Tops and QCD

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Weizmann Institute, May 29, 2011

Outline

- Factorization and NLO calculations
- Beyond NLO
- Applications

Factorization for inclusive production

Factorization for $h_1 h_2 \to t\bar{t}X$:

$$d\sigma^{t\bar{t}X}_{h_1,h_2} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F) d\hat{\sigma}_{ij}\left(\hat{s}, m_t, \dots, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

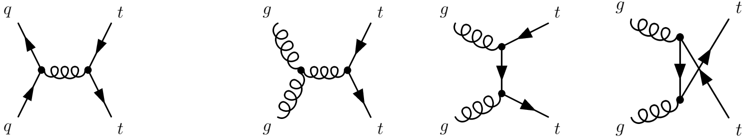
$$s = \left(p_{h_1} + p_{h_2}\right)^2 \,,\, \hat{s} = x_1 x_2 s$$

Strategy:

- take PDFs from data (PDF set collaborations)
- calculate partonic cross sections $d\hat{\sigma}_{ij}$ in QCD (Feynman diagrams)

$$d\hat{\sigma}_{ij} = \alpha_s^2 \hat{\sigma}_{ij}^{(0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(1)} + \dots$$

Born level:



- $q\bar{q}$ dominant at Tevatron ($\sim$ 90% of cross section)
- gg dominant at LHC ($\sim$ 75% of cross section at 7 TeV)

Higher-order corrections:

- virtual corrections and real emission
- $(qg, \bar{q}g) \to t\bar{t}X$ (numerically small)

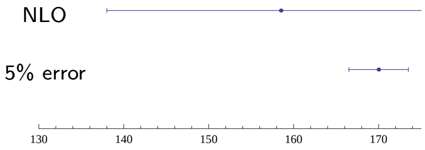
NLO calculations

NLO calculations of total and differential cross sections known for 20 years
Nason, Dawson, Ellis ('88-'90); Beenakker, Kujif, van Neerven, Smith, Schuler ('89-'91); Mangano, Nason, Ridolfi ('92), Czakon and Mitov ('08)

- implemented in numerical parton MC programs; MCFM, MadGraph
- or including parton showers; MC@NLO, etc.

NLO calculations have roughly 15% factorization and renormalization scale uncertainties, to make full use of LHC data should go beyond them.

- For illustration, compare $\sigma(\sqrt{s}= 7$ TeV) at NLO with 5% exp. error



Beyond NLO

Two routes:

1) NNLO in fixed order

2) soft gluon resummation

- extended from NLL to NNLL in last few years
- a priori useful for distributions in certain corners of phase space (e.g. large pair invariant mass)
- when used for total cross section, introduces uncertainties beyond just scale variation

NNLO in fixed order

$$\sigma_{t\bar{t}+X}^{\rm NNLO} = \sigma^{\rm VV} + \sigma^{\rm RV} + \sigma^{\rm RR}$$

Many partial results in fixed order

- $\sigma^{\rm VV}$: Czakon, Mitov, Moch; Bonciani, Ferroglia, Gehrmann; Neubert, BP, Yang; Kniehl, Korner, Merebashvili, Rogal ...
- $\sigma^{\rm RV}$ (1-loop $t\bar{t}+j$): Dittmaier, Uwer, Weinzierl '07; Bevilacqua,Czakon, Papadolpoulos, Worek '10; Melnikov, Schulze '10
- $\sigma^{\rm RR}$: Czakon '11; Abelof, Gehrmann-De Ridder '11

Need to combine the different pieces, now looks feasible

- Time frame?
- Differential cross sections?

Example: differential cross section at large pair invariant mass

$$\frac{d\sigma}{dM_{t\bar{t}}} = \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z}\, ff_{ij}(\tau/z,\mu_f)\, \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}}$$

$$\sim \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z}\, ff_{ij}(\tau/z,\mu_f) \left[\delta(1-z)C_0^{ij} + \sum_m \sum_{n\leq 2m-1} \alpha_s^m d_{mn}^{ij} \left[\frac{\ln^n(1-z)}{1-z}\right]_+ + \dots\right]$$

- when $\tau = M_{t\bar{t}}^2/s \to 1$, logs give large corrections, fixed order expansion fails
- large logs in $z = M_{t\bar{t}}^2/\hat{s} \to 1$ limit related to soft gluon emission and can be resummed using effective theory and RG techniques

$ff_{ij}(y,\mu_f) = \int_y^1 \frac{dx}{x} f_{i/h_1}(x,\mu_f)\, f_{j/h_2}(y/x,\mu_f)$ are parton luminosities

