

The Particle World

CERN Summer School Lectures 2024

David Tong

Further Reading

Particle Physics

CERN Lectures

David Tong

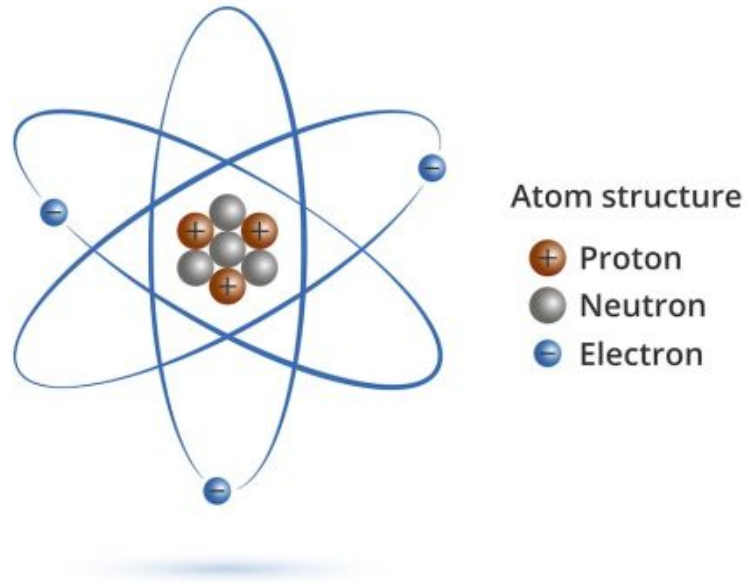
*Department of Applied Mathematics and Theoretical Physics,
Centre for Mathematical Sciences,
Wilberforce Road,
Cambridge, CB3 0BA, UK*

<http://www.damtp.cam.ac.uk/user/tong/particle.html>

d.tong@damtp.cam.ac.uk

around 240 pages! (Sorry)

What are we made of?



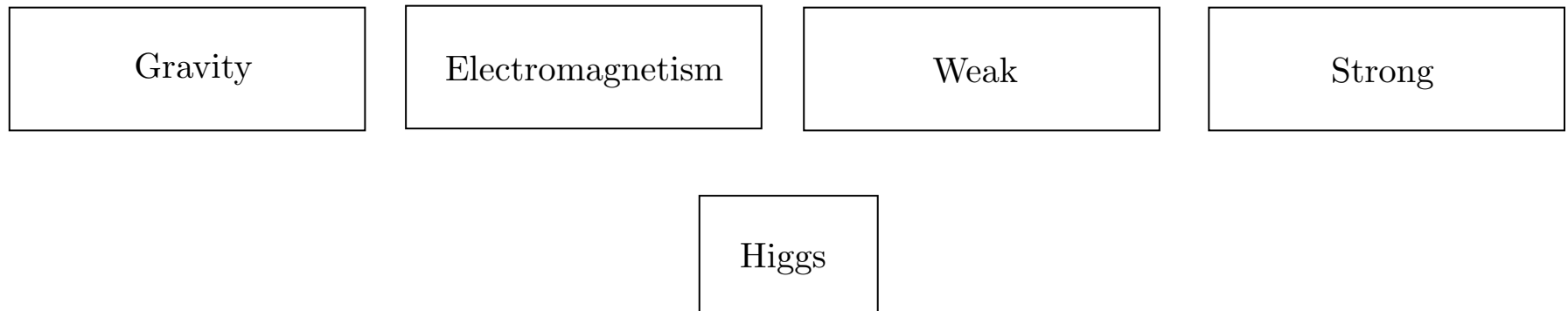
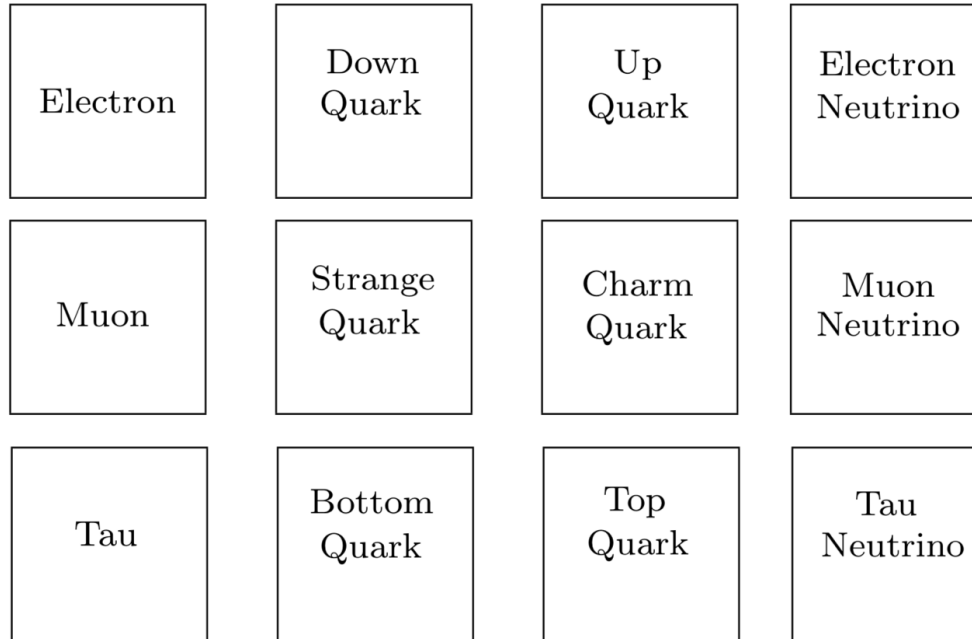
"If we consider protons and neutrons as elementary particles, we would have three kinds of elementary particles [p,n,e].... This number may seem large but, from that point of view, two is already a large number."

