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New insights into the nature of the $\Lambda(1380)$ and $\Lambda(1405)$ resonances away from the SU(3) limit

in collaboration with
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Hadron spectroscopy with strangeness
@ Glasgow, UK, 4.4.2024,



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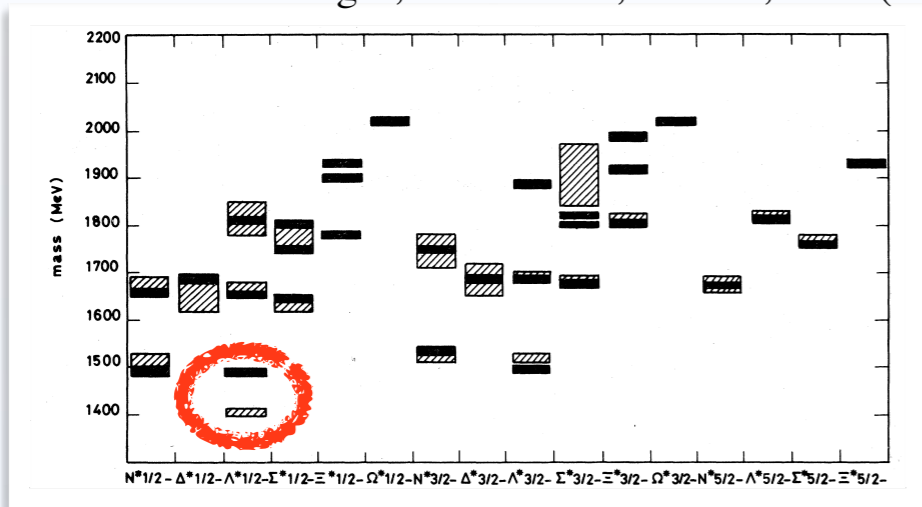
- Introduction $\sim \Lambda$ resonances and $\bar{K}N$ interaction \sim
- Chiral SU(3) model with extrapolation to unphysical point
- Results: pole trajectories
- Summary

$\bar{K}N$ interaction and $\Lambda(1405)$

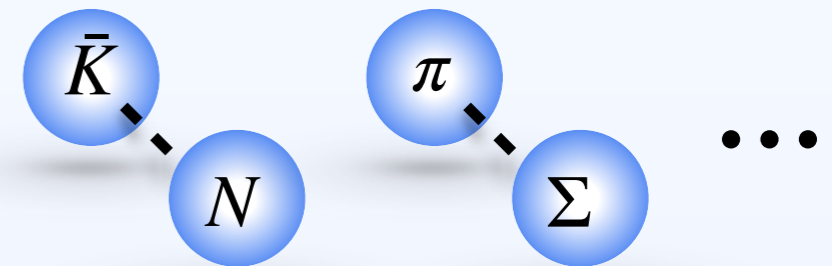
- $\Lambda(1405)$

- Large discrepancy from quark model

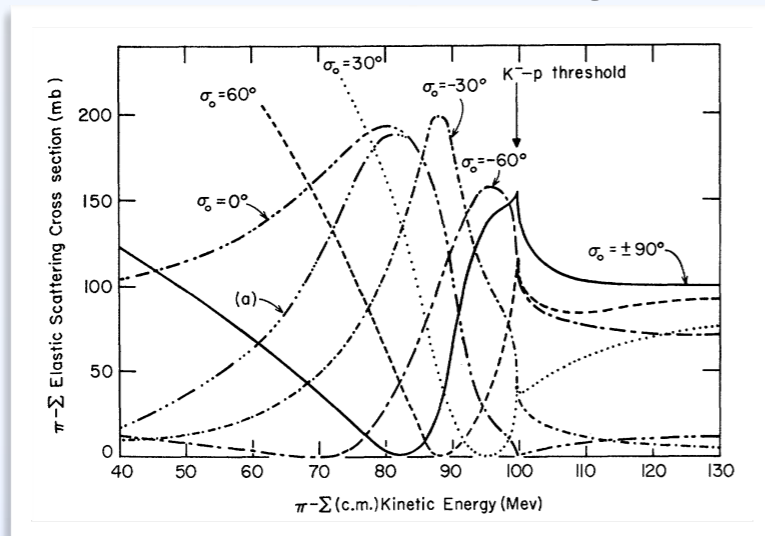
N. Isgur, and G. Karl, PRD18, 4187 (1978)



- Exotics candidate
- Quest for internal structure



- Predicted in the $\pi\Sigma$ scattering below $\bar{K}N$



→ Detailed analysis of $\pi\Sigma$ - $\bar{K}N$ interaction

R. H. Dalitz and S. F. Tuan, PRL2, 425 (1959)

R. H. Dalitz and S. F. Tuan, Annals Phys. 10, 307 (1960).

$\bar{K}N$ interaction and $\Lambda(1405)$

- $\bar{K}N$ interaction data and $\Lambda(1405)$ resonance

- Experimental data for $\pi\Sigma$ - $\bar{K}N$ system

- K^-p total cross sections
- Branching ratios
- K^-p scattering length from Kaonic nuclei
- $\pi\Sigma$ mass spectra
- K^-p femtoscopy

- Recent analysis with chiral SU(3) dynamics

- Two pole structure

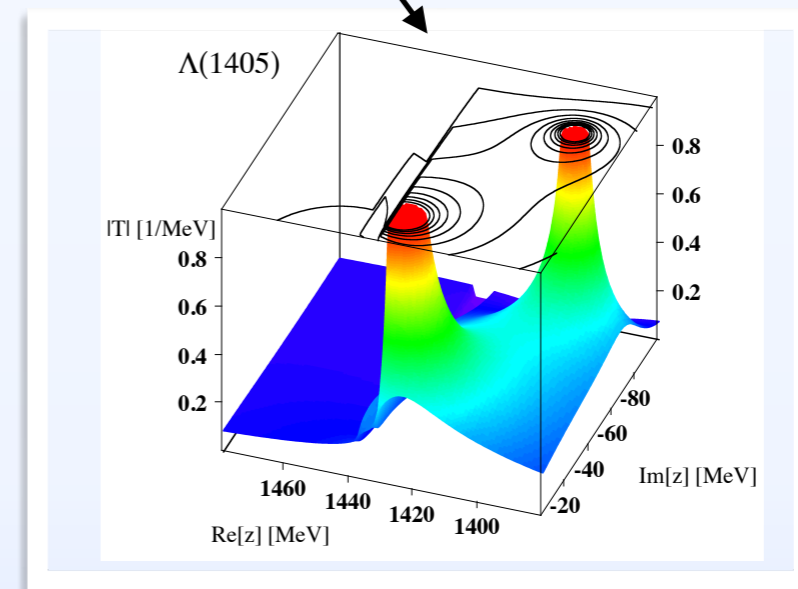
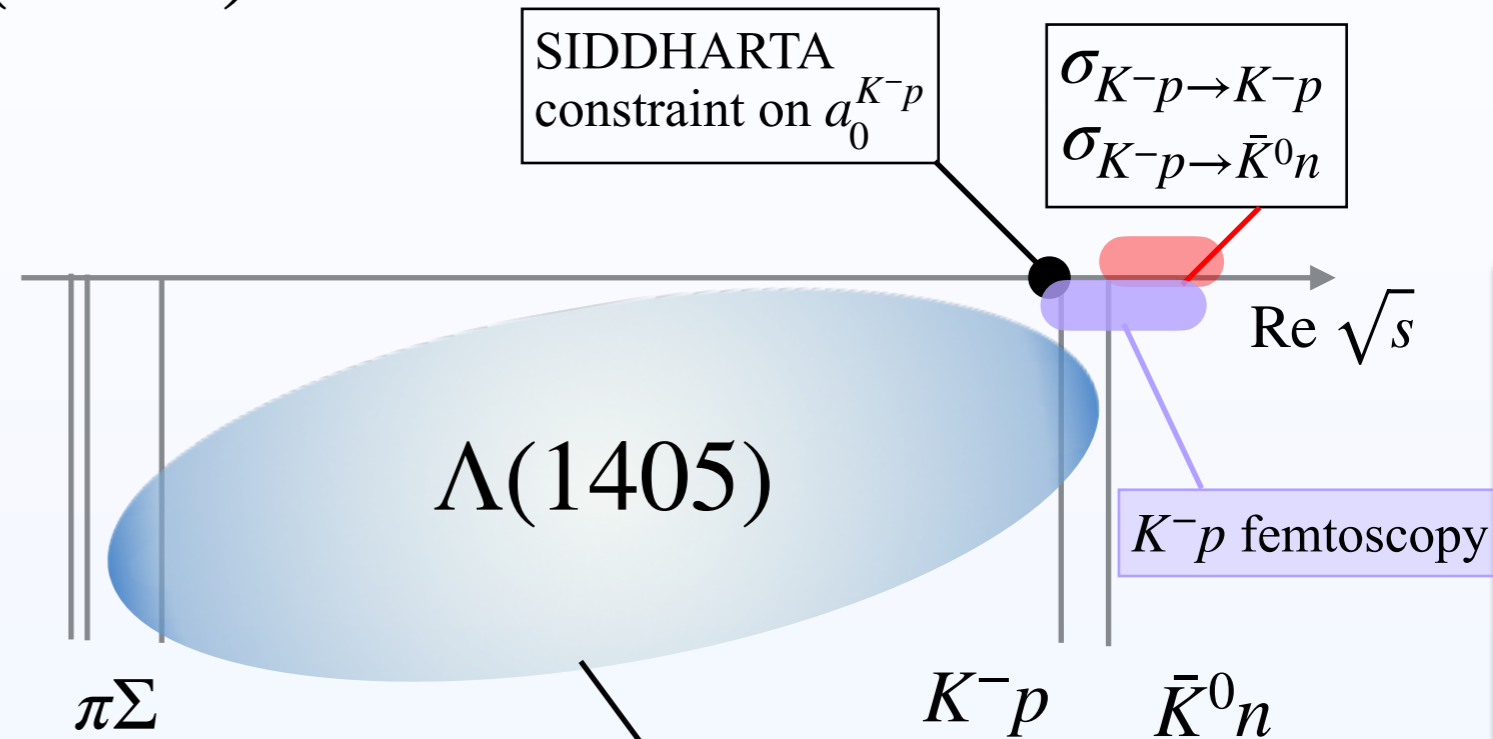
J. A. Oller and U. G. Meißner, PLB500, 263 (2001)

