Hadron spectroscopy from lattice QCD

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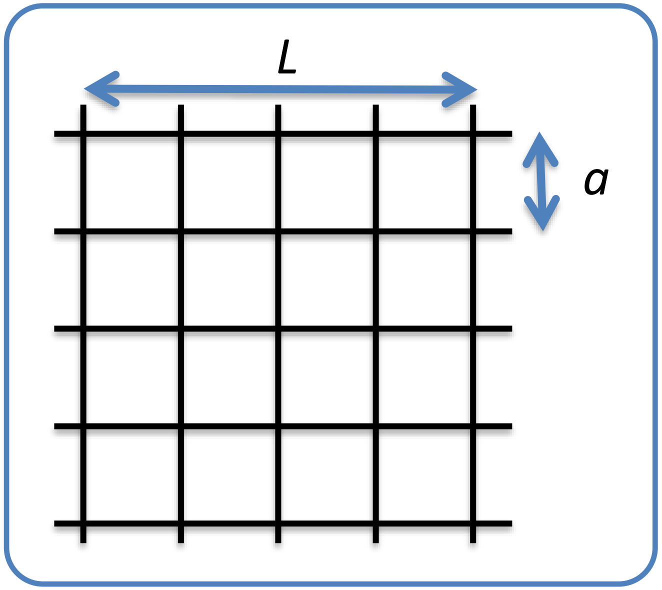
Workshop on "Hadron Spectroscopy with Strangeness",
Glasgow, 3 – 5 April 2024

UNIVERSITY OF CAMBRIDGE

hadspec

Lattice QCD spectroscopy

Systematically-improvable
first-principles calculations



- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically
  (Euclidean time, $t \rightarrow i\,t$)

Note:

- Finite a and L
- Possibly heavy u, d quarks
  ($\rightarrow$ unphysical m_π)

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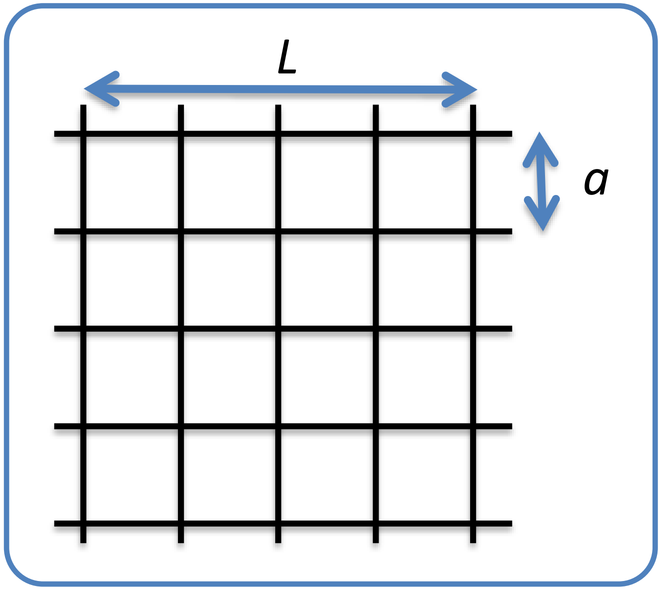

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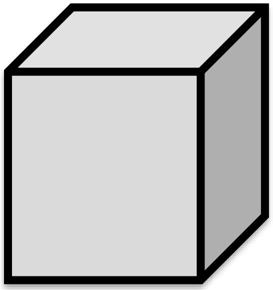
Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$



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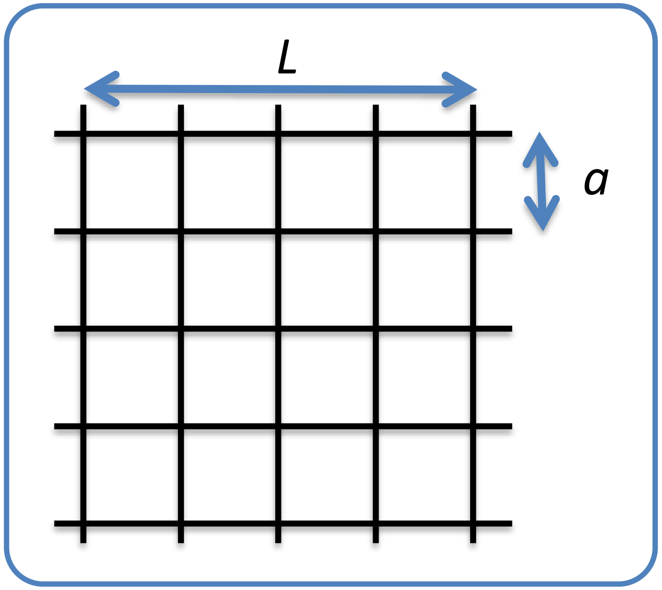

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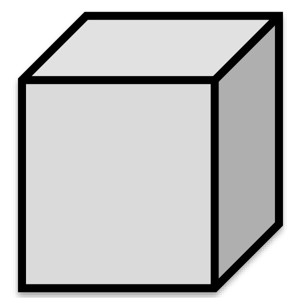
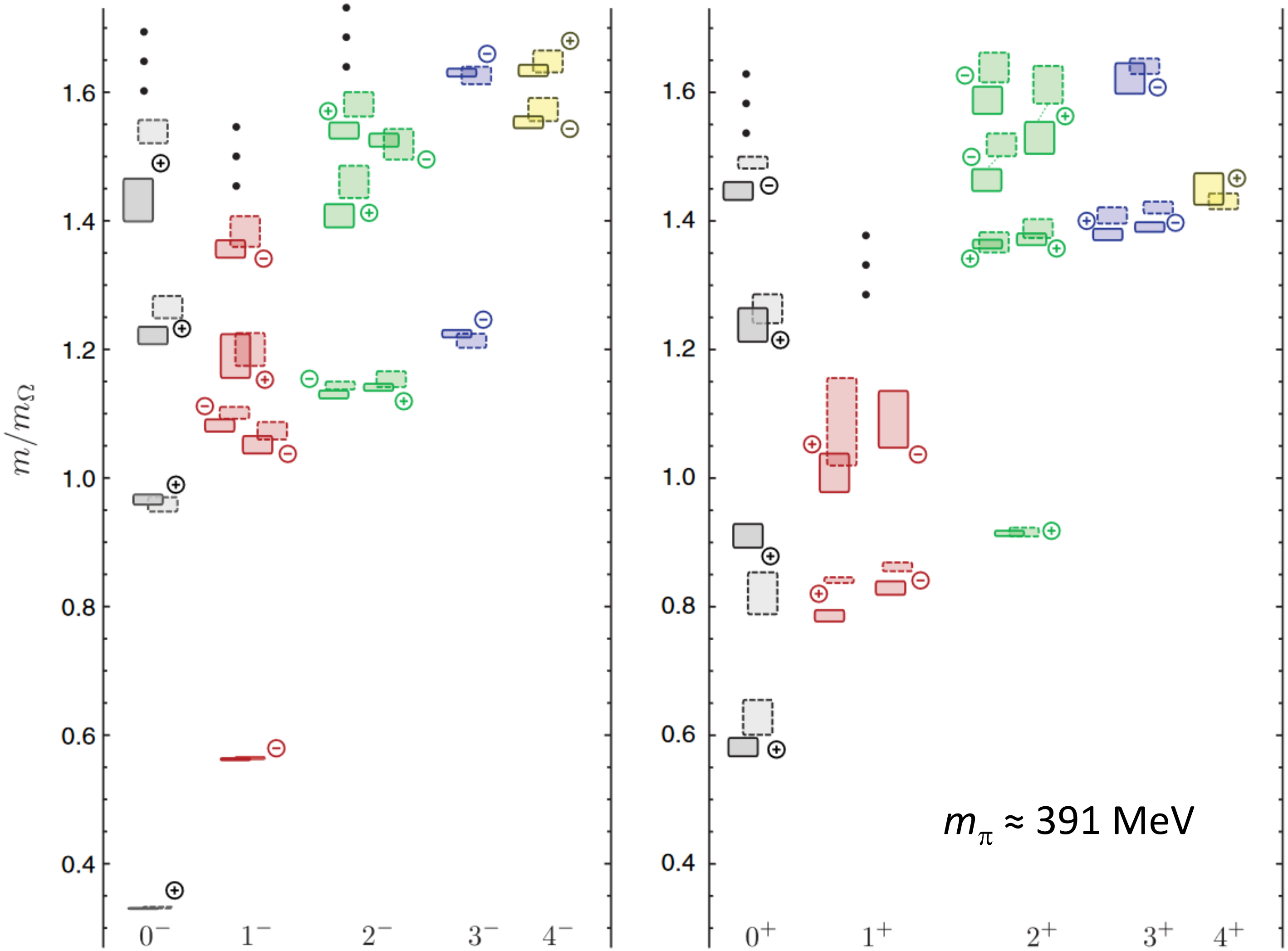


Excited spectra: large bases of operators with appropriate structures

Excited kaons

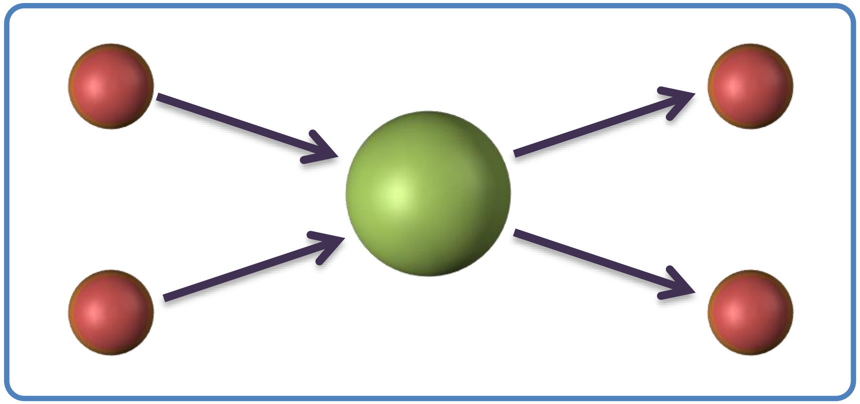
[Dudek, Edwards, Peardon, Richards, CT (HadSpec), 1004.4930 (PRD)]

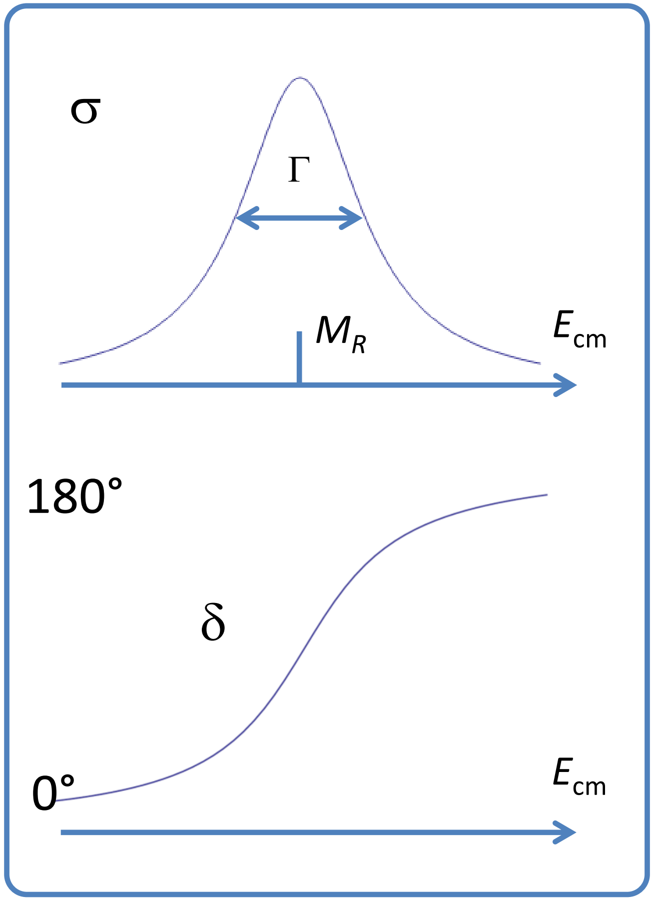


Large bases of only fermion-bilinear ops $\sim \bar{\psi}\Gamma D\ldots\psi$ (also other m_π)

