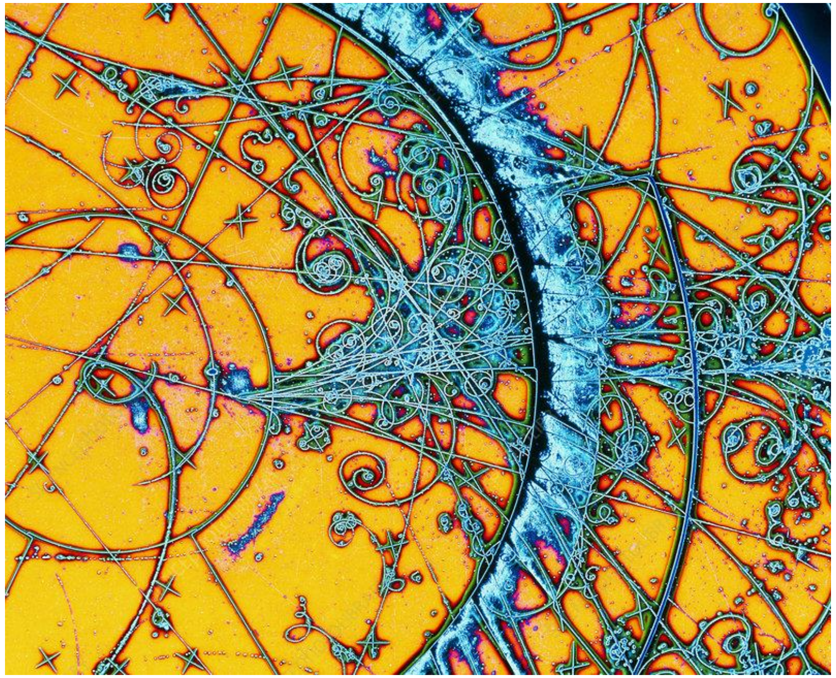
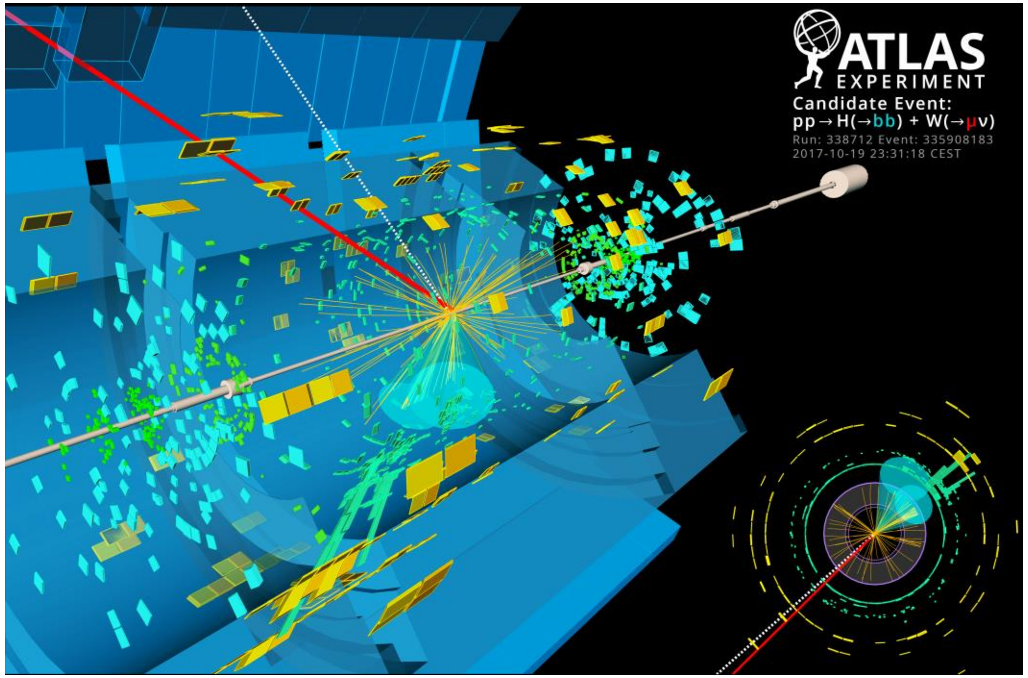
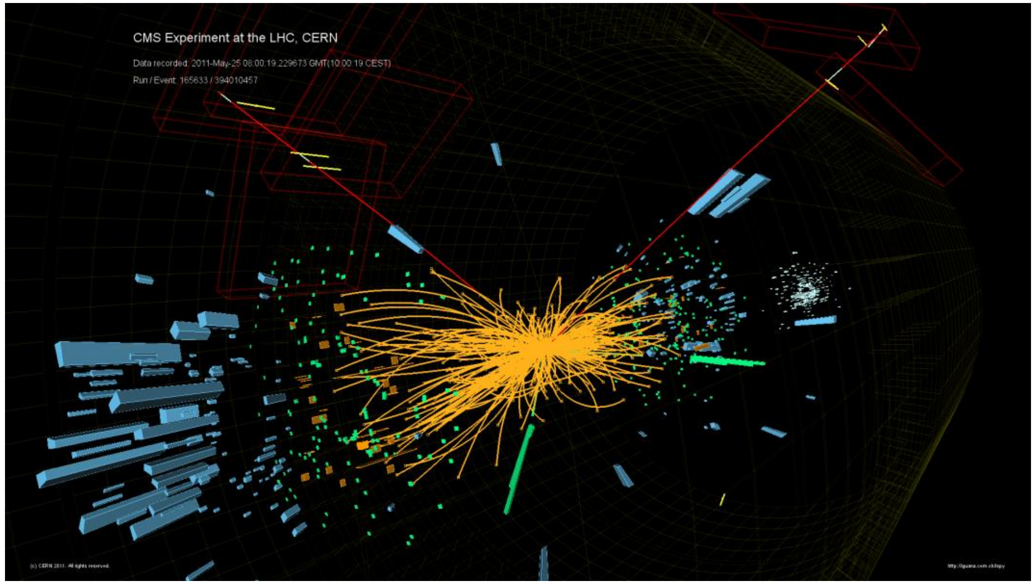



ATLAS
EXPERIMENT
Candidate Event:
$pp \to H(\to b\bar{b}) + W(\to \mu\nu)$
Run: 338712 Event: 335908183
2017-10-19 23:31:18 CEST


CMS Experiment at the LHC, CERN
Data recorded: 2011-May-25 08:00:19.229673 GMT(10:00:19 CEST)
Run / Event: 165633 / 394010457

https://indico.cern.ch/event/1426610/

Thu, 4 Jul 2024

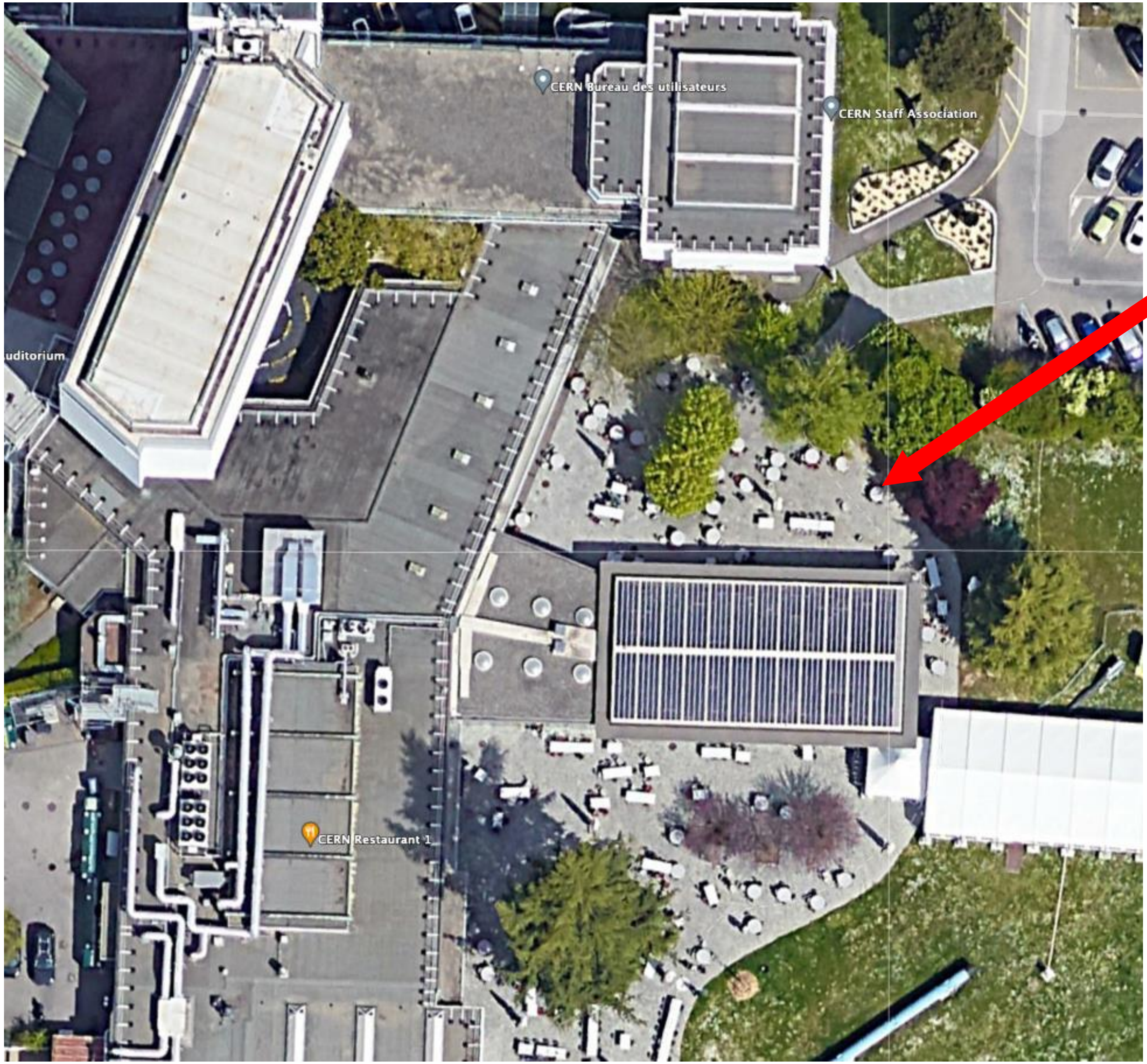
Time and duration : Doors open at 7.00 p.m. Event starts at 7.30 p.m. Duration: 2h

- **Location**: Auditorium Sergio Marchionne, CERN Science Gateway
- **Admission:** Free of charge, but registration is required for in-person attendance.
- **Refreshments**: At the Big Bang Café until 7.30 p.m.

https://indico.cern.ch/event/1426610/registrations/106599/

Oliver:

My question is whether or not systematic miss classification of events is a large problem in particle detectors. And if so, what the major sources of miss classification are ?



Tomorrow (Friday) 14:30

Particle Detectors

Summer Student Lectures 2024

Werner Riegler, CERN, werner.riegler@cern.ch

History of Instrumentation ↔ History of Particle Physics

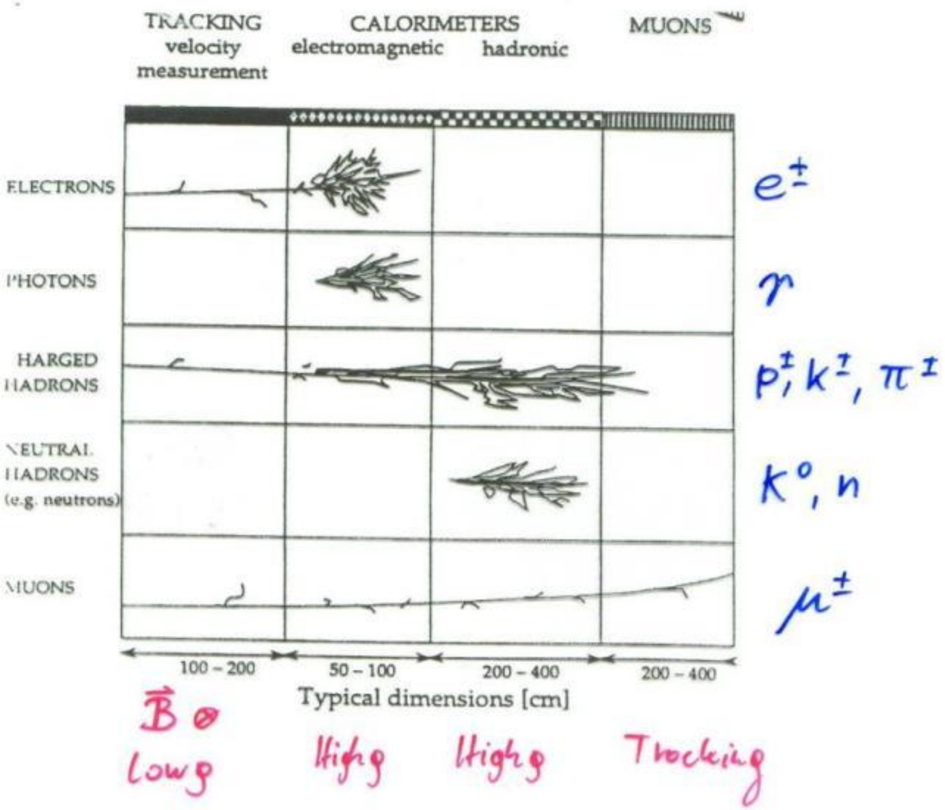
The 'Real' World of Particles

Interaction of Particles with Matter

Tracking Detectors, Calorimeters, Particle Identification

Detector Systems

We have to know the details about all interactions to design our detectors !