Factorization and resummation in the soft limit

Momentum scales at $z = M_{t\bar{t}}^2/\hat{s} \to 1$:

$$\hat{s}, M_{t\bar{t}}^2, m_t^2 \gg E_s^2 \sim \hat{s}(1-z)^2 \gg \Lambda_{\rm QCD}^2$$

Factorization:

$$\frac{d\hat{\sigma}}{dM_{t\bar{t}}\, d\cos\theta} = \mathrm{Tr}\left[\mathbf{H}(M_{t\bar{t}}, m_t, \cos\theta, \mu_f)\, \mathbf{S}\left(\frac{2E_s(z)}{\mu_f}, v_i \cdot v_j, \ldots, \alpha_s(\mu_f)\right)\right] + \mathcal{O}(1-z)$$

Kidonakis, Sterman ('97)

- $\mathbf{H}_{ij}$ are hard matrices (related to virtual corrections)
- $\mathbf{S}_{ij}$ are soft matrices. They depend on z through $\delta(1-z)$ or

$$\alpha_s^n \left[\frac{\ln^m(2E_s(z)/\mu)}{1-z}\right]_+ ; \quad m = 0, \cdots, 2n-1$$

- use RG equations to sum logs, need soft anomalous dimension $\mathbf{\Gamma}$

Resummed partonic cross section

All-orders resummation formula ($\lambda \sim 1 - z$):

$$d\hat{\sigma}(\lambda, \mu_f) \propto \exp\left[4a_{\gamma^\phi}(\mu_s, \mu_f)\right] \mathrm{Tr}\Big[\mathbf{U}_\lambda(\mu_h, \mu_s)\, \mathbf{H}(\mu_h)$$
$$\mathbf{U}_\lambda^\dagger(\mu_h, \mu_s)\, \tilde{\mathbf{s}}_\lambda(\partial_\eta, \alpha_s(\mu_s))\Big] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \left[\frac{1}{\lambda}\left(\frac{2E_s(\lambda)}{\mu_s}\right)^{2\eta}\right]_+$$
$$+\mathcal{O}(\lambda)$$

Perturbative ingredients for NNLL

γ_{cusp}	$\mathbf{\Gamma}$	$\mathbf{H}$, $\tilde{\mathbf{s}}_\lambda$
3-loop	2-loop	1-loop

Ahrens, Ferroglia, Neubert, BP, Yang ('09, '10)

- when matched with fixed order to NLO+NNLL accuracy, state of the art for invariant mass distribution at large $M_{t\bar{t}}$

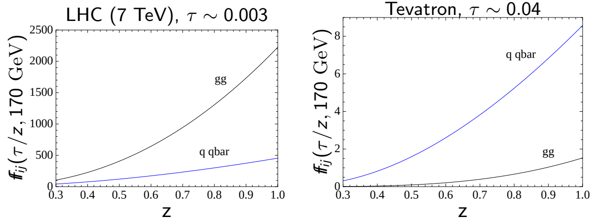
Important question: is this also useful if $M_{t\bar{t}}$ is not large?

Dynamical threshold enhancement

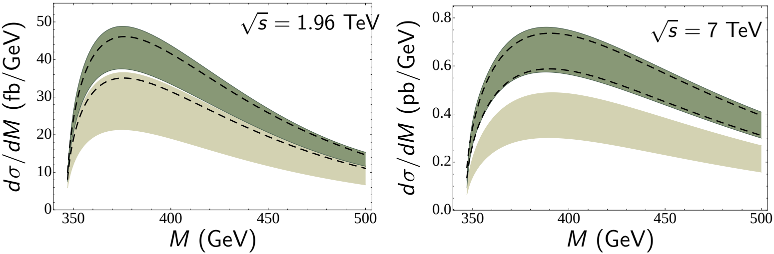
$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_{i,j} \int_\tau^1 \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f) \left[\delta(1-z) C_0^{ij} + \sum_m \sum_{n \le 2m-1} \alpha_s^m d_{mn}^{ij} \left[\frac{\ln^n(1-z)}{1-z} \right]_+ + \dots \right]$$

Leading terms in $z \to 1$ limit dominant if:

- $\tau = M_{t\bar{t}}^2/s \to 1$ (high invariant mass)
- $ff_{ij}(\tau/z, \mu)$ largest as $z \to 1$, even if τ not close to 1 ("dynamical threshold enhancement")



Dominance of soft gluon corrections at NLO



- green band = exact fixed order at NLO ($\mu_f = 200$, 800 GeV)
- dashed lines = leading terms for $z \to 1$ at NLO ($\mu_f = 200$, 800 GeV)