Peak in mass spectra

→ two poles in complex E -plane $\left(\begin{array}{l} \Lambda(1380) \\ \Lambda(1405) \end{array} \right)$



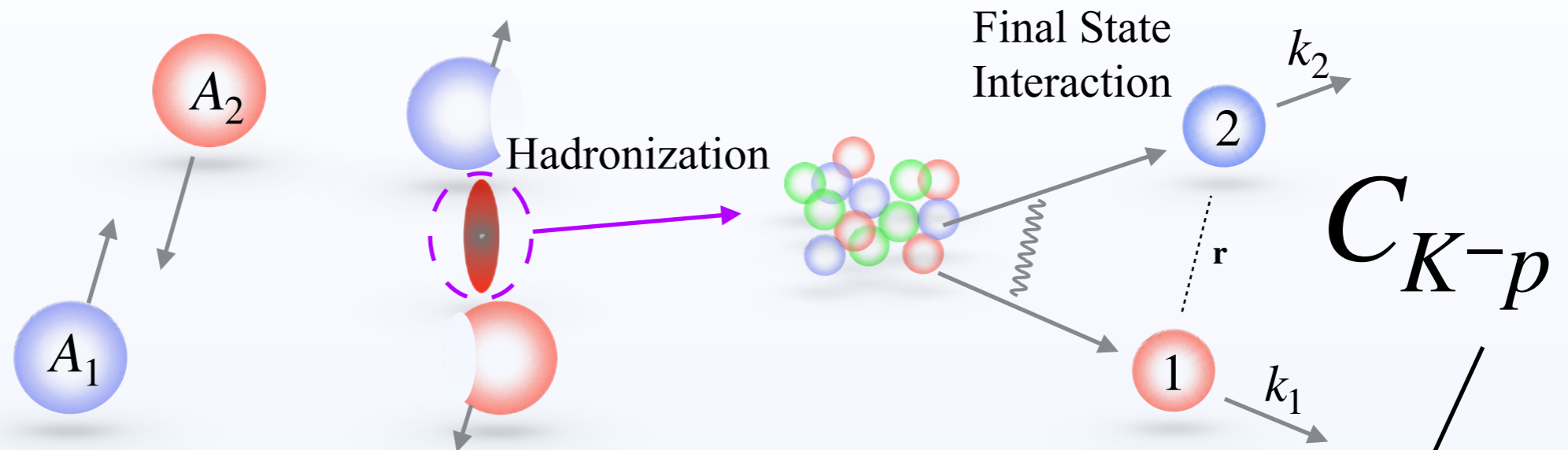
Detailed nature of two Λ resonances



T. Hyodo, D. Jido - PPNP 67 (2012) 55

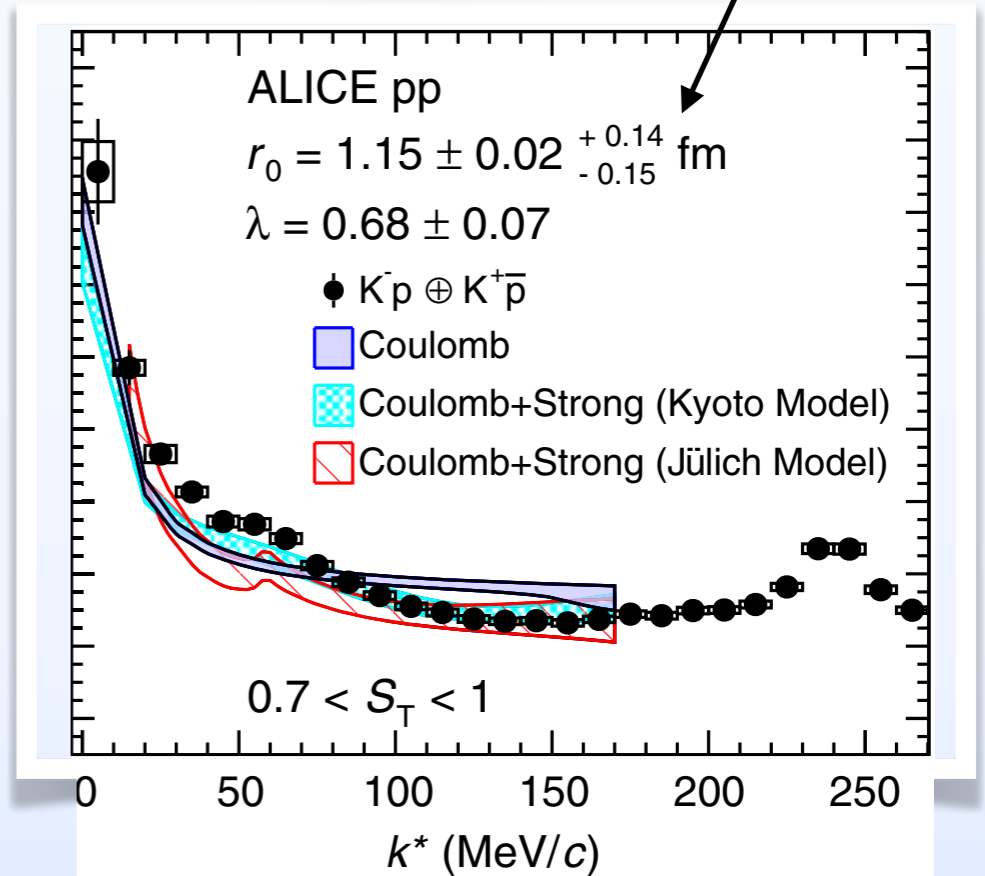
$\bar{K}N$ interaction and K^-p correlation

- K^-p correlation function in high-energy collisions



- Good resolution thanks to high statistics
- Sensitive to $k^* \lesssim 200$ MeV/c
- Detailed coupled-channel effect

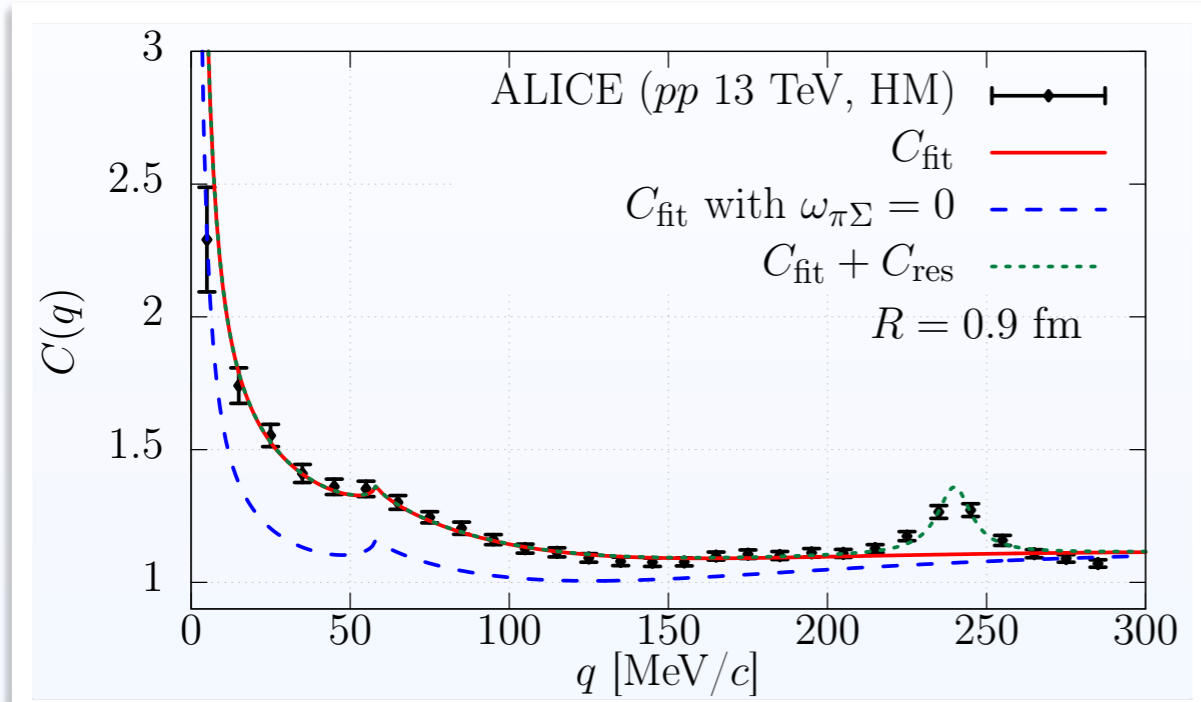
➔ Complementary data for $\bar{K}N$ interaction



