Constraining the $\pi\Sigma - \bar{K}N$ models with the $\pi\Sigma$ photoproduction data

A. Cieplý

Nuclear Physics Institute, Řež, Czechia

1. Chirally motivated $\pi\Sigma - \bar{K}N$ interactions (results overview)
2. $\pi\Sigma$ photoproduction mass spectra
   - Formalism
   - Results
   - Current development
3. Summary

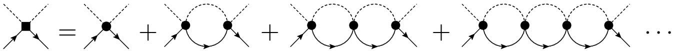
collaboration with P. C. Bruns, partly with M. Mai (Bonn) and J. Smejkal (Prague)

Chirally motivated $\bar{K}N$ interactions

$\bar{K}N - \pi\Sigma$ system (+ add-ons, mostly more MB)
meson octet - baryon octet coupled channels interactions

involved channels	$\pi\Lambda$	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
thresholds (MeV)	1250	1330	1435	1660	1740	1810

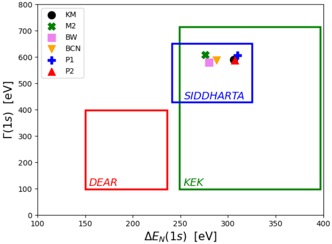
- strongly interacting multichannel system with an s-wave resonance, the $\Lambda(1405)$, just below the $K^- p$ threshold
- modern theoretical treatment based on effective chiral Lagrangians
- effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series

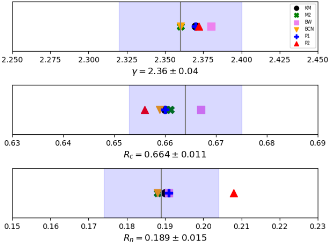


N. Kaiser, P.B. Siegel, W. Weise - Nucl. Phys. A 594 (1995) 325

$K^- p$ data fits: low energy x-sections, threshold BRs, kaonic hydrogen 1s level

Experimental data reproduction





left - kaonic hydrogen characteristics:
1s level energy shift $\Delta E_N(1s)$ and absorption width $\Gamma(1s)$

right - threshold branching ratios:

$$\gamma = \frac{\Gamma(K^- p \to \pi^+\Sigma^-)}{\Gamma(K^- p \to \pi^-\Sigma^+)} \, ,$$

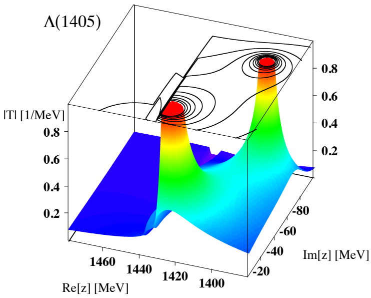
$$R_n = \frac{\Gamma(K^- p \to \pi^0\Lambda)}{\Gamma(K^- p \to \text{neutral states})} \, ,$$

$$R_c = \frac{\Gamma(K^- p \to \pi^+\Sigma^-, \pi^-\Sigma^+)}{\Gamma(K^- p \to \text{inelastic channels})}$$

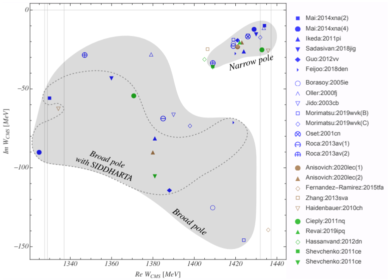
$\Lambda(1405)$ resonance

- long history since the prediction by Dalitz and Tuan in 1959
- $\Lambda(1405)\,1/2^-$ is much lighter than $N^*(1535)$ and a potential spin-orbit partner $\Lambda(1520)\,3/2^-$ which is difficult to explain within a standard constituent quark model
- hadronic molecule, a loosely bound $\bar{K}N$ state? a pentaquark?
- most common interpretation - $\bar{K}N$ quasi-bound state submerged in $\pi\Sigma$ continuum, a result of coupled channels $\pi\Sigma - \bar{K}N$ dynamics
- unitary coupled channels approaches based on effective chiral Lagrangian generate two poles related to $\Lambda(1405)$ (Oller, Meißner in 2001)
- topics related to $\Lambda(1405)$ include $\bar{K}$-nuclei and role of strangeness in dense nuclear matter (e.g. neutron stars)

