hadron spectrum collaboration
hadspec.org

J^{--} meson resonances in QCD

Jozef Dudek

… with focus on those containing strange quarks …

CHARTERED 1693

Thomas Jefferson National Accelerator Facility

$J^{-(-)}$ mesons in the PDG

isovectors

1^- **$\rho(1700)$**
Mass $m = 1720 \pm 20$ MeV [h]
Full width $\Gamma = 250 \pm 100$ MeV

3^- **$\rho_3(1690)$**
Mass $m = 1688.8 \pm 2.1$ MeV
Full width $\Gamma = 161 \pm 10$ MeV

no 2^- candidates

1^- **$\rho(1450)$**
Mass $m = 1465 \pm 25$ MeV [h]
Full width $\Gamma = 400 \pm 60$ MeV [h]

1^- **$\rho(770)$**
Mass (Breit-Wigner) $= 775.26 \pm 0.23$ MeV
Full width (Breit-Wigner) $= 149.1 \pm 0.8$ MeV

strange mesons

2^- **$K_2(1820)$** [ff]
Mass $m = 1819 \pm 12$ MeV
Full width $\Gamma = 264 \pm 34$ MeV

3^- **$K_3^*(1780)$**
Mass $m = 1779 \pm 8$ MeV
Full width $\Gamma = 161 \pm 17$ MeV

2^- **$K_2(1770)$** [ff]
Mass $m = 1773 \pm 8$ MeV
Full width $\Gamma = 186 \pm 14$ MeV

1^- **$K^*(1680)$**
Mass $m = 1718 \pm 18$ MeV
Full width $\Gamma = 322 \pm 110$ MeV

1^- **$K^*(1410)$**
Mass $m = 1414 \pm 15$ MeV
Full width $\Gamma = 232 \pm 21$ MeV

1^- **$K^*(892)$**
$K^*(892)^0$ mass $m = 895.55 \pm 0.20$ MeV
$K^*(892)^0$ full width $\Gamma = 47.3 \pm 0.5$ MeV

$J^{-(-)}$ mesons in the PDG

isoscalars

3^- **$\phi_3(1850)$**
Mass $m = 1854 \pm 7$ MeV
Full width $\Gamma = 87^{+28}_{-23}$ MeV

3^- **$\omega_3(1670)$**
Mass $m = 1667 \pm 4$ MeV
Full width $\Gamma = 168 \pm 10$ MeV

1^- **$\phi(1680)$**
Mass $m = 1680 \pm 20$ MeV [h]
Full width $\Gamma = 150 \pm 50$ MeV [h]

no 2^- candidates

1^- **$\omega(1650)$** [k]
Mass $m = 1670 \pm 30$ MeV [h]
Full width $\Gamma = 315 \pm 35$ MeV [h]

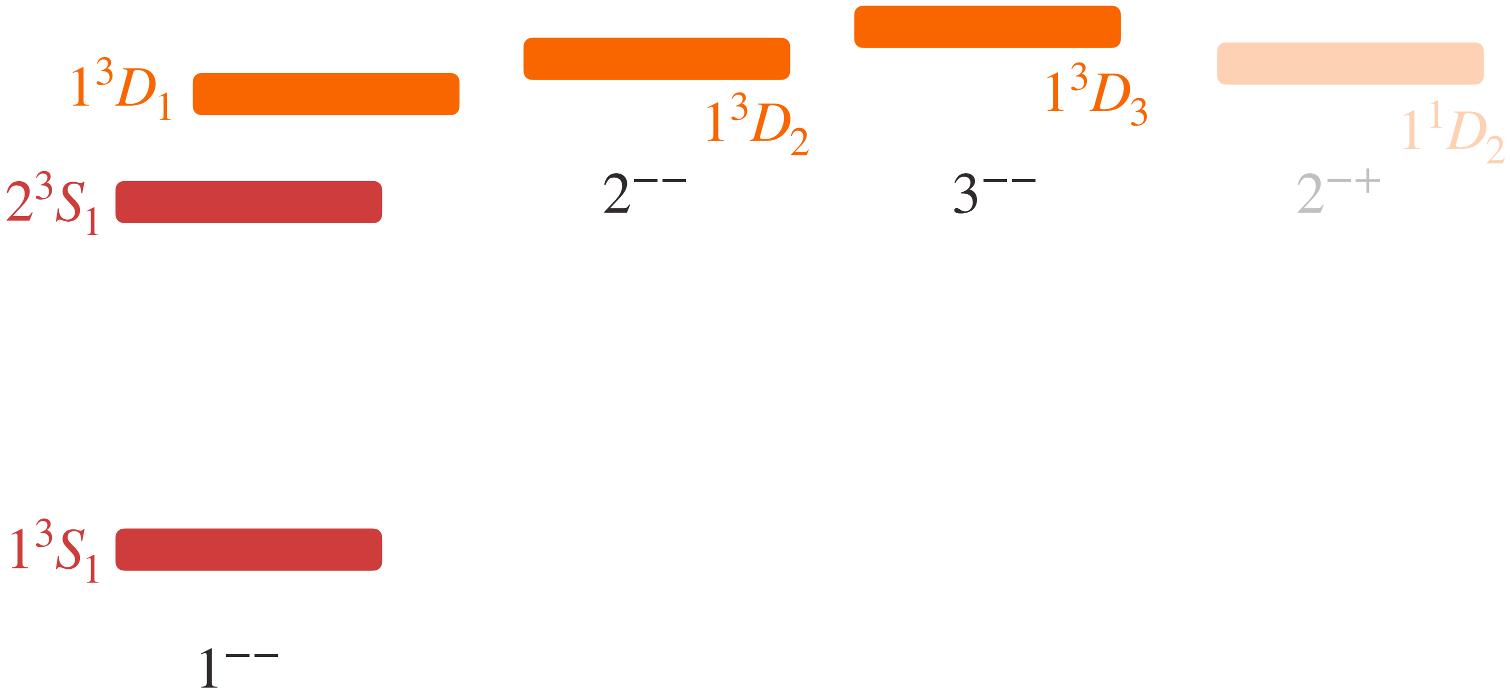
1^- **$\omega(1420)$** [j]
Mass $m = 1410 \pm 60$ MeV [h]
Full width $\Gamma = 290 \pm 190$ MeV [h]

1^- **$\phi(1020)$**
Mass $m = 1019.461 \pm 0.016$ MeV
Full width $\Gamma = 4.249 \pm 0.013$ MeV

1^- **$\omega(782)$**
Mass $m = 782.66 \pm 0.13$ MeV
Full width $\Gamma = 8.68 \pm 0.13$ MeV

phenomenology

$q\bar{q}$ quark model picture is simple



1^3D_1

2^3S_1

1^3S_1

1^{--}

1^3D_2

2^{--}

1^3D_3

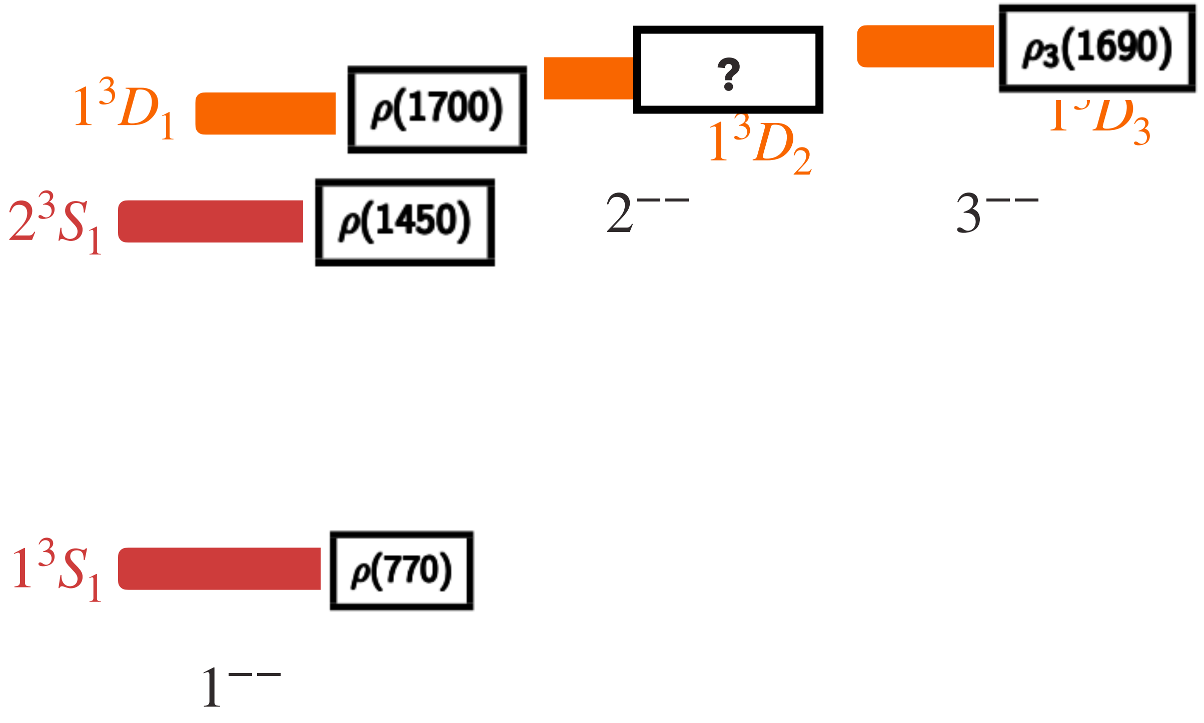
3^{--}

1^1D_2

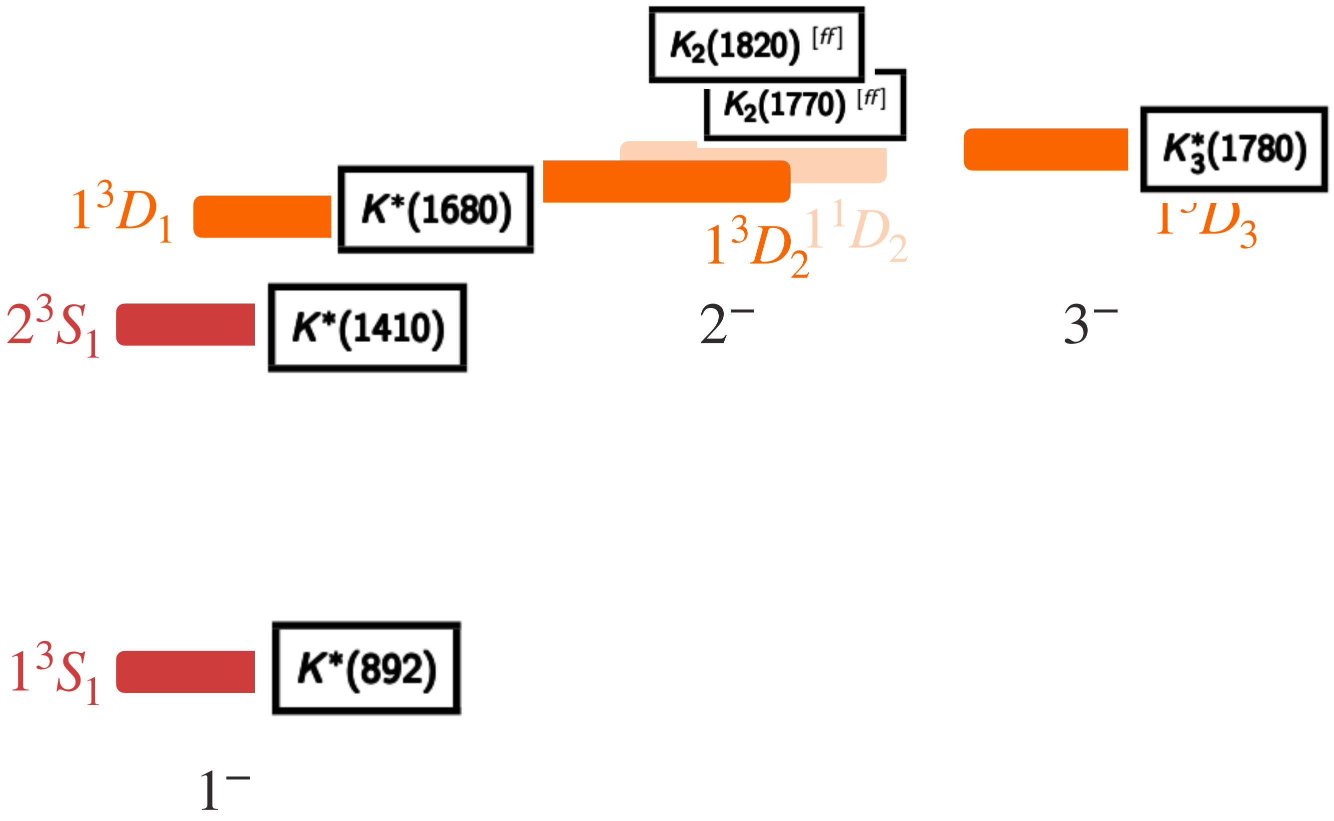
2^{-+}

phenomenology

isovectors



strange mesons



1^- states could be 2^3S_1, 1^3D_1 admixtures in this model

phenomenology

ω, ϕ names typically assigned assuming **OZI rule** holds

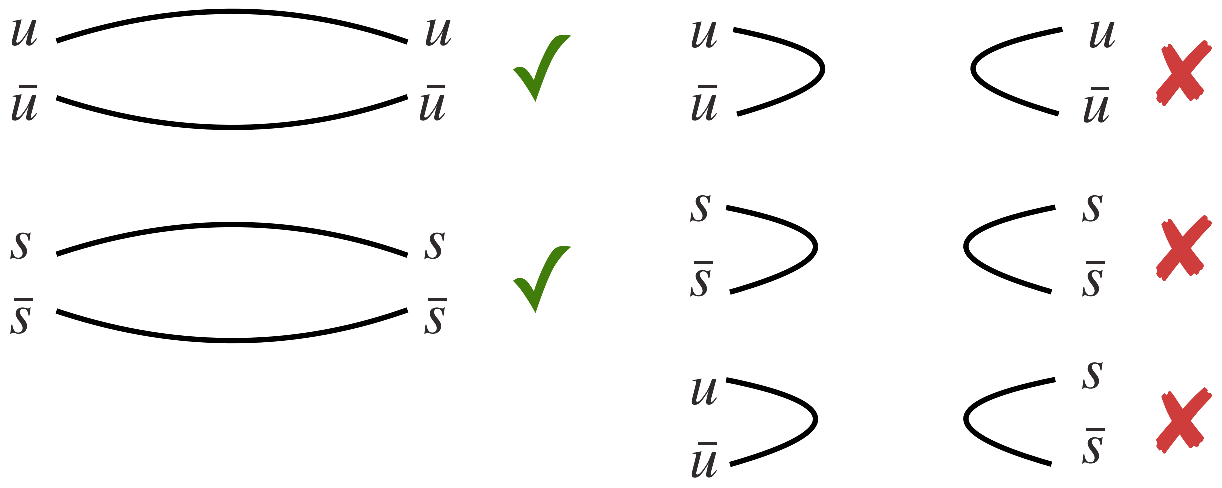
ideal flavor mixing

$\omega \sim u\bar{u} + d\bar{d}$

$\phi \sim s\bar{s}$

OZI rule (generalized)

"connected diagrams favored over disconnected"



phenomenology

ω, ϕ names typically assigned assuming **OZI rule** holds

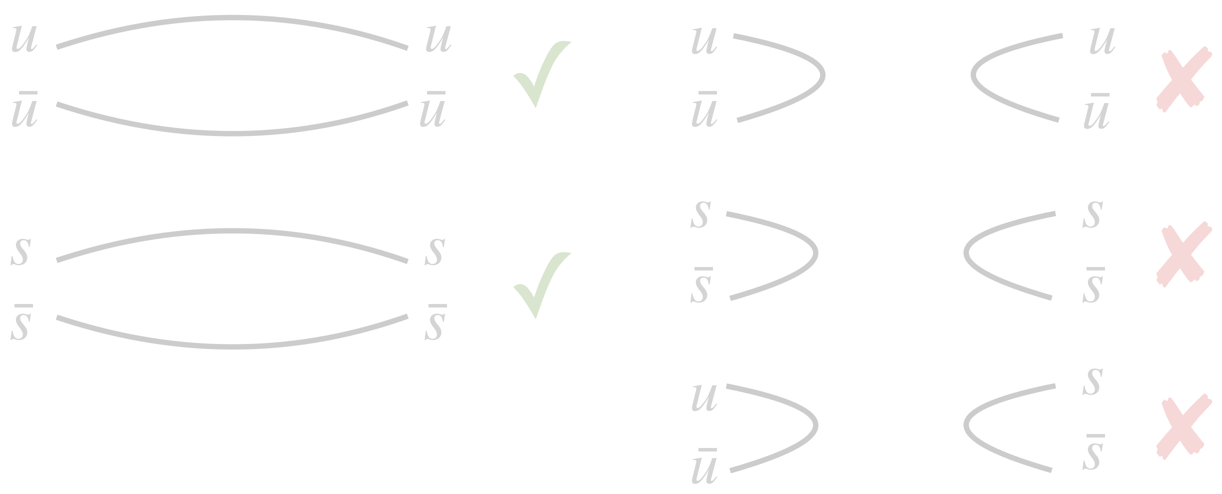
ideal flavor mixing

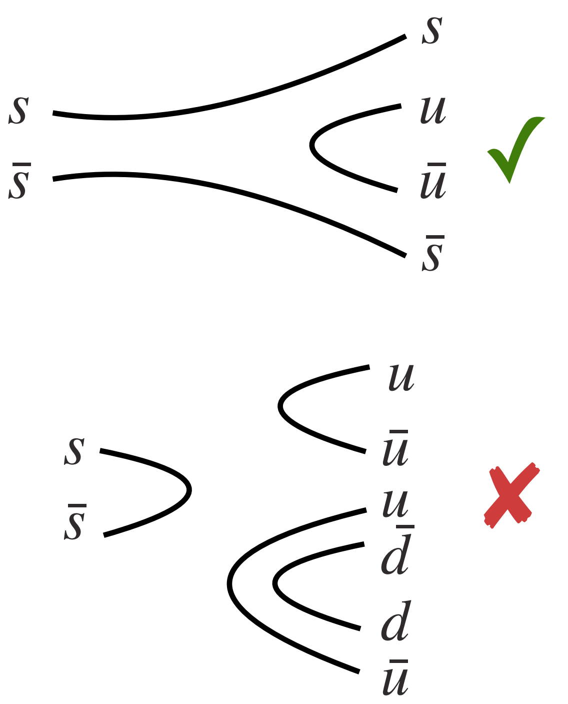
$\omega \sim u\bar{u} + d\bar{d}$

$\phi \sim s\bar{s}$

OZI rule (generalized)

"connected diagrams favored over disconnected"





$\phi \to K\bar{K}$

$\phi \to \pi\pi\pi$

phenomenology

ω, ϕ names typically assigned assuming **OZI rule** holds

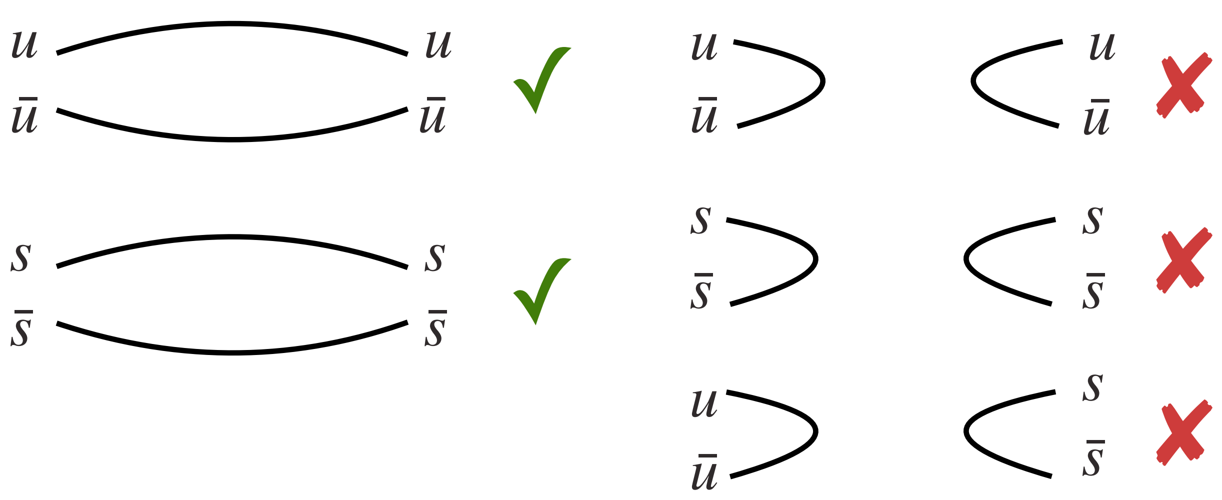
ideal flavor mixing

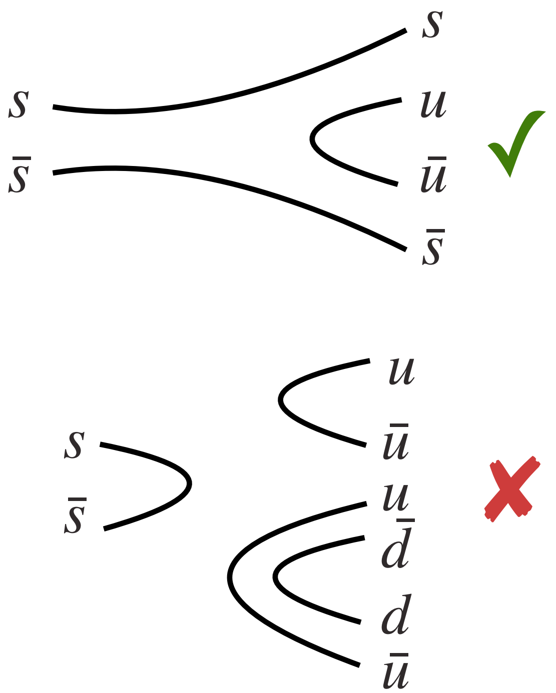
$\omega \sim u\bar{u} + d\bar{d}$

$\phi \sim s\bar{s}$

OZI rule (generalized)

"connected diagrams favored over disconnected"





$\phi \to K\overline{K}$

$\phi \to \pi\pi\pi$

essentially empirical

famous strong evidence: $\Gamma(\phi(1020) \to K\bar{K}) \gg \Gamma(\phi(1020) \to \pi\pi\pi)$

$$m_{\rho(770)} \approx m_{\omega(782)}$$

but doesn't work at all for pseudoscalars

$\eta \sim u\bar{u} + d\bar{d} - 2s\bar{s}$

$\eta' \sim u\bar{u} + d\bar{d} + s\bar{s}$

maybe axials not
ideally mixed either

phenomenology

ω, ϕ names typically assigned assuming OZI rule holds

ideal flavor mixing
$\omega \sim u\bar{u} + d\bar{d}$
$\phi \sim s\bar{s}$

OZI rule (generalized)

"connected diagrams favored over disconnected"

essentially empirical

famous strong evidence: $\Gamma(\phi(1020) \to K\bar{K}) \gg \Gamma(\phi(1020) \to \pi\pi\pi)$

$$m_{\rho(770)} \approx m_{\omega(782)}$$

but doesn't work at all for pseudoscalars $\eta \sim u\bar{u} + d\bar{d} - 2s\bar{s}$
$\eta' \sim u\bar{u} + d\bar{d} + s\bar{s}$

maybe axials not ideally mixed either

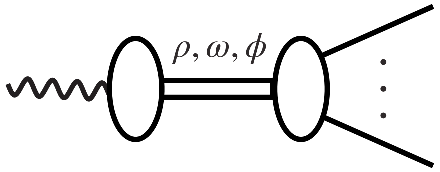
we shouldn't assume 'strangeonium' is how QCD chooses to manifest isoscalar mesons in all J^P, and at all energies

it might, but we should establish it …

(this talk is not going to achieve this)

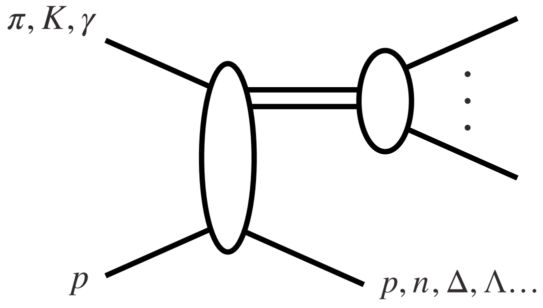
experimental evidence

e^+e^- annihilation



1^{--} only, absolute normalization (cross-section)

peripheral production

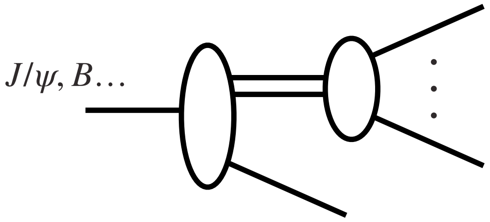


all partial-waves,
have to be separated

e.g. LASS (1980s)

$$K^- p \to \begin{array}{l} K^-\pi^+\, n \\ K^-\eta\, p \\ \overline{K}^0\pi^+\pi^-\, n \\ K^-\omega\, p \end{array}$$

heavy flavor decays



Dalitz plot analysis

model multiple pairs simultaneously

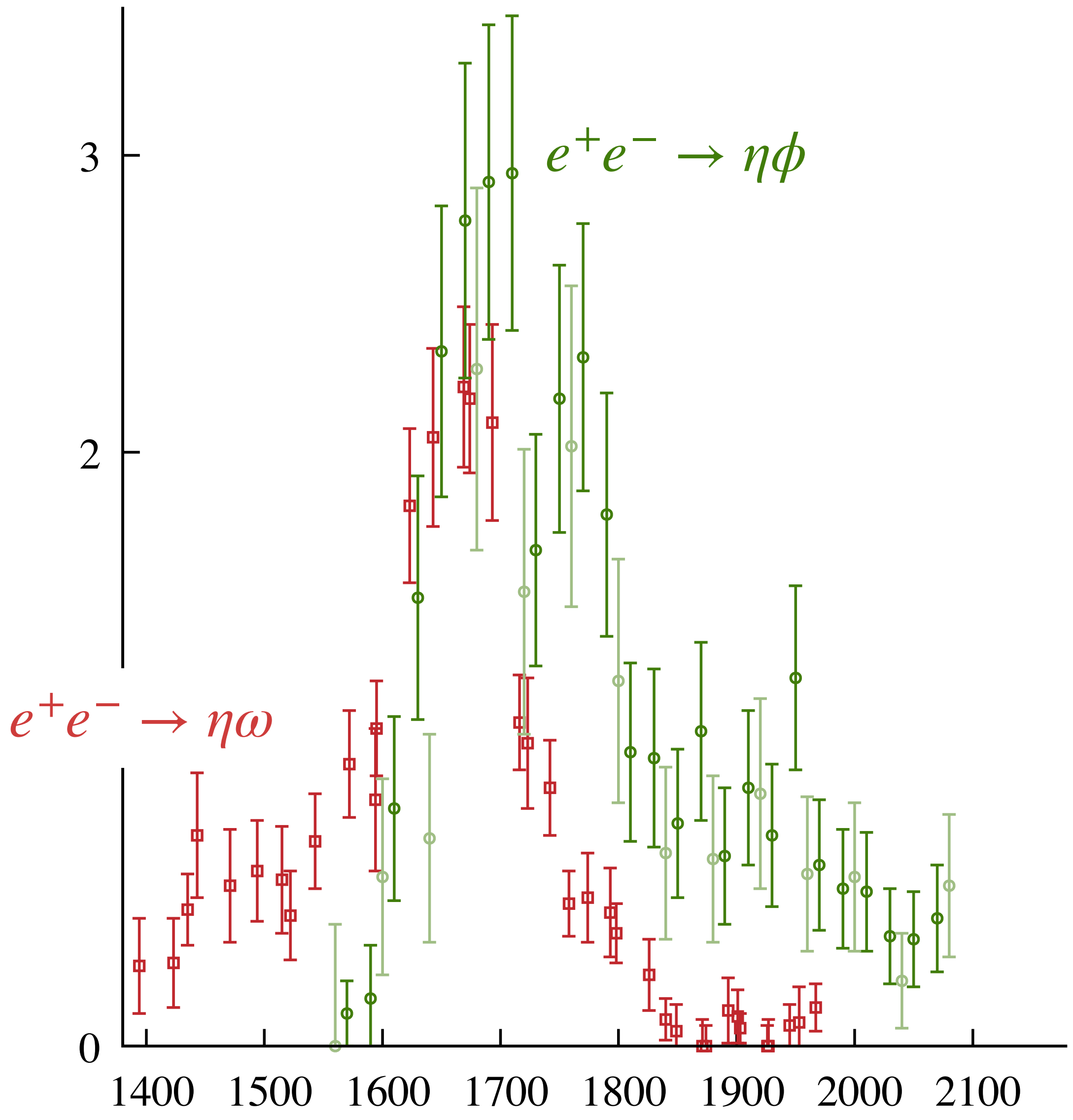
e.g. LHCb

$B \to J/\psi K\phi$

put in all PDG $K^\star$ resonances, assuming they are well established and consistent

experimental evidence

but even the 'simple' cases prove to be tricky:

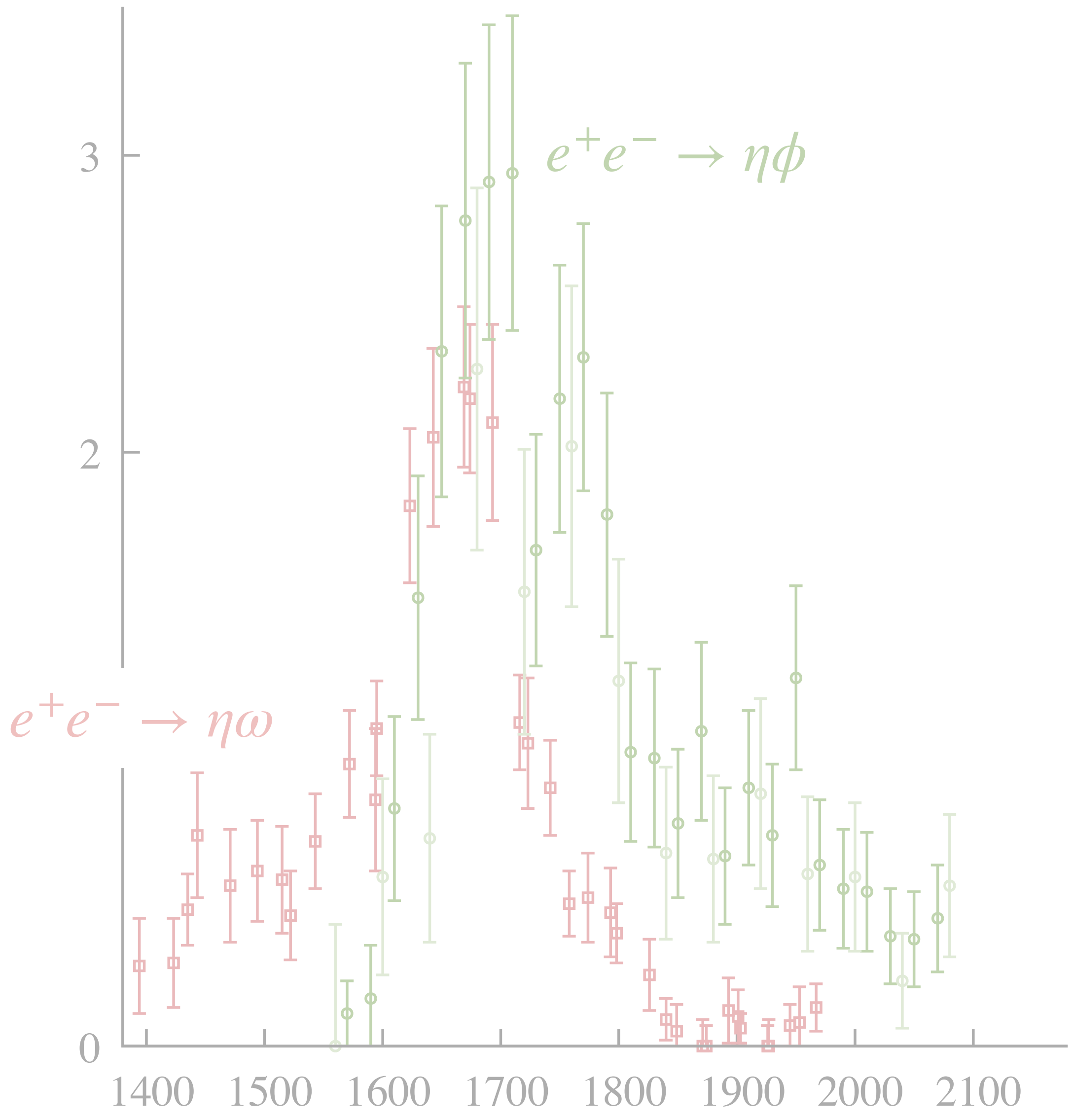


published analyses of these data have **no resonances in common !**

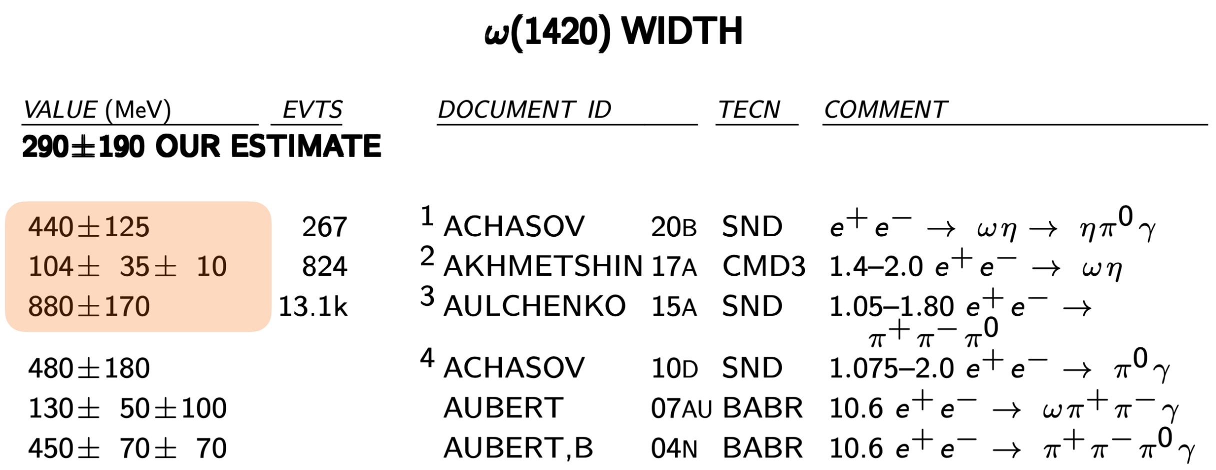
in principle expect **four** resonances could contribute to each of these final states

experimental evidence

but even the ‘simple’ cases prove to be tricky:

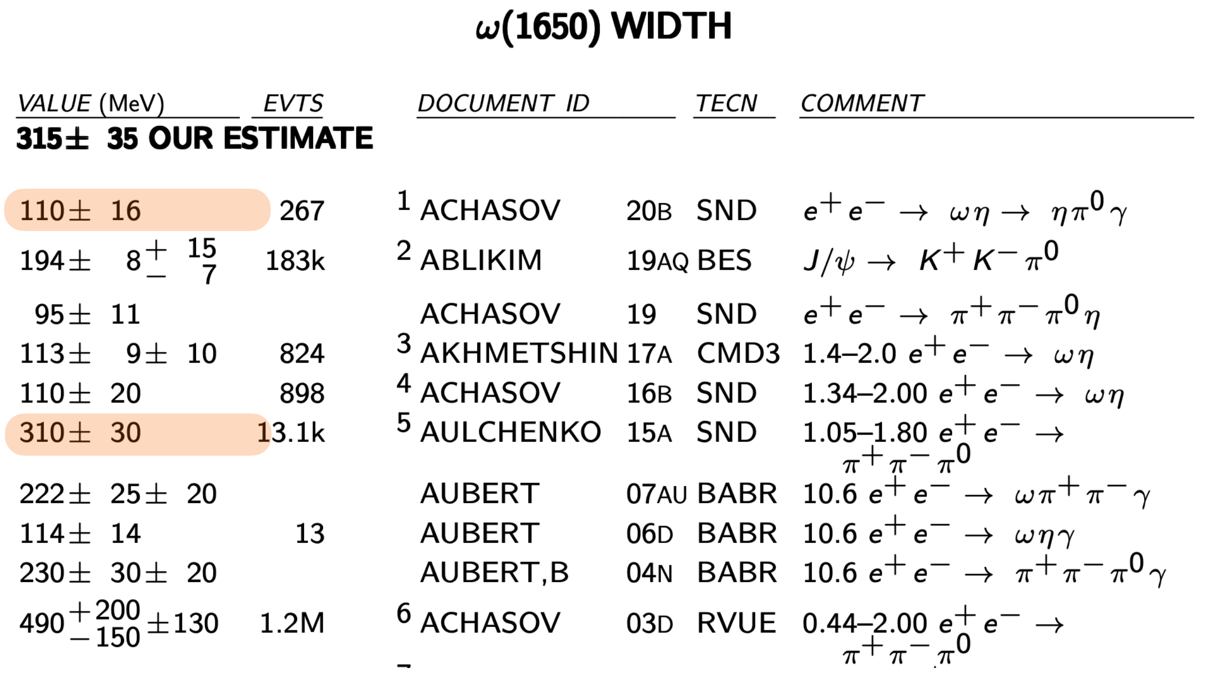


and it’s not clear resonance identifications are really consistent, e.g.



