Sharp departure beyond PPN formalism

in Brans-Dicke gravity

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Non-Euclidean Geometry in Modern Physics and Mathematics, Budapest 2024

Joint work with Bertrand Chauvineau (Univ. & Observatory of Côte d'Azur)
arXiv:2404.13887 [gr-qc], arXiv:2404.00094 [gr-qc], arXiv:2402.14076 [gr-qc]

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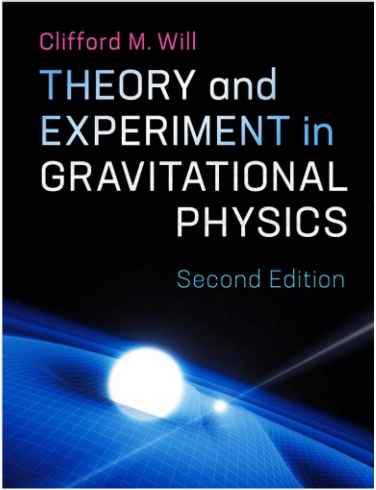
Aim of this talk



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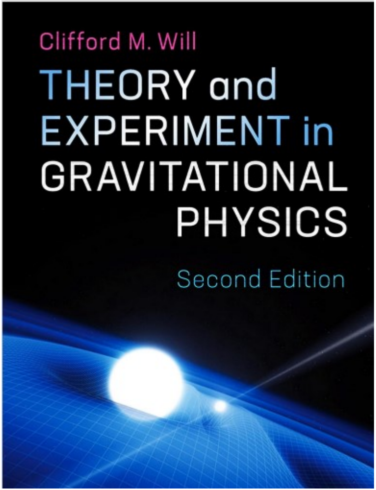

Brans-Dicke gravity

$$\gamma_{\text{PPN}}^{(\text{BD})} = \frac{\omega + 1}{\omega + 2}$$

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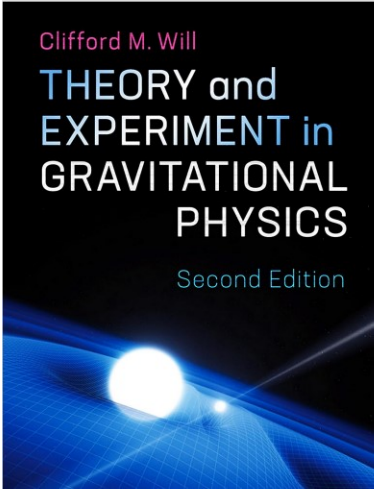

Brans-Dicke gravity

$$\gamma_{\text{PPN}}^{(\text{BD})} = \frac{\omega + 1}{\omega + 2}$$

Or something else more general ?

Parametrized Post-Newtonian formalism

On the unreasonable effectiveness of the post-Newtonian approximation in gravitational physics

Clifford M. Will[1]

McDonnell Center for the Space Sciences, Department of Physics, Washington University, St. Louis, MO

This contribution is part of the special series of Inaugural Articles by members of the National Academy of Sciences elected in 2007.

Contributed by Clifford M. Will, February 25, 2011 (sent for review February 17, 2011)

The post-Newtonian approximation is a method for solving Einstein's field equations for physical systems in which motions are slow compared to the speed of light and where gravitational fields are weak. Yet it has proven to be remarkably effective in describing certain strong-field, fast-motion systems, including binary pulsars containing dense neutron stars and binary black hole systems inspiraling toward a final merger. The reasons for this effectiveness are largely unknown. When carried to high orders in the post-Newtonian sequence, predictions for the gravitational-wave signal from inspiraling compact binaries will play a key role in gravitational-wave detection by laser-interferometric observatories.

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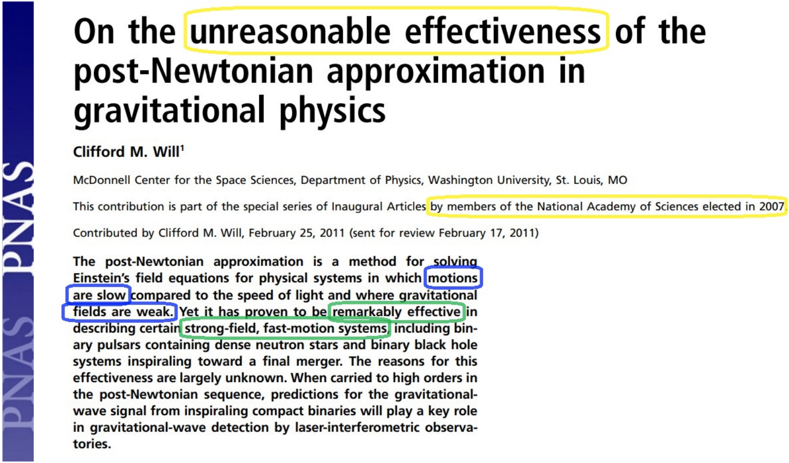

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Parametrized Post-Newtonian formalism

PHYSICAL REVIEW LETTERS **120**, 191101 (2018)

Editors' Suggestion Featured in Physics

New General Relativistic Contribution to Mercury's Perihelion Advance

Clifford M. Will[1,2,*]