Soft gluon corrections dominate cross section even at low $M_{t\bar{t}}$

Resummation in three soft limits

$$p_i(p_1) + p_j(p_2) \to t(p_3) + \bar{t}(p_4) + X(k) \qquad (p_i, p_j \in \{q, g\})$$

Name	Observable	Soft limit
production threshold	$\hat{\sigma}$	$\beta = \sqrt{1 - 4m_t^2/\hat{s}} \to 0$
single-particle-inclusive (1PI)	$d\hat{\sigma}/dp_T dy$	$s_4 = (p_4 + k)^2 - m_t^2 \to 0$
pair-invariant-mass (PIM)	$d\hat{\sigma}/dM_{t\bar{t}} d\theta$	$(1 - z) = 1 - M_{t\bar{t}}^2/\hat{s} \to 0$

$$d\hat{\sigma} = d\hat{\sigma}^{\text{leading, soft}} + \hat{R}$$

- can derive NLO+NNLL resummation for $d\hat{\sigma}^{\text{leading, soft}}$ in each case
- power corrections contained in $\hat{R}$ are different in each soft limit

Phase space in PIM and production threshold limits

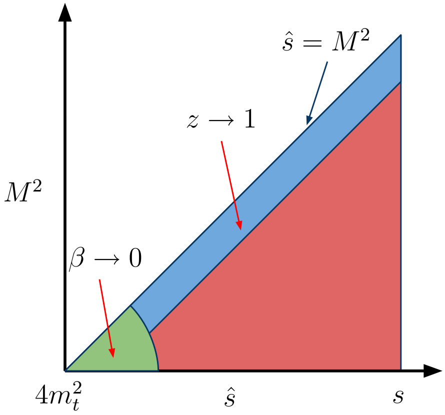


Figure: Phase space in the $(\hat{s}, M^2)$ plane. In the blue region along the diagonal threshold singularities arise, and the cross sections receives its main contributions. In the small green region near the origin Coulomb singularities appear and the small-β expansion applies.

NNLL CALCULATIONS

1) production threshold ($\beta \to 0$, total cross section only)

- Langenfeld, Moch, Uwer '08, '09; Czakon, Mitov, Sterman '09
- note: method for simultaneous resummation of soft gluon and Coulomb terms ($\sim 1/\beta$) in developed in Beneke, Falgari, Schwinn '09

2) 1PI kinematics (p_T and y distributions)

- Kidonakis '10
- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '11

3) PIM kinematics ($M_{t\bar{t}}$ distributions)

- in SCET Ahrens, Ferroglia, Neubert, BP, Yang '10

Will compare results for total cross section using "approximate NNLO formulas"

Approximate NNLO formulas

$$\hat{\sigma}_{ij}^{\mathrm{NNLO\,approx.}}(\beta,\mu_f) = \alpha_s^2\hat{\sigma}_{ij}^{(0)} + \alpha_s^3\hat{\sigma}_{ij}^{(1)} + \alpha_s^4\hat{\sigma}_{ij}^{(2),\,\mathrm{approx}}$$

$$\hat{\sigma}_{ij=q\bar{q},\,gg}^{(2)\,\mathrm{approx\,p.t.}} = \sum_{m=1}^{4} d_{2m}^{\mathrm{p.t.}} \ln^m\beta + \frac{1}{\beta}\left(c_{22}^{\mathrm{p.t.}}\ln^2\beta + c_{11}^{\mathrm{p.t.}}\ln\beta + c_{10}^{\mathrm{p.t.}}\right) + \frac{c_{20}^{\mathrm{p.t.}}}{\beta^2} + \hat{R}'^{\,\mathrm{p.t.}}(\beta)$$

$$\hat{\sigma}_{ij=q\bar{q},gg}^{(2)\,\mathrm{approx.\,1PI}} = \int dp_T dy \left\{\sum_{m=0}^{3} d_m^{\mathrm{1PI}}\left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4}\right]_+ + c^{\mathrm{1PI}}\delta(s_4) + \hat{R}'^{\,\mathrm{1PI}}(s_4)\right\}$$

$$\hat{\sigma}_{ij=q\bar{q},gg}^{(2)\,\mathrm{approx.\,PIM}} = \int dM_{t\bar{t}}d\theta \left\{\sum_{m=0}^{3} d_m^{\mathrm{PIM}}\left[\frac{\ln^m(2E_s(z)/\mu_f)}{1-z}\right]_+ + c^{\mathrm{PIM}}\delta(1-z) + \hat{R}'^{\,\mathrm{PIM}}(z)\right\}$$

- pieces in blue determined exactly from NNLL (or Coulomb res. for p.t.)
- in 1PI and PIM, the logs depend on soft energy E_s. Two schemes

