# Quantum rotating black holes

## (recovering geometry in a quantum world)


Roberto Casadio

D.I.F.A. “A. Righi”

Bologna University

INFN


Bolyai-Gauss-Lobacewski conference

03/05/2024

Theory and Phenomenology of Fundamental Interactions

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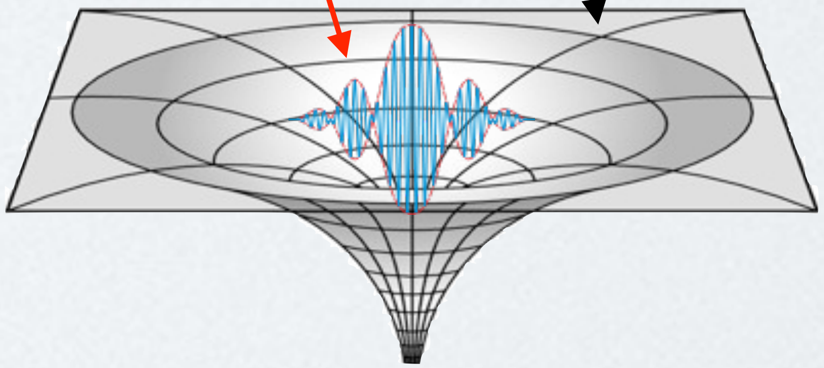
# Preamble - QG and Planck scale

Planck scale:

$$m = m_{\rm p} \equiv \sqrt{\frac{c\,\hbar}{G_{\rm N}}} \sim 10^{-8}\,{\rm kg}$$

$$\lambda_{\rm C} = \ell_{\rm p} \equiv \sqrt{\frac{\hbar\,G_{\rm N}}{c^3}} \sim 10^{-35}\,{\rm m}$$

$$\frac{\hbar}{m\,c} \equiv \lambda_{\rm C} \sim R_{\rm H} \equiv \frac{2\,G_{\rm N}\,m}{c^2}$$

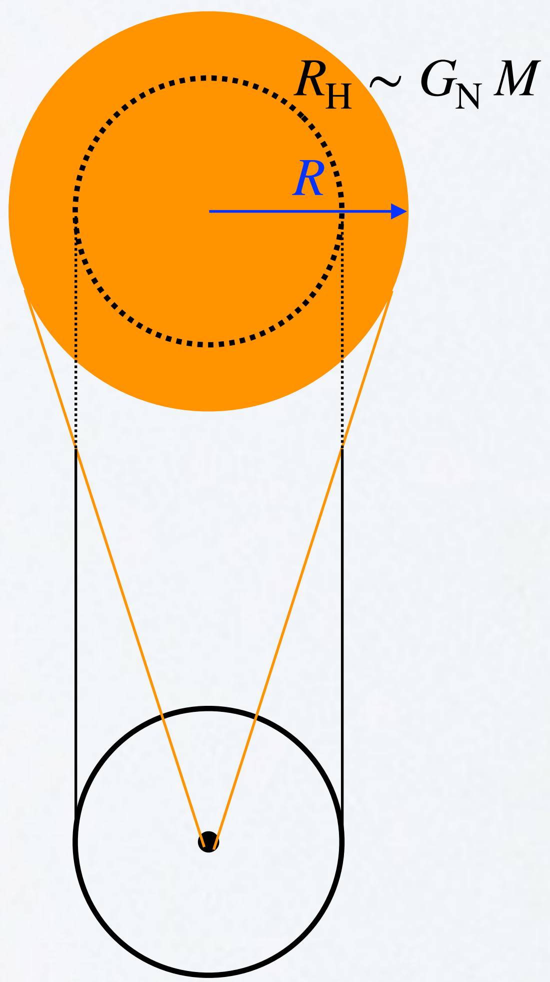


Is Quantum Gravity at the Planck scale $\left(\ell_{\rm p} \sim m_{\rm p}\right)$ or compactness $\frac{G_{\rm N}\,M}{R} = \frac{M\,\ell_{\rm p}}{m_{\rm p}\,R} \sim 1$?*

* Analogy: what is the scale of QED? Electron Compton length << atom << BEC size << neutron star

# Preamble - QG and gravitational collapse

- BH form from collapse - the classical view:




- $|\mathrm{matter}\rangle$ ~ very large number of SM particles ( $M_\odot \sim 10^{57}$ neutrons )
- $|\mathrm{gravity}\rangle$ ~ very large number of gravitons* ( $N_G \sim M_\odot^2 \sim 10^{76}$ )