Paul Dirac 1933 Solvay Conference

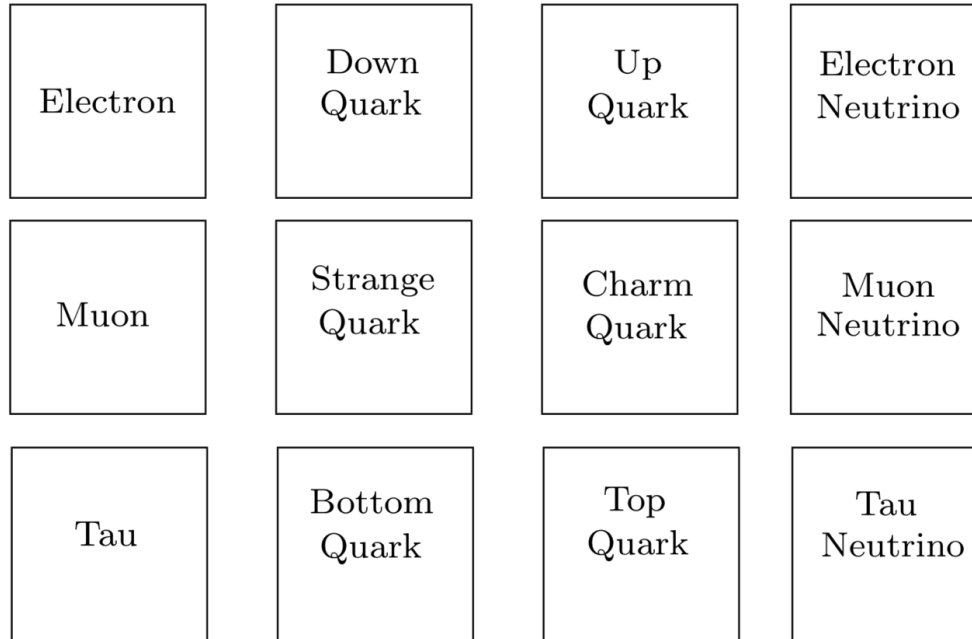
The Standard Model

12 particles + 4 forces + Higgs boson

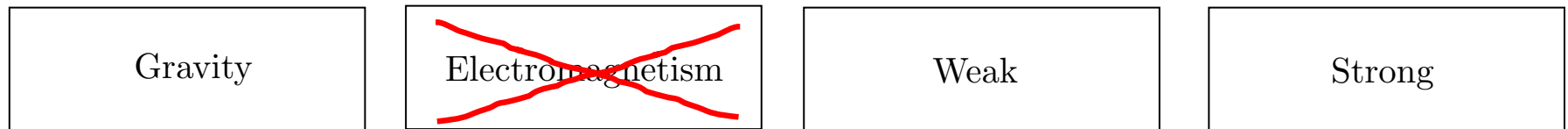
The Standard Model



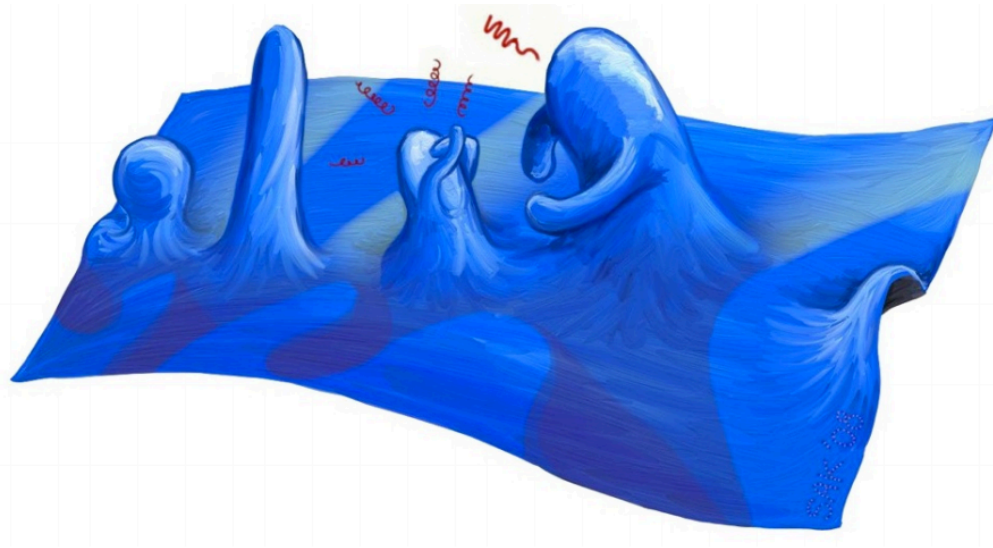
The Standard Model



Hypercharge

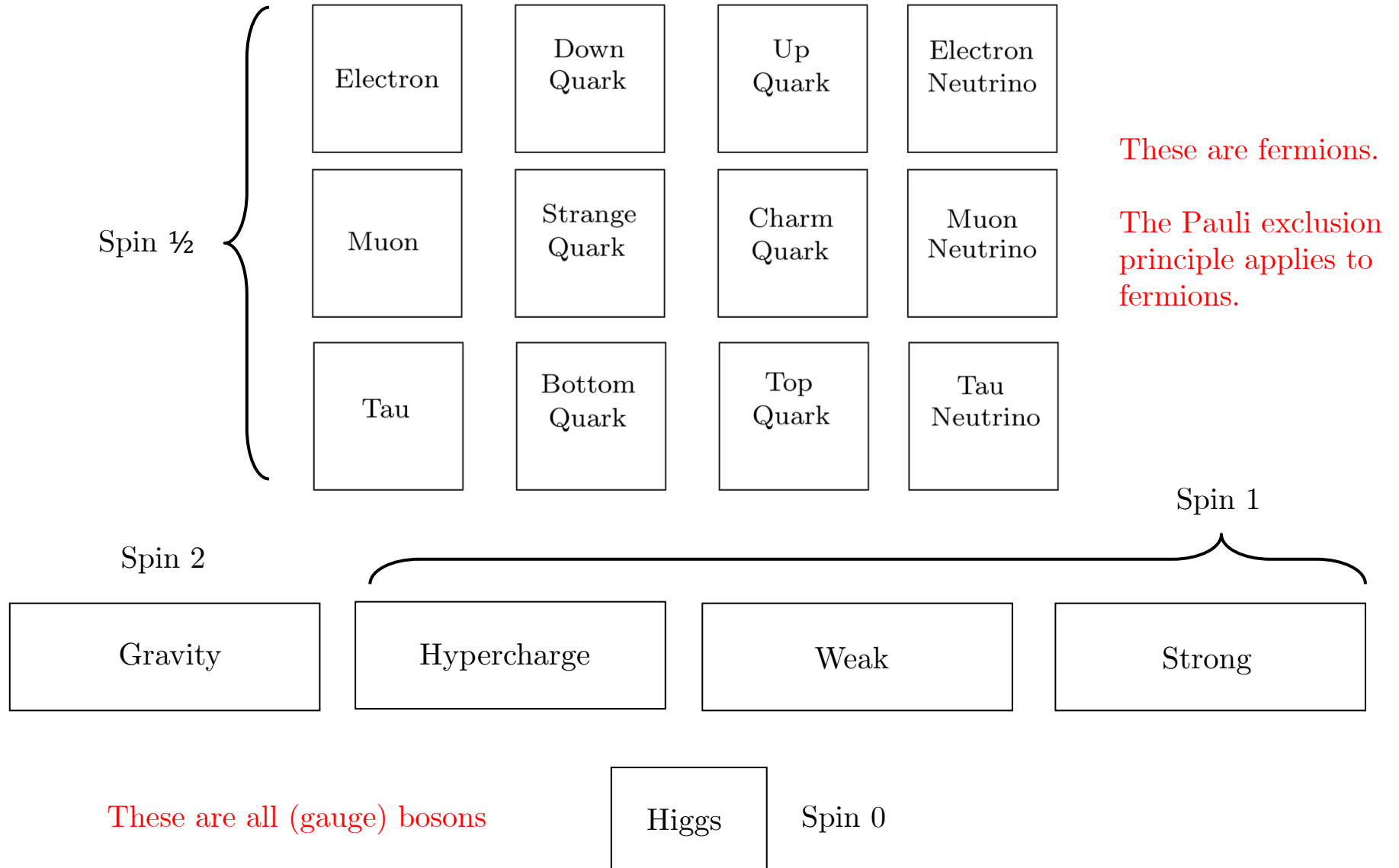


Our First Unification: Fields



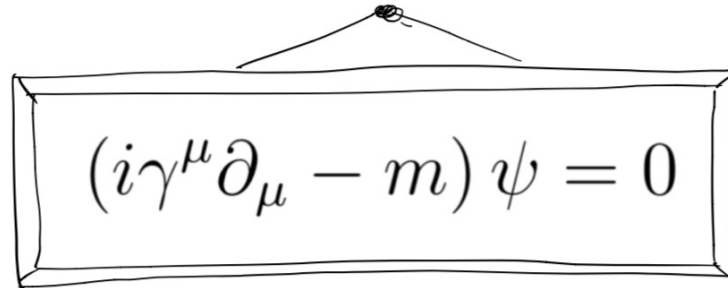
- Particles are excitations of underlying quantum fields
- Forces are also due to fields and have associated particles
 - Electromagnetism = photon
 - Strong = gluon
 - Weak = W and Z bosons
 - Gravity = graviton

Intrinsic Angular Momentum = Spin



A Remarkable Fact

All spin $\frac{1}{2}$ particles are described by the same equation, discovered by Dirac


$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

Here m is the mass.

Consequence: all matter particles come with anti-particles

Mass

| | | | |