$e^\pm$ $m_e = 0.511\ \text{MeV}$

$\mu^\pm$ $m_\mu = 105.7\ \text{MeV} \sim 200\, m_e$

γ $m_\gamma = 0\ ,\ Q = 0$

} EM

$\pi^\pm$ $m_\pi = 139.6\ \text{MeV} \sim 270\, m_e$

$K^\pm$ $m_K = 493.7\ \text{MeV} \sim 1000\, m_e$ $\quad \sim 3.5\, m_\pi$

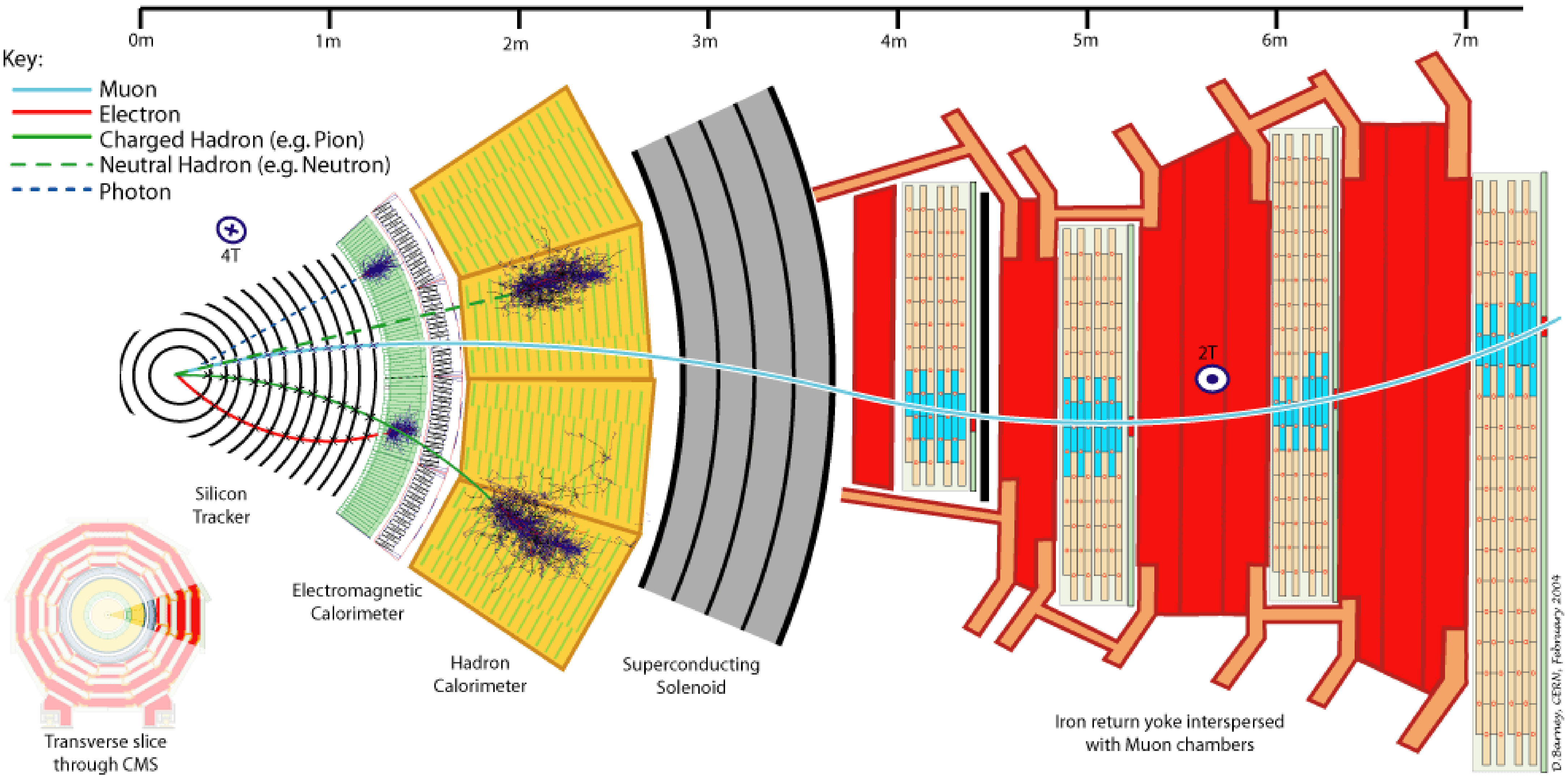
$p^\pm$ $m_p = 938.3\ \text{MeV} \sim 2000\, m_e$

} EM, Strong

K^0 $m_{K^0} = 497.7\ \text{MeV}$ $\quad Q = 0$

n $m_n = 939.6\ \text{MeV}$ $\quad Q = 0$

} Strong


0m
1m
2m
3m
4m
5m
6m
7m
Key:
Muon
Electron
Charged Hadron (e.g. Pion)
Neutral Hadron (e.g. Neutron)
Photon
4T
2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with Muon chambers
Transverse slice through CMS
D.Barney, CERN, February 2004

Detector Physics

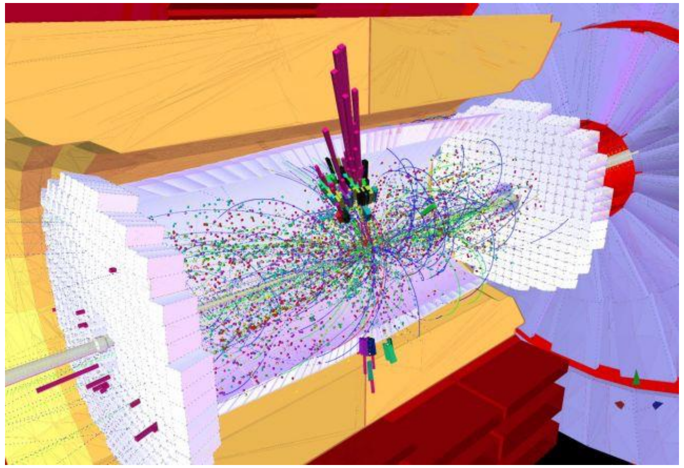
Precise knowledge of the processes leading to signals in particle detectors is necessary.

The detectors are nowadays working close to the limits of theoretically achievable measurement accuracy – even in large systems.

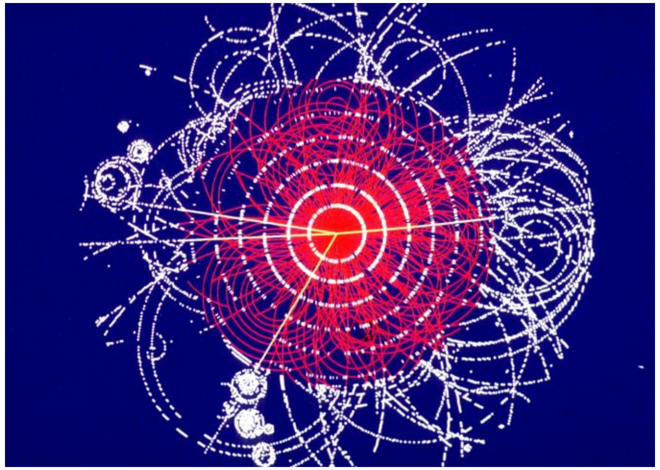
Due to available computing power, detectors can be simulated to within 5-10% of reality, based on the fundamental microphysics processes (atomic and nuclear cross-sections).

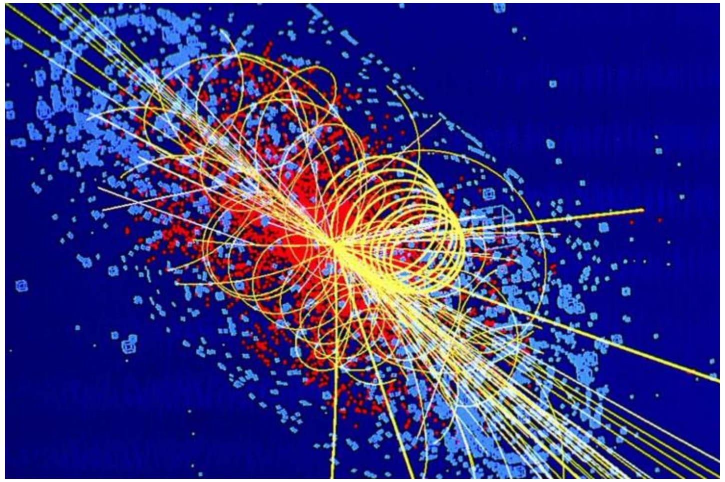
Particle Detector Simulation

CMS detector simulation GEANT

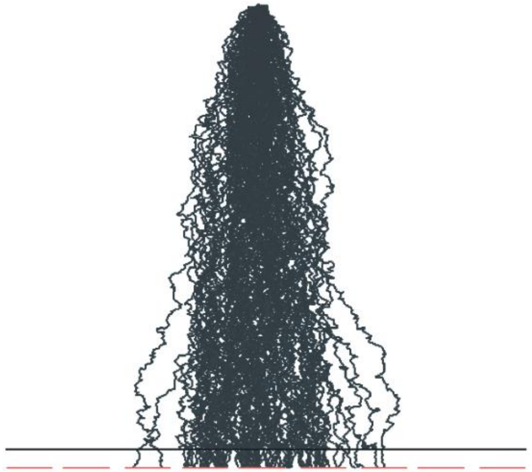


ATLAS detector simulation GEANT

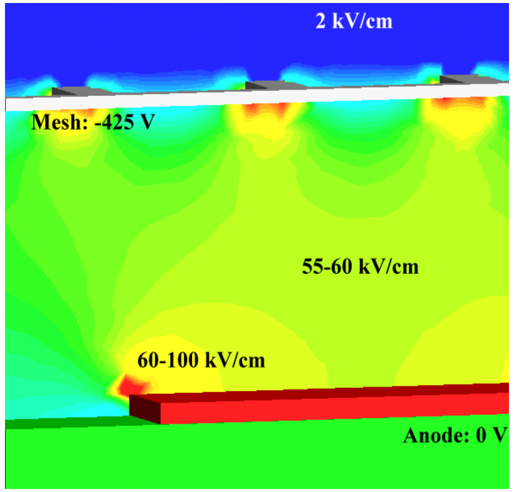




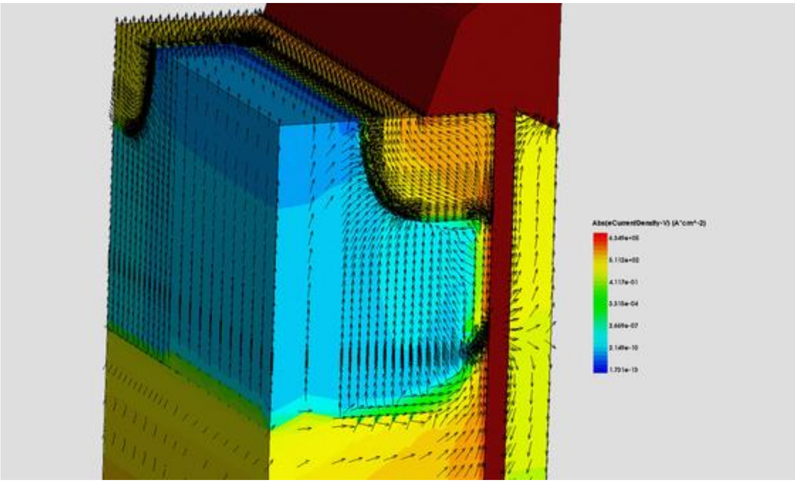
Electrons avalanche multiplication, GARFIELD++



Electric Fields in a Micromega detector, e.g. COMSOL



Silicon sensor simulation, TCAD

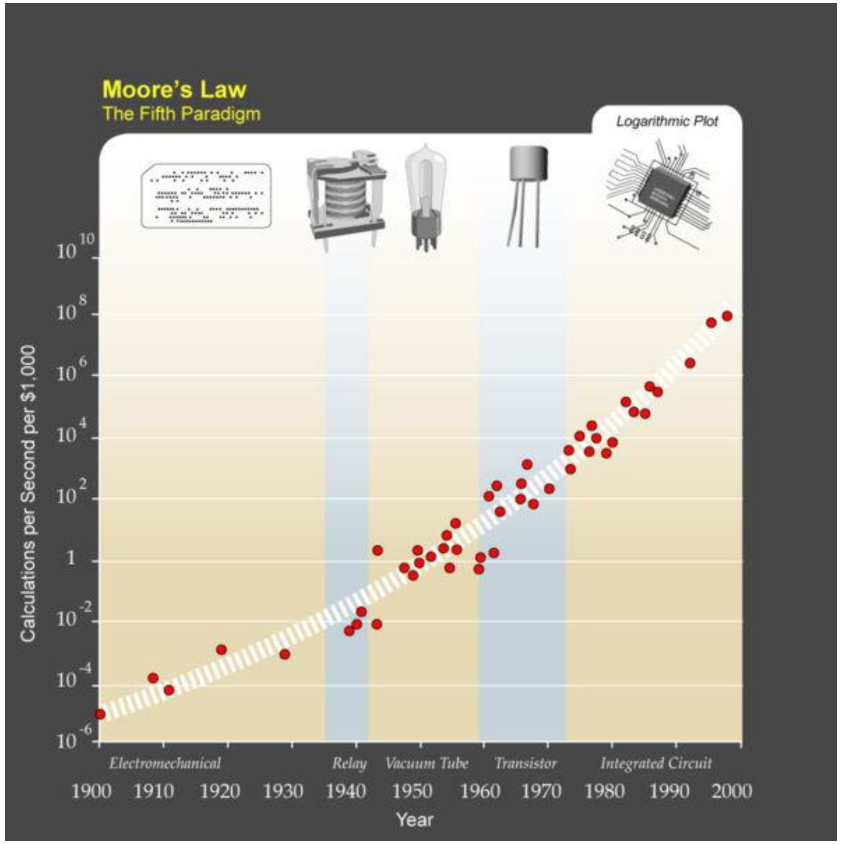


Particle Detector Simulation

I) C. Moore's Law:
Computing power doubles 18 months.

II) W. Riegler's Law:
The use of brain power for solving a problem is inversely proportional to the available computing power.

→ I) + II) = ...



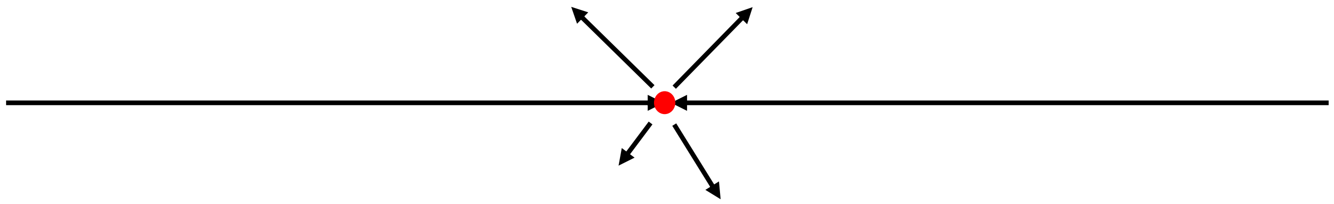
Knowing the basics of particle detectors is essential …

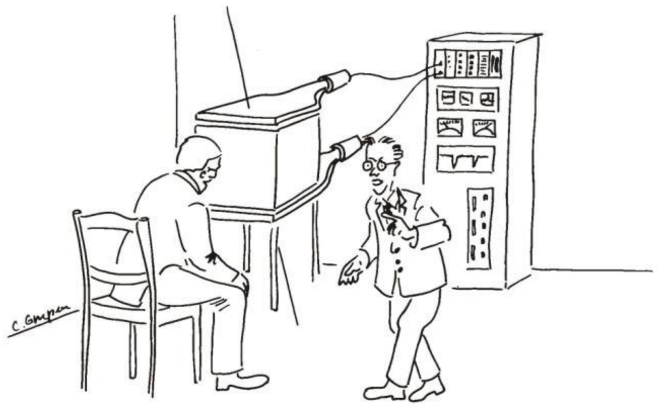
Interaction of Particles with Matter

Any device that is to detect a particle must interact with it in some way → almost …

In many experiments neutrinos are measured by missing transverse momentum.

E.g. e^+e^- collider. $P_{tot}=0$,
If the $\Sigma\, p_i$ of all collision products is $\neq 0$ → neutrino escaped.

