Correlation functions for the channels of the N*(1535), KΛ, KΣ, ηp

Eulogio Oset, Raquel Molina, Chu Wen Xiao and Wei Hong Liang,

IFIC, CSIC- Universidad de Valencia, Guangxi Normal University

Correlation functions

Generation of the N*(1535) from the interaction of coupled channels KΛ, KΣ, ηp

Correlation functions for the coupled channels KΛ, KΣ, ηp

Inverse problem of determination of observables from the correlation functions

Femtoscopic correlation functions

In heavy ion collisions or p p collisions one observes pairs of particles and defines the correlation function as the ratio of probabilities to see the pair to the product of observing individually each particle. Under certain assumptions one finds

$$C(\vec{p}) = \int d^3\vec{r} S_{12}(\vec{r}) |\Psi(\vec{r},\vec{p})|^2$$

$$S_{12}(\vec{r}) = S_{12}(r) = \frac{1}{(\sqrt{4\pi})^3 R^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

$$C(\vec{p}) = 1 + 4\pi \int_0^{+\infty} dr r^2 S_{12}(r) \times \left(\sum_j w_j |\widetilde{\Psi}_j(\vec{r},\vec{p})|^2 - j_0^2(pr) \right)$$

$$\Psi_j(\vec{r},\vec{p}) = \delta_{ij} j_0(pr) + T_{ji}(E)\theta(q_{\max} - |\vec{p}\,|) \times \int_{|\vec{q}\,|<q_{\max}} d^3\vec{q} \frac{j_0(qr)}{E - \omega_1^{(j)}(q) - \omega_2^{(j)}(q) + i\eta}$$

For the case of the observation of channel i

$$\Psi = \Phi + \frac{1}{E-H_0} V\Psi \Rightarrow \Psi = \Phi + \frac{1}{E-H_0} T\Phi$$

$$T\Phi \equiv V\Psi.$$

T = V + VGT

$$V(\vec{p},\vec{p}\,') = V\theta(q_{\text{max}} - |\vec{p}\,|)\theta(q_{\text{max}} - |\vec{p}\,'|)$$
$$T(E;\vec{p},\vec{p}\,') = T(E)\theta(q_{\text{max}} - |\vec{p}\,|)\theta(q_{\text{max}} - |\vec{p}\,'|).$$

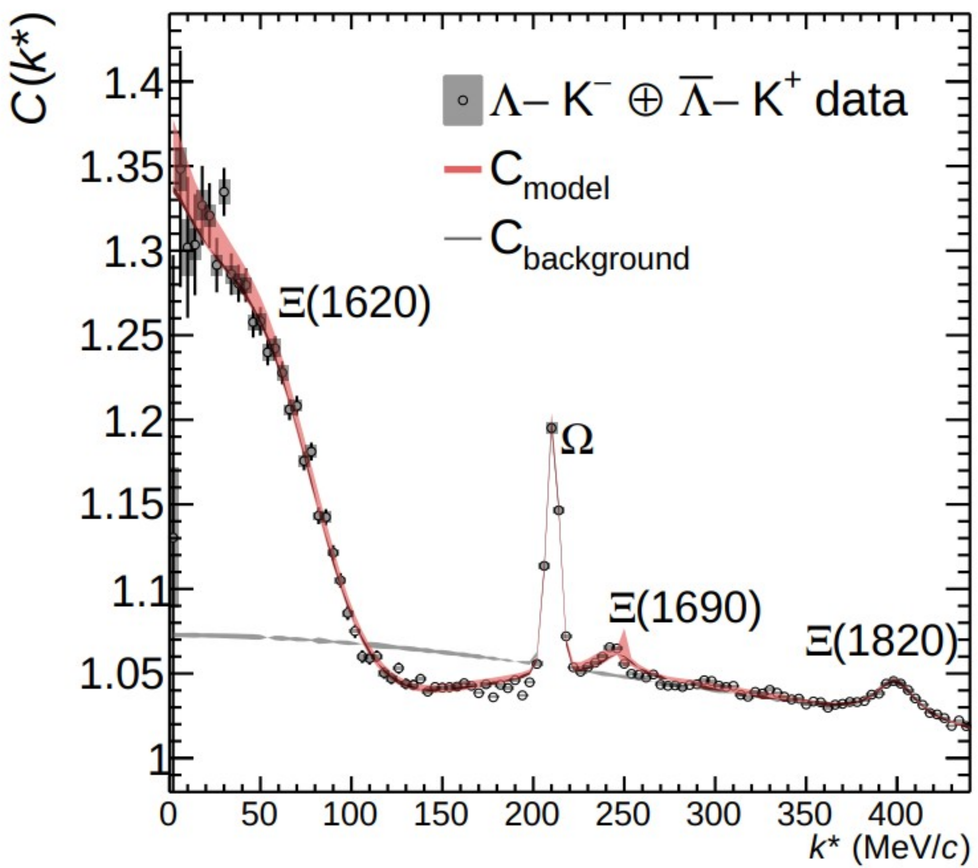
T = V + VGT algebraic eqn.