$\bar{K}N$ interaction and K^-p correlation

- ALICE pp collision data

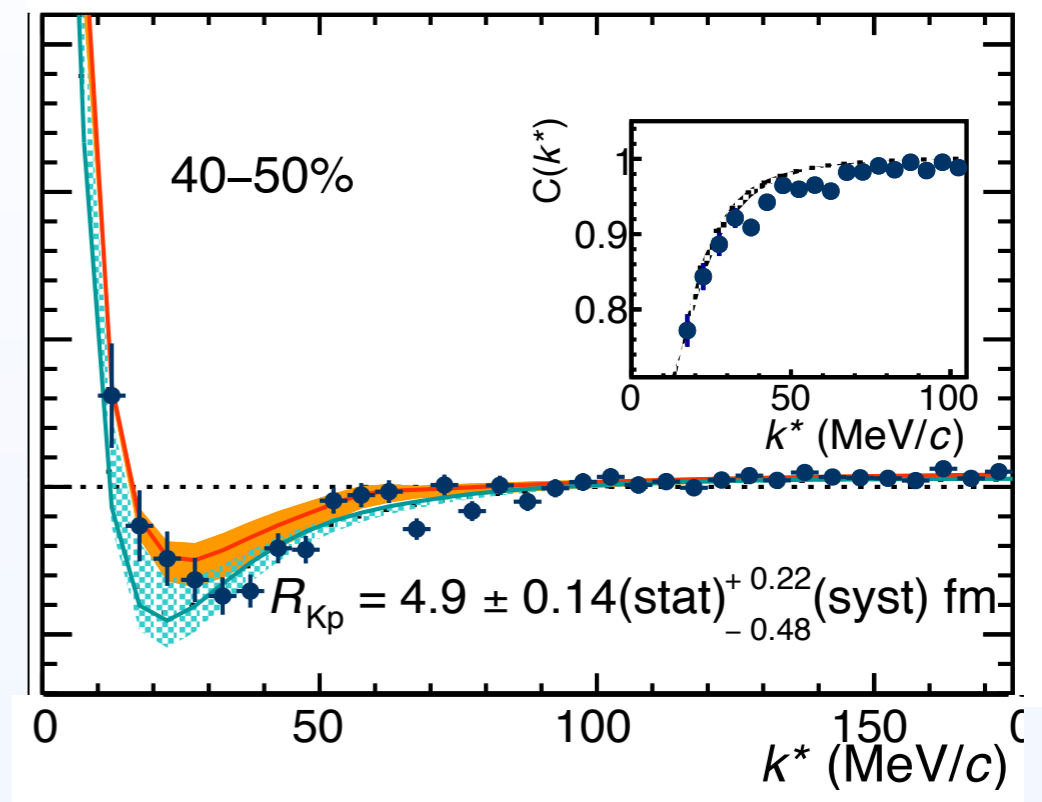
ALICE PRL 124, 092301 (2020)



YK et al, PRL 124 (2020) 13, 132501

- ALICE PbPb collision data

ALICE PLB 822 (2021) 136708



- Small source
- Clear \bar{K}^0n cusp structure
- Sizable contribution from coupled-channel source required to reproduce data

- Large source
- Weaker cusp
- Scattering length fitting
—> Consistent with Kaonic hydrogen

• Chiral SU(3) model reproduces both source data

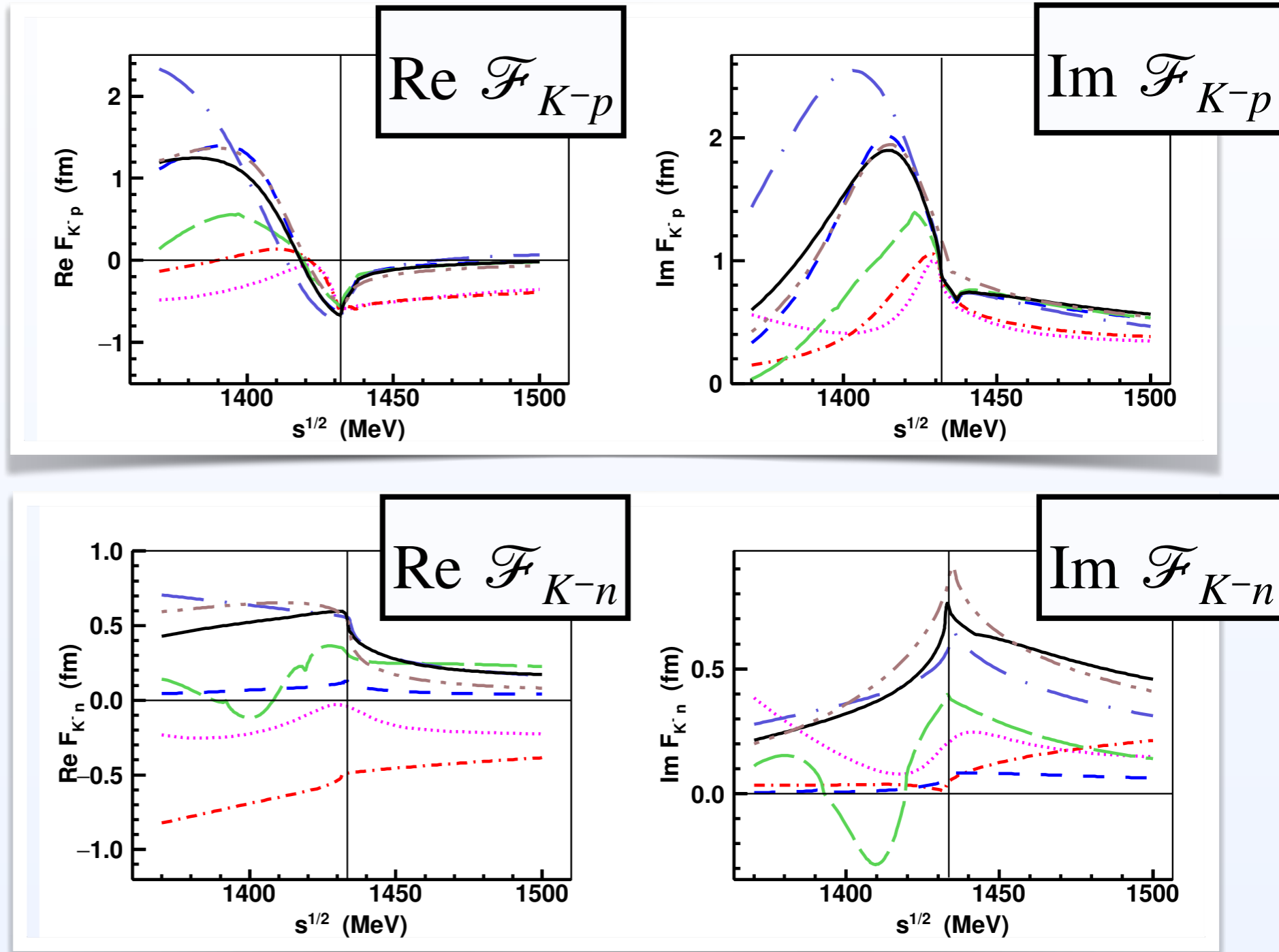
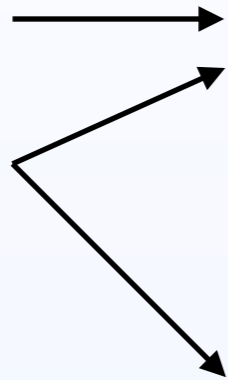
• Chiral SU(3) dynamics describes the low-energy $\bar{K}N$ scattering

$\bar{K}N-\pi\Sigma$ interaction and two pole structure

- $\bar{K}N$ interaction

$$\mathcal{F}_{\bar{K}N, I=0}$$

$$\mathcal{F}_{\bar{K}N, I=1}$$



B2, B4: Mai, Meißner, EPJA 51 (2015)

M1, MII: Guo, Oller, PRC 87 (2013)

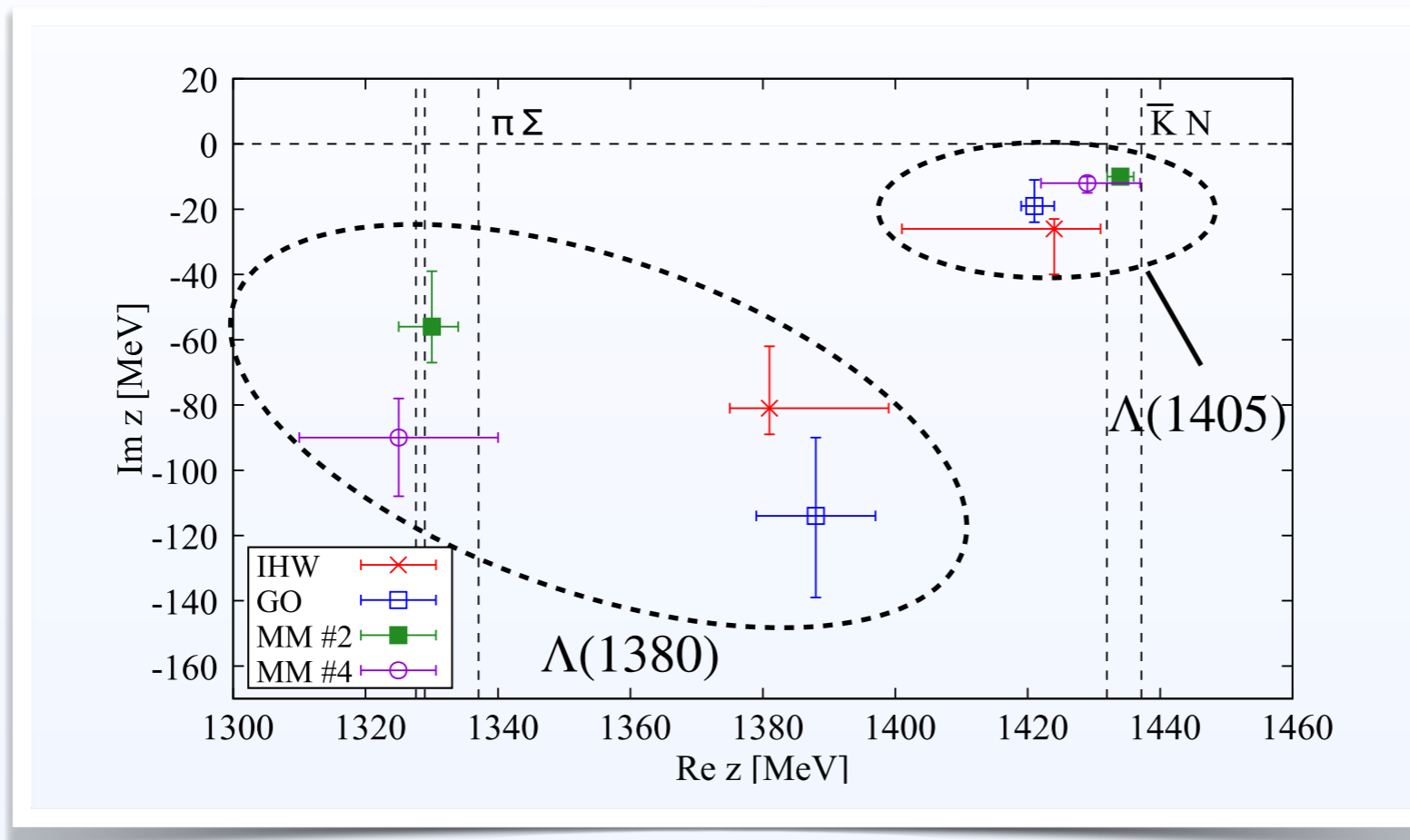
PNLO: Cieplý, Smejkal, NPA 881 (2012)