Ahrens et al '10,'11	$\mathrm{PIM_{SCET}}$: $2E_s(z) = \frac{M_{t\bar{t}}(1-z)}{\sqrt{z}}$	$\mathrm{1PI_{SCET}}$: $2E_s(s_4) = \frac{s_4}{\sqrt{m_t^2+s_4}}$
Kidonakis et al '01	PIM: $2E_s(z) \approx M_{t\bar{t}}(1-z)$	1PI: $2E_s(s_4) \approx s_4/m_t$

Results from soft gluon resummation

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL

1) production threshold (result from HATHOR Aliev et. al. '10)

	Tevatron	LHC (7 TeV)
$\sigma_{\rm NLO}$ (pb)	$6.72^{+0.36\,+0.37}_{-0.76\,-0.24}$	$159^{+20\,+8}_{-21\,-9}$
$\sigma_{\rm NNLO\,approx}$	$7.11^{+0.30\,+0.4}_{-0.40\,-0.3}$	$164^{+3\,+9}_{-9\,-9}$

2) 1PI threshold (m_t=173 GeV) Kidonakis '10

$\sigma_{\rm NNLO\,approx,\,1PI}$	$7.08^{+0.00\,+0.36}_{-0.24\,-0.24}$	$163^{+7\,+9}_{-5\,-9}$

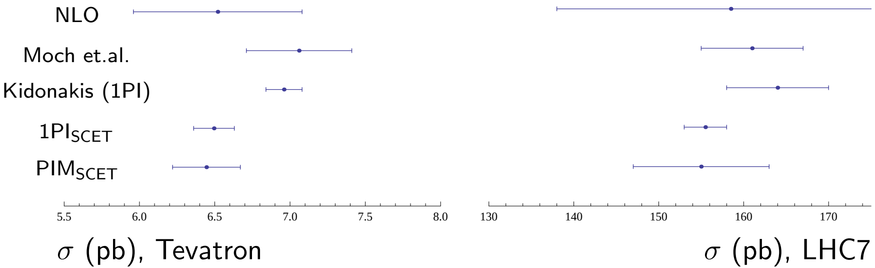
3) PIM and 1PI threshold in SCET Ahrens et.al. '10, '11

$\sigma_{\rm NNLO\ approx,\ 1PI_{SCET}}$	$6.63^{+0.00\,+0.33}_{-0.27\,-0.24}$	$155^{+3\,+8}_{-2\,-9}$
$\sigma_{\rm NNLO\ approx,\ PIM_{SCET}}$	$6.62^{+0.05\,+0.33}_{-0.40\,-0.24}$	$155^{+8\,+8}_{-8\,-9}$
$\sigma_{\rm NLO+NNLL,\ 1PI_{SCET}}$	$6.55^{+0.16\,+0.32}_{-0.14\,-0.24}$	$150^{+7\,+8}_{-7\,-8}$
$\sigma_{\rm NLO+NNLL,\ PIM_{SCET}}$	$6.46^{+0.18\,+0.32}_{-0.19\,-0.24}$	$147^{+7\,+8}_{-6\,-8}$

Note: large discrepancy between PIM and 1PI kinematics observed in Kidonakis et. al. '01 not present in SCET calculation.

NNLO approximations at Tevatron and LHC7

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008



- scale variation not necessarily good indication of uncertainties from subleading terms in soft limits
- study of NLO cross sections indicates that PIM$_{SCET}$ and 1PI$_{SCET}$ receive smallest power corrections away from soft limit (backup slides)

Outline

- Factorization and NLO calculations
- Beyond NLO
- Applications

Top production: some key observables

- total cross section
- $t\bar{t}$ invariant mass distribution
- forward-backward asymmetry

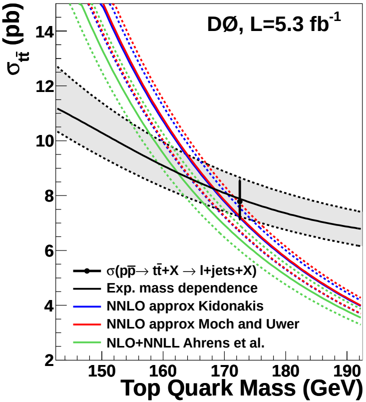


Figure from D0 in lepton+jets, arXiv:1101.0124

Can extract pole mass through cross section

- NNLO would be very useful
- can also use $\overline{\text{MS}}$ mass [Langenfeld, Moch, Uwer '09] or other short-distance masses [Ahrens et. al.]