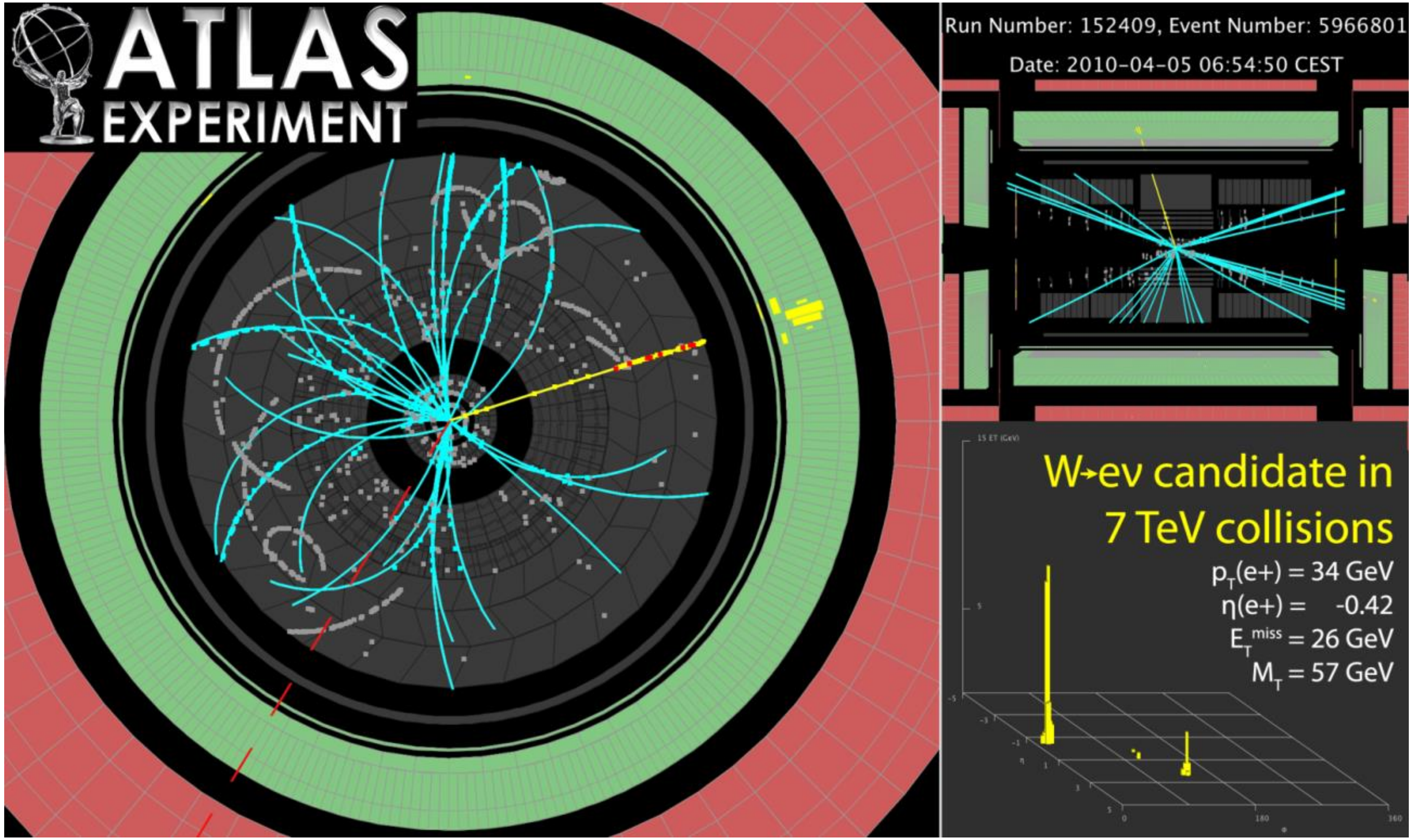




"Did you see it?"
"No nothing."
"Then it was a neutrino!"

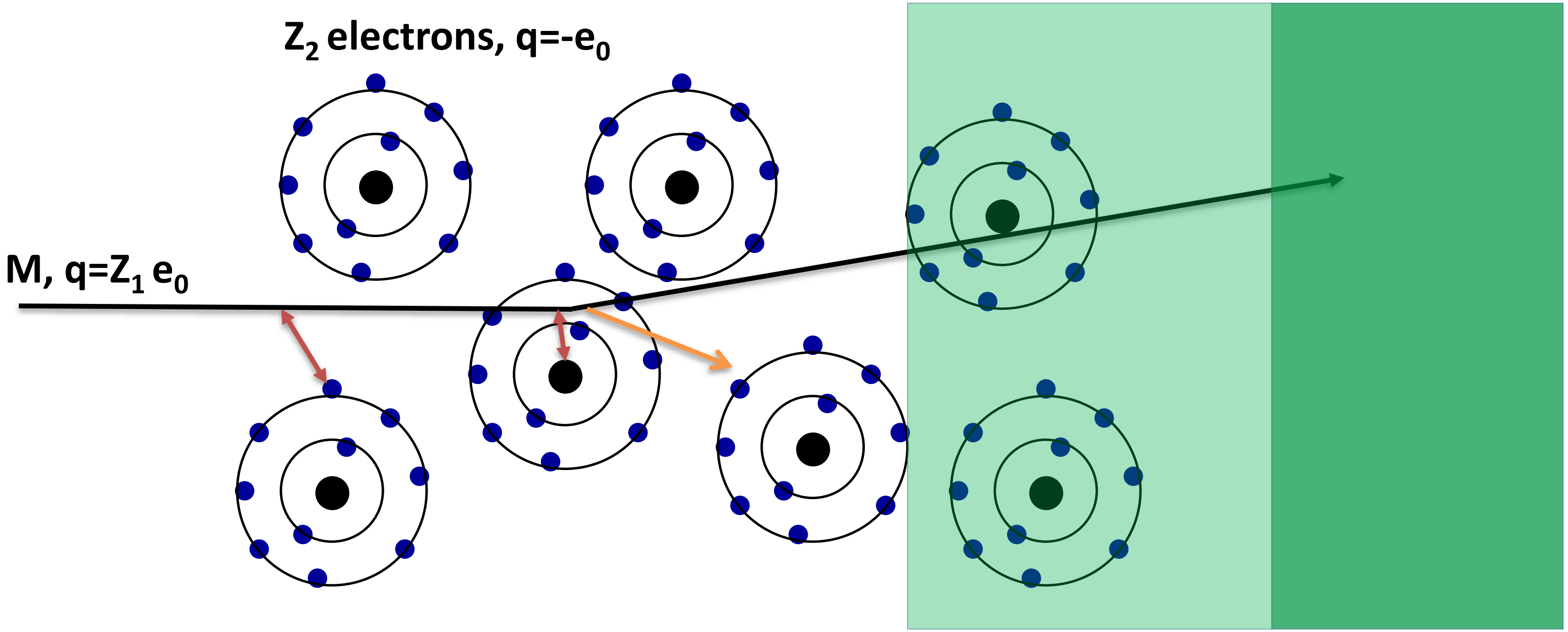
Claus Grupen, Particle Detectors, Cambridge University Press, Cambridge 1996 (455 pp. ISBN 0-521-55216-8)

Neutrinos and other invisible particles



They are seen by missing momentum vectors – if you are sure your detector is hermetic !

Electromagnetic Interaction of Particles with Matter

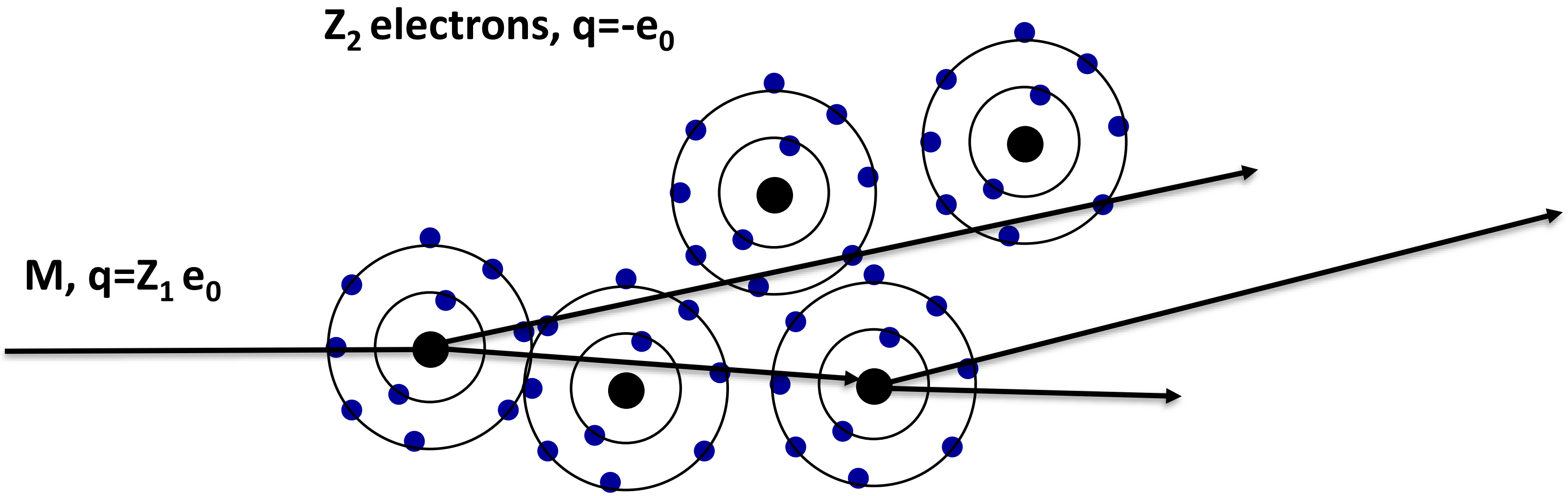


Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.

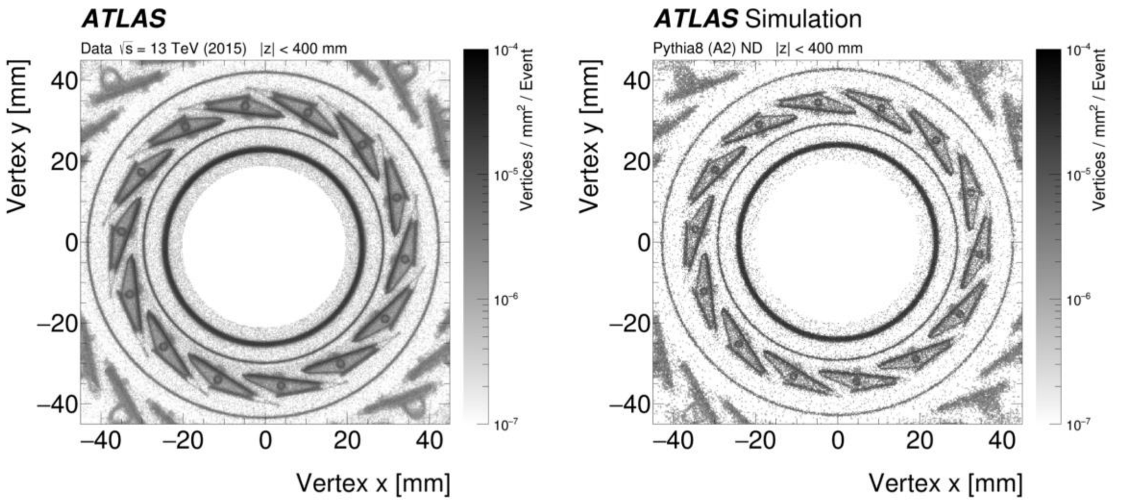
Hadronic Interaction of Particles with Matter



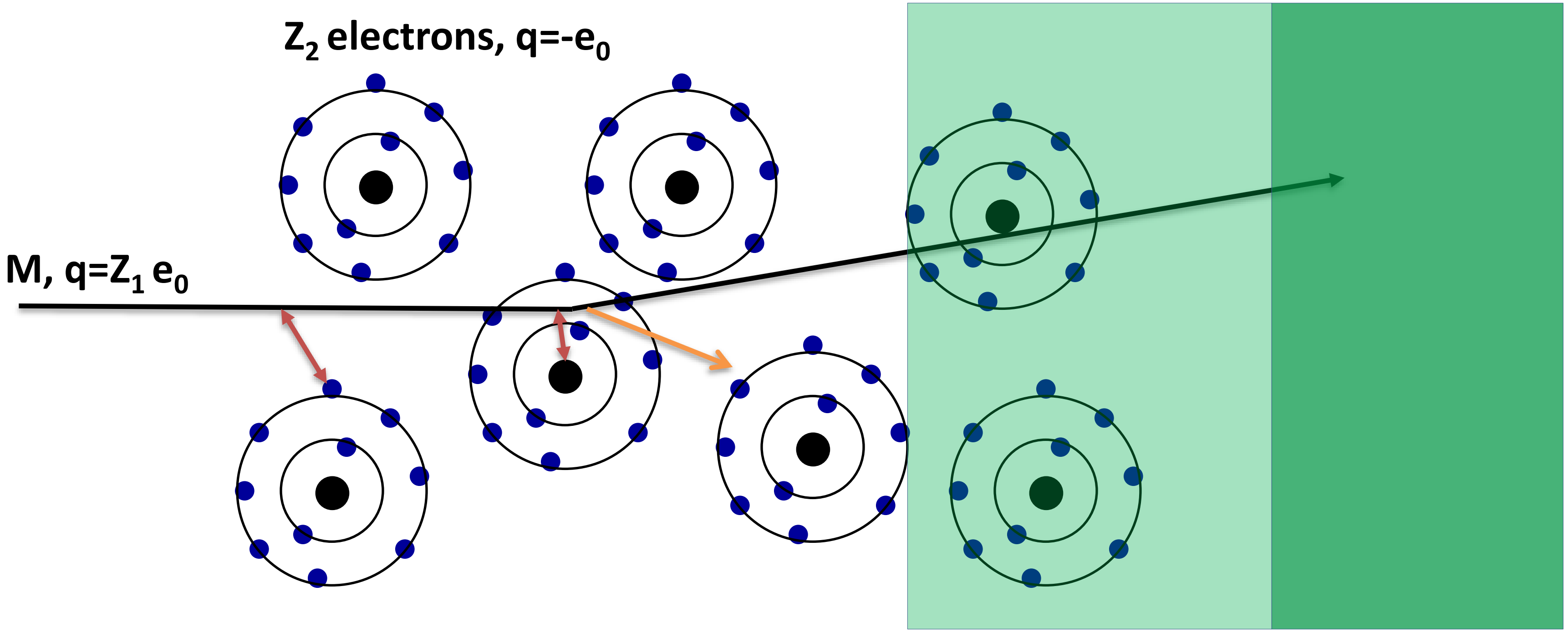
Hadronic interactions with the nuclei produce a complex sequence of secondary particles. This is at the basis of hadron calorimetry.

Hadronic interactions taking place in the trackers are of course having bad an impact on the tracker measurements !

Tracing back these secondary particles to their origin gives the place of interaction. Making a map of these points we can image our detectors !



Electromagnetic Interaction of Particles with Matter



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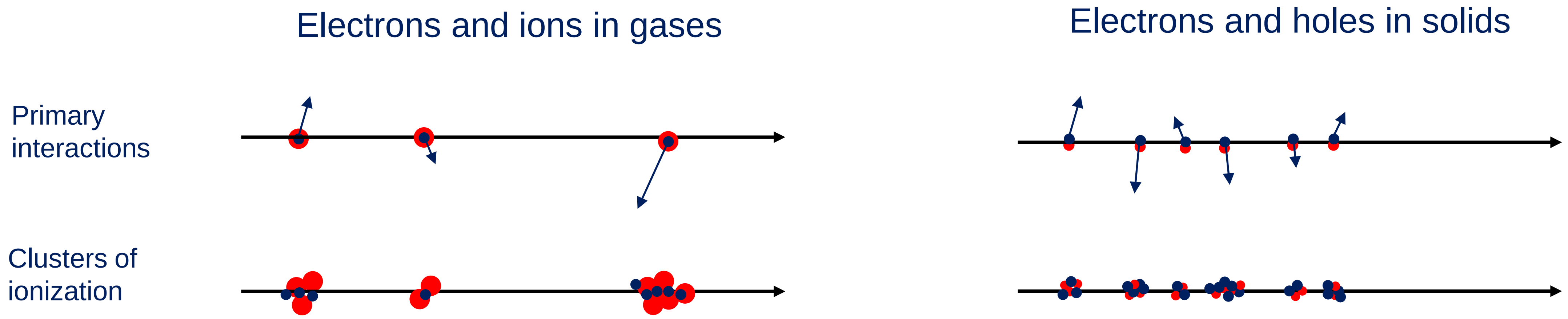
Ionization and Excitation

The charged particles leave a trail of excited and ionized atoms.