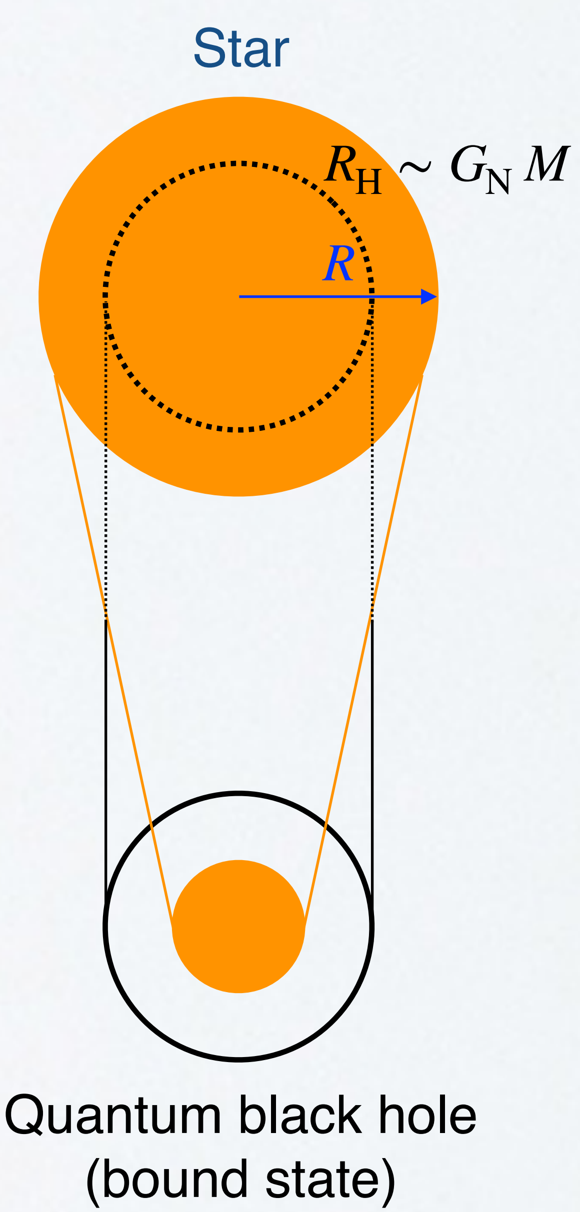
Reduce and simplify**: the Oppenheimer-Snyder model

* J.D. Bekenstein, PRD 7 (1973) 2333

** Rovelli, Thiemann,… Relational approach: "Quantise only what you measure"… But this is just what physics is about

# Preamble - QG and gravitational collapse

- BH form from collapse - the quantum view:




- $|\text{matter}\rangle$ ~ very large number of SM particles ( $M_\odot \sim 10^{57}$ neutrons )
- $|\text{gravity}\rangle$ ~ very large number of gravitons* ( $N_G \sim M_\odot^2 \sim 10^{76}$ )

- $|\text{gravity}\rangle$ always entangled with $|\text{matter}\rangle$ $\Longleftrightarrow$ "quantum hair" [1,2]

Dynamics

$$|\mathbf{g}\ \phi\rangle = \sum_{ij} C_{ij} |\mathbf{g}_i\rangle |\phi_j\rangle \quad \not\Rightarrow \quad \left(\sum_{ab} c_{ab} |\mathbf{g}_a\rangle |\phi_b\rangle\right)\left(\sum_{AB} c_{AB} |\mathbf{g}_A\rangle |\phi_B\rangle\right)$$

$$\hat{H}^\mu |\mathbf{g}\ \phi\rangle = 0$$

BH interior     BH exterior

[1] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]
[2] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

# 1 - Coherent state for classical geometry

- SdS geometry [1]:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

$$f = 1 + 2\,V_{\rm SdS} = 1 - \frac{2\,G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{3}$$

$$V_{\rm SdS} = V_M + V_\Lambda = -\frac{G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

- Horizons ( $f = 0 \leftrightarrow 2\,V = -1$ ):