The total cross section $\sigma^{t\bar{t}X}(s = 7\ \mathrm{TeV})$ and PDF determinations

	σ (pb)	δσ (pb)	comment
ABKM09	139.55	7.96	combined PDF and α_s
CTEQ6.6	156.2	8.06	combined PDF and α_s *
GJR08	169	6	PDF only
HERAPDF1.0	147.31	+5.18 -13.76	combined PDF and α_s**
MSTW08	168.1	+7.2-6.0	combined PDF and α_s***
NNPDF2.0	169	7	combined PDF and α_s ****

m_{top}= 171.3 GeV
zero width approximation,
no branching ratios
68% cl uncertainties
scales $\mu_F = \mu_R = m_{top}$

* ±6.63 (PDF) ±4.59 (α_s)
** expt.+model+param.+α_s , see report for details
*** +4.7-5.6 (PDF) +3.8-4.6 (α_s)
**** ±6 (PDF) ±4 (α_s)

PDF4LHC Working Group Interim Report, arXiv:1101.0536 (January 2011) 37

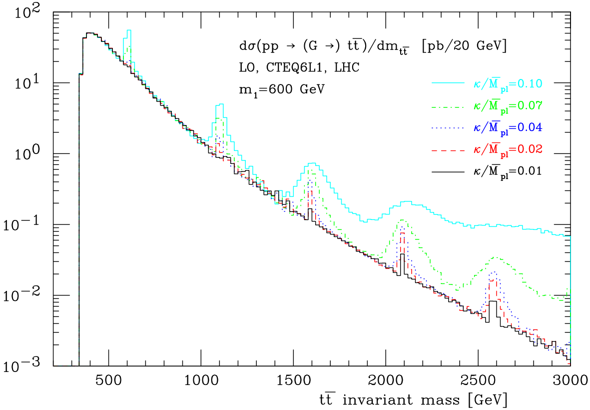
Stirling, Heavy Particles at LHC, Zurich '11

- predictions range between 131-175pb at 68% CL (difference due mainly to gluon distribution at $x \sim 2m_t/\sqrt{s}$)
- measurement of $\sigma^{t\bar{t}X}(s = 7\ \mathrm{TeV})$ important for discriminating PDF sets

- total cross section
- $t\bar{t}$ invariant mass distribution
- forward-backward asymmetry

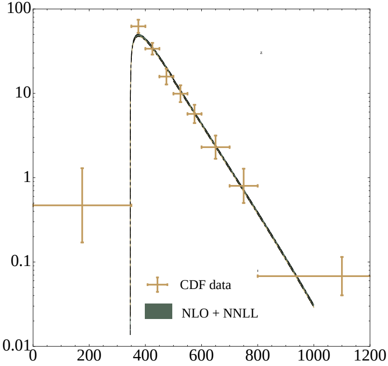
The $t\bar{t}$ invariant mass distribution

The distribution in the invariant mass $M_{t\bar{t}}^2 = (p_t + p_{\bar{t}})^2$ can be used to search for s-channel heavy resonances



Frederix and Maltoni ('07)

The $t\bar{t}$ invariant mass distribution at Tevatron



- good agreement with between theory and data at Tevatron

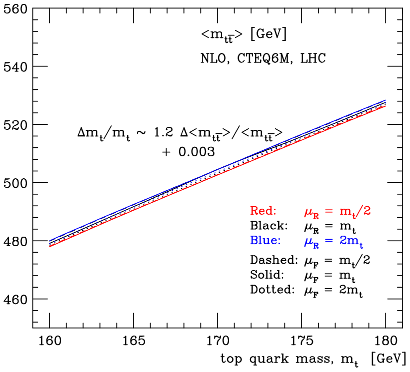
- Tevatron result: no $t\bar{t}$ resonances to 900 GeV
- LHC will extend reach to higher energies

$d\sigma/dM_{t\bar{t}}$ at 400 and 1000 GeV

Tevatron	$M = 400\,\text{GeV}$ [fb/GeV]	$M = 1\,\text{TeV}$ [fb/GeV]
NLO, leading	$36.5^{+5.1\,+1.9}_{-5.0\,-1.4}$	$(23.2^{+4.2\,+1.9}_{-4.0\,-1.4}) \cdot 10^{-3}$
NNLL	$41.3^{+1.3\,+1.9}_{-1.2\,-1.4}$	$(29.6^{+1.0\,+2.2}_{-1.1\,-1.6}) \cdot 10^{-3}$

LHC ($\sqrt{s} = 7\,\text{TeV}$)	$M = 400\,\text{GeV}$ [fb/GeV]	$M = 1\,\text{TeV}$ [fb/GeV]
NLO, leading	$656^{+72\,+26}_{-76\,-27}$	$7.17^{+1.19\,+0.69}_{-1.10\,-0.69}$
NNLL	$775^{+39\,+30}_{-47\,-31}$	$10.83^{+0.84\,+1.01}_{-0.87\,-1.03}$