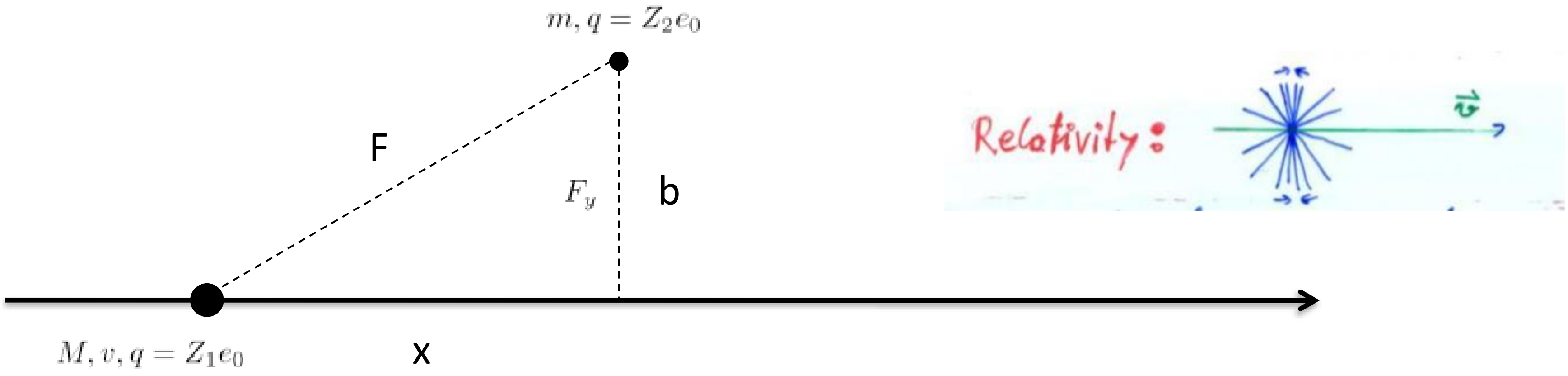
The electrons and holes produced in these interactions do themselves have sufficient energy ionize the gas or solid, but because there energy is small the loose all their energy over a very small distance, so in we get a trail of ionization-clusters along the way.

The de-excitation of the excited atoms and the movement of the electrons/holes/ions in electric field are the basis for signals in our particle detectors.

Let's make a more quantitative assessment of this …



Ionization and Excitation



While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer

$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 (b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}} \qquad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter

$$F_y = \frac{\gamma Z_1 Z_2 e_0^2 b}{4\pi\varepsilon_0 (b^2 + \gamma^2 v^2 t^2)^{3/2}} \qquad \Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The transferred energy is then $\quad \Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2}$

$$\Delta E(electrons) = Z_2 \frac{1}{m_e} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \Delta E(nucleus) = \frac{Z_2^2}{2 Z_2 m_p} \frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \frac{\Delta E(electrons)}{\Delta E(nucleus)} = \frac{2 m_p}{m_e} \approx 4000$$

→ The incoming particle transfer energy only (mostly) to the atomic electrons !

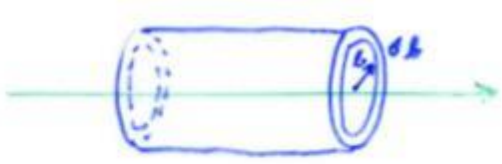
Ionization and Excitation

Target material: mass A, Z_2, density ρ [g/cm^3], Avogadro number N_A

A gramm → N_A Atoms: Number of atoms/cm^3 $n_a = N_A\,\rho/A$ [1/cm^3]

Number of electrons/cm^3 $n_e = N_A\,\rho Z_2/A$ [1/cm^3]

$$\Delta E(electrons) = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2}\frac{e_0^4}{(4\pi\varepsilon_0 m_e c^2)^2} = \frac{2Z_2 Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$



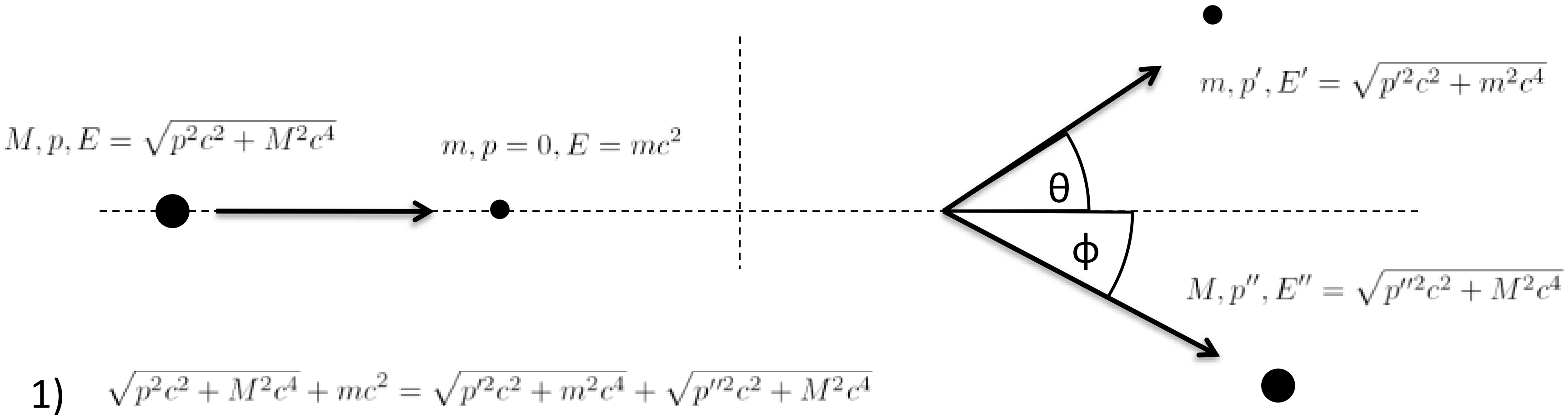
$$dE = -\int_{b_{min}}^{b_{max}} n_e \Delta E dx 2b\pi db = -\frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2}\frac{N_A \rho}{A}\int_{b_{min}}^{b_{max}}\frac{db}{b}$$

With $\Delta E(b)$ → $db/b = -1/2\; dE/E$ → $E_{max} = \Delta E(b_{min})$ $E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2}\frac{N_A Z_2 \rho}{A}\int_{E_{min}}^{E_{max}}\frac{dE}{E} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2}\frac{N_A Z_2 \rho}{A}\ln\frac{E_{max}}{E_{min}}$$

$E_{min} \approx I$ (Ionization Energy)

Relativistic Collision Kinematics, E_{max}



1) $$\sqrt{p^2c^2 + M^2c^4} + mc^2 = \sqrt{p'^2c^2 + m^2c^4} + \sqrt{p''^2c^2 + M^2c^4}$$

2) $$p = p' \cos\theta + p'' \cos\phi$$
$$0 = p' \sin\theta + p'' \sin\phi$$
$$p''^2 = p'^2 + p^2 - 2pp' \cos\theta$$

1+2) $$E^{k'} = \sqrt{p'^2c^2 + m^2c^4} - mc^2 = \frac{2mc^2\, p^2c^2 \cos^2\theta}{\left[mc^2 + \sqrt{p^2c^2 + M^2c^4}\right]^2 - p^2c^2\cos^2\theta}$$

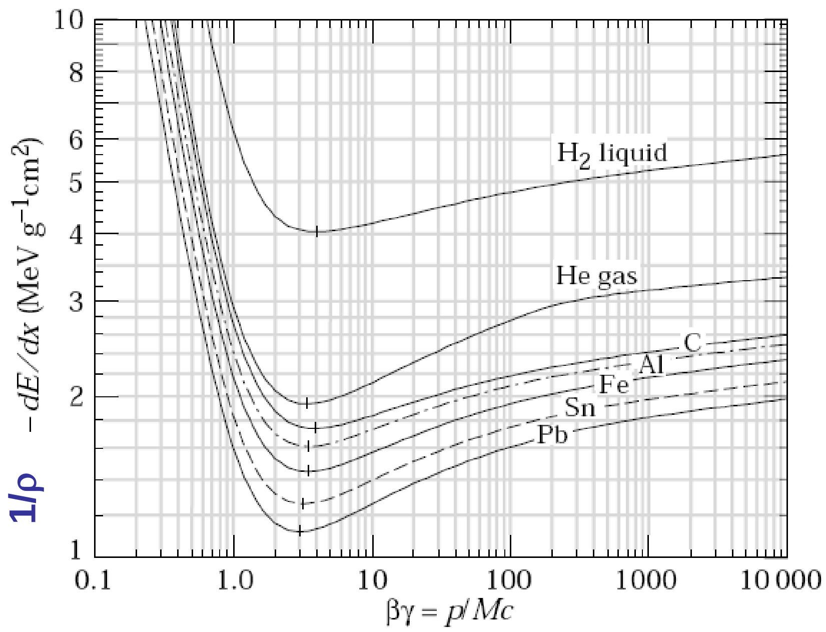
$$E^{k'}_{max} = \frac{2mc^2p^2c^2}{(m^2 + M^2)c^4 + 2m\sqrt{p^2c^2 + M^2c^4}} = 2mc^2\beta^2\gamma^2 F \qquad F = \left(1 + \frac{2m}{M}\sqrt{1 + \beta^2\gamma^2} + \frac{m^2}{M^2}\right)^{-1}$$

Classical Scattering on Free Electrons

$$\frac{1}{\rho}\frac{dE}{dx} = -2\pi r_e^2\, m_e c^2\, \frac{Z_1^2}{\beta^2}\, N_A \frac{Z}{A} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 F}{I}$$

This formula is up to a factor 2 and the density effect identical to the precise QM derivation →

Bethe Bloch Formula



$$\frac{1}{\rho}\frac{dE}{dx} = \underline{-4}\pi r_e^2\, m_e c^2\, \frac{Z_1^2}{\beta^2}\, N_A \frac{Z}{A} \left[\ln \frac{2 m_e c^2 \beta^2 \gamma^2 F}{I} - \beta^2 - \underline{\frac{\delta(\beta\gamma)}{2}}\right]$$

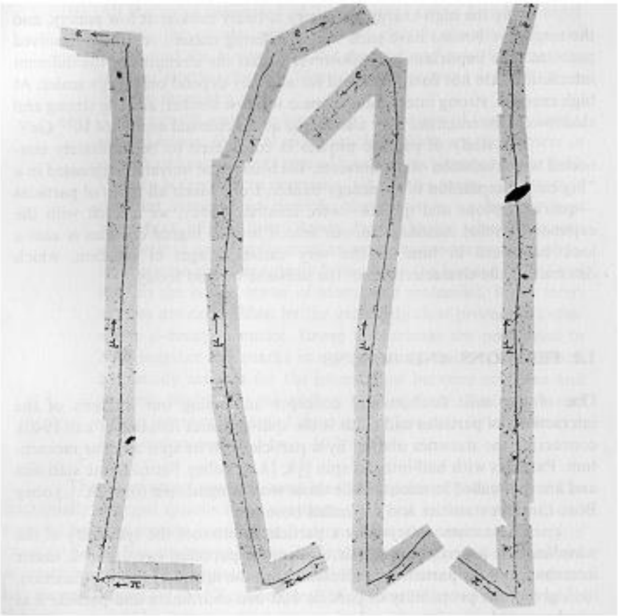
Electron Spin

$$\delta(\beta\gamma) = \ln \hbar\omega_p/I + \ln \beta\gamma - \frac{1}{2}$$

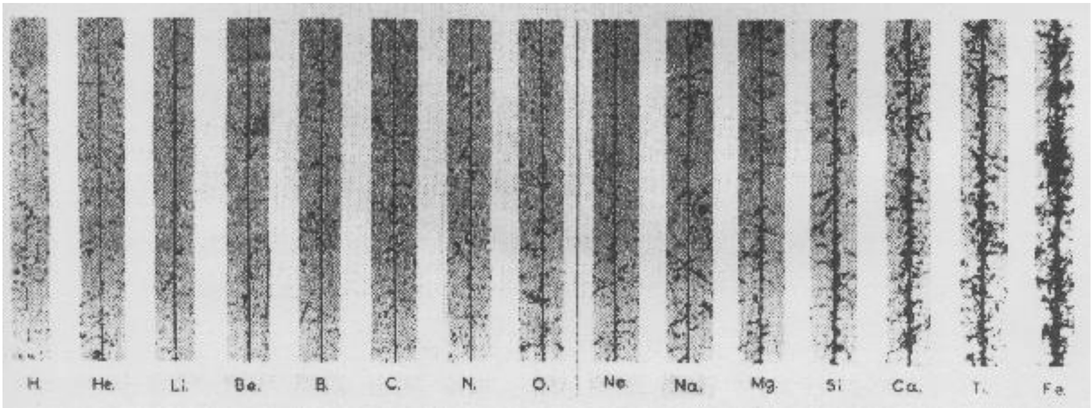
Density effect. Medium is polarized Which reduces the log. rise.

Bethe-Bloch formula

Small energy loss
→ Fast Particle



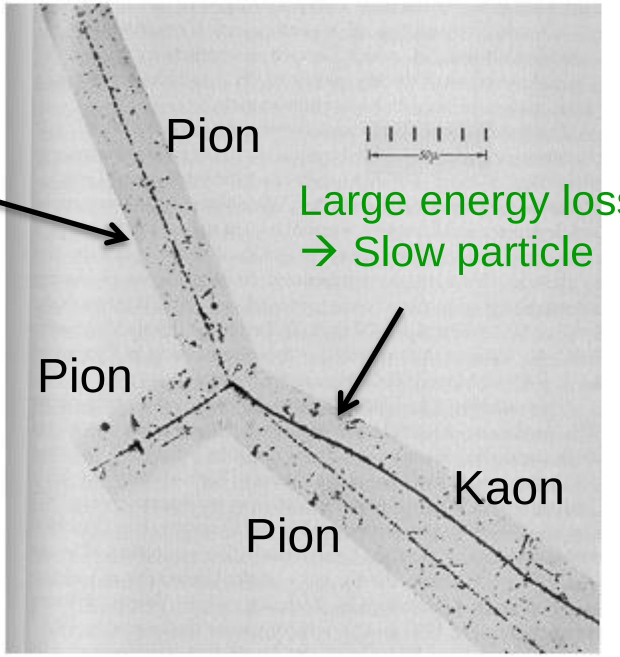
Discovery of muon and pion



Cosmis rays: dE/dx α Z^2

Small energy loss
→ Fast particle

Large energy loss
→ Slow particle



Bethe-Bloch formula

Bethe Bloch Formula, a few Numbers:

For Z ≈ 0.5 A

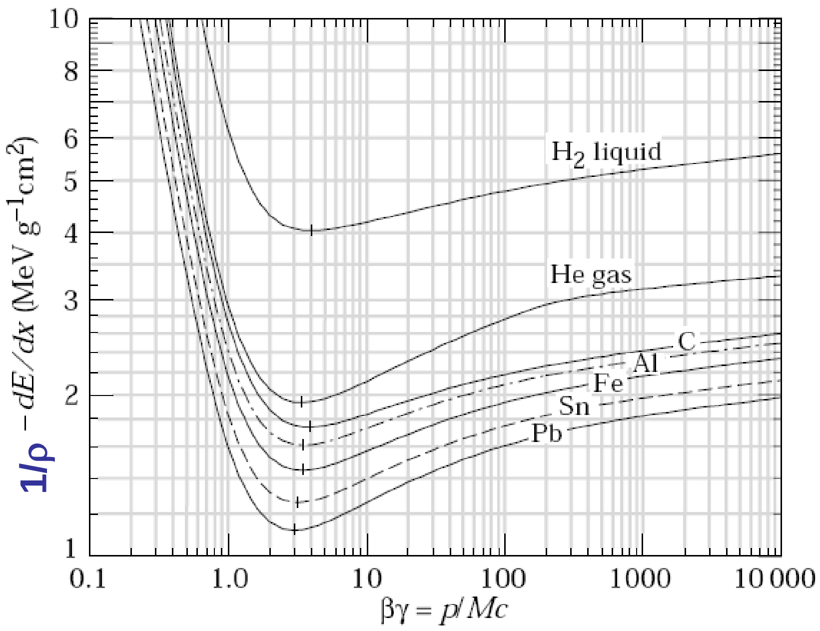
1/ρ dE/dx ≈ 1.4 MeV cm ^{2}/g for ßγ ≈ 3

Example:

Iron: Thickness = 100 cm; ρ = 7.87 g/cm^3

dE ≈ 1.4 * 100* 7.87 = 1102 MeV

→ A 1 GeV Muon can traverse 1m of Iron



Range of Particles in Matter

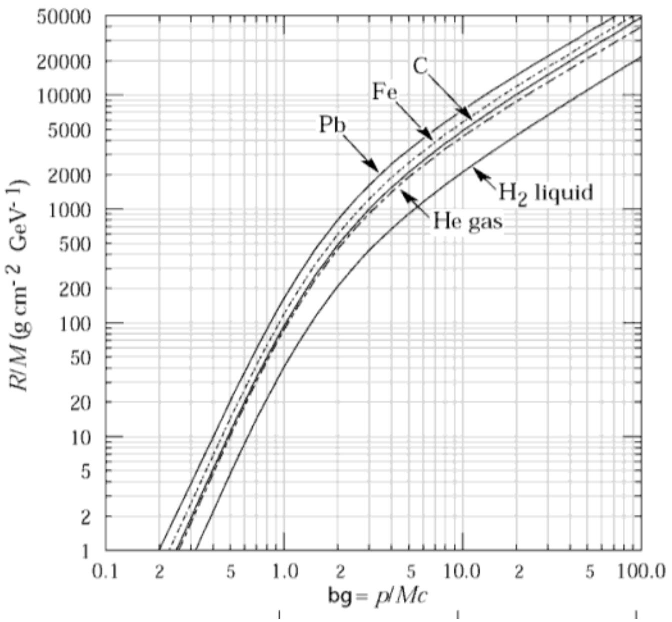
Particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{-1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0\gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

≈ independent of the material

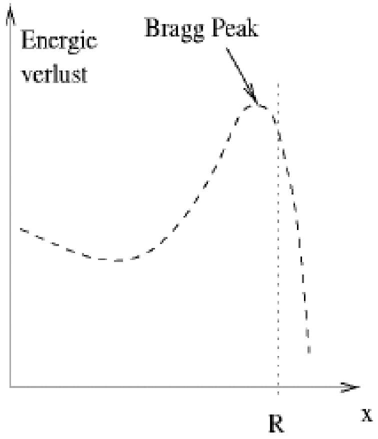


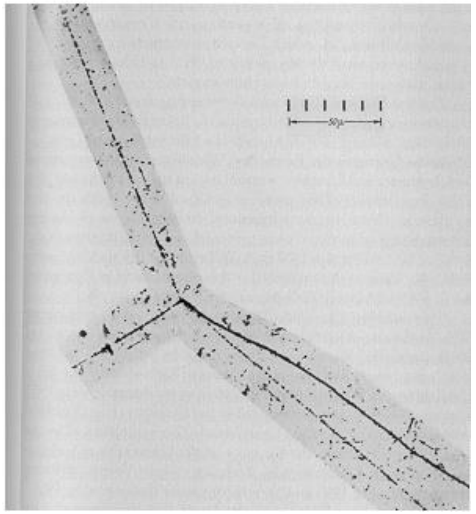
Bragg Peak:

For βγ>3 the energy loss is ≈ constant (Fermi Plateau)

If the energy of the particle falls below βγ=3 the energy loss rises as $1/\beta^2$

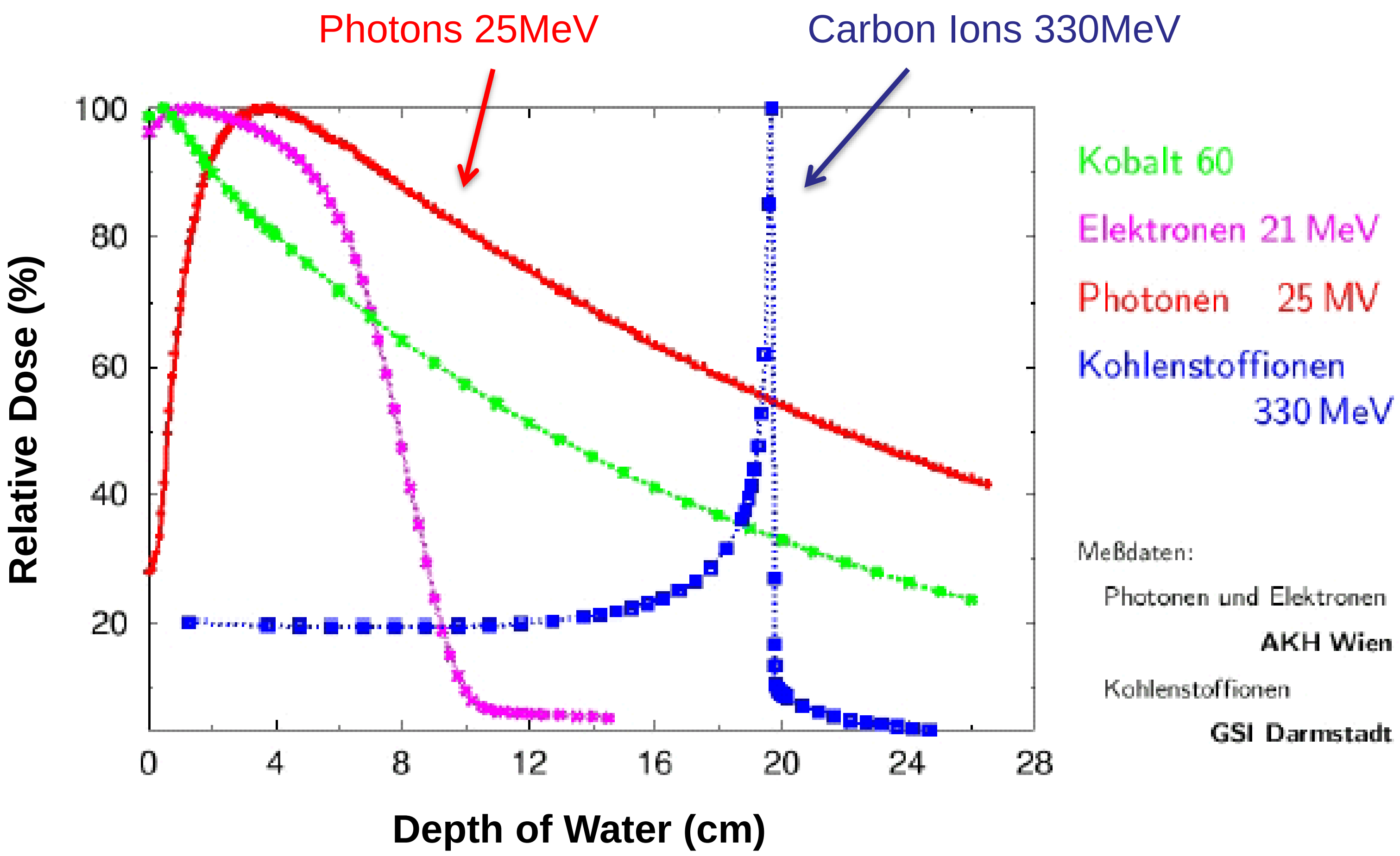
Towards the end of the track the energy loss is largest → Cancer Therapy.





Range of Particles in Matter

Average Range:
Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



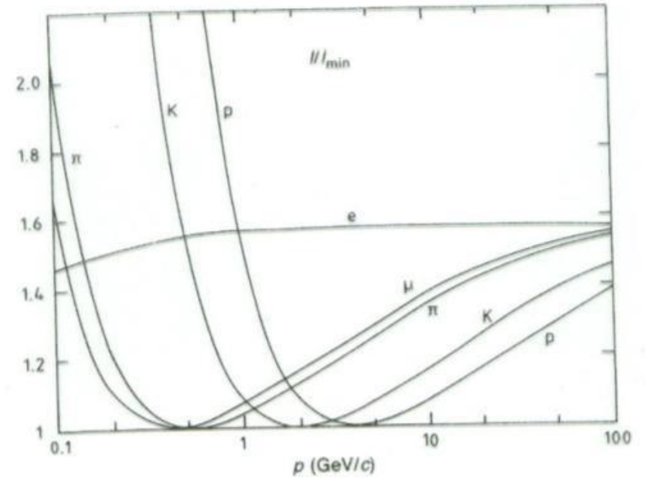
Energy loss as a function of the momentum

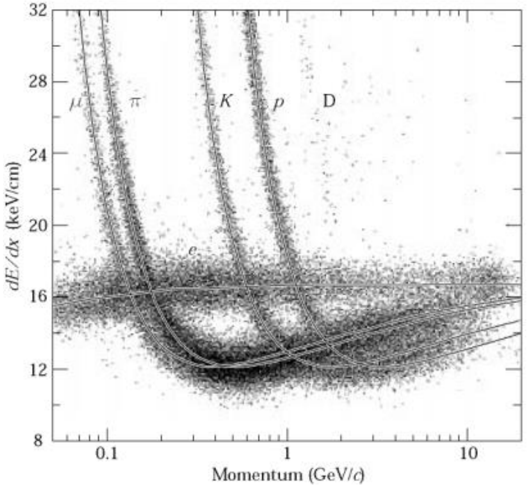
Energy loss depends on the particle velocity and is ≈ independent of the particle's mass M.

The energy loss as a function of particle momentum p = Mcβγ IS however depending on the particle's mass

By measuring the particle momentum (deflection in the magnetic field) and measurement of the energy loss on can measure the particle mass

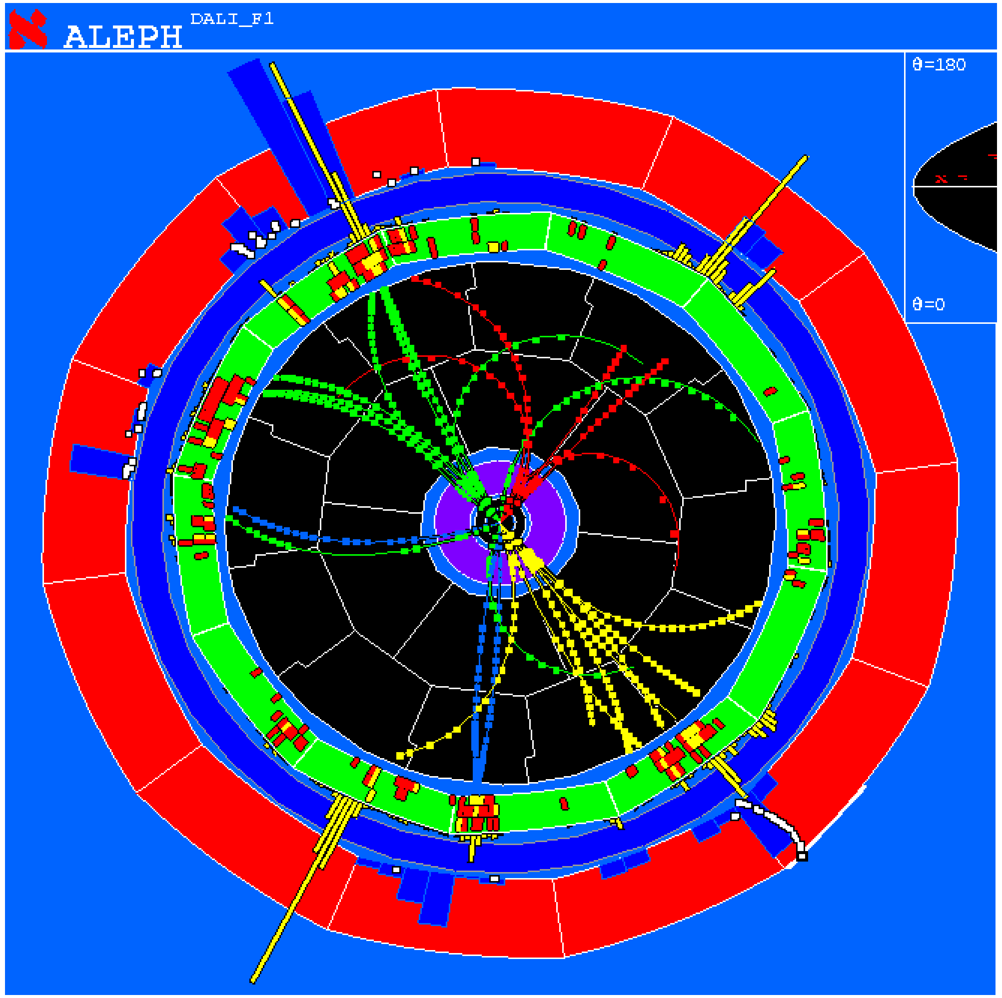
→ Particle Identification !





$$\frac{1}{\rho}\frac{dE}{dx} = -4\pi r_e^2\, m_e c^2\, Z_1^2 \frac{p^2+M^2c^2}{p^2}\, N_A \frac{Z}{A}\left[\ln\frac{2m_e c^2 F}{I}\frac{p^2}{M^2c^2} - \frac{p^2}{p^2+M^2c^2}\right]$$

Energy loss as a function of the momentum



Measure momentum by curvature of the particle track.

Find dE/dx by measuring the deposited charge along the track.

→Particle ID

Landau Distribution

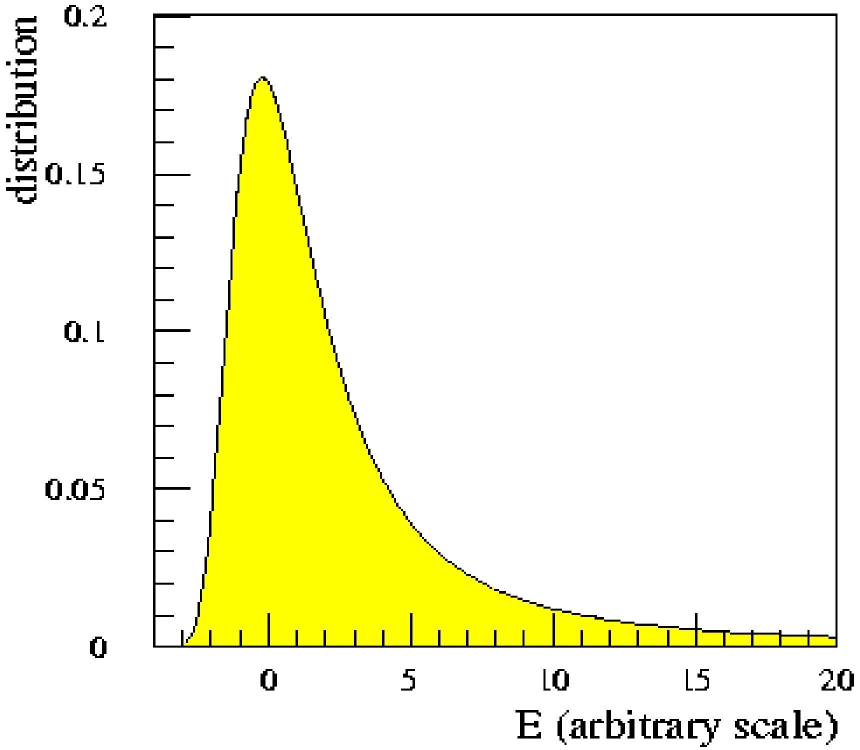
P(Δ): Probability for energy loss Δ in matter of thickness D.