|---------------|-------------------------|-------------------------|--|
| Electron 1 | Down Quark 9 | Up Quark 4 | Electron Neutrino $\sim 10^{-6}$ |
| Muon 207 | Strange Quark 186 | Charm Quark 2495 | Muon Neutrino $\sim 10^{-6}$ |
| Tau 3483 | Bottom Quark 8180 | Top Quark 340,000 | Tau Neutrino $\sim 10^{-6}$ |

An aside: In the Standard Model, the masses of all particles are determined by the strength of interaction with the Higgs field.

Units for Mass

We measure mass in terms of energy, using $E=mc^2$. The unit of choice is the electronvolt

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

Or $\text{MeV} = 10^6 \text{ eV}$ or $\text{GeV} = 10^9 \text{ eV}$ or $\text{TeV} = 10^{12} \text{ eV}$.

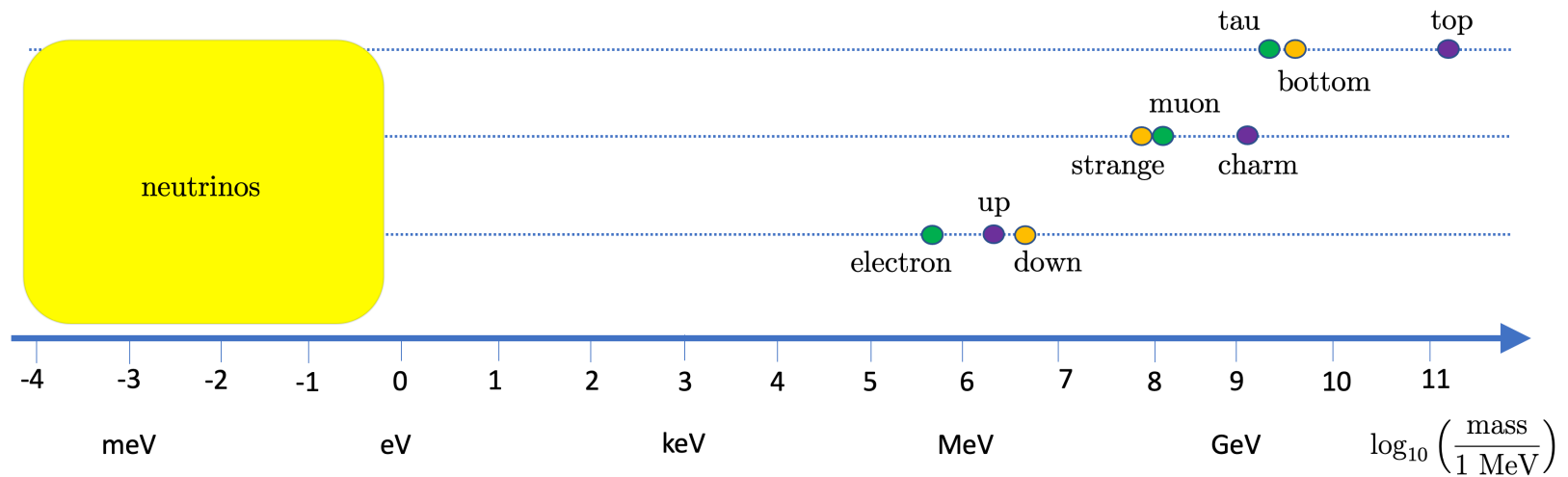
More confusingly, we also measure distance in terms of inverse energy, using

$$\lambda = \frac{\hbar c}{E}$$

For a particle of mass E , this is the “Compton wavelength”, or the size of the particle.