Scattering and resonances

Most hadrons are resonances and decay strongly to lighter hadrons





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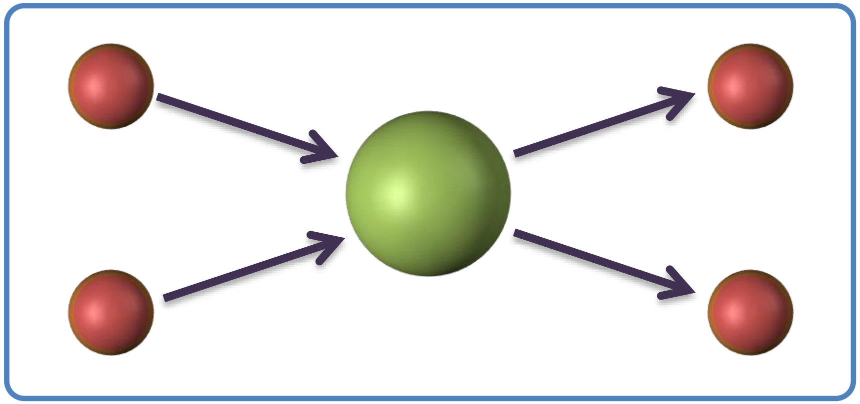
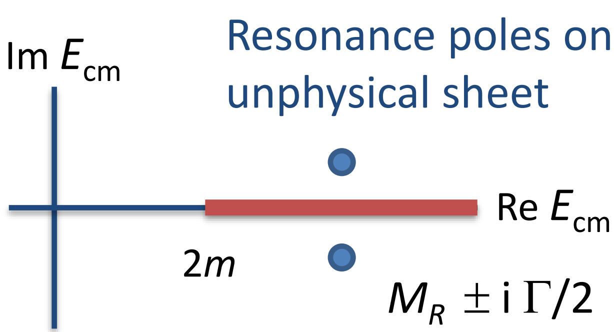
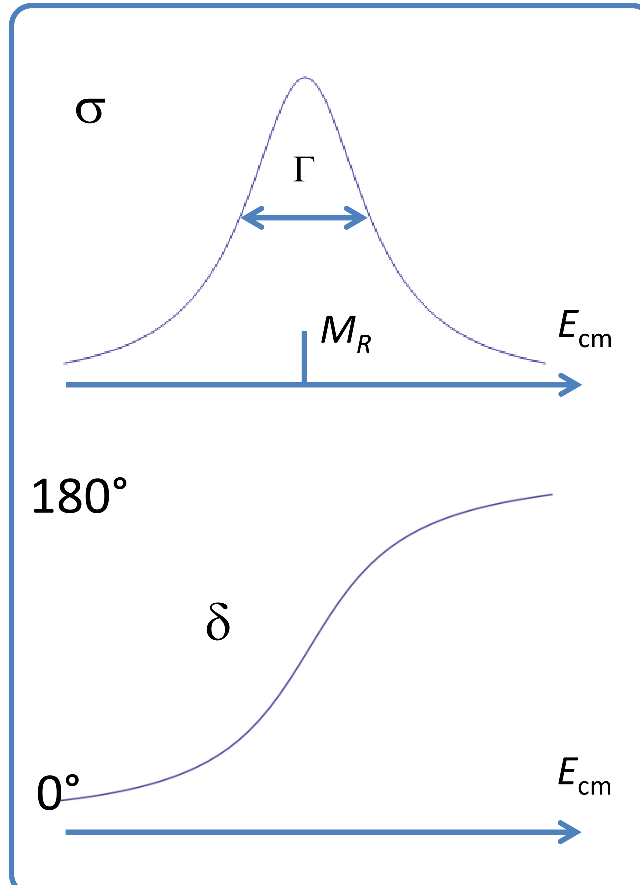


Singularity structure of scattering matrix (poles → state content)





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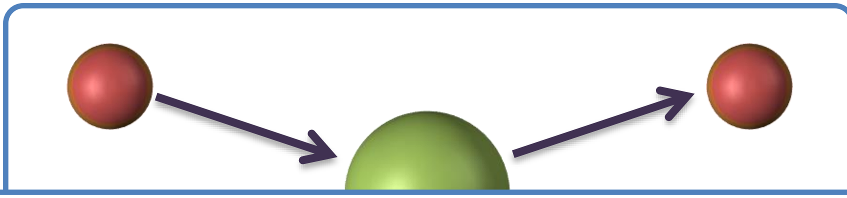
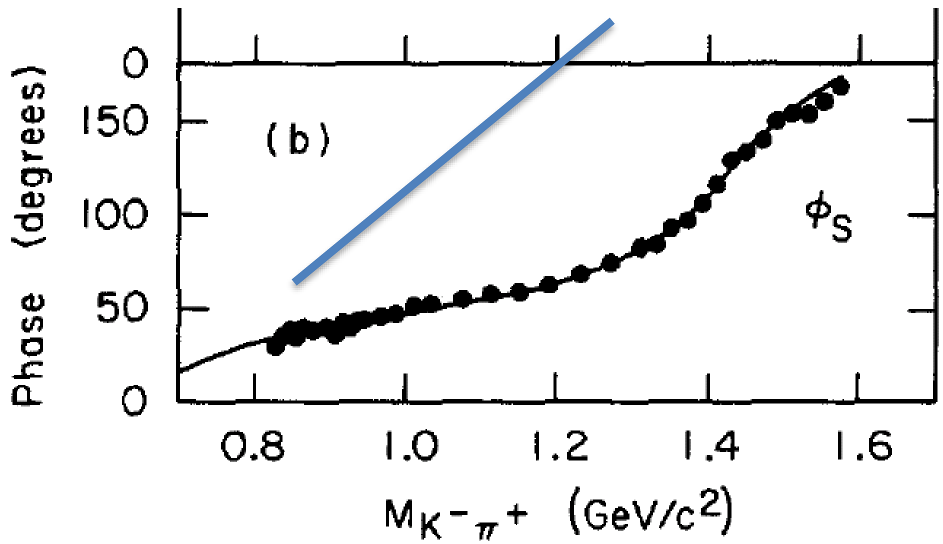
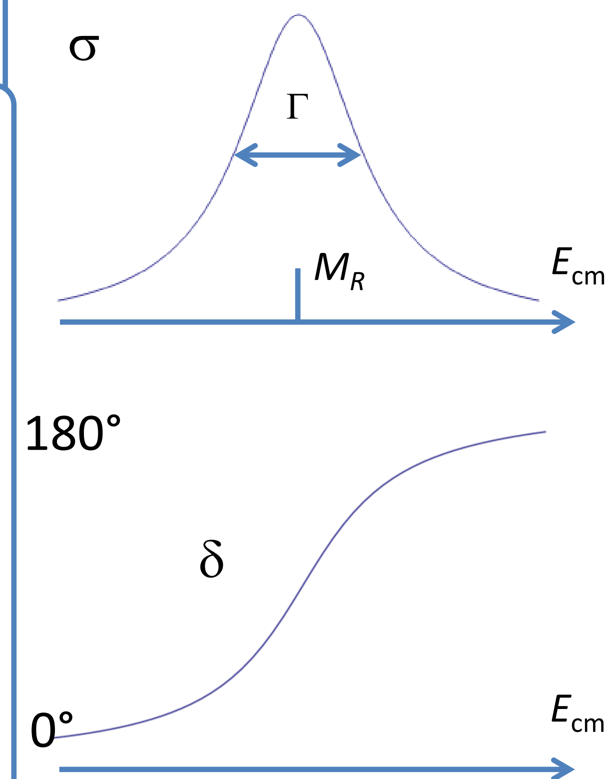


S-wave (0^+) $K\pi$

$\kappa/K_0^*(700)$



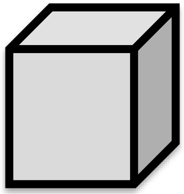
Aston *et al* (LASS) [NP B296, 493 (1988)]



Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

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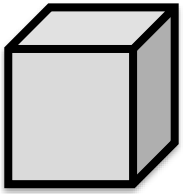

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Elastic scattering: one-to-one mapping $E_{\mathrm{cm}} \leftrightarrow t(E_{\mathrm{cm}})$

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

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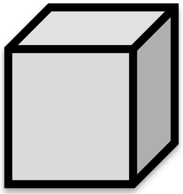

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Coupled channels: under-constrained problem
(each E_{cm} constrains t-matrix at that E_{cm})
Param. $t(E_{cm})$ using various forms, e.g. K-matrix (unitarity)
[see e.g. review Briceño, Dudek, Young, Rev. Mod. Phys. 90, 025001 (2018)]

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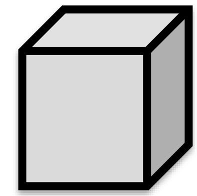

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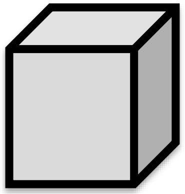

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Demonstrated in calcs. of ρ, light scalars, b_1, charm mesons, …

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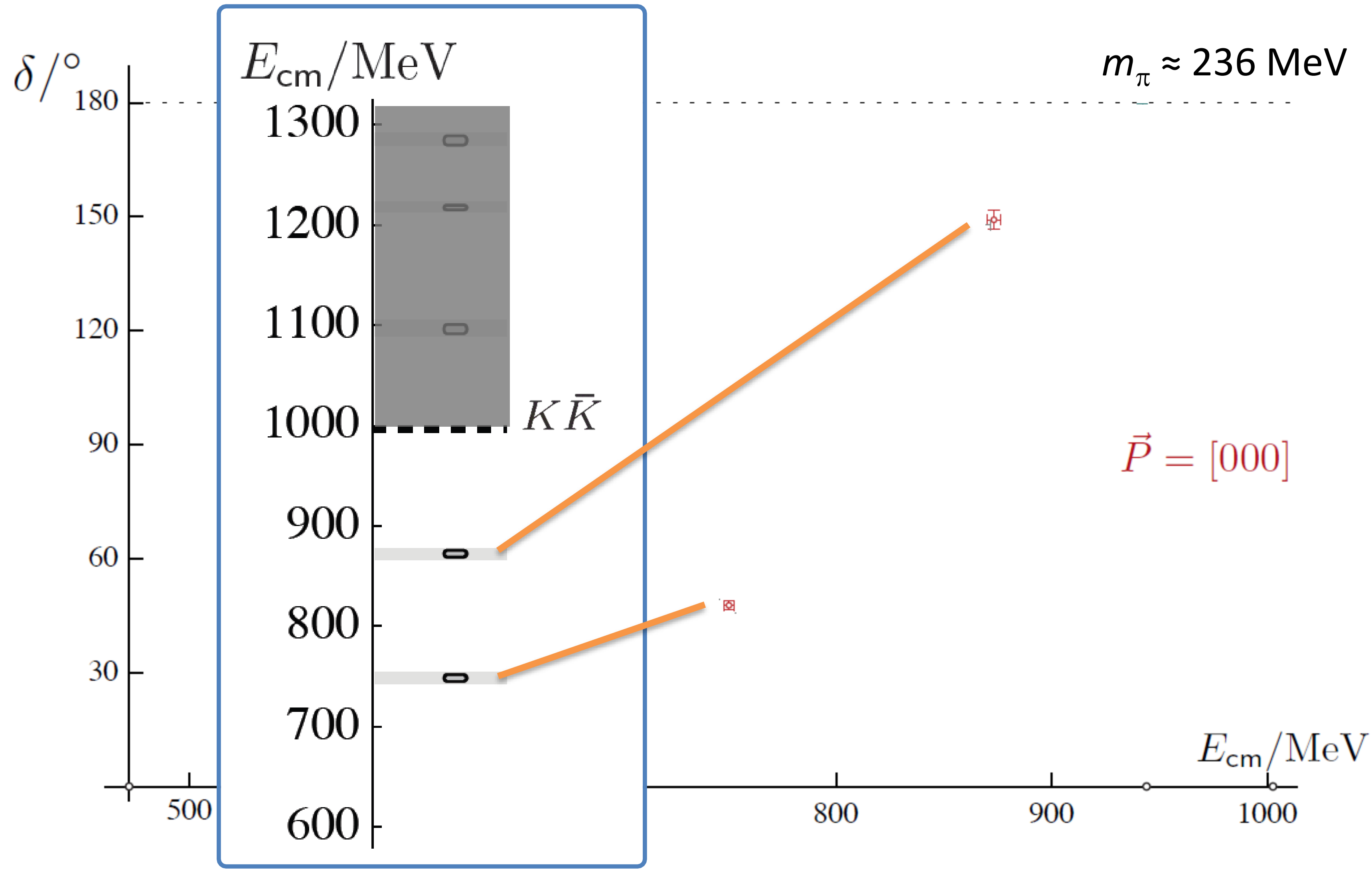
The ρ resonance: elastic P-wave ππ scattering



