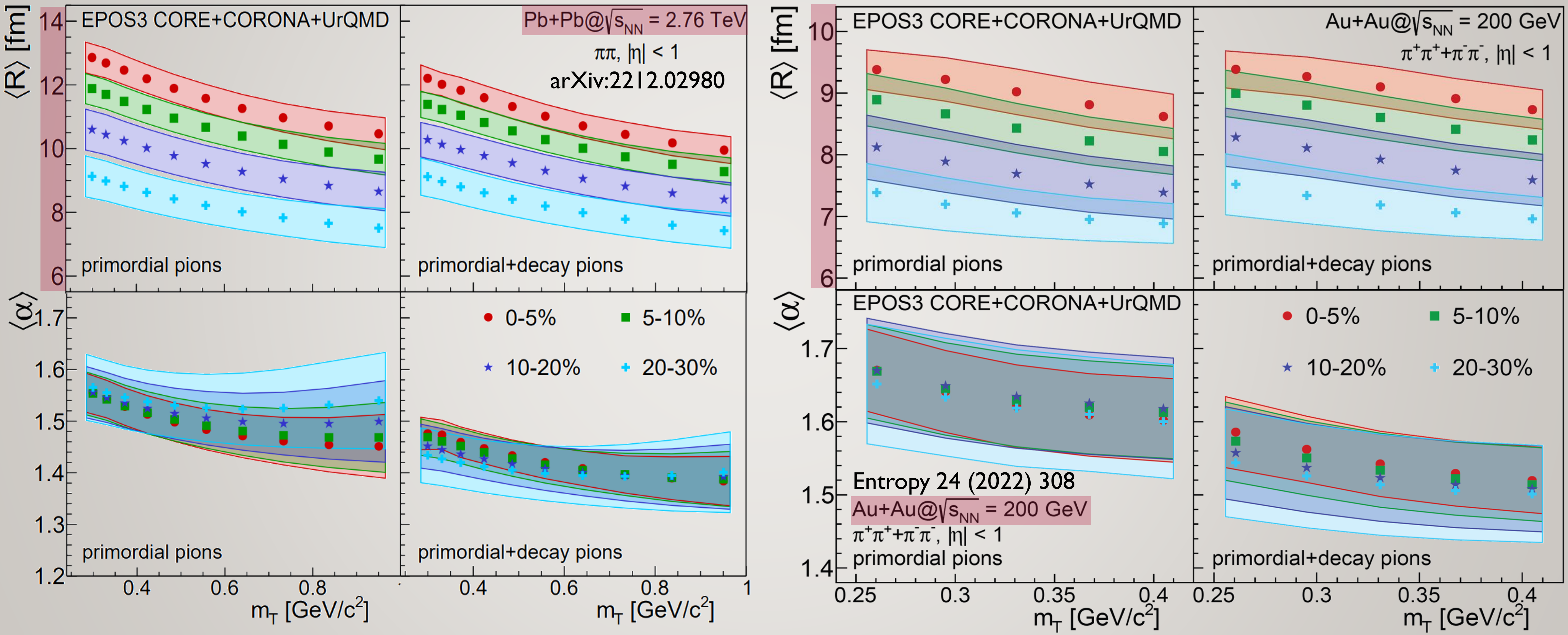


EPOS SUMMARY

- D(r) calculated in EPOS evt-by-evt
- Lévy fits done evt-by-evt
- Non-Gaussianity in single events
- Extracting mean, & std.dev. of R, α
- m_T & centrality dependence