$\omega(1420)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
290±190 OUR ESTIMATE					
440±125	267	[1] ACHASOV	20B	SND	$e^+e^- \to \omega\eta \to \eta\pi^0\gamma$
104± 35± 10	824	[2] AKHMETSHIN	17A	CMD3	1.4–2.0 $e^+e^- \to \omega\eta$
880±170	13.1k	[3] AULCHENKO	15A	SND	1.05–1.80 $e^+e^- \to \pi^+\pi^-\pi^0$
480±180		[4] ACHASOV	10D	SND	1.075–2.0 $e^+e^- \to \pi^0\gamma$
130± 50±100		AUBERT	07AU	BABR	10.6 $e^+e^- \to \omega\pi^+\pi^-\gamma$
450± 70± 70		AUBERT,B	04N	BABR	10.6 $e^+e^- \to \pi^+\pi^-\pi^0\gamma$



$\omega(1650)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
315± 35 OUR ESTIMATE					
110± 16	267	[1] ACHASOV	20B	SND	$e^+e^- \to \omega\eta \to \eta\pi^0\gamma$
$194 \pm 8^{+15}_{-7}$	183k	[2] ABLIKIM	19AQ	BES	$J/\psi \to K^+K^-\pi^0$
95± 11		ACHASOV	19	SND	$e^+e^- \to \pi^+\pi^-\pi^0\eta$
113± 9± 10	824	[3] AKHMETSHIN	17A	CMD3	1.4–2.0 $e^+e^- \to \omega\eta$
110± 20	898	[4] ACHASOV	16B	SND	1.34–2.00 $e^+e^- \to \omega\eta$
310± 30	13.1k	[5] AULCHENKO	15A	SND	1.05–1.80 $e^+e^- \to \pi^+\pi^-\pi^0$
222± 25± 20		AUBERT	07AU	BABR	10.6 $e^+e^- \to \omega\pi^+\pi^-\gamma$
114± 14	13	AUBERT	06D	BABR	10.6 $e^+e^- \to \omega\eta\gamma$
230± 30± 20		AUBERT,B	04N	BABR	10.6 $e^+e^- \to \pi^+\pi^-\pi^0\gamma$
$490^{+200}_{-150} \pm 130$	1.2M	[6] ACHASOV	03D	RVUE	0.44–2.00 $e^+e^- \to \pi^+\pi^-\pi^0$

first principles QCD ?

lattice QCD

… using the physical light quark mass very challenging here

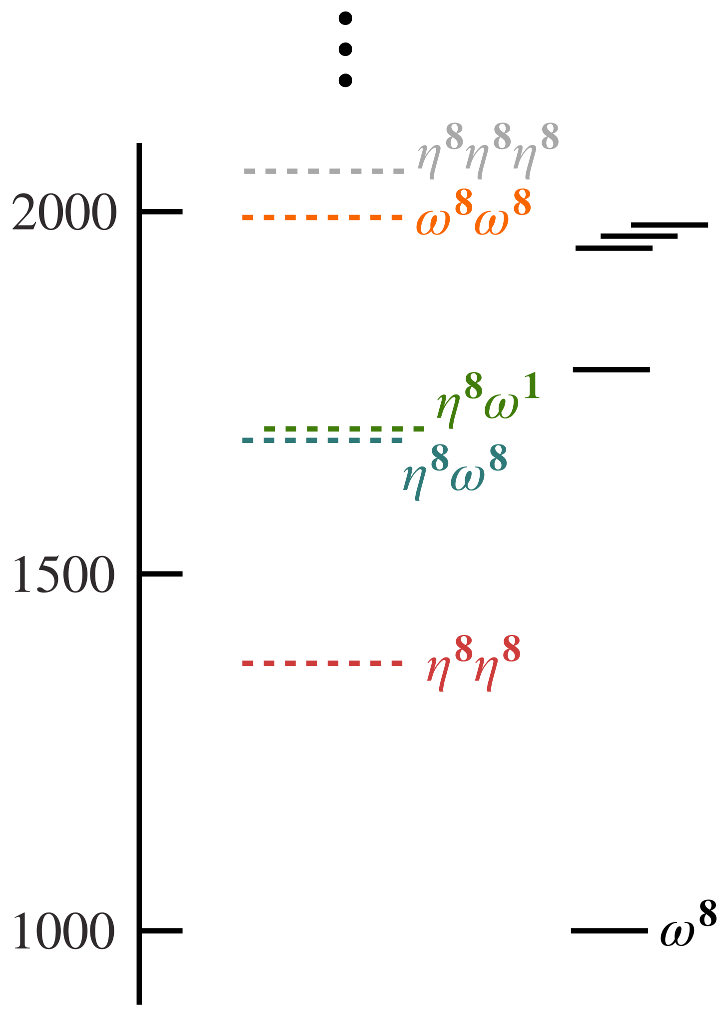
begin with a simpler case, still QCD but take $m_{u,d} \to m_s$

ensures an **exact** $SU(3)$ flavor symmetry – mesons in either octet $(\mathbf{8})$ or singlet $(\mathbf{1})$ representations

ρ, ω_8, K^* ω_1

lightest pseudoscalar becomes heavy, $m(\eta^{\mathbf{8}}) \sim 700\,\mathrm{MeV}$

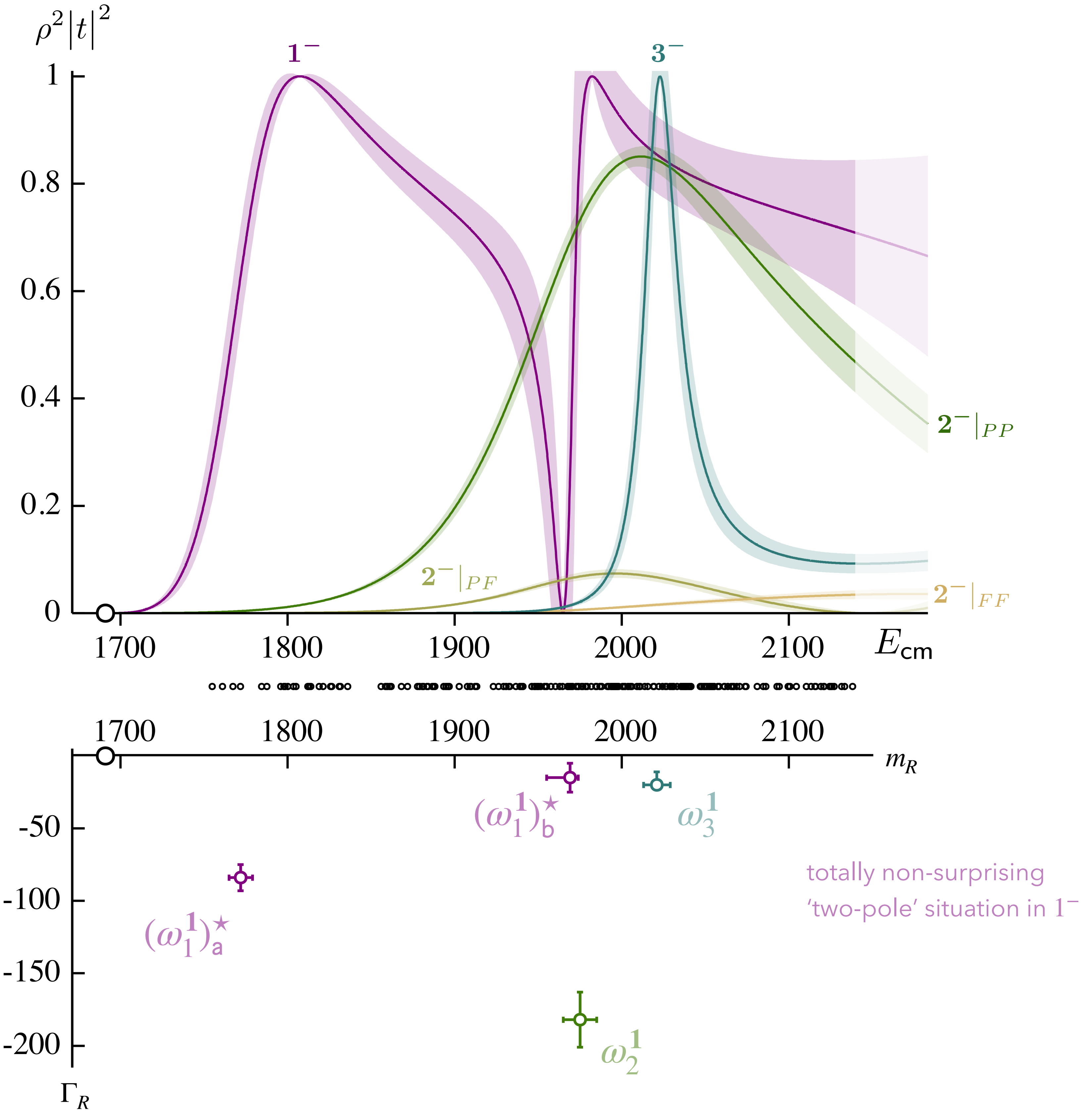
and only meson-meson decays kinematically accessible to the resonances of interest



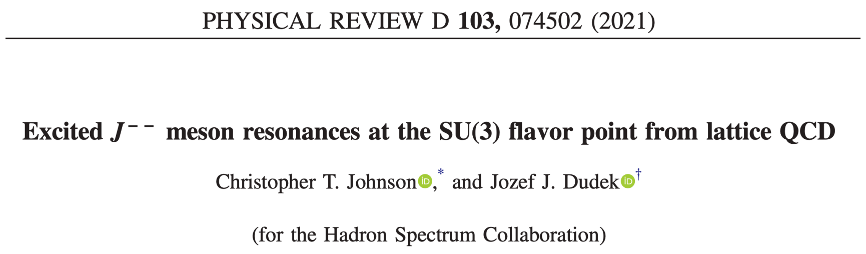
singlet $1^-, 2^-, 3^-$

singlet case computed in 2021

just **elastic** $\eta^8\omega^8$ scattering



totally non-surprising 'two-pole' situation in 1^-



PHYSICAL REVIEW D **103,** 074502 (2021)

Excited J^{--} meson resonances at the SU(3) flavor point from lattice QCD

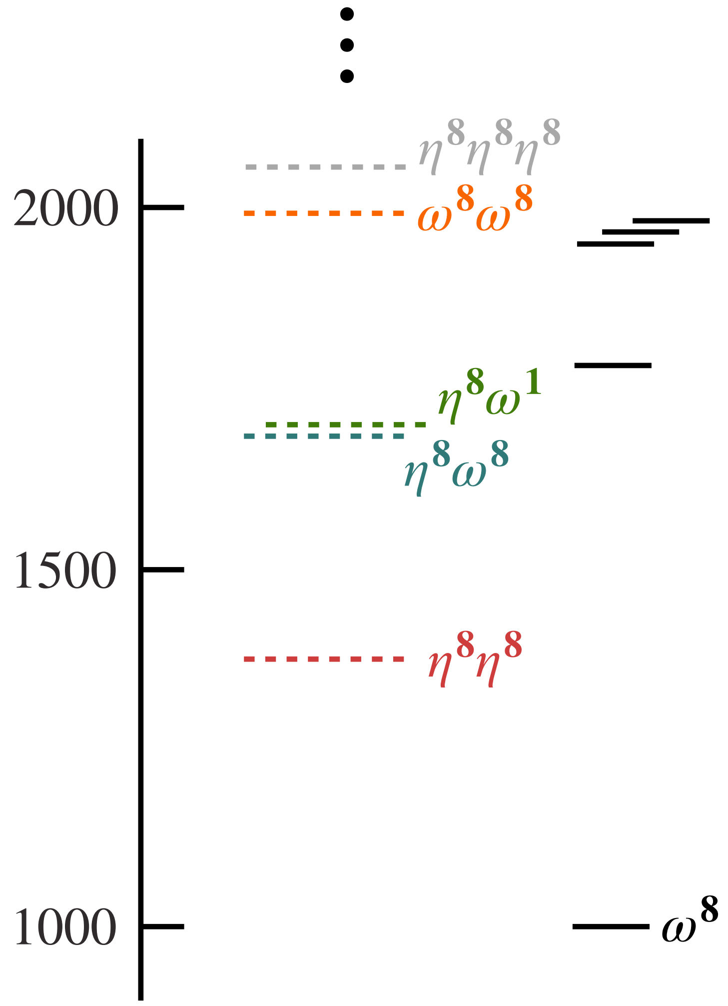
Christopher T. Johnson,* and Jozef J. Dudek†

(for the Hadron Spectrum Collaboration)

octet $1^-, 2^-, 3^-$

octet case is new, presenting here for the first time

coupled-channel $\eta^8\eta^8, \eta^8\omega^8, \eta^8\omega^1$ scattering

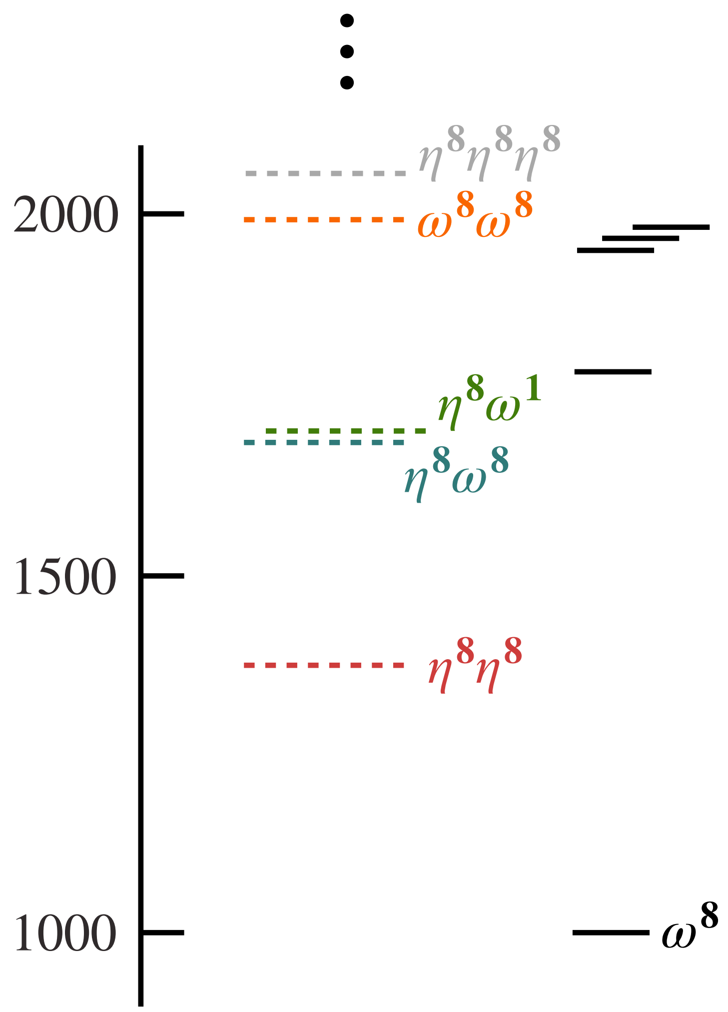


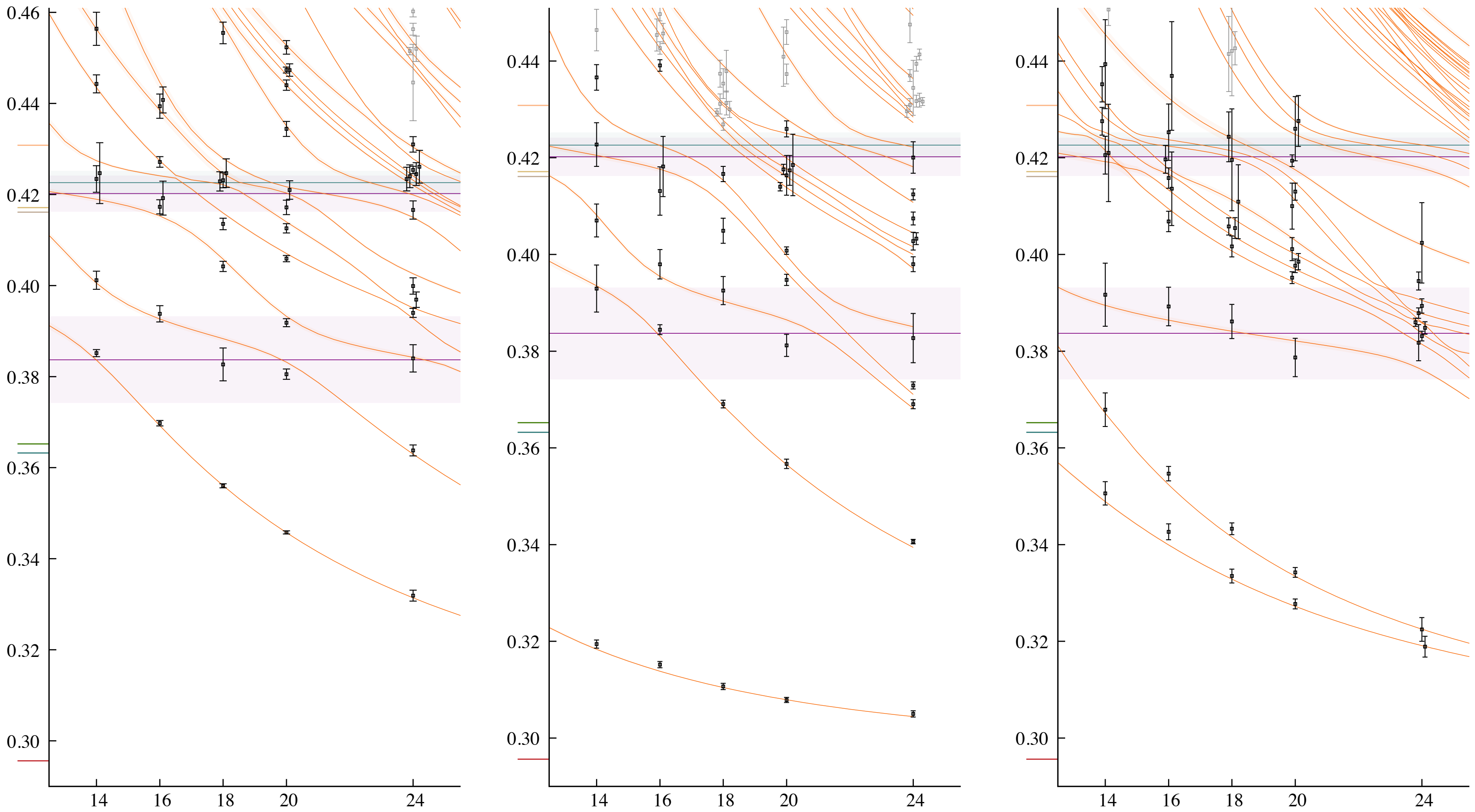
octet $1^-, 2^-, 3^-$

octet case is new, presenting here for the first time

coupled-channel $\eta^8\eta^8, \eta^8\omega^8, \eta^8\omega^1$ scattering

computed in **five lattice volumes**,
hundreds of energy levels constraining
the scattering amplitudes

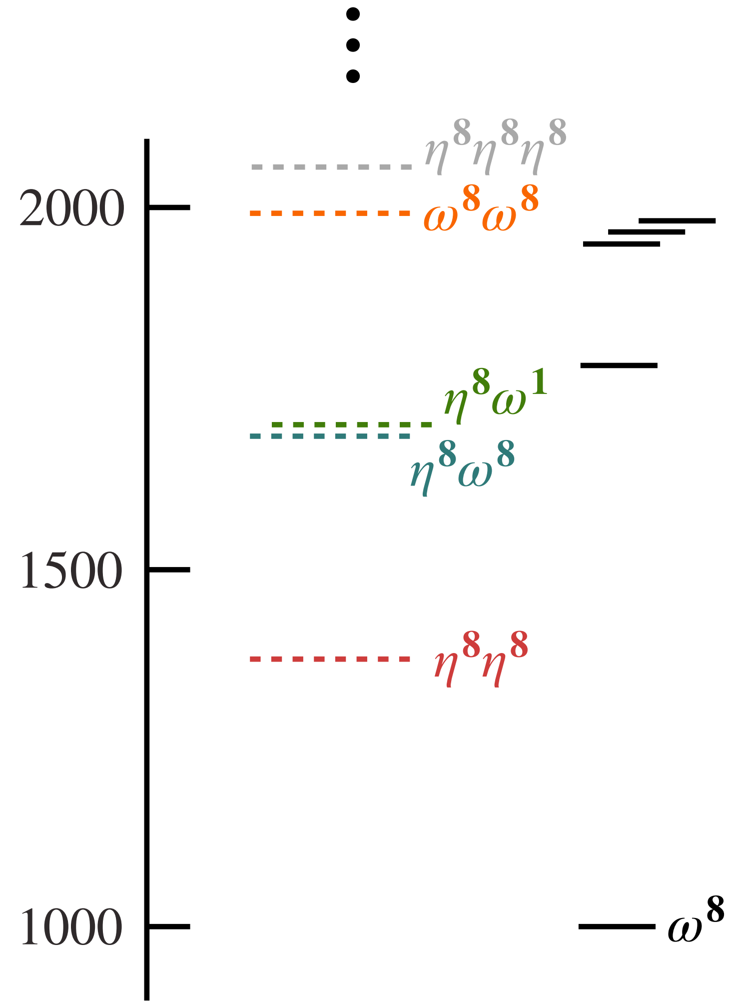


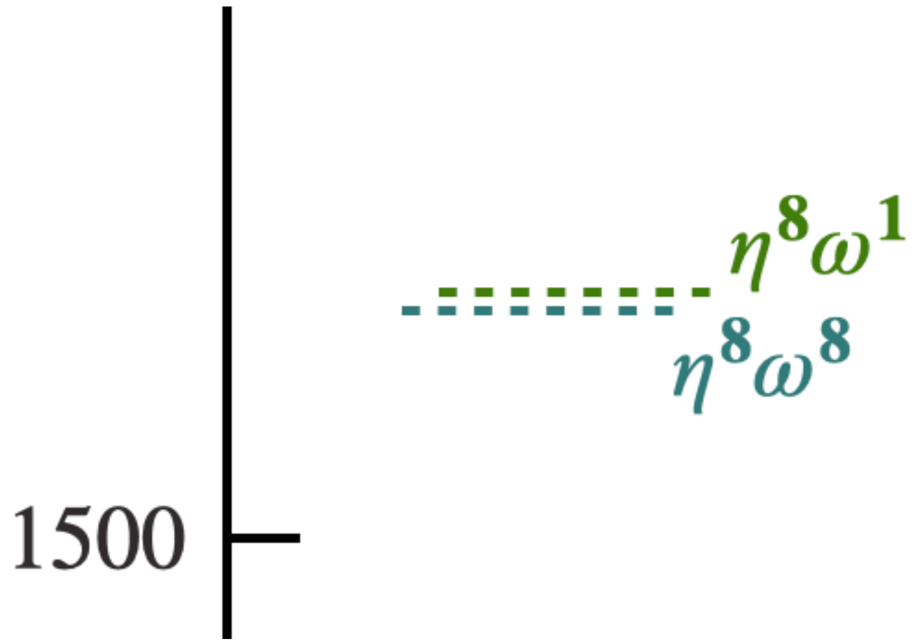


octet $1^-, 2^-, 3^-$

octet case is new, presenting here for the first time

coupled-channel $\eta^8\eta^8, \eta^8\omega^8, \eta^8\omega^1$ scattering

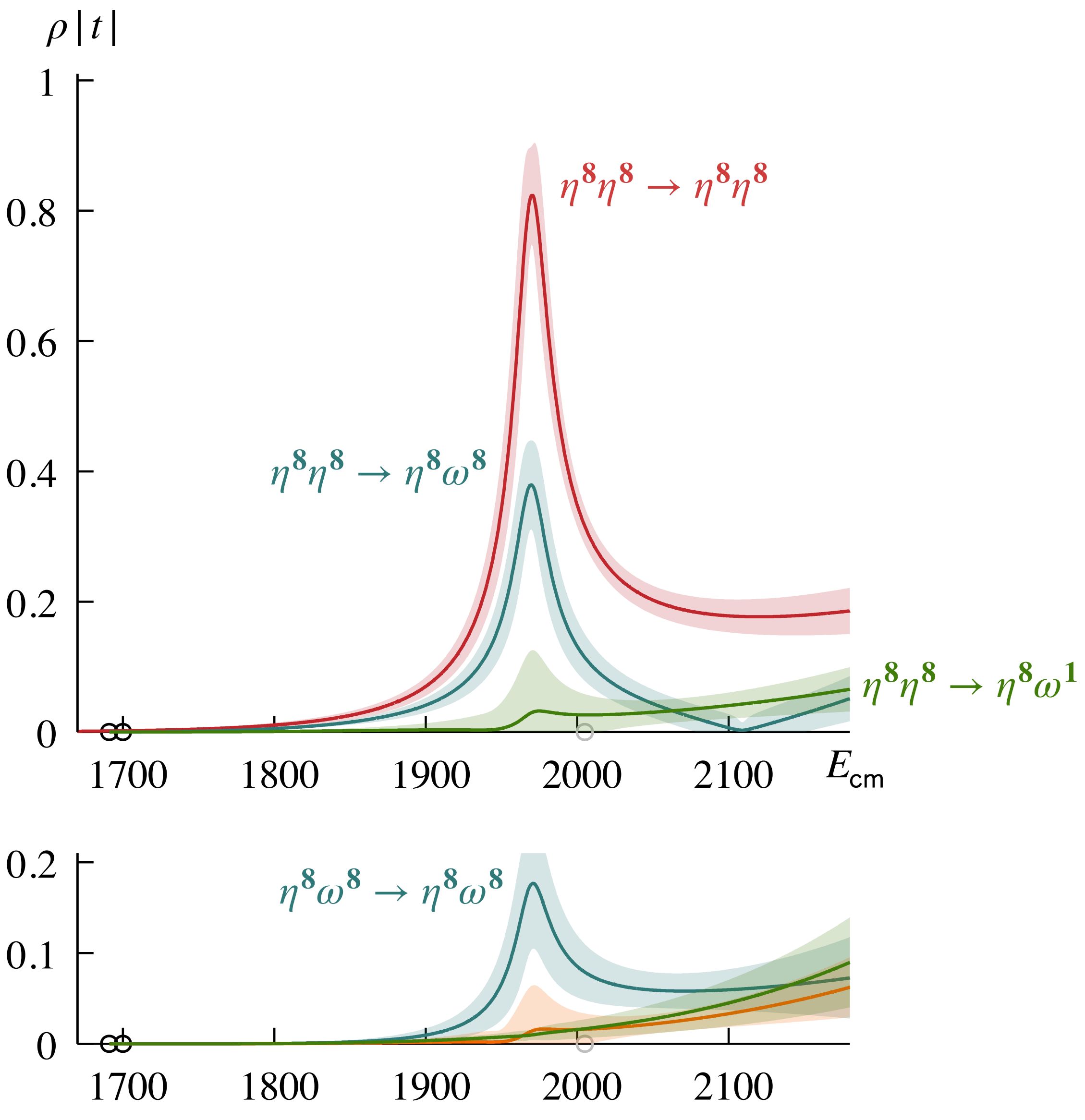




very similar masses for octet and singlet vector mesons makes separation of these channels challenging in finite volume

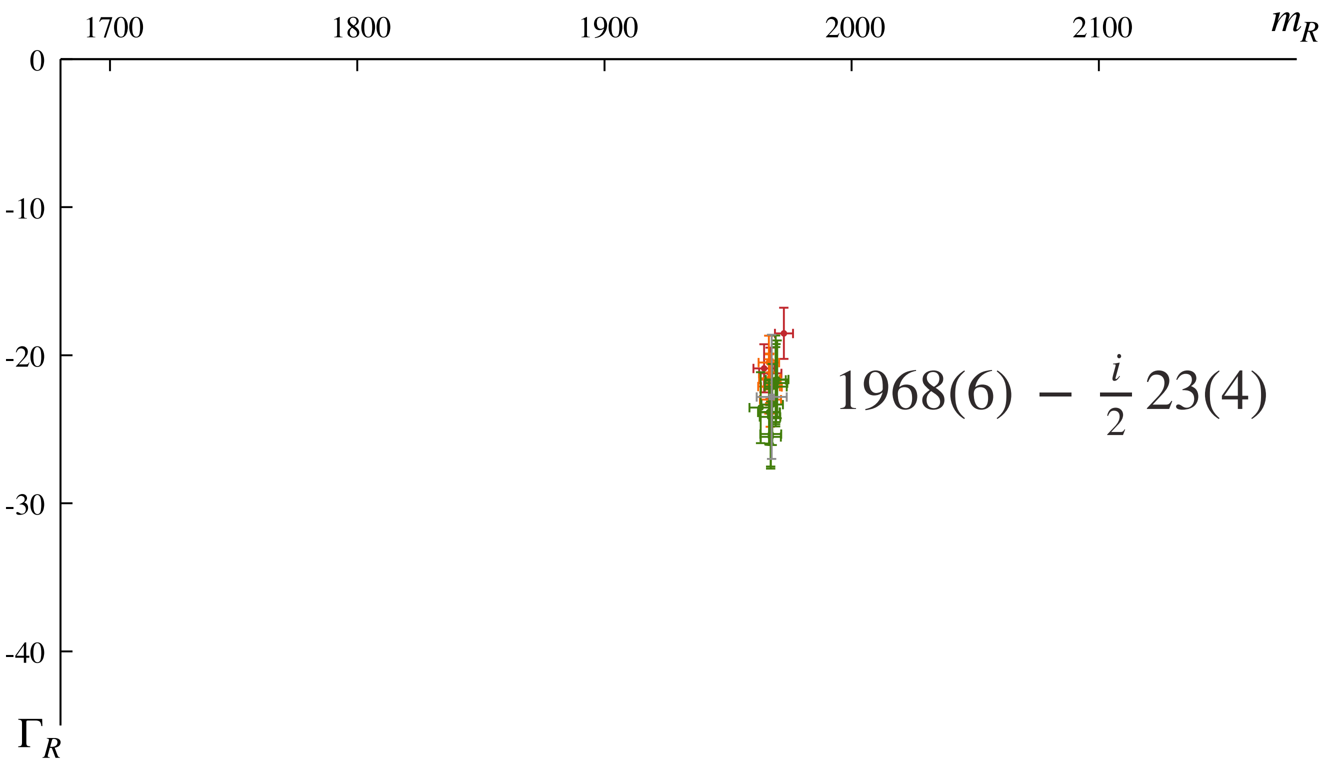
finite-volume approach works via energy shifts

octet 3^-



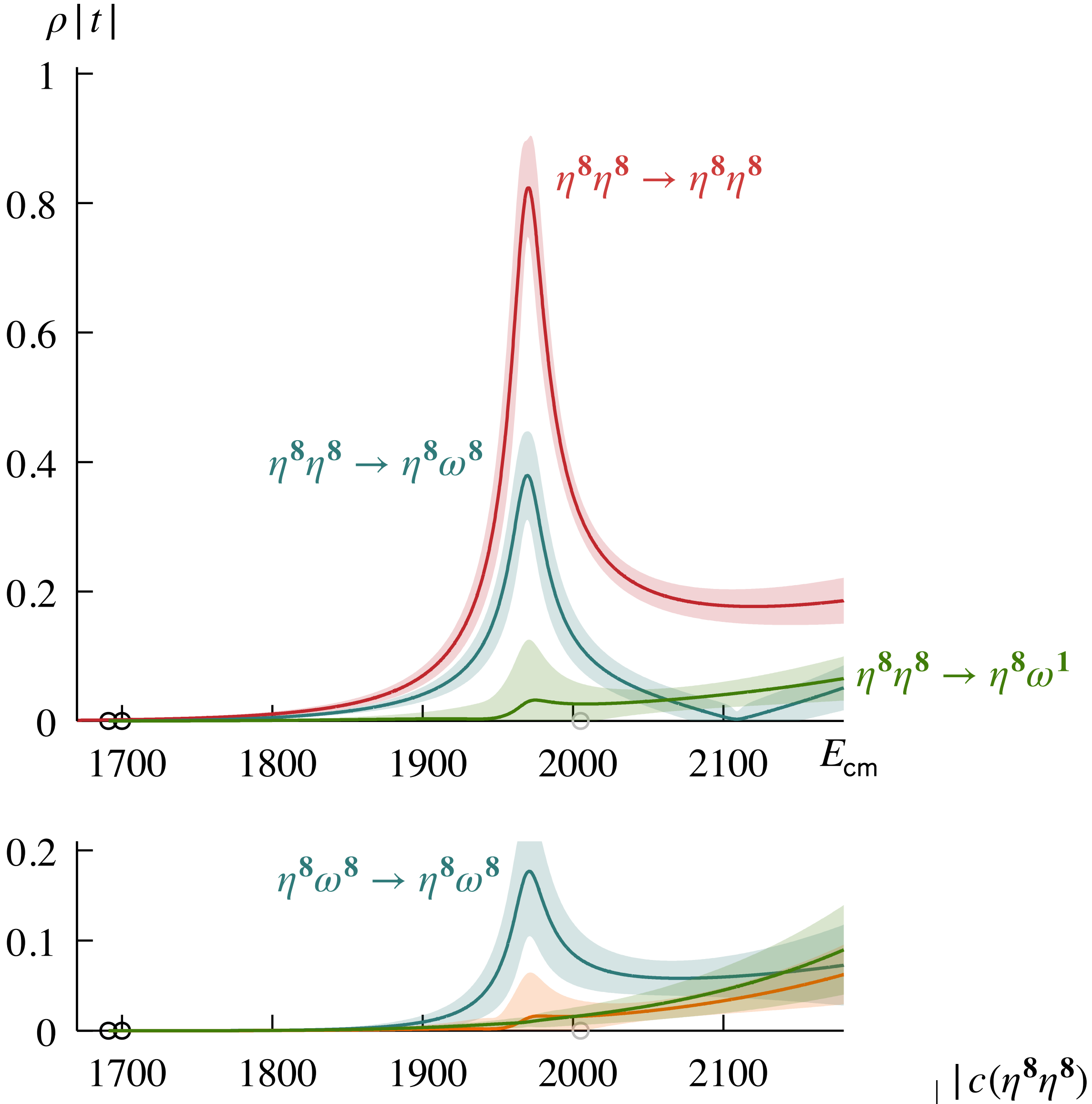
$$t_{ij}(s \sim s_R) \sim \frac{c_i\, c_j}{s_R - s}$$

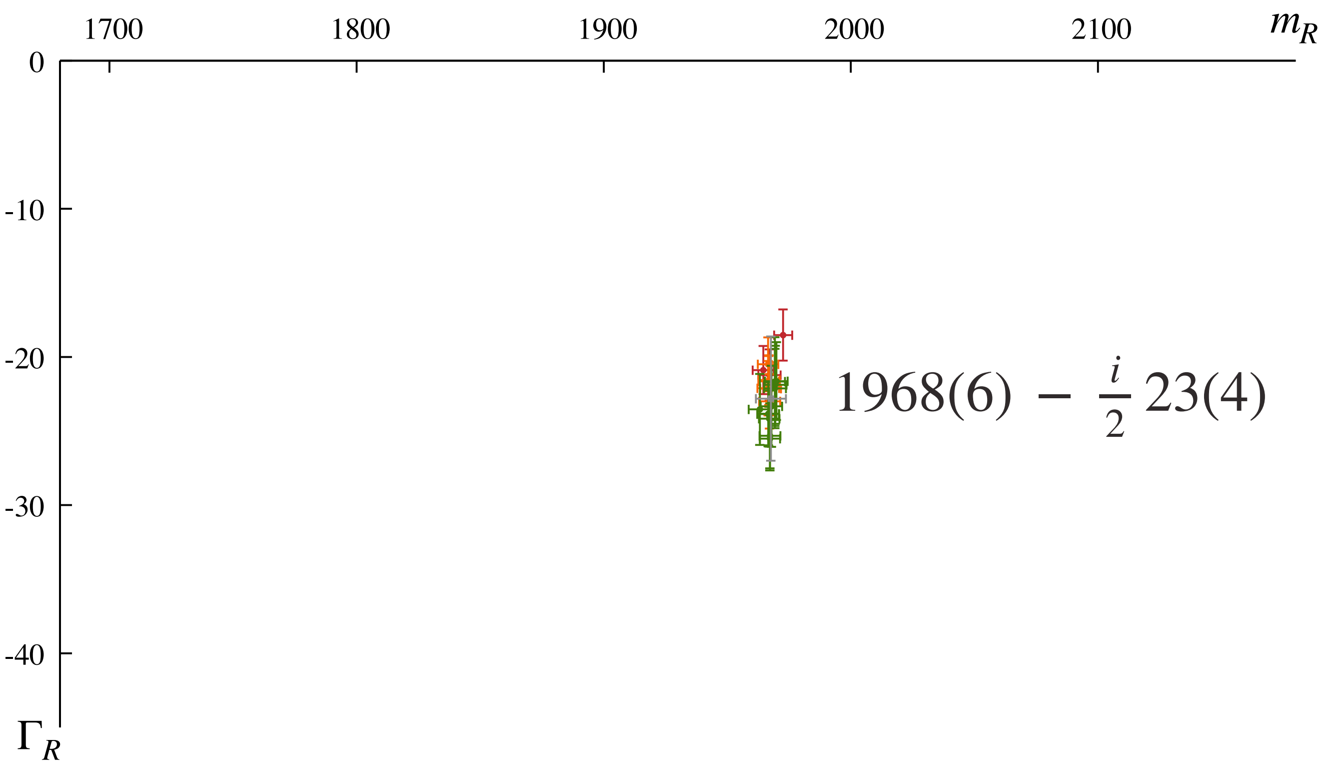
$$\sqrt{s_R} = m_R - \frac{i}{2}\Gamma_R$$