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

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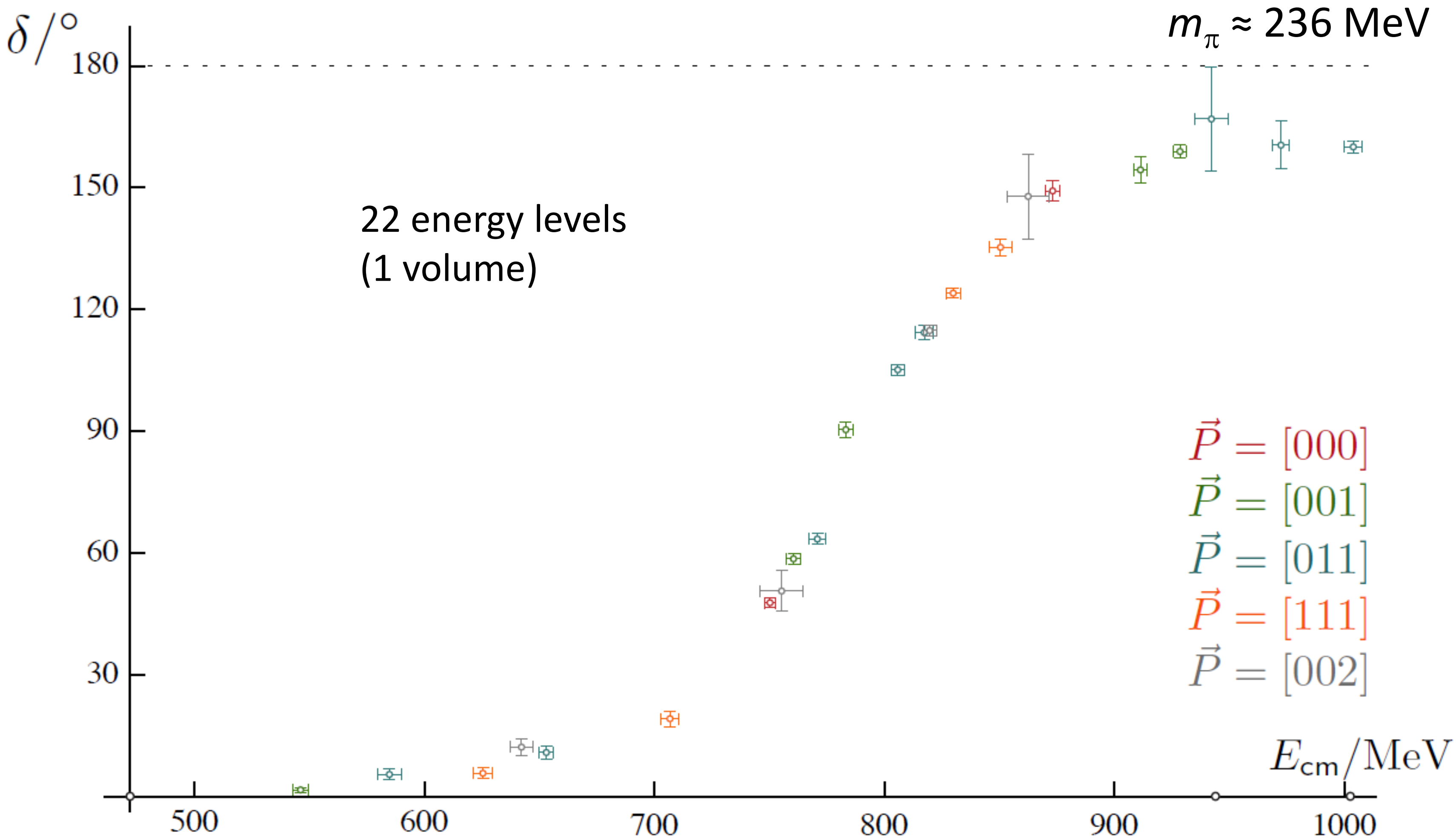

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Kπ (I=1/2)

$m_\pi \approx$ 239, 284,
327, 391 MeV
(28, 21, 18, 36 energies)

[Wilson, Briceño, Dudek, Edwards, CT (HadSpec), 1904.03188 (PRL)]

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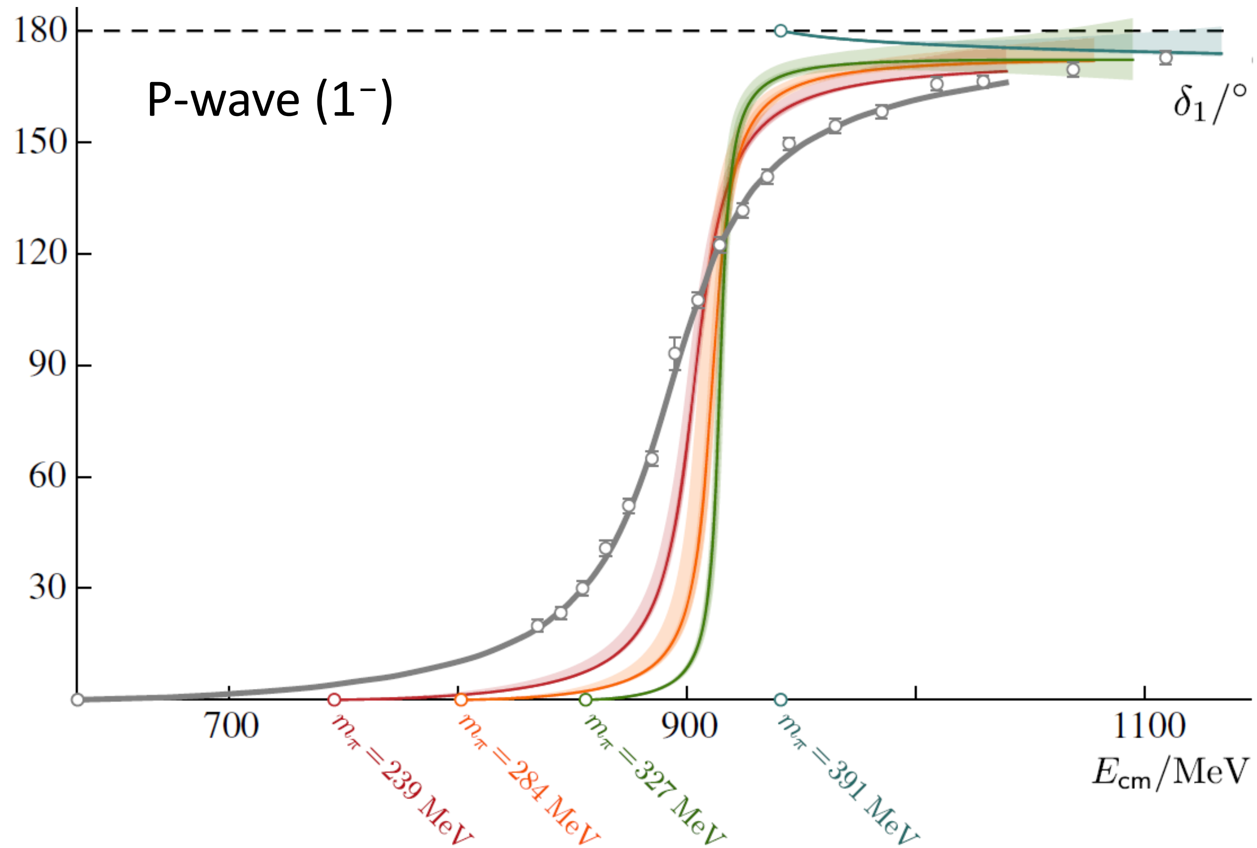

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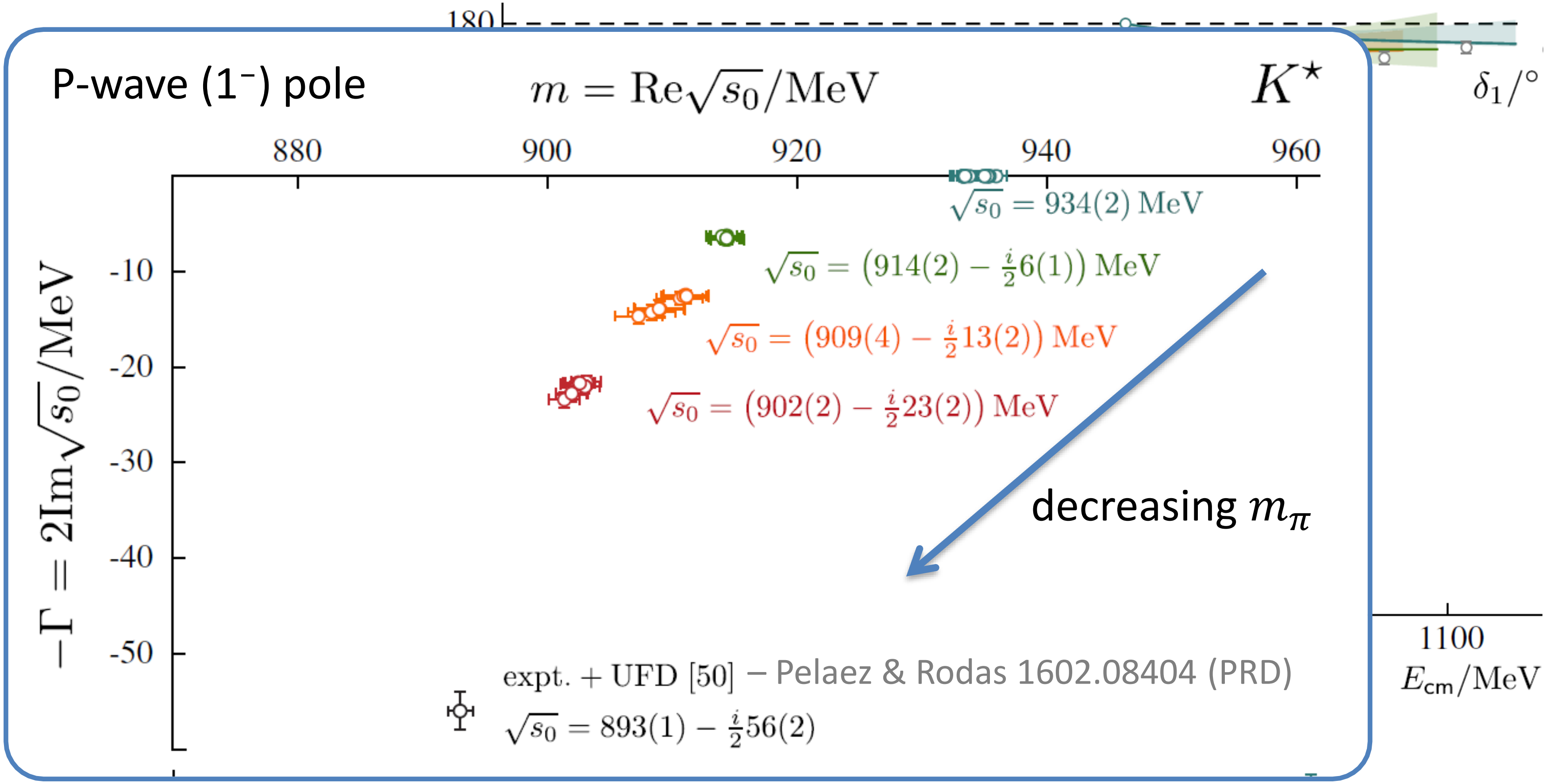

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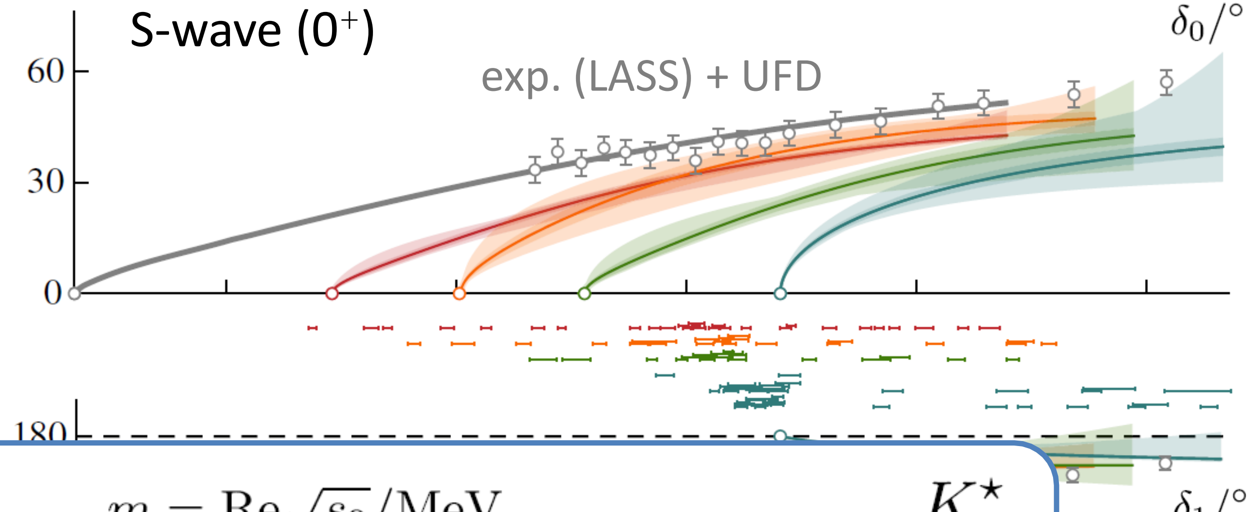