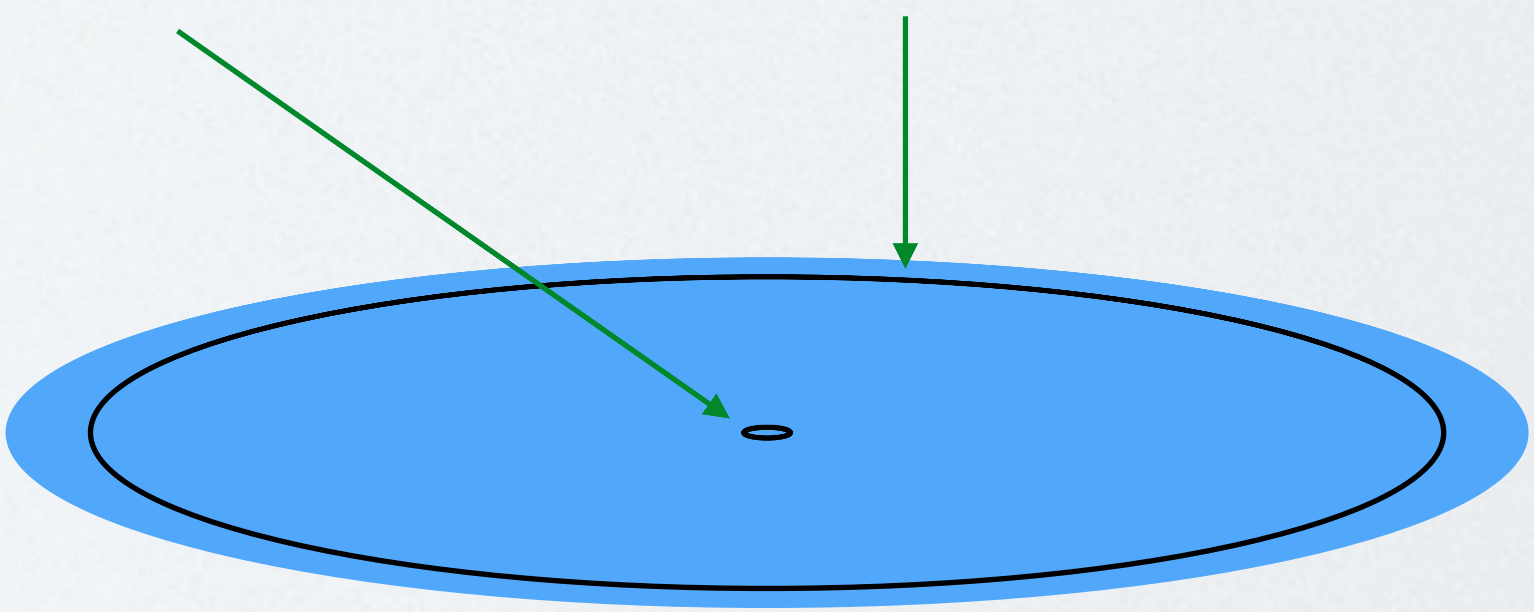
$1/\sqrt{\Lambda} \gg G_{\rm N}\,M$

Localised source gravitational radius

$R_{\rm H} \simeq 2\,G_{\rm N}\,M$

Cosmological horizon

$H^{-1} = L \simeq \sqrt{\frac{3}{\Lambda}}$



[1] A. Giusti, S. Buffa, L. Heisenberg, R.C., *A quantum state for the late* Universe, PLB 826 (2022) 136900 [arXiv:2108.05111]

# 1 - Coherent state for classical geometry

- "Effective" massless scalar field in Minkowski ( ~ true QG vacuum $\hat{a}_k\,|\,0\rangle = 0$ *):

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right]\Phi(t,r) = 0$$

$$u_k(t,r) = e^{-i\,k\,t}\,j_0(k\,r)$$

$$4\,\pi\int_0^\infty r^2\,dr\,j_0(k\,r)\,j_0(p\,r) = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Normal mode expansion of operators:

$$\hat{\Phi}(t,r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar}{2\,k}}\left[\hat{a}_k\,u_k(t,r) + \hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{\Phi}(t,r),\hat{\Pi}(t,s)\right] = \frac{i\,\hbar}{4\,\pi\,r^2}\,\delta(r-s)$$

$$\hat{\Pi}(t,r) = i\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar\,k}{2}}\left[\hat{a}_k\,u_k(t,r) - \hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{a}_k,\hat{a}_p^\dagger\right] = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Coherent state: $\hat{a}_k\,|\,g\rangle = g(k)\,e^{i\,\gamma_k(t)}\,|\,g\rangle$

$$\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{2\,\ell_p\,m_p}{k}}\,g(k)\,\cos[\gamma_k(t) - k\,t]\,j_0(k\,r)$$

* Metric ~ causal structure ~ gravity emerges from "excitations" along with matter ( ~ LQG, Regge calculus, etc…)

# 1 - Coherent state for classical geometry

- "Classical" coherent state: $$\sqrt{\frac{\ell_{\rm p}}{m_{\rm p}}}\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = V(r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,\tilde{V}(k)\,j_0(k\,r)$$

Single mode occupation number: $$g(k) = \sqrt{\frac{k}{2}}\,\frac{\tilde{V}(k)}{\ell_{\rm p}}$$

$$\gamma_k = k\,t$$ ← virtual non-propagating modes *

- Total occupation number (must be finite!) ~ distance from vacuum:

$$|\,g\rangle = e^{-N_{\rm G}/2}\,\exp\left\{\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g(k)\,\hat{a}_k^\dagger\right\}|\,0\rangle$$

$$N_{\rm G} = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g^2(k) < \infty$$

$$\langle k\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,k\,g^2(k) < \infty$$

* "Potentials" in QFT are generated by virtual / non propagating modes

# 2 - Coherent state for classical Schwarzschild geometry

- Localised source: $V_M = -\frac{G_N M}{r}$ $\qquad \tilde{V}_M = -4\pi G_N \frac{M}{k^2}$

- Mass scaling [1]:

Divergences

$$N_M = 4\frac{M^2}{m_p^2}\int_0^\infty \frac{dk}{k} \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} \frac{dk}{k} = 4\frac{M^2}{m_p^2}\ln\left(\frac{k_{UV}}{k_{IR}}\right)$$

- Compton length scaling:

$$\langle k \rangle = 4\frac{M^2}{m_p^2}\int_0^\infty dk \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} dk = 4\frac{M^2}{m_p^2}(k_{UV} - k_{IR})$$

- Quantum core (BH = ground state): $k_{UV}^{-1} \simeq R_s \simeq G_N M$

* Cut-offs = existence condition for quantum state: $g(k < k_{IR}) = g(k > k_{UV}) \simeq 0$ !

[1] R.C., *Geometry and thermodynamics of coherent quantum black holes,* IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

# 2 - Coherent state for classical Schwarzschild geometry

- Localised source: $V_M = -\frac{G_N M}{r}$

- Localised source in dS: $k_{UV} \simeq 1/R_s$

  $k_{IR} \simeq 1/L$

$$V_{QM} \equiv \sqrt{\frac{\ell_p}{m_p}} \langle g | \hat{\Phi}(t,r) | g \rangle \simeq V_M(r)$$

(*Observable?*) quantum hair

$$V_{QM} = V_M(r) \left\{ 1 - \left[ 1 - \frac{2}{\pi} \mathrm{Si}\left( \frac{r}{R_s} \right) \right] \right\}$$

$$\mathrm{Si}(x) = \int_0^x j_0(z)\, dz$$

- Excited (coherent) states $\Longleftrightarrow$ deformations (Love numbers…)