Landau distribution is very asymmetric.

Average and most probable energy loss must be distinguished !

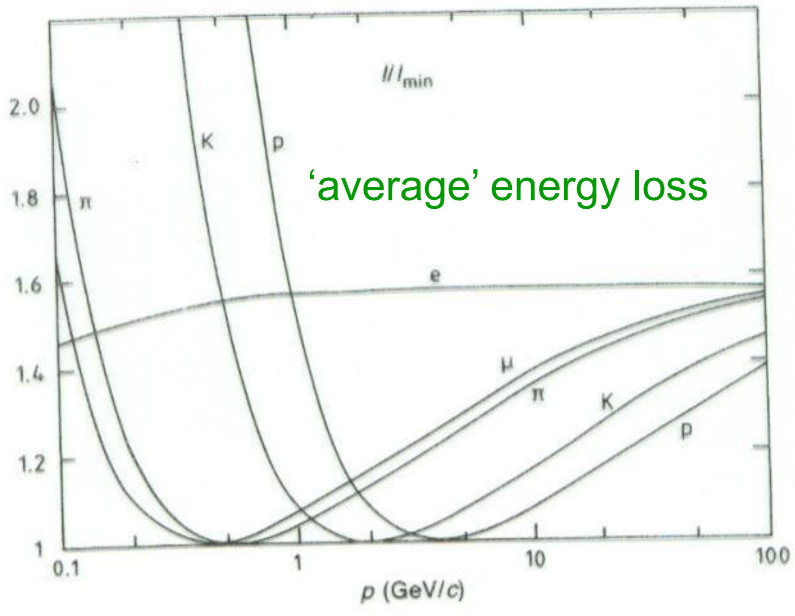
Measured Energy Loss is usually smaller that the real energy loss:

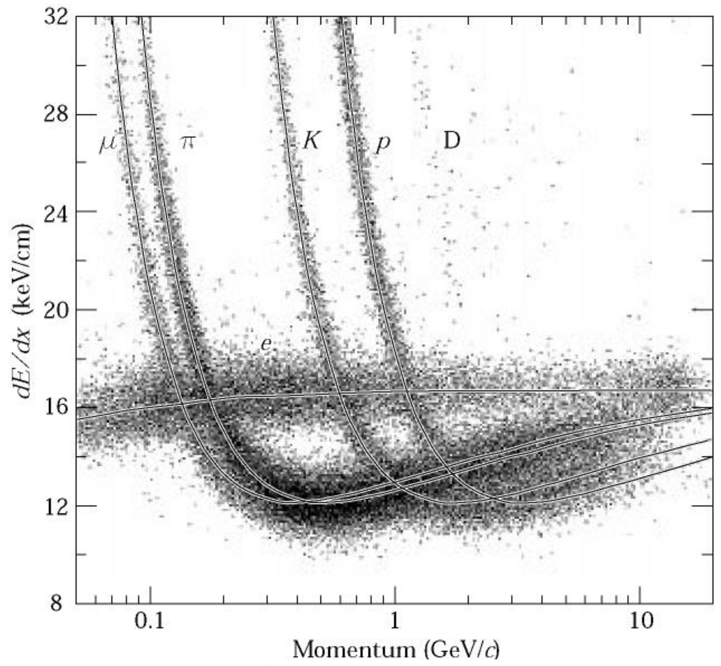
3 GeV Pion: E'_{max} = 450MeV → A 450 MeV Electron usually leaves the detector.

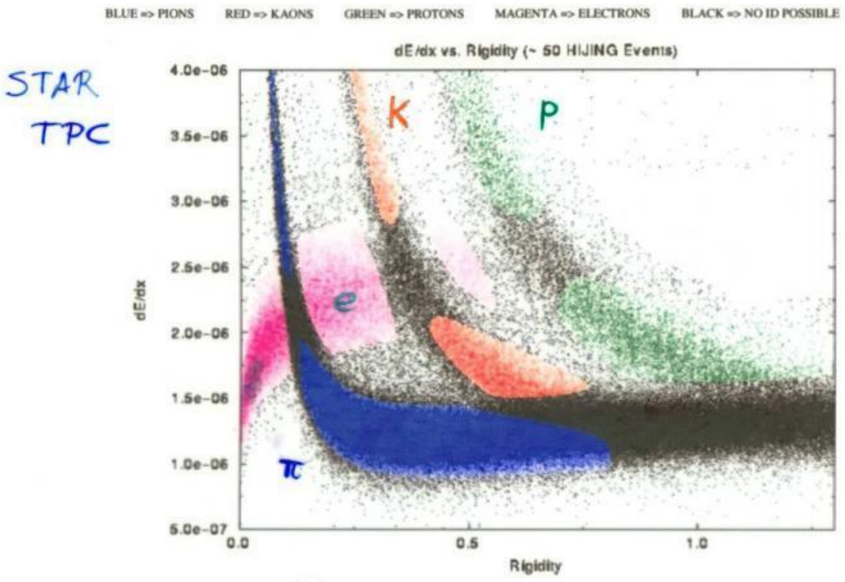


Particle Identification

Measured energy loss

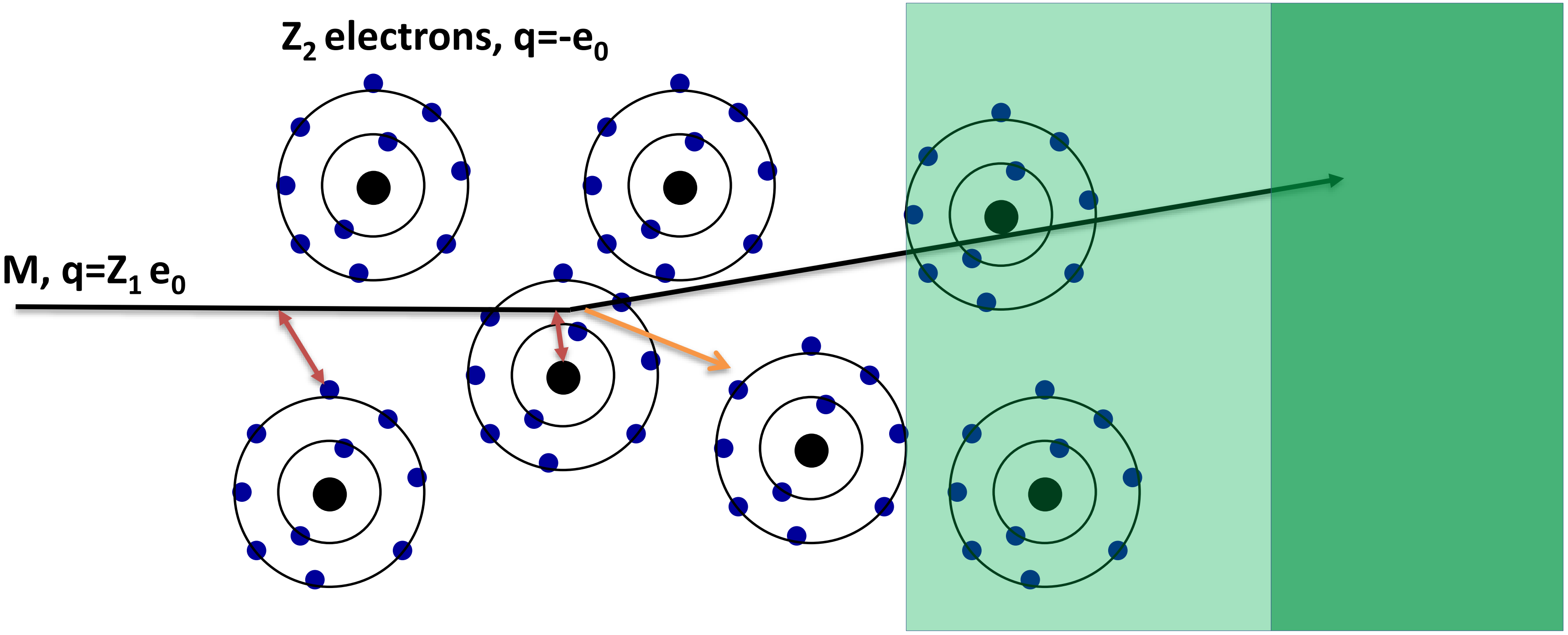






In certain momentum ranges, particles can be identified by measuring the energy loss.

Electromagnetic Interaction of Particles with Matter



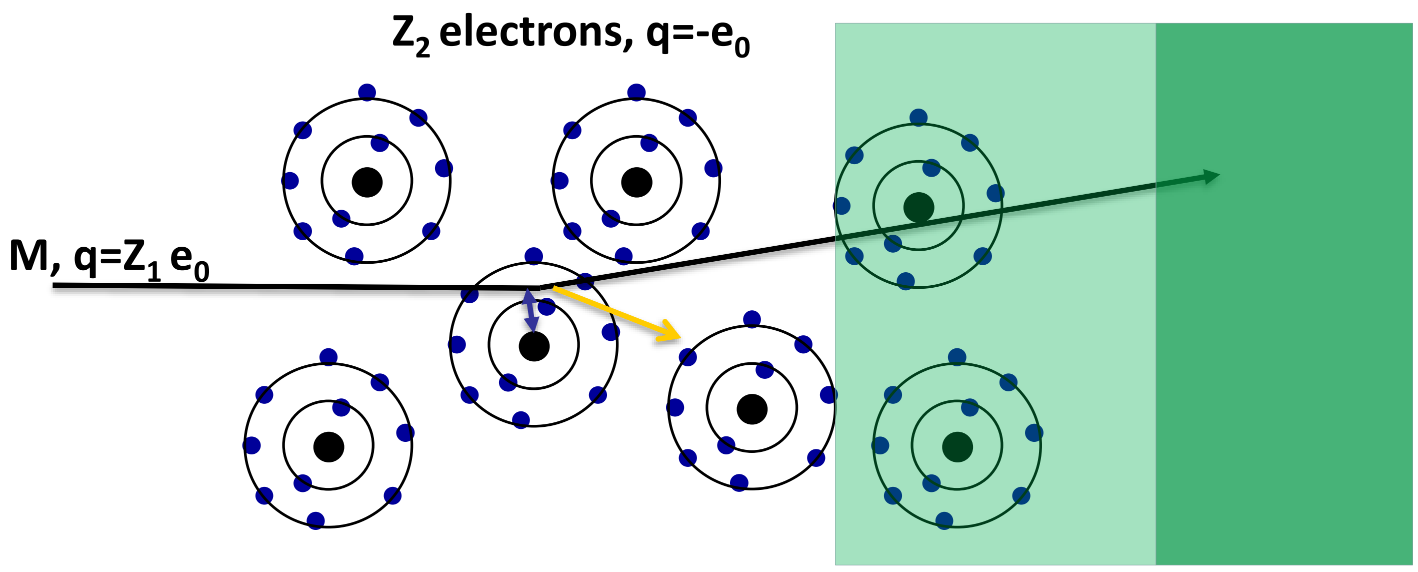
Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

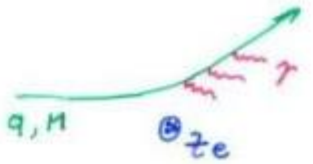
In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.

Bremsstrahlung

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated → Bremsstrahlung.



Bremsstrahlung, Classical



$q = Z_1 \cdot e$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 Z_1 Z_2 e^2}{4\pi\varepsilon_0 \, p \cdot v}\right)^2 \frac{1}{(2\sin\frac{\theta}{2})^4} \qquad p = M\gamma v$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer $Q^2 = 2p^2(1-\cos\theta)$

$$\frac{d\sigma}{dQ} = 8\pi \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \beta c}\right)^2 \cdot \frac{1}{Q^3}$$

$Q = |\vec{p} - \vec{p}\,'|$

→ From Maxwell's Eq (Jackson)

$$\lim_{\omega \to 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\varepsilon_0} Q^2$$ Radiated Energy between $\omega, \omega + d\omega$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \int_0^{\omega_{max}} d\omega \int_{Q_{min}}^{Q_{max}} dQ \, \frac{dI}{d\omega} \cdot \frac{d\sigma}{dQ} \qquad , \omega_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \frac{16}{3} \alpha \cdot Z^2 \cdot \left(\frac{Z_1^2 e^2}{4\pi\varepsilon_0 M c^2}\right)^2 \cdot E \cdot \ln \frac{Q_{max}}{Q_{min}}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \sim \frac{1}{137}$$

A charged particle of mass M and charge $q = Z_1 e$ is deflected by a nucleus of Charge Ze.

Because of the acceleration the particle radiated EM waves → energy loss.

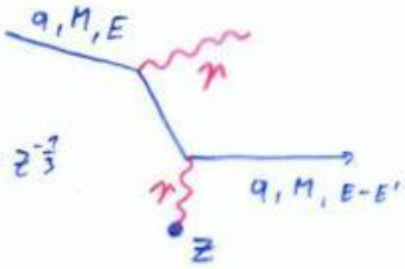
Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

→ dE/dx

Bremsstrahlung, QM

2b Bremsstrahlung Q.M.



$q = Z_1 e, \quad E + Mc^2 \gg 137\, Mc^2 Z^{-\frac{1}{3}}$

→ Highly Relativistic :

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{Mc^2}\right)^2 \frac{1}{E'} F(E,E')$$

$$F(E,E') = \left[1 + \left(1-\frac{E'}{E+Mc^2}\right)^2 - \frac{2}{3}\left(1-\frac{E'}{E+Mc^2}\right)\right] \ln 183 Z^{-\frac{1}{3}} + \frac{1}{9}\left(1-\frac{E'}{E+Mc^2}\right)$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A}\int_0^E E' \frac{d\sigma}{dE'} dE' \approx 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{Mc^2}\right)^2 E \left[\ln 183 Z^{-\frac{1}{3}} + \frac{1}{18}\right]$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A} 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{Mc^2}\right)^2 E \ln\left(183 Z^{-\frac{1}{3}}\right)$$

$$E(x) = E_0 e^{-\frac{x}{X_0}} \qquad X_0 = \frac{A}{4\alpha N_A \rho Z^2 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{Mc^2}\right)^2 \ln 183 Z^{-\frac{1}{3}}}$$

X_0 ... Radiation length

Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional to ρ of the material.

Proportional $1/M^2$ of the incoming particle.