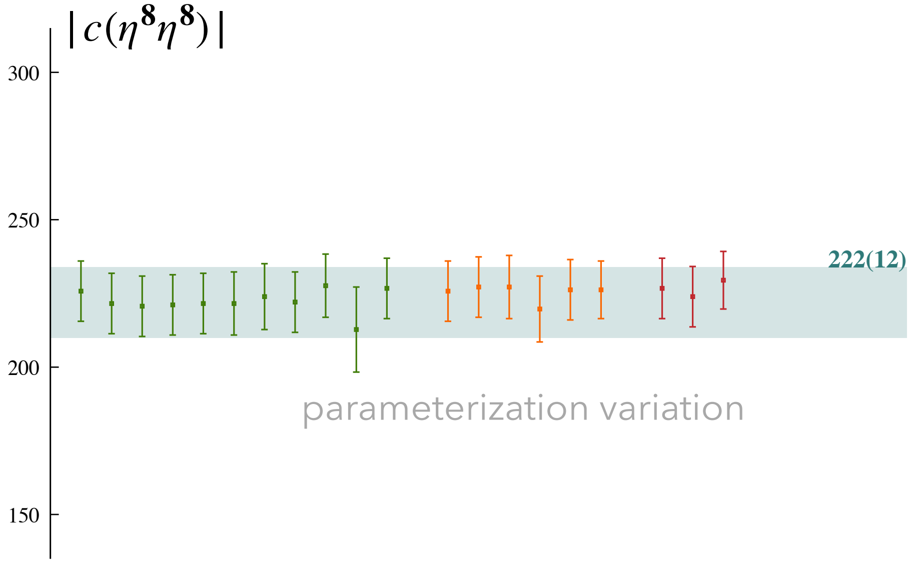


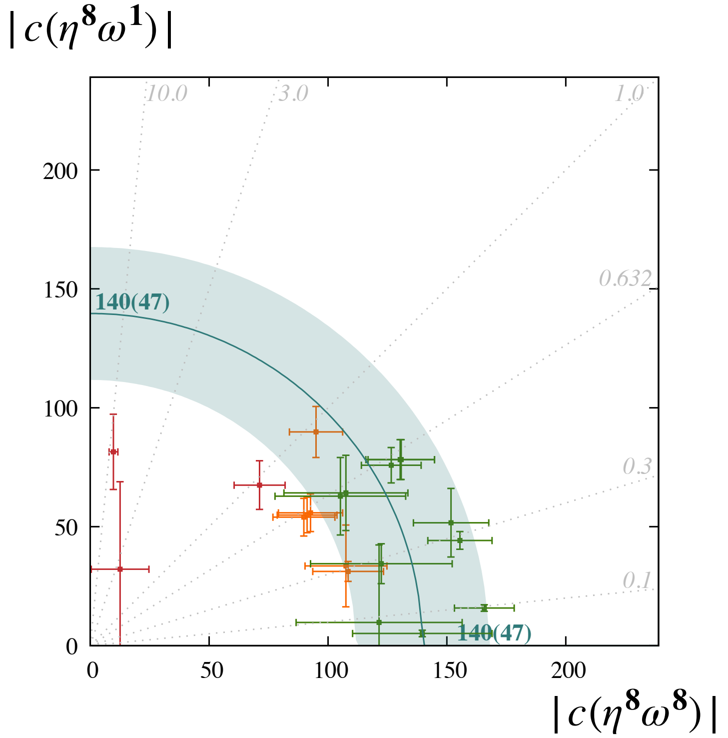
octet 3^-

$\{\eta^8\eta^8|_F, \eta^8\omega^8|_F, \eta^8\omega^1|_F\}$

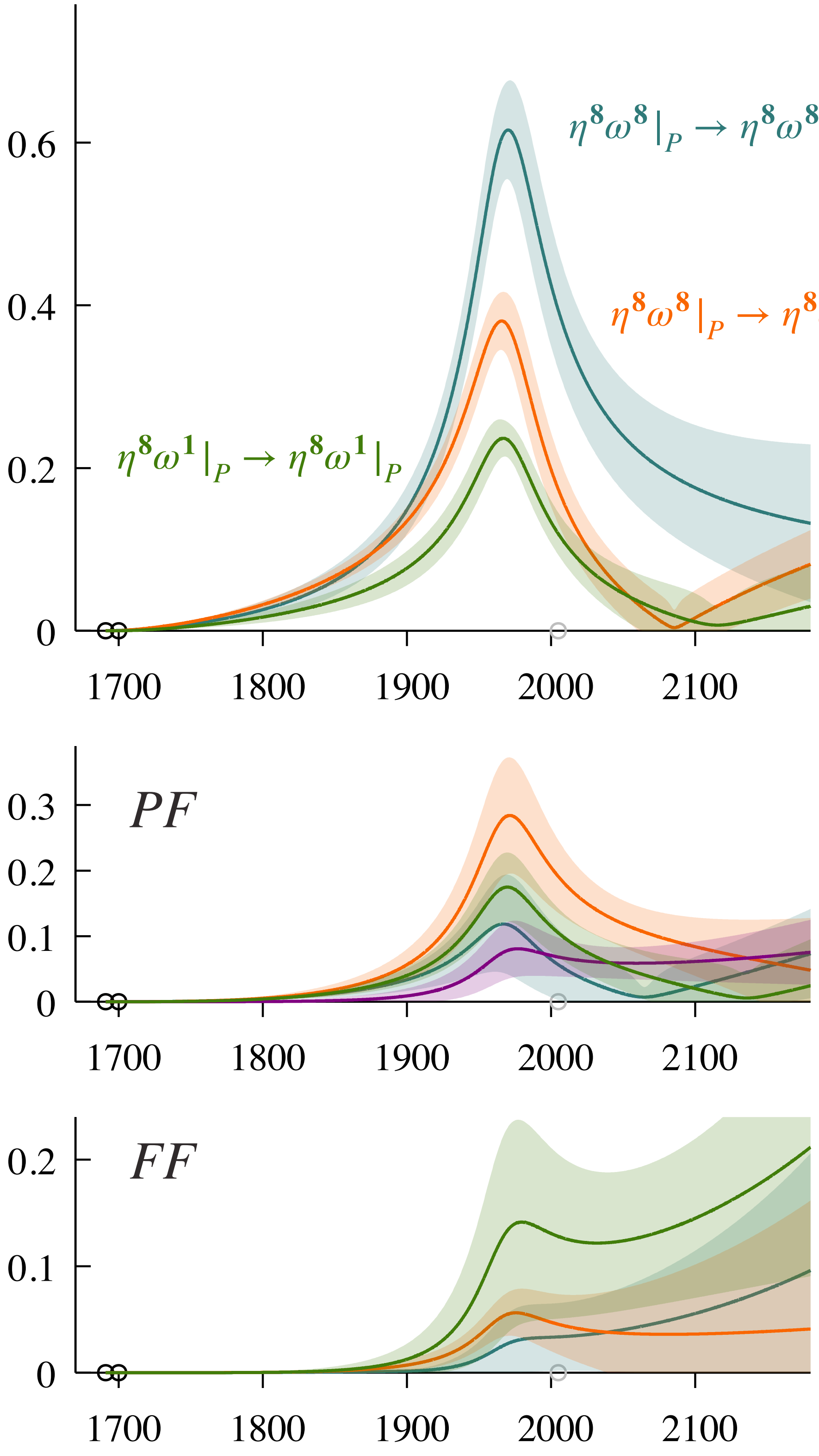


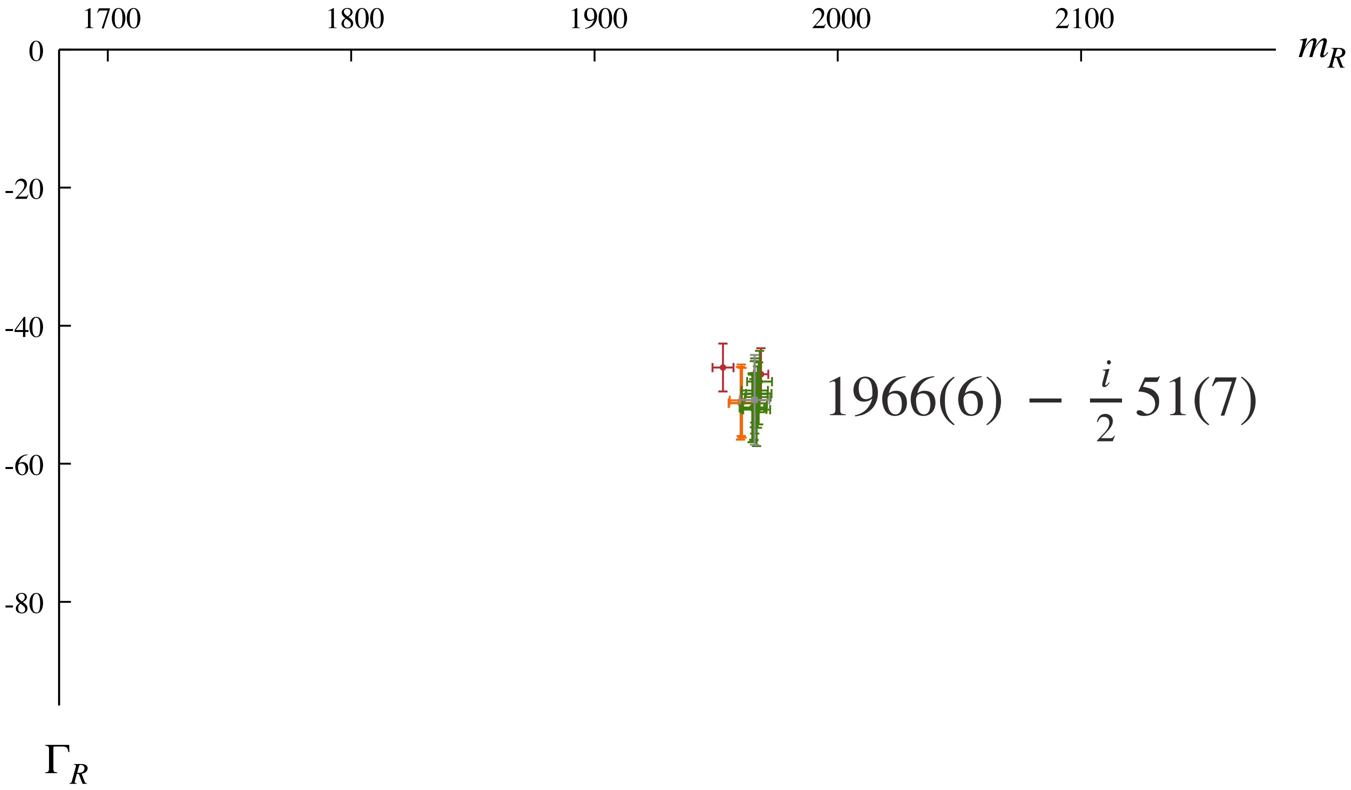


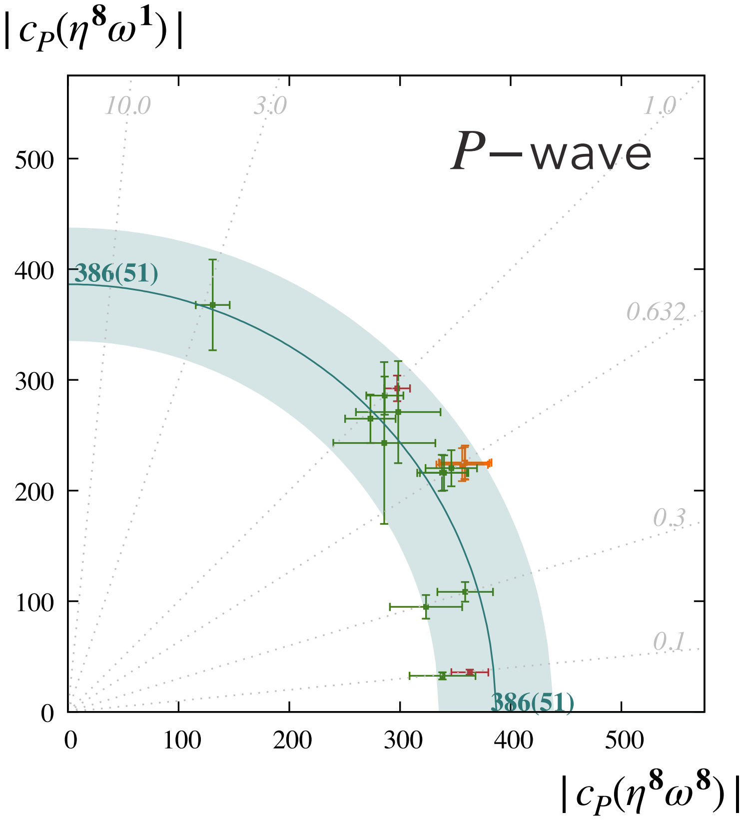


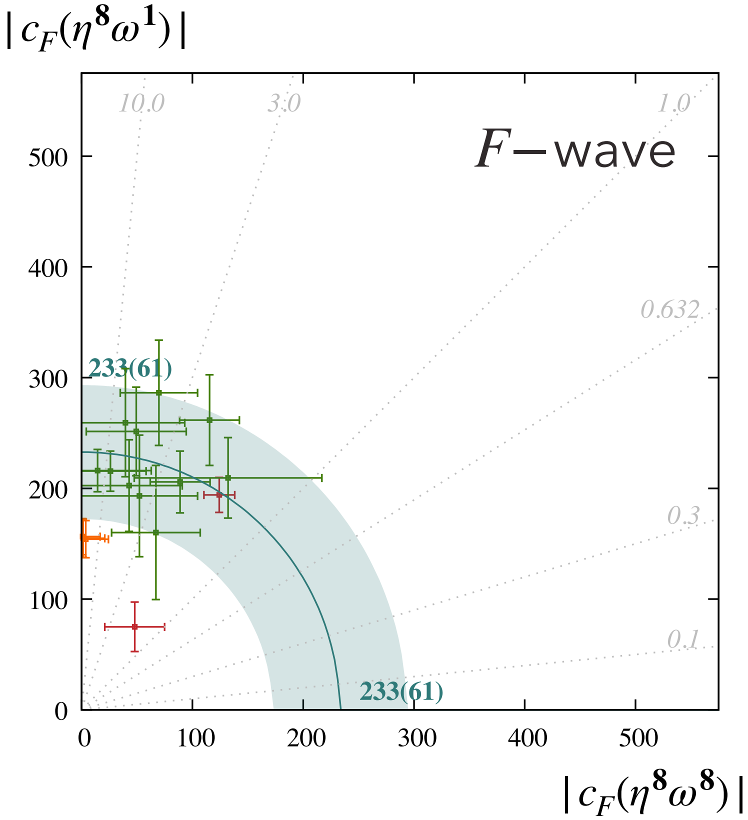


octet 2^{-}

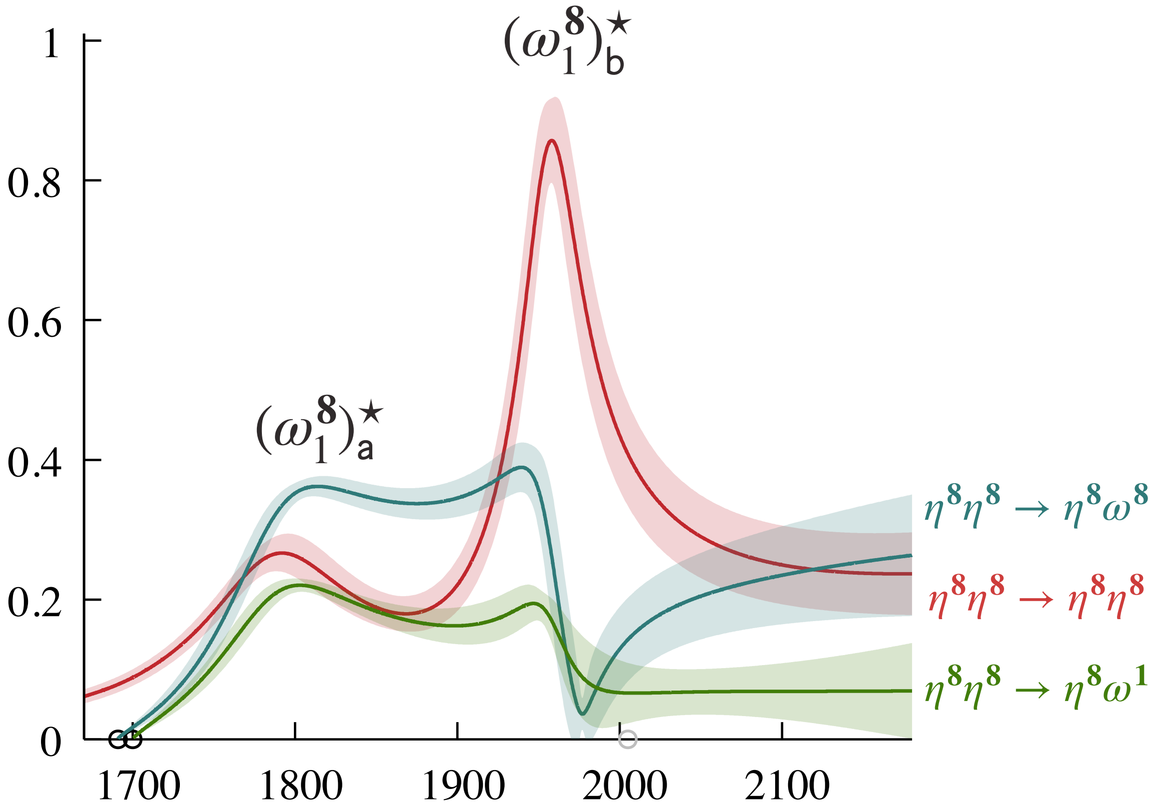

$\eta^8\omega^8|_P \to \eta^8\omega^8|_P$
$\eta^8\omega^8|_P \to \eta^8\omega^1|_P$
$\eta^8\omega^1|_P \to \eta^8\omega^1|_P$
0.6
0.4
0.2
0
1700
1800
1900
2000
2100
PF
0.3
0.2
0.1
0
FF
0.2
0.1
0

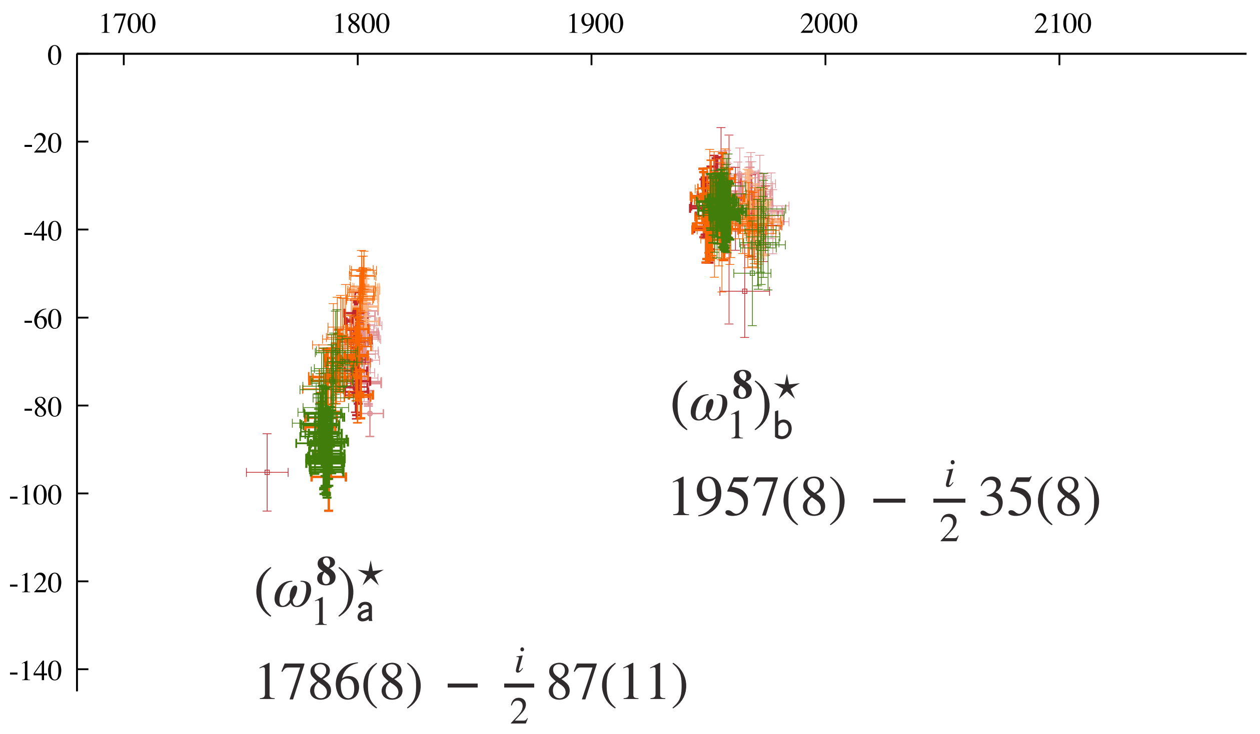

1700
1800
1900
2000
2100
m_R
0
-20
-40
-60
-80
Γ_R
$1966(6) - \frac{i}{2}\,51(7)$

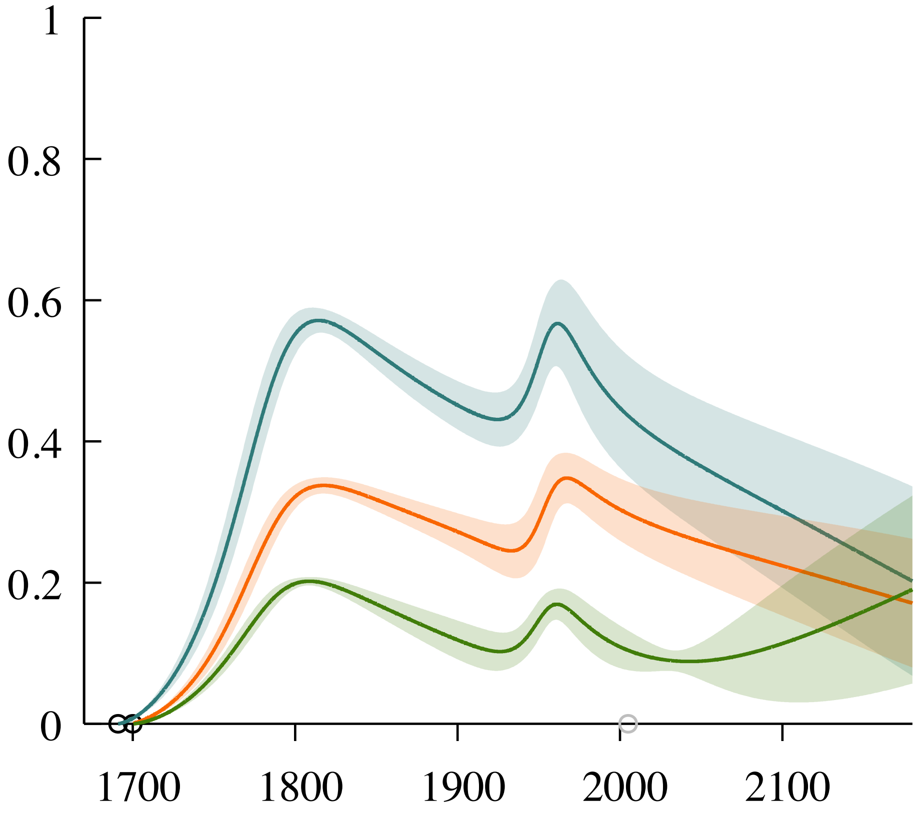

$|c_P(\eta^8\omega^1)|$
$P-$wave
10.0
3.0
1.0
0.632
0.3
0.1
386(51)
386(51)
0
100
200
300
400
500
$|c_P(\eta^8\omega^8)|$


$|c_F(\eta^8\omega^1)|$
$F-$wave
10.0
3.0
1.0
0.632
0.3
0.1
233(61)
233(61)
0
100
200
300
400
500
$|c_F(\eta^8\omega^8)|$

octet 1^-

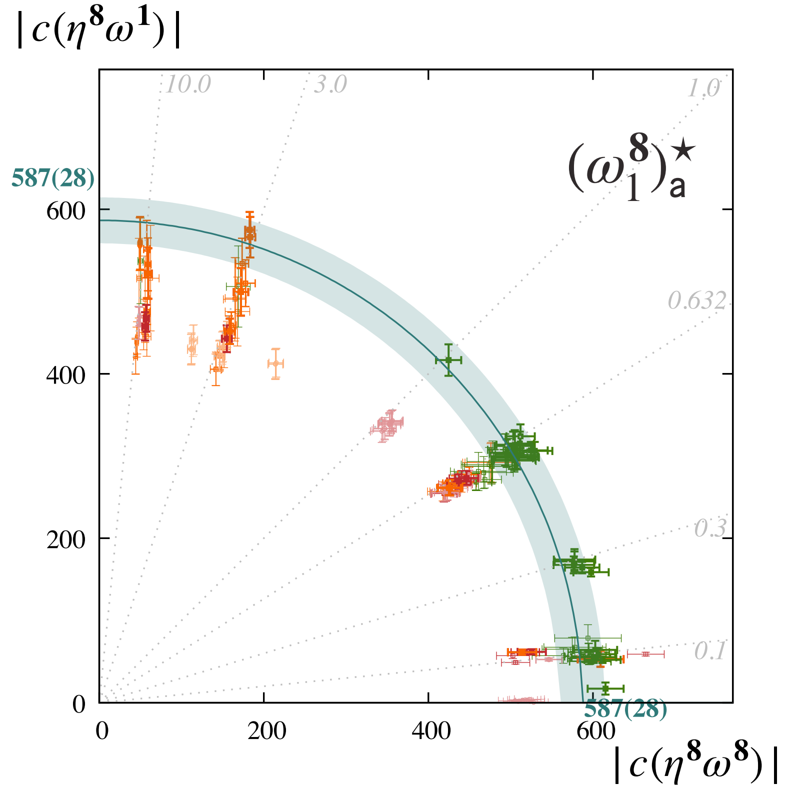


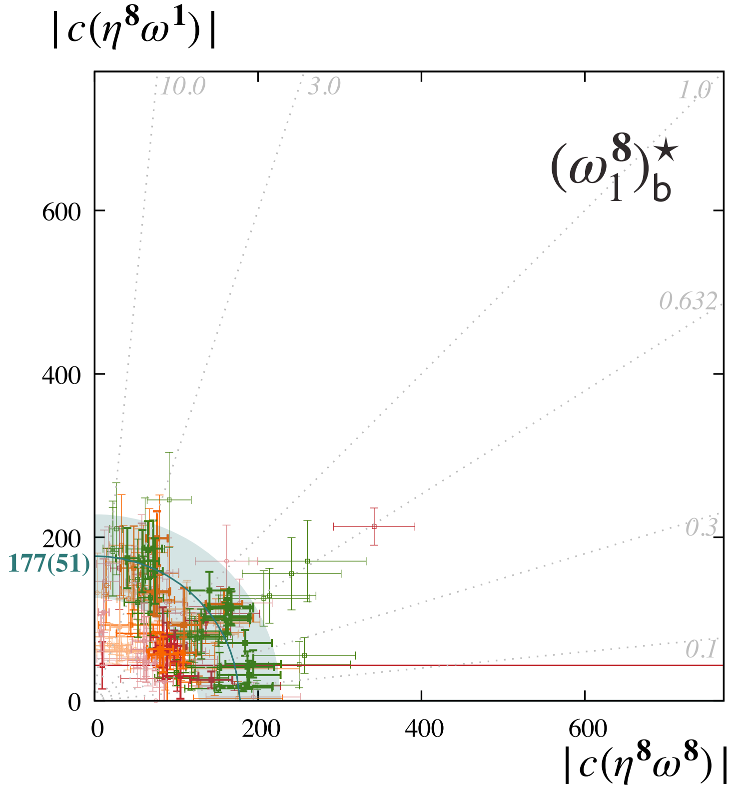




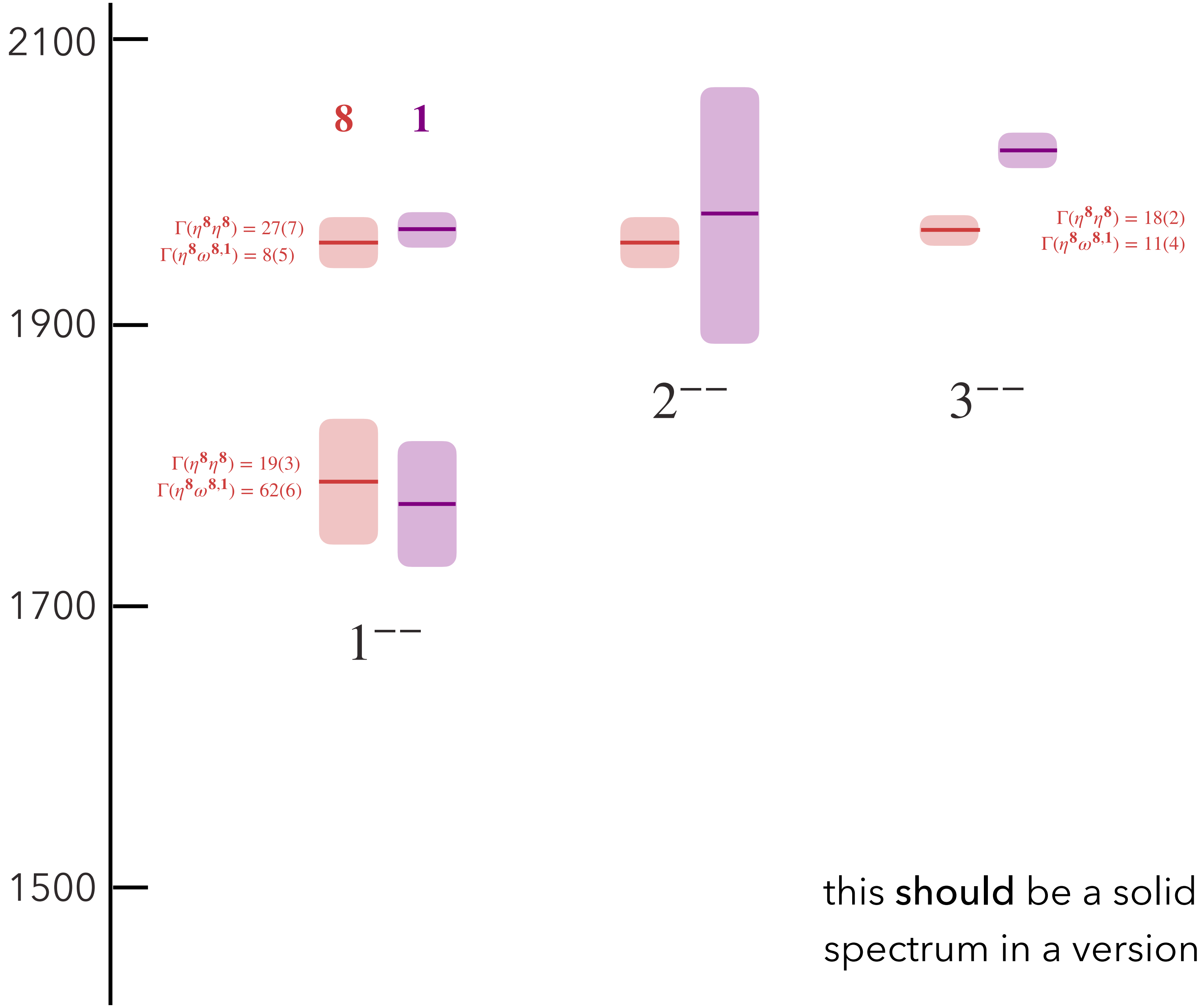
$|c(\eta^8\eta^8)_a| = 233(19)$

$|c(\eta^8\eta^8)_b| = 270(33)$





$SU(3)$ flavor point resonance spectrum in QCD



this **should** be a solid prediction for the spectrum in a version of QCD with $m_u = m_d = m_s \approx m_s^{\text{phys}}$

up to probably modest discretization effects
or blunders

crude (?) extrapolation scheme

decompose $SU(3)$ into (I, I_z, S) states

e.g. $|\mathbf{8}\,(I=0, I_z=0, S=0)\rangle = \frac{1}{\sqrt{20}}\left(K^+\overline{K}^{*-} + K^-\overline{K}^{*+} - K^0\overline{K}^{*0} - \overline{K}^0 K^{*0}\right) - \frac{1}{\sqrt{5}}\left(\pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right)$

assume ideally mixed ω, ϕ

assume $\eta = \eta_8$

assume couplings are constant with changing light quark mass apart from angular-momentum barrier:

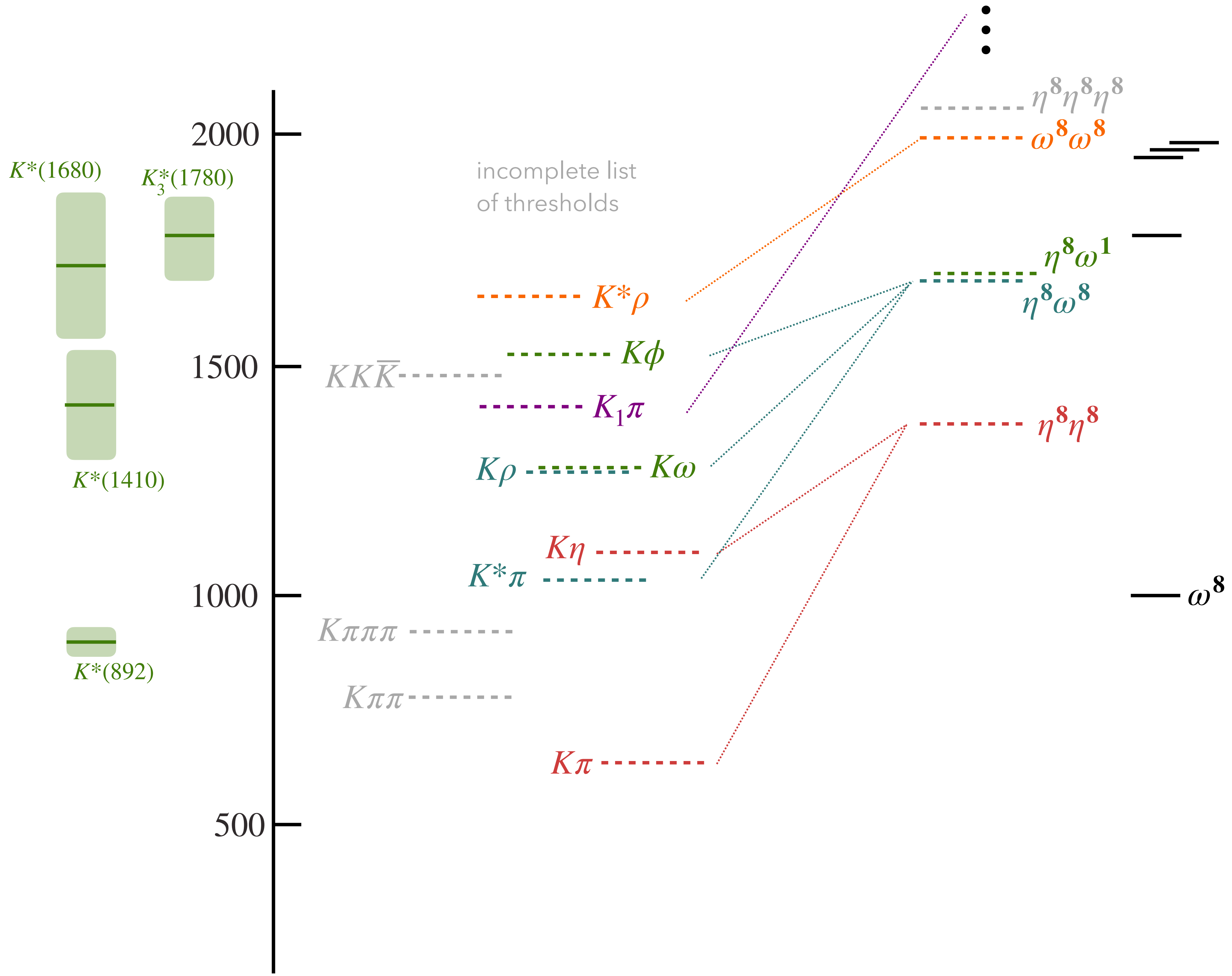
$$|c_{\text{phys}}| = \frac{k^\ell_{\text{phys}}}{k^\ell_{\text{lat}}}\,|c_{\text{lat}}|$$

use the PDG masses to set the kinematics

construct partial widths via $\Gamma = \frac{\rho}{M}|c|^2$

everything shown after this point is a MODEL, not a direct result of a QCD calculation

extrapolation to physical kaons



incomplete list
of thresholds

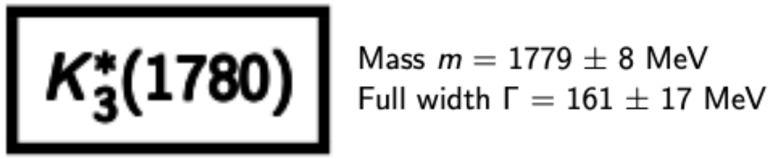
extrapolated $K^\star$ resonances

$\boxed{K_3^*(1780)}$

$\Gamma(K\pi) \sim 30 \quad \Gamma(K\eta) \sim 13$

$\Gamma(K\rho) \sim 9 \quad \Gamma(K^*\pi) \sim 14 \quad \Gamma(K^*\eta) \sim 0 \quad \Gamma(K\omega) \sim 3 \quad \Gamma(K\phi) \sim 1$