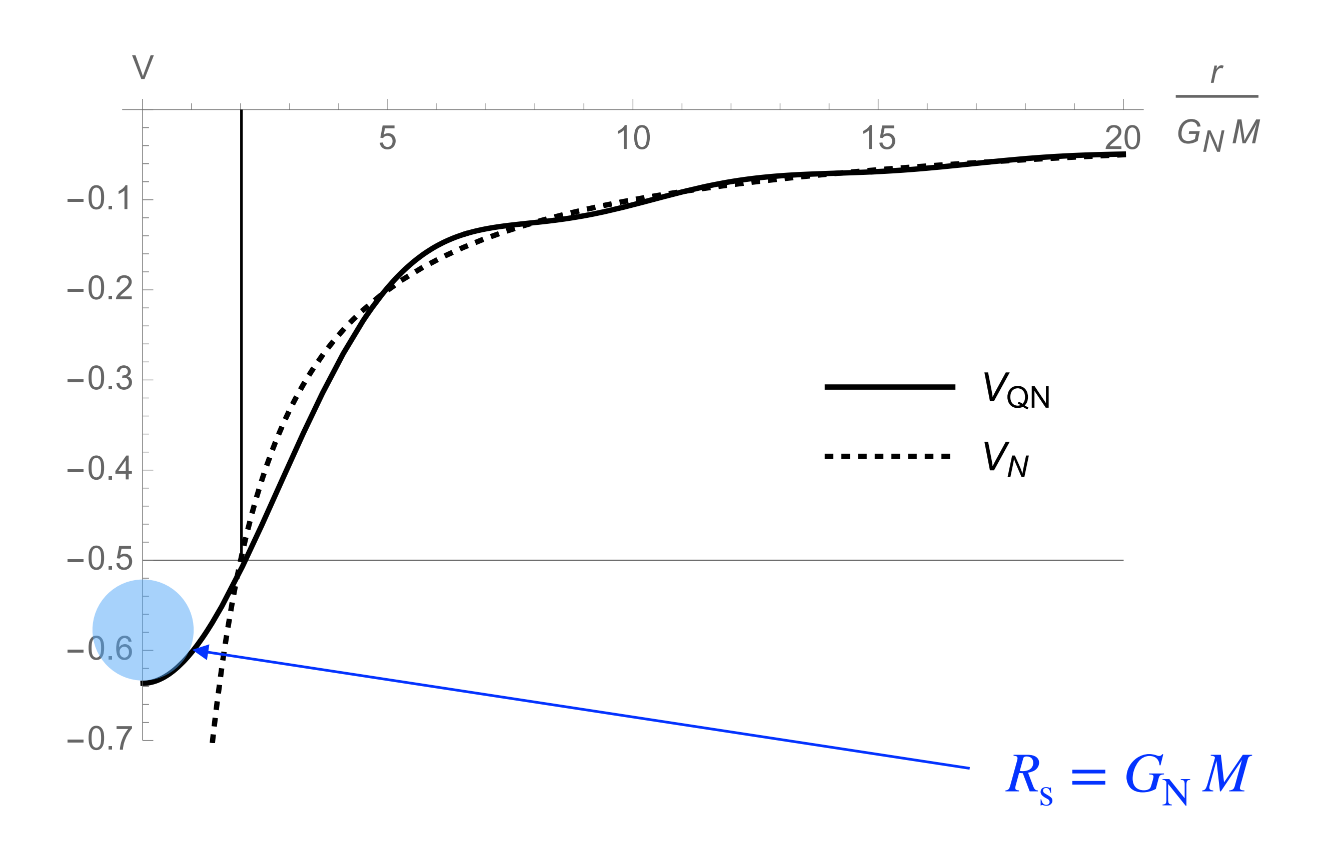
# 3 - Quantum integrable black holes

- Corpuscular scaling laws*:

$$N_M \sim \frac{M^2}{m_{\rm p}^2} \ln\left(\frac{R_\infty}{R_{\rm s}}\right)$$

$$\lambda_M \simeq \frac{N_M}{\langle k \rangle} \sim R_{\rm H} \sim G_{\rm N}\, M$$

cutoffs → proper $g(k)$ ~ "quantum hair"