Model predictions - Λ(1405) resonance



Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55



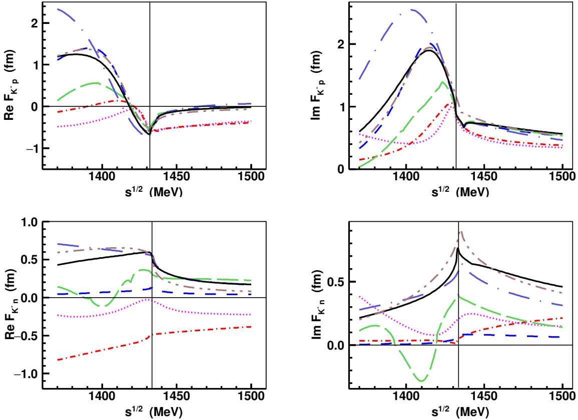
all recent (year $\geq$ 2000) predictions

M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

- the *higher* pole around 1425 MeV couples more strongly to $\bar{K}N$, the *lower* pole is much further from the real axis and has larger coupling to $\pi\Sigma$
- all models tend to agree on the position of the $\bar{K}N$ related pole
- the K^-p reactions data are not very sensitive to the position of the $\pi\Sigma$ related pole

Model predictions - $K^- N$ amplitudes

$K^- p$ and $K^- n$ elastic amplitudes

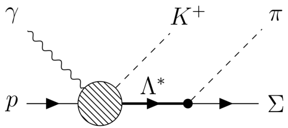


models: Kyoto-Munich, Prague, Bonn, Murcia, Barcelona
updated, original in A.C., M.Mai, U.G.Meißner, J.Smejkal - NPA 954 (2016) 17

Chiral models in need of more data

- $\pi\Sigma - \bar{K}N$ models fitted to experimental data available at energies from $K^- p$ threshold up, provide varied theoretical predictions for subthreshold energies and in the isovector sector.
- In the $K^- p$ reactions data, the $\Lambda(1405)$ is hidden below the threshold. The resonance can be seen in processes, where $\pi\Sigma$ re-scatter in the final state, e.g.

$$\gamma p \longrightarrow K^+ \pi\Sigma$$

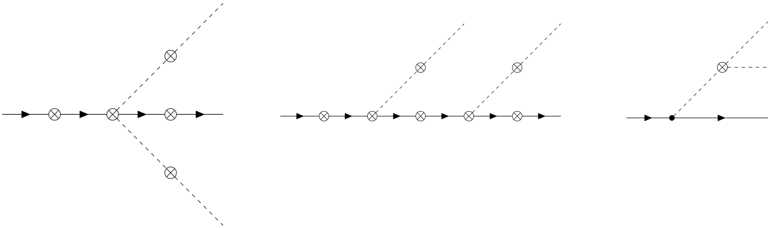


In this two meson protoproduction reaction the K^+ meson carries away momentum, enabling a scan in the invariant mass of the $\pi\Sigma$ system down to its production threshold.

$\pi\Sigma$ photoproduction: Formalism

formalism outlined in: P. C. Bruns - arXiv:2012.11298 [nucl-th] (2020)
application to $\pi\Sigma$ mass spectra predictions:
P. C. Bruns, A. C., M. Mai - Phys. Rev. D 106 (2022) 7, 074017

leading-order BχPT used to derive expressions for the photoproduction amplitude $\mathcal{M}$, constructed from tree level graphs:



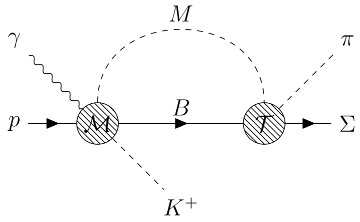
Weinberg-Tomozawa (WT) Born term (BT) - B1, B2 anomalous (AN)

lines: directed - baryons, dashed - pseudoscalar mesons; $\otimes$ symbols - photon insertions