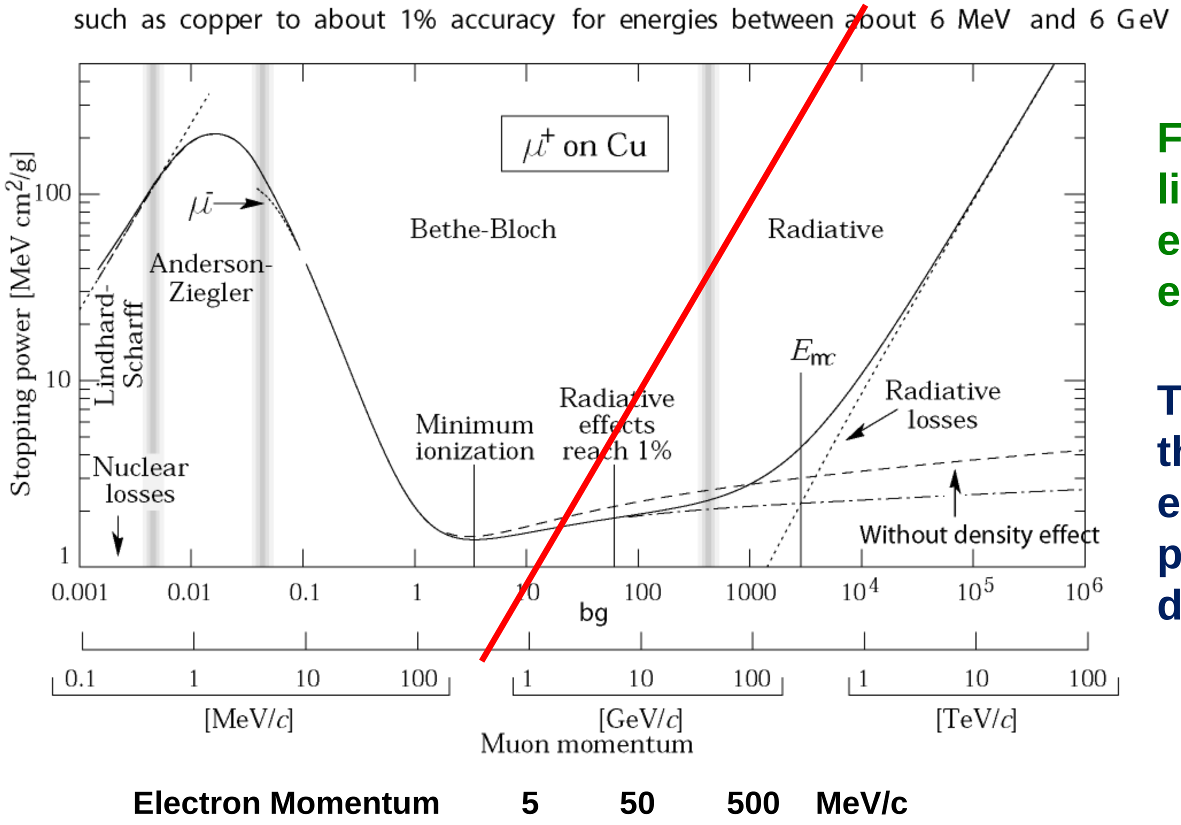
Proportional to the Energy of the Incoming particle →

$E(x)=Exp(-x/X_0)$ – 'Radiation Length'

$X_0 \propto M^2 A / (\rho\, Z_1^4\, Z^2)$

X_0: Distance where the Energy E_0 of the incoming particle decreases $E_0 Exp(-1)=0.37 E_0$.

Critical Energy



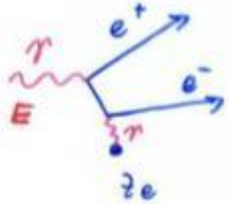
For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

The EM Bremsstrahlung is therefore only relevant for electrons at energies of past and present detectors.

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

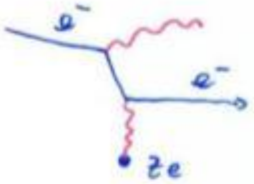
Myon in Copper: p ≈ 400GeV
Electron in Copper: p ≈ 20MeV

Pair Production, QM



$\gamma + \text{Nucl.} \rightarrow e^+ + e^- + \text{Nucl}$

The Diagram is very similar to Bremsstrahlung



$e^- + \text{Nucl} \rightarrow \gamma + e^- + \text{Nucl}$

'Crossing Symmetry': bring particle to the other side and make it the antiparticle → 'some' correction ...

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 r_e^2 \frac{1}{E} \cdot G(E,E') \qquad E \gg 137 m_e c^2 Z^{-\frac{1}{3}}$$

$$G(E,E') = \left[\left(\frac{E'+m_ec^2}{E}\right)^2 + \left(1-\frac{E'+m_ec^2}{E}\right)^2 + \frac{2}{3}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\ln Z^{-\frac{1}{3}} - \frac{1}{9}\frac{E'+m_ec^2}{E}\left(1-\frac{E'+m_ec^2}{E}\right)\right]$$

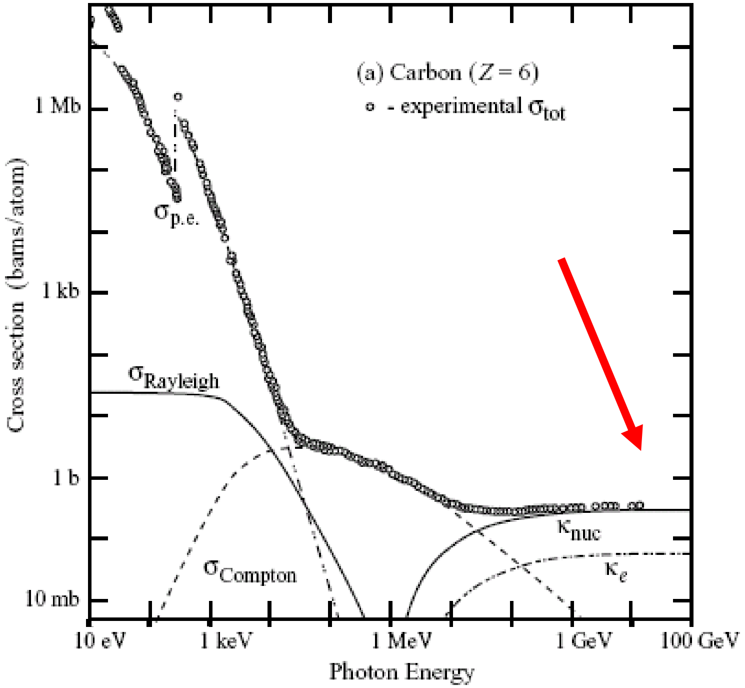
$$\sigma = \int_0^{E-2m_ec^2} \frac{d\sigma}{dE'} dE' = 4\alpha Z^2 r_e^2 \cdot \frac{7}{9} \ln 183 Z^{-\frac{1}{3}}$$

$$P(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \qquad \lambda = \frac{A}{\rho N_A \sigma} = \frac{9}{7} X_0$$

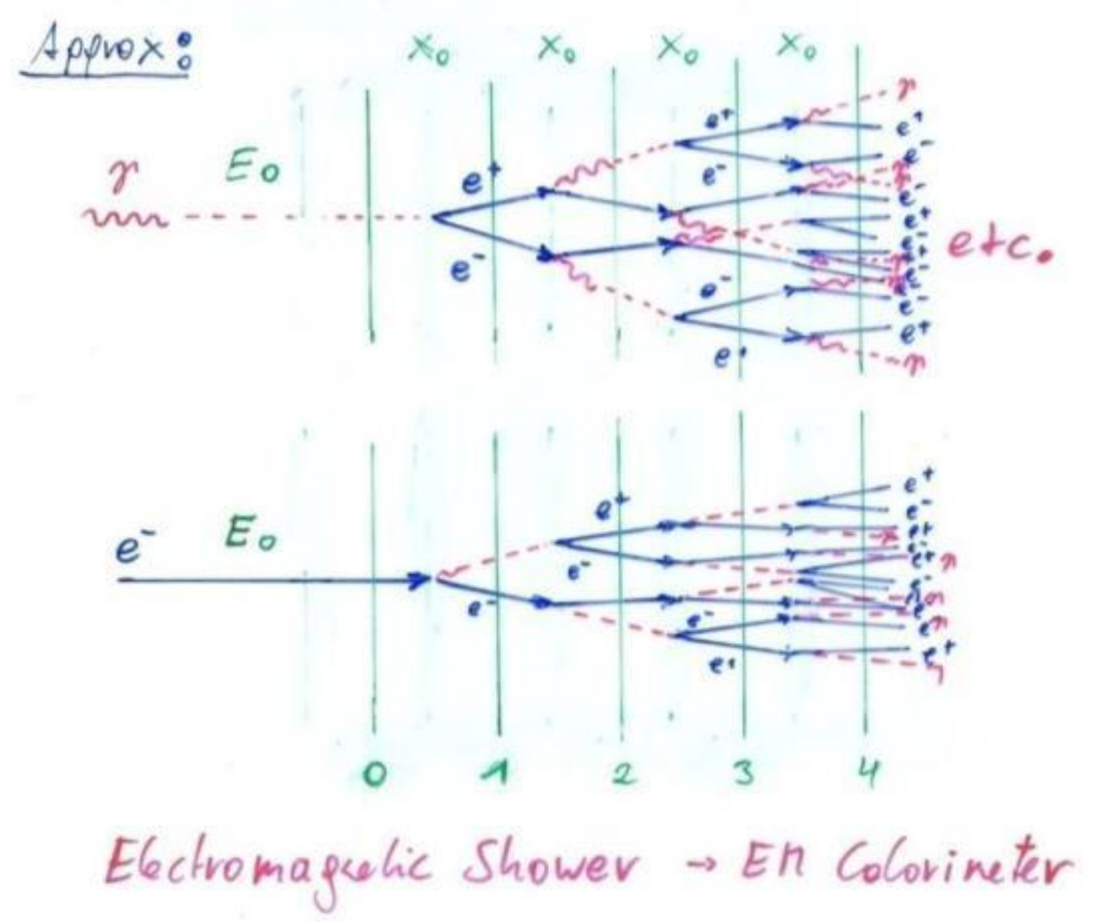
↳ Probability that Photon converts to $e^+ e^-$ after a distance x.

For $E\gamma \gg m_e c^2 = 0.5$MeV : $\lambda = 9/7 X_0$

Average distance a high energy photon has to travel before it converts into an $e^+ e^-$ pair is equal to 9/7 of the distance that a high energy electron has to travel before reducing it's energy from E_0 to E_0*Exp(-1) by photon radiation.



Bremsstrahlung + Pair Production → EM Shower



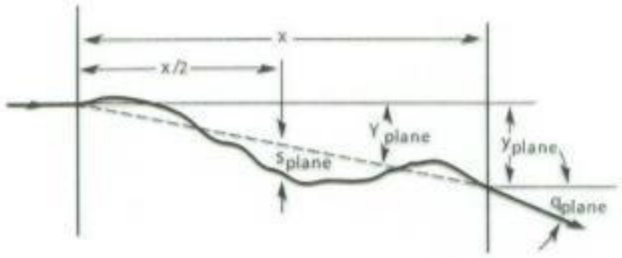
Multiple Scattering

Statistical (quite complex) analysis of multiple collisions gives:

Probability that a particle is defected by an angle θ after travelling a distance x in the material is given by a Gaussian distribution with sigma of:

$$\Theta_0 = \frac{0.0136}{\beta c p [\mathrm{GeV/c}]} Z_1 \sqrt{\frac{x}{X_0}}$$

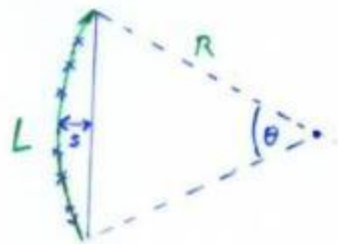
X_0 ... Radiation length of the material
Z_1 ... Charge of the particle
p ... Momentum of the particle



Multiple Scattering

Magnetic Spectrometer: A charged particle describes a circle in a magnetic field:

$\vec{B} \otimes$



$$p = q \cdot R \cdot B$$

$$p\left[\frac{GeV}{c}\right] = 0.3\, R[m]\, B[T]$$

$$L = R \cdot \theta$$

$$s = R\left(1 - \cos\frac{\theta}{2}\right) \approx R\frac{\theta^2}{8} = \frac{L^2}{8R} \rightarrow R = \frac{L^2}{8s}$$

$$\Delta p = 0.3\, B\, \Delta R = 0.3\, B\, \frac{L^2}{8s^2}\Delta s$$

$$\Delta s = \frac{\sigma_x}{\sqrt{N}}$$ σ_x ... point resolution, N ... Measurement Points

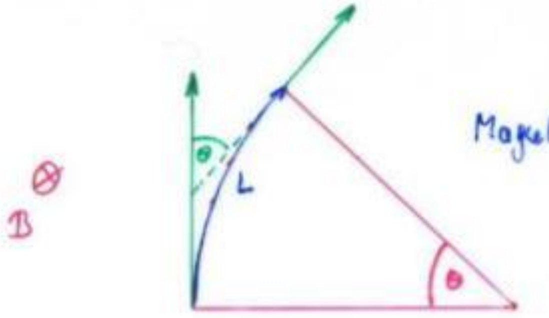
$$\boxed{\frac{\Delta p}{p} = \frac{\Delta s}{s} = \frac{\sigma_x[m]}{\sqrt{N}} \cdot \frac{3.3 \cdot 8\, p\left[\frac{GeV}{c}\right]}{B[T] \cdot L^2[m^2]}}$$

E.g.: $p = 10\,\frac{GeV}{c}$, $B = 1T$, $L = 1m$, $\sigma = 200\mu m$, $N = 25$

$$\frac{\Delta p}{p} = 0.01 \rightarrow 1\%$$

Limit → Multiple Scattering

Multiple Scattering



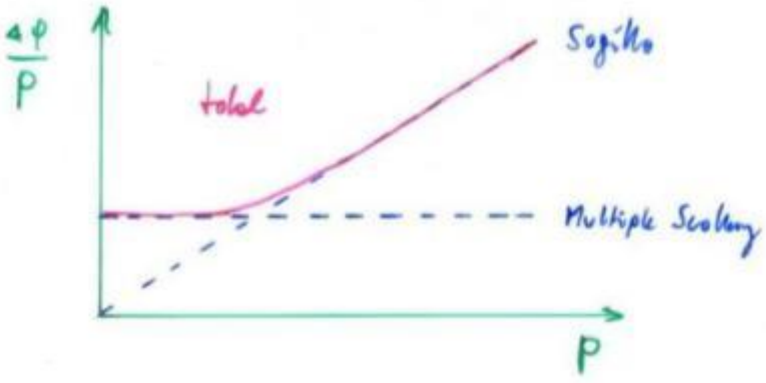
$$p\left[\frac{GeV}{c}\right] = 0.3\, R[m]\, B[T]$$

$$\theta = \frac{L}{R} = \frac{L}{p} \cdot 0.3\, B$$

$$\frac{\Delta p}{p} = \frac{\Delta\theta}{\theta} = \frac{\theta_0}{\theta} \sim \frac{0.05}{\beta B[T] L[m]} \sqrt{\frac{L}{X_0}}$$

→ Independent of p

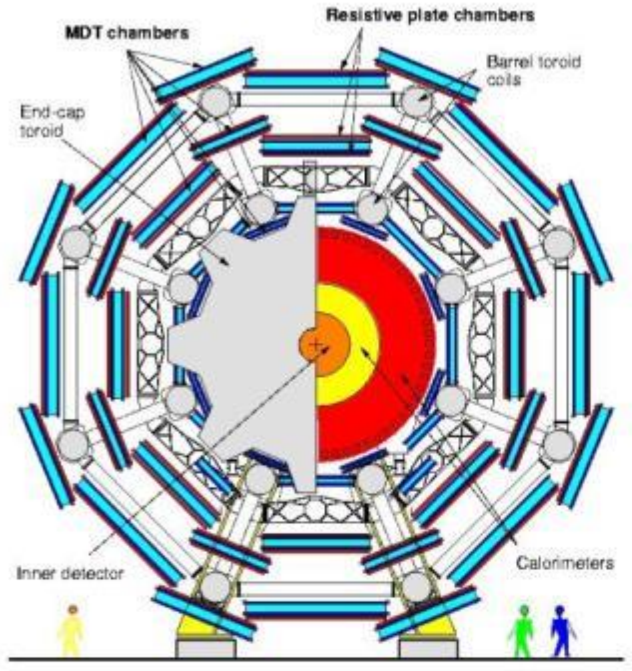
$$\left.\frac{\Delta p}{p}\right|_{tot} = \sqrt{\left(\left.\frac{\Delta p}{p}\right|_{sag}\right)^2 + \left(\left.\frac{\Delta p}{p}\right|_{ms}\right)^2}$$