KMNLO: Ikeda, Hyodo Weise NPA 881 (2012)

Cieplý and Mai, EPJ Web Conf. 130, 02001 (2016)

$\bar{K}N-\pi\Sigma$ interaction and two pole structure

- Pole structure with chiral NLO models



- Ikeda, Hyodo Weise NPA 881 (2012)
- Mai, Meißner, EPJA 51 (2015)
- Guo, Oller, PRC 87 (2013)

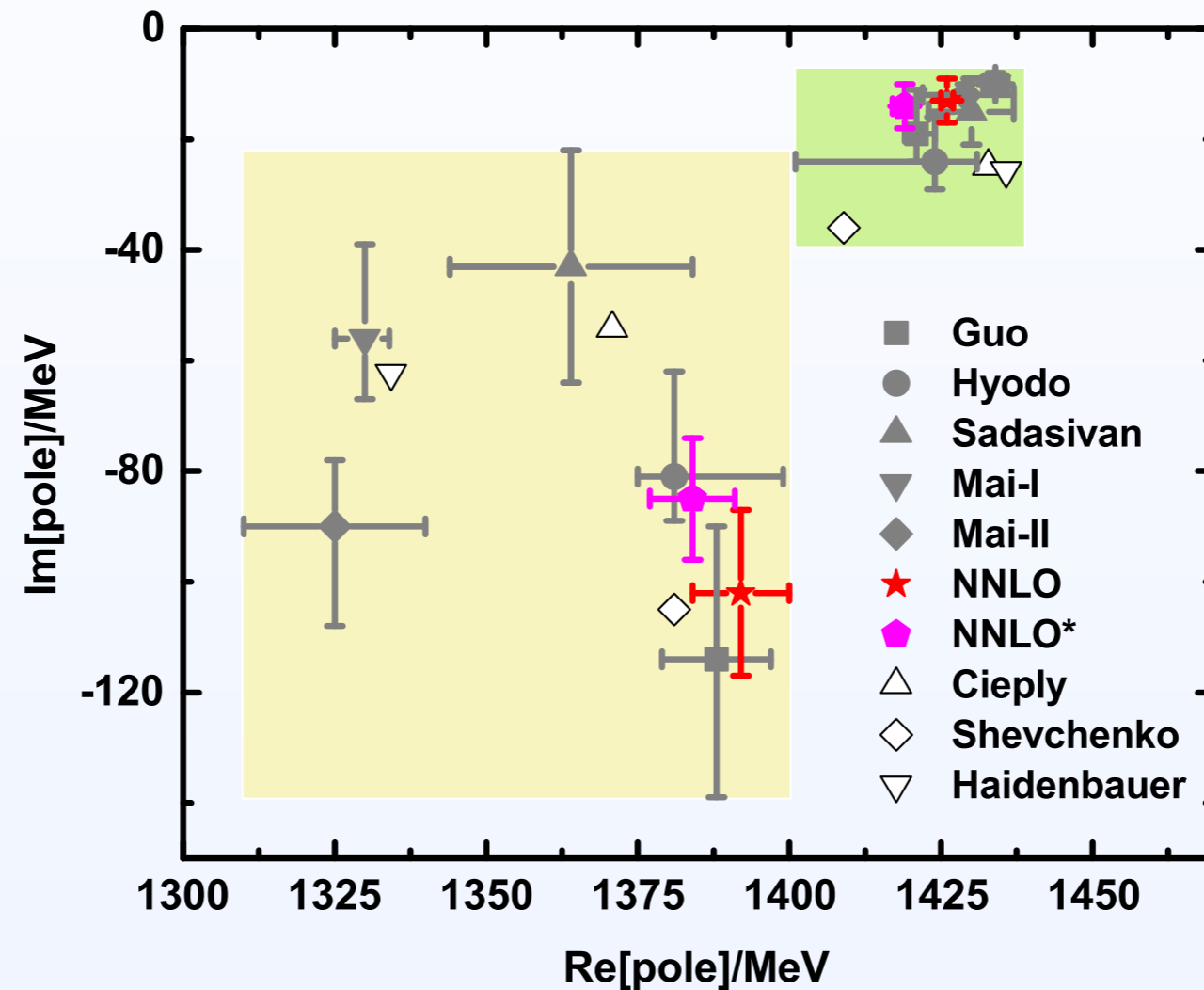
- $\Lambda(1405)$: Good consensus on $\bar{K}N$ threshold pole
 - Strong constraint from SIDDHARTA data
- $\Lambda(1380)$: lying close to $\pi\Sigma$ but the poles are scattered
 - Further constraint needed...
 - Are they stable with the higher-terms of chiral order?
 - What are their origin?



$\bar{K}N-\pi\Sigma$ interaction and two pole structure

- New chiral NNLO analysis

J.-X. Lu, L.-S. Geng, M. Doering, M. Mai, PRL 130 (7) (2023) 071902.



- Two pole structure is found in the latest NNLO analysis
- $\Lambda(1405)$ pole is consistent in NLO and NNLO models
- $\Lambda(1380)$ is also in the range of errors

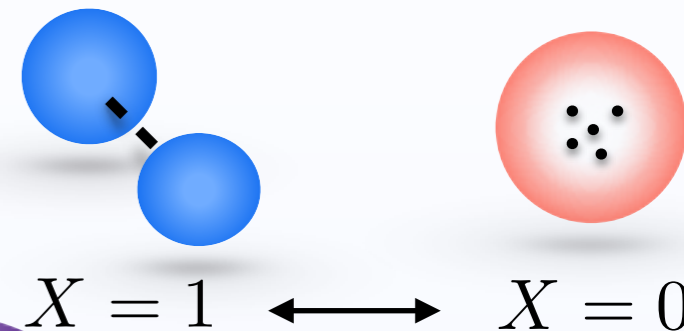
Origin of poles ~Compositeness~

- Compositeness for bound state

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}(R_{\text{typ}}/R) \right\}$$

S. Weinberg, Phys. Rev. 137, B672 (1965)

$$R = \frac{1}{\sqrt{2\mu B}}$$



Observables

a_0 : Scattering length
B : Binding energy

Indication of structure

X : Compositeness

- Extension to unstable states and application $\Lambda(1405)$

- Extended weak binding relation for unstable states

- Interpretation of complex value of X

$$a_0 = R \left[\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right) \right] \Rightarrow \tilde{X} \equiv \frac{1 - |Z| + |X|}{2}$$

- Applications to $\Lambda(1405)$ with chiral models

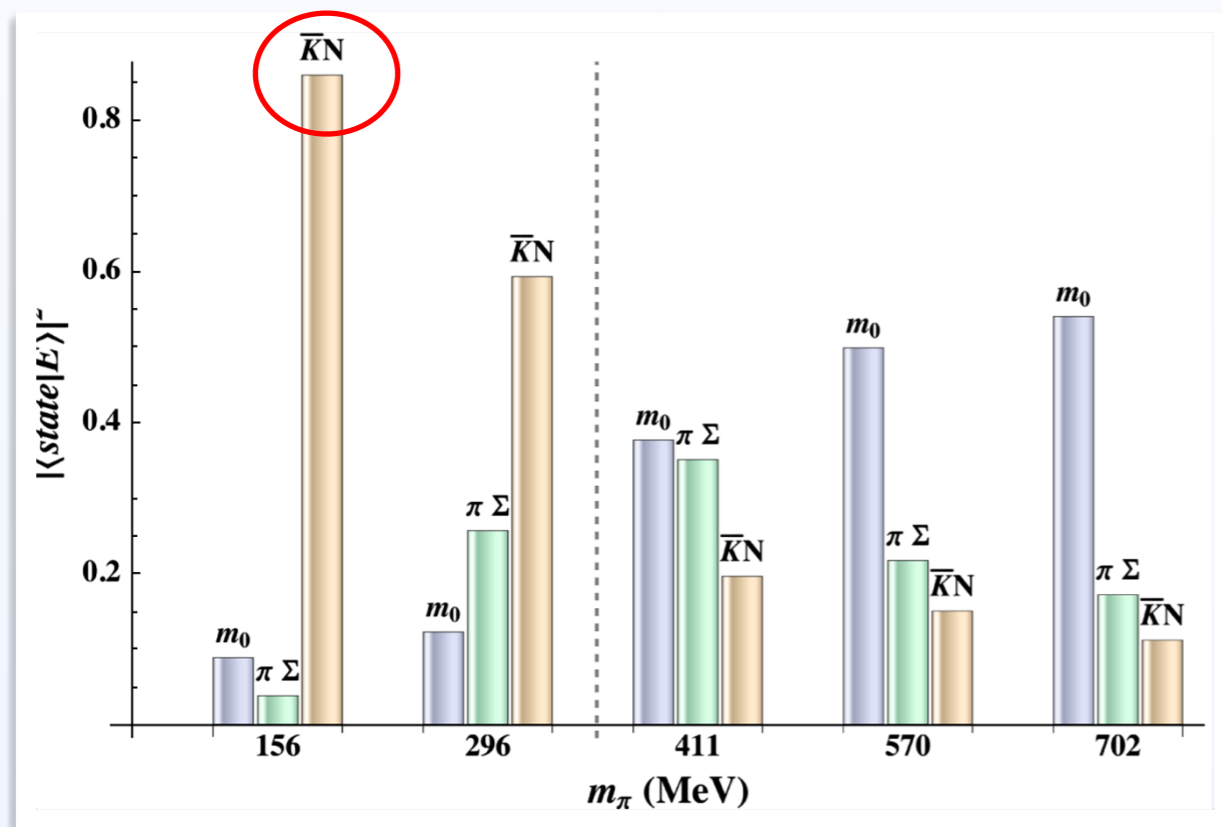
Models	B [MeV]	a_0 [fm]	X	\tilde{X}	U/2
IHW	-10-i26	1.39-i0.85	1.3+i0.1	1.0	0.3
MM(2013)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.0
GO	-13-i20	1.30-i0.85	0.9-i0.2	0.9	0.1
MM(2015)	2-i10	1.21-i1.47	0.6+i0.0	0.6	0.0
MM(2015)	-3-i12	1.52-i1.85	1.0+i0.5	0.8	0.3