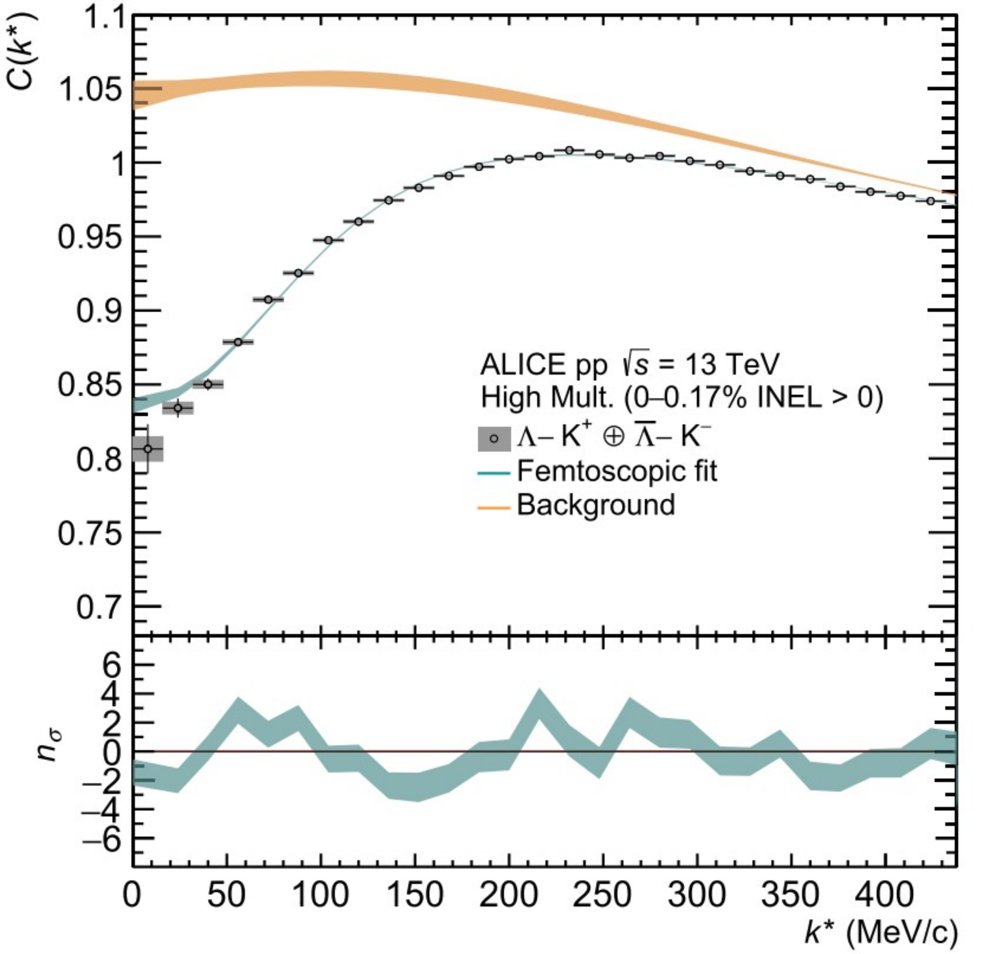
$$G(E) = \int_{|\vec{q}\,|<q_{\text{max}}} \frac{d^3\vec{q}}{E-\omega_1(q)-\omega_2(q)+i\eta}$$