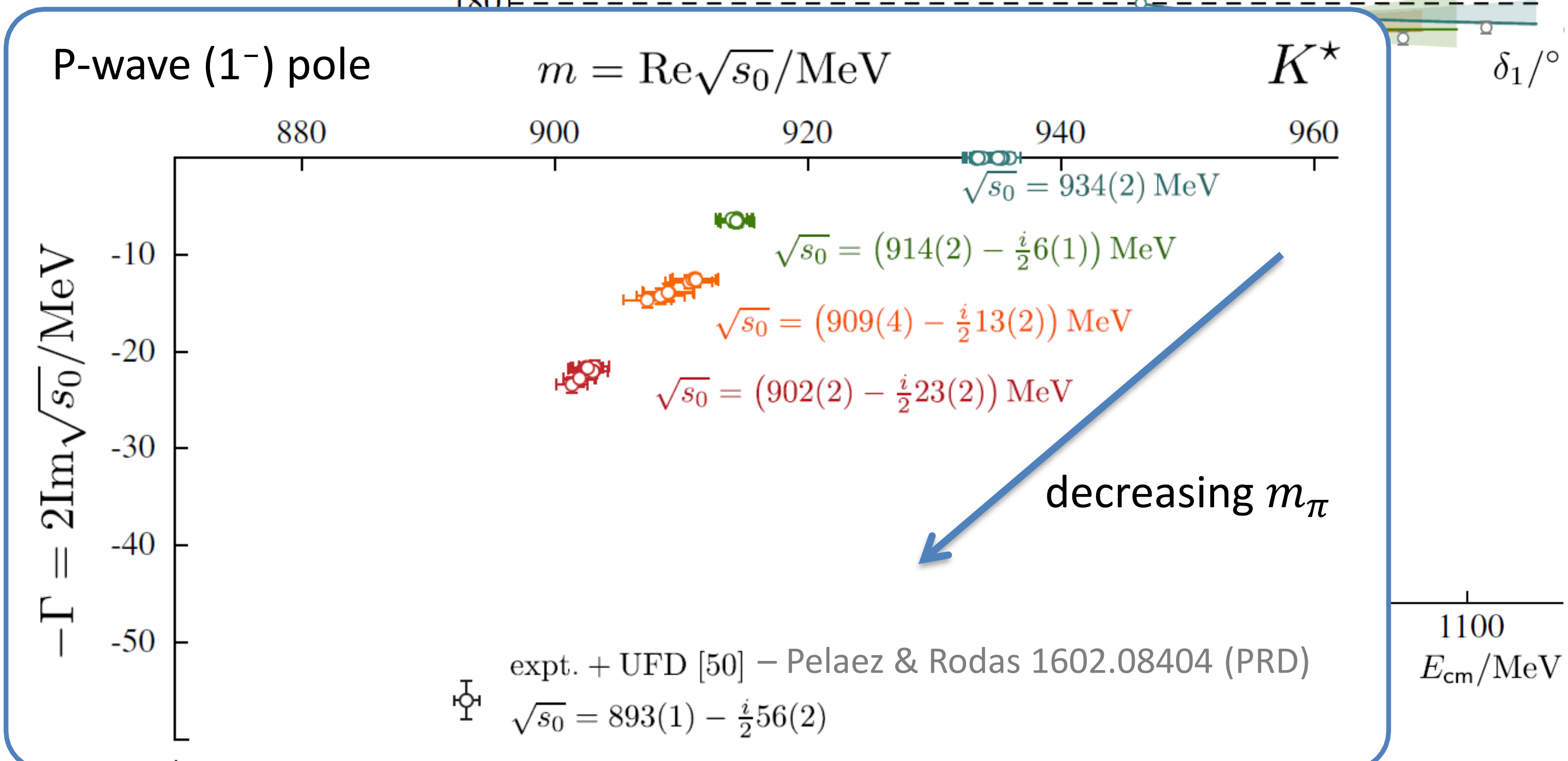

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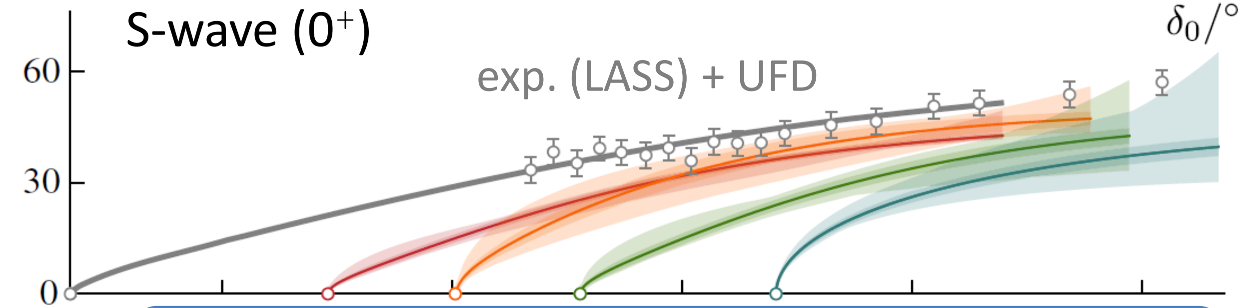
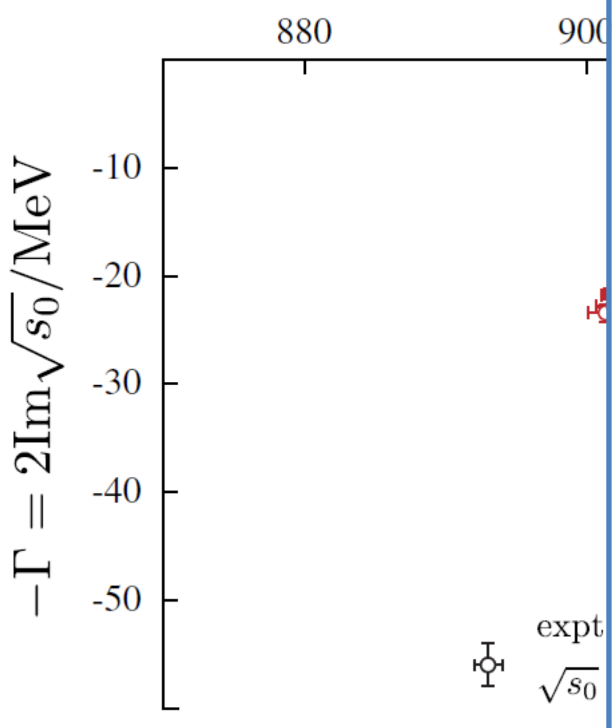
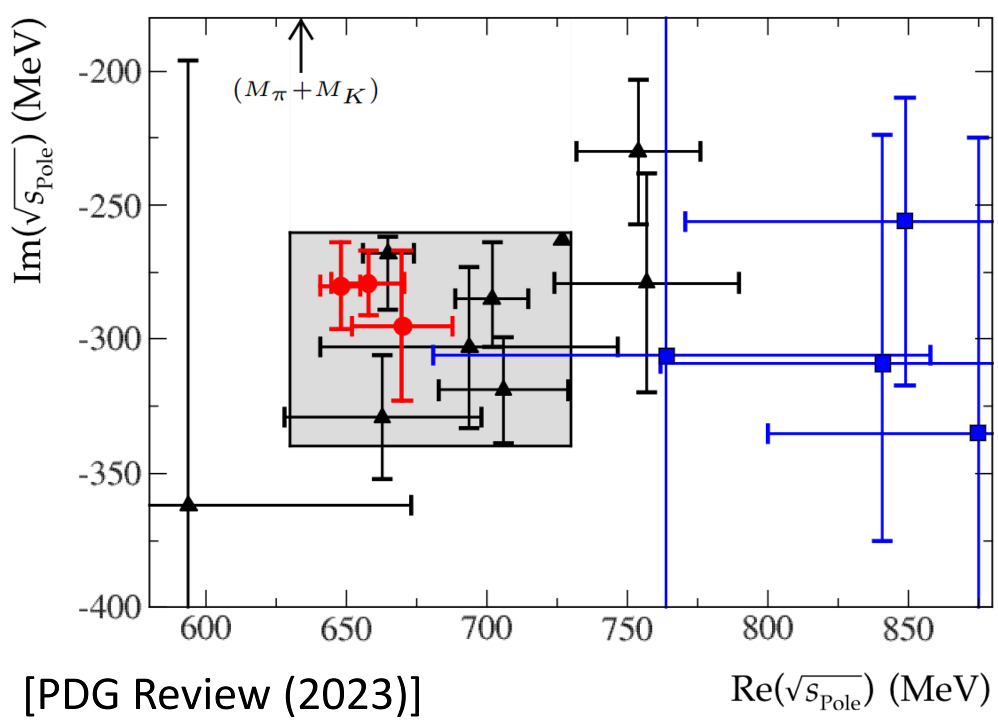


Need more sophisticated amplitude analysis techniques to pin down 0^+ state ($\kappa/K_0^*(700)$)



c.f. analysis of experimental data

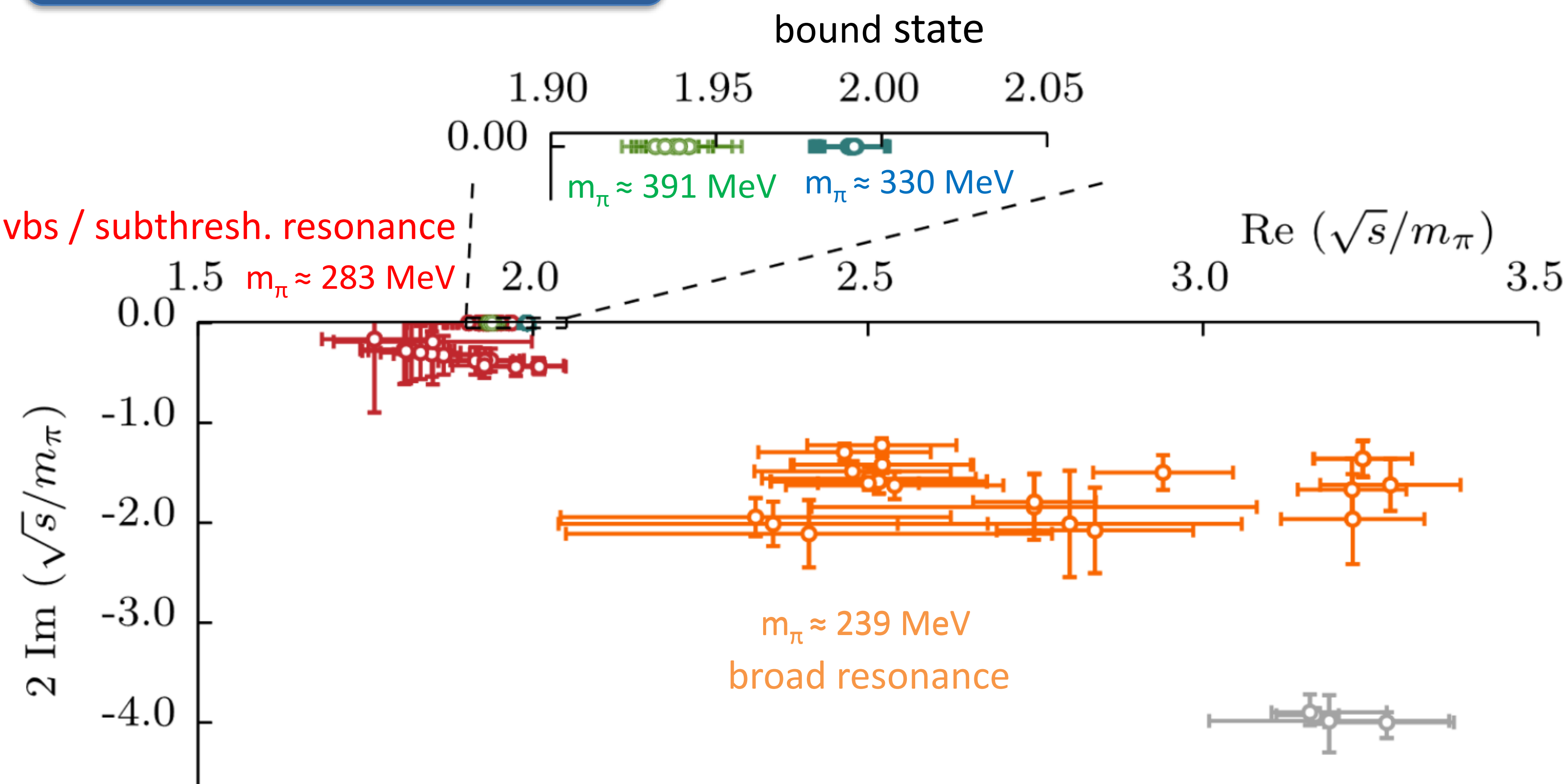


[PDG Review (2023)]