Origin of poles ~lattice QCD analysis~

Component of $\Lambda(1405)$

J.M.M.Hall, et al., PRL 114 (13) (2015)



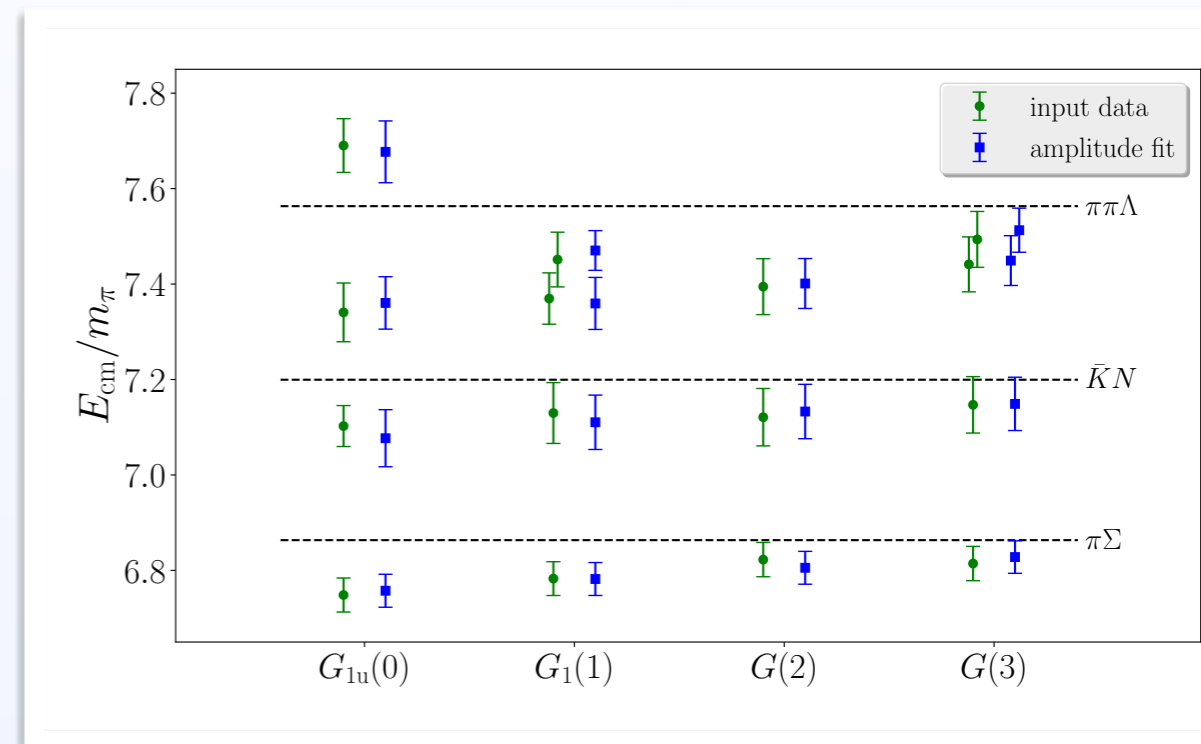
- Structure identification by strange magnetic form factor
- $\bar{K}N$ component dominant at the near physical point



Nature of Λ s can be accessed in lattices

Two pole structure

J.Bulava PRL. 132 (2024) 5, 051901



- Two pole structure is found at $m_\pi \approx 200$ MeV
 $\text{Re } E_1 = 1325 - 1380$ MeV
 $\text{Re } E_2 = 1421 - 1434$ MeV.

Origin of poles ~Representations~

- **Extrapolation of chiral model** D. Jido, J.A. Oller, E. Oset, A. Ramos, U.-G. Meißner, NPA 725 (2003)

- Weinberg-Tomozawa term and extrapolation

$$V_{ij}^{WT} = -\frac{C_{ij}^{WT}}{8F^2} \mathcal{N}_i \mathcal{N}_j (2\sqrt{s} - m_i - m_j)$$

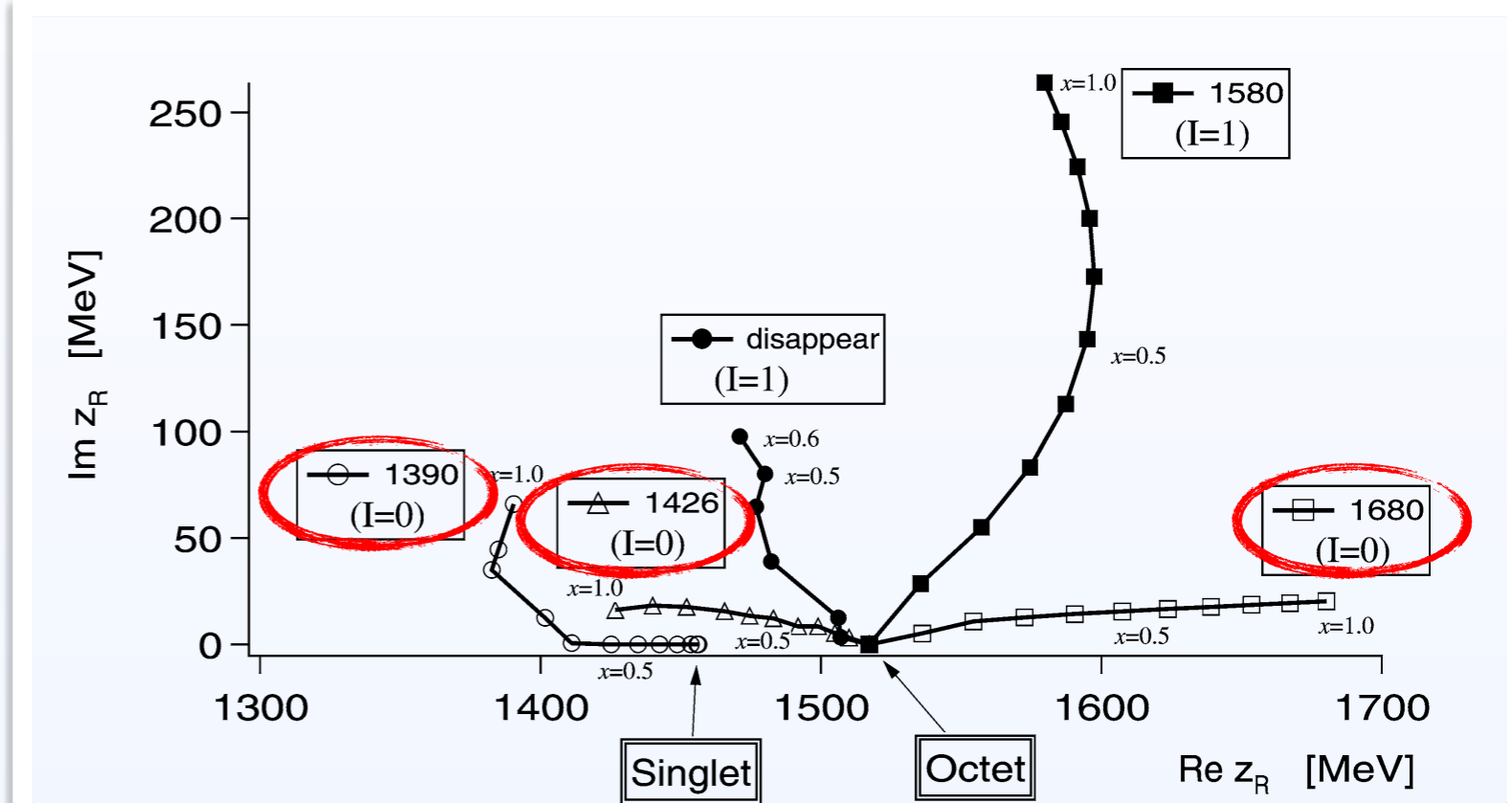
$$M_i(x) = M_0 + x(M_i - M_0), \dots$$

$x = 0$: physical point

$x = 1$: SU(3) limit

- Projection to multiplets decomposition of multiplets (1,8,8',27) in SU(3) limit

$$C_{ij}^{WT} \xrightarrow{U} C_{\alpha\beta} = \begin{pmatrix} \textcircled{6} & 0 & 0 & 0 \\ 0 & \textcircled{3} & 0 & 0 \\ 0 & 0 & \textcircled{3} & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$



Origin of Int.	SU(3) limit	Physical point
Singlet (1)	Deep bound state	$\Lambda(1380)$
Octet (8, 8')	Weakly bound states (degenerated)	$\Lambda(1405), \Lambda(1680)$

- Resonant poles are related to the interaction multiplets
- Trajectories depend on extrapolation detail \rightarrow More detailed extrapolation towards lattice?
- How trajectories change with NLO terms?