- at $M = 400$ GeV, subleading terms in soft limit potentially important
- at $M = 1000$ GeV, subleading terms from soft limit less important, PDF uncertainties go up (due to large x)



Frederix and Maltoni ('07)

Mean invariant mass

$$\langle M_{t\bar{t}} \rangle = \int^{M_{\rm cutoff}} \mathrm{d}M_{t\bar{t}}\, M_{t\bar{t}} \frac{d\sigma}{dM_{t\bar{t}}}\bigg|_{\rm norm.}$$

- theory errors can be reduced compared to total cross section
- example: 1% measurement of $\langle M_{t\bar{t}} \rangle \Rightarrow \delta m_t / m_t \sim 1.3\%$

- total cross section
- $t\bar{t}$ invariant mass distribution
- forward-backward asymmetry

Forward-backward asymmetry in $pp(\bar{p}) \to t\bar{t}X$

$$A_{\rm FB}^i = \frac{N_t(y^i > 0) - N_t(y^i < 0)}{N_t(y^i > 0) + N_t(y^i < 0)} = \frac{\Delta\sigma_{\rm FB}}{\sigma}$$

- measured in $i = p\bar{p}$ or $t\bar{t}$ rest frame
- Tevatron: $N_{\bar{t}}(y) = N_t(-y)$, so is a charge symmetry

QCD predictions:

- Tevatron: $A_{\rm FB}^i \sim 5\%$ from NLO (α_s^3) contribution to $\Delta\sigma_{\rm FB}$ from $q\bar{q}$ channel (gg is charge symmetric)
- LHC: $A_{\rm FB}^i = 0$, since initial pp state is symmetric

TOTAL ASYMMETRY IN $t\bar{t}$ FRAME

Experiment

$$A_{\rm FB} = 0.158 \pm 0.072\ \text{stat} \pm 0.017\ \text{syst}\ (\text{CDF},\ 5.3\text{pb}^{-1})$$

$$A_{\rm FB} = 0.08 \pm 0.08\ \text{stat} \pm 0.01\ \text{syst}\ (\text{D0},\ 4.3\text{pb}^{-1})$$

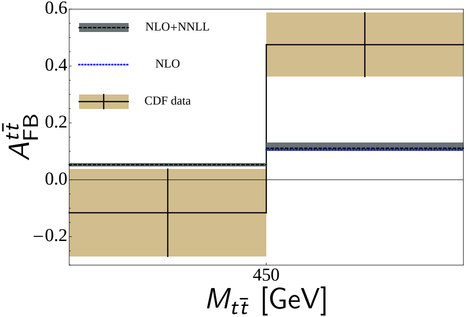
Theory ($m_t = 173.1$ GeV, $m_t/2 < \mu_f < 2m_t$)

	MSTW2008		CTEQ6.6	
	$\Delta\sigma_{\rm FB}$ [pb]	$A^t_{\rm FB}$ [%]	$\Delta\sigma_{\rm FB}$ [pb]	$A^t_{\rm FB}$ [%]
NLO	$0.400^{+0.215\,+0.028}_{-0.129\,-0.022}$	$7.41^{+0.70\,+0.17}_{-0.58\,-0.21}$	$0.394^{+0.208}_{-0.126}$	$7.23^{+0.68}_{-0.56}$
NLO+NNLL	$0.452^{+0.082\,+0.033}_{-0.071\,-0.026}$	$7.32^{+1.08\,+0.25}_{-0.68\,-0.25}$	$0.465^{+0.084}_{-0.074}$	$7.24^{+1.06}_{-0.70}$

- resummation greatly improves scale dependence $\Delta\sigma_{\rm FB}$, but NLO result for $A_{\rm FB}$ rather stable
- PDF uncertainties roughly halved in $A_{\rm FB}$ compared to $\Delta\sigma_{\rm FB}$
- earlier results at NLO+NLL (in moment space) had similar findings [Almeida, Sterman, Vogelsang 2008]
- theory and experiment agree at about 1σ

