The pole nature of the $\Lambda(1405)$: A lattice QCD calculation

Hadron Spectroscopy with Strangeness Workshop
University of Glasgow

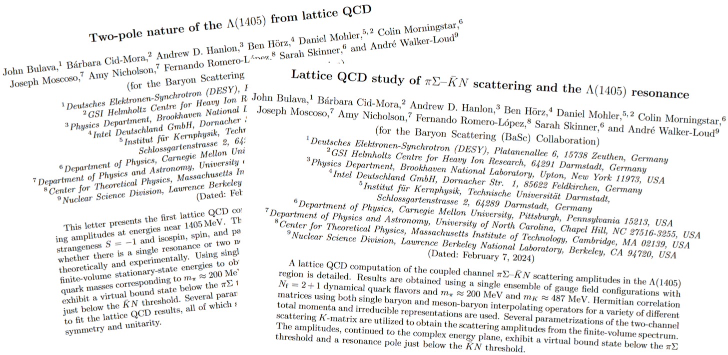
Bárbara Cid-Mora

April 3-5, 2024

HIM
HELMHOLTZ
Helmholtz-Institut Mainz

TECHNISCHE UNIVERSITÄT DARMSTADT

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research



Two-pole nature of the $\Lambda(1405)$ from lattice QCD

John Bulava,[1] Bárbara Cid-Mora,[2] Andrew D. Hanlon,[3] Ben Hörz,[4] Daniel Mohler,[5,2] Colin Morningstar,[6]
Joseph Moscoso,[7] Amy Nicholson,[7] Fernando Romero-L... Sarah Skinner,[6] and André Walker-Loud[9]
(for the Baryon Scattering ...

This letter presents the first lattice QCD co... ing amplitudes at energies near 1405 MeV. Th... strangeness $S=-1$ and isospin, spin, and pa... whether there is a single resonance or two n... theoretically and experimentally. Using singl... finite-volume stationary-state energies to ob... quark masses corresponding to $m_\pi \approx 200$ Me... exhibit a virtual bound state below the $\pi\Sigma$ ... just below the $\bar{K}N$ threshold. Several param... to fit the lattice QCD results, all of which ... symmetry and unitarity.

Lattice QCD study of $\pi\Sigma$–$\bar{K}N$ scattering and the $\Lambda(1405)$ resonance

John Bulava,[1] Bárbara Cid-Mora,[2] Andrew D. Hanlon,[3] Ben Hörz,[4] Daniel Mohler,[5,2] Colin Morningstar,[6]
Joseph Moscoso,[7] Amy Nicholson,[7] Fernando Romero-López,[8] Sarah Skinner,[6] and André Walker-Loud[9]
(for the Baryon Scattering (BaSc) Collaboration)

[1] *Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, 15738 Zeuthen, Germany*
[2] *GSI Helmholtz Centre for Heavy Ion Research, 64291 Darmstadt, Germany*
[3] *Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[4] *Intel Deutschland GmbH, Dornacher Str. 1, 85622 Feldkirchen, Germany*
[5] *Institut für Kernphysik, Technische Universität Darmstadt, Schlossgartenstrasse 2, 64289 Darmstadt, Germany*
[6] *Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[7] *Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27516-3255, USA*
[8] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[9] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
(Dated: February 7, 2024)

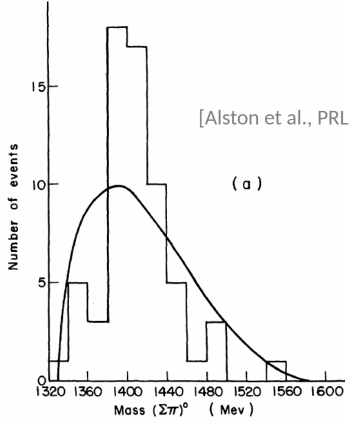
A lattice QCD computation of the coupled channel $\pi\Sigma$–$\bar{K}N$ scattering amplitudes in the $\Lambda(1405)$ region is detailed. Results are obtained using a single ensemble of gauge field configurations with $N_f = 2+1$ dynamical quark flavors and $m_\pi \approx 200$ MeV and $m_K \approx 487$ MeV. Hermitian correlation matrices using both single baryon and meson-baryon interpolating operators for a variety of different total momenta and irreducible representations are used. Several parametrizations of the two-channel scattering K-matrix are utilized to obtain the scattering amplitudes from the finite-volume spectrum. The amplitudes, continued to the complex energy plane, exhibit a virtual bound state below the $\pi\Sigma$ threshold and a resonance pole just below the $\bar{K}N$ threshold.