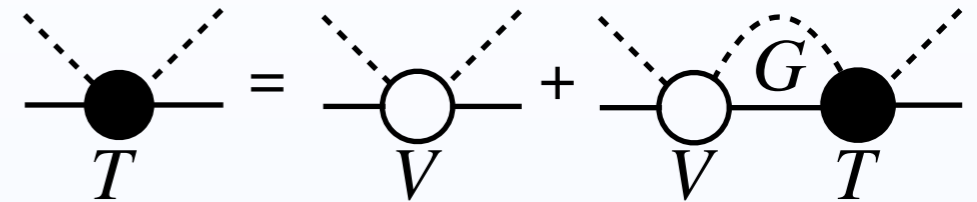
Motivation of this study

Model extrapolation to unphysical point

- Chiral unitary dynamics ~ Weinberg-Tomozawa term model ~

- Bethe-Salpeter equation

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj} \quad i, j, k \in \{\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi\}$$



- Interaction kernel: Weinberg-Tomozawa term

$$V_{ij}^{WT} = -\frac{C_{ij}}{8F^2} \mathcal{N}_i \mathcal{N}_j (2\sqrt{s} - m_i - m_j) \quad \mathcal{N}_i = \mathcal{N}_i(m_i, M_i)$$

- m_i, M_i, F ; Depends on the bare pion/Kaon masses ($M_{0\pi}, M_{0K}$)

→ Relations $(m_i, M_i, F) \longleftrightarrow (M_{0\pi}, M_{0K})$ required → (next page)

- Loop function

$$G_i(\sqrt{s}) = \frac{2m_i}{16\pi^2} \left\{ a_i(\mu) + \ln \frac{m_i^2}{\mu^2} \frac{M_i^2 - m_i^2 + s}{2s} \ln \frac{M_i^2}{m_i^2} + (\log \text{ terms}) \right\}$$

- Natural normalization scheme M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A 700 (2002) 193–308,
T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C 78 (2008) 025203

$$G(\sqrt{s} = m_B; a(\mu)) = 0 \iff T(\mu) = V(\mu) \quad \text{with } \mu = m_B$$

Given the hadron masses m_B , $a(\mu)$ can be determined.

Model extrapolation to unphysical point

- Quark mass dependence of NG bosons and octet baryons

Hadron masses are needed for calculations at unphysical point with $m_{0,\pi,K}$

← Relations from chiral perturbation models

- NG bosons J. Gasser, H. Leutwyler, NPB 250 (1985) 465–516

$$M_\pi^2 = M_{0\pi} \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16M_{0K}^2}{F_0^2} (2L_6^r - L_4^r) + \frac{8M_{0\pi}^2}{F_0^2} (2L_6^r + 2L_8^r - L_4^r - L_5^r) \right]$$

- $M_\pi \leftrightarrow M_{0,\pi,K}, F_0$

Decay const. in chiral limit

- LECs (L_i^r): determined with the lattice meson-meson scattering data

- Octet baryons M. Frink, U.-G. Meißner, JHEP 07 (2004) 028.

$$m_B = \underbrace{m_0}_{\text{chiral limit mass}} + m^{(2)}(\underbrace{b_0, b_D, b_F}_{\text{LECs}})$$

- Determination of LECs and bare parameters

$F_0, b_0, b_D, b_F \rightarrow$ fitted to the hadron masses at the physical point

$m_0 \rightarrow$ hadron mass at the unphysical point (lattice) [$M_\phi = 659.4$ MeV, $m_B = 1444.2$ MeV]

J.M.M.Hall, et al, PRL114 (13) (2015) 132002.

Chiral SU(3) amplitude and poles can be calculated

at any unphysical point (M_π, M_K) with systematic model

$$M_\pi, M_K \rightarrow (M_{0,\pi}, M_{0,K}) \rightarrow m_N, m_\Lambda, m_\Sigma, F \rightarrow \mathcal{F}^{\text{chiral}} \rightarrow E_{\Lambda^*}$$



Pole trajectory with WT model

- Extrapolation from physical point to SU(3) limit

- Poles at physical point

$$E_{\Lambda(1380)}^{\text{LO}} = 1403.3 - i 80.3 \text{ MeV},$$

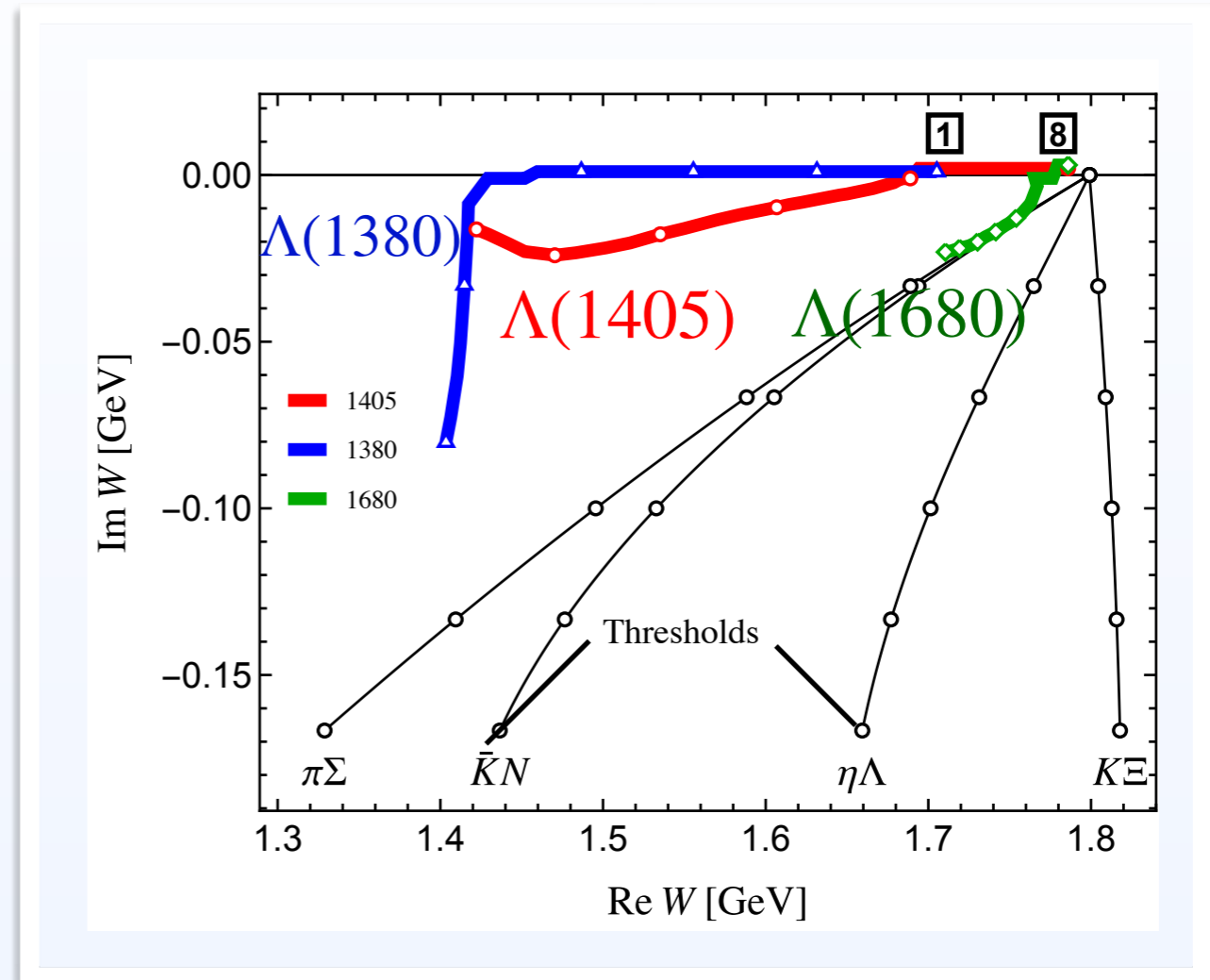
$$E_{\Lambda(1405)}^{\text{LO}} = 1422.7 - i 16.2 \text{ MeV},$$

$$E_{\Lambda(1680)}^{\text{LO}} = 1717.4 - i 22.9 \text{ MeV},$$

Consistent with the latest chiral models
 → Model describes the Λ^* s well

- Extrapolation to unphysical point

$$\underbrace{(m_{0,\pi}^{\text{phys}}, m_{0,K}^{\text{phys}})}_{\text{Physi. point}} \rightarrow \underbrace{(m_{0,K}^{\text{phys}}, m_{0,K}^{\text{phys}})}_{\text{SU(3) limit}}$$



F-K, Guo, YK, M Mai, U-G Meißner, PL B 846 (2023) 138264

- Pole trajectories

- $\Lambda(1380)$ connected to singlet pole
- $\Lambda(1405)/\Lambda(1680)$ connected to degenerated octet pole

Consistent with simple extrapolation
 D. Jido, et al, NPA 725 (2003)



- Pole origins are well related to representations of interactions with detailed extrapolation