$$\Psi(\vec{r},\vec{p}\,) = e^{i\vec{p}\cdot\vec{r}} + \theta(q_{\text{max}} - |\vec{p}\,|)\, T(E) \times \int_{|\vec{q}\,|<q_{\text{max}}} \frac{d^3\vec{q}\, e^{i\vec{q}\cdot\vec{r}}}{E-\omega_1(q)-\omega_2(q)+i\eta}$$

S-wave projected

$$\Psi_j(\vec{r},\vec{p}\,) = \delta_{ij}\, j_0(pr) + T_{ji}(E)\theta(q_{\text{max}} - |\vec{p}\,|) \times \int_{|\vec{q}\,|<q_{\text{max}}} d^3\vec{q}\, \frac{j_0(qr)}{E-\omega_1^{(j)}(q)-\omega_2^{(j)}(q)+i\eta}$$

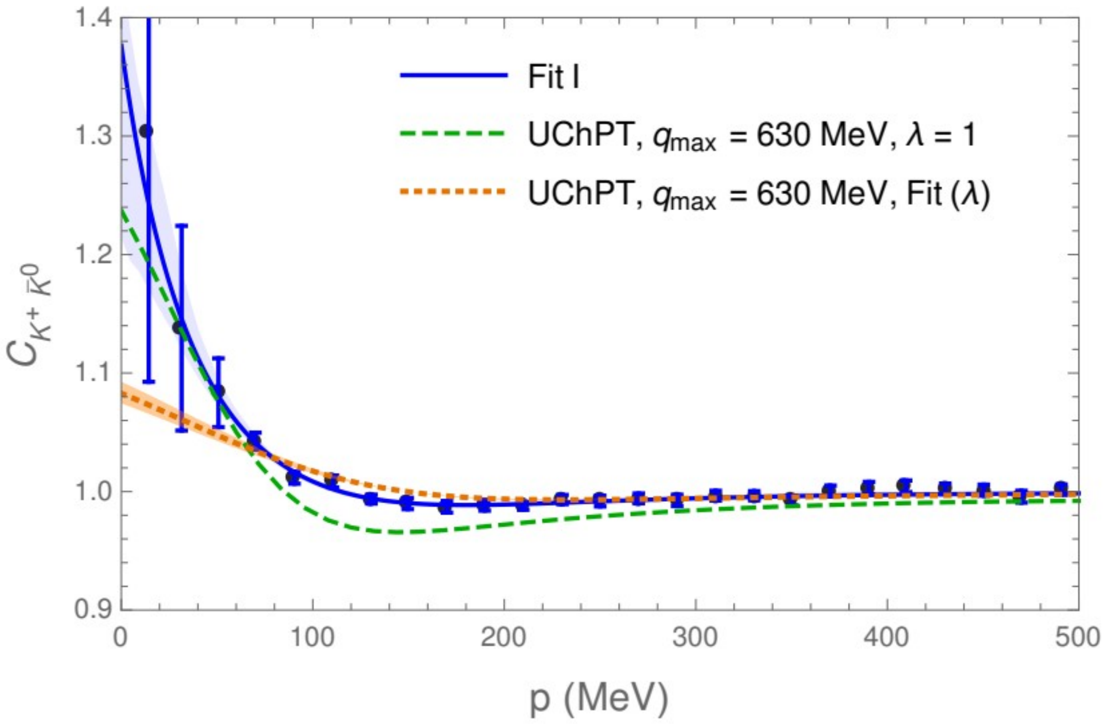




V.~M.~Sarti, A.~Feijoo, I.~Vida\~na, A.~Ramos, F.~Giacosa, T.~Hyodo and Y.~Kamiya, 2309.08756

A model is used by the authors based on chiral unitary theory and the parameters are fitted to the data