The two-pole nature of the $\Lambda(1405)$ from Lattice QCD *

Lattice QCD study of $\pi\Sigma - \bar{K}N$ scattering and the $\Lambda(1405)$ resonance **

* Letter: PRL **132** (2024) 5 $\quad$ ** Long paper: PRD **109** (2024) 1

Table of Contents



[Alston et al., PRL **6** (1961) 698]

$$\Lambda(1405) \rightarrow \quad I(J^P) = 0(\tfrac{1}{2}^-) \quad S = -1$$

- Theoretical Prediction $K^- p - \pi\Sigma$
  [Dalitz & Tuan, PRL **2** (1959) 425]
- Experimental evidence of resonance ($\pi\Sigma$ mass spectrum)
  [Alston et al., PRL **6** (1961) 698]
- Meson-Baryon comp. $\Lambda(1405)$ (Chiral sym.)
  [Veit et al., PLB **137** (1984) 415]
  [Jennings, PLB **176** (1986) 229]
- First time two-pole picture
  [Fink et al., PRC **41** (1990) 2720]
- Chiral dynamics: coupled-channel
  [Kaiser et al., NPA **594** (1995) 325]
  [Oset & Ramos, NPA **635** (1997) 99]
- SIDDHARTA at DAΦNE: $K^- p$ Scattering Length det.
  [Bazzi et al., PLB **704** (2011) 113]
- Spin & Parity by CLAS Collab.
  [Moriya et al., PRC **87** (2013) 035206]
  [Moriya et al., PRL **112** (2014) 082004]

One or two-pole picture?

Λ resonances

[PDG, PTEP **2022** (2022) 083C01]

Hadron	J^P	status
$\Lambda(1116)$	$1/2^+$	$(****)$
$\Lambda(1380)$	$1/2^-$	$(**)$
$\Lambda(1405)$	$1/2^-$	$(****)$
$\vdots$		

Is $\Lambda(1380)$ a second pole of the scattering amplitude in the complex energy plane in the $\Lambda(1405)$ region?

[Isgur & Karl, PRD **18** (1978) 4187]

[Oller & Meißner, PLB **500** (2001) 263]

[Roca & Oset, PRC **88** (2013) 055206]

[Mai & Meißner, EPJA **51** (2015) 30]

[Anisovich et al., EPJA **56** (2020) 139]

[Scheluchin et al., PLB **833** (2022) 137375]

[Wickramaarachchi et al., EPJ **271** (2022) 07005]

[Aikawa et al., PLB **837** (2023) 137637]

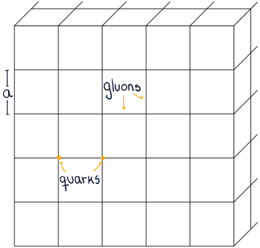
[Acharya et al., EPJC **83** (2023) 340]

⋇ [Bulava et al., PRL **132** (2024) 5]

⋇ [Bulava et al., PRD **109** (2024) 1]

Table of Contents

Our approach: Lattice



- Quarks and gluons in a finite size discretized grid
- Observables estimated by sampling gauge configurations
- Correlation functions are computed
- Finite-volume energy spectrum extraction

Importance of lattice QCD to study the $\Lambda(1405)$

→ Predictions once quark masses and couplings fixed

→ Facilitates exploration of the elastic region $\pi\Sigma - \bar{K}N$

→ Resulting motion of poles under variation of quark masses

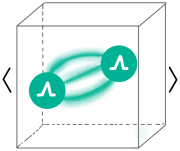
Lattice details

Details of the $D200$ ensemble generated by the Coordinated Lattice Simulations consortium (CLS) [Bruno et al., JHEP **02** (2015) 043]:

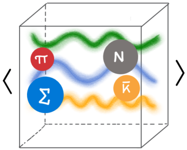
$a[fm]$	$(L/a)^3 \times (T/a)$	m_π	m_K
$0.0633(4)(6)$	$64^3 \times 128$	≈ 200 MeV	≈ 487 MeV