Model parameters

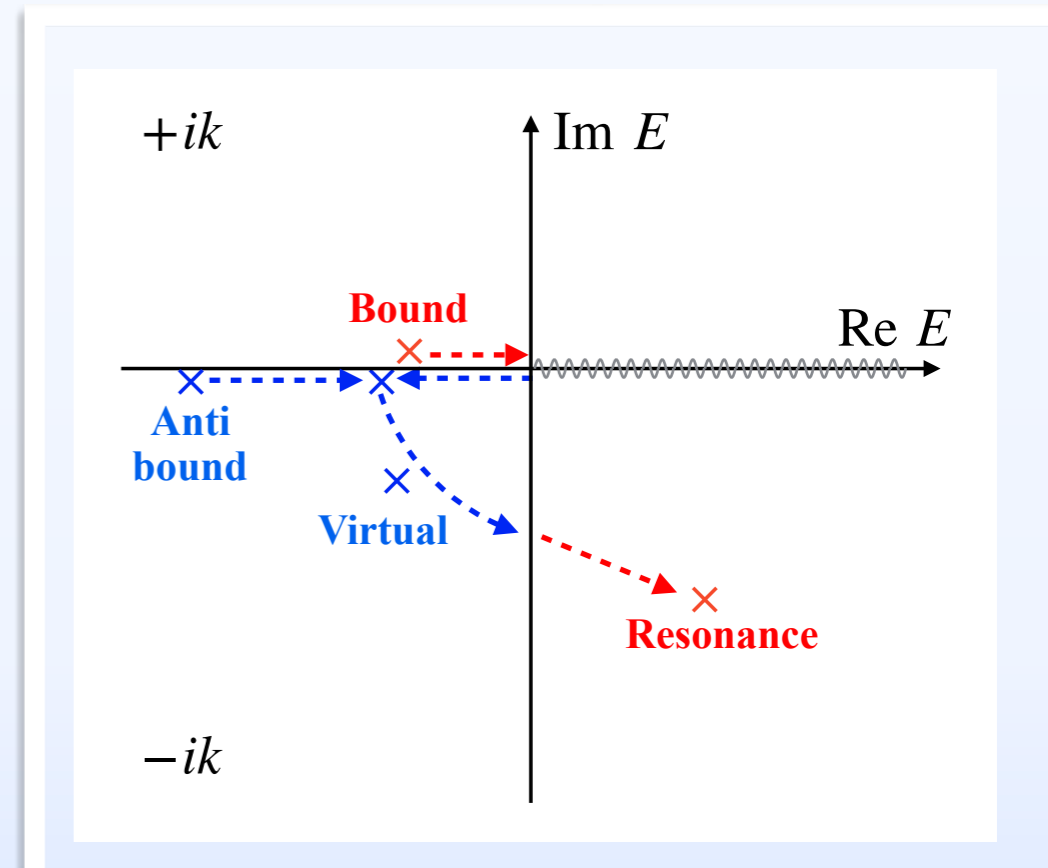
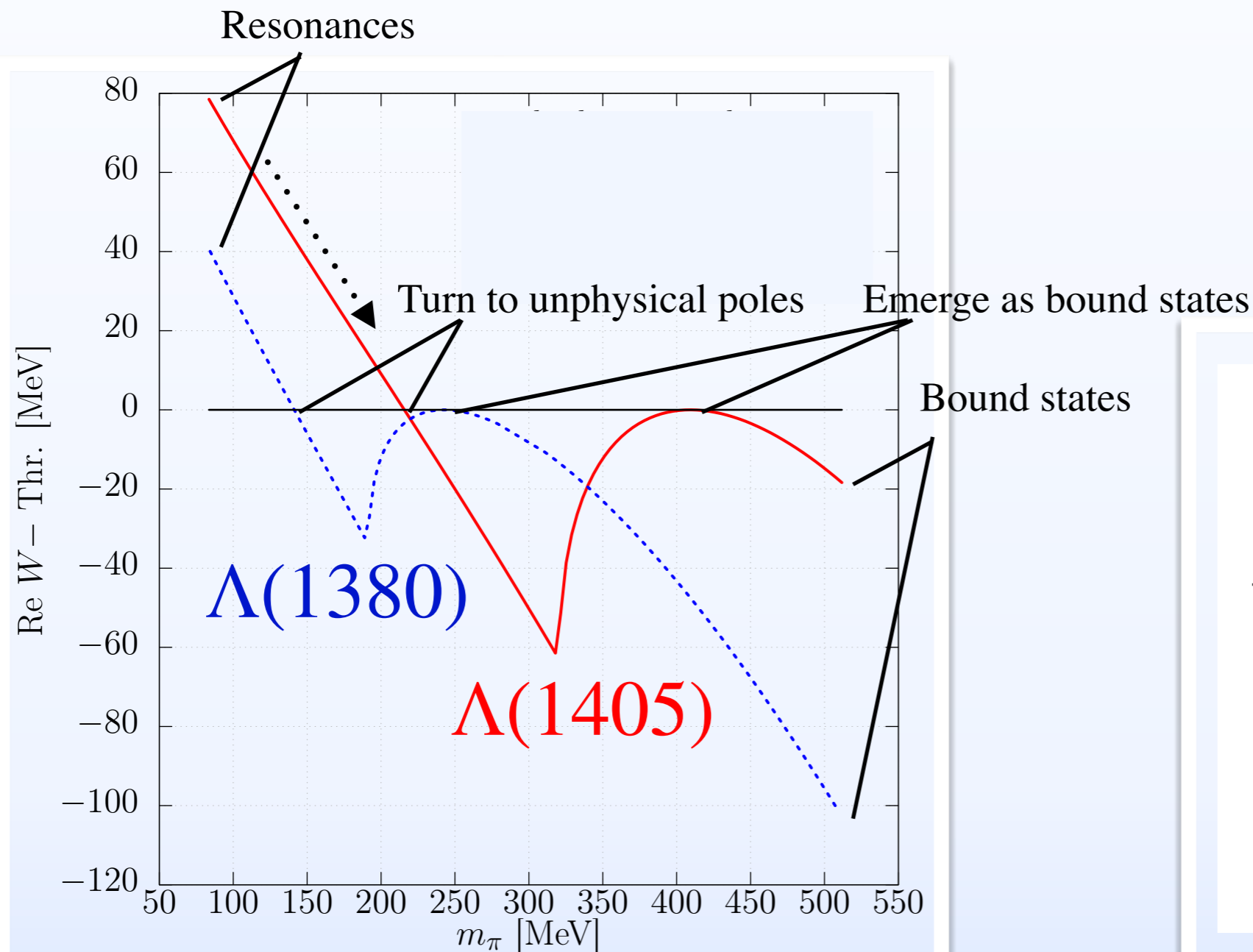
- Trajectory of SU(3) limit (Re E vs. pion mass)

With WT model

$(m_{0,\pi}, m_{0,K} = m_{0,\pi}) \rightarrow$ Vary $m_{0,\pi}$

In SU(3) limit,

\Rightarrow Two (physical, unphysical) Riemann sheets exit

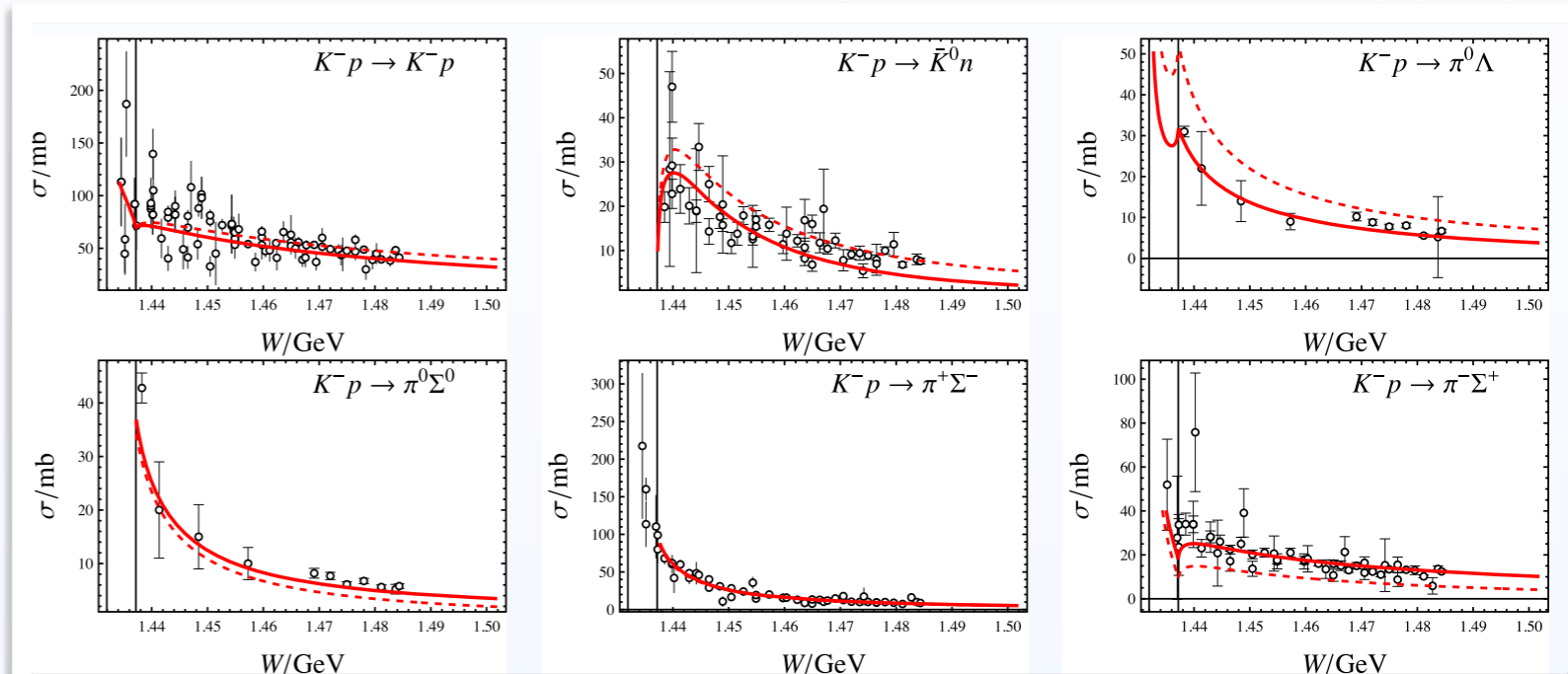


Addition of NLO terms

• NLO interaction

$$V_{ij}^{\text{NLO}}(\sqrt{s}) = \frac{\mathcal{N}_i \mathcal{N}_j}{F^2} \left(C_{ij}^{\text{NLO1}} - 2C_{ij}^{\text{NLO2}} \left(E_i E_j + \frac{q_i^2 q_j^2}{3\mathcal{N}_i \mathcal{N}_j} \right) \right)$$