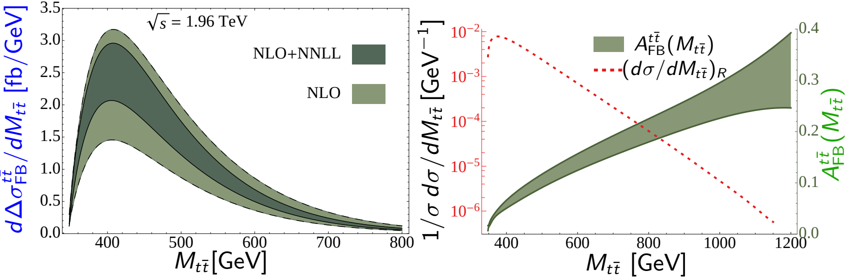
Comparison with binned CDF result

$$A_{\rm FB}^{t\bar{t}}(m_1, m_2) = \frac{\int_{m_1}^{m_2} dM_{t\bar{t}} \left[\left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_F - \left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_B \right]}{\int_{m_1}^{m_2} dM_{t\bar{t}} \left[\left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_F + \left(\frac{d\sigma}{dM_{t\bar{t}}} \right)_B \right]}$$



	$M_{t\bar{t}} \leq 450$ GeV		$M_{t\bar{t}} > 450$ GeV	
	$\Delta\sigma_{\rm FB}^{tt}$ [pb]	$A_{\rm FB}^{tt}$[%]	$\Delta\sigma_{\rm FB}^{tt}$ [pb]	$A_{\rm FB}^{tt}$[%]
CDF		$-11.6^{+15.3}_{-15.3}$		$47.5^{+11.2}_{-11.2}$
NLO	$0.17^{+0.08+0.02}_{-0.05-0.00}$	$5.3^{+0.3+0.1}_{-0.4-0.1}$	$0.21^{+0.12+0.02}_{-0.07-0.00}$	$10.6^{+1.1+0.3}_{-0.8-0.1}$
NLO+NNLL	$0.21^{+0.04+0.02}_{-0.03-0.00}$	$5.2^{+0.7+0.1}_{-0.5-0.0}$	$0.24^{+0.05+0.02}_{-0.04-0.00}$	$11.1^{+1.9+0.3}_{-1.0-0.0}$

NLO+NNLL numbers from [Ahrens,Ferroglia,Neubert,BP,Yang], in progress



$$A_{\rm FB}^{t\bar{t}}(M_{t\bar{t}}) = \frac{\left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_F - \left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_B}{\left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_F + \left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_B} \equiv \frac{\frac{d\Delta\sigma_{\rm FB}^{t\bar{t}}}{dM_{t\bar{t}}}}{\frac{d\sigma}{dM_{t\bar{t}}}}$$

- Figure from [Ahrens,Ferroglia,Neubert,BP,Yang], in progress

CDF, 5.3fb^{-1}

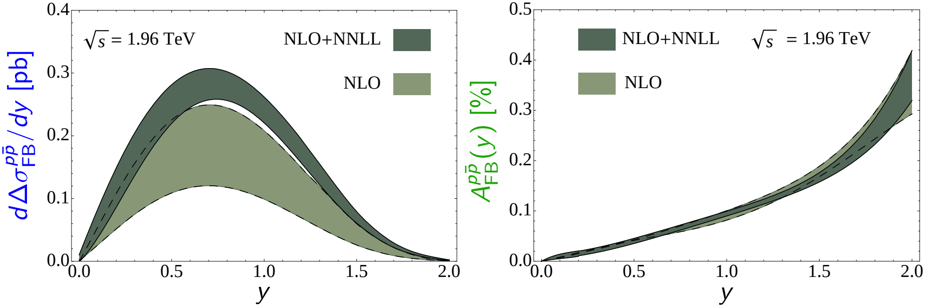
$$A_{\rm FB}(\rm lab) = 0.150 \pm 0.050 \text{ stat} \pm 0.024 \text{ syst}$$

Theory ($m_t = 173.1$ GeV, $m_t/2 < \mu_f < 2m_t$)

	MSTW2008		CTEQ6.6	
	$\Delta\sigma_{\rm FB}$ [pb]	$A^t_{\rm FB}$ [%]	$\Delta\sigma_{\rm FB}$ [pb]	$A^t_{\rm FB}$ [%]
NLO	$0.260^{+0.141\,+0.020}_{-0.084\,-0.014}$	$4.81^{+0.45\,+0.13}_{-0.39\,-0.13}$	$0.256^{+0.135}_{-0.082}$	$4.69^{+0.44}_{-0.38}$
NLO+NNLL	$0.312^{+0.027\,+0.023}_{-0.035\,-0.019}$	$4.88^{+0.20\,+0.17}_{-0.23\,-0.18}$	$0.319^{+0.026}_{-0.037}$	$4.79^{+0.17}_{-0.25}$

- resummation improves $\Delta\sigma_{\rm FB}$ more than A_{FB}, for which NLO is quite stable
- theory and experiment agree at about 2σ