[Wilson, Briceño, Dudek, Edwards, CT (HadSpec), 1904.03188 (PRL)]

$f_0(500)/\sigma$ in $\pi\pi$ – poles

$J^P = 0^+, I = 0$



Reduce m_π: bound state → virtual bound state or subthreshold resonance
→ broad resonance

[Briceño, Dudek, Edwards, Wilson (HadSpec), 1607.05900 (PRL);
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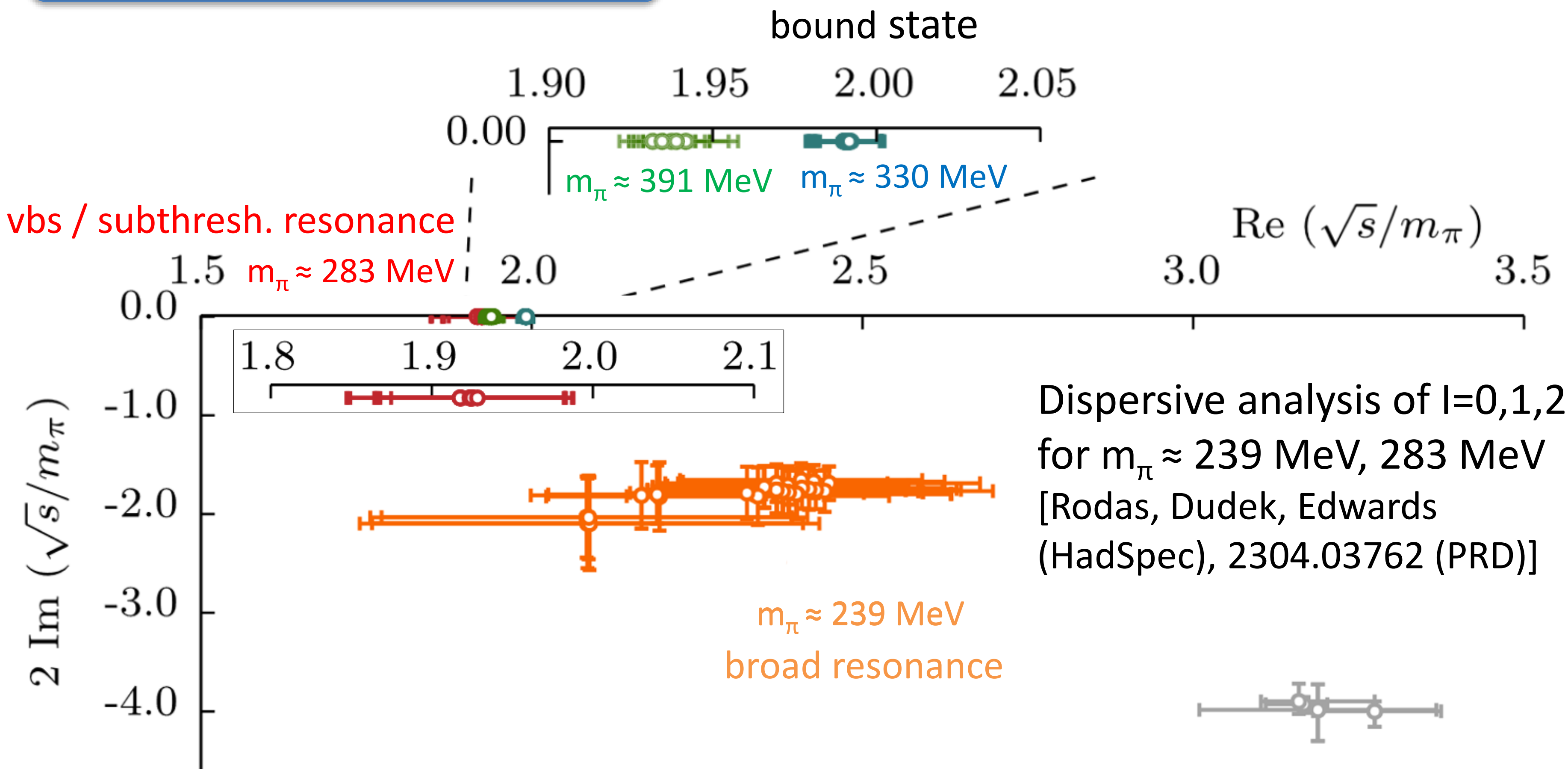

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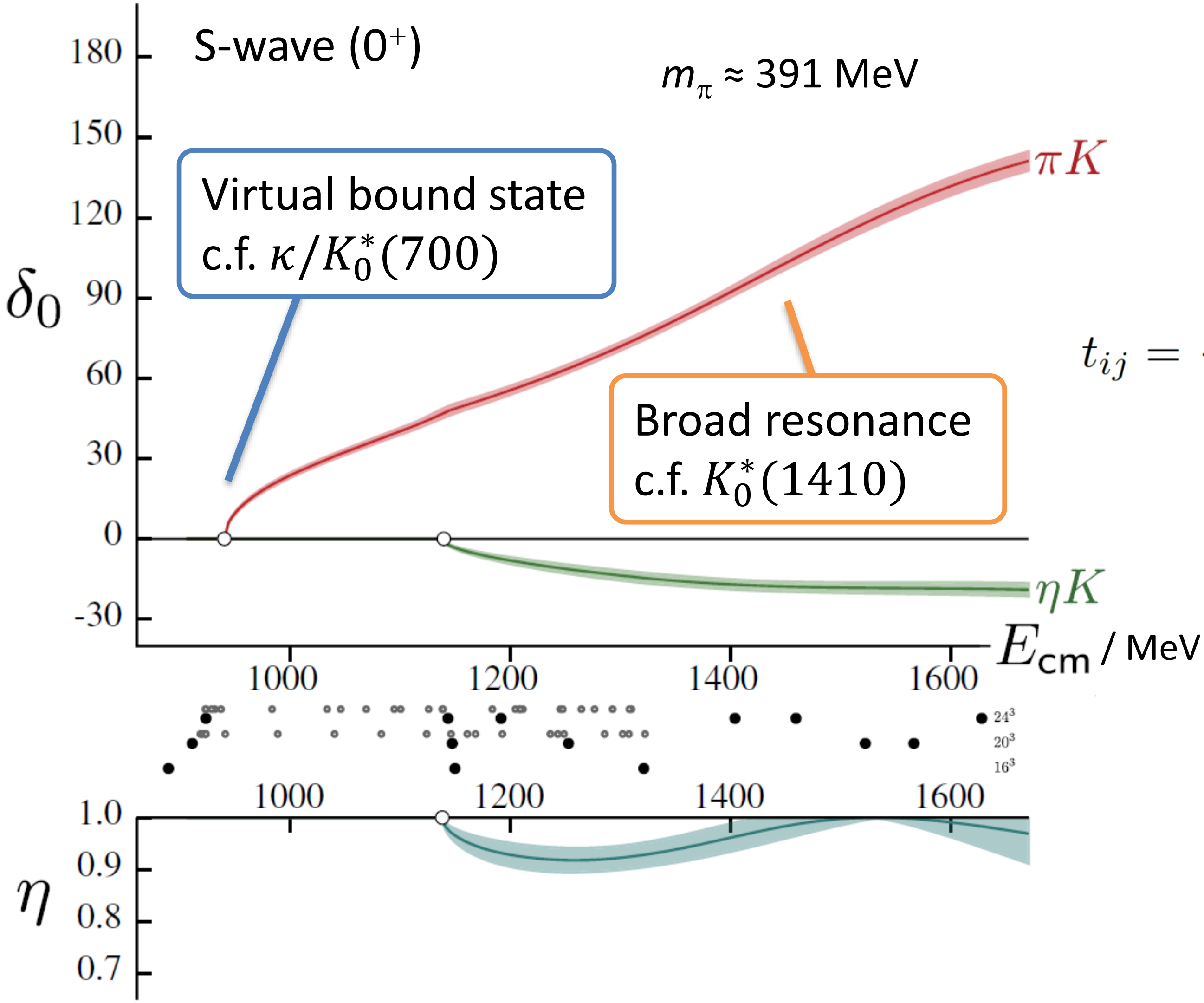
Kπ, Kη (I=1/2)

[Wilson, Dudek, Edwards, CT (HadSpec), 1406.4158 (PRL); 1411.2004 (PRD)]



$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i=j) \\ \dfrac{\sqrt{1-\eta^2}\; e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

80 energy levels

(3 volumes,
$L \sim 2-3$ fm,
$m_\pi L \sim 4-6$)

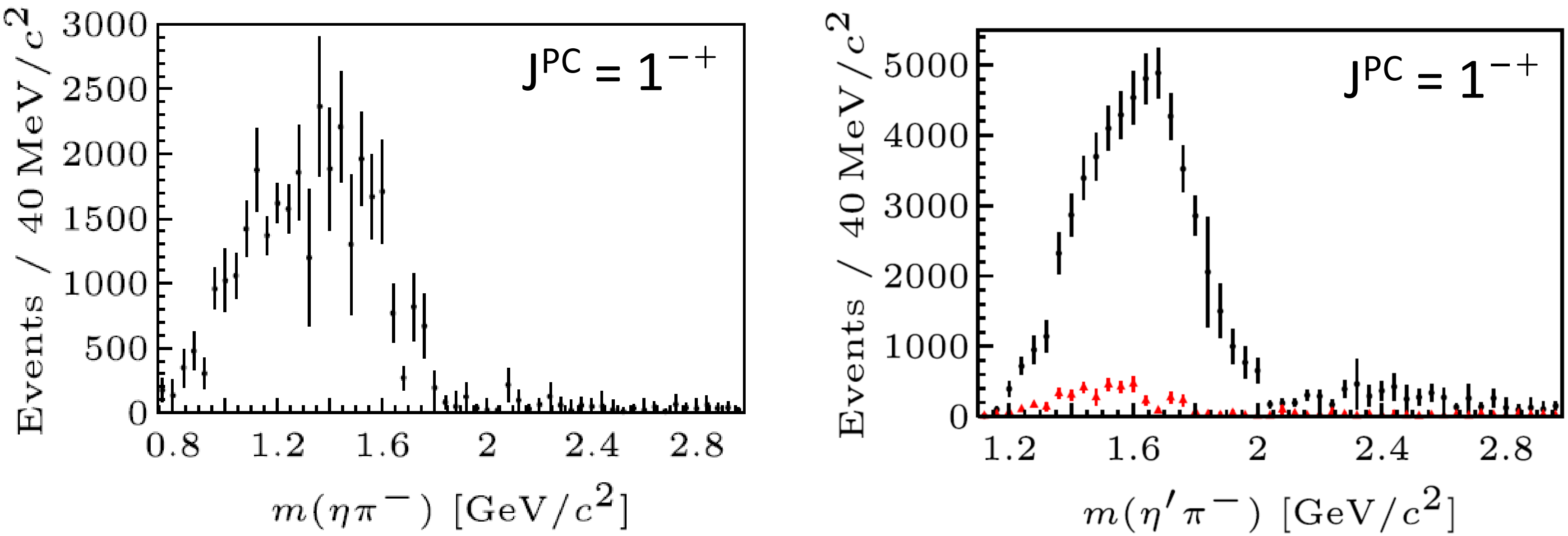
Exotic 1^{-+} meson

[PL B 740, 303 (2015)]

$\eta^{(\prime)}\pi^-$ in $\pi^- p \to \eta^{(\prime)}\pi^- p$

$\pi_1(1400)/\pi_1(1600)$?