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(Received 22 February 2018; revised manuscript received 12 March 2018; published 8 May 2018)

We point out the existence of a new general relativistic contribution to the perihelion advance of Mercury that, while smaller than the contributions arising from the solar quadrupole moment and angular momentum, is 100 times larger than the second-post-Newtonian contribution. It arises in part from relativistic "crossterms" in the post-Newtonian equations of motion between Mercury's interaction with the Sun and with the other planets, and in part from an interaction between Mercury's motion and the gravitomagnetic field of the moving planets. At a few parts in 10^6 of the leading general relativistic precession of 42.98 arcseconds per century, these effects are likely to be detectable by the BepiColombo mission to place and track two orbiters around Mercury, scheduled for launch around 2018.

Robertson γ parameter

Generally, at 1PN level, ten PPN parameters.

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Asymptotically flat static spherisymmetric vacuum

$$ds^2 = -\left(1 - 2\frac{M}{r} + 2\beta\frac{M^2}{r^2} + \dots\right)dt^2 + \left(1 - 2\gamma\frac{M}{r} + \dots\right)dr^2 + r^2 d\Omega^2 \quad (1)$$

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Eddington-Robertson-Schiff parameters:

$$\begin{aligned} M &: \text{active gravitational mass of the source} \\ \beta &: \text{How much nonlinearity in superposition of gravity ?} \\ \gamma &: \text{How much \textbf{spatial} curvature produced by mass ?} \end{aligned} \quad (2)$$

γ *is directly measurable* via:

- Light deflection
- Shapiro time delay

Brans-Dicke gravity

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$$\int d^4x\sqrt{-g}\left[\phi\,\mathcal{R} - \frac{\omega}{\phi}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi\right]$$

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Using PPN approximation (weak field and slow motions)

$$\gamma^{(\text{BD})}_{\text{PPN}} = \frac{\omega + 1}{\omega + 2} \quad (5)$$

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Using PPN approximation (weak field and slow motions)

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In Solar System, $\gamma_{\text{Solar System}} \approx 1 \pm 10^{-5} \implies |\omega| > 40,000$

Pressure in neutron stars

Slow motion assumption of PPN means:

1. Macroscopic objects are in slow motion
2. Microscopic constituents are in slow motion too

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K. Y. Ekşi, *Neutron stars: compact objects with relativistic gravity*, arXiv:2404.00094 [gr-qc]

Einstein's field equations lead to the Tolman-Oppenheimer-Volkov (TOV) equations

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1} \qquad (2.6)$$

and

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \qquad (2.7)$$

where $\rho = \rho(r)$ is the density, $P = P(r)$ is the pressure and $m = m(r)$ is the mass within radial coordinate r. The terms in parentheses in Equation 2.6 are relativistic corrections. In general relativity not only mass but all forms of energy act as a source of gravity.

Pressure in neutron stars

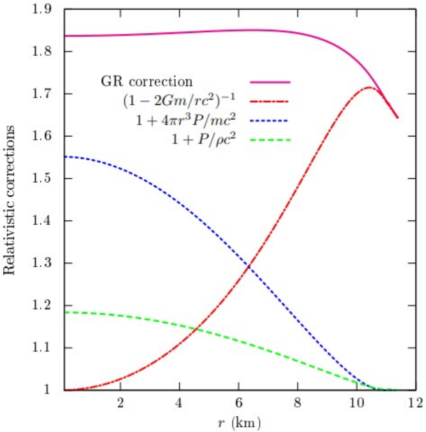

Figure 1. Relativistic corrections within a neutron star with equation of state AP4 [132] for a central pressure of $P_c = 1.73 \times 10^{35}$ dyne cm^{-2}. The mass and radius of the star are $M = 1.51 M_\odot$

$\Longrightarrow$ Pressure in neutron stars is significant !

Pressure and γ

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Claim:

$$\boxed{\gamma_{\text{exact}}^{(\text{BD})} = \frac{\omega + 1 + (\omega + 2)\,\frac{3P}{E}}{\omega + 2 + (\omega + 1)\,\frac{3P}{E}}}$$

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Brans-Dicke

BD field equations

$$R_{ab} - \frac{\omega}{\Phi^2}\partial_a\Phi\partial_b\Phi - \frac{1}{\Phi}\partial_a\partial_b\Phi + \Gamma^c_{ab}\,\partial_c \ln\Phi = \frac{8\pi}{\Phi}\left(T_{ab} - \frac{\omega+1}{2\omega+3}T\,g_{ab}\right) \tag{6}$$

$$\partial_a\left(\sqrt{-g}\,g^{ab}\partial_b\Phi\right) = \frac{8\pi}{2\omega+3}T\sqrt{-g} \tag{7}$$

Energy-momentum tensor: $$T_a^b = \text{diag}\left(-\epsilon, p, p, p\right) \tag{8}$$

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Q: Are they most general? Are they "unique"?

Brans-Dicke

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Q: Are they most general? Are they “unique”?