EPOS @ LHC VS RHIC

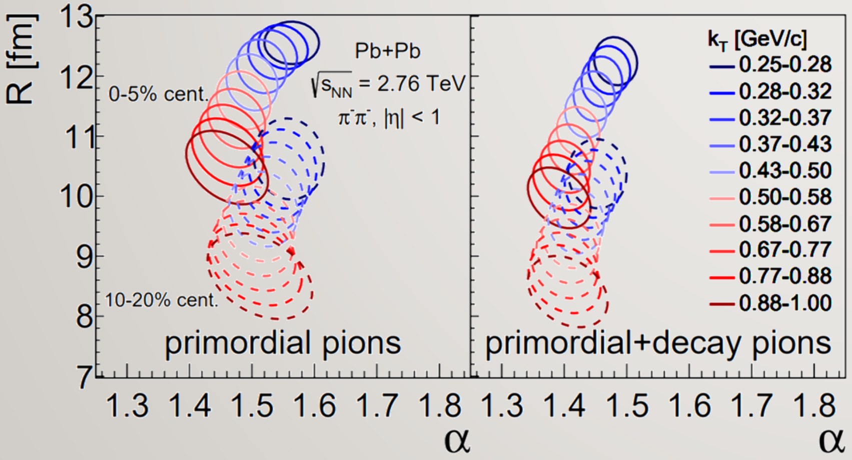


EPOS IN 2.76 TEV PBPB, EVENT-BY-EVENT

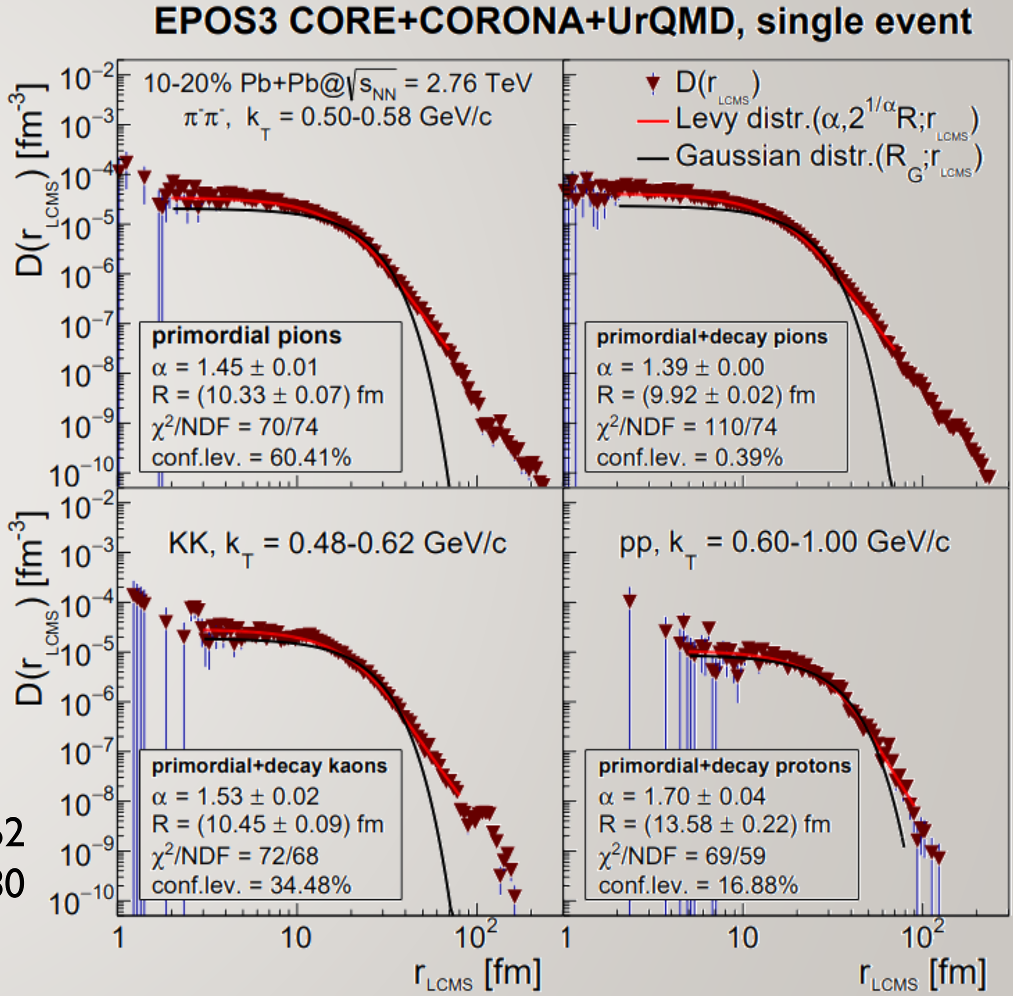
- Pion and kaon pair distributions calculated in individual EPOS events

$$D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$$

- Lévy source parameters determined for 800k events separately
  - Fit limits: from 2-5 fm to 70-100 fm
  - Criterion for acceptance: confidence level > 0.1%
  - Strongly non-Gaussian shapes observed
- Separately for various centrality and k_T classes, primordial or decays



arXiv:2201.07962
arXiv:2212.02980



A CROSS-CHECK: THREE-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\mathrm{core}}/N_{\mathrm{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