COMPASS



Rodas *et al* (JPAC) [PRL 122, 042002 (2019)]: single resonance, *m* = 1564(24)(86) MeV, Γ = 492(54)(102) MeV

Kopf *et al* [EPJ C81, 12 (2021)] CB & COMPASS data: single resonance, *m* = $(1561.6 \pm 3.0\,^{+6.6}_{-2.6})$ MeV , Γ = $(388.1 \pm 5.4\,^{+0.2}_{-14.1})$ MeV

1^{-+} channel with $SU(3)_F$ flavour sym

[**Woss**, Dudek, Edwards, Thomas, Wilson, 2009.10034 (PRD)]

$SU(3)_F$ symmetry ($m_u = m_d = m_s$), 6 lattice volumes

$m_\pi \approx 700$ MeV, $m_\rho \approx 1000$ MeV, $m_{\eta'} \approx 940$ MeV

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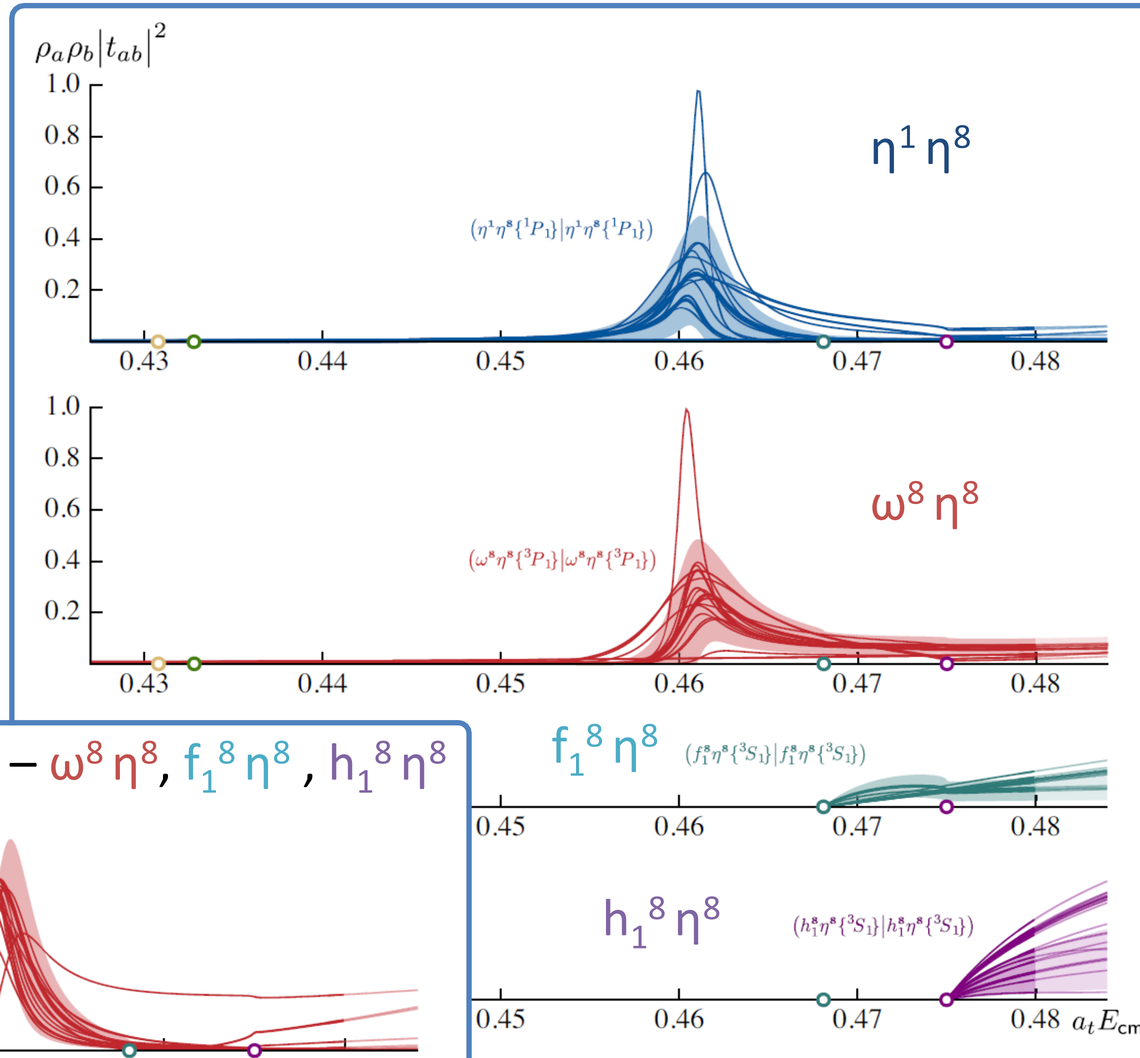
Constrain eight 1^{-+} $SU(3)_F$ octet coupled partial waves with 53 energy levels

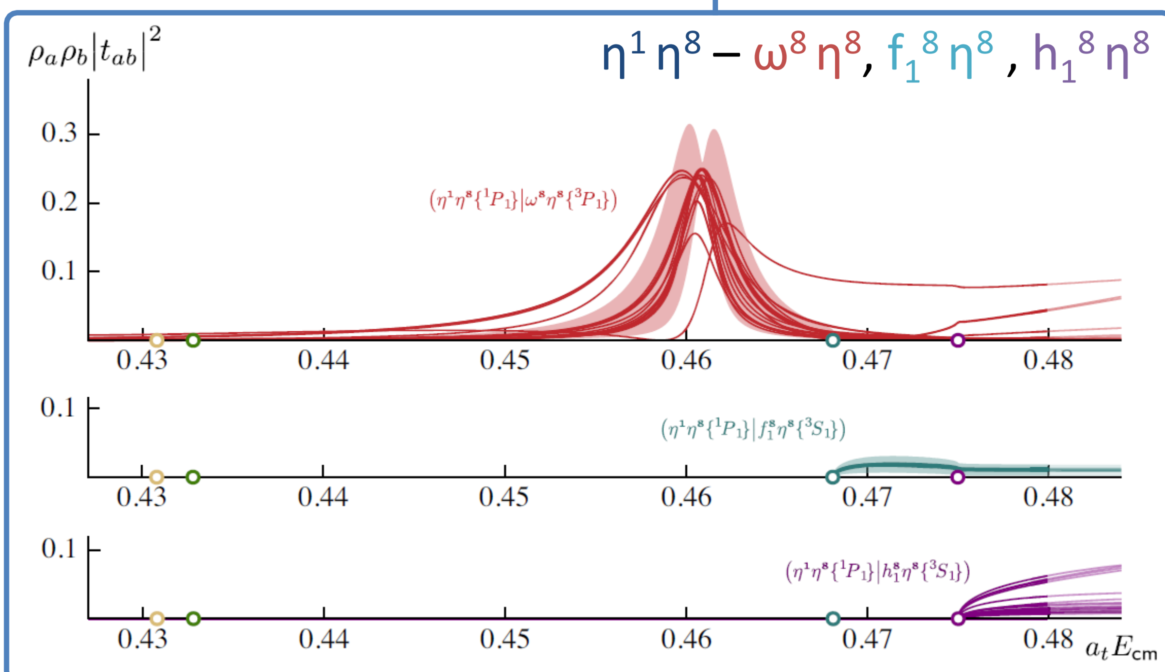
$$\eta^{\mathbf{1}}\eta^{\mathbf{8}}\{{}^1P_1\}$$
$$\omega^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3P_1\}$$
$$\omega^{\mathbf{8}}\omega^{\mathbf{8}}\{{}^3P_1\},\ \omega^{\mathbf{1}}\omega^{\mathbf{8}}\{{}^1P_1, {}^3P_1, {}^5P_1\}$$
$$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3S_1\},\ h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{{}^3S_1\}$$

(Another 8 energy levels constrain three 3^{-+} partial waves.)

Scattering amps

27 parameterisations with $\chi^2/N_{\mathrm{dof}} \leq 1.25$





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Pole and couplings

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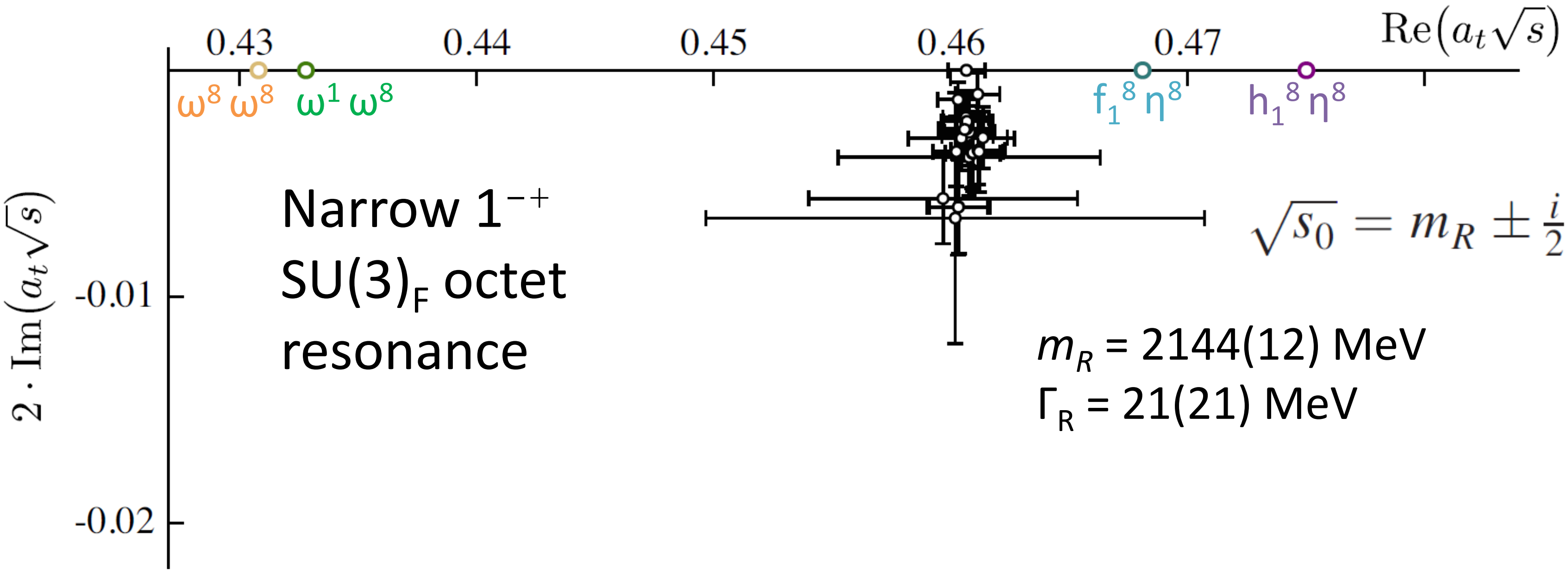

Narrow 1^{-+}
$SU(3)_F$ octet
resonance

$$\sqrt{s_0} = m_R \pm \tfrac{i}{2}\Gamma_R$$

m_R = 2144(12) MeV
Γ_R = 21(21) MeV

Strongly coupled to $h_1^8\eta^8$

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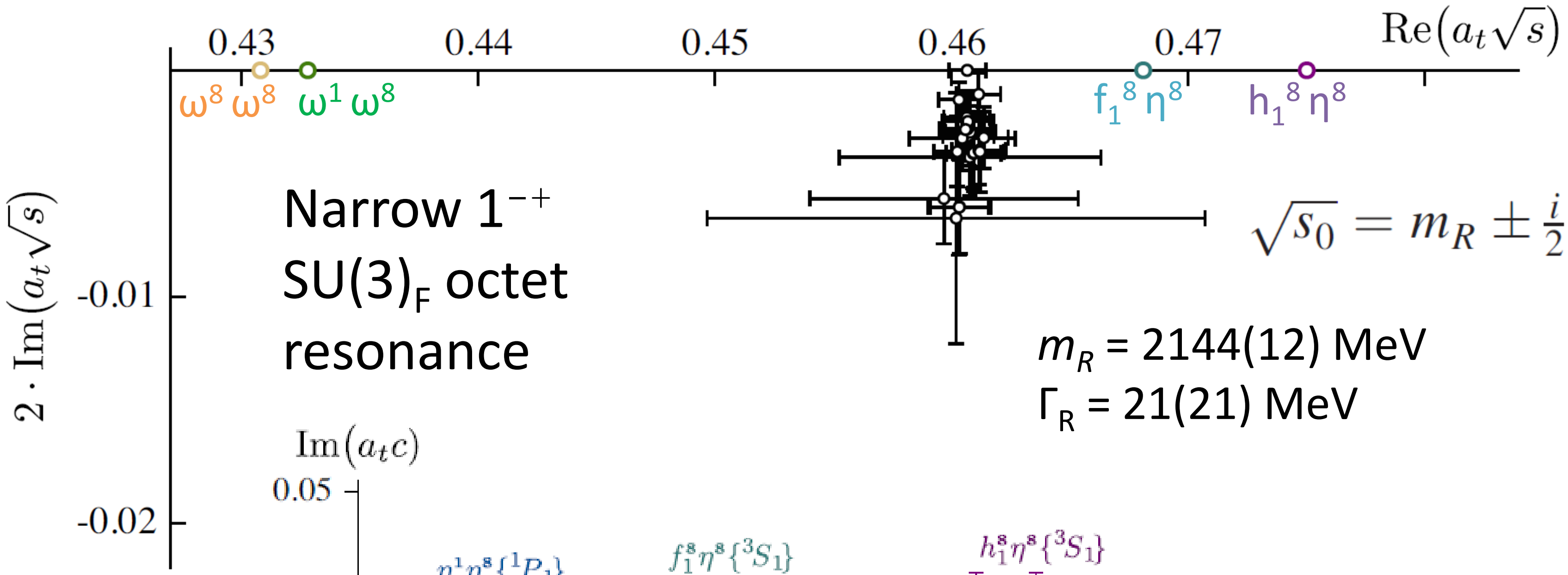