S. Acharya et al. (ALICE Collaboration), Phys. Lett. B 845, 138145 (2023)



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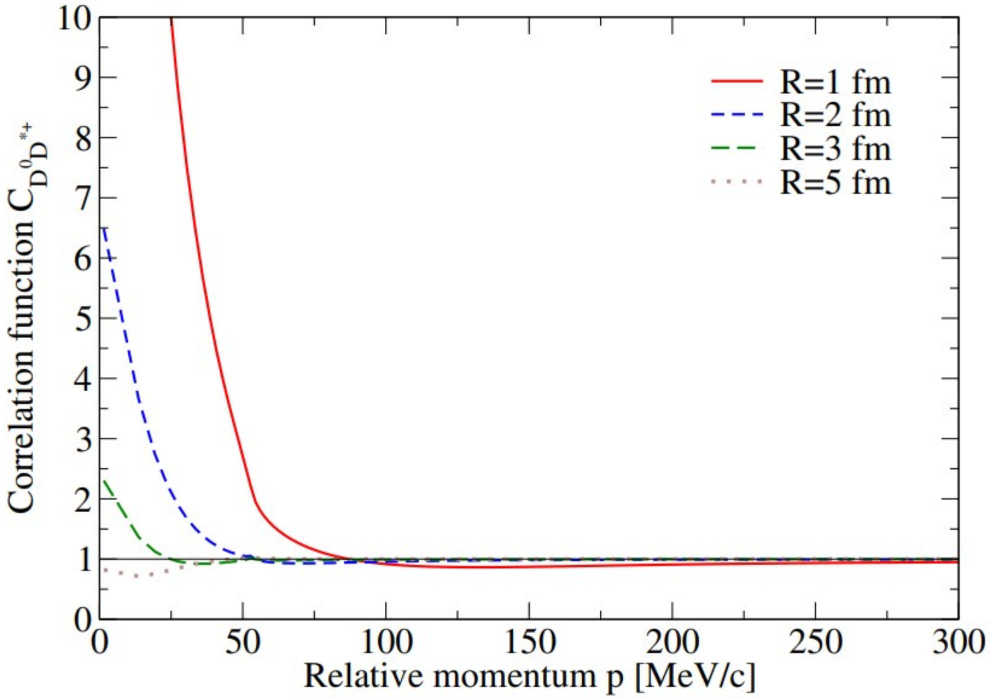