- Born terms are neglected.
small contribution for s -wave
- Additional LECs (d_1, d_2, d_3, d_4)
→ fitted to the experimental data
 $\chi^2/\text{d.o.f} \approx 2.1$



- LECs and subt. const. are consistent with WT model

F-K, Guo, YK, M Mai, U-G Meißner, PL B 846 (2023) 138264

• Projection to representations

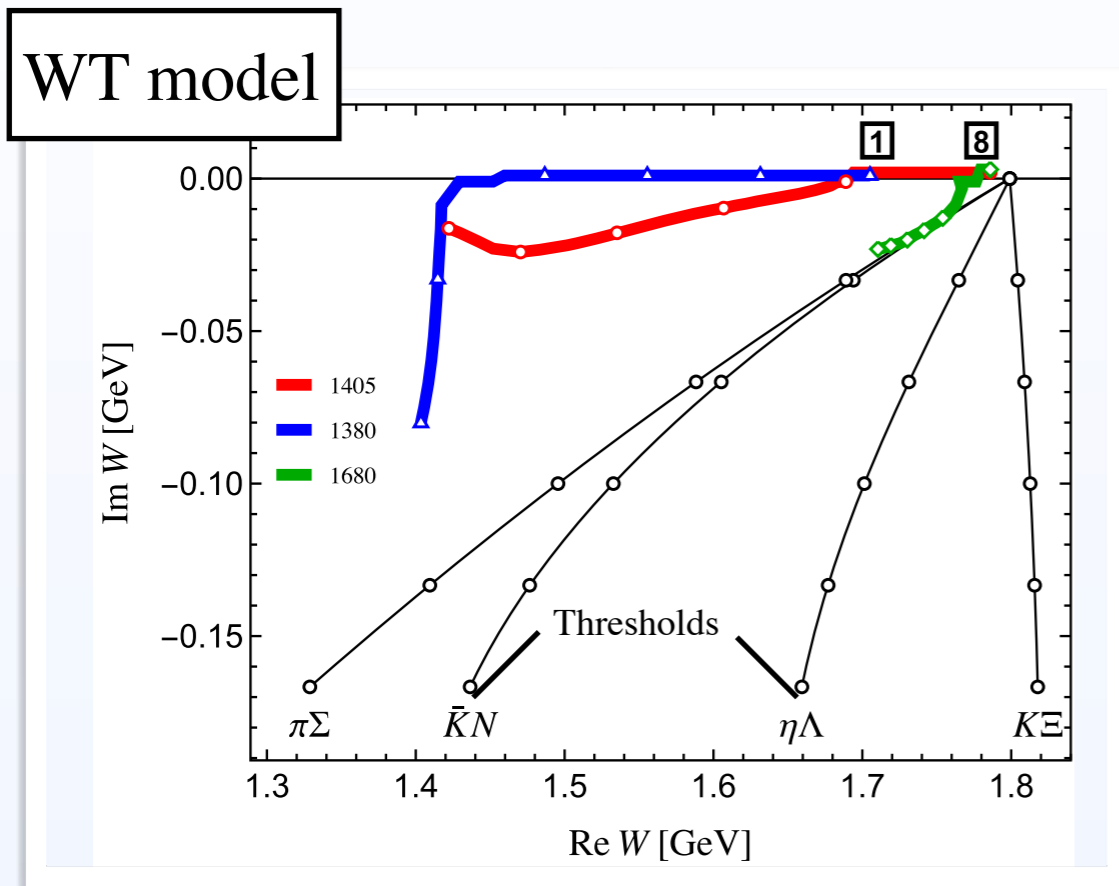
$$\begin{array}{l}
 C_{ij}^{\text{NLO1}} \\
 C_{ij}^{\text{NLO2}}
 \end{array}
 \xrightarrow{U}
 \begin{pmatrix}
 \frac{4}{3}(3b_0 + 7b_D)m_q & 0 & 0 & 0 \\
 0 & \frac{2}{3}(6b_0 + b_D)m_q & -\sqrt{20}b_F m_q & 0 \\
 0 & -\sqrt{20}b_F m_q & 2(2b_0 + 3b_D)m_q & 0 \\
 0 & 0 & 0 & 4(b_0 + b_D)m_q \\
 \\
 -3d_2 + \frac{9}{2}d_3 + d_4 & 0 & 0 & 0 \\
 0 & \frac{1}{2}(-3d_2 + d_3 + 2d_4) & -\frac{\sqrt{5}}{2}d_1 & 0 \\
 0 & -\frac{\sqrt{5}}{2}d_1 & \frac{1}{2}(9d_2 - d_3 + 2d_4) & 0 \\
 0 & 0 & 0 & \frac{1}{2}(2d_2 + d_3 + 2d_4)
 \end{pmatrix}.$$

- Two octet representations ($8, 8'$) are mixed.

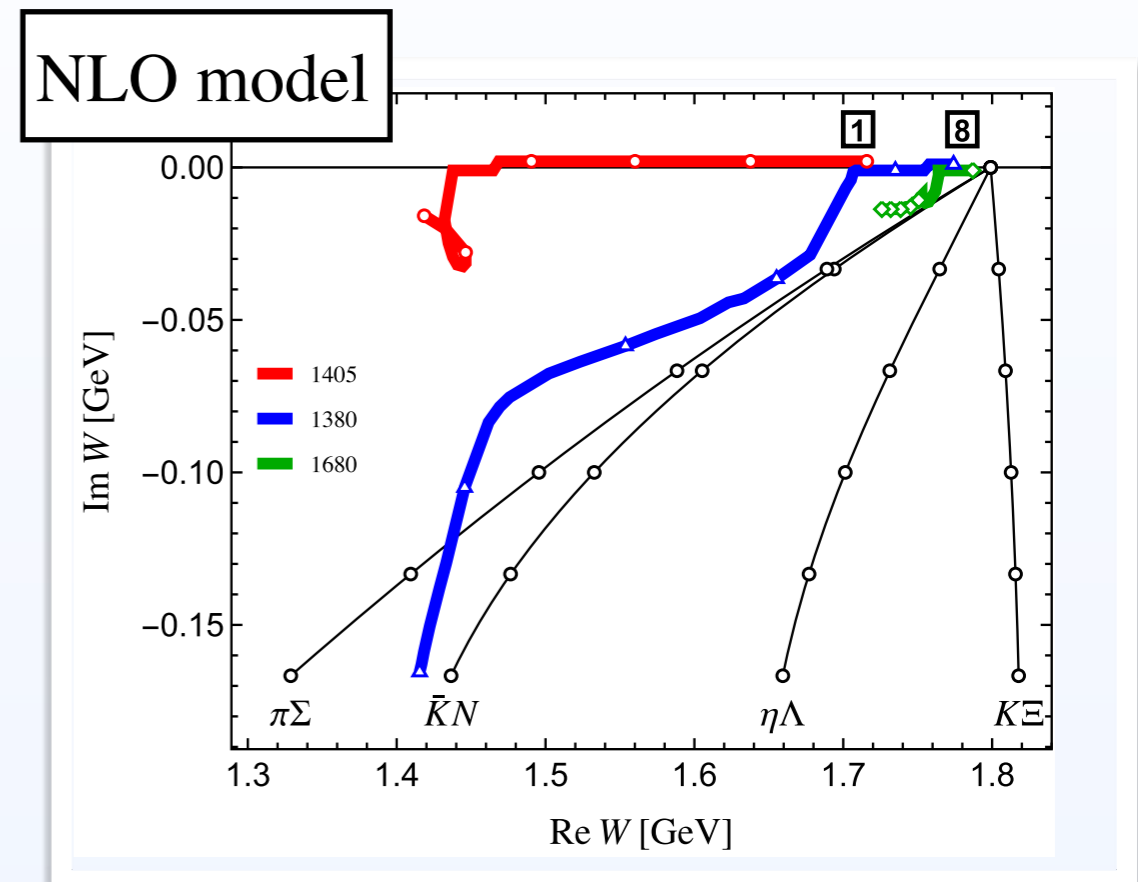
→ Degeneration of $8, 8'$ poles?

Addition of NLO terms

- Pole trajectories with NLO model



Addition of
NLO terms

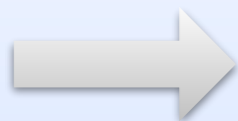


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- Trajectories with NLO terms

- Two octet poles are not degenerated and remained as two separated poles
- $\Lambda(1405)$ is now connected to singlet and $\Lambda(1380)$ is connected to octet

Model	E_1	E_8	$E_{8'}$
WT	1704	1788	1788
NLO	1716	1772	1787



- Connection to SU(3) limit poles may be exchanged by the detailed interaction
- Mixing effect of 8 and 8' interaction is moderate

Summary

- For understanding of nature of $\Lambda(1380)/\Lambda(1405)$, detailed analysis of $\pi\Sigma-\bar{K}N$ interaction is required.
- Extrapolation to the unphysical point was performed with the quark mass dependence relation and the natural renormalization scheme.
- With WT model, $\Lambda(1380)/\Lambda(1405)$ are connected to singlet/octet.
- With NLO model, $\Lambda(1380)/\Lambda(1405)$ are connected to octet/singlet and two octets state in SU(3) are separated due to breaking term in V_{NLO}

Thank you for your attention!

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Thank you!

Addition of NLO terms

- Pole trajectory change by NLO terms