$\sum_i \Gamma_i \sim 70$



$K_3^*(1780)$

Mass $m = 1779 \pm 8$ MeV
Full width $\Gamma = 161 \pm 17$ MeV

$\boxed{\text{"}K_2(1780)\text{"}}$

$\Gamma(K\rho) \sim 55 \quad \Gamma(K^*\pi) \sim 74 \quad \Gamma(K^*\eta) \sim 3 \quad \Gamma(K\omega) \sim 17 \quad \Gamma(K\phi) \sim 9$

$\sum_i \Gamma_i \sim 157$

$K_2(1820)$ [ff]

Mass $m = 1819 \pm 12$ MeV
Full width $\Gamma = 264 \pm 34$ MeV

$K_2(1770)$ [ff]

Mass $m = 1773 \pm 8$ MeV
Full width $\Gamma = 186 \pm 14$ MeV

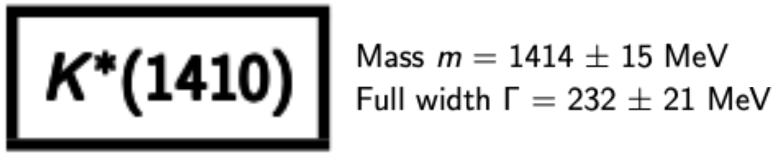
don't yet have the 2^{-+} state to admix

$\boxed{K^*(1410)}$

$\Gamma(K\pi) \sim 19 \quad \Gamma(K\eta) \sim 9$

$\Gamma(K\rho) \sim 41 \quad \Gamma(K^*\pi) \sim 95 \quad \Gamma(K^*\eta) \sim 0 \quad \Gamma(K\omega) \sim 12$

$\sum_i \Gamma_i \sim 176$



$K^*(1410)$

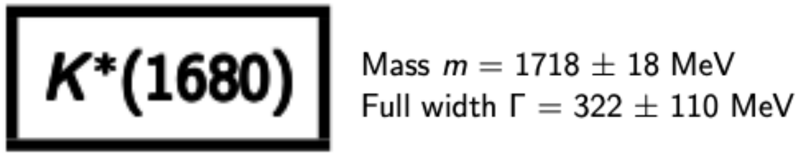
Mass $m = 1414 \pm 15$ MeV
Full width $\Gamma = 232 \pm 21$ MeV

$\boxed{K^*(1680)}$

$\Gamma(K\pi) \sim 24 \quad \Gamma(K\eta) \sim 16$

$\Gamma(K\rho) \sim 5 \quad \Gamma(K^*\pi) \sim 7 \quad \Gamma(K^*\eta) \sim 0 \quad \Gamma(K\omega) \sim 2 \quad \Gamma(K\phi) \sim 1$

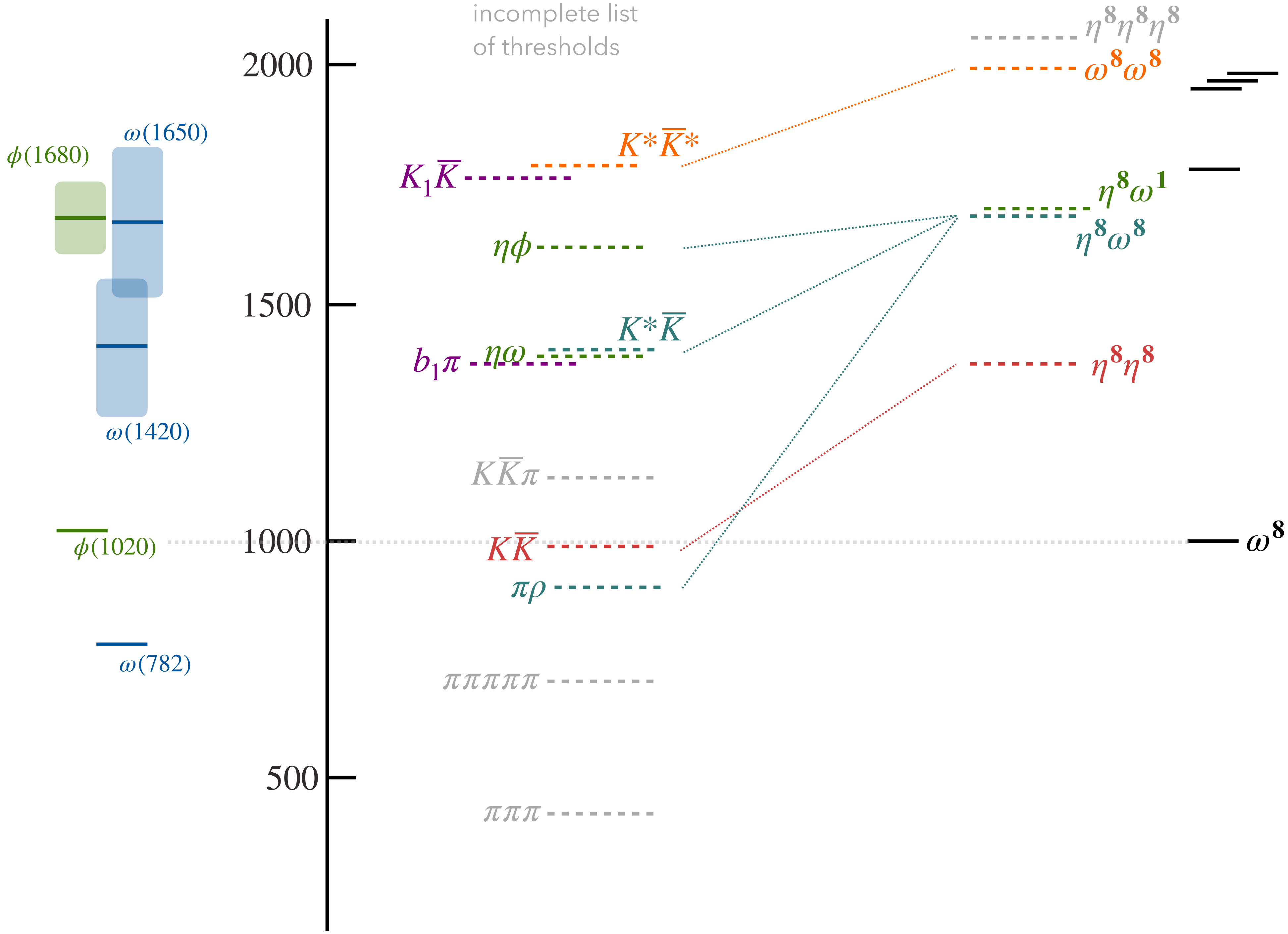
$\sum_i \Gamma_i \sim 56$



$K^*(1680)$

Mass $m = 1718 \pm 18$ MeV
Full width $\Gamma = 322 \pm 110$ MeV

extrapolation to physical isoscalars



extrapolation to physical isoscalars

in this case states are superpositions of octet and singlet

$SU(3)$ point calculation can't tell us the mixing angle – leave as a free parameter

$$|\omega^\star\rangle = \cos\theta|\omega_8\rangle - \sin\theta|\omega_1\rangle$$

$$|\phi^\star\rangle = \sin\theta|\omega_8\rangle + \cos\theta|\omega_1\rangle$$

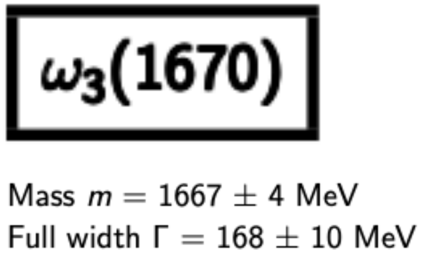
ideal flavor $\theta = -54.74°$,
$\cos\theta = \sqrt{\frac{1}{3}},\ \sin\theta = -\sqrt{\frac{2}{3}}$

extrapolation to physical isoscalars – $\omega_3(1670)$, $\phi_3(1850)$

ideal flavor mixing:

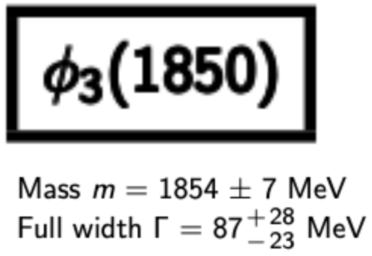
$$\Gamma(\omega^\star \to K\overline{K}) \sim 6$$
$$\Gamma(\omega^\star \to \pi\rho) \sim 61$$
$$\Gamma(\omega^\star \to K\overline{K}^*) \sim 2$$
$$\Gamma(\omega^\star \to \eta\omega) \sim 1$$
$$\Gamma(\omega^\star \to \eta\phi) \sim 0$$
$$\sum_i \Gamma_i \sim 70$$

$$\Gamma(\phi^\star \to K\overline{K}) \sim 29$$
$$\Gamma(\phi^\star \to \pi\rho) \sim 0$$
$$\Gamma(\phi^\star \to K\overline{K}^*) \sim 20$$
$$\Gamma(\phi^\star \to \eta\omega) \sim 0$$
$$\Gamma(\phi^\star \to \eta\phi) \sim 3$$
$$\sum_i \Gamma_i \sim 51$$



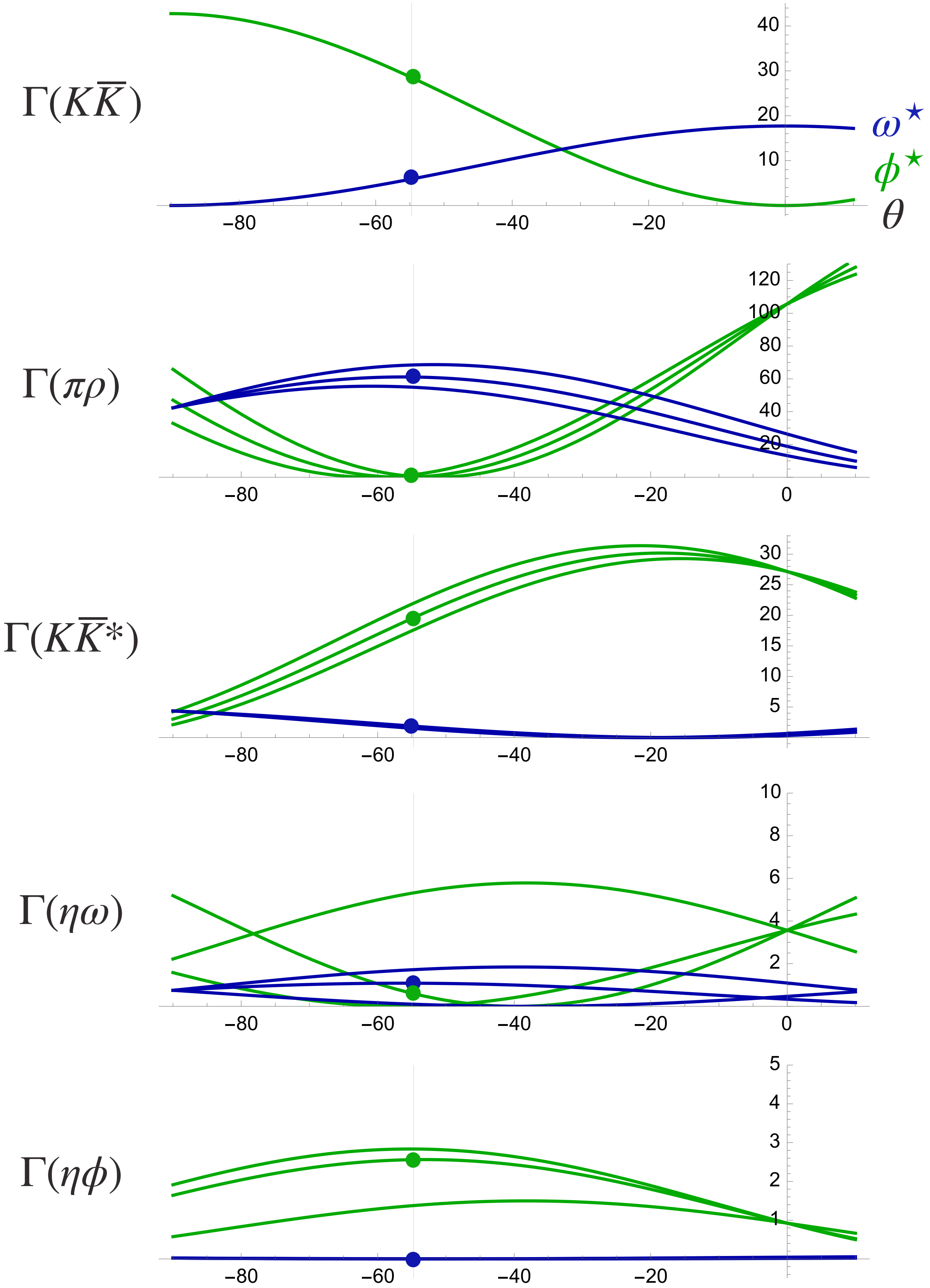
$\omega_3(1670)$

Mass $m = 1667 \pm 4$ MeV
Full width $\Gamma = 168 \pm 10$ MeV



$\phi_3(1850)$

Mass $m = 1854 \pm 7$ MeV
Full width $\Gamma = 87^{+28}_{-23}$ MeV



extrapolation to physical isoscalars – "$\omega_2(1670)$, $\phi_2(1850)$"

ideal flavor mixing:

P-wave, F-wave

$\Gamma(\omega^\star \to \pi\rho) \sim 237{,}110$

$\Gamma(\omega^\star \to K\overline{K}^*) \sim 38{,}2$

$\Gamma(\omega^\star \to \eta\omega) \sim 12{,}2$

$\Gamma(\omega^\star \to \eta\phi) \sim 0$

$\sum_i \Gamma_i \sim 401$

! likely to be very broad !

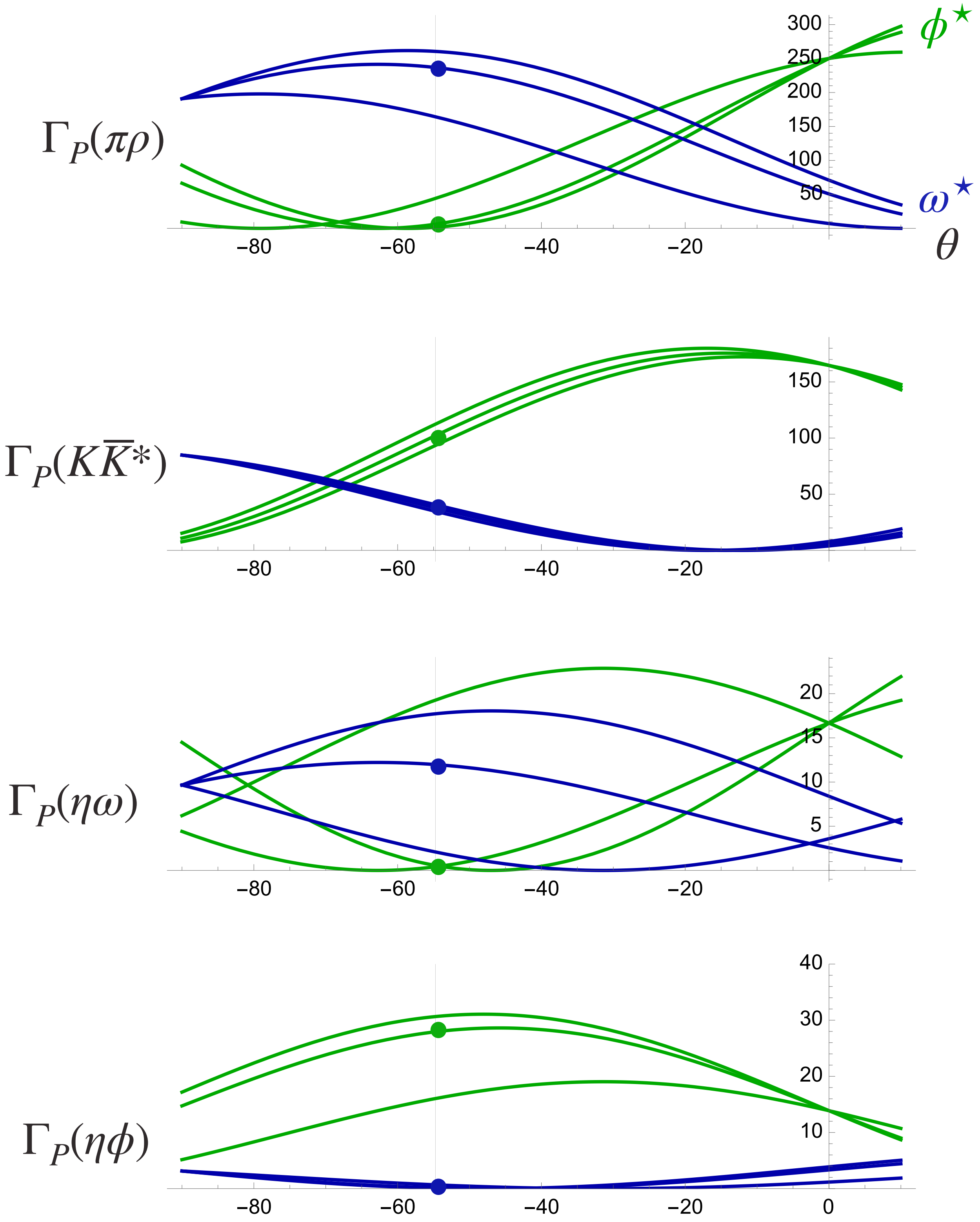
$\Gamma(\phi^\star \to \pi\rho) \sim 6{,}6$

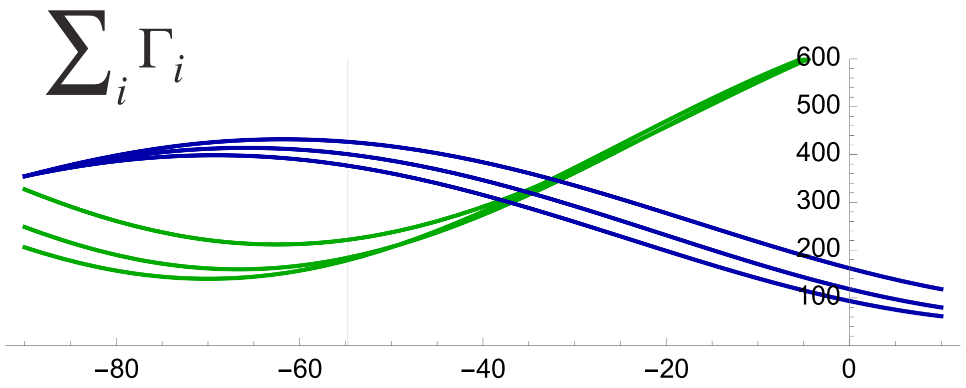
$\Gamma(\phi^\star \to K\overline{K}^*) \sim 102{,}35$

$\Gamma(\phi^\star \to \eta\omega) \sim 0{,}0$

$\Gamma(\phi^\star \to \eta\phi) \sim 28{,}6$

$\sum_i \Gamma_i \sim 184$





extrapolation to physical isoscalars – $\omega(1420)$, $\phi(1680)$

ideal flavor mixing:

$\Gamma(\omega^\star \to K\bar{K}) \sim 7$

$\Gamma(\omega^\star \to \pi\rho) \sim 468$

$\Gamma(\omega^\star \to K\bar{K}^*) \sim 2$

$\Gamma(\omega^\star \to \eta\omega) \sim 6$

$\Gamma(\omega^\star \to \eta\phi) \sim 0$

$\sum_i \Gamma_i \sim 482$

$\Gamma(\phi^\star \to K\bar{K}) \sim 25$

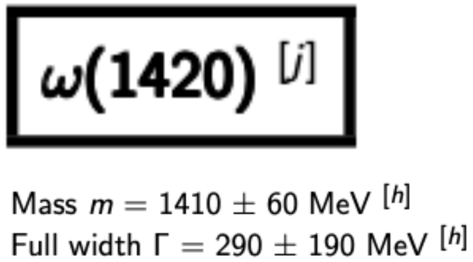
$\Gamma(\phi^\star \to \pi\rho) \sim 10$

$\Gamma(\phi^\star \to K\bar{K}^*) \sim 191$

$\Gamma(\phi^\star \to \eta\omega) \sim 0$

$\Gamma(\phi^\star \to \eta\phi) \sim 37$

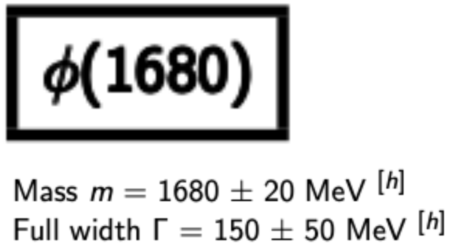
$\sum_i \Gamma_i \sim 263$



$\omega(1420)$ [j]

Mass $m = 1410 \pm 60$ MeV [h]

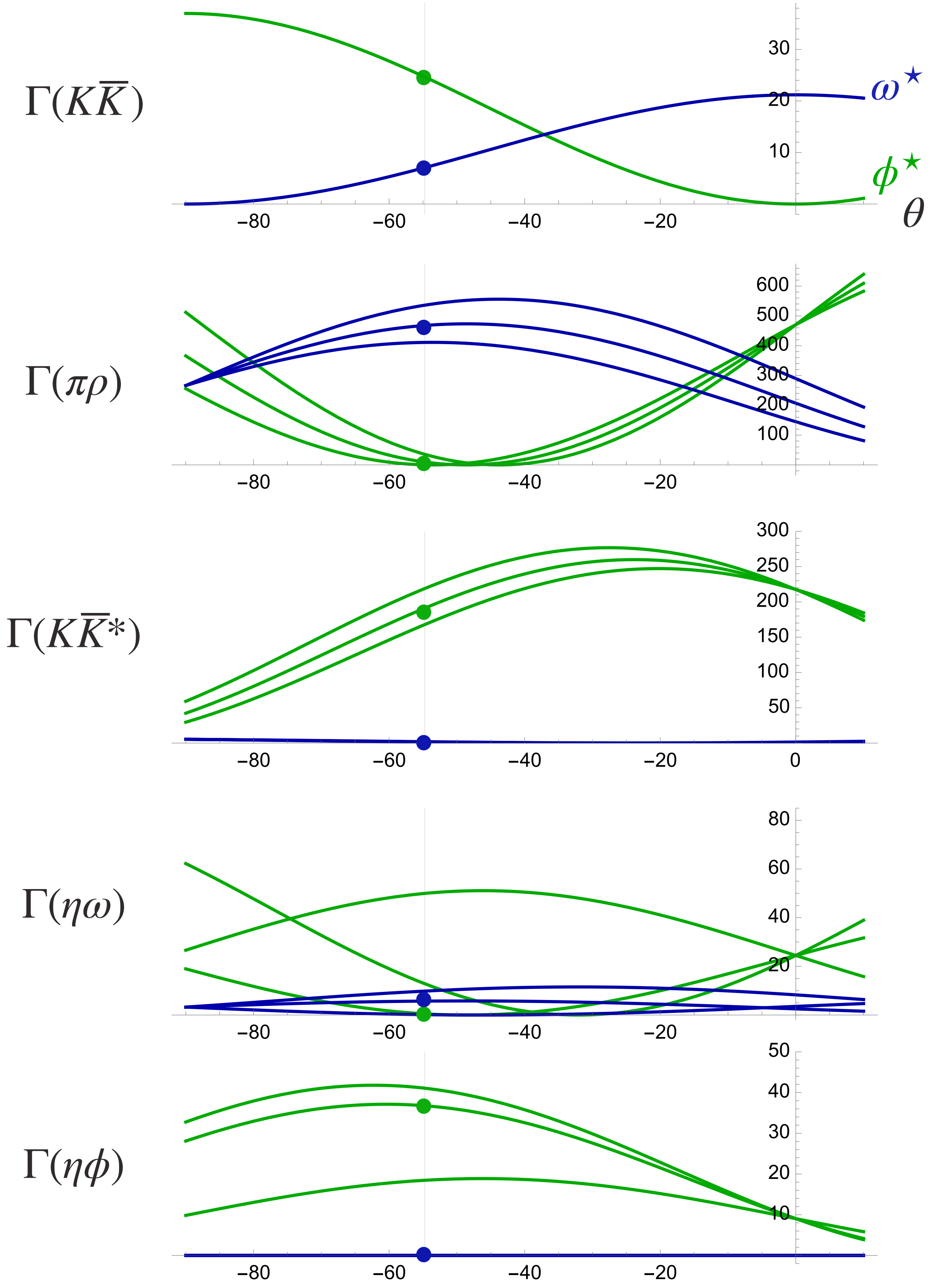
Full width $\Gamma = 290 \pm 190$ MeV [h]

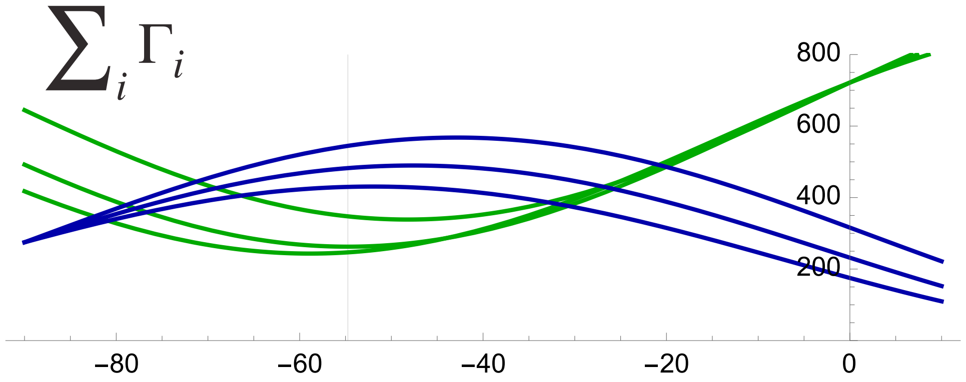


$\phi(1680)$

Mass $m = 1680 \pm 20$ MeV [h]

Full width $\Gamma = 150 \pm 50$ MeV [h]



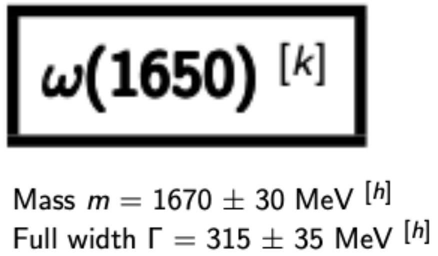


extrapolation to physical isoscalars – $\omega(1650)$, “$\phi(1850)$”

ideal flavor mixing:

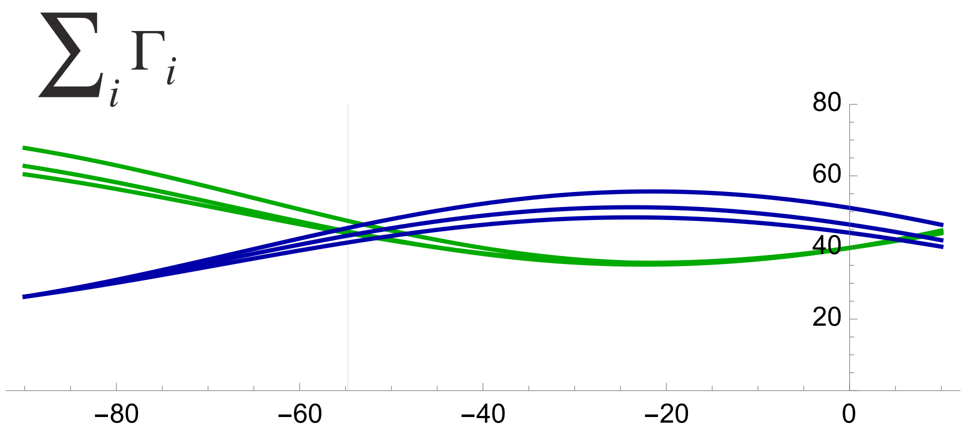
$$\Gamma(\omega^\star \to K\overline{K}) \sim 11$$
$$\Gamma(\omega^\star \to \pi\rho) \sim 28$$
$$\Gamma(\omega^\star \to K\overline{K}^*) \sim 3$$
$$\Gamma(\omega^\star \to \eta\omega) \sim 1$$
$$\Gamma(\omega^\star \to \eta\phi) \sim 0$$
$$\sum_i \Gamma_i \sim 43$$

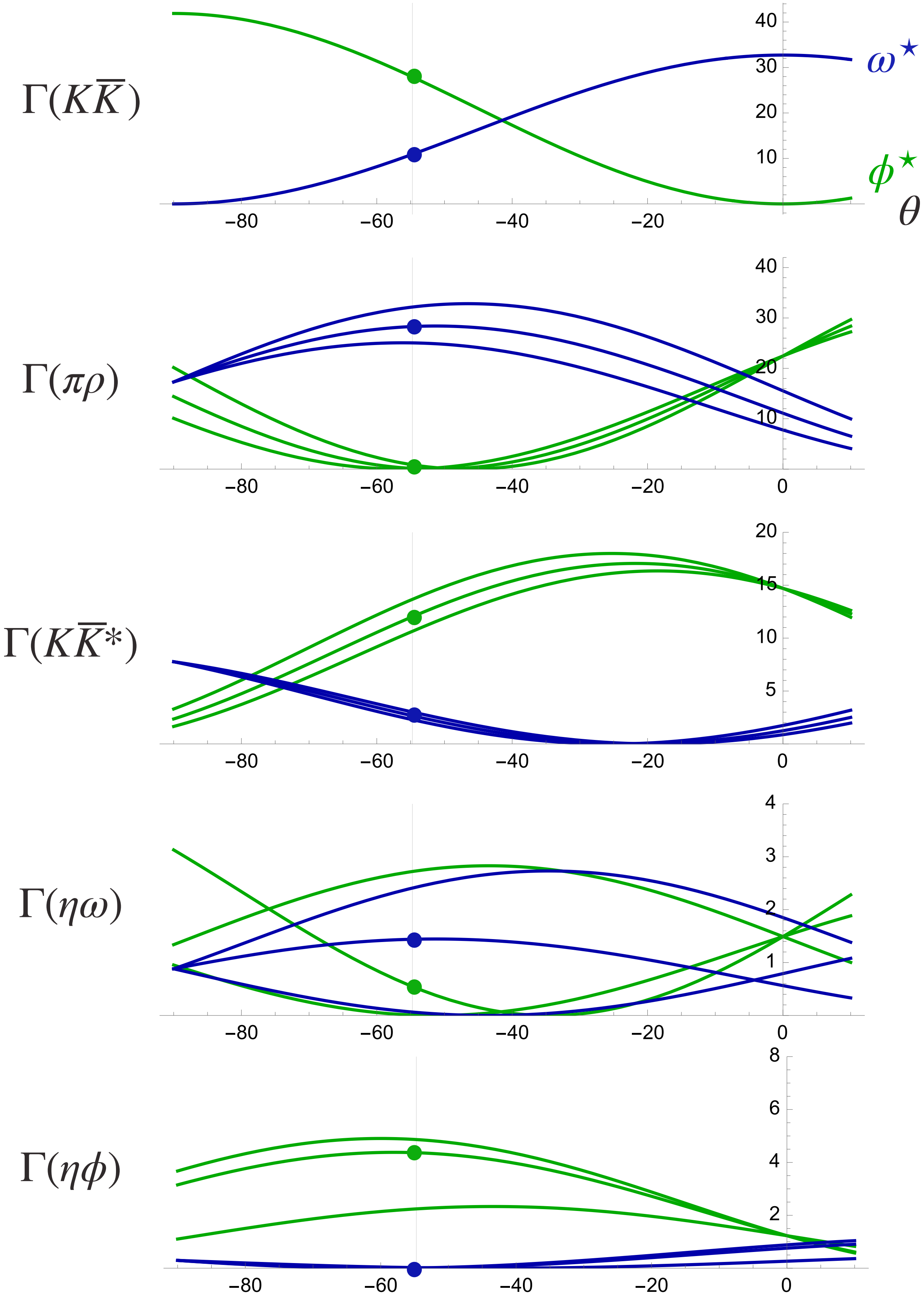
$$\Gamma(\phi^\star \to K\overline{K}) \sim 28$$
$$\Gamma(\phi^\star \to \pi\rho) \sim 0$$
$$\Gamma(\phi^\star \to K\overline{K}^*) \sim 12$$
$$\Gamma(\phi^\star \to \eta\omega) \sim 0$$
$$\Gamma(\phi^\star \to \eta\phi) \sim 4$$
$$\sum_i \Gamma_i \sim 44$$



ω(1650) [k]

Mass m = 1670 ± 30 MeV [h]
Full width Γ = 315 ± 35 MeV [h]





conclusions

not sure I have any yet … but I can summarize some observations :

SU(3) point calculation doesn't indicate any large deviations from OZI expectations

2^{--} **isoscalars** likely to be **very broad** (even before $2^{+}0^{-}$ decays are considered)

the **lighter** 1^{-} **state** has large $0^{-}1^{-}$ decay branches – can lead to a large total width

the **heavier** 1^{-} **state** has modest $0^{-}0^{-}$, $0^{-}1^{-}$ decay branches
– if the PDG widths are right, there would need to be modes not considered here

possible next steps for these states in lattice calcs ...