- 2000 gauge configurations
- Open temporal boundary conditions

Hadrons in the lattice



Single hadron operator in the latice (Λ)



Multihadron operators in the lattice ($\pi\Sigma$ and $\bar{K}N$)

- **Operators**
  $\rightarrow$ Single and meson-baryon
  - $\Lambda[\vec{P}]$
  - $\pi[\vec{P}_1]\ \Sigma[\vec{P}_2]$
  - $\bar{K}[\vec{P}_1]\ N[\vec{P}_2]$

$\Lambda(\mathbf{d}^2)$	Operators
$G_{1g}(0)$	$\Lambda[G_{1g}(0)]$ $\bar{K}[A_2(1)]\ N[G_1(1)]$ $\pi[A_2^-(1)]\ \Sigma[G_1(1)]$

Correlation functions

- **Correlation matrices** $\rightarrow$ Stochastic LapH method (sLaph)

  [Peardon et al., PRD **80** (2009) 054506] (Original distillation)

  [Morningstar et al., PRD **83** (2011) 114505]

$$C(t) = \langle \mathcal{O}_1(t)\bar{\mathcal{O}}_2(0)\rangle = \sum_n A_n e^{-tE_n}$$

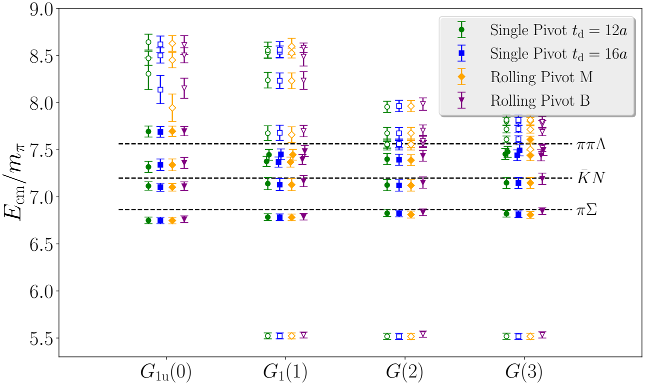
- **Extraction of energy spectra** $\rightarrow$ Solving the GEVP

  [Michael & Teasdale, NPB **215** (1983) 433]

  [Blossier et al., JHEP **04** (2009) 094]

$$C(t_\mathrm{d})\vec{v}_n(t_o, t_\mathrm{d}) = \lambda_n(t_o, t_\mathrm{d})\; C(t_o)\; \vec{v}_n(t_o, t_\mathrm{d})$$

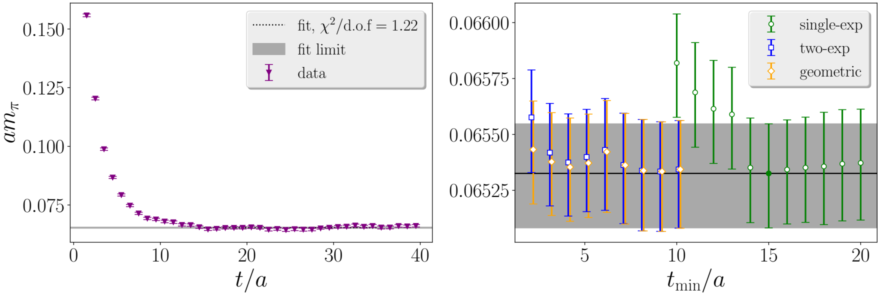
Single Pivot & Rolling Pivot



Center-of-mass finite-volume energy spectra results under variation of implementation of the GEVP method.

Table of Contents

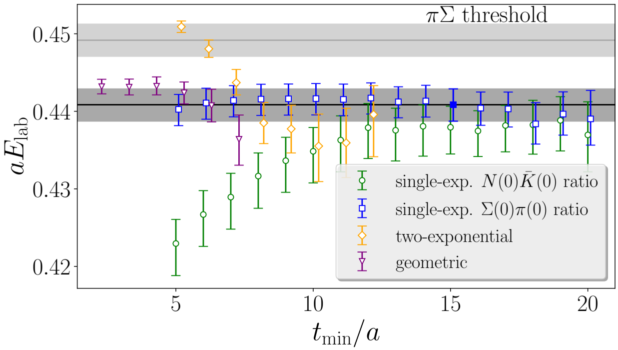
Single hadrons energy



Single hadrons results: π effective mass and variety of fits to Lattice data using different values of t_{min}.