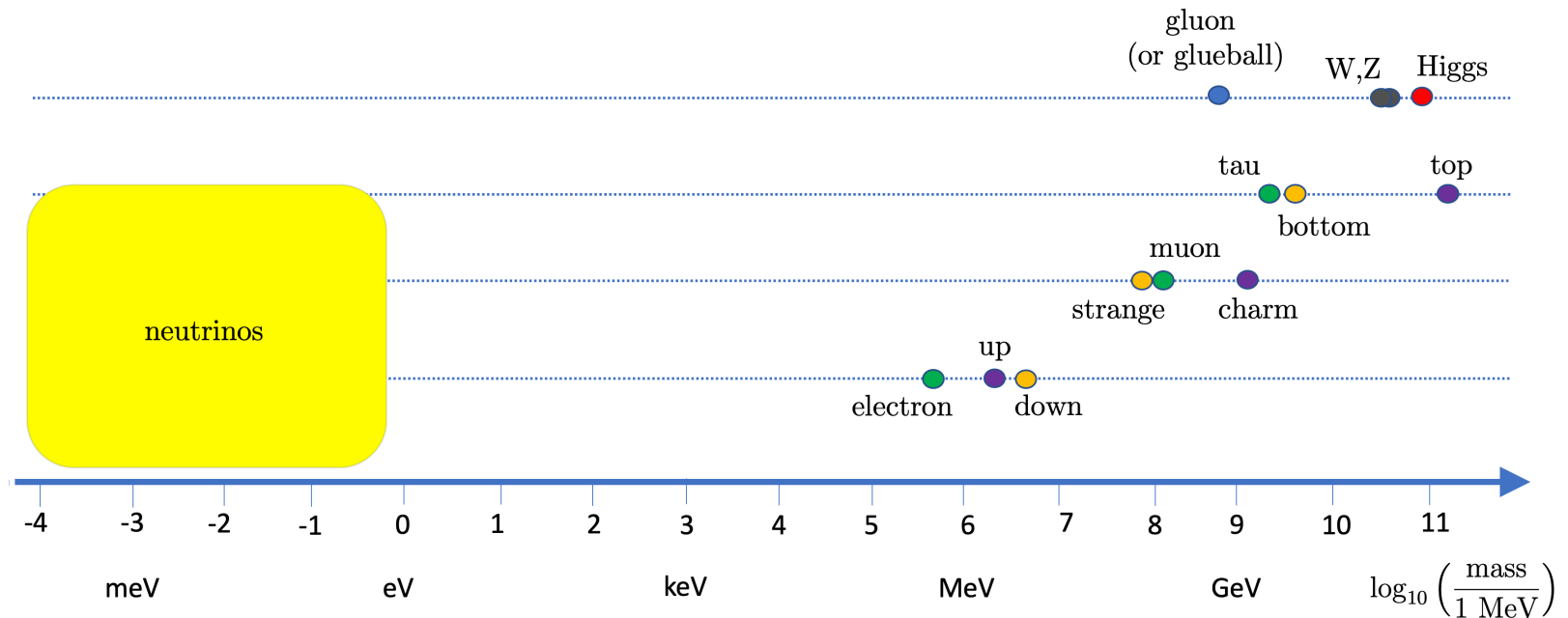
Note: heavier particles are smaller!

The Masses of Particles



Compton wavelength of electron $\lambda_e \approx 2 \times 10^{-12} \text{ m}$

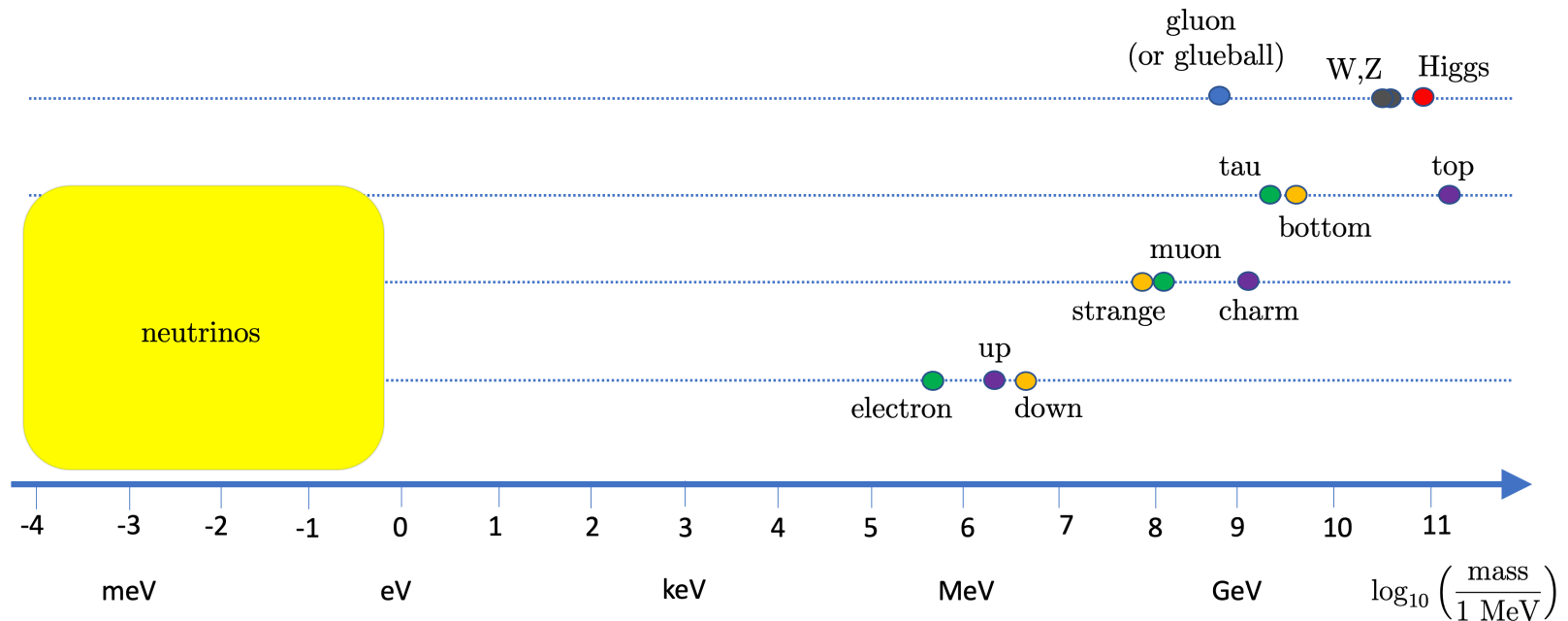
The Masses of Particles



Note: photon and graviton both massless.

(The gluon is a little subtle...see later.)