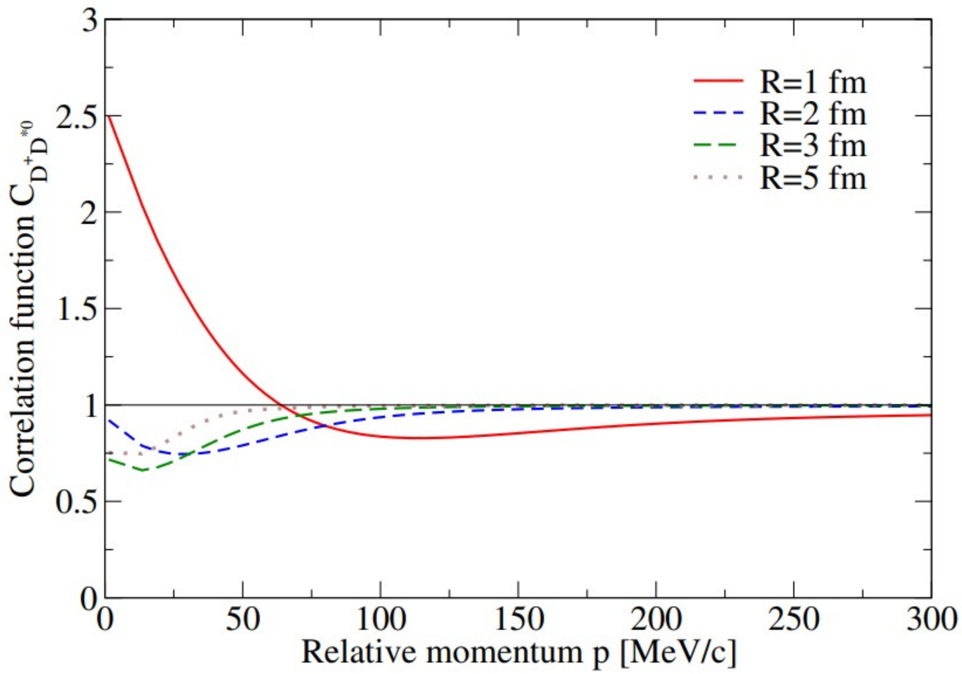

Note: the correlation functions start from threshold of each channel. Can we induce the Tcc and its properties from there, if these c. f. are measured?

The N*(1535) saga

Molecular

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q qbar + pentaquark

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Need for a large s sbar component in addition to 3 q

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Lattice

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[18] C. D. Abell, D. B. Leinweber, Z. W. Liu, A. W. Thomas, and J. J. Wu, Phys. Rev. D **108**, 094519 (2023).

3 q and ηN and πN channels

$K\Lambda$ channel in addition

Conclude that N*(1535) is mostly 3 q state

The chiral unitary approach for the N*(1535)

N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A594, 325 (1995).

N. Kaiser, T. Waas, and W. Weise, Nucl. Phys. A612, 297 (1997).

T.~Inoue, E.~Oset and M.~J.~Vicente Vacas, Phys Rev C 65,035204,(2002)

To get a bound state at 1535 MeV, we can neglect the π N channels since they are 500 MeV below

R.~Molina, C.~W.~Xiao, W.~H.~Liang and E.~Oset
Phys.Rev.D 109 (2024) 5, 054002

Coupled channels

$$K^+\Sigma^-, K^0\Sigma^0, K^0\Lambda, \pi^- p, \pi^0 n, \eta n.$$

$$K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \pi^+ n, \pi^0 p, \eta p.$$

$$V_{ij} = -\frac{1}{4f^2} C_{ij}(k^0 + k'^0); \qquad f = 93 \text{ MeV},$$

$$T = [1 - VG]^{-1} V,$$

$$G_i(s) = \int_{|\vec{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_i(q) + E_i(q)}{2\omega_i(q) E_i(q)} \times \frac{2M_i}{s - [\omega_i(q) + E_i(q)]^2 + i\varepsilon},$$

TABLE I. C_{ij} coefficients of Eq. (3).