Bulava et al., PRD **109** (2024) 1

Multi-hadron energy spectra



Multihadron results: Variety of fit forms to lattice data vs t_{min} in the energy laboratory frame. (Lowest level of the $G_{1u}(0)$ irrep)

Bulava et al., PRL **132** (2024) 5

Energy spectra

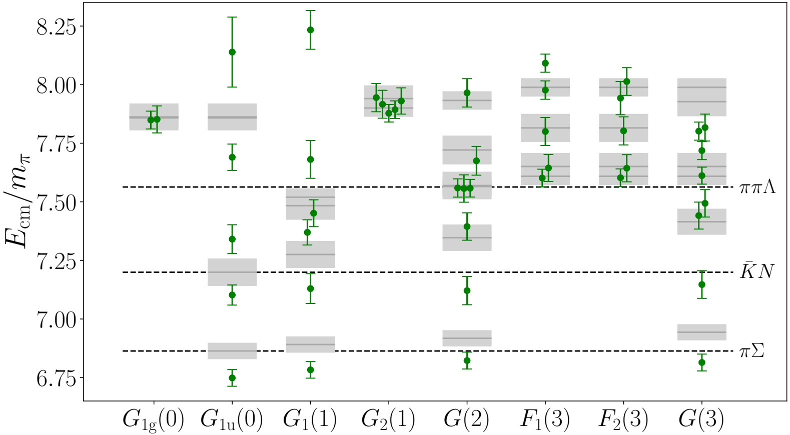
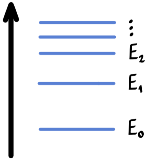


Table of Contents

Lüscher formalism

The recipe for scattering amplitudes goes as

[M. Lüscher, NPB **354** (1991) 53] [M. Lüscher, NPB **364** (1991) 237; and extensions.]



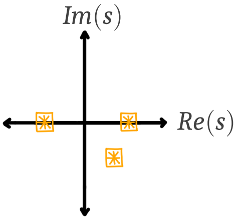
Discrete energy spectra

$$\det\left[\widetilde{K} + F^{-1}\right] = 0$$

Quantization condition

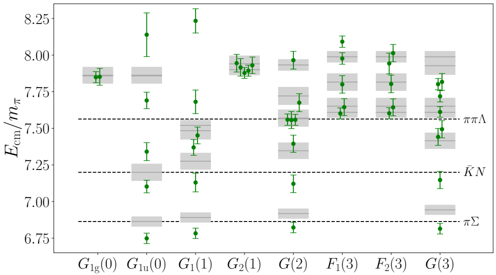
$$\mathcal{M}$$

Scattering matrix



Pole Positions

Quantization condition



Finite-volume energy spectra

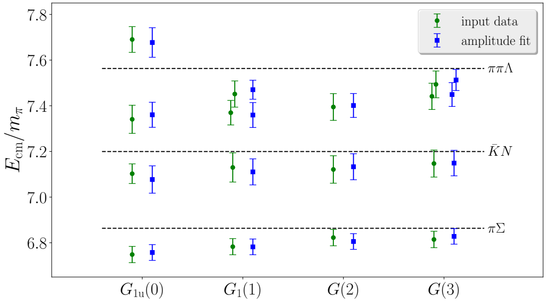
$$\det\left[\widetilde{K}+F^{-1}\right]=0$$