The Masses of Particles



← The biggest

$$H \approx 2 \times 10^{-33} \text{ eV}$$

$$L_{\text{universe}} \approx 9 \times 10^{26} \text{ m}$$

The smallest →

$$M_{\text{pl}} = \sqrt{\frac{\hbar c}{8\pi G}} \approx 2 \times 10^{18} \text{ GeV}$$

$$L_{\text{pl}} = \sqrt{\frac{8\pi\hbar G}{c^3}} \approx 8 \times 10^{-35} \text{ m}$$

Electric Charge

Charge =

| | -1 | -1/3 | +2/3 | 0 |
|--|---------------|-------------------------|-------------------------|--|
| | Electron 1 | Down Quark 9 | Up Quark 4 | Electron Neutrino $\sim 10^{-6}$ |
| | Muon 207 | Strange Quark 186 | Charm Quark 2495 | Muon Neutrino $\sim 10^{-6}$ |
| | Tau 3483 | Bottom Quark 8180 | Top Quark 340,000 | Tau Neutrino $\sim 10^{-6}$ |

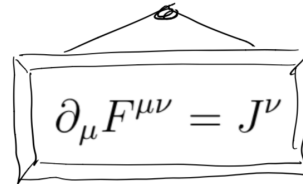
The electric charge characterizes the (relative) strength of the electromagnetic interaction

Electromagnetism (or QED)

The Maxwell Equations

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} & , & & \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & , & & \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)\end{aligned}$$

or


$$\partial_\mu F^{\mu\nu} = J^\nu$$

This implies the Coulomb force which, in natural units, reads

$$F = \alpha \frac{Q_1 Q_2}{r^2}$$

with the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

Feynman Diagrams

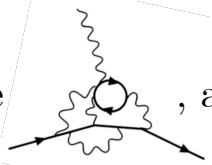
An important fact: quantum field theory is hard!

We are saved in QED because $\alpha \approx \frac{1}{137} \ll 1$. This allows us to write down an approximate solution

e.g. what is the probability for a photon to scatter off an electron in some direction?

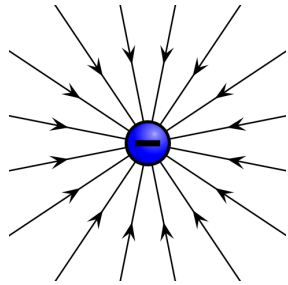
$$\text{Probability} = \left| \begin{array}{c} \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \end{array} \right|^2$$

$\swarrow \quad \searrow$
 $\mathcal{O}(\alpha) \quad \quad \quad \mathcal{O}(\alpha^2)$

More complicated diagrams, like , are less and less important.

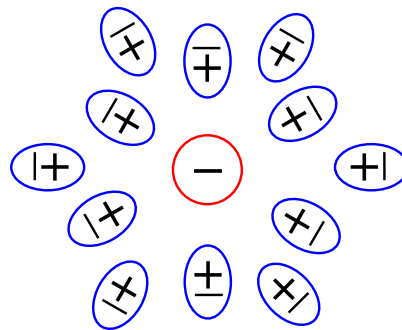
Renormalisation

Look close at the electron.

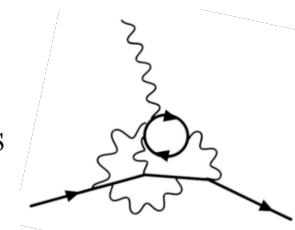


$$\mathbf{E} = \frac{e}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} \quad \Rightarrow$$

Large energy density near electron, This allows for the creation of particle-anti-particle pairs

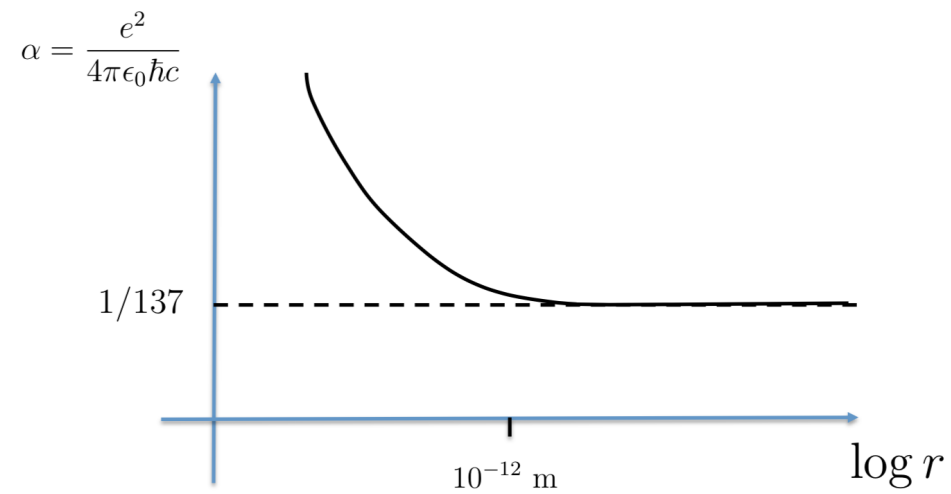


This is the physics behind the increasingly complicated diagrams like this



Renormalisation

As you look more closely, the charge of an electron gets bigger!



Constants of nature are not constant!

The Strong and Weak Force

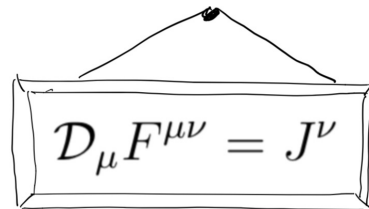
Both nuclear forces have associated “electric” and “magnetic” fields

$$\mathbf{E} = (E_x, E_y, E_z) \quad \mathbf{B} = (B_x, B_y, B_z)$$

But each component is now itself a matrix.

- 1 x 1 matrix \implies Electromagnetism
 - 2 x 2 matrix \implies Weak force
 - 3 x 3 matrix \implies Strong force
- or $U(1) \times SU(2) \times SU(3)$

These fields are governed by the Yang-Mills equations


$$\mathcal{D}_\mu F^{\mu\nu} = J^\nu$$

The Strong Force (or QCD)

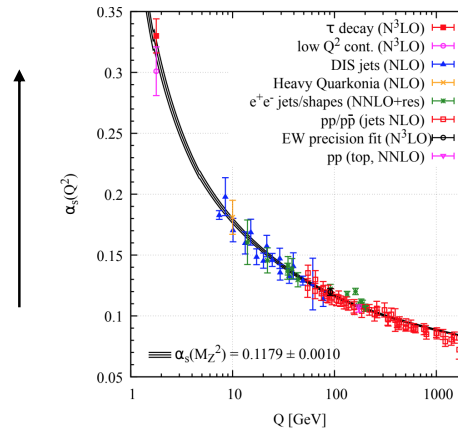
| No | Yes | Yes | No |
|---------------|-------------------------|-------------------------|--|
| Electron 1 | Down Quark 9 | Up Quark 4 | Electron Neutrino $\sim 10^{-6}$ |
| Muon 207 | Strange Quark 186 | Charm Quark 2495 | Muon Neutrino $\sim 10^{-6}$ |
| Tau 3483 | Bottom Quark 8180 | Top Quark 340,000 | Tau Neutrino $\sim 10^{-6}$ |

Each quark comes in three *colours*, which we take to be red, green and blue.