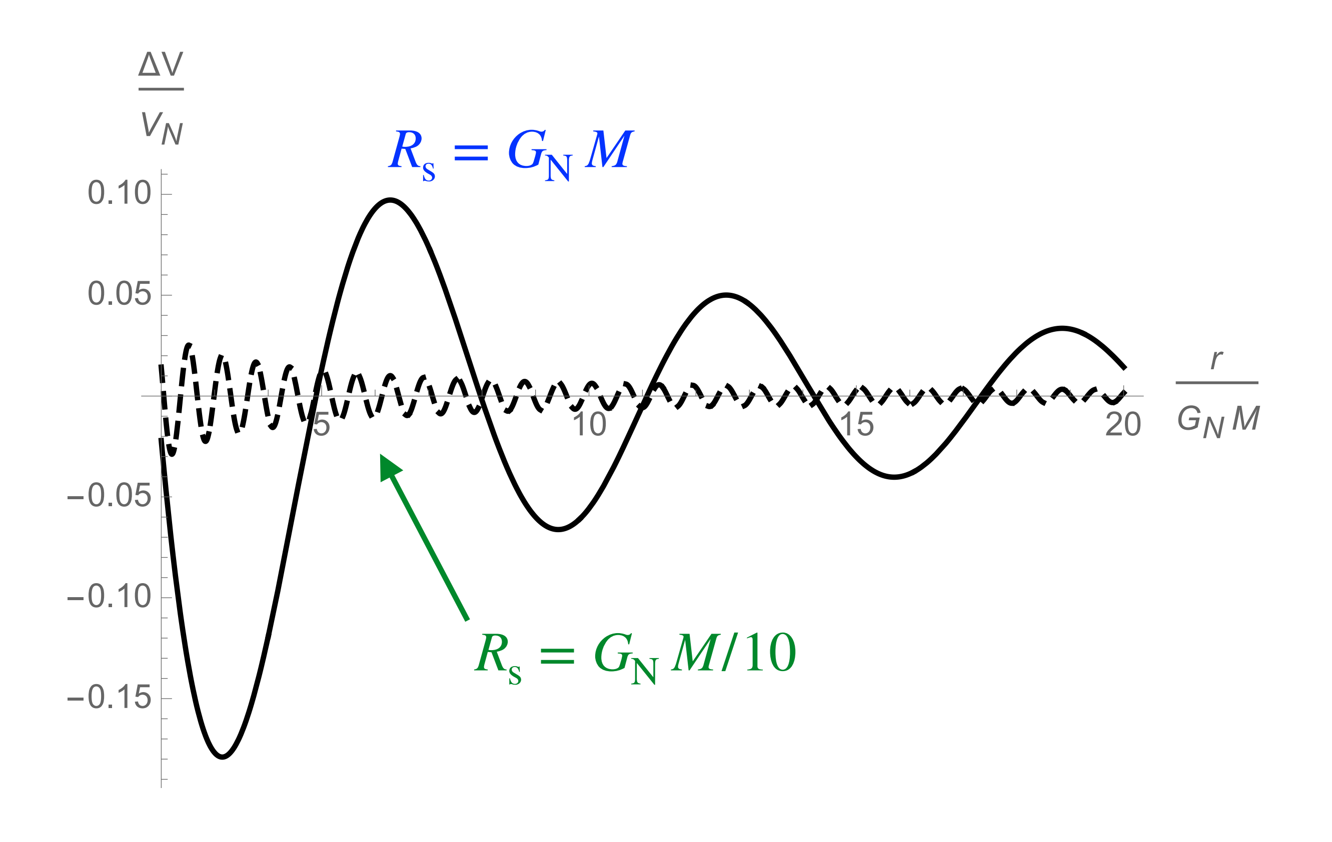



- Quantum metric [1]:

$$ds^2 \simeq -\left(1 + 2\, V_{\rm QM}\right) dt^2 + \frac{dr^2}{1 + 2\, V_{\rm QM}} + r^2\, d\Omega^2$$

*Integrable singularity*** without inner horizon

$$R^2 \sim R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \sim r^{-4}$$




* G. Dvali, C. Gomez, Fortsch. Phys. 61 (2013) 742 [arXiv:1112.3359]
** V.N. Lukash, V.N. Strokov, IJMPA 28 (2013) 1350007 [arXiv:1301.5544]
[1] R.C., *Geometry and thermodynamics of coherent quantum black holes,* IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

# 3 - Quantum integrable black holes

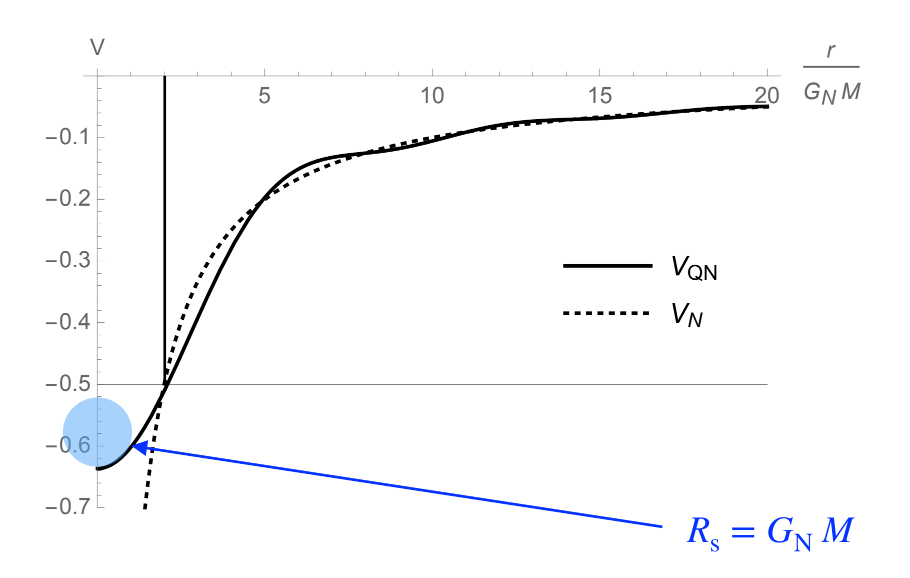
- Spherical *integrable singularity* without inner horizon [1,2]

$$\rho_{\rm eff} \sim |\Psi^2(r)| \sim r^{-2} \qquad m(r) \sim \int_0^r \rho(x)\, x^2\, dx < \infty$$