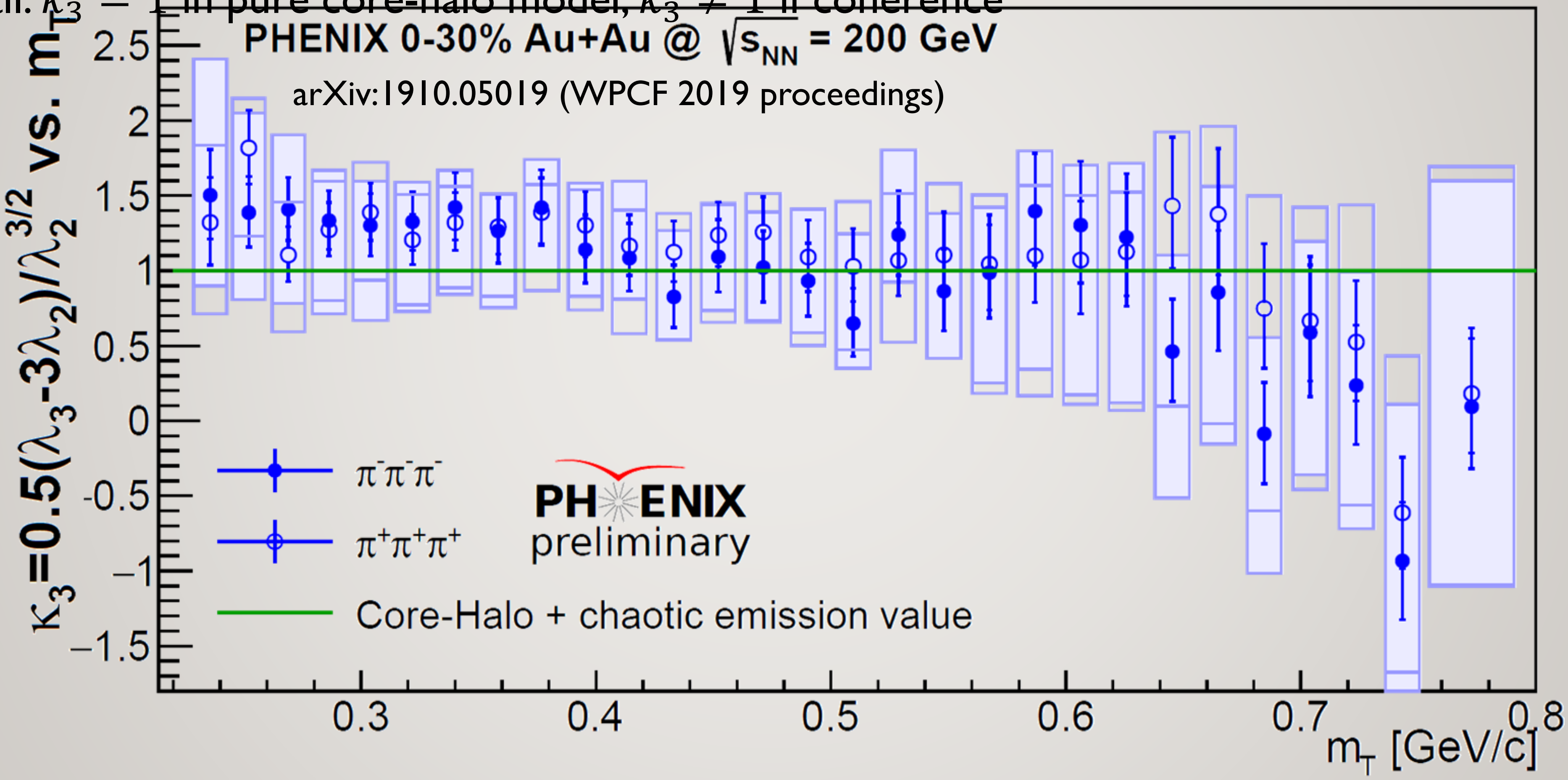
$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
  - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