Quantization condition

Scattering amplitude parametrization

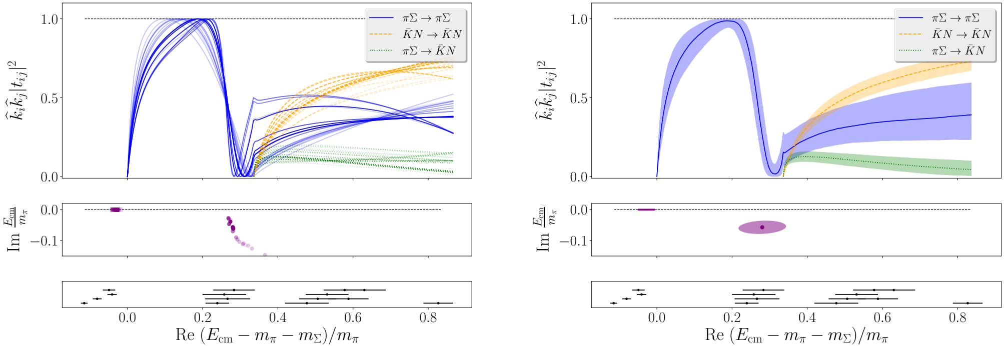
$$\det\left[\begin{pmatrix}\widetilde{K}_{\pi\Sigma\to\pi\Sigma} & \widetilde{K}_{\pi\Sigma\to\bar{K}N}\\ \widetilde{K}_{\bar{K}N\to\pi\Sigma} & \widetilde{K}_{\bar{K}N\to\bar{K}N}\end{pmatrix}+\begin{pmatrix}F^{-1}_{\pi\Sigma}(E_n,\vec{P},L) & 0\\ 0 & F^{-1}_{\bar{K}N}(E_n,\vec{P},L)\end{pmatrix}\right]$$

→ $\widetilde{K}$ – matrix parametrization



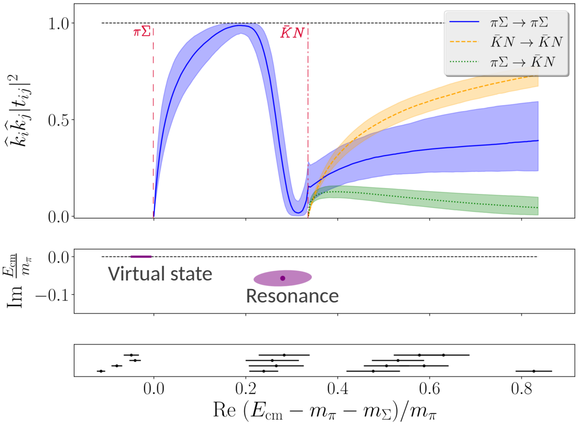
$$t^{-1}=\widetilde{K}^{-1}-i\hat{k}$$

$$t=\frac{m_\pi}{E_{\rm cm}-E_{\rm pole}}\begin{pmatrix}c^2_{\pi\Sigma} & c_{\pi\Sigma}c_{\bar{K}N}\\ c_{\pi\Sigma}c_{\bar{K}N} & c^2_{\bar{K}N}\end{pmatrix}+\dots$$



(Left) Scattering amplitude results based on different parametrizations

(Right) "Preferred" parametrization of the scattering amplitude



Virtual bound state

$$E_1 = 1392(9)_{\mathrm{st}}(2)_{\mathrm{md}}(16)_{\mathrm{a}}\,\mathrm{MeV}$$

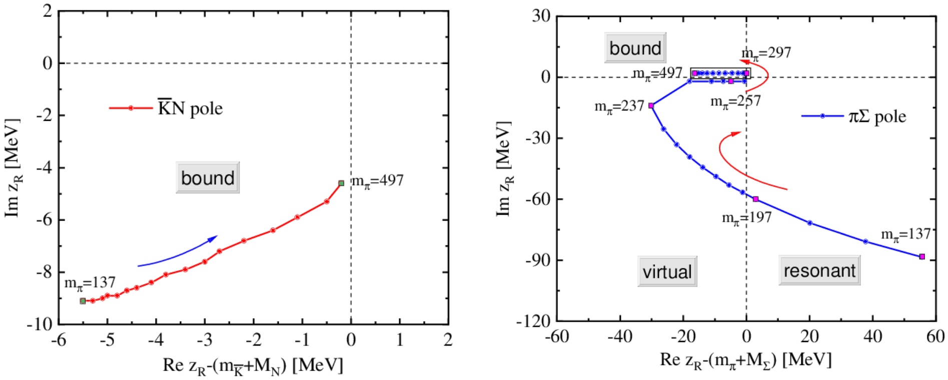
$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\mathrm{st}}(6)_{\mathrm{md}}$$

Resonance

$$E_2 = \left[1455(13)_{\mathrm{st}}(2)_{\mathrm{md}}(17)_{\mathrm{a}} - i11.5(4.4)_{\mathrm{st}}(4.0)_{\mathrm{md}}(0.1)_{\mathrm{a}}\right]\mathrm{MeV}$$