$5 + (2 \times 7) + 1 = 20$ tree graphs, 16 independent $\mathcal{M}_j$ structure functions

$\pi\Sigma$ photoproduction: Formalism

Final state interaction of the MB pair needs to be accounted for:



$\pi\Sigma - \bar{K}N$ coupled channels models provide the $f_{\ell\pm}^{c',c}(M_{\pi\Sigma})$ amplitudes, that describe the scattering from channel c to channel c' ($c, c' = \pi\Lambda,\ \pi\Sigma,\ \bar{K}N,\ \eta\Lambda, \ldots$)

unitarized amplitudes for $\gamma p \to K^+ MB$ will be taken as the coupled-channel vector

$$[\mathcal{A}^{i}_{0+}] = [\mathcal{A}^{i(\mathrm{tree})}_{0+}] + [f_{0+}]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[\mathcal{A}^{i(\mathrm{tree})}_{0+}]$$

gauge invariance guarantied by the $\mathcal{A}^{i(\mathrm{tree})}_{0+}$ amplitudes construction

The second term represents the final-state MB rescattering and $G(M_{\pi\Sigma})$ is a diagonal channel-space matrix with entries given by regularized loop integrals.

$\pi\Sigma$ photoproduction: Formalism

There are four independent structure functions $\mathcal{A}^i_{0+}(s, M^2_{\pi\Sigma}, t_K)$ constructed from $\mathcal{M}_j$, projected on s-wave and satisfying the partial-wave unitarity relation

$$\mathrm{Im}(\mathcal{A}^i_{0+}) = (f_{0+})^\dagger(|\vec{p}^{\,*}|)(\mathcal{A}^i_{0+})\,, \quad i = 1, \ldots, 4\,.$$

Neglecting $\ell > 0$ contributions, we get

$$\frac{d^2\sigma}{d\Omega_K dM_{\pi\Sigma}} = \frac{|\vec{q}_K||\vec{p}^{\,*}_\Sigma|}{(4\pi)^4 s|\vec{k}|}|\mathcal{A}|^2\,,$$

$$\begin{aligned}
4|\mathcal{A}|^2 &= (1-z_K)\left|\mathcal{A}^1_{0+} + \mathcal{A}^2_{0+}\right|^2 + (1+z_K)\left|\mathcal{A}^1_{0+} - \mathcal{A}^2_{0+}\right|^2 \\
&+ (1-z_K)\left|\mathcal{A}^1_{0+} + \mathcal{A}^2_{0+} + \frac{2|\vec{q}_K|(1+z_K)}{M_K^2 - t_K}\left((\sqrt{s}+m_N)\mathcal{A}^3_{0+} + (\sqrt{s}-m_N)\mathcal{A}^4_{0+}\right)\right|^2 \\
&+ (1+z_K)\left|\mathcal{A}^1_{0+} - \mathcal{A}^2_{0+} - \frac{2|\vec{q}_K|(1-z_K)}{M_K^2 - t_K}\left((\sqrt{s}+m_N)\mathcal{A}^3_{0+} - (\sqrt{s}-m_N)\mathcal{A}^4_{0+}\right)\right|^2,
\end{aligned}$$

with $z_K \equiv \cos\theta_K$, θ_K being the angle between $\vec{q}_K$ and $\vec{k}$ in the overall c.m. frame.