$$p^r_{\rm eff} \sim -\rho_{\rm eff} \sim -r^{-2}$$

$$p^t_{\rm eff} \sim r^0$$

$m(r) \sim r$




$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$

$$\Delta(r \sim 0) \leq 0$$

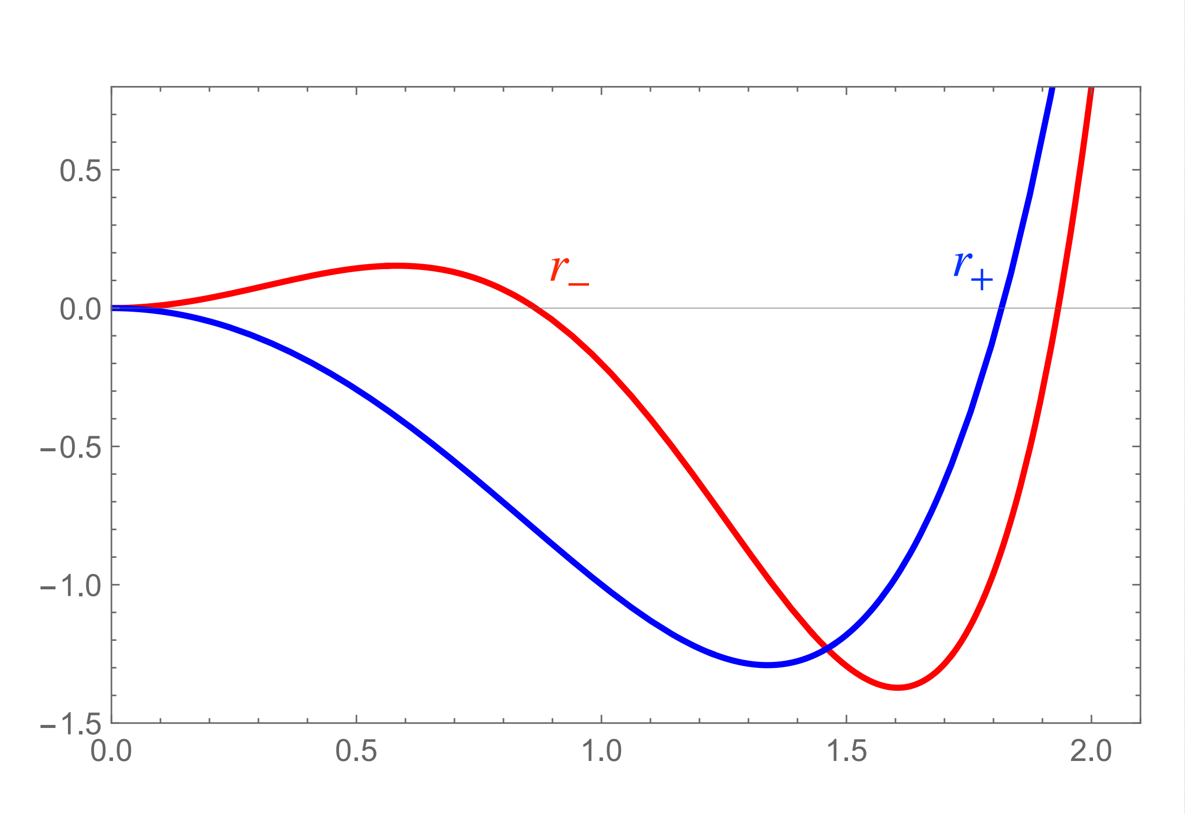
↕

Regular (classical) black holes:

$$\rho \sim r^0 \implies m \sim r^3$$

$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$

$$\Delta(r \sim 0) \sim r^2 \geq 0$$





[1] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]
[2] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]

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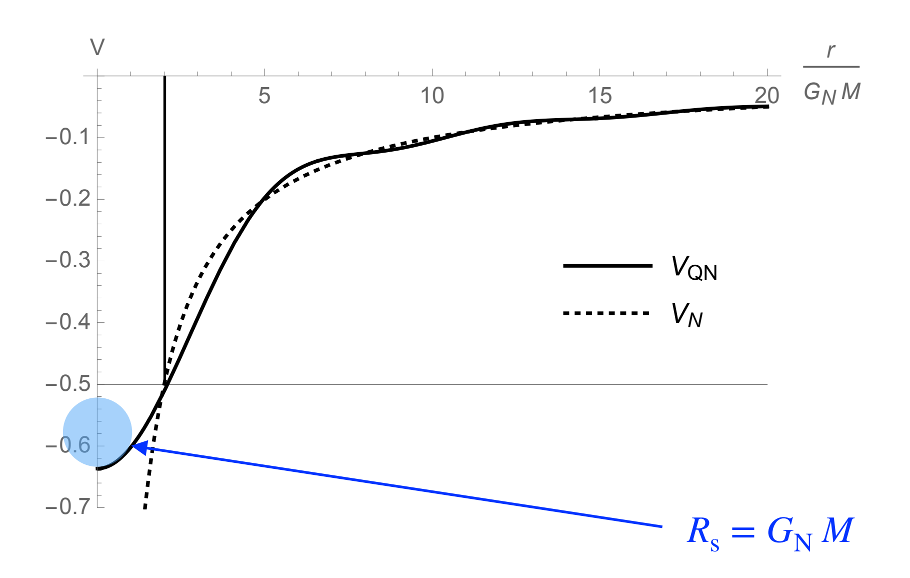
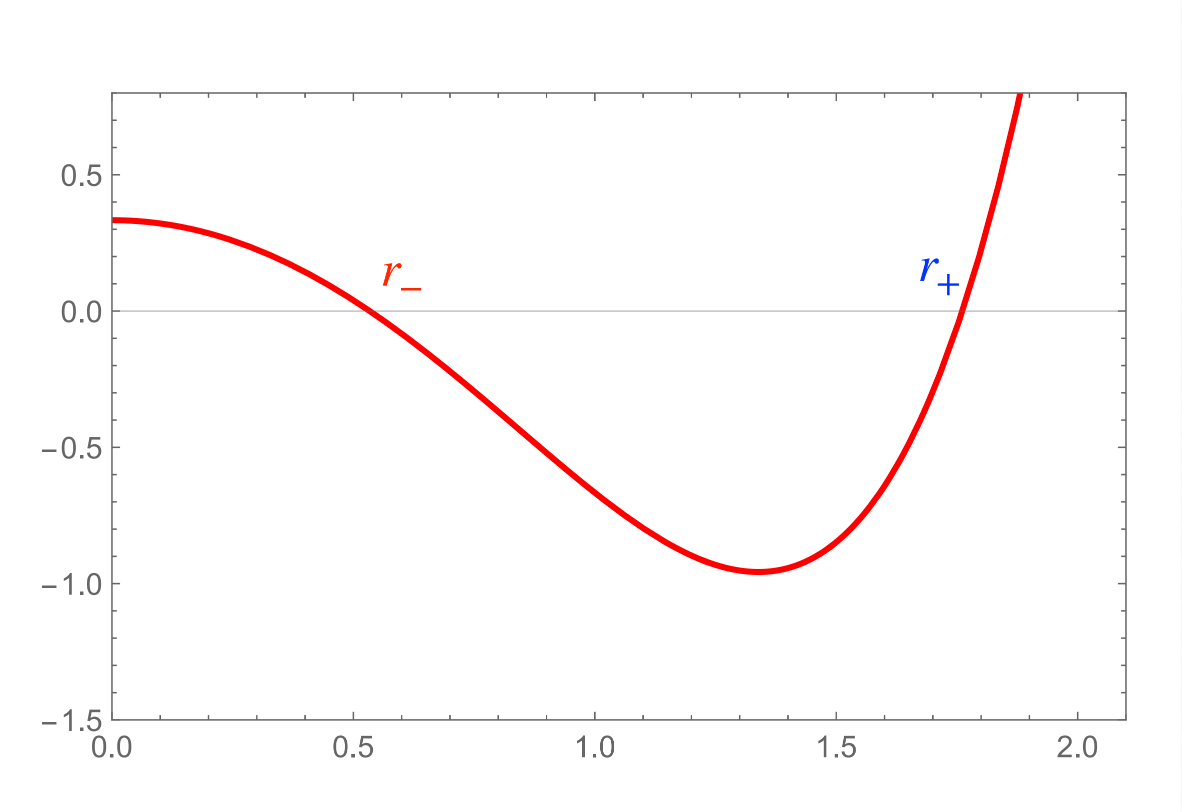