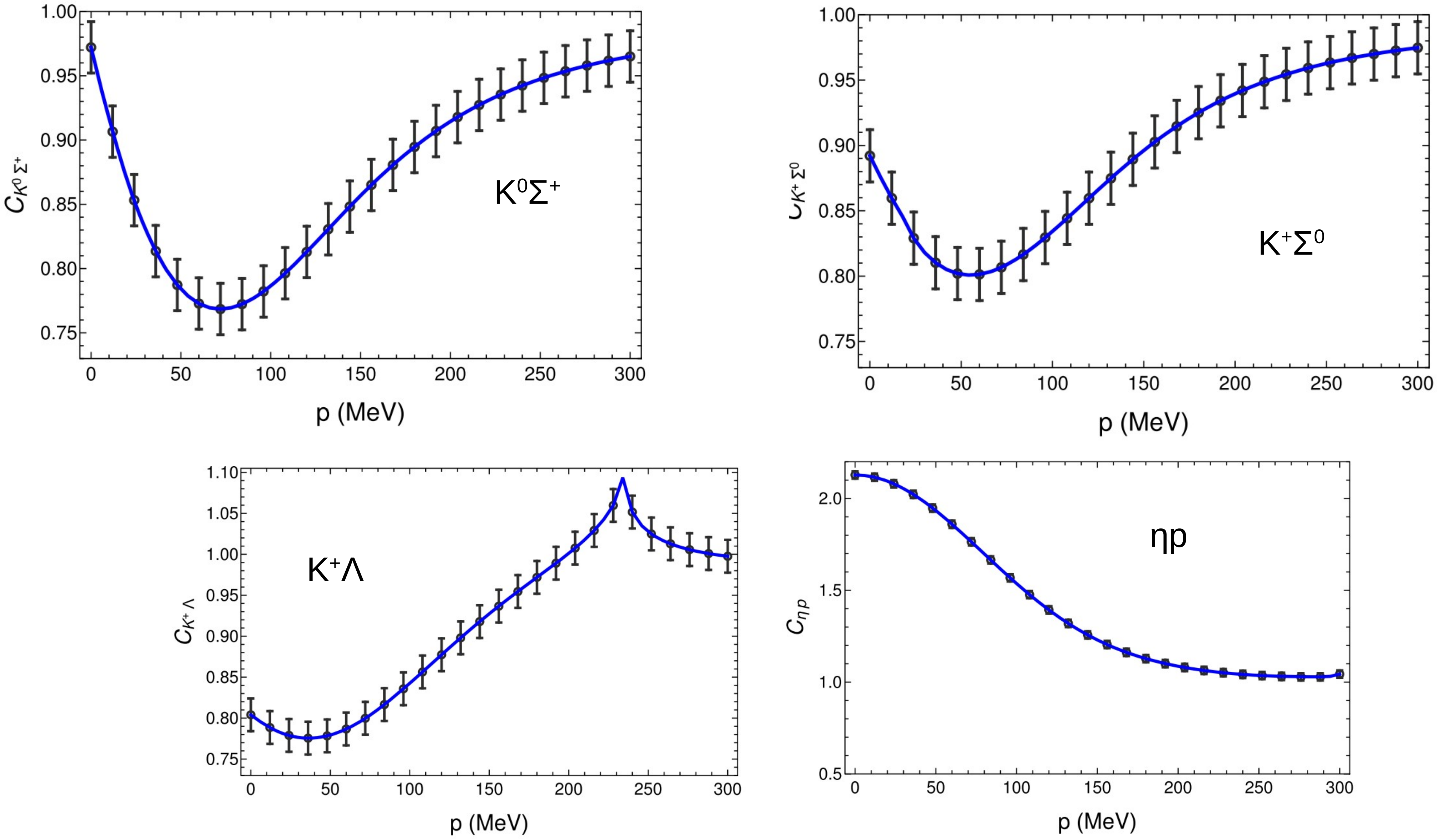
C_{ij}	$K^0\Sigma^+$	$K^+\Sigma^0$	$K^+\Lambda$	$\pi^+ n$	$\pi^0 p$	ηp
$K^0\Sigma^+$	1	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{2}}$	$-\sqrt{\frac{3}{2}}$
$K^+\Sigma^0$		0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$K^+\Lambda$			0	$-\sqrt{\frac{3}{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$
$\pi^+ n$				1	$\sqrt{2}$	0
$\pi^0 p$					0	0
ηp						0

$$
\begin{aligned}
C_{K^0\Sigma^+}(p_{K^0}) = 1 + 4\pi\theta(q_{\max} - p_{K^0}) \int drr^2 S_{12}(r) \cdot \{ & |j_0(p_{K^0} r) + T_{K^0\Sigma^+,K^0\Sigma^+}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 \\
& + |T_{K^+\Sigma^0,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 \\
& + |T_{\eta p,K^0\Sigma^+}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^0} r)\},
\end{aligned}
$$

$$
\begin{aligned}
C_{K^+\Sigma^0}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{ & |j_0(p_{K^+} r) + T_{K^+\Sigma^0,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 \\
& + |T_{K^0\Sigma^+,K^+\Sigma^0}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Lambda,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 \\
& + |T_{\eta p,K^+\Sigma^0}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+} r)\},
\end{aligned}
$$

$$
\begin{aligned}
C_{K^+\Lambda}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{ & |j_0(p_{K^+} r) + T_{K^+\Lambda,K^+\Lambda}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 \\
& + |T_{K^0\Sigma^+,K^+\Lambda}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,K^+\Lambda}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 \\
& + |T_{\eta p,K^+\Lambda}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+} r)\},
\end{aligned}
$$

$$
\begin{aligned}
C_{\eta p}(p_\eta) = 1 + 4\pi\theta(q_{\max} - p_\eta) \int drr^2 S_{12}(r) \cdot \{ & |j_0(p_\eta r) + T_{\eta p,\eta p}(E)\tilde{G}^{(\eta p)}(r;E)|^2 \\
& + |T_{K^0\Sigma^+,\eta p}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,\eta p}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,\eta p}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 - j_0^2(p_\eta r)\},
\end{aligned}
$$