$\pi\Sigma$ photoproduction: Formalism

loop function integral:

$$\mathrm{i}\, G^{c=bj}(M_{\pi\Sigma}) = \int_{\mathrm{reg.}} \frac{d^4 l}{(2\pi)^4} \frac{1}{((p_b + q_j - l)^2 - m_b^2 + \mathrm{i}\epsilon)(l^2 - M_j^2 + \mathrm{i}\epsilon)}$$

two coupled channels approaches to generate the f_{0+} amplitudes:

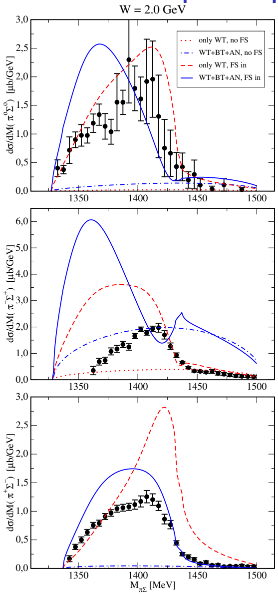
Bonn B2, B4 models - M.Mai, U.-G.Meißner, Eur. Phys. J. A 51 (2015) 30
BW model - D.Sadasivan, M.Mai, M.Döring, Phys. Lett. B 789 (2019) 329–335
dimensional regularization used in $G(M_{\pi\Sigma})$, mass scales μ_c

Prague P model - P.C.Bruns, A.C., Nucl. Phys. A 1019 (2022) 122378
Yamaguchi form factors used in $G(M_{\pi\Sigma})$, inverse ranges α_c

CLAS data: K. Moriya et al. - Phys. Rev. C 87 (2013) 035206

Result: different models provide varied predictions of the $\pi\Sigma$ mass spectra

$\pi\Sigma$ photoproduction: FSI impact



CLAS data (2013) by Moriya et al.

c.m. energy $W = \sqrt{s} = 2.0\ \mathrm{GeV}$

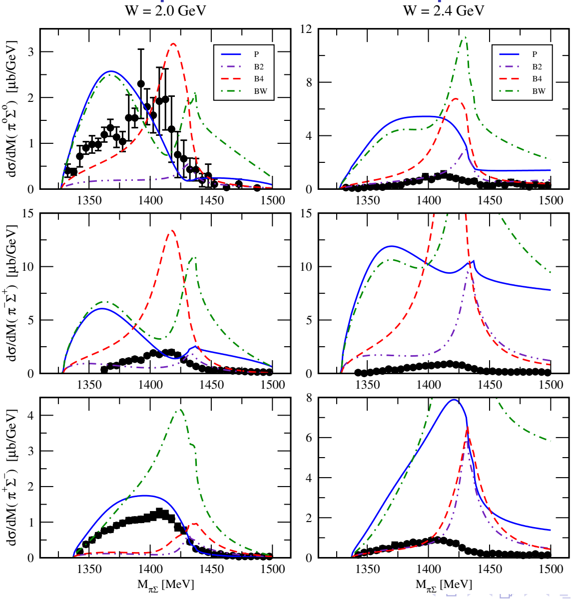
P model used for the *MB* amplitudes

- only WT, no FSI:
  small (or zero for $\pi^+\Sigma^-$) cross sections
- WT+BT+AN, no FSI:
  the cross sections remain flat, the $\pi^-\Sigma^+$ one reaches magnitude comparable with the data
- addition of FSI:
  MB rescattering is responsible for the peak structure
  $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$ reproduced rather well
  Born terms move the peak to lower energies

parameter-free predictions!

no adjustment to the f_{0+} amplitudes

πΣ mass spectra - model dependence



New results - FSI not fixed

First time fits to combined $K^- p$ reactions and $\pi\Sigma$ photoproduction data with the FSI no longer fixed at a particular $\pi\Sigma - \bar{K}N$ model parameters setup.
A. C., P. C. Bruns - Nucl. Phys. A 1043 (2024) 122819

- The P model used for the $MB \to \pi\Sigma$ amplitude represented by the MB rescattering $\mathcal{T}$ bubble:
  6 regularization scales α_c, 6 NLO couplings (b_0, b_F, $d_{1...4}$)
- Tree level photoproduction amplitudes representing the $\mathcal{M}$ bubble multiplied by MB and K^+ form factors: 6 regularization scales β_c (3 fixed), and β_K
- Yamaguchi forms adopted for all form factors $g_c(k^*) = 1/[1 + (k^*/\alpha_c)^2]$ etc.
- fitted data: kaonic hydrogen characteristics, $K^- p$ threshold branching ratios, $K^- p$ reaction cross sections, $\pi\Sigma$ photoproduction mass distributions at $\sqrt{s} = 2.1$ GeV