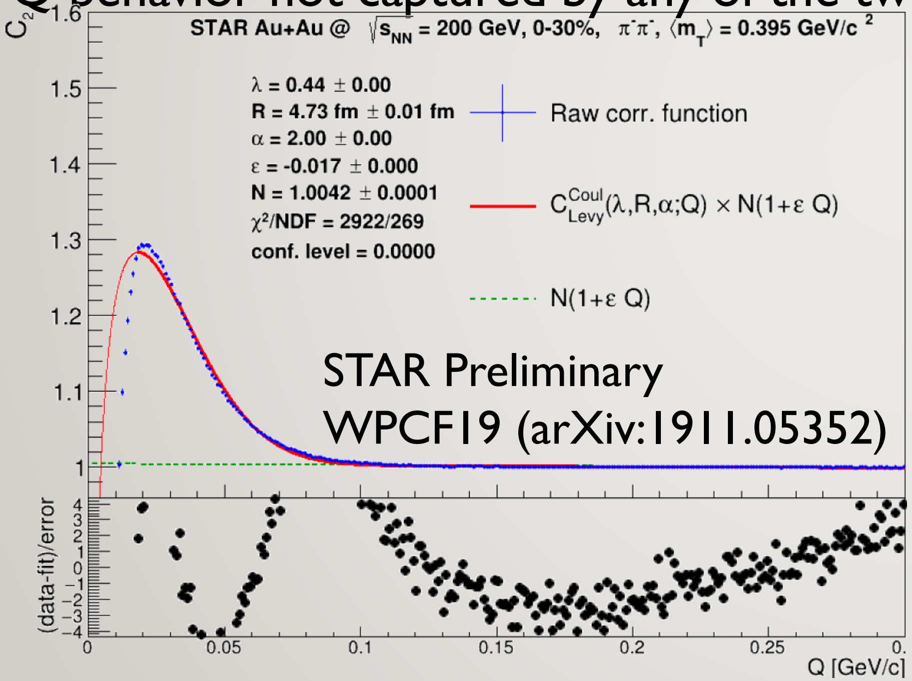
TEST OF CORE-HALO MODEL / COHERENCE

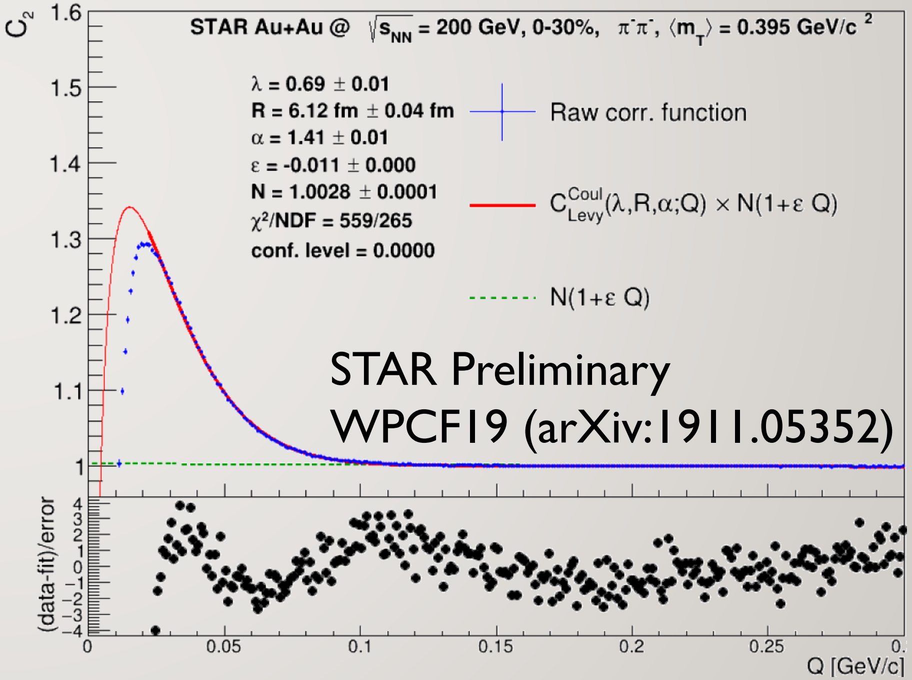
- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



SHAPE ANALYSIS AT STAR

- Gaussian fit: unacceptable description
- Levy fit somewhat better, but still additional effects present
- Low Q behavior not captured by any of the two