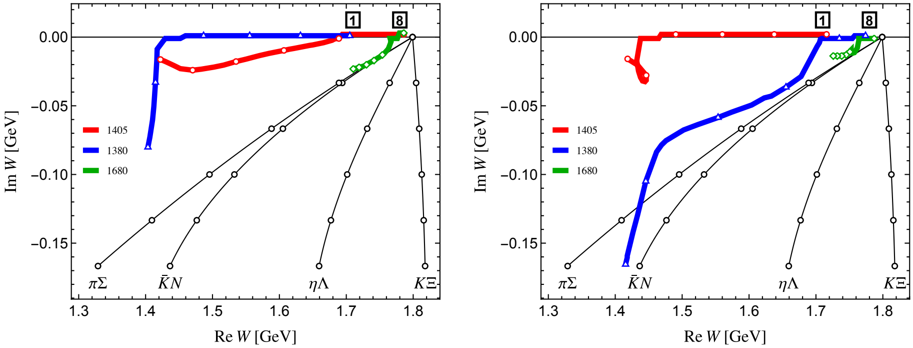
$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2}_{\bar{K}N}}\right| = 0.53(9)_{\mathrm{st}}(10)_{\mathrm{md}}$$

Example I of motion of the poles vs m_π



Trajectories of the two poles of $\Lambda(1405)$ as functions of the pion mass m_π from 137 MeV to 497 MeV. Critical masses are labeled by solid squares, between which the points are equally spaced. ($z_R = m_R - i\Gamma_R/2$) [Xie et al., PRD **108** (2023) 11]

Example II of motion of the poles vs m_π



Motion of the poles from the $SU(3)$ limit to the physical values of the particle masses. Blue, red and green lines denote the $\Lambda(1380)$, $\Lambda(1405)$ and $\Lambda(1680)$, respectively. Left panel: Weinberg-Tomozawa term; Right panel: includes NLO. [Guo et al., PLB **846** (2023) 138264]

Table of Contents

Summary

- First Lattice QCD study of coupled-channel $\pi\Sigma - \bar{K}N$ in the $\Lambda(1405)$ region
- Every parametrization used found two poles in this region
  - **NOTE:** These parametrizations could accommodate zero, one or two poles
- Our results show qualitative agreement with phenomenological extractions [See PDG, section 83]

Lower Pole: $E_1 = 1392(9)_{\text{stat}}(2)_{\text{model}}(16)_{\text{a}}\ \text{MeV}$

Higher Pole: $E_2 = [1455(13)_{\text{stat}}(2)_{\text{model}}(17)_{\text{a}} - i11.5(4.4)_{\text{stat}}(4.0)_{\text{model}}(0.1)_{\text{a}}]\ \text{MeV}$

Reference Results: $\mathcal{Re}\,(E_1) = 1325 - 1380\ \text{MeV};\quad \mathcal{Re}\,(E_2) = 1421 - 1434\ \text{MeV}$

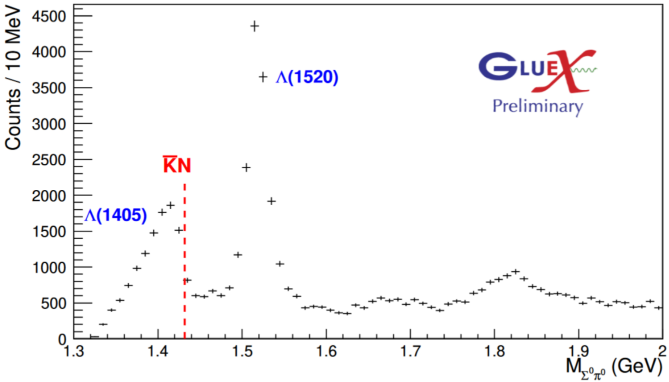
- **Future work:**
  - Explore quark masses dependence of the poles: strange mass dependence
  - Explore impact of three particle operators
  - Study lattices with a closer to physical m_π

Thanks

Table of Contents

GlueX Preliminary analysis

Back-up



Wickramaarachchi et al., EPJ Web Conf. 271 (2022) 07005, [e-Print: 2209.06230]

Lattice details

Ensemble **D200** generated by CLS was used. Its properties are:

- Dynamical mass-degenerate $u-$ and $d-$quarks (heavier than physical), and $s-$quarke (lighter than physical).
- Tree-level improved Lüscher-Weisz gauge action.
- Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermion action.