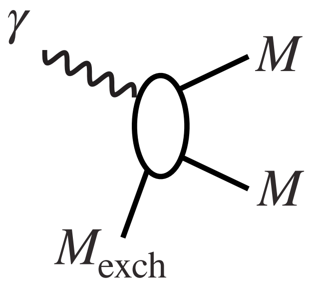
depart from SU(3) symmetry point, by reducing $m_{u,d}$:
- establish pattern of octet, singlet mixing
- investigate reasonableness of extrapolation scheme

maturation of three-body analysis techniques will allow analysis at closer to physical light quark masses

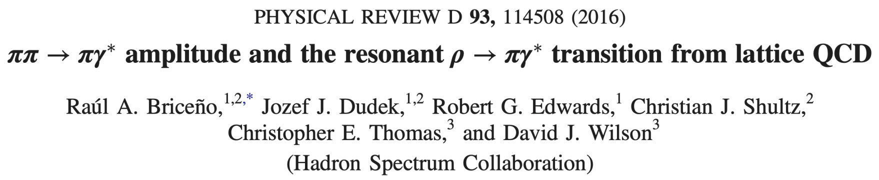
compute amplitudes closer to those experimentally measured :

- in 1^{--} case can compute $\langle 0 | \bar{\psi}\gamma^\mu\psi | MM \rangle$ to get $A(e^+e^- \to MM)$

- can calculate contributions to photoproduction treating exchange meson as stable and on-shell

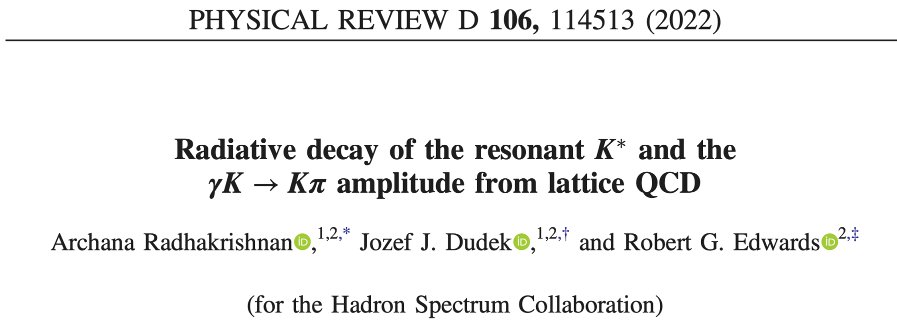


previously done for $\gamma\pi \to \pi\pi,\ \gamma K \to \pi K$


PHYSICAL REVIEW D **93,** 114508 (2016)

$\pi\pi \to \pi\gamma^*$ amplitude and the resonant $\rho \to \pi\gamma^*$ transition from lattice QCD

Raúl A. Briceño,[1,2,*] Jozef J. Dudek,[1,2] Robert G. Edwards,[1] Christian J. Shultz,[2]
Christopher E. Thomas,[3] and David J. Wilson[3]
(Hadron Spectrum Collaboration)


PHYSICAL REVIEW D **106,** 114513 (2022)

Radiative decay of the resonant K^* and the
$\gamma K \to K\pi$ amplitude from lattice QCD

Archana Radhakrishnan,[1,2,*] Jozef J. Dudek,[1,2,†] and Robert G. Edwards[2,‡]

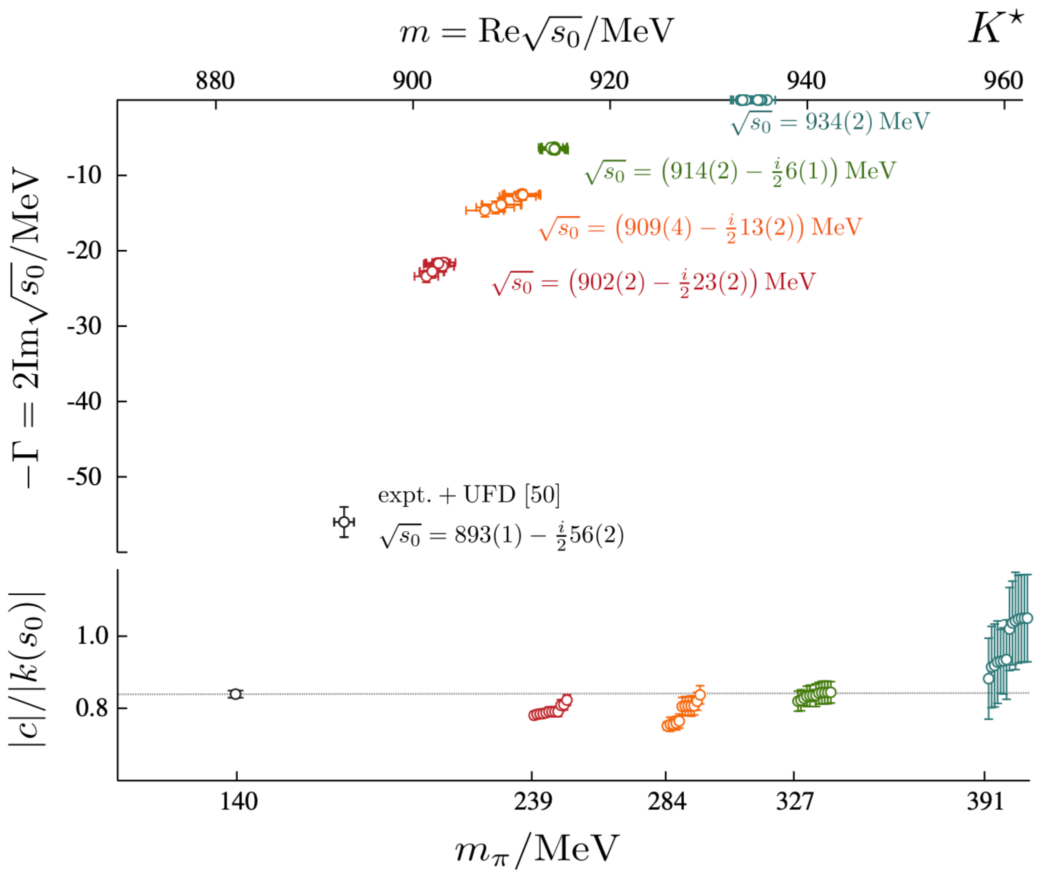
(for the Hadron Spectrum Collaboration)

coupling extrapolation

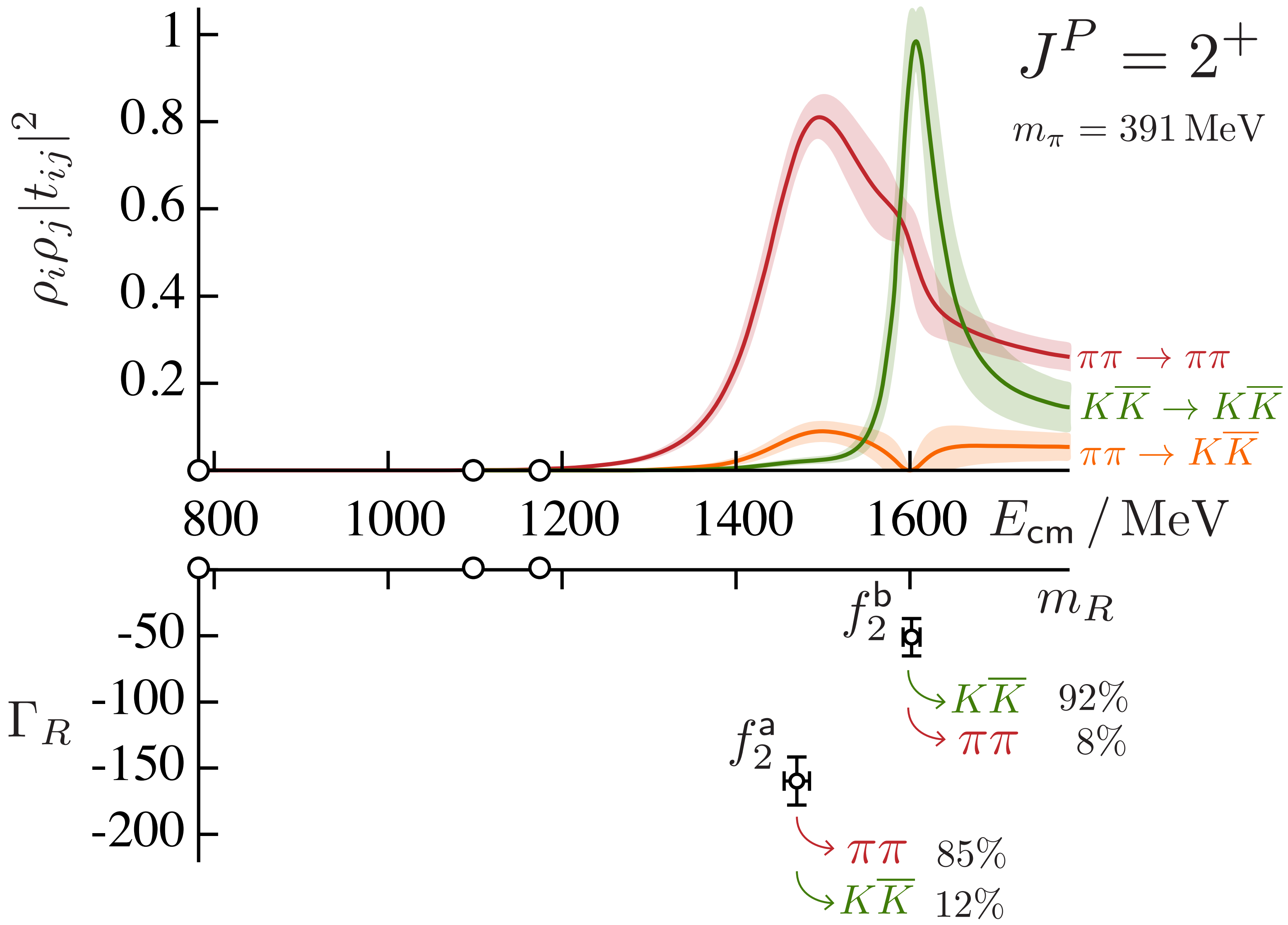
This approach is motivated by observations made in lattice calculations of the decays of $b_1 \to \omega\pi$ dominantly in S-wave [68], ρ to $\pi\pi$ in P-wave [46], K^* to $K\pi$ in P-wave [93] and f_2, f_2' decays to $\pi\pi$ and $K\bar{K}$ in D-wave [66], which appear to show quark-mass independence when treated this way. For example in the b_1 case, the coupling computed in [68] at $m_\pi \sim 391$ MeV is $|c| = 564(114)$ MeV, in good agreement with the coupling $|c|^{\mathrm{phys}} = 556(17)$ MeV extracted from the experimental b_1 decay width. In the P-wave ρ decay, an explicit factor of k is required for the scaling to work, as presented in Ref. [46]. In addition, as shown in Fig. 4 of Ref. [93], the K^* coupling scaled in this way is approximately constant for four different light-quark masses corresponding to $m_\pi = 239$ to 391 MeV, even when the K^* is a shallow bound state, and is in agreement with the experimentally measured coupling. Scaling the f_2, f_2' D-wave couplings computed at $m_\pi \sim 391$ MeV in [66] gives, in comparison to values extracted from the Particle Data Group (PDG) review [90],

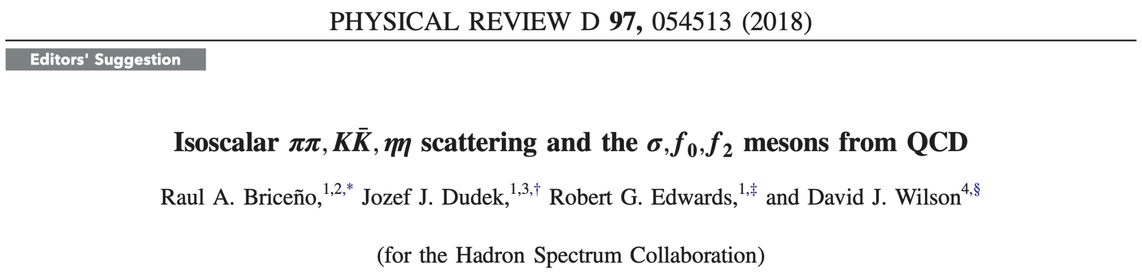
	Scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	488(28)	453^{+9}_{-4},
$\lvert c(f_2 \to K\bar{K})\rvert$	139(27)	132(7),
$\lvert c(f_2' \to \pi\pi)\rvert$	103(32)	33(4),
$\lvert c(f_2' \to K\bar{K})\rvert$	321(50)	389(12),

which is quite a reasonable agreement given the large extrapolation in quark mass.^{13}



coupled $\pi\pi$, $K\bar{K}$ scattering with $m_\pi \sim 391$ MeV





PHYSICAL REVIEW D **97,** 054513 (2018)

Editors' Suggestion

Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]

(for the Hadron Spectrum Collaboration)

$f_2^{\mathsf{a}} \sim u\bar{u} + d\bar{d}$ $\quad f_2^{\mathsf{b}} \sim s\bar{s}$

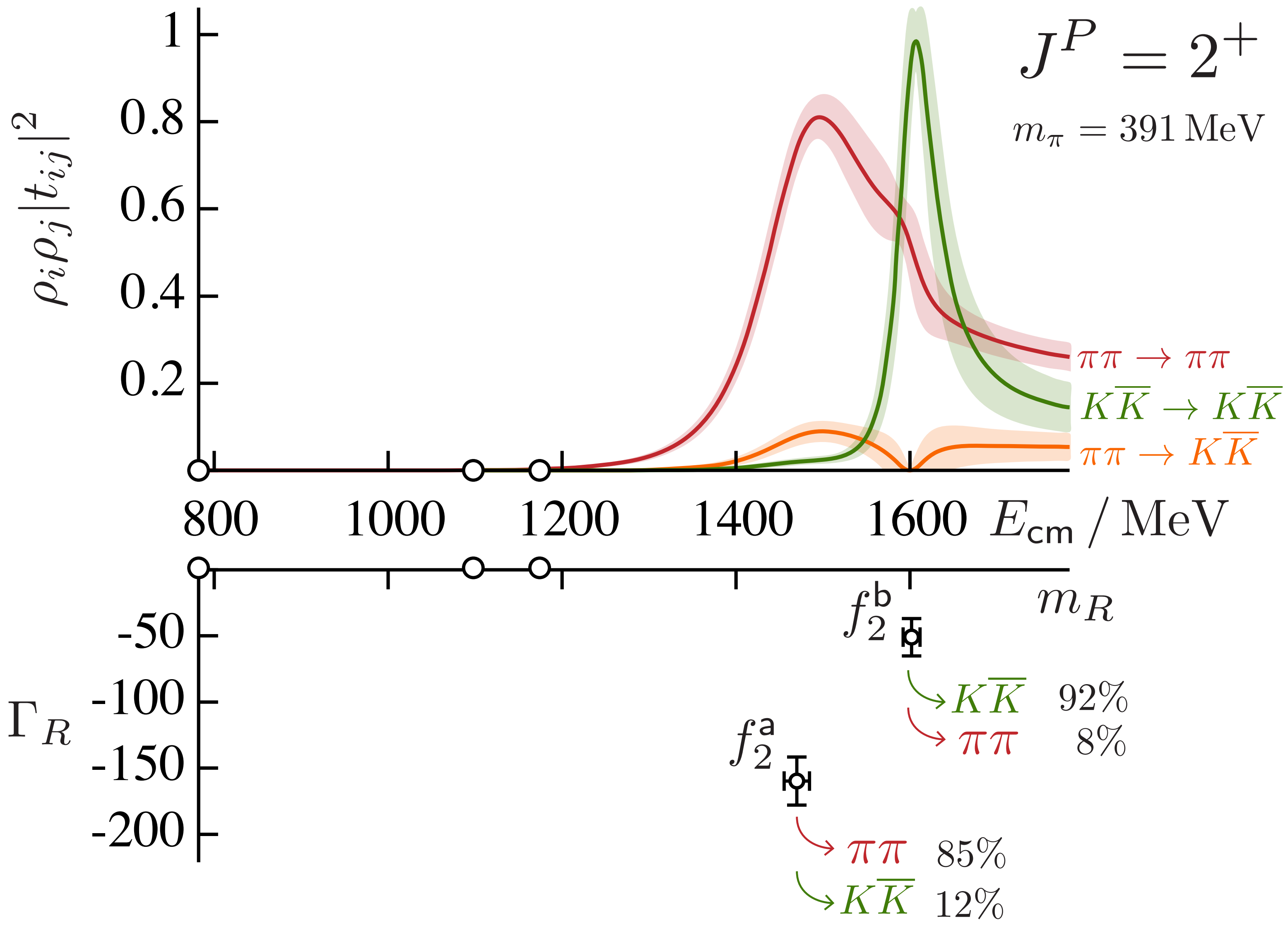
couplings from pole residue

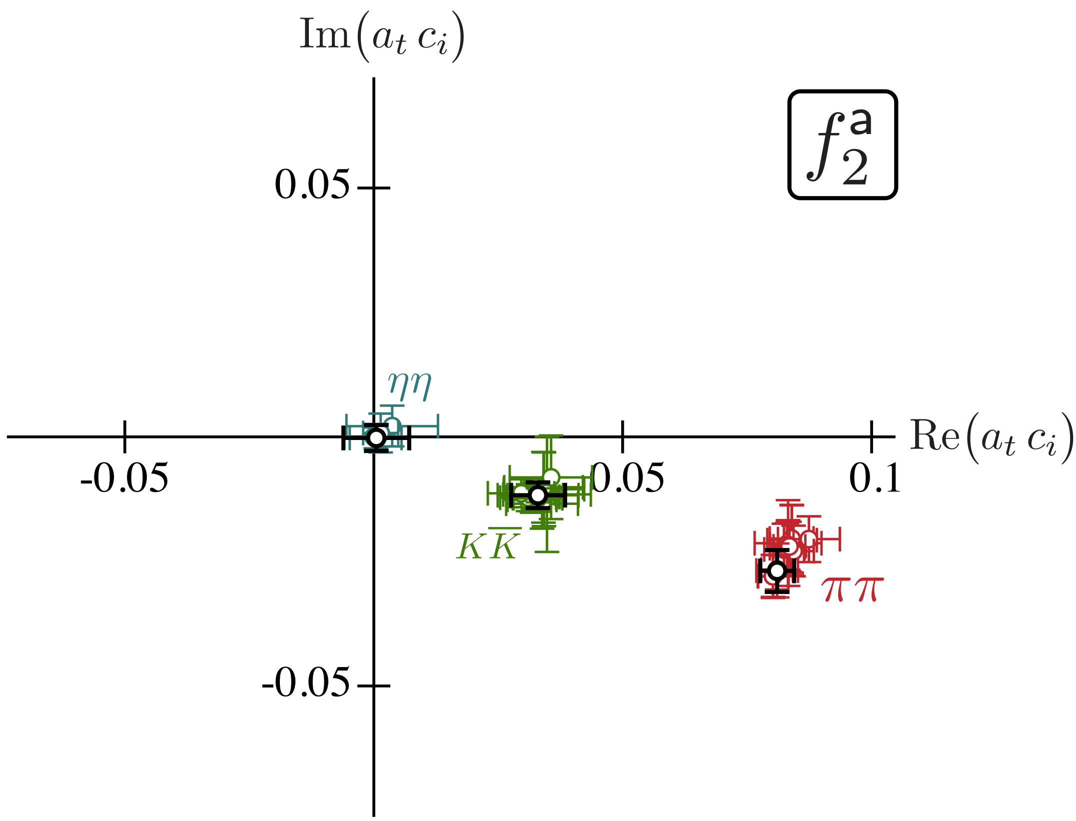
	$\frac{a_t \lvert c_{\pi\pi} \rvert}{(a_t k_{\pi\pi})^2}$	$\frac{a_t \lvert c_{K\bar{K}} \rvert}{(a_t k_{K\bar{K}})^2}$
f_2^{a}	$7.1(4)$	$4.8(9)$
f_2^{b}	$1.0(3)$	$5.5(8)$

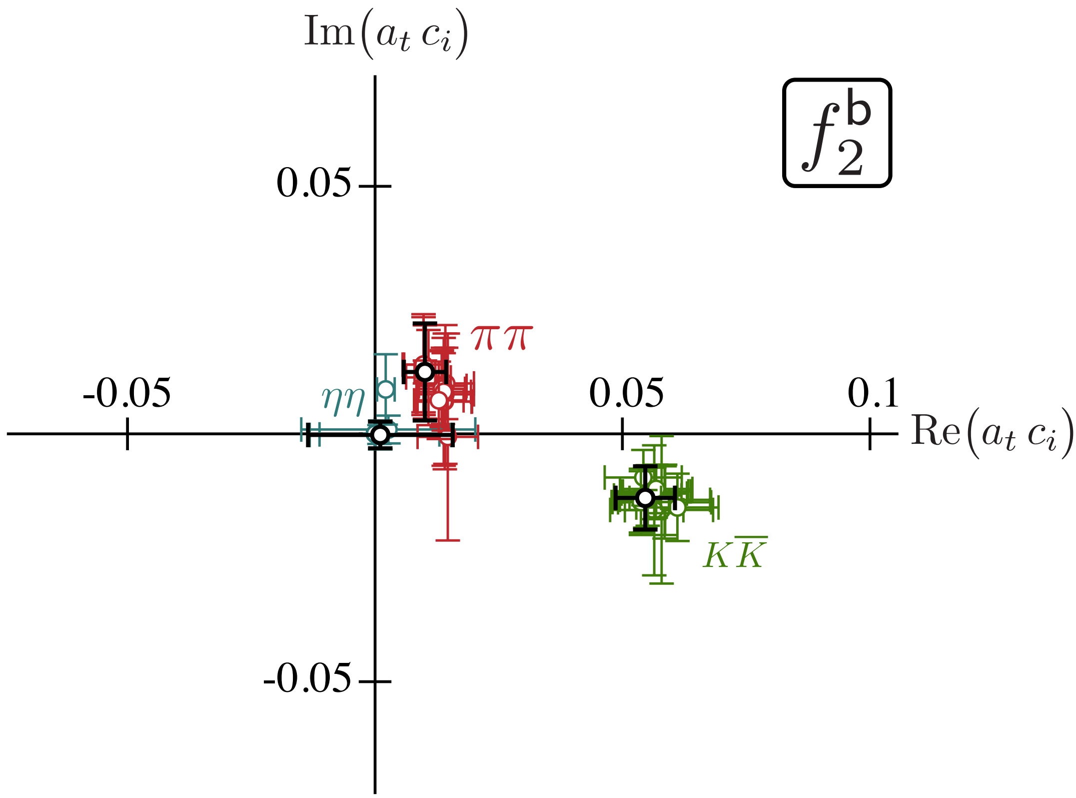
zero in 'OZI' limit

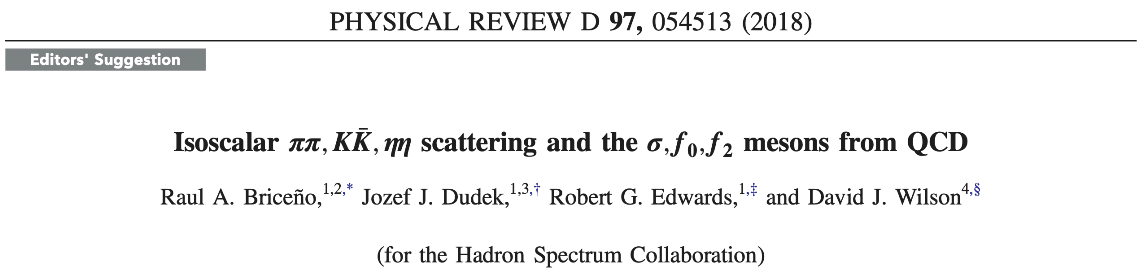
– requires $s\bar{s}$ annihilation

coupled $\pi\pi, K\overline{K}$ scattering with $m_\pi \sim 391$ MeV








PHYSICAL REVIEW D **97,** 054513 (2018)

Editors' Suggestion

Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

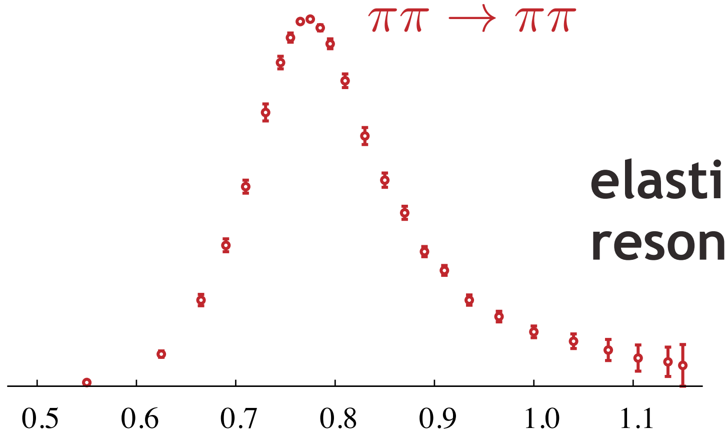
Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]

(for the Hadron Spectrum Collaboration)

coupled-channel resonances

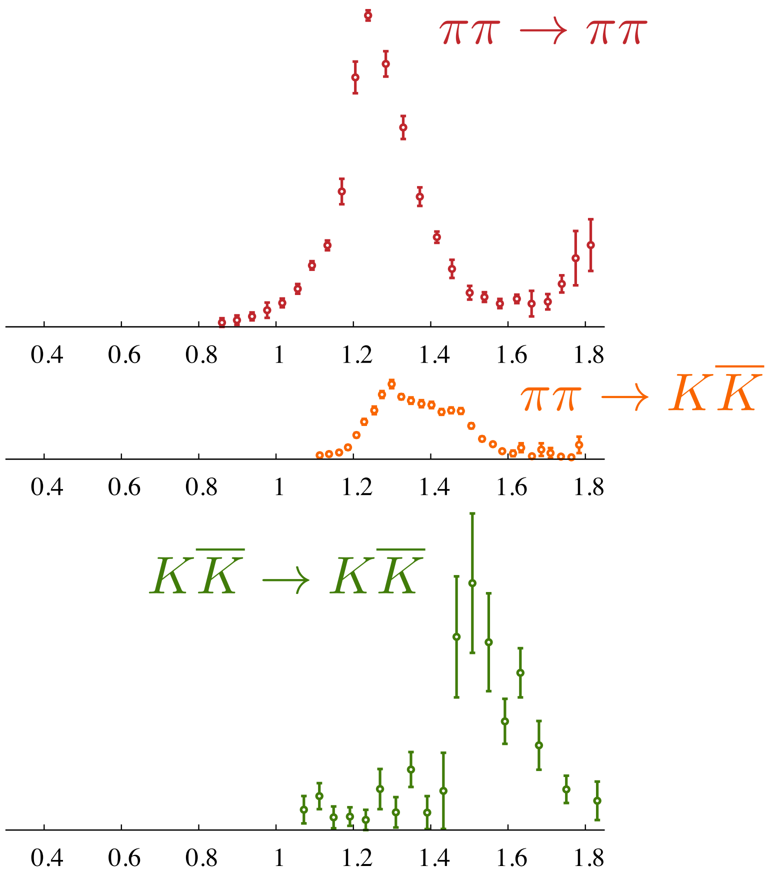
most resonances can decay into more than one final state

e.g. $\overbrace{\pi\pi}^{\rho(770)}$



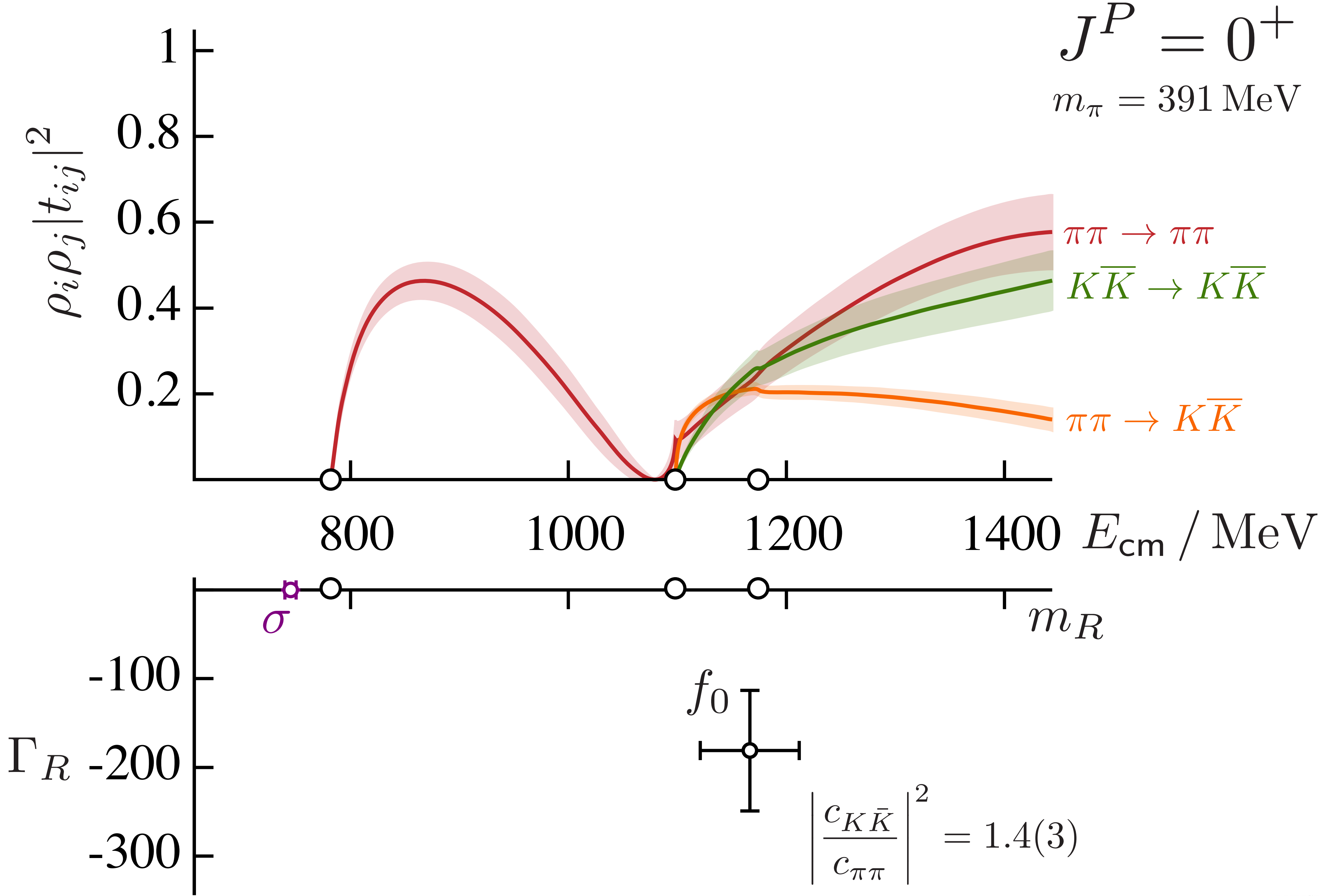
elastic resonance

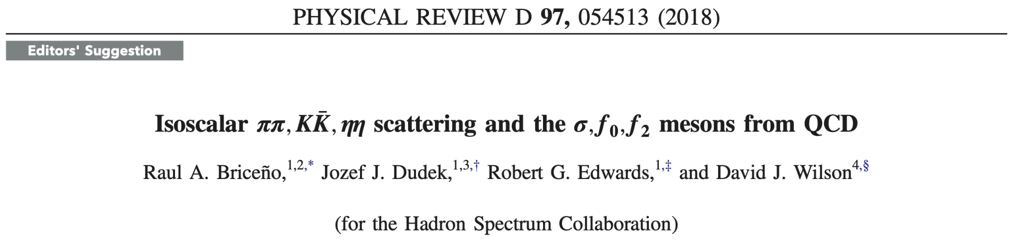
$\overbrace{\pi\pi, \pi\pi\pi\pi, K\overline{K}, \eta\eta}^{f_2(1270)}$ $\overbrace{\pi\pi, K\overline{K}, \eta\eta}^{f_2(1525)}$



coupled-channel resonances

coupled $\pi\pi, K\bar{K}$ scattering with $m_\pi \sim 391$ MeV




PHYSICAL REVIEW D **97,** 054513 (2018)

Editors' Suggestion

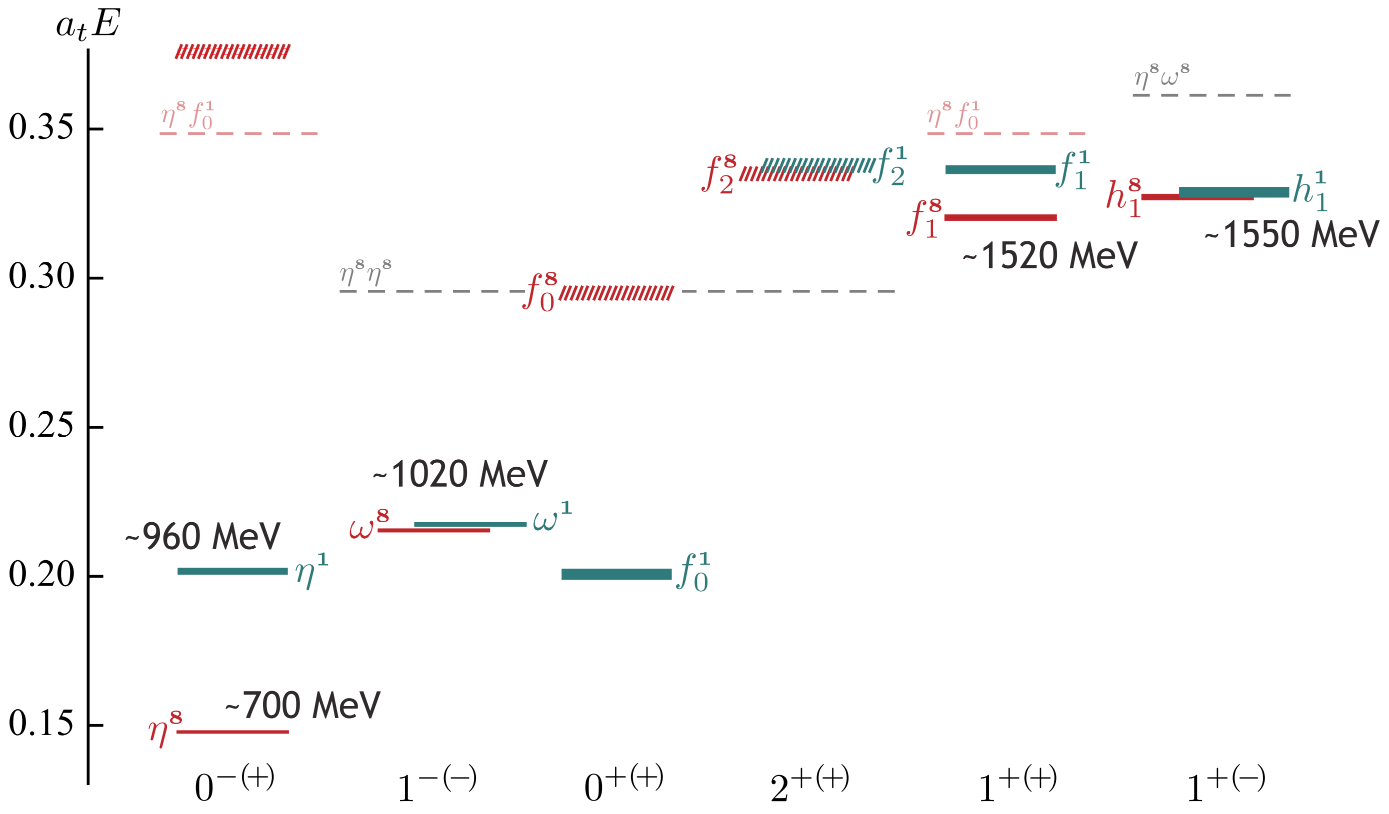
Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]

(for the Hadron Spectrum Collaboration)

$m_u=m_d=m_s$ SU(3)$_F$ point

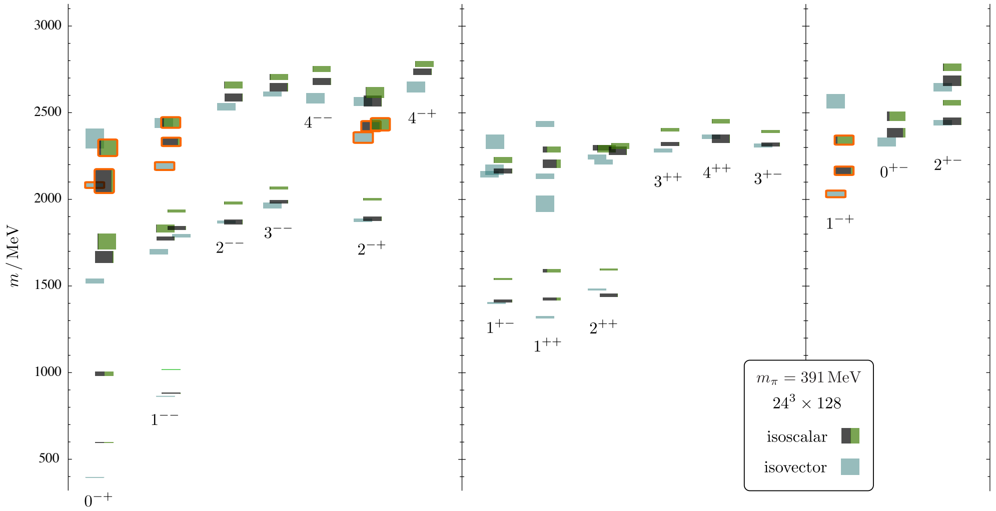
several stable mesons:

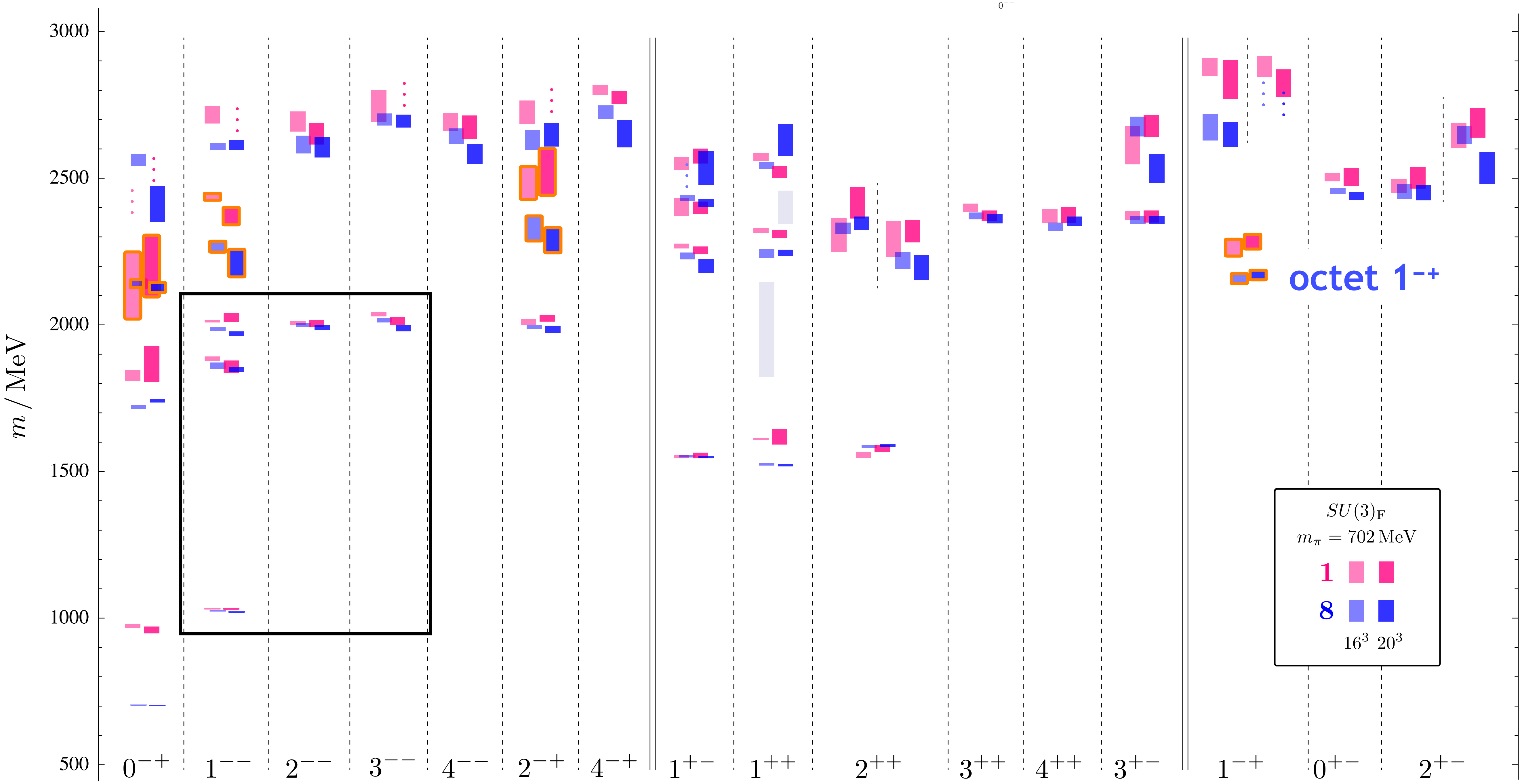


$m_u = m_d = m_s$ SU(3)$_F$ point

increase the light quark mass to the strange quark mass …

(incomplete) **lattice spectrum calculation** PRD 88 094505 (2013)



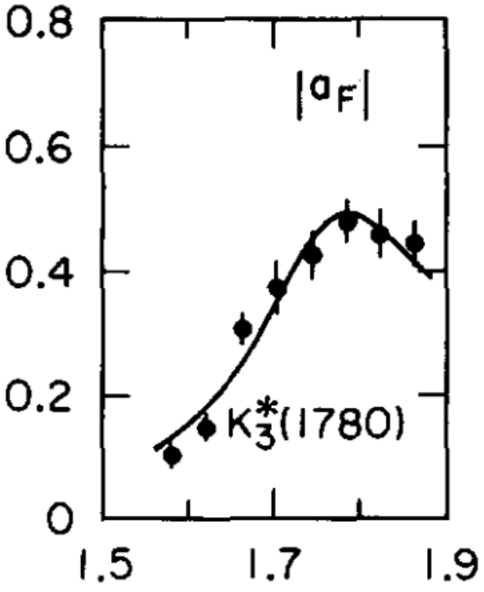


$K^\star$ resonances (in LASS)

$K_3^*(1780)$

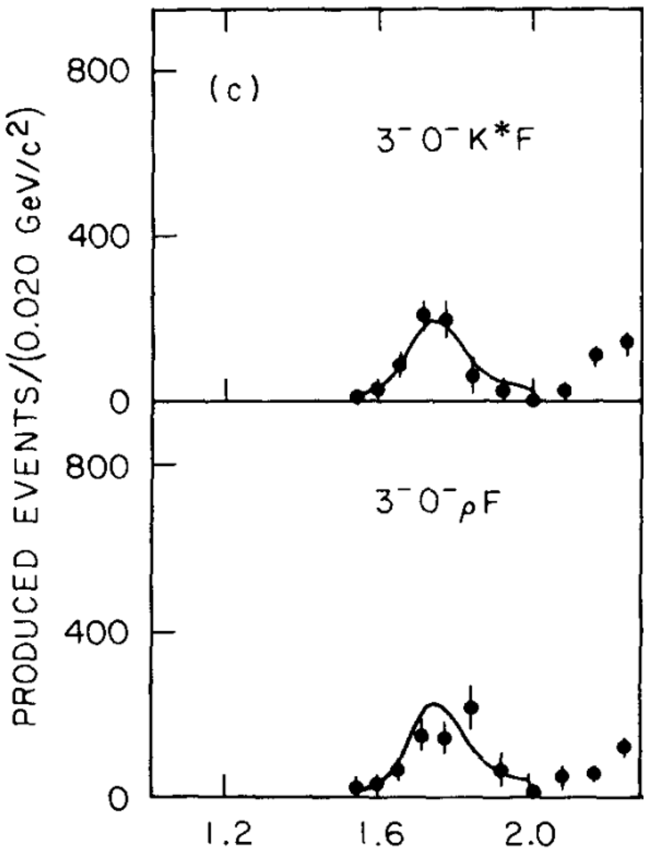
Mass $m = 1779 \pm 8$ MeV
Full width $\Gamma = 161 \pm 17$ MeV

$K\pi$



$\mathrm{br}(K\pi) \sim 0.19(1)$

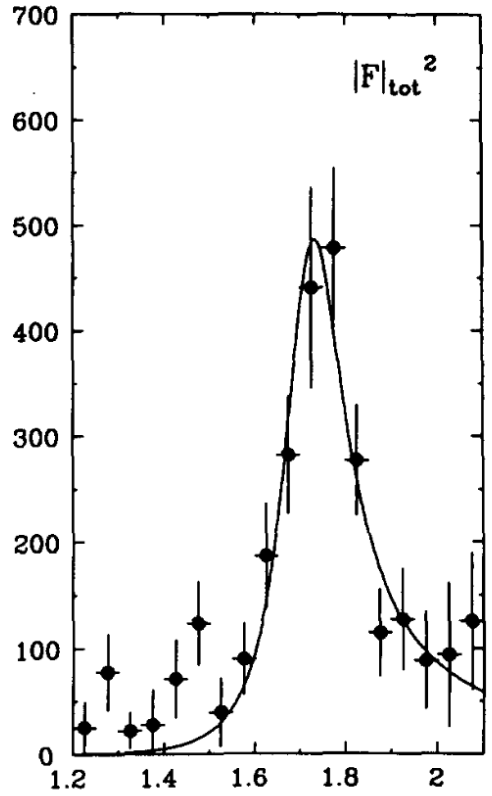
$K\pi\pi$



$$\frac{\Gamma(K\rho)}{\Gamma(K^*\pi)} \sim 1.5(2)$$

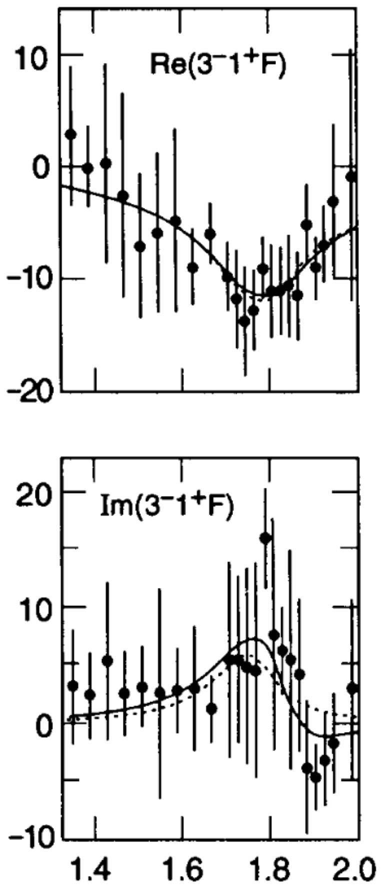
$$\frac{\Gamma(K_2^*\pi)}{\Gamma(K^*\pi)} \lesssim 0.8$$

$K\eta$

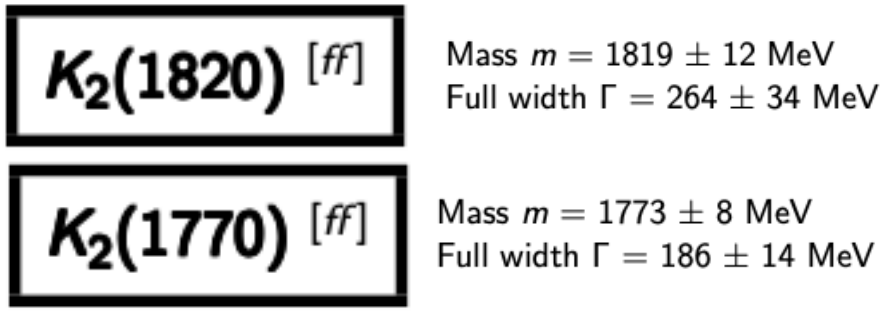


$$\frac{\Gamma(K\eta)}{\Gamma(K\pi)} \sim 0.5(2)$$

$K\omega$



$K^\star$ resonances (in LASS & ACCMOR)



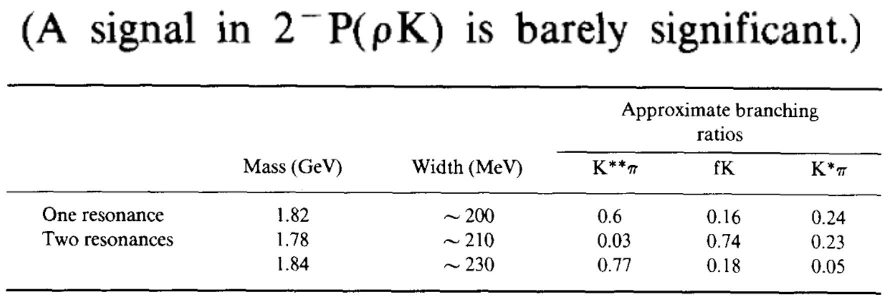
$K_2(1820)$ [ff]

Mass $m = 1819 \pm 12$ MeV
Full width $\Gamma = 264 \pm 34$ MeV

$K_2(1770)$ [ff]

Mass $m = 1773 \pm 8$ MeV
Full width $\Gamma = 186 \pm 14$ MeV

ACCMOR data suggests significant branches into $K_2^*\pi$, Kf_2 which are closed at the SU(3) flavor point



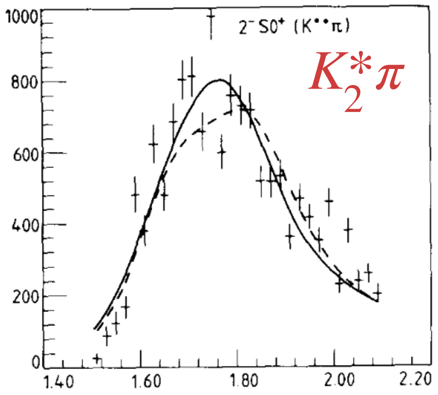
(A signal in 2^-P(ρK) is barely significant.)

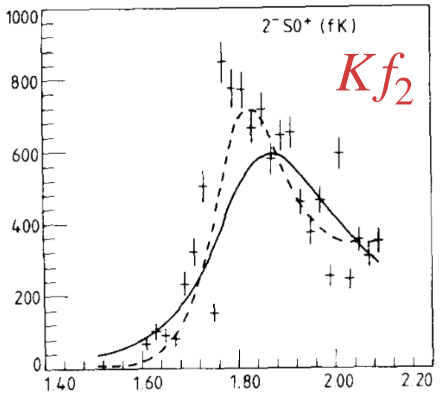
	Mass (GeV)	Width (MeV)	Approximate branching ratios		
			K**π	fK	K*π
One resonance	1.82	~ 200	0.6	0.16	0.24
Two resonances	1.78	~ 210	0.03	0.74	0.23
	1.84	~ 230	0.77	0.18	0.05

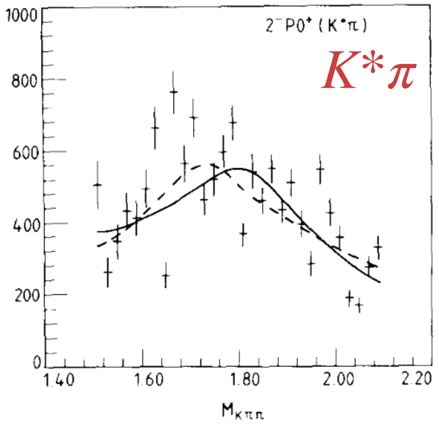
$K\pi\pi$

2^- not seen in LASS (dominance of pion exchange ?)

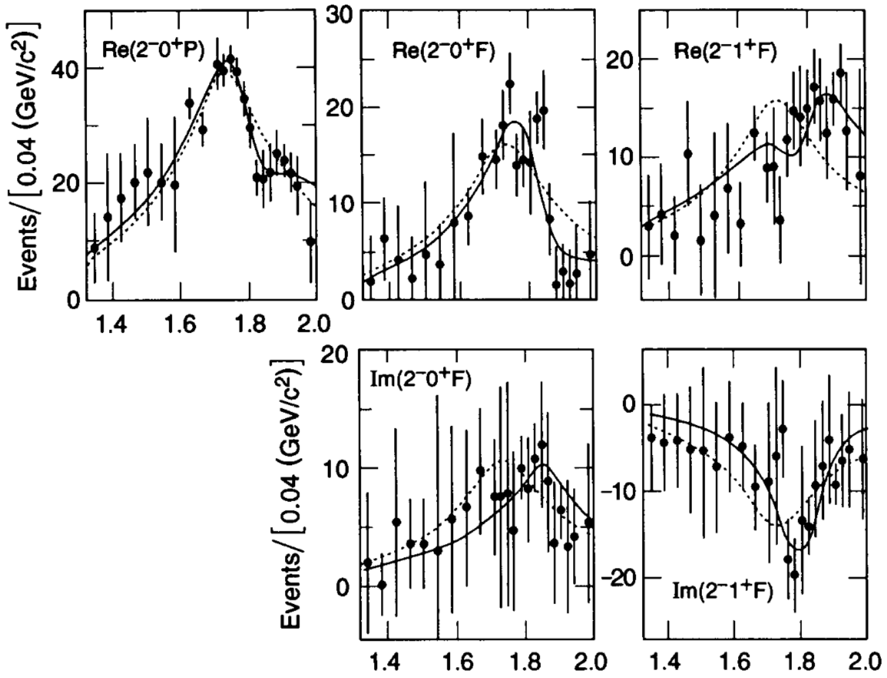
ACCMOR 1981 (higher beam energy)
$K^\pm p \to K^\pm \pi^+ \pi^- p$







$K\omega$



slight χ^2 improvement here is the main evidence for **two** 2^- resonances

COMPASS data to make two 2^- states firm?

$K^\star$ resonances (in LASS)

$K^*(1680)$

Mass $m = 1718 \pm 18$ MeV
Full width $\Gamma = 322 \pm 110$ MeV

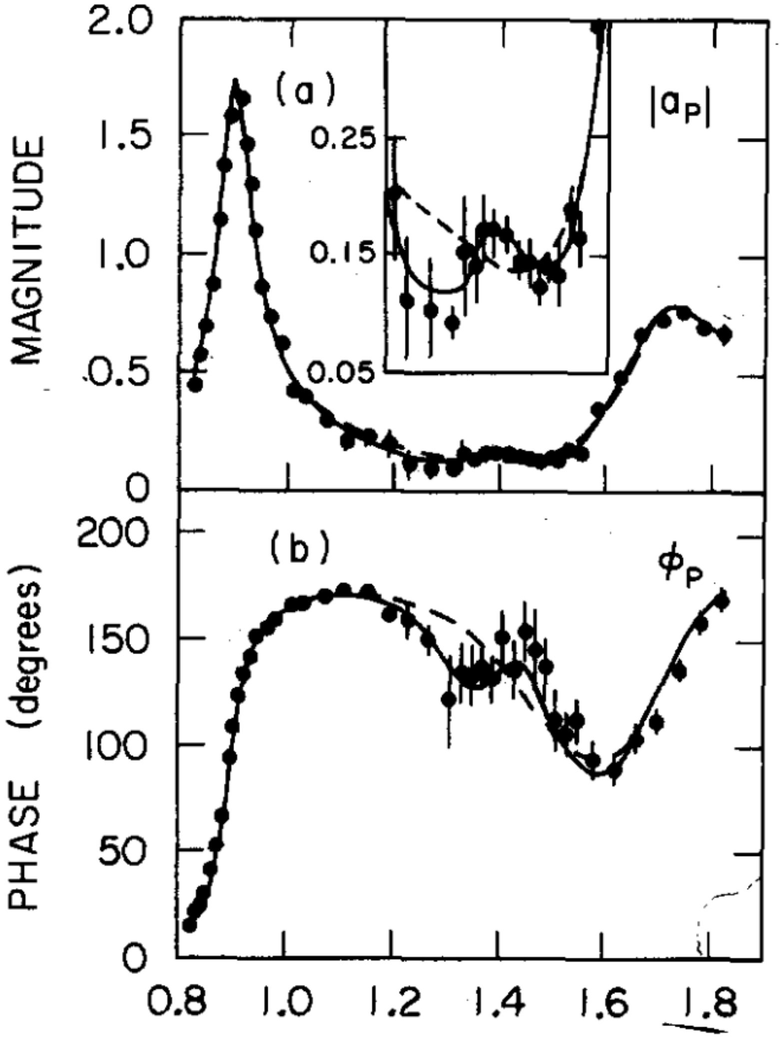
205(16)(34) in $K\pi$, 423(18)(30) in $K\pi\pi$ compatible ?

$K^*(1410)$

Mass $m = 1414 \pm 15$ MeV
Full width $\Gamma = 232 \pm 21$ MeV

176(52)(22) in $K\pi$, 240(18)(12) in $K\pi\pi$

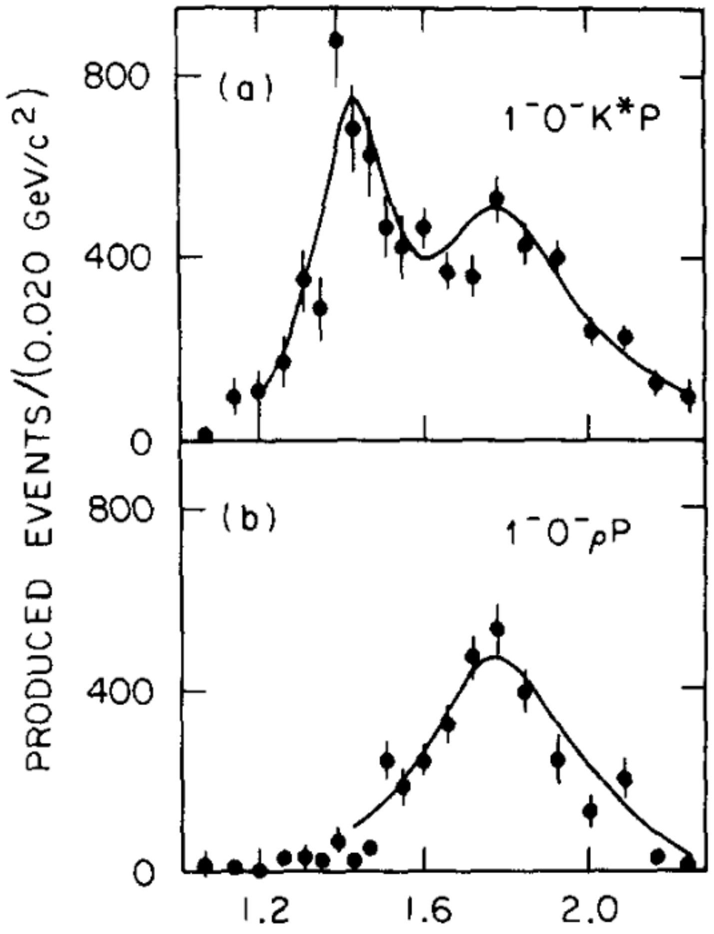
$K\pi$



very weak contribution from $K^*(1410)$

$$\mathrm{br}(K^*(1410) \to K\pi) \sim 0.07(1)$$
$$\mathrm{br}(K^*(1680) \to K\pi) \sim 0.39(3)$$

$K\pi\pi$

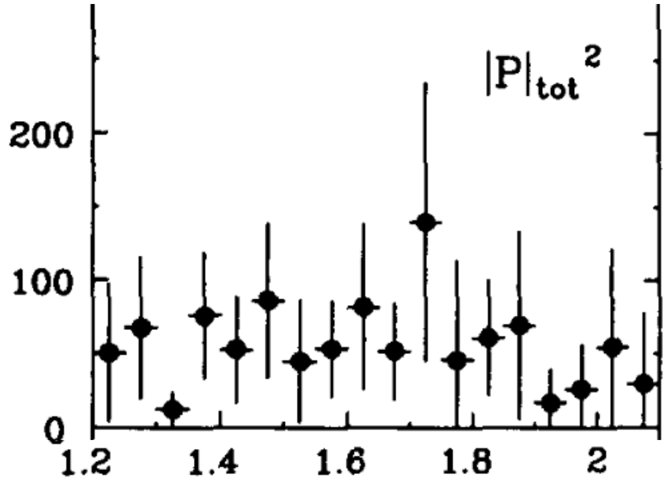


no $K^*(1410)$ in $K\rho$?

$$\frac{K^*(1680) \to \Gamma(K\rho)}{K^*(1680) \to \Gamma(K^*\pi)} \sim 0.97(10)$$

ACCMOR 1981
(higher beam energy)
$K^\pm p \to K^\pm \pi^+ \pi^- p$
has no 1^- waves

$K\eta$



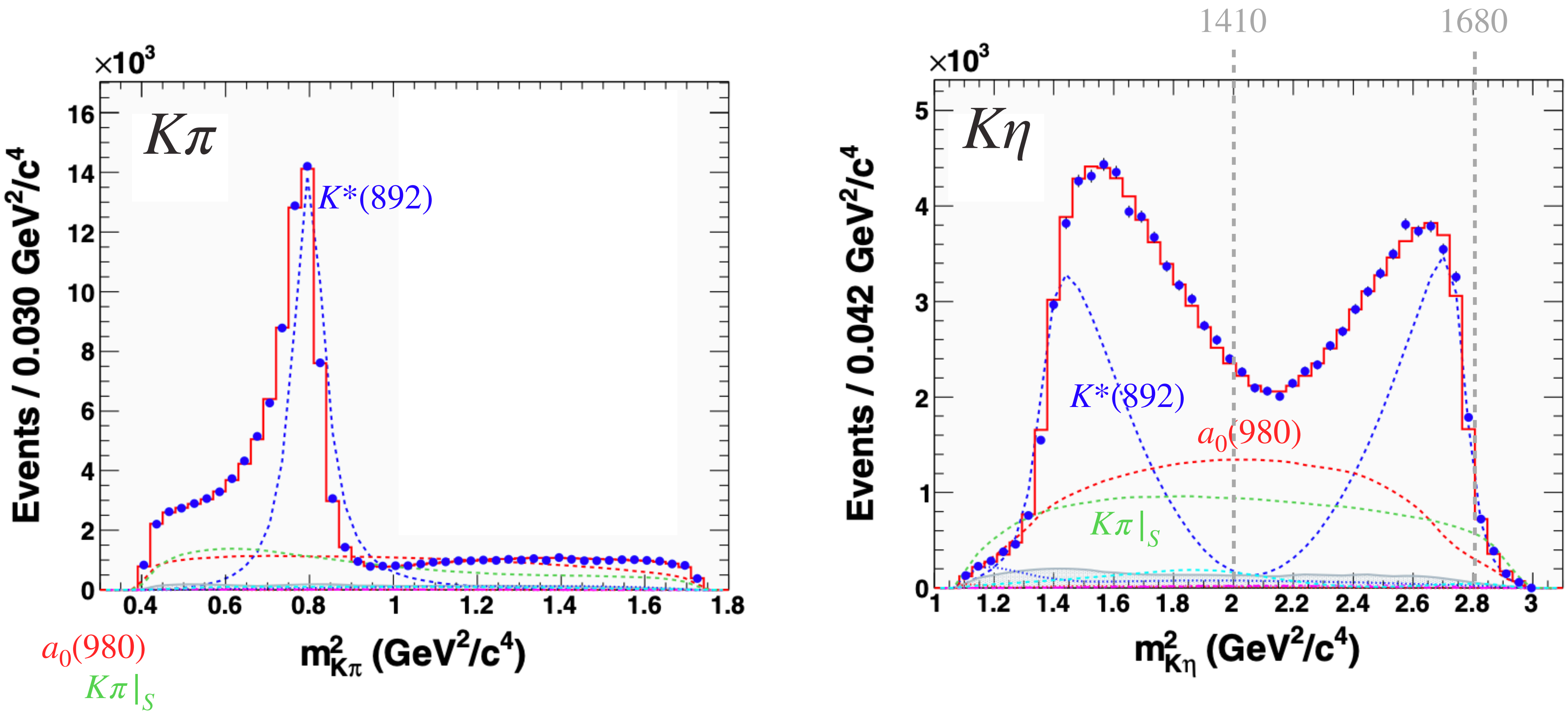
no 1^- signal

$K\omega$

ambiguous 1^- wave
(only in a thesis)

$K^\star$ resonances (in D-decays)

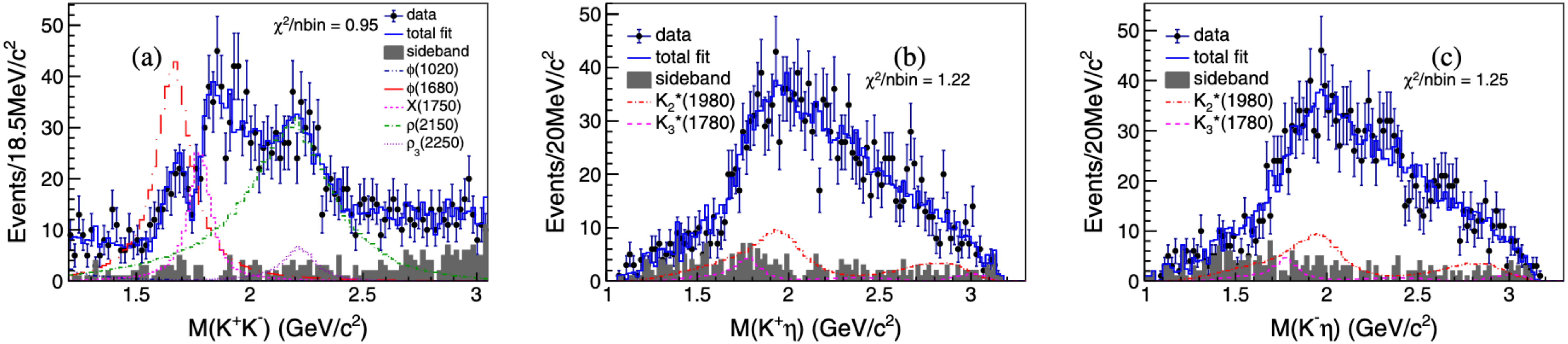
Belle Dalitz plot analysis of $D^0 \to K^-\pi^+\eta$



other contributions smaller than combinatoric background

$K^\star$ resonances (in ψ'-decays)

BES Dalitz plot analysis of $\psi' \to K^- K^+ \eta$

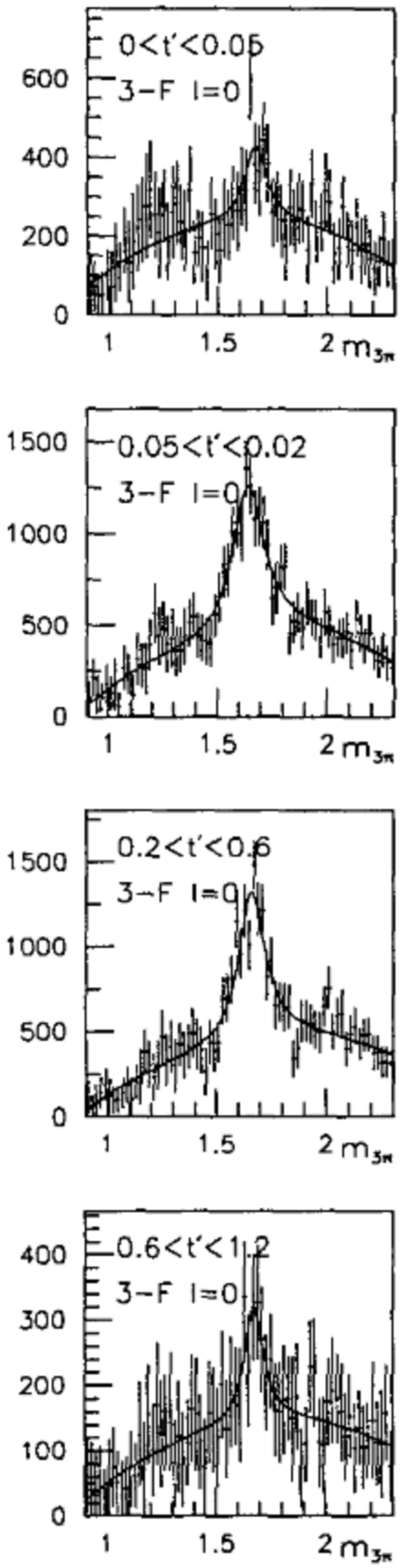


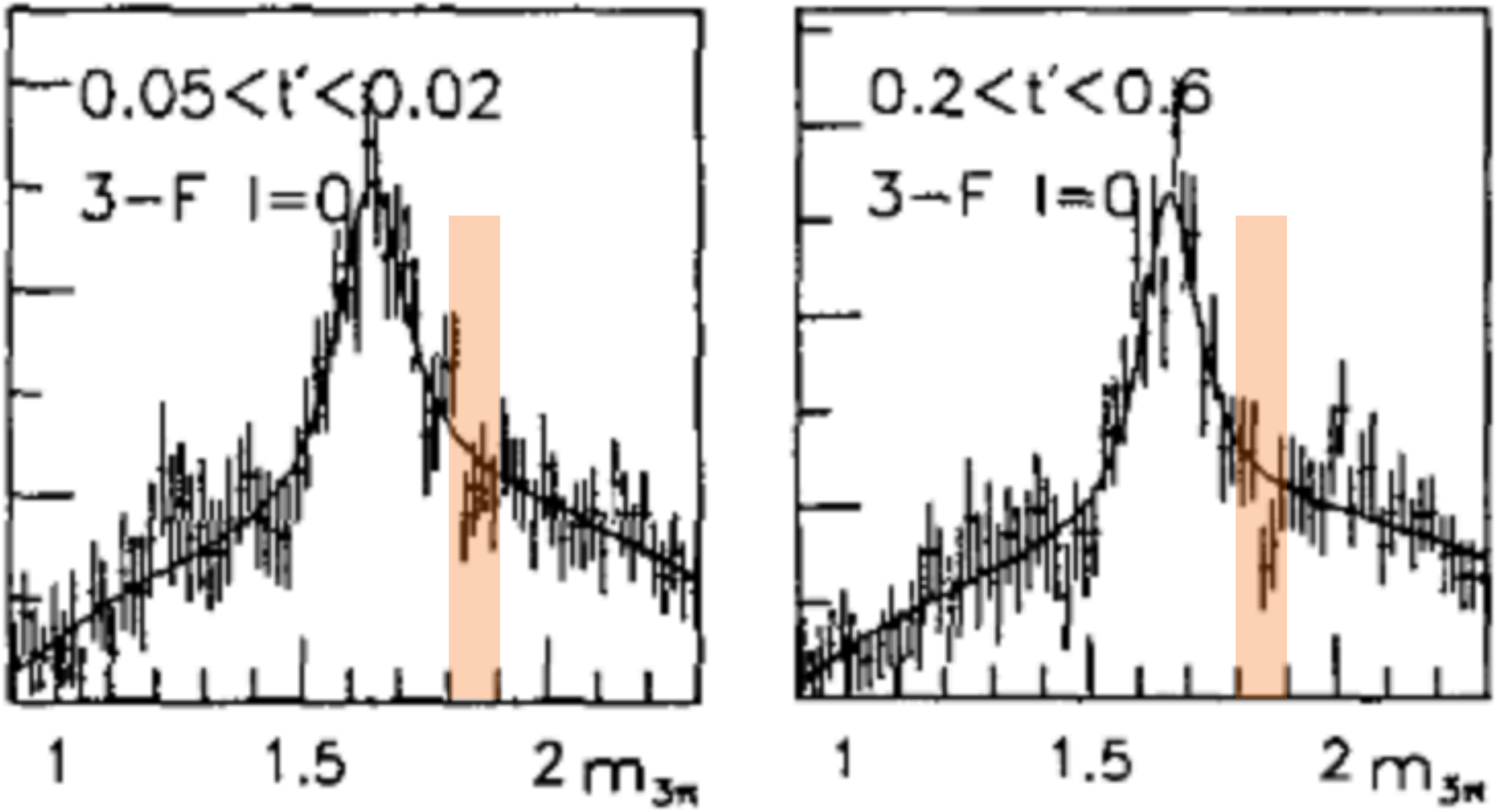
claim of rather small $K_3^*(1780)$ contributing in $K\eta$

$\omega_3(1670)$ in $\pi\rho$ (and $\phi_3(1850)$?)

VES $\pi^- p \to \pi^+\pi^-\pi^0\, n$ @ 36 GeV

only 'modern' high statistics evidence for this state

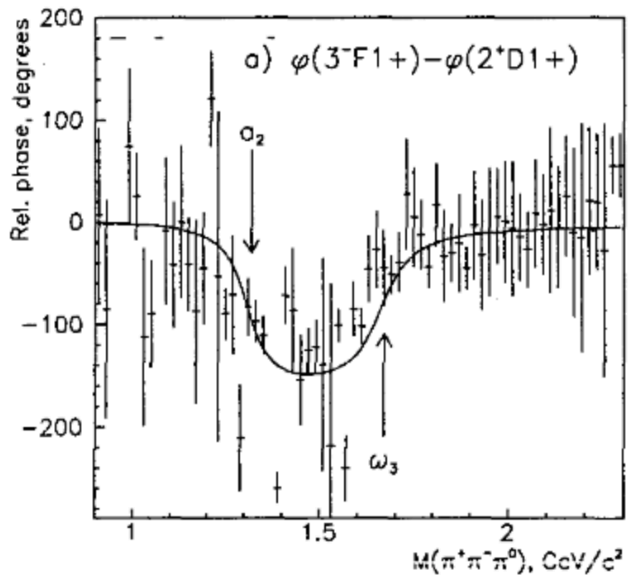




could there be a weak $\phi_3(1850)$ present in $\pi\rho$?