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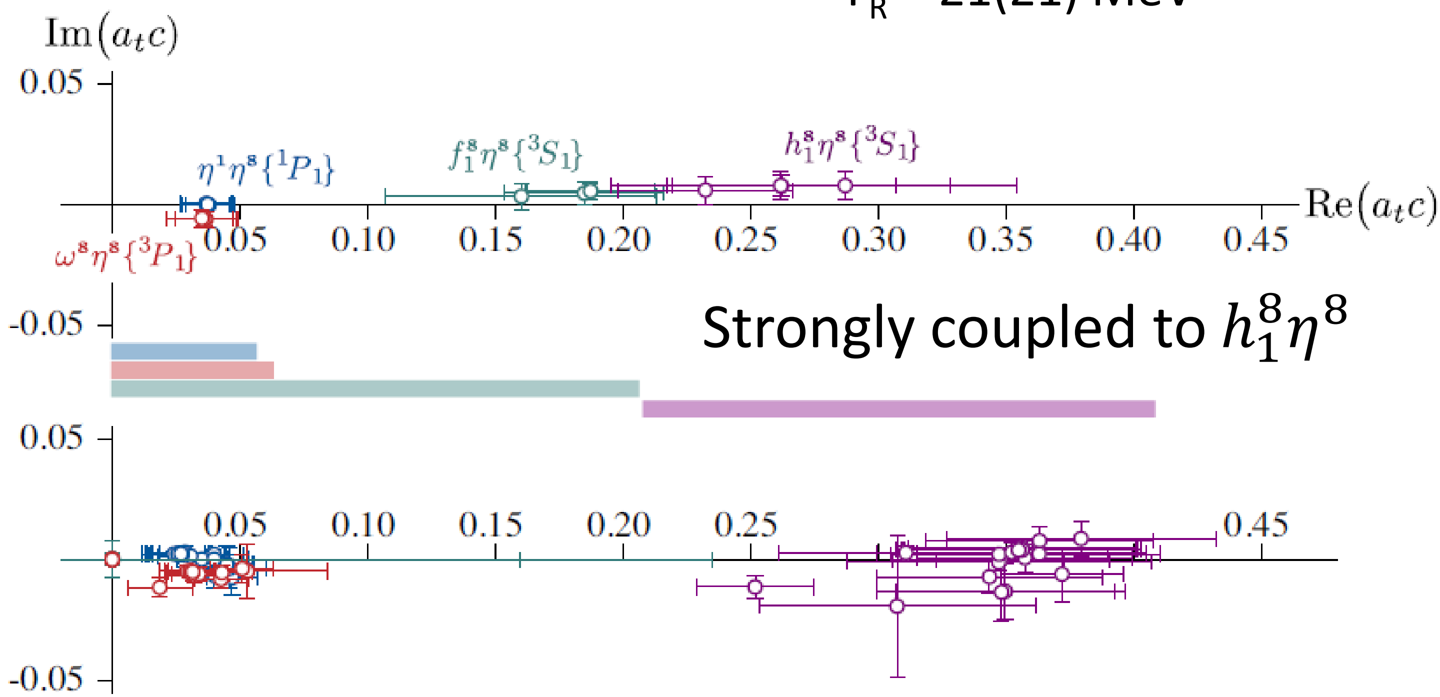

Strongly coupled to $h_1^8\eta^8$

$$t_{ij} \sim \frac{c_i\, c_j}{s_0 - s}$$

Extrapolation of couplings

[2009.10034 (PRD)]

Attempt crude extrapolation to physical masses (break $SU(3)_F$ symmetry).

Assume couplings scale with appropriate barrier factor k^{ℓ}.

Use PDG masses and m_R = 1564 MeV.

$\Gamma = \Sigma_i \Gamma_i$ = 139 – 590 MeV

c.f. JPAC: Γ = 492(54)(102) MeV
Kopf *et al*: Γ = $(388.1 \pm 5.4\,^{+0.2}_{-14.1})$ MeV

	Γ_i/MeV
$\eta\pi$	$0 \to 1$
$\rho\pi$	$0 \to 20$
$\eta'\pi$	$0 \to 12$
$b_1\pi$	$139 \to 529$
$K^*\overline{K}$	$0 \to 2$
$f_1(1285)\pi$	$0 \to 24$
$\rho\omega\{^1P_1\}$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	$\lesssim 0.09$
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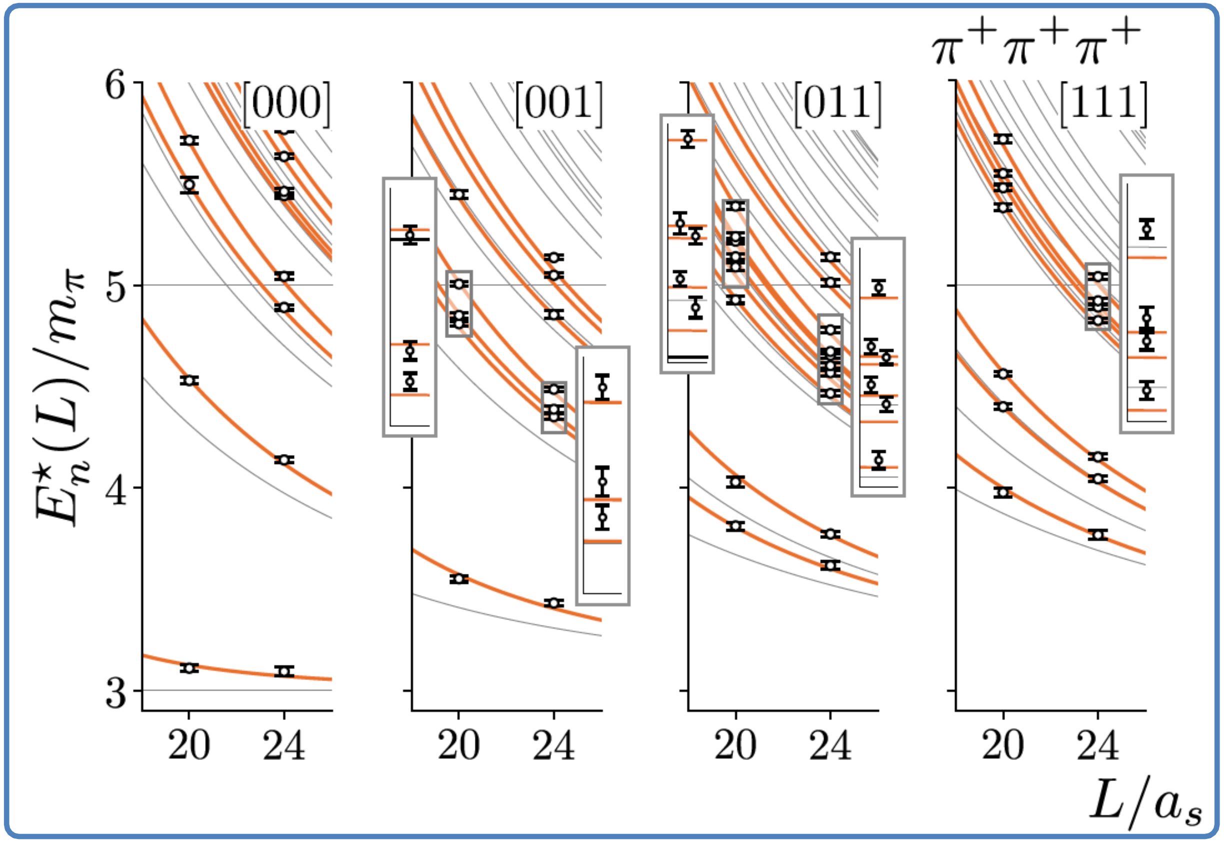
LQCD calc. in McNeile & Michael [PR D73, 074506 (2006)]: consider setup with $m_\pi \approx$ 500 MeV, $m_{\pi_1} = m_{b_1} + m_\pi$

Isospin-3 πππ

[Hansen et al (HadSpec), 2009.04931 (PRL)]



$m_\pi \approx 391$ MeV

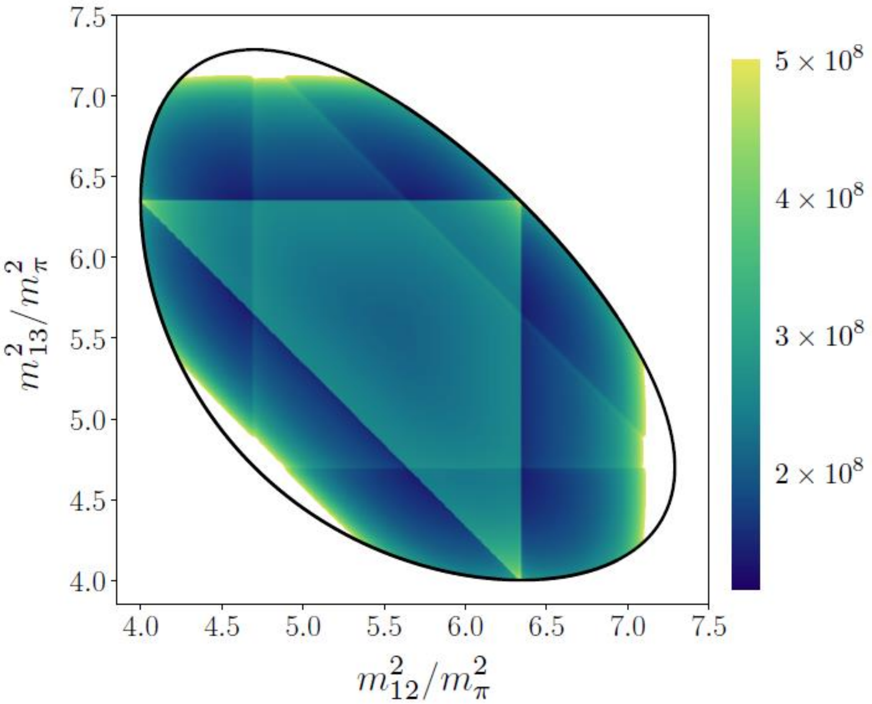
Isospin-3 $\pi\pi\pi$

[Hansen et al (HadSpec), 2009.04931 (PRL)]