A: For $\omega > -3/2$, Brans Class I is ***the unique*** SSS vacuum solution

Brans Class I vacuum solution

$$ds^2 = -A(r)\,dt^2 + B(r)\left(dr^2 + r^2 d\Omega^2\right) \tag{9}$$

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$$\begin{cases} \kappa := (1+\Lambda)^2 - \Lambda\left(1 - \frac{\omega}{2}\Lambda\right) \in \mathbb{R} \\ \kappa > 0 : \text{ Brans Class I, “physical”} \\ \kappa \leq 0 : \text{ Brans Classes II \& III \& IV “pathological”} \end{cases}$$

$\Longrightarrow$ **Unifying** 4 Brans Classes into one !

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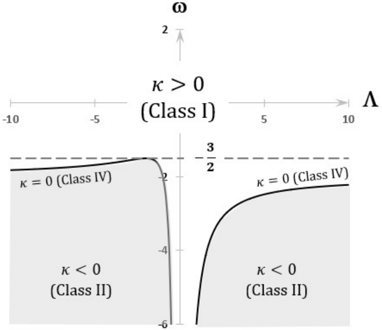

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(Robertson) Large distance expansion

$$ds^2 \simeq -\left(1 - 2\frac{M}{r} + 2\frac{M^2}{r^2}\right) dt^2 + \left(1 + 2\left(1 + \Lambda\right)\frac{M}{r}\right) dr^2 + r^2 d\Omega^2 \tag{11}$$

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From (1): $ds^2 \simeq -\left(1 - 2\frac{M}{r} + 2\beta\frac{M^2}{r^2}\right) dt^2 + \left(1 + 2\gamma\frac{M}{r}\right) dr^2 + r^2 d\Omega^2$

$$\Longrightarrow \begin{cases} \beta_{\text{exact}} & = 1 \\ \gamma_{\text{exact}} & = 1 + \Lambda \end{cases} \tag{12}$$

Solving for Λ...

The scalar eqn and 00−component of metric eqn give

$$\left(r^2\sqrt{AB}\,\Phi'\right)' = \frac{8\pi}{2\omega+3}\Big[-\epsilon+3p\Big]\,r^2\sqrt{AB^3} \tag{13}$$

$$\left(r^2\,\Phi\sqrt{\frac{B}{A}}\,A'\right)' = 16\pi\left[\epsilon+\frac{\omega+1}{2\omega+3}\left(-\epsilon+3p\right)\right]r^2\sqrt{AB^3} \tag{14}$$

everywhere, interior and exterior...

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Define the energy and pressure integrals, with r^* the star's radius

$$E = 4\pi\int_0^{r_*} dr\, r^2\sqrt{AB^3}\,\epsilon \;\sim\; \text{Total energy of star} \qquad (15)$$

$$P = 4\pi\int_0^{r_*} dr\, r^2\sqrt{AB^3}\,p \;\sim\; \text{Total pressure of star} \qquad (16)$$

Solving for Λ...

Integrate (13) & (14) from star's center to r ***outside*** of the star ($r > r^*$), then use $A(r)$ & $B(r)$ in (10):

$$\Lambda\, M = \frac{4\pi}{2\omega+3}\left(-E+3P\right) \tag{17}$$

$$2\, M = \frac{4\pi}{2\omega+3}\left[(\omega+2)E+3(\omega+1)P\right] \tag{18}$$

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$$\xrightarrow{\text{Eq (11)}} \qquad \gamma_{\text{exact}}^{(\text{BD})} \;=\; 1 + \Lambda$$

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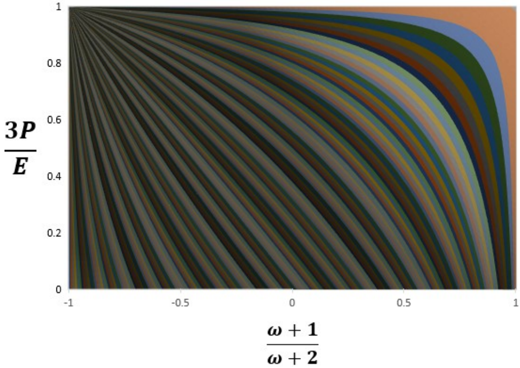

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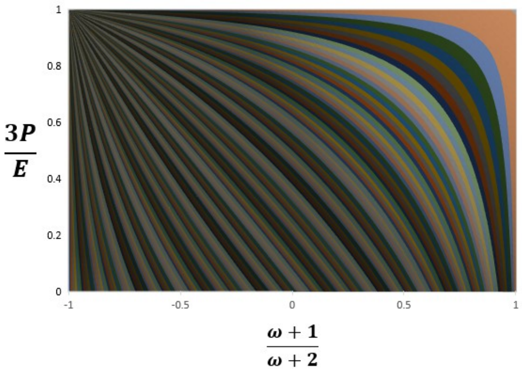

$$\omega = 2;\ \frac{3P}{E} = 0.8 \ \Rightarrow\ \gamma^{(\mathrm{BD})}_{\mathrm{exact}} = 0.96875 \ \text{ whereas } \ \gamma^{(\mathrm{BD})}_{\mathrm{PPN}} = 0.75$$

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