- Rotating *integrable singularity* without inner horizon [3]

$$m(r) \sim r$$

$$\Delta = a^2 - 2\, r_\pm m(r_\pm) + r_\pm^2 = 0$$

$$\Delta(0) = a^2 > 0$$





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[2] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]
[3] R.C., A. Giusti, J. Ovalle, *Quantum rotating black holes*, JHEP 05 (2023) 118 [arXiv:2303.02713]

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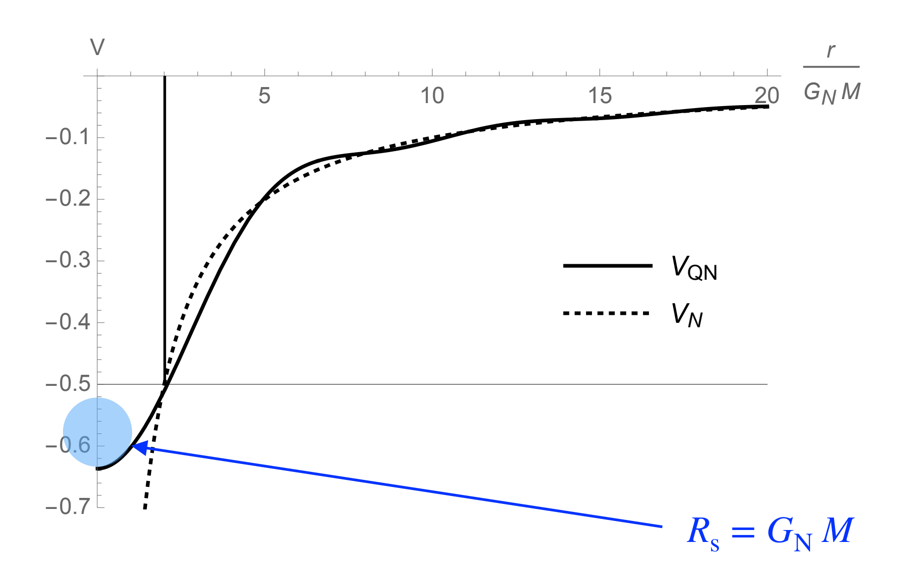