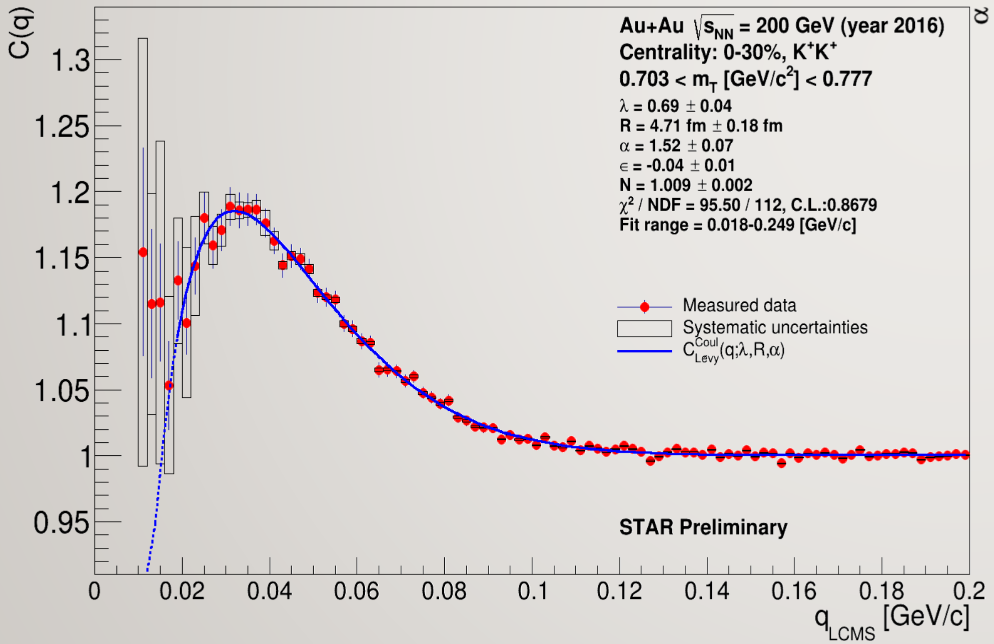


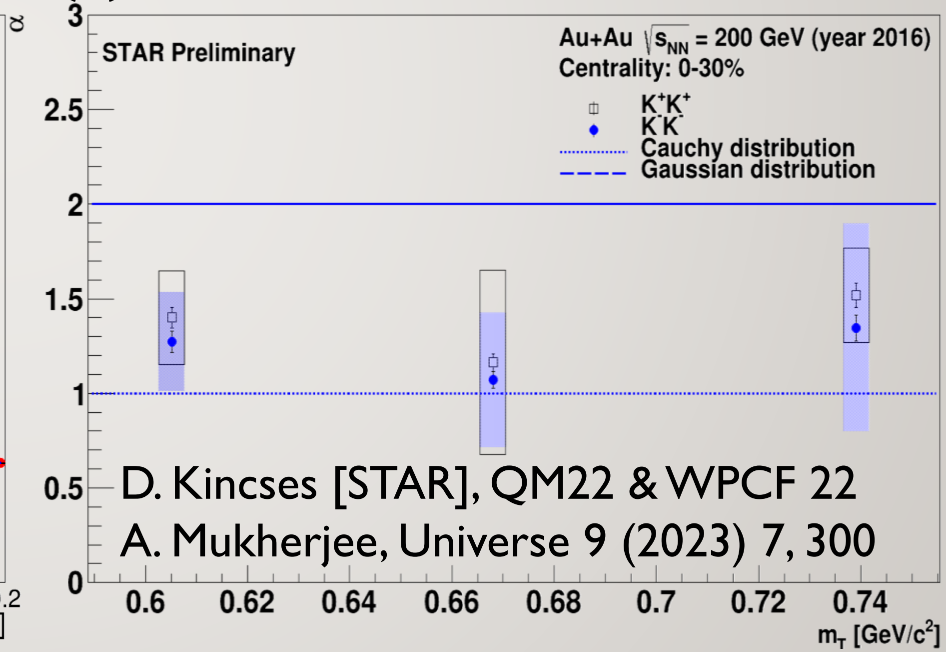


D. Kincses, Phys. Part. Nuc. 51 (2020) 267–269

KAON ANALYSIS AT STAR

- Data successfully described by Lévy fits
- Lévy-stability parameter α between 1 and 2
- Kaon and pion source of same shape at the same m_T?
- Unlike anomalous diffusion expectation of $\alpha(K) < \alpha(\pi)$





D. Kincses [STAR], QM22 & WPCF 22
A. Mukherjee, Universe 9 (2023) 7, 300