Effective masses and correlated-χ^2

Back-up

The effective mass is calculated as:

$$m_{eff}(t+1/2) = \ln\left(\frac{C(t)}{C(t+1)}\right)$$

$$\chi^2 = \sum_{t,t'=t_{min}}^{t_{max}} (C(t) - f(t)) \frac{1}{Cov_N(t,t')} (C(t') - f(t'))$$

where

$$f(t) = A_i e^{-E_i t}$$

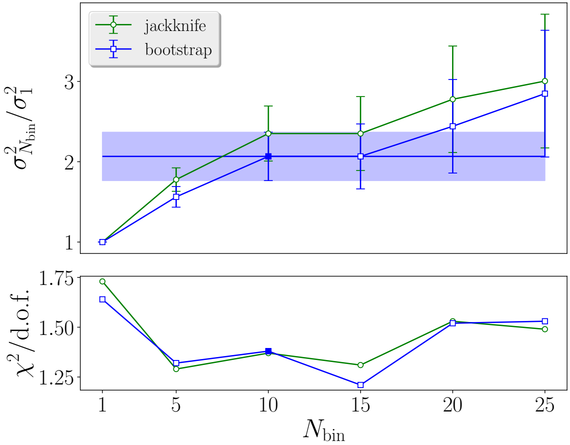
and

$$Cov_N(t,t') = \frac{1}{N-1} \langle (C(t) - \langle C(t) \rangle_N)(C(t') - \langle C(t') \rangle_N) \rangle_N$$

t, t' : lattice time E_i : Energy N : Nr. samples $\langle \dots \rangle_N$: statistical average

Binning results

Back-up



(Top) Ratios of variances for fits to m_π versus $N_{\rm bin}$ for jackknife and bootstrap resampling.
(Bottom) Correlated-χ^2 of two-exponential fit to m_π versus $N_{\rm bin}$. In both panels, the final binning choice is illustrated as a blue solid square.

Correlation function fits

Back-up

Several fit forms used to extract the finite-volume energy spectra:

$$C(t) = A_n e^{-tE_n} \text{ (Single exponential)}$$

$$C(t) = A_n e^{-tE_n} + A_1 e^{-tD^2} \text{ (Two-exponential)}$$

$$C(t) = \frac{A_n e^{-tE_n}}{1 - Be^{-Mt}} \text{ (Geometric)}$$

$$R_n(t) = \frac{D_n(t)}{C_A(\mathbf{d}_A^2, t) C_B(\mathbf{d}_B^2, t)} = A_n e^{-t\Delta E_n} \text{ (Ratio of correlators)}$$

Single Hadron results

Back-up

am_π	$0.06533(25)$	am_K	$0.15602(16)$	am_N	$0.3143(37)$
am_Λ	$0.3634(14)$	am_Σ	$0.3830(19)$	am_Ξ	$0.41543(96)$

Table: Summary of hadron masses in Lattice units.

am_π	~ 200	am_K	~ 487	am_N	~ 980
am_Λ	~ 1120	am_Σ	~ 1194	am_Ξ	~ 1295

Table: Summary of hadron masses in MeV units.

Basically one searches for the zero's of the following equation, using the finite-volume energy spectra as constrain.

$$\det\left[\underbrace{\begin{pmatrix}\widetilde{K}_{\pi\Sigma\to\pi\Sigma} & \widetilde{K}_{\pi\Sigma\to\bar{K}N} \\ \widetilde{K}_{\bar{K}N\to\pi\Sigma} & \widetilde{K}_{\bar{K}N\to\bar{K}N}\end{pmatrix}}_{\text{Multi-channel Matrix}} + \underbrace{\begin{pmatrix}F^{-1}_{\pi\Sigma}(E_n,\vec{P},L) & 0 \\ 0 & F^{-1}_{\bar{K}N}(E_n,\vec{P},L)\end{pmatrix}}_{\text{Zeta Function}}\right] = 0$$