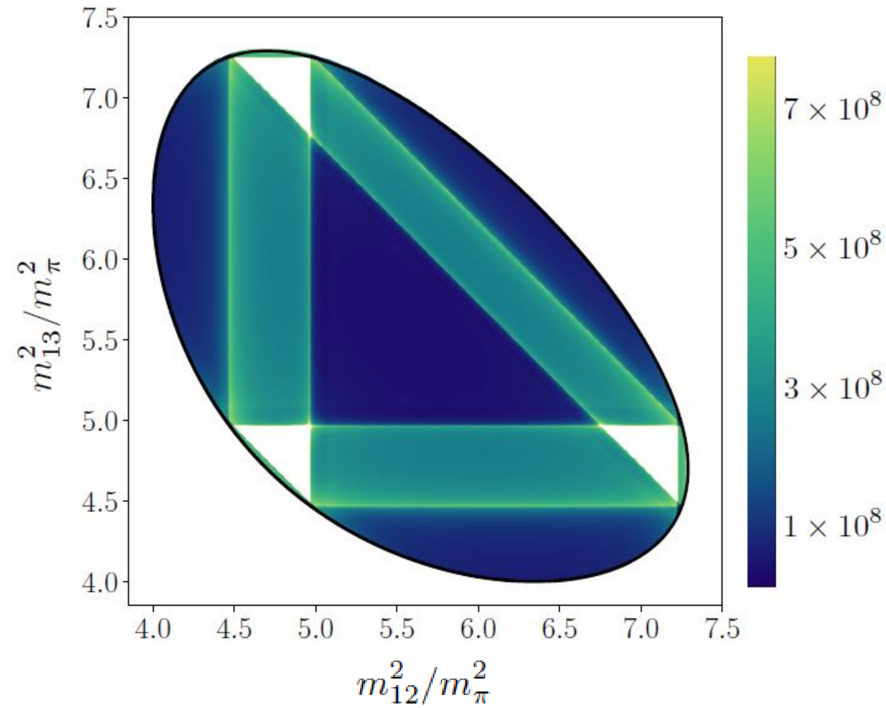
$m_\pi \approx 391$ MeV

$m_\pi^4 |\mathcal{M}_3|^2$

$\sqrt{s_3} = 3.7 m_\pi$



$m_{12}'^2 = 2.1 m_\pi,\ m_{13}'^2 = 2.25 m_\pi$

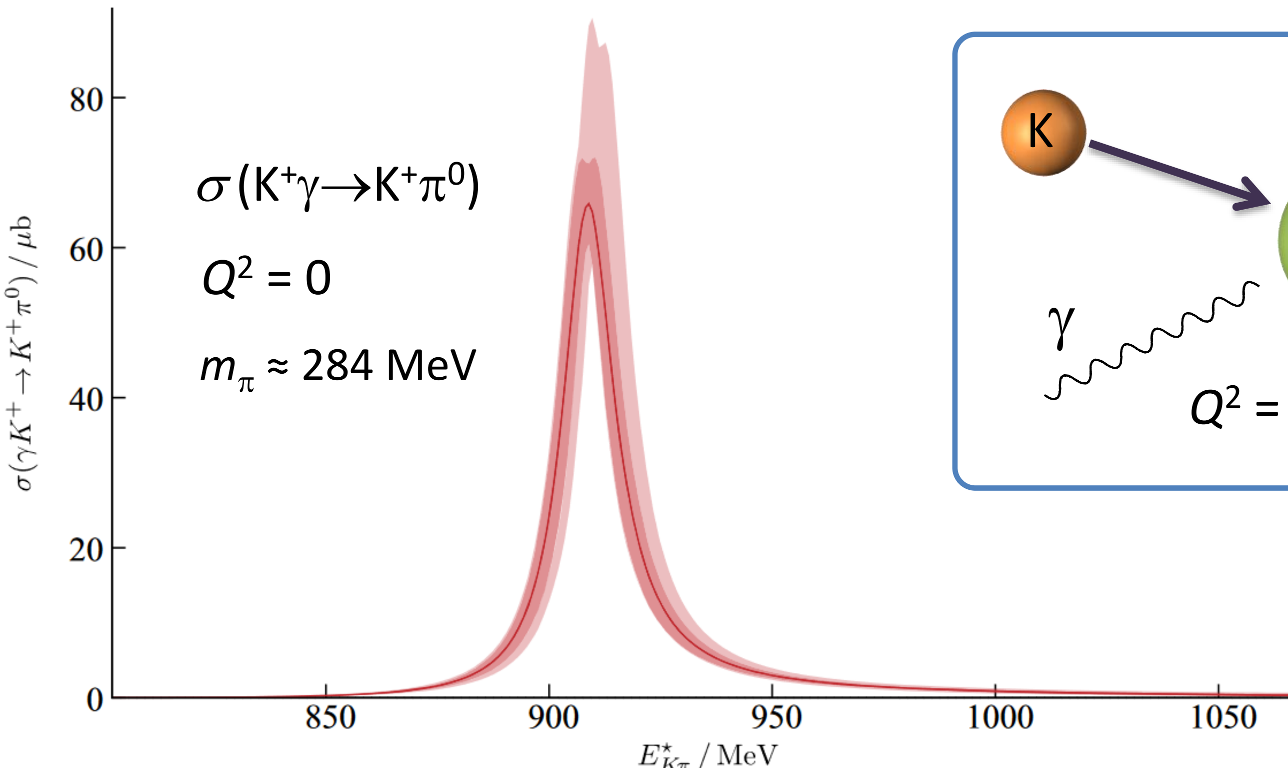


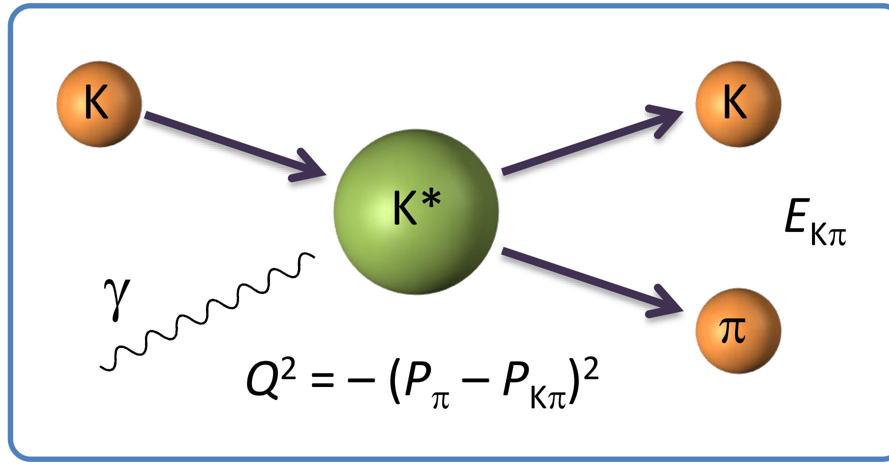
$m_{12}'^2 = m_{12}^2,\ m_{13}'^2 = m_{13}^2$

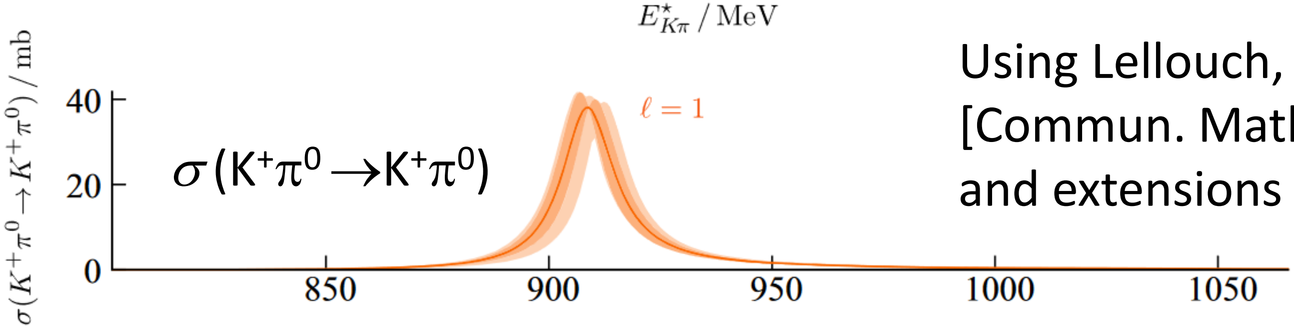
Resonant $K^+ \gamma \to K^{*+} \to K^+ \pi^0$ amplitude

[Radhakrishnan, Dudek, Edwards (HadSpec), 2208.13755 (PRD)]

Need: $\langle 0 \left| \mathcal{O}_i(t_f) \;\; \bar{\psi}(t)\gamma^\mu\psi(t) \;\; \mathcal{O}_j(t_i) \right| 0 \rangle$





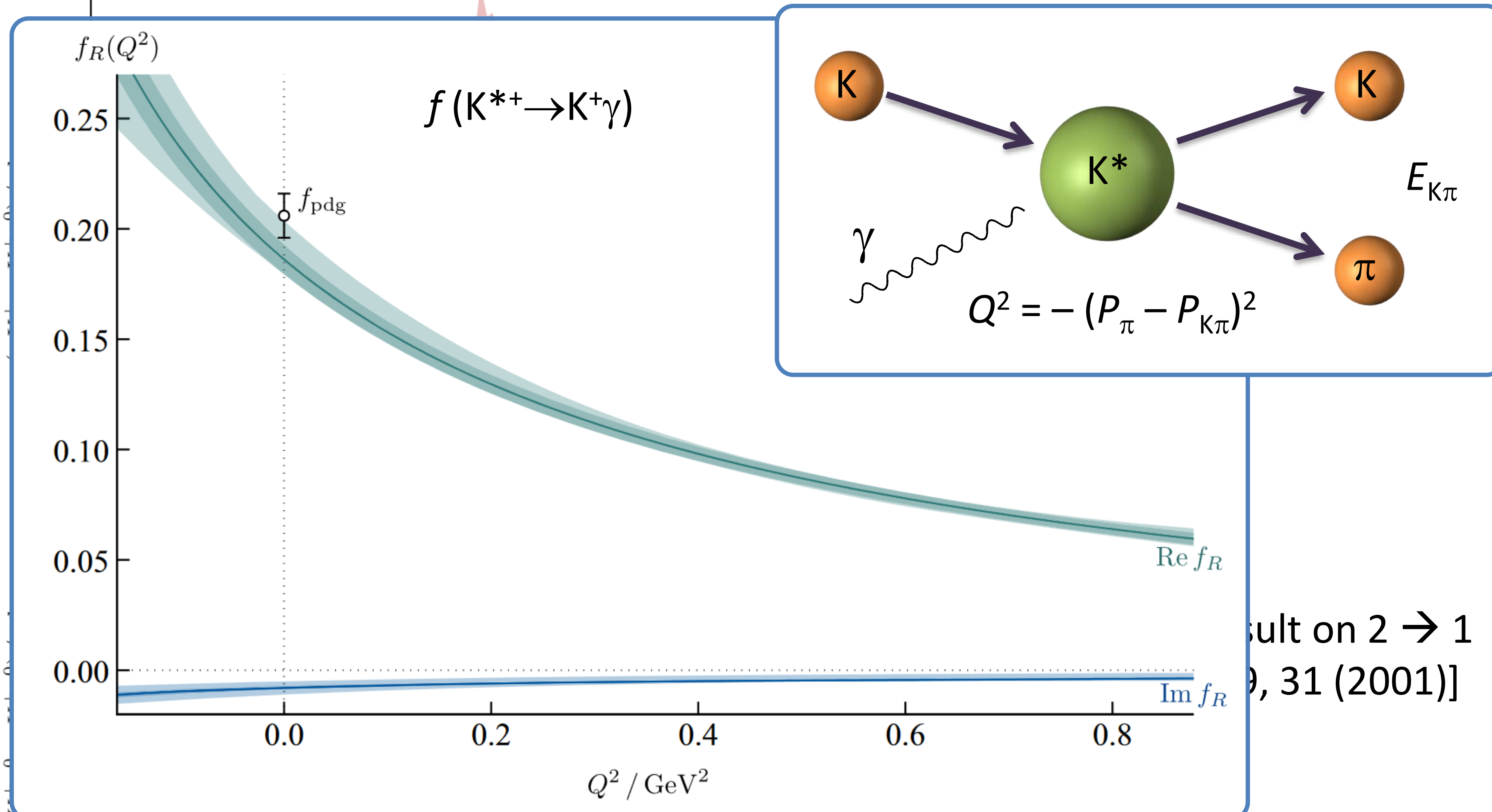


Using Lellouch, Lüscher result on 2 → 1 [Commun. Math. Phys. 219, 31 (2001)] and extensions

Resonant $K^+ \gamma \rightarrow K^{*+} \rightarrow K^+ \pi^0$ amplitude

[Radhakrishnan, Dudek, Edwards (HadSpec), 2208.13755 (PRD)]

Need: $\langle 0 \left| \mathcal{O}_i(t_f) \;\; \bar{\psi}(t)\gamma^\mu\psi(t) \;\; \mathcal{O}_j(t_i) \right| 0 \rangle$



…sult on 2 → 1
…9, 31 (2001)]

Summary

- Significant progress in using lattice QCD to map out scattering amplitudes and study resonances etc. in recent years
- Presented some examples (there are lots more)
- Study evolution of phenomena as vary light-quark masses

- More sophisticated analysis techniques (c.f. analysis of experimental data)
- Three (or more!?) hadron scattering
- Probe structure, e.g. transitions and form factors

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Acknowledgements

Hadron Spectrum Collaboration

[*www.hadspec.org*]

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UK: University of Cambridge: CT, David Wilson, Nelson Lachini, *Daniel Yeo*

Edinburgh: Max Hansen

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