CLAS data			$K^- p$ threshold		$K^- p$ cross sections								all
$\pi^0\Sigma^0$	$\pi^-\Sigma^+$	$\pi^+\Sigma^-$	atom	BRs	$\pi^0\Lambda$	$\pi^0\Sigma^0$	$\pi^-\Sigma^+$	$\pi^+\Sigma^-$	$K^- p$	$\bar{K}^0 n$	$\eta\Lambda$	$\eta\Sigma^0$	total
34	30	32	2	3	4	4	31	32	27	22	24	7	252

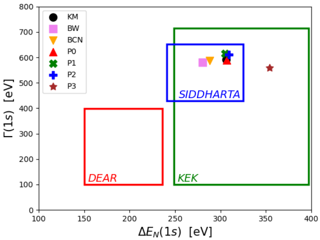
$$\chi^2/\mathrm{dof} = \frac{\sum_i N_i}{N_{obs}(\sum_i N_i - N_{par})} \sum_i \frac{\chi_i^2}{N_i}\,, \quad N_{obs} = 13\,, \quad N_{par} = 16$$

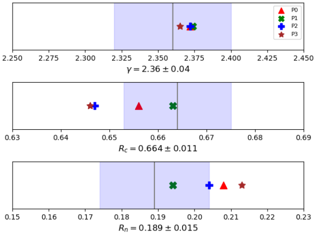
New results - FSI not fixed

results for 4 selected solutions (local χ^2 minima):

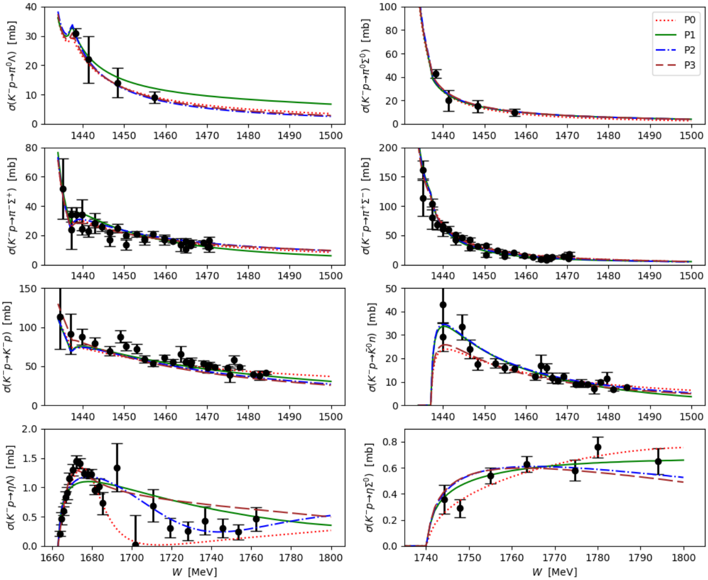
- P0 $\chi^2/\mathrm{dof} \approx 5.40$, *MB* FSI sector fixed to the P model setting, 4 parameters
- P1 $\chi^2/\mathrm{dof} \approx 3.34$, both, FSI and tree level photoproduction sectors varied, 16 par.
- P2 $\chi^2/\mathrm{dof} \approx 4.41$, same as for P1
- P3 $\chi^2/\mathrm{dof} \approx 4.72$, same as for P1

$K^- p$ threshold data:

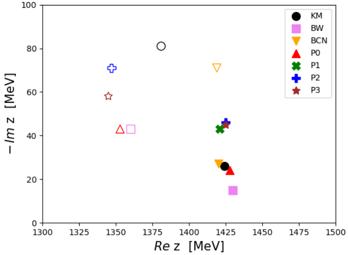




$K^- p \to MB$ total cross sections



Λ(1405) poles predictions

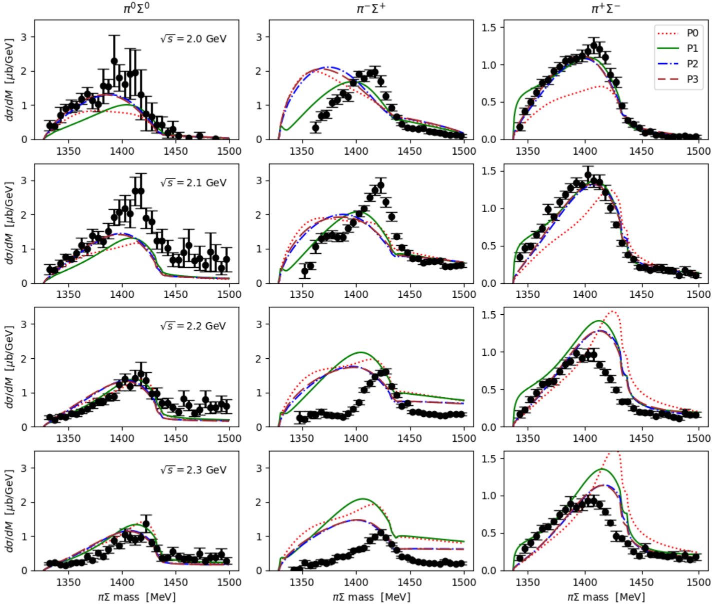


Λ(1405) - z_1 and z_2; Λ(1670) - z_3

model	z_1 [MeV]	z_2 [MeV]	z_3 [MeV]
P0	(1353,-43)	(1428,-24)	(1677,-14)
P1	—	(1421,-43)	—
P2	(1347,-71)	(1425,-46)	(1725,-57)
P3	(1345,-58)	(1425,-45)	(1665,-7.1)

- The P1 model provides the best χ^2 but was found unphysical due to generating an extremely narrow $I = 1$ resonance close to $\pi\Sigma$ threshold. It is also missing the *lower mass* Λ(1405) and the Λ(1670) poles.
- Apparently, the $K^- p$ reaction (and to some extend threshold) data are not very sensitive to the pole positions. Similar observations by Revai (with a non-chiral model), Shevchenko (phenomenology), or Anisovich (partial wave analysis), all indicating good reproduction of the $K^- p$ data with one-pole Λ(1405) models.
- All our models agree on the position of the *higher mass* Λ(1405) pole with Im z_2 much larger than in the fits based merely on the $K^- p$ reactions data. The *lower mass* Λ(1405) pole seems to be constrained around $z_1 \approx (1350, -60)$ MeV. Model dependence should be checked!

$\pi\Sigma$ mass spectra predictions



$\pi\Sigma$ mass spectra predictions

- Despite being rather simple, the model reproduces reasonably well the $\pi^0\Sigma^0$ and $\pi^+\Sigma^-$ mass distributions.
- The energy dependence seems to be under control thanks to introducing the g_{K^+} form-factor in the tree level photoproduction amplitudes.
- Our model fails to reproduce the $\pi^-\Sigma^+$ mass distributions.
- χ^2 applied selectively on the K^-p reactions and $\pi\Sigma$ photoproduction sectors:

model	P0	P1	P2	P3
χ^2/dof	5.40	3.34	4.41	4.72
$\chi^2/\mathrm{dof}\ (K^-p)$	1.52	1.93	2.06	2.74
$\chi^2/\mathrm{dof}\ (\pi\Sigma)$	19.2	9.14	13.9	12.9

  the $\pi\Sigma - \bar{K}N$ models fitted exclusively to the K^-p data can achieve $\chi^2/\mathrm{dof} \sim 1$
- The applied $\pi\Sigma$ photoproduction formalism may not be realistic: *p*-waves, vector meson contributions?. Thus, the results should be taken with caution and may change once a more advanced photo-kernel is implemented.