One fits with respect to the energy shifts of the non-interacting energies:

$$\Delta E_i = E^{latt}_{\rm cm} - E^{free}_{\rm cm}$$

Where one minimize correlated χ^2:

$$\delta_i = \Delta E_{\rm cm,i} - \Delta E^{\rm QC}_{\rm cm,i}$$

And the preferred fit is based on lowest Akaike Information Criterion:

$$\text{AIC} = \chi^2 - 2n_{dof}$$

Amplitude Analysis

Back-up

The following quantity is defined proportional to the scattering transition amplitude and to $\widetilde{K}$ as:

$$t^{-1} = \widetilde{K}^{-1} - i\hat{k}$$

where $\hat{k} = \mathrm{diag}(k_{\pi\Sigma}, k_{\bar{K}N})$, and

$$k^2_{\pi\Sigma} = \frac{1}{E^2_{\mathrm{cm}}}\lambda_K\left(E^2_{\mathrm{cm}}, m^2_\pi, m^2_\Sigma\right)$$
$$k^2_{\bar{K}} = \frac{1}{E^2_{\mathrm{cm}}}\lambda_K\left(E^2_{\mathrm{cm}}, m^2_{\bar{K}}, m^2_N\right)$$

where λ_K is the Källén function. Which is equivalent to searching for the ∞'s of

$$t = \frac{m_\pi}{E_{\mathrm{cm}} - E_{\mathrm{pole}}}\begin{pmatrix} c^2_{\pi\Sigma} & c_{\pi\Sigma}c_{\bar{K}N} \\ c_{\pi\Sigma}c_{\bar{K}N} & c^2_{\bar{K}N}\end{pmatrix} + \ldots$$

1. An effective range expansion (ERE) of the form

$$\widetilde{K}_{ij} = \frac{m_\pi}{E_{\rm cm}} \Big(A_{ij} + B_{ij} \Delta_{\pi\Sigma}(E_{\rm cm}) \Big). \tag{1}$$

2. A variation of the first parametrization without the factor of $m_\pi / E_{\rm cm}$:

$$\widetilde{K}_{ij} = \widehat{A}_{ij} + \widehat{B}_{ij} \Delta_{\pi\Sigma}(E_{\rm cm}). \tag{2}$$

3. An ERE of $\widetilde{K}^{-1}$ of the form

$$\widetilde{K}^{-1}_{ij} = \frac{E_{\rm cm}}{m_\pi} \left(\widetilde{A}_{ij} + \widetilde{B}_{ij} \Delta_{\pi\Sigma}(E_{\rm cm}) \right) . \tag{3}$$

1. A Blatt-Biederharn parametrization:

$$\widetilde{K} = C\,F\,C^{-1}, \tag{4}$$

where

$$C = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix}, \tag{5}$$

$$F = \begin{pmatrix} f_0(E_{\rm cm}) & 0 \\ 0 & f_1(E_{\rm cm}) \end{pmatrix}, \tag{6}$$

and

$$f_i(E_{\rm cm}) = \frac{m_\pi}{E_{\rm cm}}\,\frac{a_i + b_i\Delta_{\pi\Sigma}(E_{\rm cm})}{1 + c_i\Delta_{\pi\Sigma}(E_{\rm cm})}. \tag{7}$$

Scattering amplitude parametrization

Back-up

$$\det\left[\begin{pmatrix}\widetilde{K}_{\pi\Sigma\to\pi\Sigma} & \widetilde{K}_{\pi\Sigma\to\bar{K}N}\\ \widetilde{K}_{\bar{K}N\to\pi\Sigma} & \widetilde{K}_{\bar{K}N\to\bar{K}N}\end{pmatrix}+\begin{pmatrix}F^{-1}_{\pi\Sigma}(E_n,\vec{P},L) & 0\\ 0 & F^{-1}_{\bar{K}N}(E_n,\vec{P},L)\end{pmatrix}\right]=0$$

$\widetilde{K}$ – matrix parametrization

$$\begin{cases}\widetilde{K}_{ij}=\frac{m_\pi}{E_{cm}}\left(A_{ij}+B_{ij}\Delta_{\pi\Sigma}\right)\\ \widetilde{K}^{-1}_{ij}=\frac{E_{cm}}{m_\pi}\left(\widetilde{A}_{ij}+\widetilde{B}_{ij}\Delta_{\pi\Sigma}\right)\\ \widetilde{K}=CFC^{-1}\\ \widetilde{K}_{ij}=\widehat{C}_{ij}(2E_{cm}-M_i-M_j)\end{cases}$$