(Note: a better counting is that each generation contains $1+3+3+1=8$ particles.)

Why is the Strong Force Strong?

strong coupling constant



energy = 1/distance

At high energy, say $E=100$ GeV, we have $\alpha_s \approx 0.1$. But the strong force gets stronger as we go to larger distances. (Asymptotic freedom.)

Taken naively, $\alpha_s \rightarrow \infty$ at the energy scale:

$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$$

This corresponds to a distance scale $R_{\text{QCD}} = \frac{1}{\Lambda_{\text{QCD}}} \approx 5 \times 10^{-15} \text{ m}$

Confinement

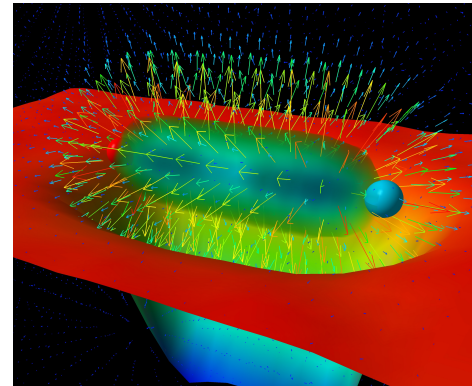
At short distances, $F(r) \sim \frac{\alpha_s}{r^2}$ but at long distances $F(r)$ becomes constant.

In terms of the potential energy, $V(r) \sim -\frac{\alpha_s}{r}$ at short distances, but at long distances

$$V(r) \sim \Lambda_{\text{QCD}}^2 r$$

This is *confinement*. We don't see isolated quarks.

Also, the force carrying field is not massless. The gluons stick together to form glueballs, with mass around $m_{\text{gluon}} \approx \Lambda_{\text{QCD}}$. This is the “mass gap” problem.

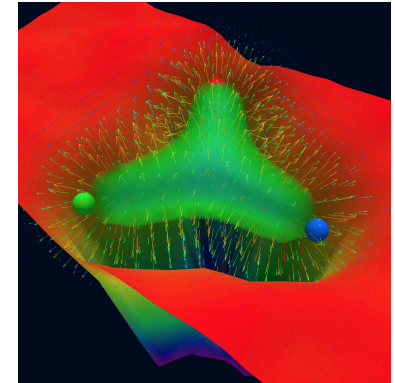


Hadrons (Stuff Made of Quarks)

- Baryons: three quarks. For example

$$n (ddu) \quad m_n \approx 939.57 \text{ MeV}$$

$$p (uud) \quad m_p \approx 938.28 \text{ MeV}$$



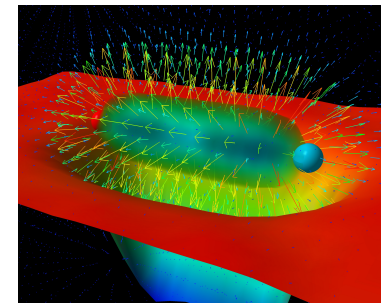
A puzzle: $m_{\text{down}} = 5 \text{ MeV}$ and $m_{\text{up}} = 2 \text{ MeV}$. Where does the mass come from?

- Mesons: quark-anti-quark pair. For example, pions

$$\pi^+ (\bar{d}u) \quad m \approx 139 \text{ MeV}$$

$$\pi^0 \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \quad m \approx 135 \text{ MeV}$$

$$\pi^- (\bar{u}d) \quad m \approx 139 \text{ MeV}$$



Note: Pions have spin 0 and so should be thought of as “force carrying” particles! So ...

Decay

All hadrons other than the proton are unstable. They decay.

- Strong decay: $\sim 10^{-22}$ to 10^{-24} seconds.
- Electromagnetic decay: $\sim 10^{-16}$ to 10^{-21} seconds.
- Weak decay: $\sim 10^{-7}$ to 10^{-13} seconds.

$$\Delta^{++} (uuu) \rightarrow p + \pi^+$$

$$\Sigma^0 (dus) \rightarrow \Lambda^0 + \gamma, \text{ with } \Lambda^0 (dus)$$

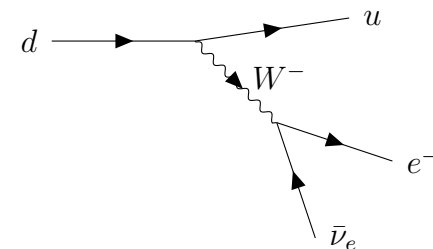
$$\pi^+ (ud) \rightarrow \mu^+ + \nu_\mu$$

The most famous weak decay is how we first discovered the weak force

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Or, if you look more closely,

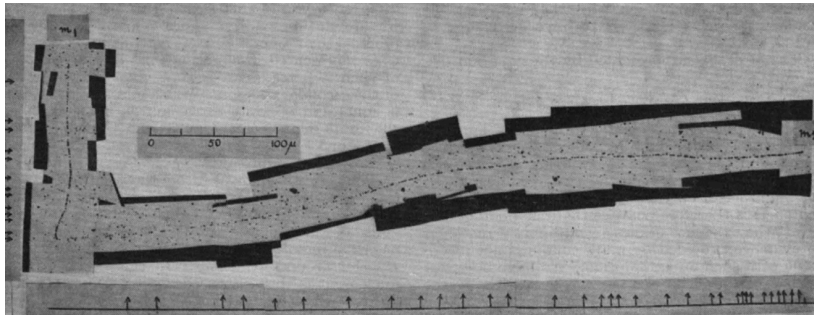
$$d \rightarrow u + e^- + \bar{\nu}_e$$



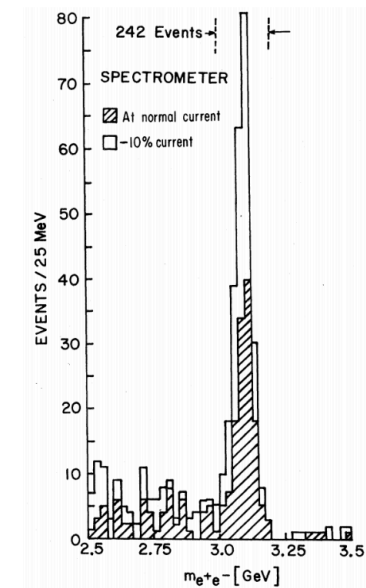
Particles vs Resonances

- Strong decay: $\sim 10^{-22}$ to 10^{-24} seconds.
- Electromagnetic decay: $\sim 10^{-16}$ to 10^{-21} seconds.
- Weak decay: $\sim 10^{-7}$ to 10^{-13} seconds.