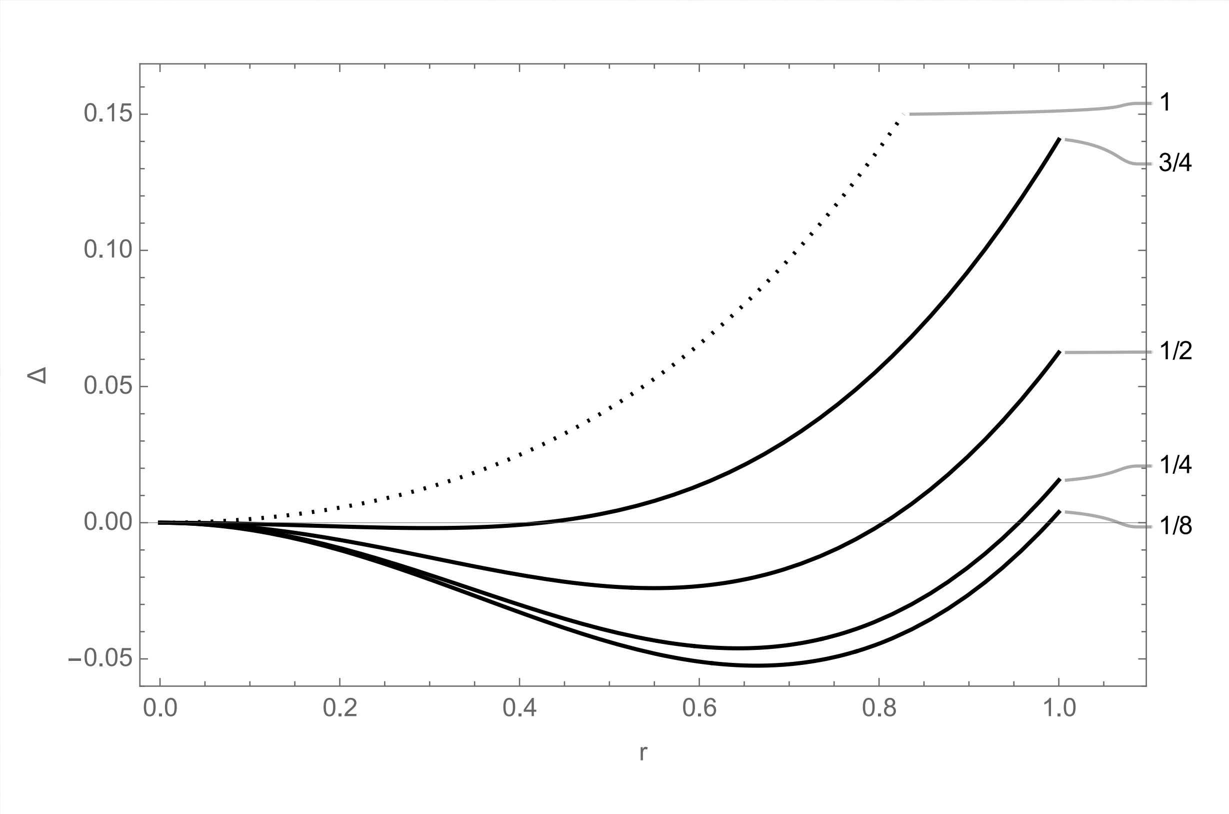

- Rotating *integrable singularity* without inner horizon [3]

$$m(r) \sim r$$

$$a(r) \sim r^{\alpha} \qquad \alpha \geq 1$$

$$\Delta = a^2(r_{\text{H}}) - 2\, r_{\text{H}}\, m(r_{\text{H}}) + r_{\text{H}}^2 = 0$$

$$\Delta(r \sim 0) \sim -(2\, m_1 - 1)\, r \leq 0$$





[1] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]
[2] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]
[3] R.C., A. Giusti, J. Ovalle, *Quantum rotating black holes*, JHEP 05 (2023) 118 [arXiv:2303.02713]

# 4 - Coherent state for slowly rotating geometry

- Slowly rotating metric [1]: $$ds^2 \simeq -\left(1 + 2\,V_M + 2\,W_a\right) dt^2 + \frac{dr^2}{1 + 2\,V_M + 2\,W_a} - \frac{4\,G_{\rm N}\,M\,a}{r}\,\sin^2\theta\,dt\,d\phi + r^2\,d\Omega^2$$

$$W_a = \frac{a^2}{2\,r^2}$$

$$\hbar \ll J = J^z = |a|\,M \ll G_{\rm N}\,M^2$$

- Angular momentum modes: $$n_k\,|k\rangle \quad \Longrightarrow \quad n_{k\ell m}\,|k,\ell,m\rangle$$

$$J_{\ell m} = \hbar\sqrt{\ell\,(\ell+1)}\;n_{k\ell m}$$

$$J^z_{\ell m} = \hbar\,m\,n_{k\ell m}$$

- Entropy of Schwarzschild geometry from $\ell = m = 0$: $$\mathcal{N}_M \sim \sum_{n=0}^{N_M} \frac{N_M!}{(N_M-n)!\,n!} = 2^{N_M}$$

$$S_M \propto \ln(\mathcal{N}_M) \sim \left(\frac{M}{m_{\rm p}}\right)^2$$

- Entropy of Schwarzschild geometry from $|m| \ll \ell \lesssim \ell_{\rm c}$: $$W_{{\rm q}a} \sim \frac{G_{\rm N}\,M}{r}\left(\frac{\ell_{\rm p}}{r}\right)^2$$

~ 1-loop corrections [2]

[1] W. Feng, R. da Rocha, R.C., *Quantum hair and entropy for slowly rotating quantum black holes,* arXiv:2401.14540

[2] M.B. Fröb., C. Rein, R. Verch, JHEP 01 (2022) 180 [arXiv:2401.14540]

# Conclusions

- Black holes as (macroscopic) quantum states (*bound states* far from vacuum)
- Singularity is not resolved (integrable “fuzzy” geometry)
- Exterior quantum hair (from core size)
- No Cauchy horizon (also for electrically charged black holes)
- No Cauchy horizon for non-rigidly rotating black holes
- Effective cosmological DM
- Test the model*: perturbations $\Longrightarrow$ binary systems $\Longrightarrow$ GW
- Test the model**: geodesic motion $\Longrightarrow$ shadow

*R. Brustein, *Quantum Love numbers,* PRD 105 (2022) 024043 [arXiv:2008.02738]

**D. Malafarina et al, in preparation