Comparison with other approaches

our photoproduction amplitude constructed from four $\mathcal{A}^i_{0+}$ amplitudes

$$[\mathcal{A}^i_{0+}(s, M_{\pi\Sigma})] = [\mathcal{A}^{i(\mathrm{tree})}_{0+}(s, M_{\pi\Sigma})] + [f_{0+}(M_{\pi\Sigma})]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[\mathcal{A}^{i(\mathrm{tree})}_{0+}(s, M_{\pi\Sigma})]$$

L. Roca, E. Oset - Phys. Rev. C 87 (2013) 055201

M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30

makeshift photoproduction amplitude $[\mathcal{A}] = [f_{0+}]\,[8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})]\,[C(\sqrt{s})]$

S.X. Nakamura, D. Jido - PTEP 2014 (2014) 023D01

similar to our approach with some non-relativistic simplifications, additional contributions from K^* exchange, phenomenological energy dependent contact terms, and adjustments to the first loop function and to the photoproduction vertex

E. Wang et al. - Phys. Rev. C 95 (2017) 015205

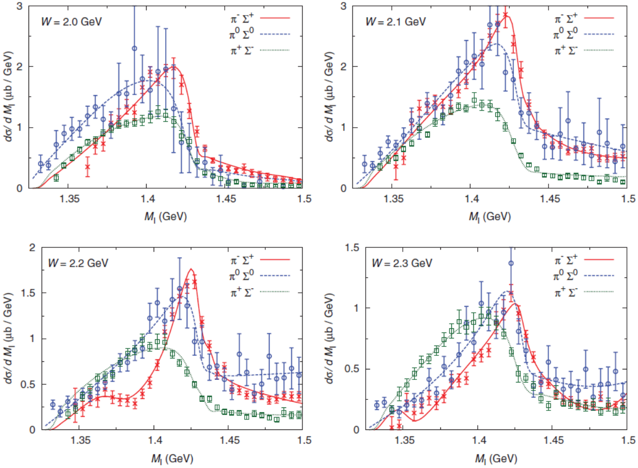
focus on triangle singularity contribution $\gamma p \to N^*(2030) \to K^*\Sigma \to K^+\Lambda(1405)$ combined with K, K^* meson exchanges and a contact term

$\mathcal{A} = \mathcal{A}^{\mathrm{tree}} = a\, t_{\mathrm{triangle}} + b\, t_{K\ \mathrm{exchange}} + c\, t_{K^*\ \mathrm{exchange}} + d\, t_{\mathrm{contact}}$

All these fit a good number of model parameters to reproduce the CLAS data.

In contrast, we just demonstrated what can be achieved with just four new parameters (originally parameter-free!) approach based on (unitarized) ChPT.

Results by Nakamura, Jido (2014)

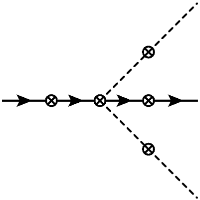
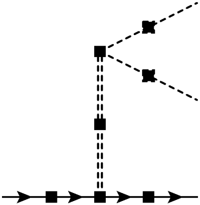
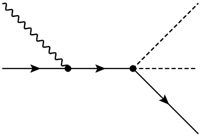
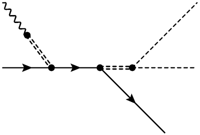


much better agreement with the CLAS photoproduction data, but 20 subtraction constants, 15 energy dependent complex couplings λ_n^j, common form factor scale $\beta_c = \beta_{K^+} = \Lambda$ (51 real parameters, 30 energy dependent!)