If a particle decays through the weak force, we can take a photograph of it!



If it decays through the strong force, or EM, then we see it more indirectly

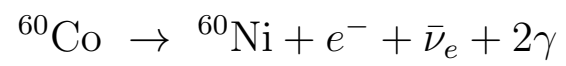


The Weak Force

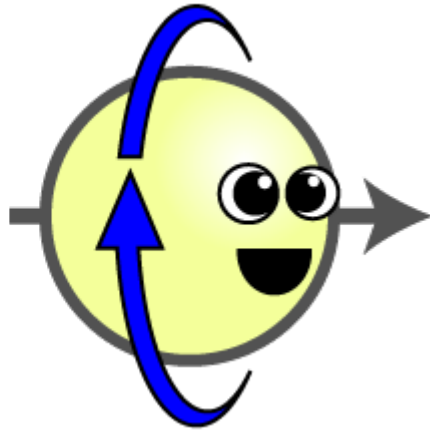
half of each particle!

| | | | |
|---------------|-------------------------|-------------------------|--|
| Electron 1 | Down Quark 9 | Up Quark 4 | Electron Neutrino $\sim 10^{-6}$ |
| Muon 207 | Strange Quark 186 | Charm Quark 2495 | Muon Neutrino $\sim 10^{-6}$ |
| Tau 3483 | Bottom Quark 8180 | Top Quark 340,000 | Tau Neutrino $\sim 10^{-6}$ |

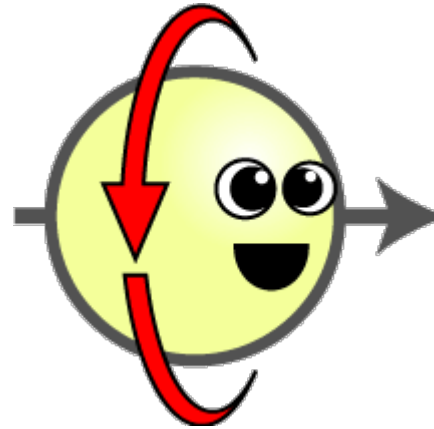
Parity Violation



Chiral Fermions



left-handed fermion

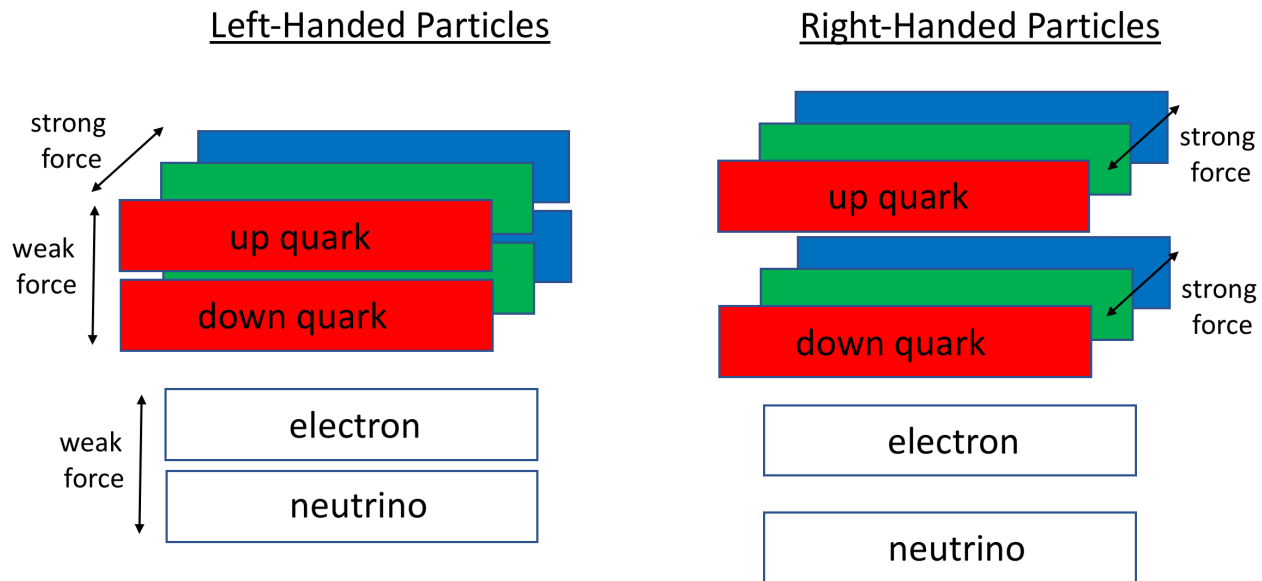


right-handed fermion

Left-handed particles experience the weak force, right-handed do not.

The Forces of the Standard Model

Each generation splits into 8x2 sets of particles



Note: We don't yet know if the right-handed neutrino exists.

The Structure of the Standard Model

$$G = SU(3) \times SU(2) \times U(1)$$

strong
weak
hypercharge

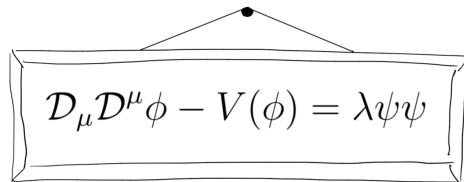
| Particles | | Strong | Weak | Hypercharge |
|--------------|------------|--------|------|-------------|
| Left-handed | quarks | yes | yes | +1/6 |
| | leptons | no | yes | -1/2 |
| Right-handed | up quark | yes | no | +2/3 |
| | down quark | yes | no | -1/3 |
| | electron | no | no | -1 |
| | neutrino | no | no | 0 |

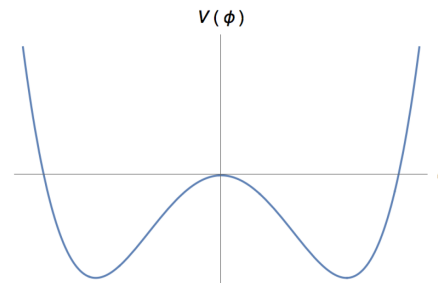
A perfect jigsaw: Anomaly cancellation means that it could hardly be any other way!