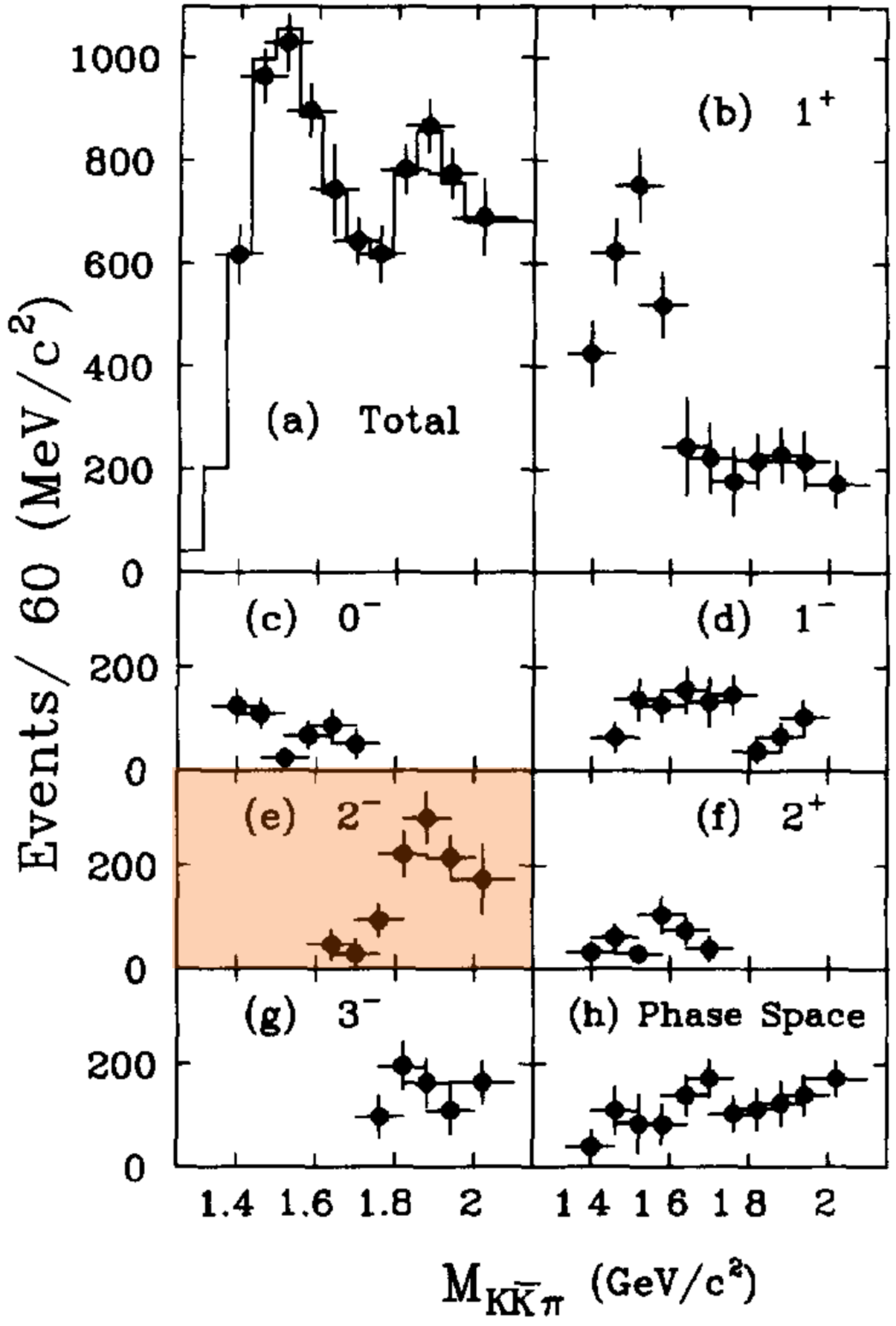
2^+ wave fit only has $a_2(1320)$ and not any $a_2(1700)$ so unclear what to conclude from this good fit to the phase difference

no sharp feature in the phase, but the ϕ_3 coupling could be small

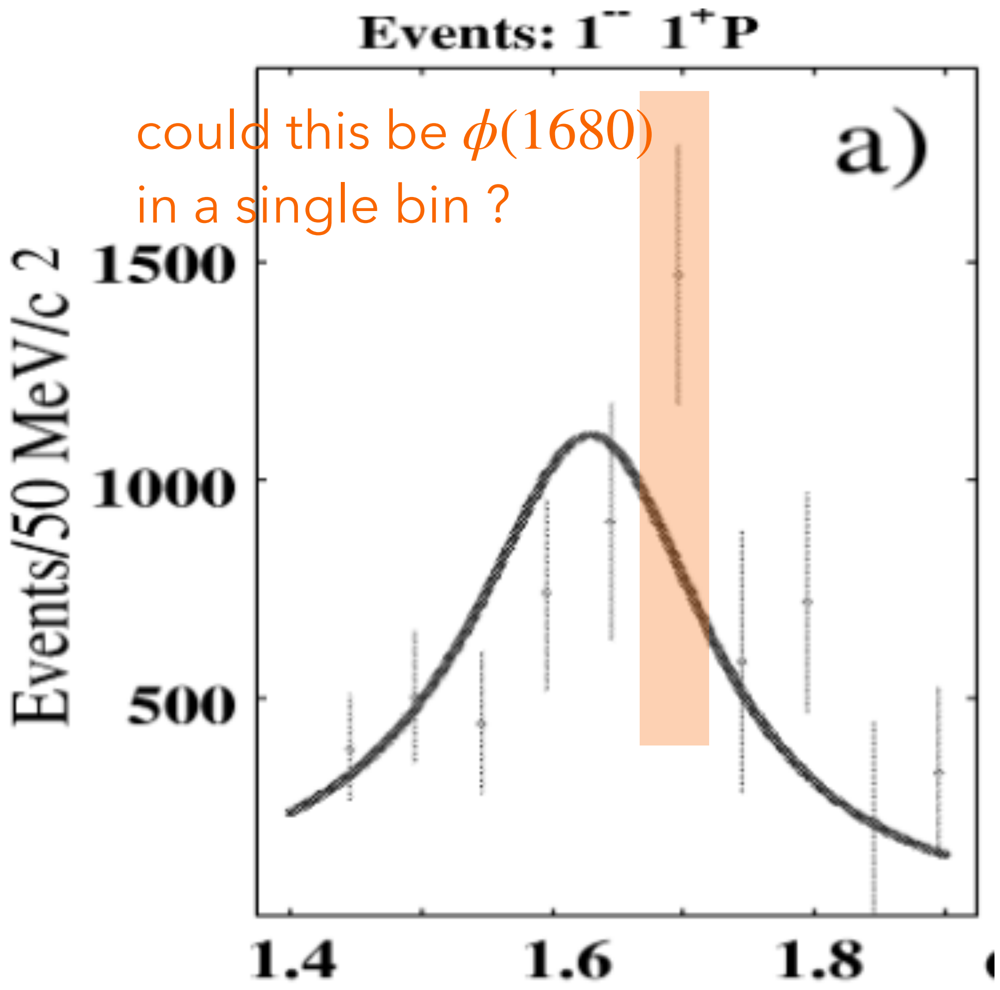


LASS $K^- p \to K_S^0 K^\pm \pi^\mp \Lambda$

could this be a $\phi_2(\sim 1900)$ with $\Gamma \sim 200$?



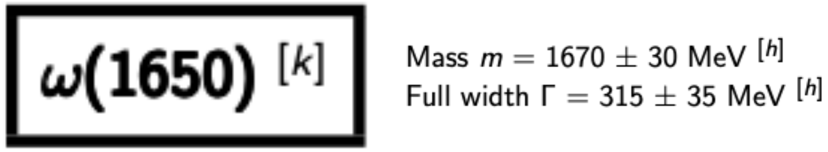
E852 $\pi^- p \to \eta\omega\, n$



could this be $\phi(1680)$ in a single bin ?

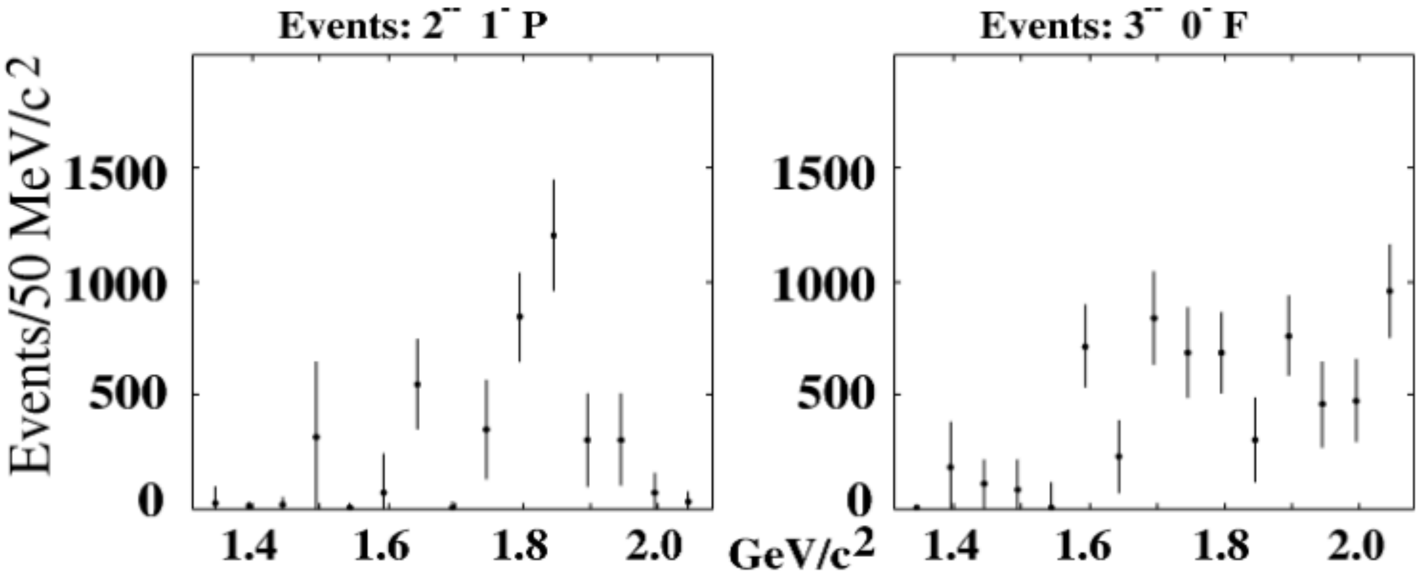
$\omega(1650)$ with fixed PDG (in 2001) params

$$m = 1649(24),\ \Gamma = 220(35)$$



$\omega(1650)$ [k]

Mass $m = 1670 \pm 30$ MeV [h]
Full width $\Gamma = 315 \pm 35$ MeV [h]



could this be a $\phi_2(1850)$?

nothing here

e^+e^- production of 1^{--} mesons

can access $I = 1, I_z = 0\,(\rho)$ and $I = 0, I_z = 0\,(\omega, \phi)$

$e^+e^- \to \pi\pi\pi$

Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ and $e^+e^- \to \omega\pi^+\pi^-$ reactions in the energy interval 1350–2400 MeV

DM2 Collaboration

$$G = - \implies I = 0$$

in principle expect four resonances could contribute here

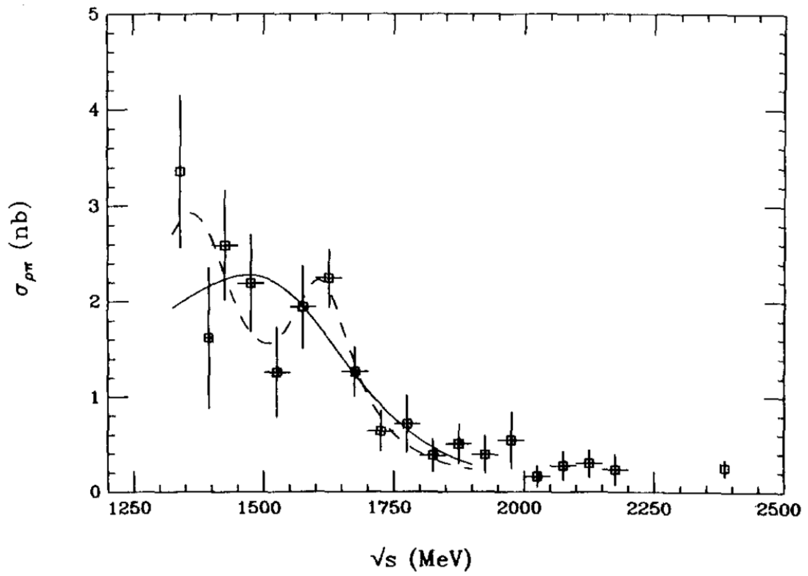
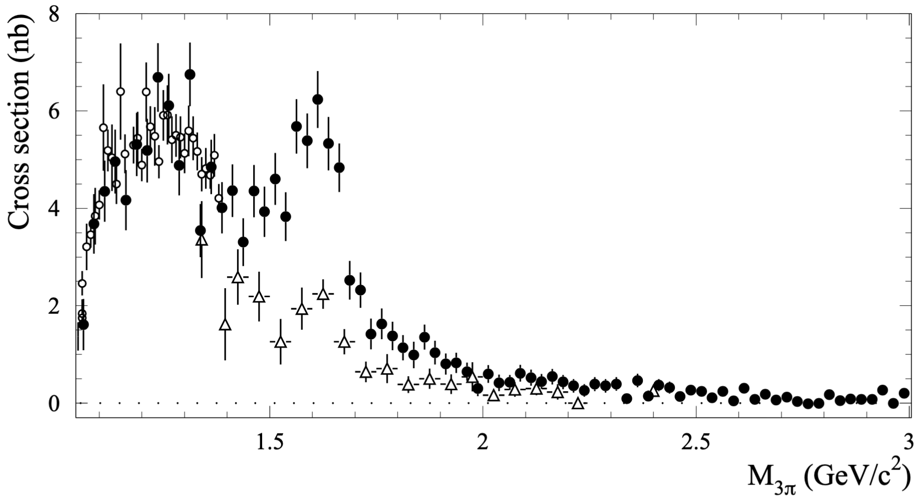


Fig. 3. $e^+e^- \to \rho\pi$ cross section. The solid and dotted lines refer respectively to the fits to a single or two Breit Wigner functions

Study of $e^+e^- \to \pi^+\pi^-\pi^0$ process using initial state radiation with BABAR



appears to be a significant tension between BaBar cross-section and DM2 above 1.5 GeV

FIG. 18: The $e^+e^- \to \pi^+\pi^-\pi^0$ cross section measured in this work (filled circles), by SND (open circles), and DM2 (open triangles).

$e^+e^- \to \omega\pi\pi$

Measurement of the $e^+e^-\to\pi^+\pi^-\pi^0$ and $e^+e^-\to\omega\pi^+\pi^-$ reactions in the energy interval 1350–2400 MeV

DM2 Collaboration

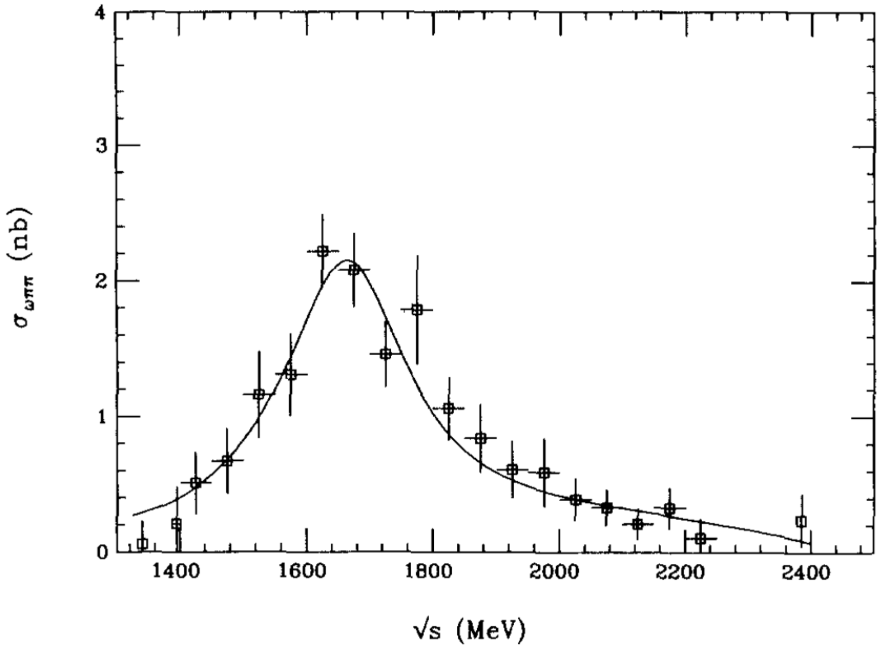


Fig. 6. $e^+e^-\to\omega\pi^+\pi^-$ cross section. The line refers to the single Breit Wigner fit

The $e^+e^-\to 2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$ and $K^+K^-\pi^+\pi^-\eta$ cross sections measured with initial-state radiation

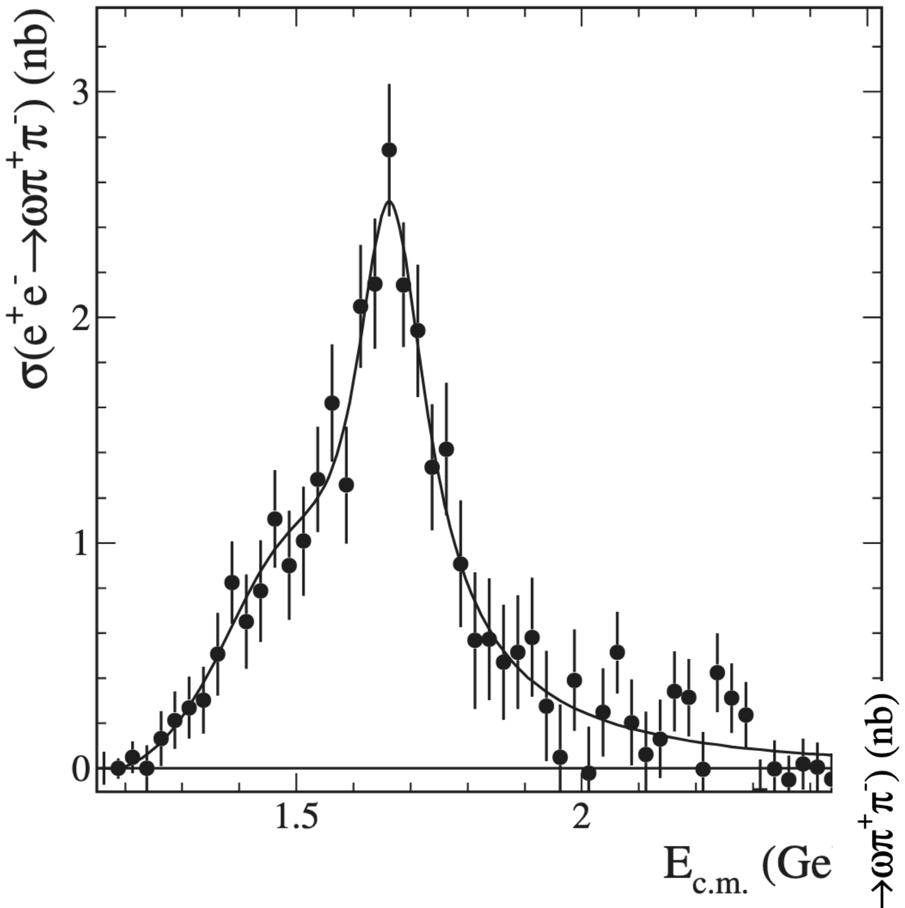


FIG. 16. The $e^+e^-\to\omega\pi^+\pi^-$ cross section excluding $\omega f_0(980)$ contribution (points). The curve shows the resul the fit of the $\omega(1420)$ and $\omega(1650)$ resonances described in text (Fit 3 in Table VI).

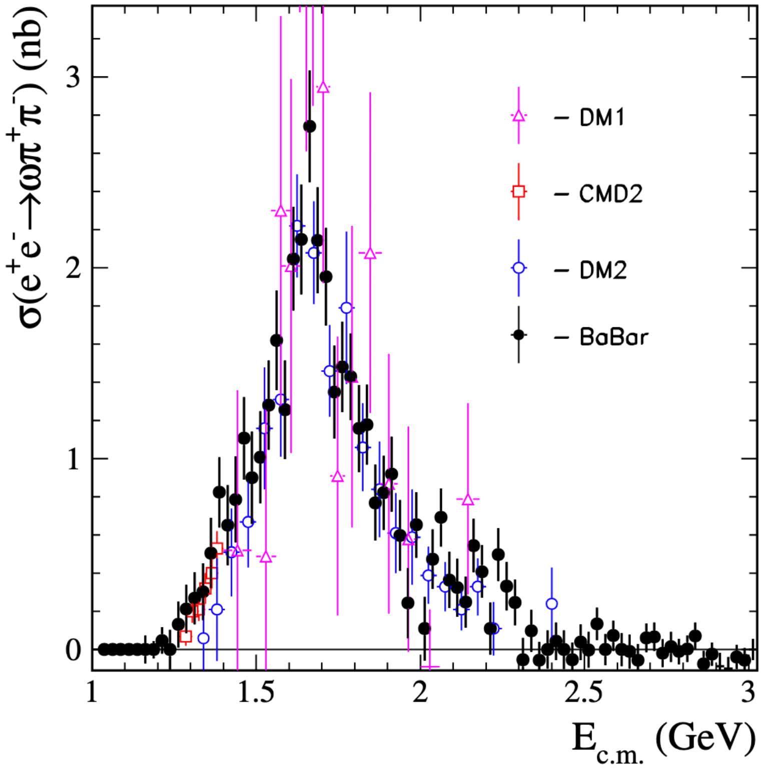


FIG. 14 (color online). The $e^+e^-\to\omega\pi^+\pi^-$ cross section as a function of c.m. energy obtained via ISR at *BABAR*. The direct measurements from DM1, DM2, and CMD2 are also shown. Only statistical errors are shown.

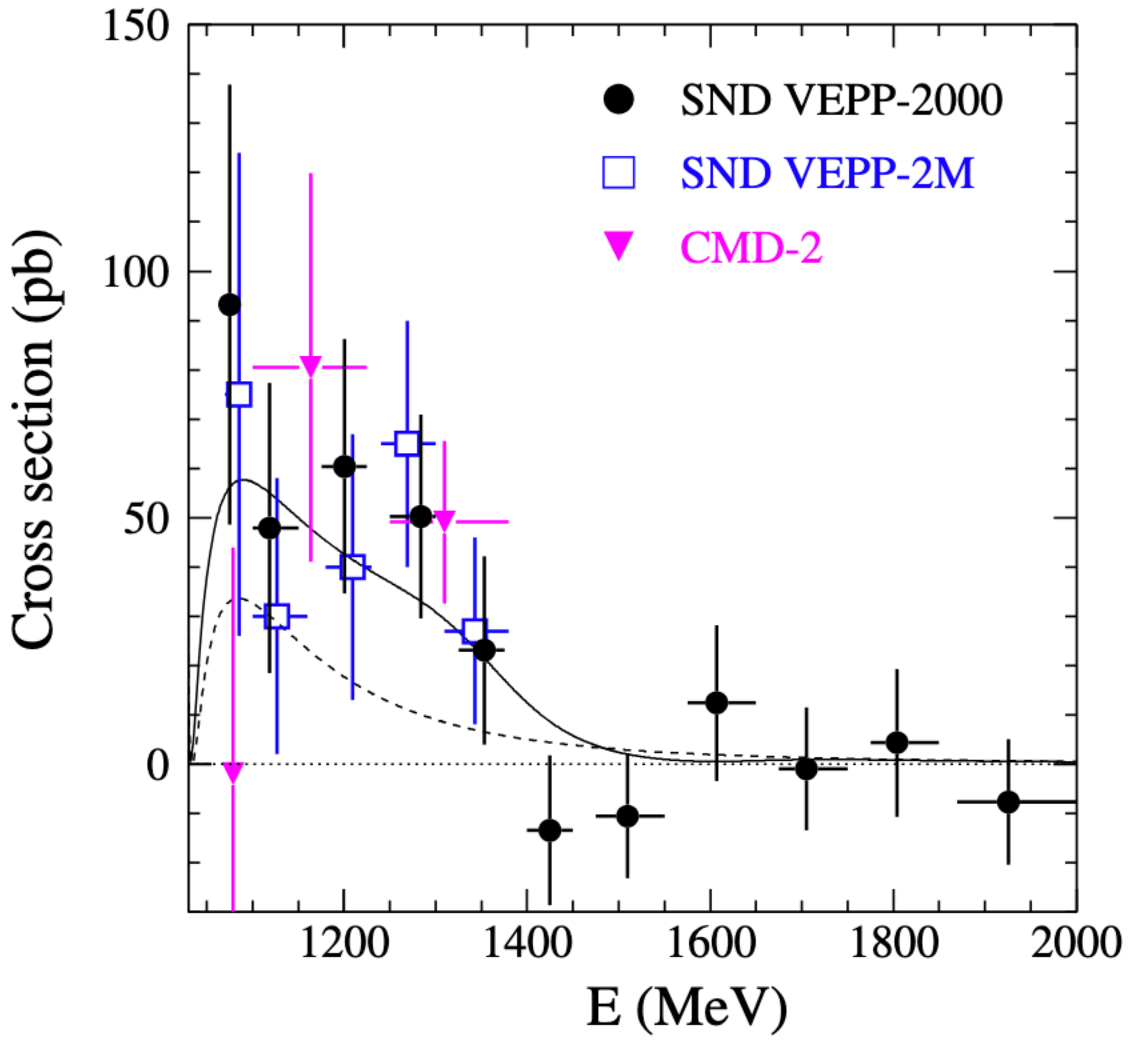
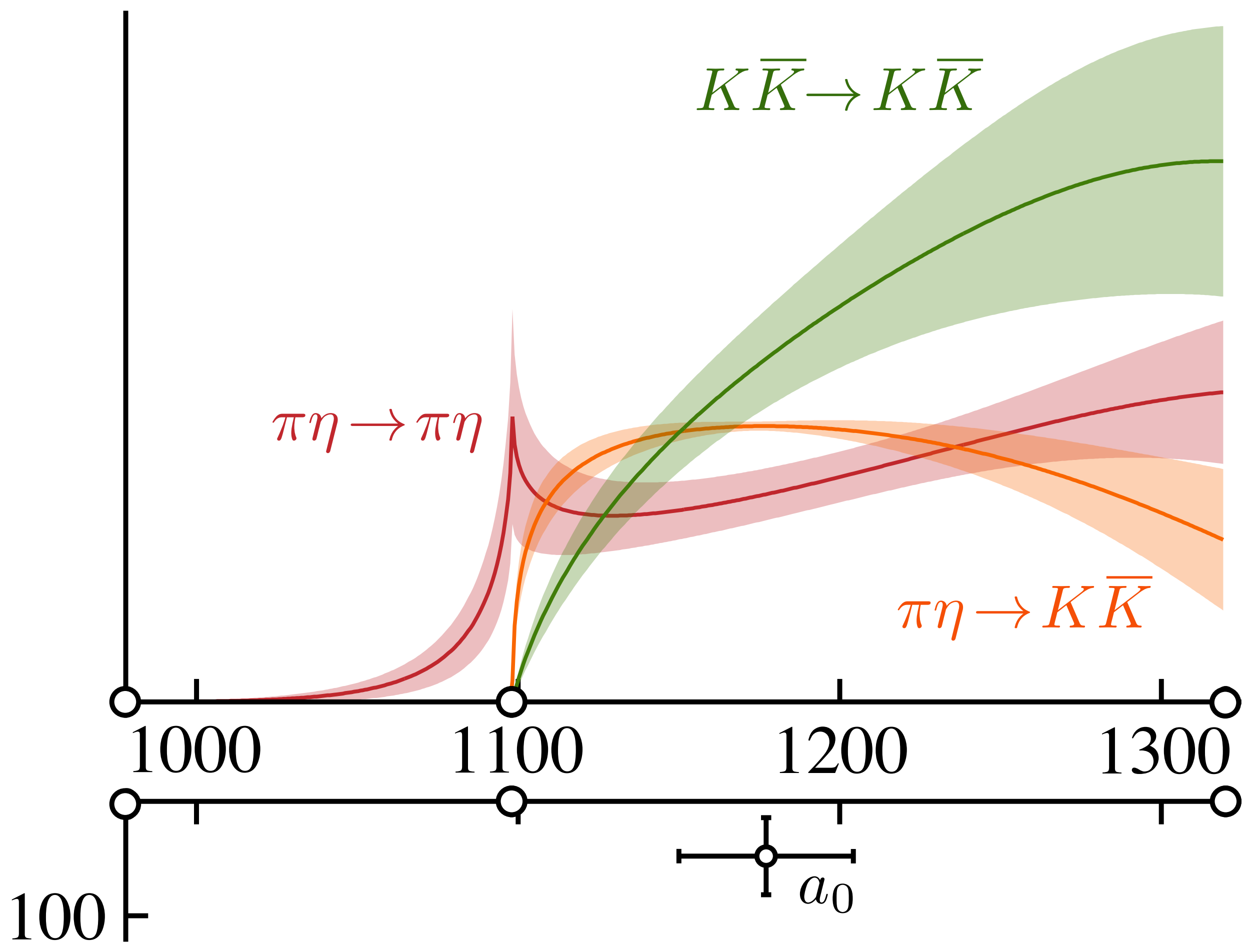
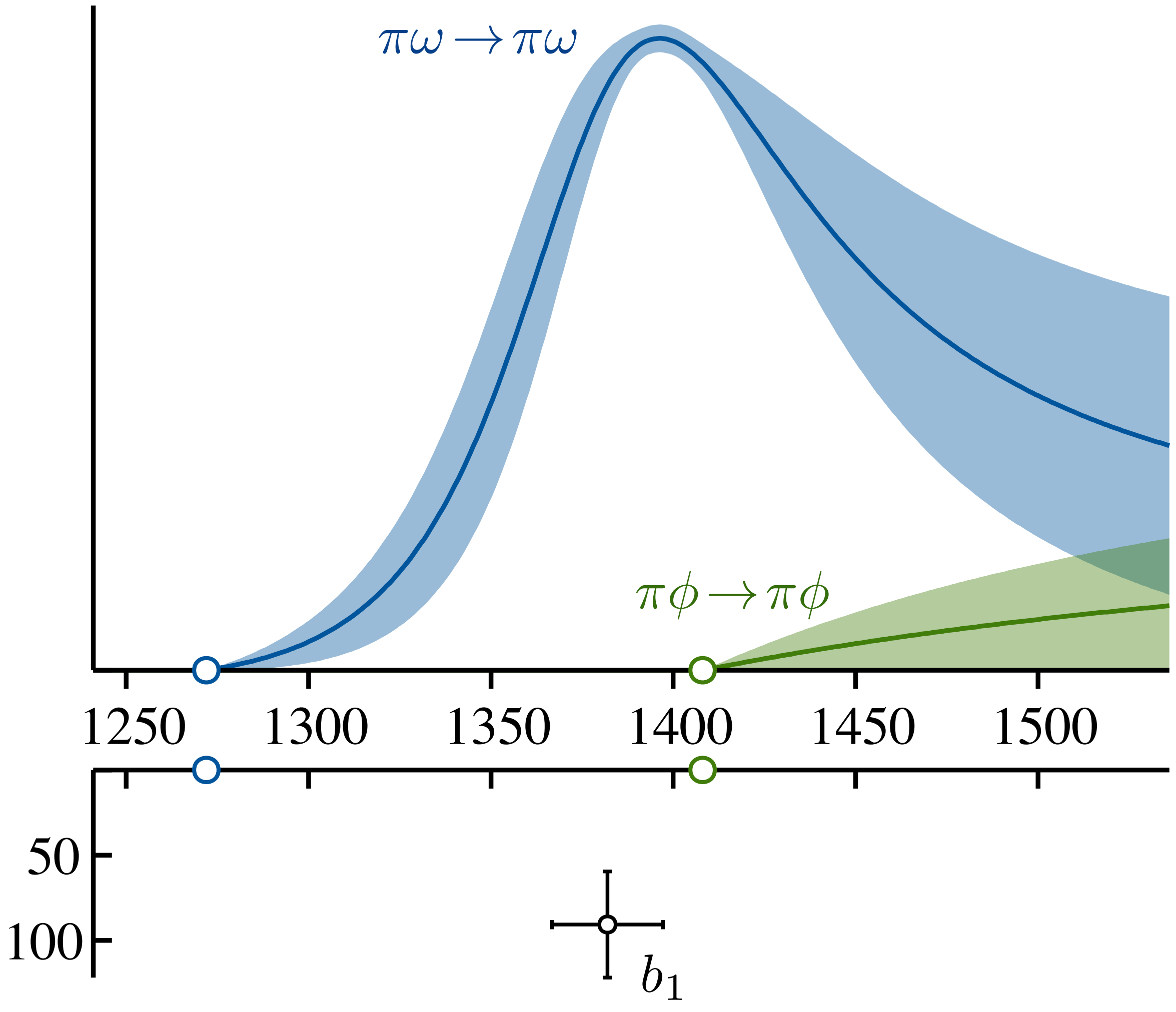


FIG. 5. The $e^+e^- \to \pi^0\gamma$ cross section measured in this work (SND@VEPP-2000) in comparison with the previous measurements: SND@VEPP-2M [4], and CMD-2 [5]. The solid curve is the result of the VMD fit with two excited resonances, V' and V''. The dashed curve represents the result of the fit with $\sigma_{V'} = \sigma_{V''} = 0$.