$C_{K^0\Sigma^+}$
$K^0\Sigma^+$
$C_{K^+\Sigma^0}$
$K^+\Sigma^0$
$C_{K^+\Lambda}$
$K^+\Lambda$
$C_{\eta p}$
ηp
p (MeV)

Inverse problem, or how to get the physical observables from the correlation functions

$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k^0 + k'^0)$$

10 parameters plus qmax and R

But we impose that V has isospin symmetry

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\middle| V \middle| K\Sigma, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = 0,$$
$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\middle| V|K^+\Lambda\right\rangle = 0,$$
$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\middle| V|\eta p\right\rangle = 0.$$

$$K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$$

$$V_{ij} = \begin{pmatrix} V_{11} & \sqrt{2}(V_{11} - V_{22}) & V_{13} & V_{14} \\ & V_{22} & \frac{1}{\sqrt{2}}V_{13} & \frac{1}{\sqrt{2}}V_{14} \\ & & V_{33} & V_{34} \\ & & & V_{44} \end{pmatrix}$$

Now 9 parameters

$C_{11}, C_{22}, C_{13}, C_{14}, \; C_{33}, C_{34}, C_{44}$ plus $q_{\max}$ and R

A fit to all the correlation functions is done. The parameters are determined and then observables are calculated.

Many parameters → correlations within the parameters

Resampling method is used: centroids are generated with a Gaussian distribution and a new fit is conducted. After each fit the observables are evaluated. At the end of many fits, the average and dispersion of the observables is calculated.

$$G^{(\mathrm{II})}(\sqrt{s}) = G(\sqrt{s}) + i\,\frac{2M}{4\pi\sqrt{s}}\,q_{\mathrm{on}},$$

$$q_{\mathrm{on}} = \frac{\lambda^{1/2}(s, m^2, M^2)}{2\sqrt{s}},$$

for channels where $\mathrm{Re}\sqrt{s} > \sqrt{s_{\mathrm{th}}}$.

$$g_j^2 = \lim_{\sqrt{s}\to\sqrt{s_p}} (\sqrt{s} - \sqrt{s_p})\,T_{jj},$$

$$g_i g_j = \lim_{\sqrt{s}\to\sqrt{s_p}} (\sqrt{s} - \sqrt{s_p})\,T_{ij},$$

$$T = -\frac{8\pi\sqrt{s}}{2M} f^{\mathrm{QM}} \simeq -\frac{8\pi\sqrt{s}}{2M} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik},$$

$$-\frac{1}{a_j} + \frac{1}{2} r_{0,j} k_j^2 \equiv -\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1} + ik_j,$$

$$-\frac{1}{a_j} = -\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1} \bigg|_{\sqrt{s_{\mathrm{th},j}}},$$

$$\begin{aligned} r_{0,j} &= 2 \frac{\partial}{\partial k^2} \left[-\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1} + ik_j \right]_{\sqrt{s_{\mathrm{th},j}}}, \\ &= \frac{1}{\mu_j} \frac{\partial}{\partial\sqrt{s}} \left[-\frac{8\pi\sqrt{s}}{2M_j} (T_{jj})^{-1} + ik_j \right]_{\sqrt{s_{\mathrm{th},j}}}, \end{aligned}$$

TABLE II. Values obtained for parameters C_{ij}, $q_{\max}$ and R. The channels are $K^0\Sigma^+(1)$, $K^+\Sigma^0(2)$, $K^+\Lambda(3)$, $\eta p(4)$.