The Higgs Field

This is both the simplest and most complicated field in the Standard Model!

| Particle | Strong | Weak | Hypercharge |
|----------|--------|------|-------------|
| Higgs | no | yes | +1/2 |


$$\mathcal{D}_\mu \mathcal{D}^\mu \phi - V(\phi) = \lambda \psi \psi$$



- Two relevant scales:
- Mass $m_H \approx 125$ GeV
 - Condensate $\langle \phi \rangle \approx 246$ GeV

It is the condensate that gives the Higgs its Midas touch: everything that it touches gets a mass

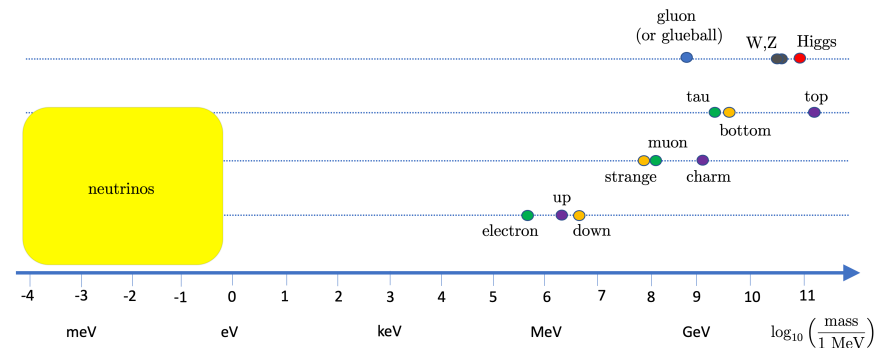
How Particles Get a Mass

In the Standard Model, all fermions and gauge bosons are obliged to be fundamentally massless

They get a mass by interaction with the Higgs.

$$\text{Mass} = g \times \langle \phi \rangle$$

some dimensionless coupling $\langle \phi \rangle \approx 246 \text{ GeV}$



- The Higgs gives mass to the W-boson and Z-boson and all fermions.
- The photon remains massless: it is the one that got away!
- Recall: the mass of the proton and neutron do not come from the Higgs!

One Last Thing: Quark Mixing

There is a misalignment between the interactions with the Higgs and the interaction with the weak force.

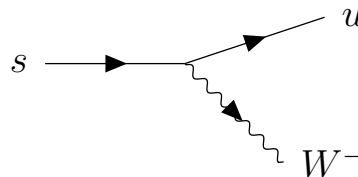
It turns out that you can choose to have the up-sector aligned. But then the down sector is not. The result is a superposition of particles.

These particles interact with weak force

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

These particles interact with Higgs, and so definite mass.

This is how, for example, mesons with strange quarks decay



One Last Thing: and Lepton Mixing

There is a similar statement for neutrinos

These particles interact with weak force and are produced in, say, beta decay

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

These particles have definite mass. These are energy eigenstates that travel unchanged through space.

This gives rise to neutrino oscillations

The Mixing Matrices

For quarks, we have the CKM matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.97 & 0.04 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$

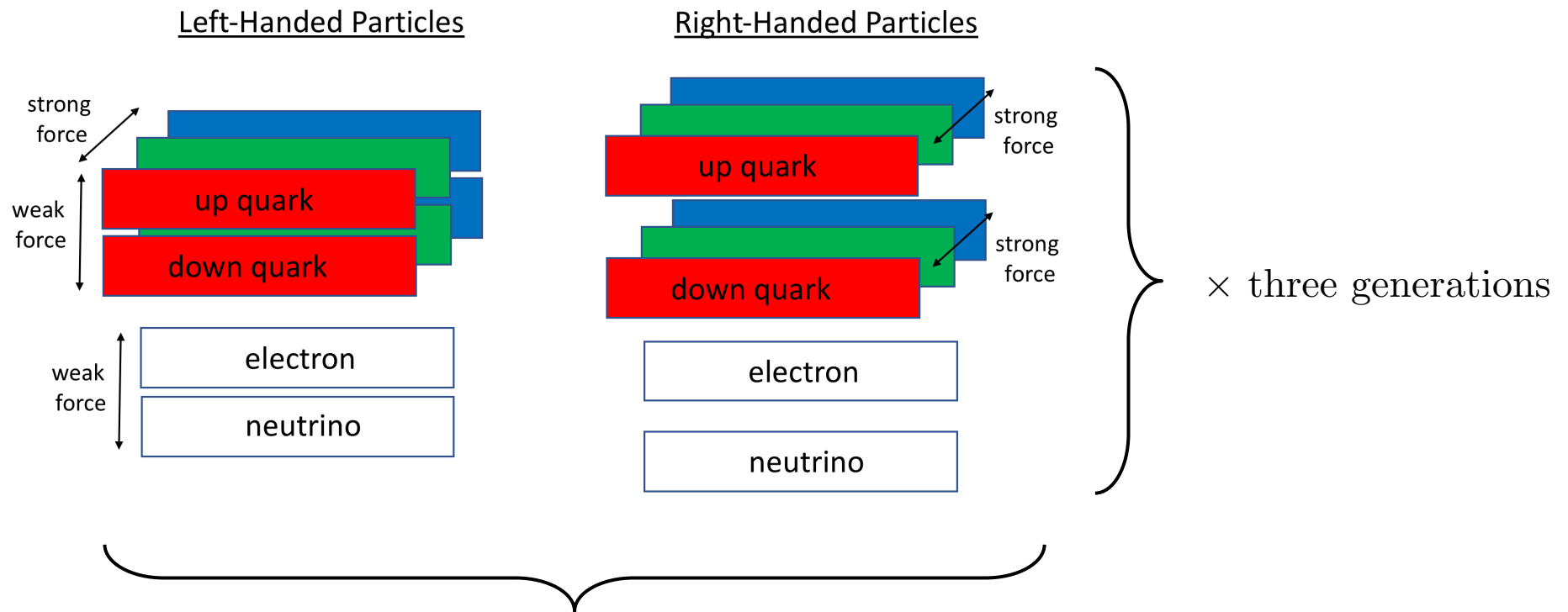
For neutrinos, we have the PMNS matrix

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{pmatrix} \approx \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.3 & 0.5 & 0.7 \\ 0.4 & 0.6 & 0.6 \end{pmatrix}$$

We only know these parameters by experimental measurement. Why do they take these values?
Why are the matrices so different?

Summary: The Greatest Theory of All Time

$$G = SU(3) \times SU(2) \times U(1)$$



with all complications coming from interactions with Higgs!