Current development

enhanced photo-kernel by including processes involving vector mesons (in particular K^*), adopting the concept of hidden local symmetries (HLS) and VM dominance, Lagrangian from Bando, Kugo, Yamawaki - Phys. Rept. 164 (1988), 217

vector mesons implementation: WT replaced by lighthouse diagrams

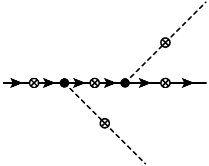
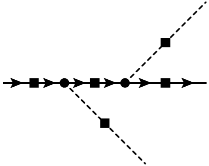
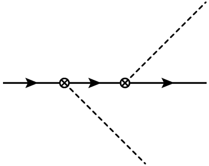
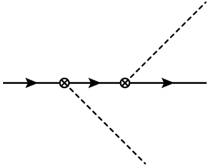
tag	LOχ (WT)	HLS (lighthouse)
all		
sample		

Current development

some comments on the WT/lighthouse graphs:

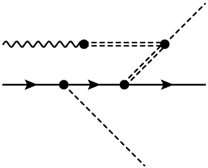
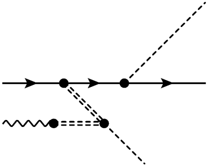
- The lighthouse graphs in which the photon/VM attaches to the baryon or meson line generate the amplitudes that match exactly those obtained in the LOχ approach for $M_V \to \infty$.
- The lighthouse graphs with the photon/VM connecting to the VM propagator are exclusive for the approach based on the HLS and the respective amplitudes vanish in the limit $M_V \to \infty$.
- Our preliminary results with just one lighthouse amplitude indicate large contributions of the K^* mesons to the $I = 1$ sector, which might be just what we need to reproduce the $\pi^-\Sigma^+$ mass spectra.
- Our evaluation of the lighthouse graphs is almost completed but it is impossible to predict what will be the final picture, especially once we introduce the BT and AN graphs in the HLS approach.

Current development - BT graphs

tag	LOχ	HLS
G		
H		

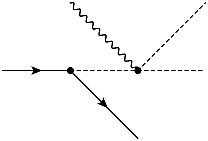
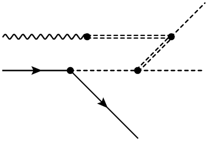
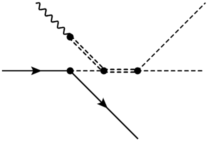
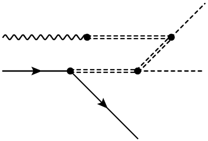
- Since we work with real photons and assume the validity of the VMD the G-graphs will provide exactly the same amplitudes in both approaches.
- As there is no BBVM vertex in the HLS Lagrangian but the BBγM one is available (unlike the BBγ one), the H-graphs lead to the same amplitudes in both approaches as well.

Current development - more BT graphs

tag	LOχ	HLS
I	—	
J	—	

- The I and J graphs, that include an anomalous VVM vertex, do not have counterparts in the LOχ approach.

Current development - AN graphs

tag	LOχ	HLS
K		
L	—	
M	—	

Current development

Very preliminary results:

including most of the graphs involving K^* improves the χ^2/dof by about 1

reproduction of $\pi\Sigma$ CLAS data still not adequate

considering decuplet baryons in the intermediate state may help

Summary

- The up-to-date (NLO) chirally motivated $\pi\Sigma - \bar{K}N$ models provide very different predictions for the *MB* amplitudes at energies below $\bar{K}N$ threshold.
- The $\Lambda(1405)$ energy region can be accessed by studying processes involving $\pi\Sigma$ rescattering in the final state. The $\pi\Sigma$ photoproduction on protons represents such a process where the *MB* rescattering plays a crucial role as our results demonstrate.
- Our approach to the two-meson photoproduction implements coupled-channel unitarity, low-energy theorems from ChPT and gauge invariance. We have revealed large variations when different models for the *MB* amplitudes are adopted.
- Our new results of fits that combine the K^-p reactions data with those on the $\pi\Sigma$ photoproduction can achieve a reasonable χ^2/dof but are not satisfactory especially for the $\pi^-\Sigma^+$ mass distributions.
- The presented models tend to limit the mass of the *lower mass* $\Lambda(1405)$ pole and yield a larger width of the pole that couples more strongly to $\bar{K}N$. A possible relation to a large absorption width found for the $\bar{K}NN$ bound state at J-PARC?
- We need a more precise experimental data on low energy K^-p reactions as the current ones are not too restrictive on the theoretical models. Our model (the photoproduction kernel) should also be improved by implementing the vector mesons (work in progress).