Rapidity-dependent asymmetry in lab frame



$$A_{\rm FB}^{p\bar{p}}(y) = \frac{\left(\frac{d\sigma}{dy}\right) - \left(\frac{d\sigma}{d\bar{y}}\right)\Big|_{\bar{y}=-y}}{\left(\frac{d\sigma}{dy}\right) + \left(\frac{d\sigma}{d\bar{y}}\right)\Big|_{\bar{y}=-y}} \equiv \frac{\frac{d\Delta\sigma_{\rm FB}^{p\bar{p}}}{dy}}{\left(\frac{d\sigma}{dy}\right) + \left(\frac{d\sigma}{d\bar{y}}\right)\Big|_{\bar{y}=-y}}$$

Summary

- Total and differential cross sections in top-pair production at hadron colliders have been known at NLO for 20 years.

- Two routes beyond NLO
  - NNLO in fixed order (in progress)
  - soft gluon resummation to NLO+NNLL $\leftrightarrow$ approximate NNLO (known in three different soft limits)

- NLO+NNLL calculations have reduced scale uncertainties compared to NLO, but care must be taken in applying to the total cross section, because of power corrections to the soft limit

- Higher-order QCD corrections do not explain current discrepancies in A_{FB} at the Tevatron backup slides

Comparing soft limits

If subleading terms in soft limit are known, the production, 1PI, and PIM formulas agree. For instance, at NLO

$$
\begin{aligned}
\hat{\sigma}^{(1)}_{ij=q\bar{q},\,gg} &= \sum_{m=1}^{2} d_m^{\rm p.t.} \ln^m \beta + \frac{C^{\rm p.t.}}{\beta} + c^{\rm p.t.} + \hat{R}^{\rm p.t.}(\beta) \\
&= \int dp_T\, dy \left\{ \sum_{m=0}^{1} d_m^{\rm 1PI} \left[\frac{\ln^m(2E_s(s_4)/\mu_f)}{s_4} \right]_+ + c^{\rm 1PI} \delta(s_4) + \hat{R}^{\rm 1PI}(s_4) \right\} \\
&= \int dM_{t\bar{t}}\, d\theta \left\{ \sum_{m=0}^{1} d_m^{\rm PIM} \left[\frac{\ln^m(2E_s(z)/\mu_f)}{1-z} \right]_+ + c^{\rm PIM} \delta(1-z) + \hat{R}^{\rm PIM}(z) \right\}
\end{aligned}
$$

By evaluating NLO corrections using only leading pieces in soft limits and comparing with exact answer, know size of $\int_{PS}[\hat{R}]$. Will compare using

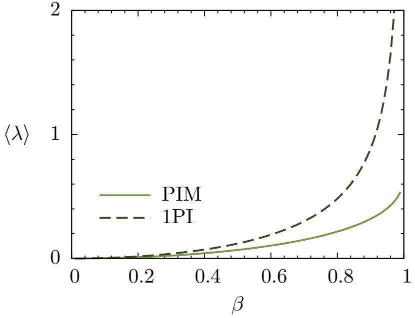
$$
\frac{d\sigma}{d\beta} = \frac{1}{s} \frac{8\beta}{(1-\beta^2)^2} \sum_{ij} ff_{ij}\left(\frac{\hat{s}}{s}, \mu_f\right) m_t^2\, \hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}}, \mu_f\right); \quad \hat{s} = 4m_t^2/(1-\beta^2)
$$

- if $\hat{R}$ is not small at NLO, weaker case for it to be at NNLO

The β-distribution and total cross section

$$\frac{d\sigma}{d\beta} = \frac{1}{s}\frac{8\beta}{(1-\beta^2)^2}\sum_{ij} ff_{ij}\left(\frac{\hat{s}}{s},\mu_f\right) m_t^2\,\hat{\sigma}_{ij}\left(\frac{4m_t^2}{\hat{s}},\mu_f\right); \quad \hat{s} = 4m_t^2/(1-\beta^2)$$

- $ff_{ij}(y,\mu_f) = \int_y^1 \frac{dx}{x}\, f_{i/h_1}(x,\mu_f)\, f_{j/h_2}(y/x,\mu_f)$ are parton luminosities
- $\hat{\sigma}_{ij} \sim \int_{\rm PS}[\mathrm{Tr}[\mathbf{H}\,\mathbf{S}]_{\rm ij} + \mathcal{O}(\lambda)], \qquad \lambda = \{E_s(z)/M,\ E_s(s_4)/m_t\}$



- for $\beta \to 0$, gluon emission is soft, 1PI, PIM, and exact QCD should agree
- for larger β, expect power corrections to be more important in 1PI kinematics than in PIM kinematics

Approximate vs. exact NLO (red = $\beta \to 0$)