C_{11} 1.10 ± 0.20	C_{22} -0.02 ± 0.20	C_{33} 0.14 ± 0.30	C_{44} 0.16 ± 0.07	C_{13} 0.13 ± 0.20
C_{14} -1.10 ± 0.20	C_{34} -1.37 ± 0.16	$q_{\max}$ (MeV) 637 ± 72	R (fm) 1.02 ± 0.02	

TABLE III. Scattering lengths for channel i (in units of fm).

a_1 $(0.46\pm0.04)-(0.64\pm0.03)i$	a_2 $(0.32\pm0.01)-(0.35\pm0.02)i$
a_3 $(0.30\pm0.02)-(0.22\pm0.04)i$	a_4 $(-0.780\pm0.013)+(0\pm0)i$

TABLE IV. Effective range parameters for channel i (in units of fm).

r_1 $(-1.1 \pm 0.2) - (2.7 \pm 0.2)i$	r_2 $(-6.2 \pm 1.4) + (8.8 \pm 0.5)i$	r_3 $(-2.8 \pm 0.3) - (0.3 \pm 0.6)i$	r_4 -1.48 ± 0.13

TABLE V. Pole position and couplings (in units of MeV).

$\sqrt{s_p}$ $(1515 \pm 6) - (89 \pm 9)i$	g_1 $(3.7 \pm 0.3) - (1.04 \pm 0.13)i$	g_2 $(2.6 \pm 0.2) - (0.74 \pm 0.10)i$
	g_3 $(3.6 \pm 0.2) - (0.28 \pm 0.05)i$	g_4 $(-2.68 \pm 0.13) + (1.4 \pm 0.2)i$

$$\mathcal{P}_i = g_i^2 \frac{\partial G_i}{\partial E}; \qquad \psi(r=0) = g_i G_i.$$

$$\mathcal{P}_1 \simeq 0.12 - 0.23i, \qquad \mathcal{P}_2 \simeq 0.06 - 0.12i,$$
$$\mathcal{P}_3 \simeq 0.22 - 0.28i, \qquad \mathcal{P}_4 \simeq -0.34 - 0.24i,$$

$$|\mathcal{P}_1| = 0.26, \qquad |\mathcal{P}_2| = 0.13,$$
$$|\mathcal{P}_3| = 0.35, \qquad |\mathcal{P}_4| = 0.42.$$

Interpretation in

F.~Aceti, L.~R.~Dai, L.~S.~Geng, E.~Oset and Y.~Zhang

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We find that the sum of these numbers, 1.16, exceeds unity, which indicates again that these are not probabilities but gives us an idea or the strength of each channel.

$$\psi_1(r=0) \simeq -26 + 14i, \quad \psi_2(r=0) \simeq -19 + 9.8i,$$
$$\psi_3(r=0) \simeq -30 + 11i, \quad \psi_4(r=0) \simeq -18 - 30i.$$

Experimental information

K Λ : S. Acharya et al. (ALICE Collaboration), Phys. Lett. B 845, 138145 (2023)

Lednický-Lyuboshits formula

$$C_{LL}(0) = 1 + \frac{2\mathrm{Re}f_0(0)}{\sqrt{\pi}R} + \frac{|f_0(0)|^2}{2R^2}.$$

$$a = (0.61 \pm 0.03 \pm 0.03) - i(0.23 \pm 0.06 \pm 0.04)\ \mathrm{fm}.$$

$$a^{(\mathrm{ours})} = (0.30 \pm 0.02) - i(0.22 \pm 0.04)\ \mathrm{fm}.$$

The experimental analysis is done with a single channel formula
Coupled channels are necessary to obtain a correct result

Conclusions

Correlation functions are emerging as a powerful tool to investigate hadron interactions

Yet, care must be taken to avoid analyses based on single channel

Based on the chiral unitary approach, the correlation functions for the channels

$K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$ associated to the N*(1535) were calculated

The inverse problem was faced: given these correlation functions, what can one learn about the N*(1535) and scattering observables of these channels?

We determined the existence and binding of the N*(1535) with high precision, as well as a and r0 for all the channels.

The measurement of these correlation functions is highly encouraged to find out the nature of the N*(1535) and its strangeness content.

Relationship of correlation functions to mass distributions measured for instance in LHCb?