$J^P = 0^+ \; I^G = 1^- \; (\eta\pi, K\bar{K})$



$J^P = 1^+ \; I^G = 1^- \; (\pi\omega, \pi\phi)$

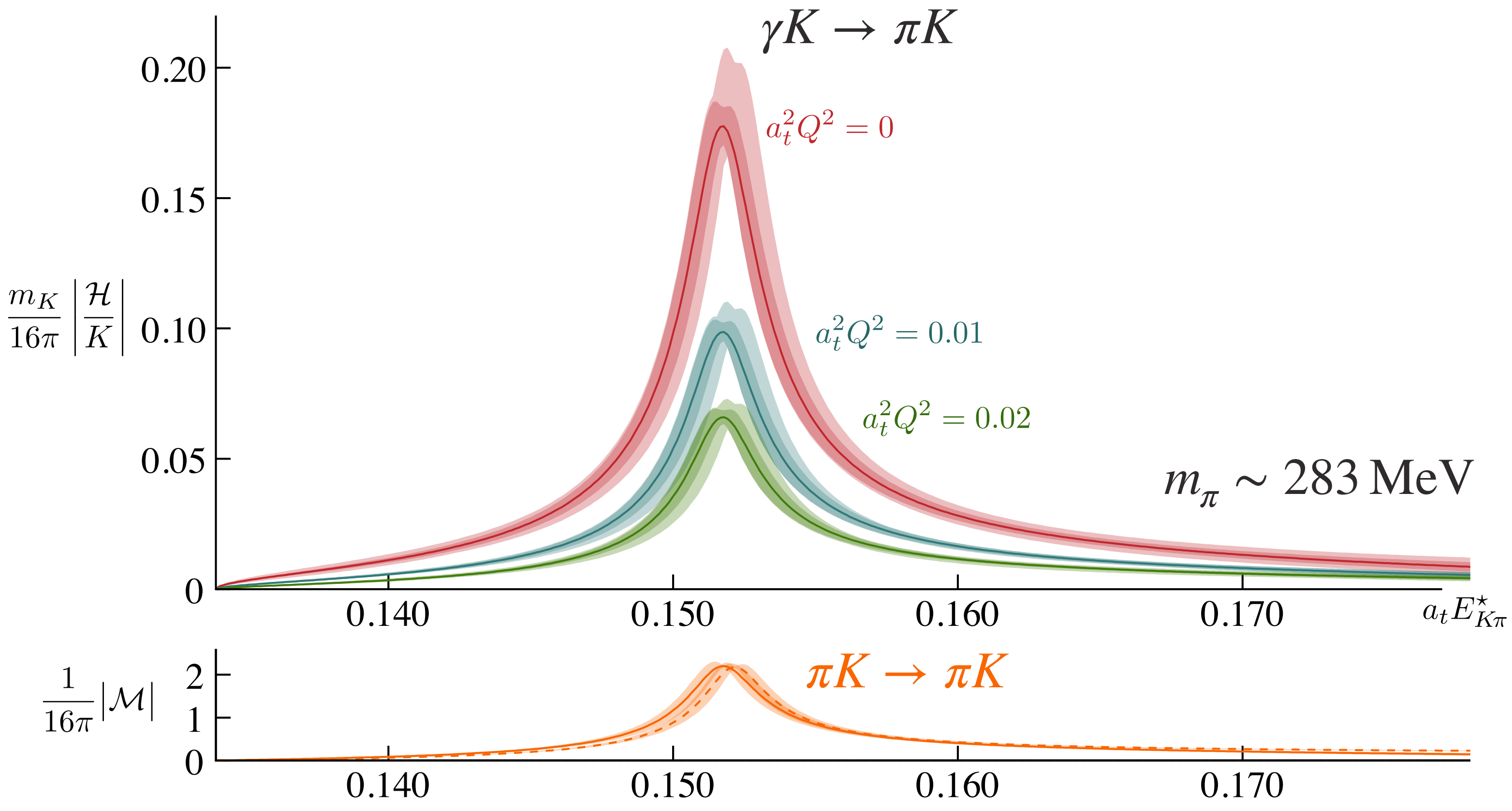


production amplitudes

can also couple to an external current, e.g. electroweak production

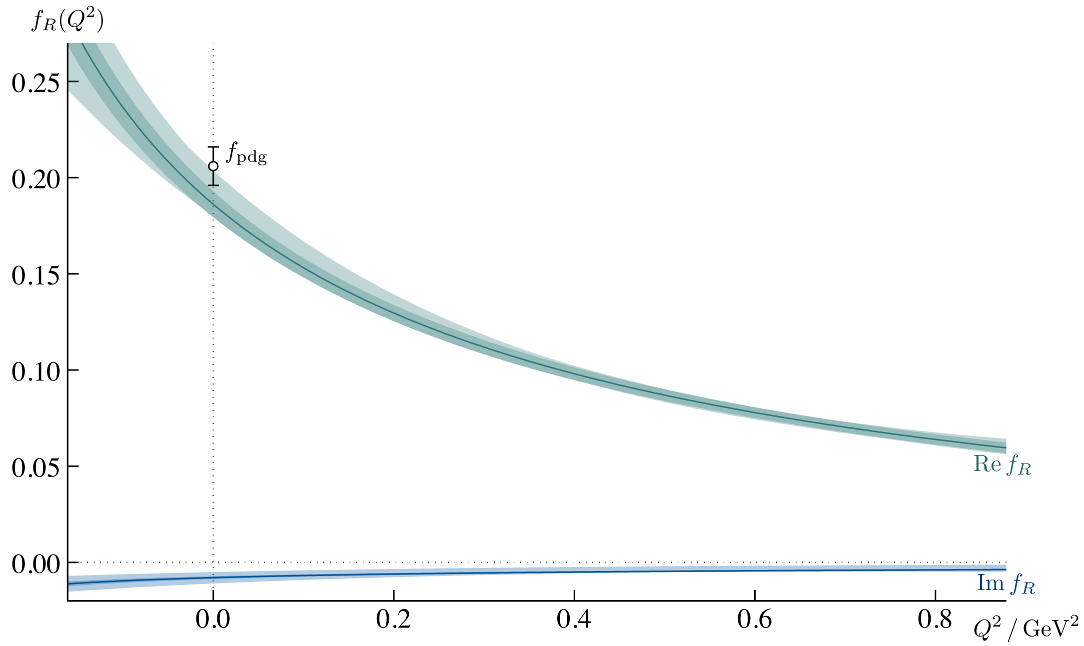
compute three-point functions with a current insertion
correct for the finite-volume

a recent example of the approach …



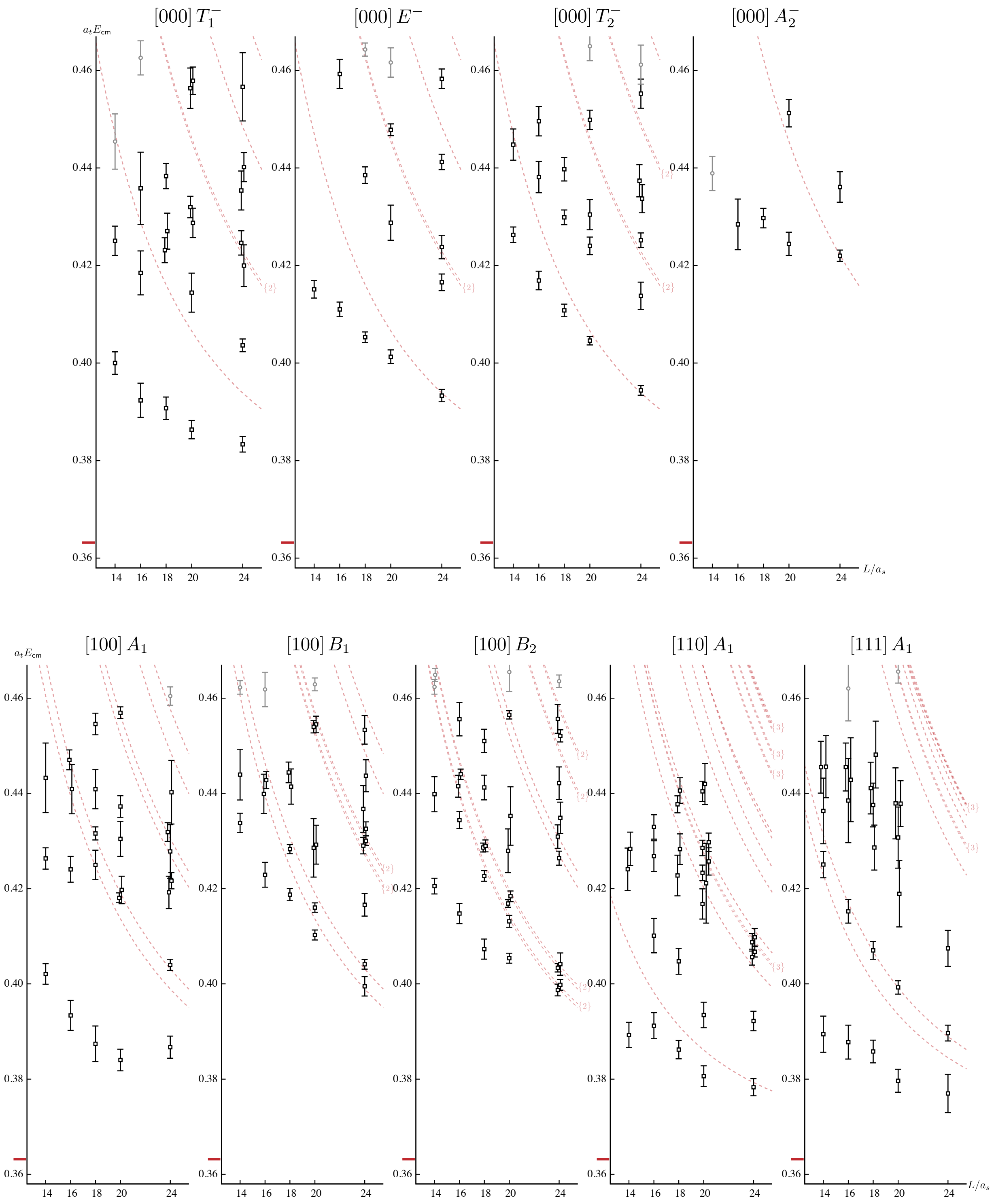
PRD 106 114513 (2022)

resonance transition form-factor $K^* \to K\gamma$

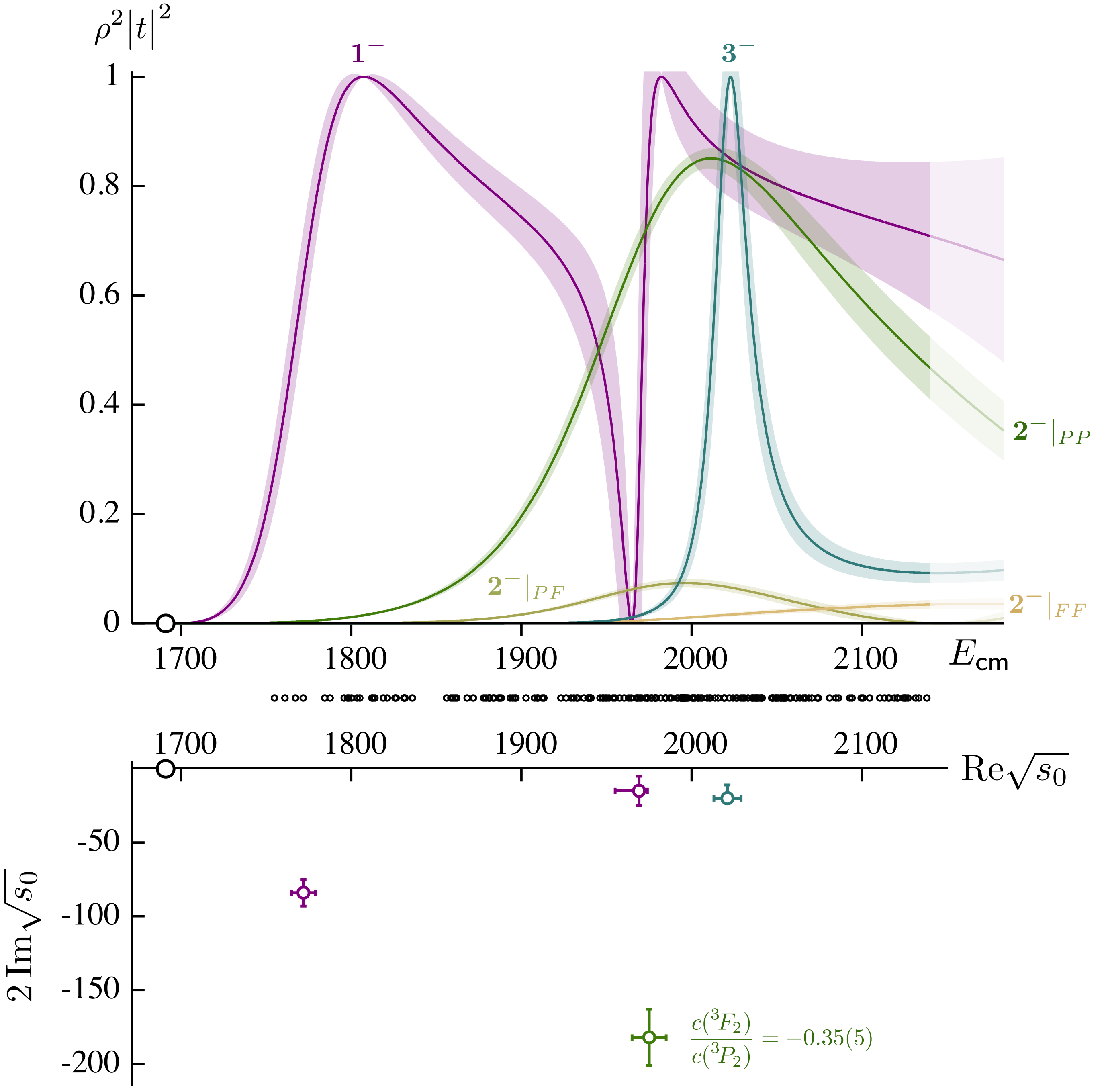


excited J^{--} meson resonances – m_π~700 MeV

exact SU(3) flavor symmetry $\omega_J^{\mathbf{1}} \to \eta^{\mathbf{8}}\omega^{\mathbf{8}}$



unprecedented number of energy levels



operator basis – I=0 $\pi\pi$, $K\bar{K}$, $\eta\eta$

operator basis: ‘single-meson’ + ‘meson-meson’

$$\bar{\psi}\mathbf{\Gamma}\psi$$

(& if you like,
tetraquark & …)

$$\sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

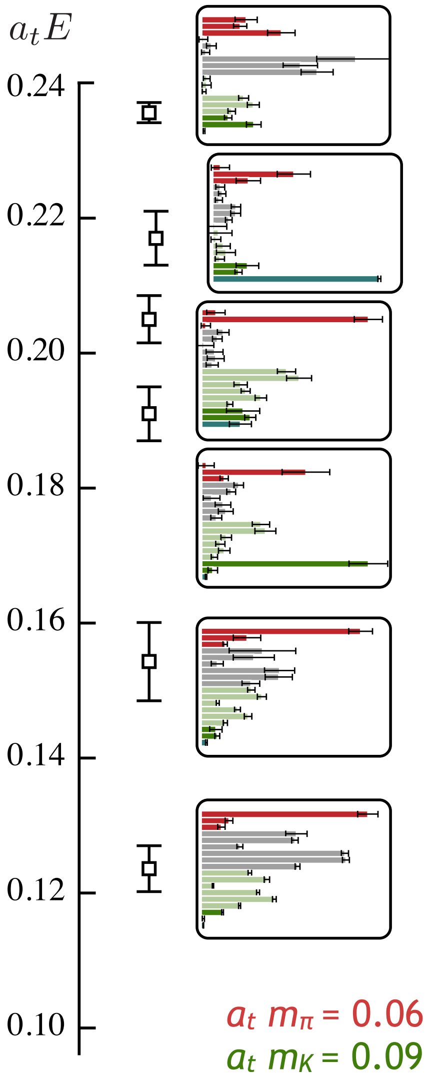
$$\mathbf{p}=\frac{2\pi}{L}[n_x,n_y,n_z]$$

maximum momentum guided by non-interacting energies

$$\sqrt{m_1^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$$

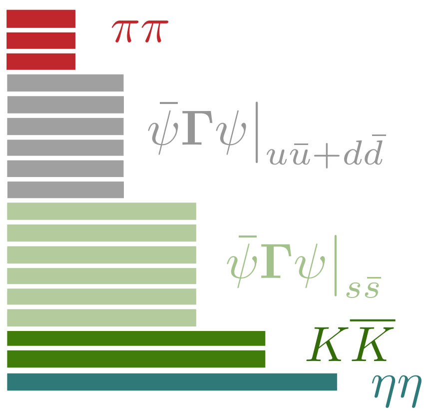
solutions of the det equation when **t** = **0**

[000] A_1^+ 24^3



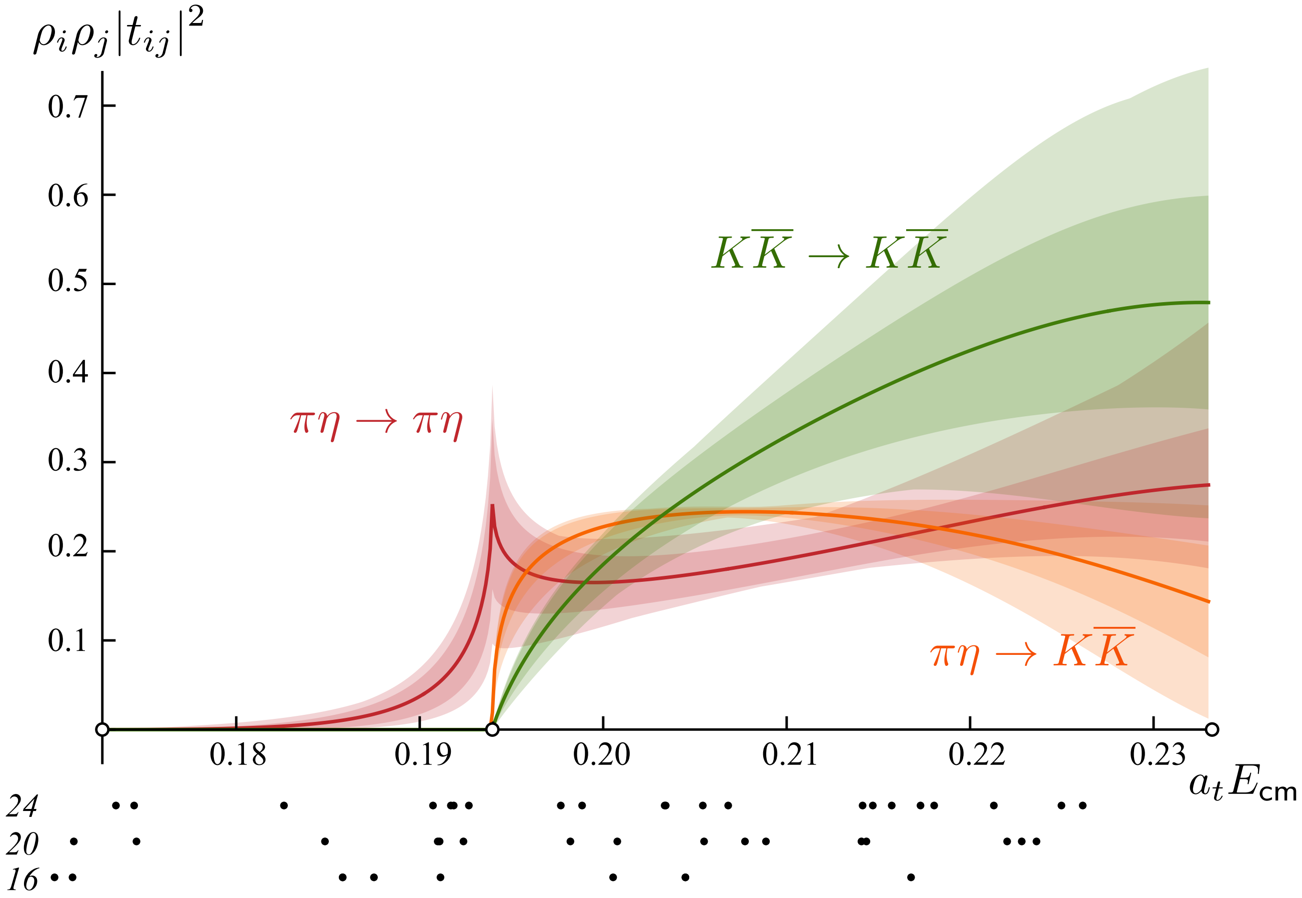
$a_t\, m_\pi$ = 0.069
$a_t\, m_K$ = 0.097
$a_t\, m_\eta$ = 0.104

operator basis

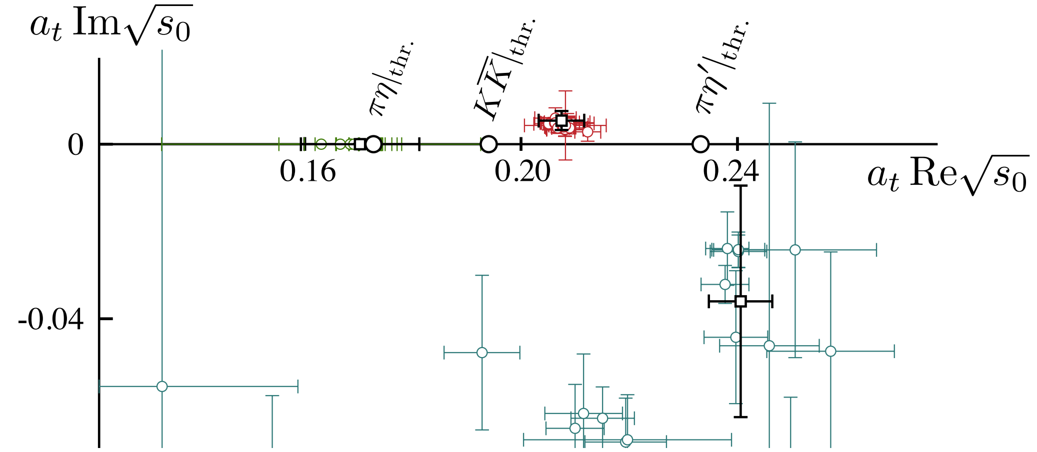


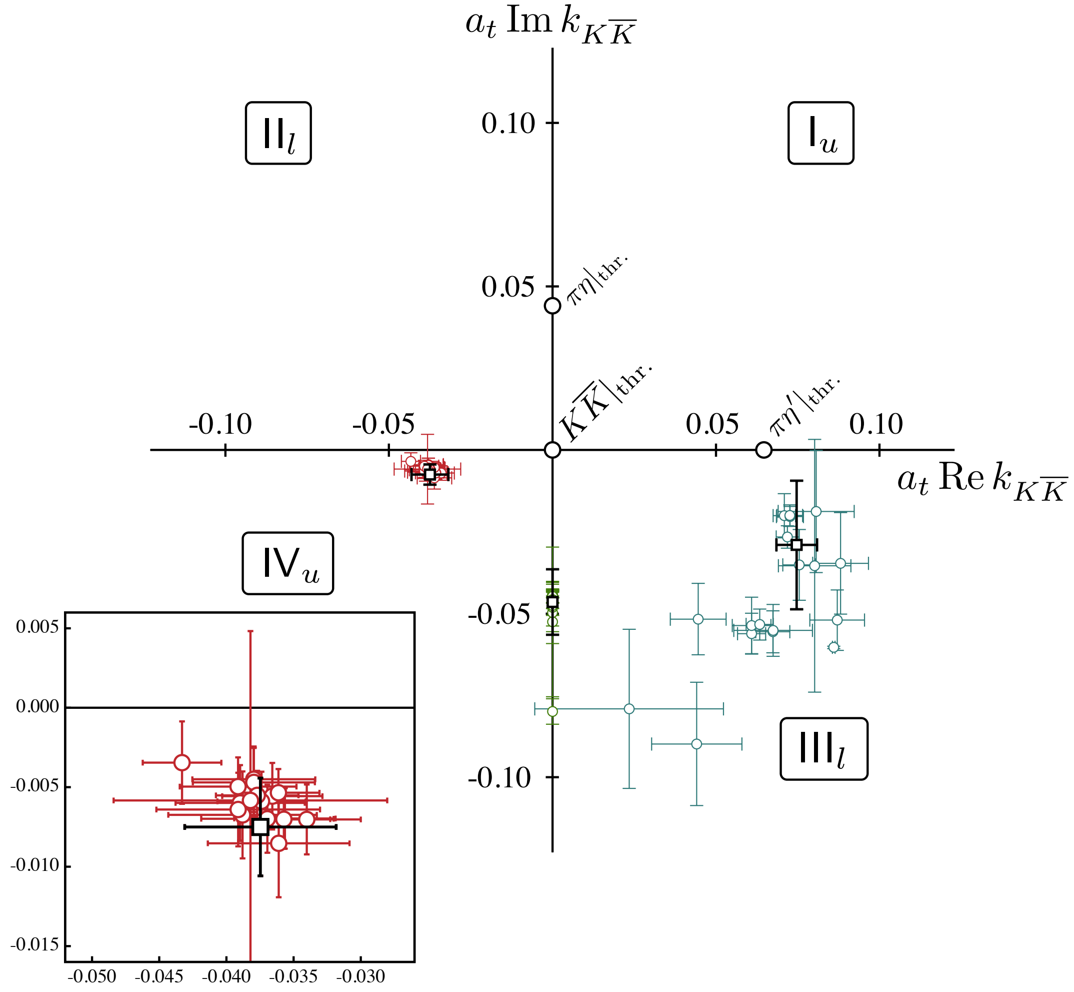
$\pi\eta$, $K\bar{K}$ scattering

S-wave amplitudes



pole singularities





a_0('980') as sheet IV pole

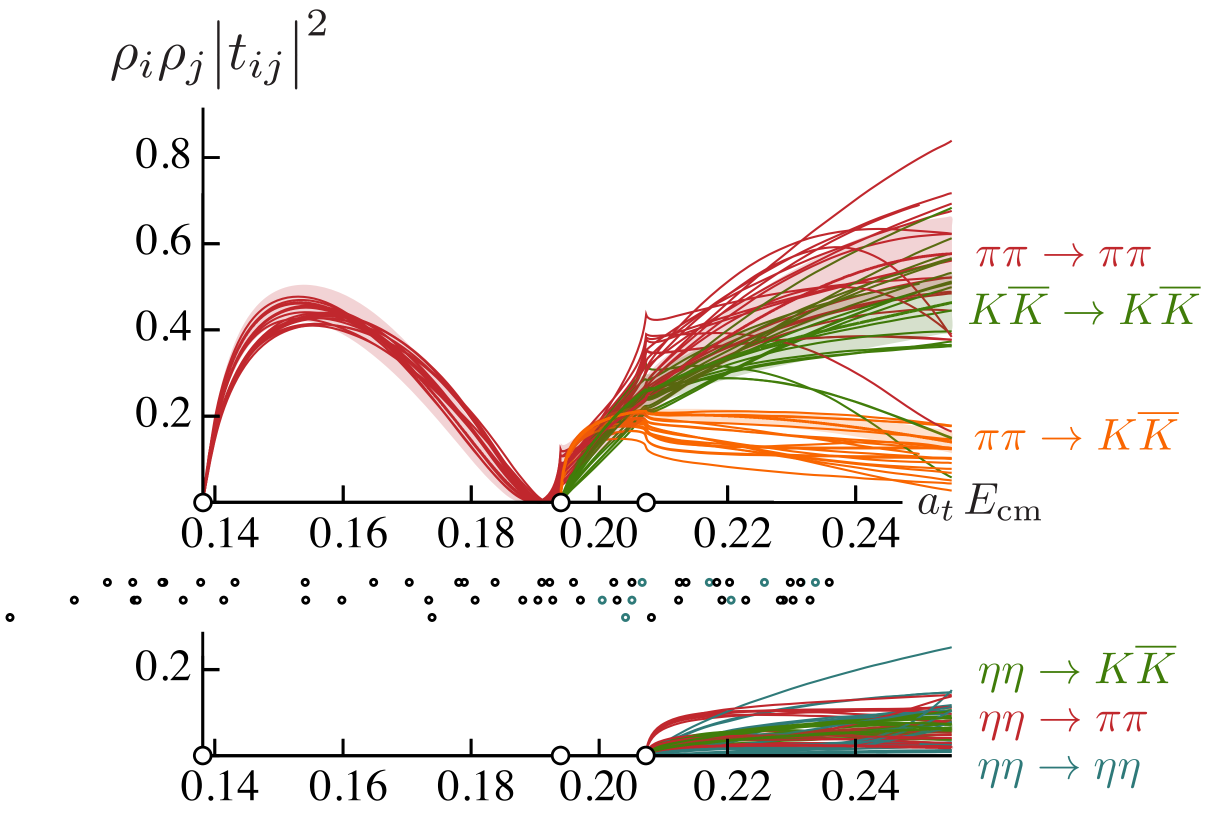
$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right| = 1.3(4)$$

$\pi\pi, K\bar{K}, \eta\eta$ scattering

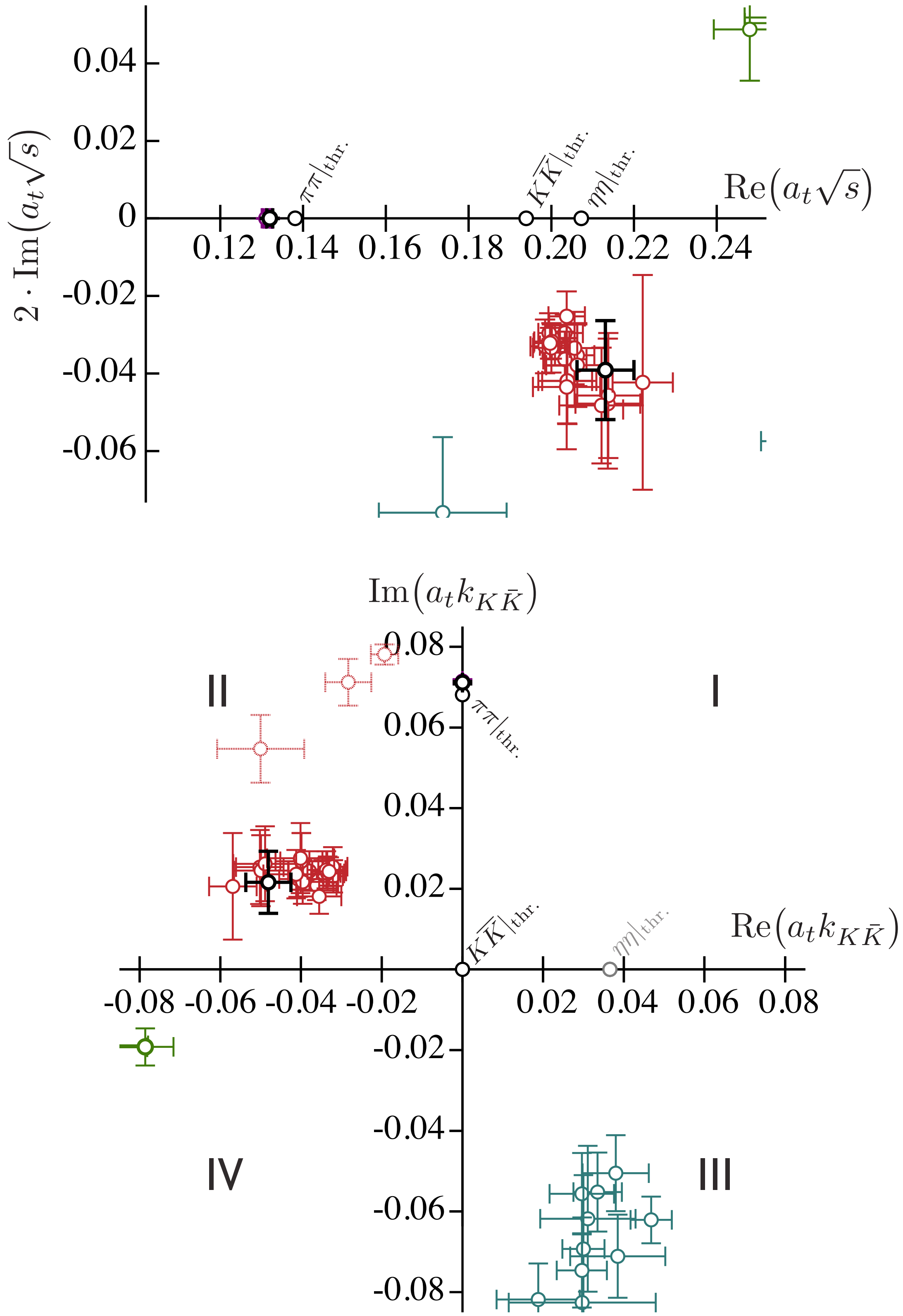
$m_\pi = 391\,\mathrm{MeV}$

PRD 97 054513 (2018)

S-wave amplitudes



pole singularities



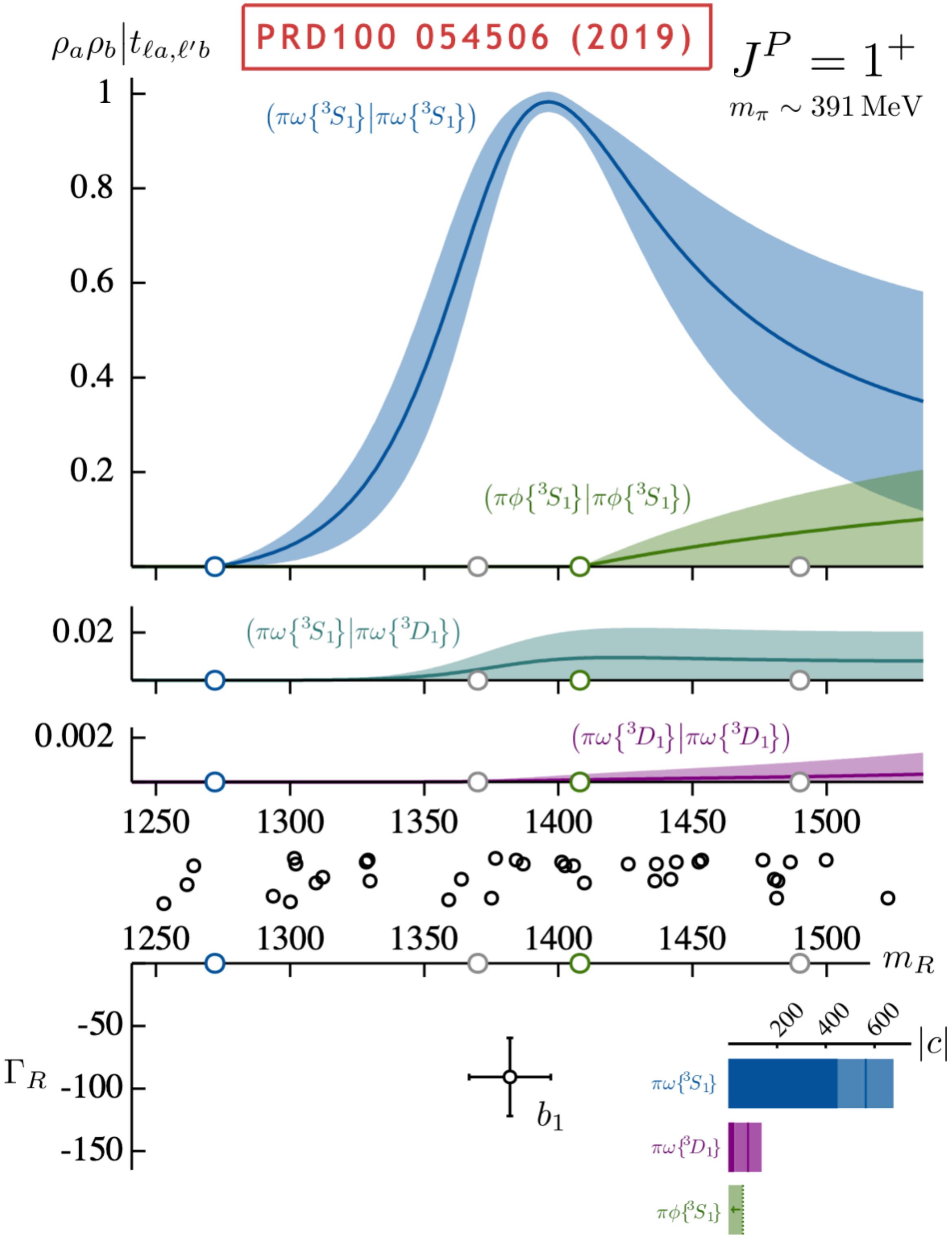
σ as a stable bound-state

f_0('980') as sheet II pole

$|c_{K\bar{K}}| \approx |c_{\pi\pi}|$

coupled-channel scattering from lattice QCD

$b_1 \to \pi\omega, \pi\phi$


PRD100 054506 (2019)

‘dropping’ ops in $\rho \to \pi\pi$

PRD92 094502 (2015)

exclude the $K\bar{K}$ operator

exclude all meson-meson

exclude all $\bar{\psi}\Gamma\psi$

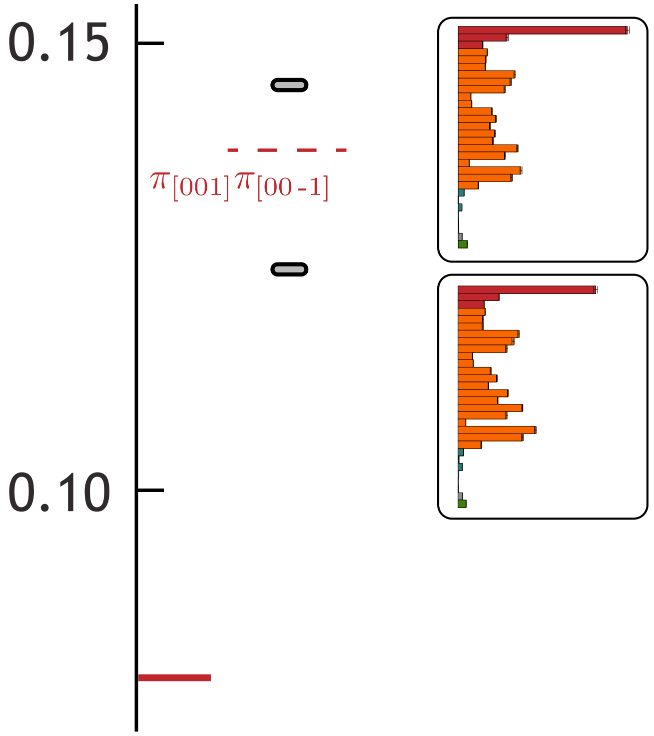
$m_\pi = 0.039$

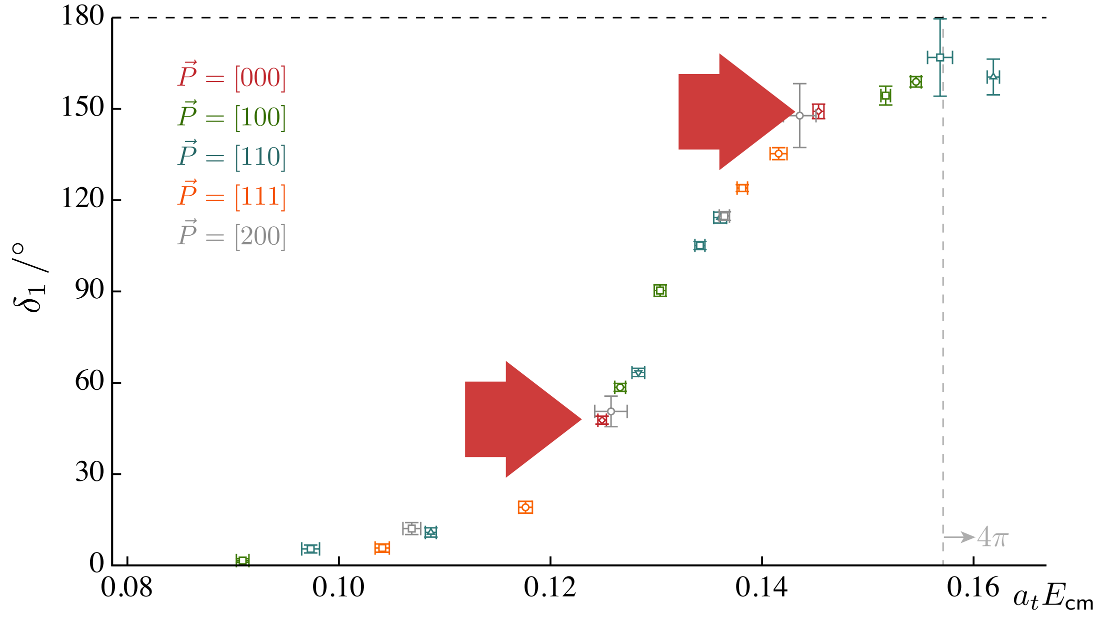
$m_K = 0.083$

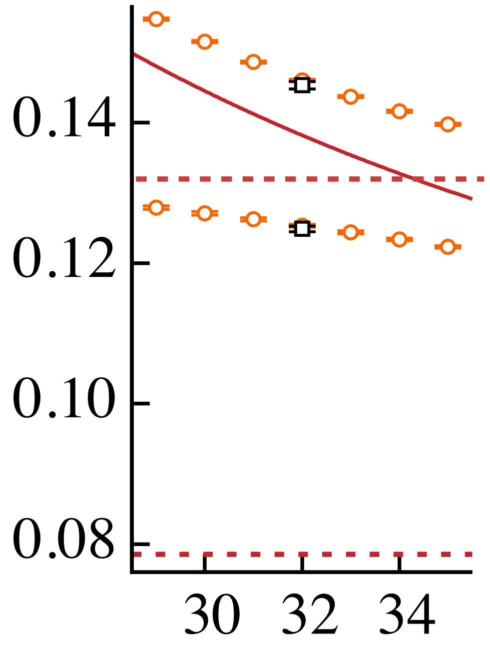
$L \sim 3.8$ fm

what’s happening here ?

focus on the lowest two states







an avoided level crossing