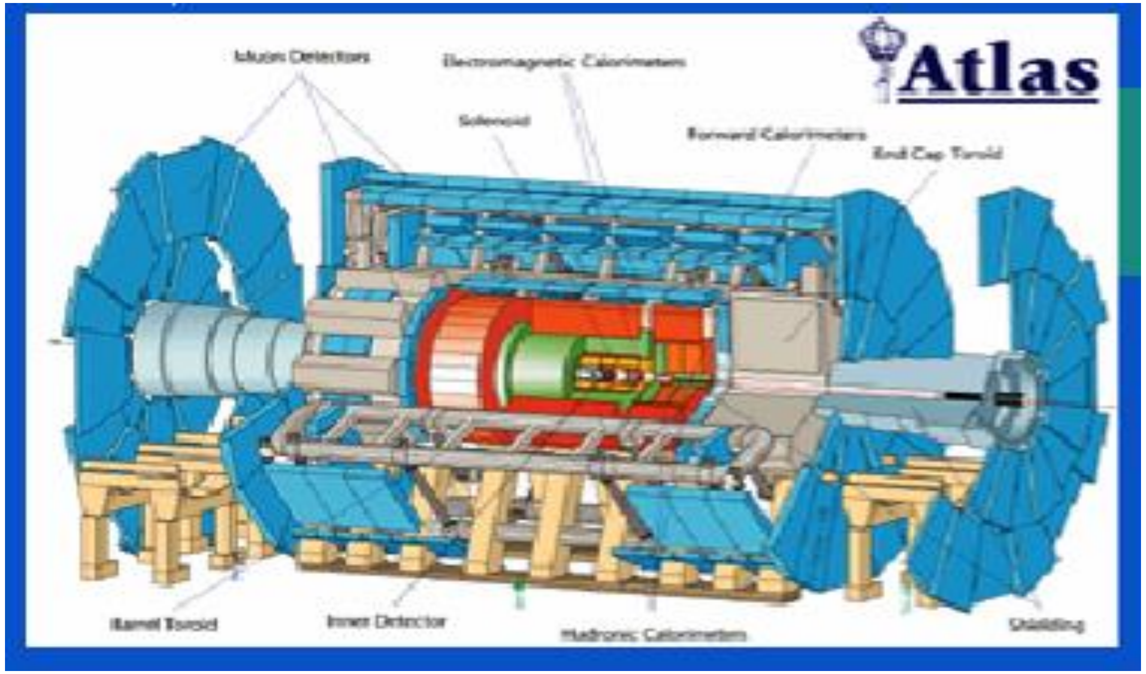


Multiple Scattering

ATLAS Muon Spectrometer:
N=3, sig=50µm, P=1TeV,
L=5m, B=0.4T

Δp/p ~ 8% for the most energetic muons at LHC





Cherenkov Radiation

If we describe the passage of a charged particle through material of dielectric permittivity ε (using Maxwell's equations) the differential energy cross-section is >0 if the velocity of the particle is larger than the velocity of light in the medium is

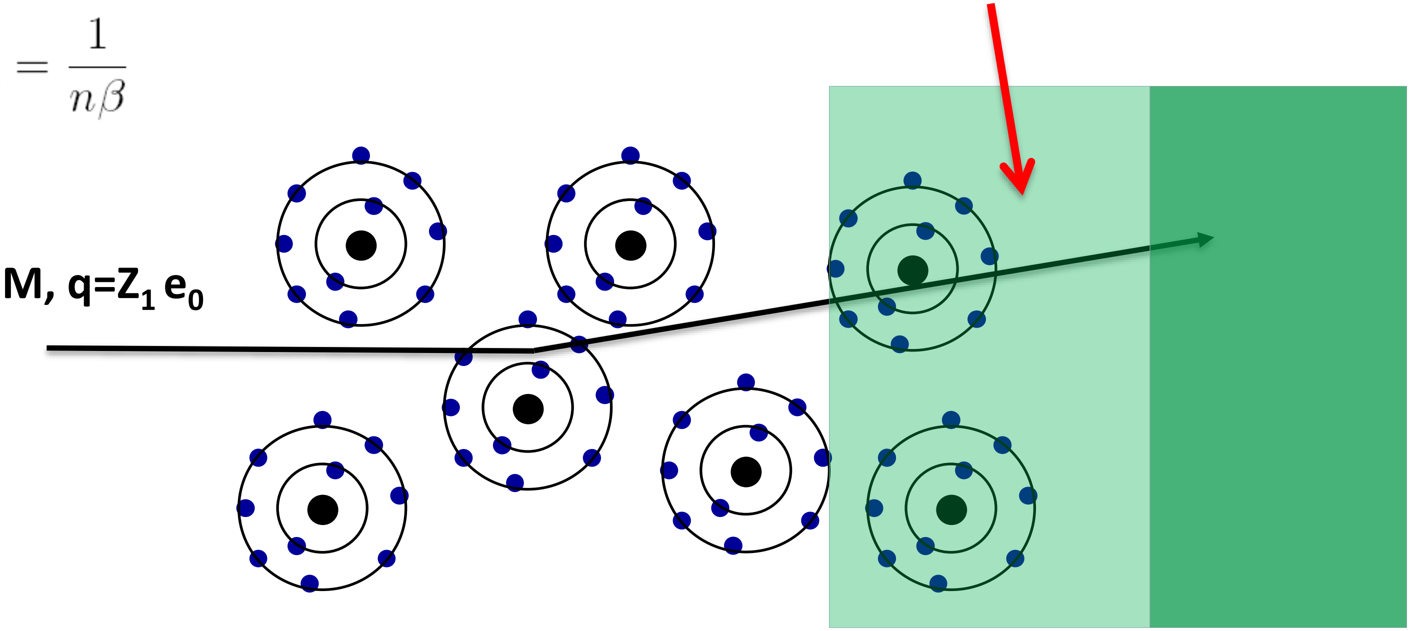
$$\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2\pi}\frac{A}{N_A\rho Z_2\hbar c}\left(\beta^2 - \frac{1}{\epsilon_1}\right) \quad \rightarrow \quad \frac{N_A\rho Z_2}{A}\frac{d\sigma}{d\omega}\frac{d\omega}{dE} = \frac{\alpha}{c}\left(1 - \frac{1}{\beta^2 n^2}\right) \qquad n = \sqrt{\epsilon_1} \qquad E = \hbar\omega$$

$$\frac{dE}{dx d\omega}\frac{1}{\hbar} = \frac{\alpha}{c}\left(1 - \frac{1}{\beta^2 n^2}\right) \quad \rightarrow \quad \frac{dN}{dx d\lambda} = \frac{2\pi\alpha}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2}\right) \qquad \omega = \frac{2\pi c}{\lambda}$$

N is the number of Cherenkov Photons emitted per cm of material. The expression is in addition proportional to Z_1^2 of the incoming particle.

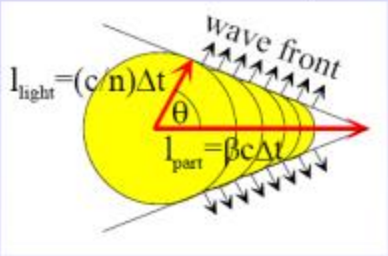
The radiation is emitted at the characteristic angle θ_c , that is related to the refractive index n and the particle velocity by

$$\cos\Theta_c = \frac{1}{n\beta}$$

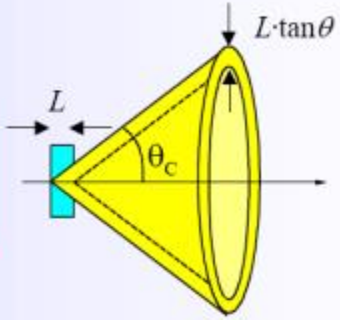


Cherenkov Radiation

with velocity $\beta \geq \beta_{thr} = \frac{1}{n}$ n: refractive index



$$\cos\theta_C = \frac{1}{n\beta}$$
with $n = n(\lambda) \geq 1$



- $\beta_{thr} = \frac{1}{n} \rightarrow \theta_C \approx 0$ Cherenkov threshold
- $\theta_{max} = \arccos\frac{1}{n}$ 'saturated' angle (β=1)

If the velocity of a charged particle is larger than the velocity of light in the medium $v > \frac{c}{n}$ (n... Refractive Index of material) it emits 'Cerenkov' radiation at a characteristic angle of $\cos\theta_c = \frac{1}{n\beta}$ $(\beta = \frac{v}{c})$

$$\frac{dN}{dx} \sim 2\pi\alpha z_1^2 \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{\lambda_2 - \lambda_1}{\lambda_2 \cdot \lambda_1}$$

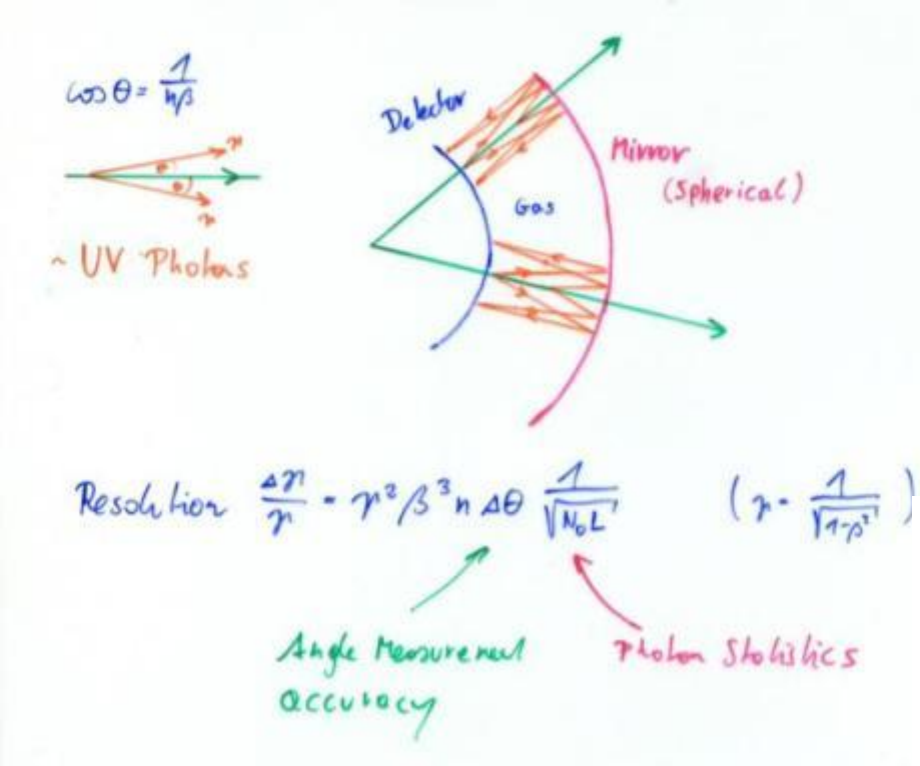
- Number of emitted Photons/length with λ between λ_1 and λ_2

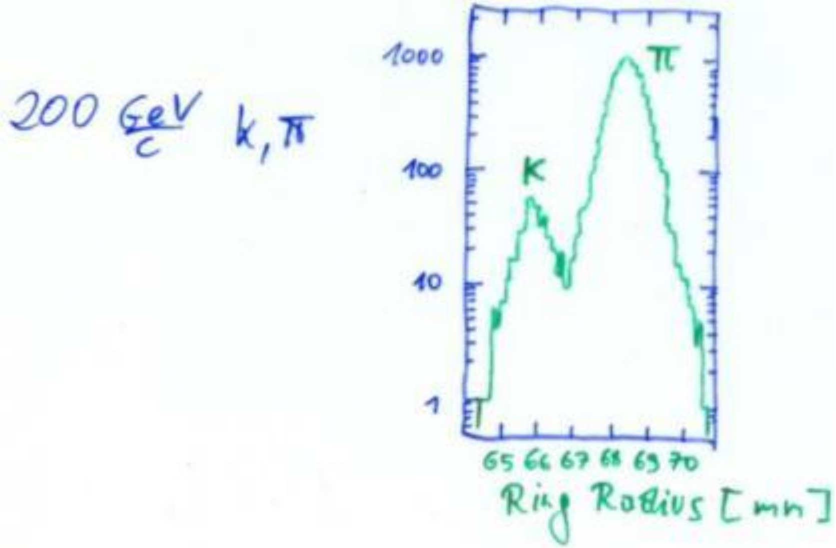
With $\lambda_1 = 400nm$ $\lambda_2 = 700nm$

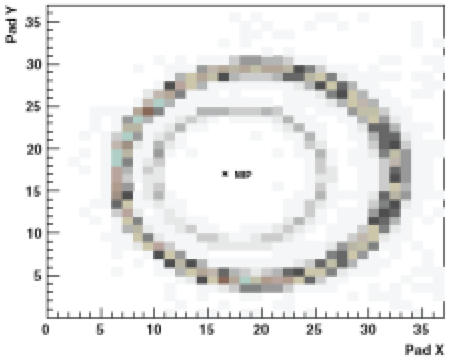
$$\frac{dN}{dx} = 490 \left(1 - \frac{1}{\beta^2 n^2}\right) \left[\frac{1}{cm}\right]$$

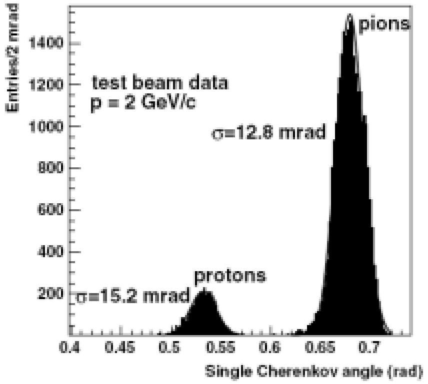
Material	n-1	β threshold	γ threshold
solid Sodium	3.22	0.24	1.029
lead glass	0.67	0.60	1.25
water	0.33	0.75	1.52
silica aerogel	0.025-0.075	0.93-0.976	2.7 - 4.6
air	$2.93 \cdot 10^{-4}$	0.9997	41.2
He	$3.3 \cdot 10^{-5}$	0.99997	123

Ring Imaging Cherenkov Detector (RICH)





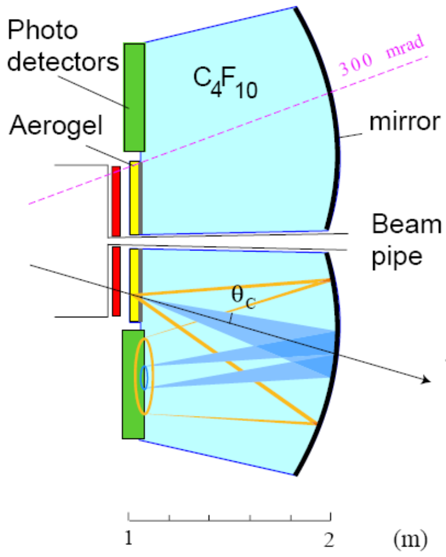


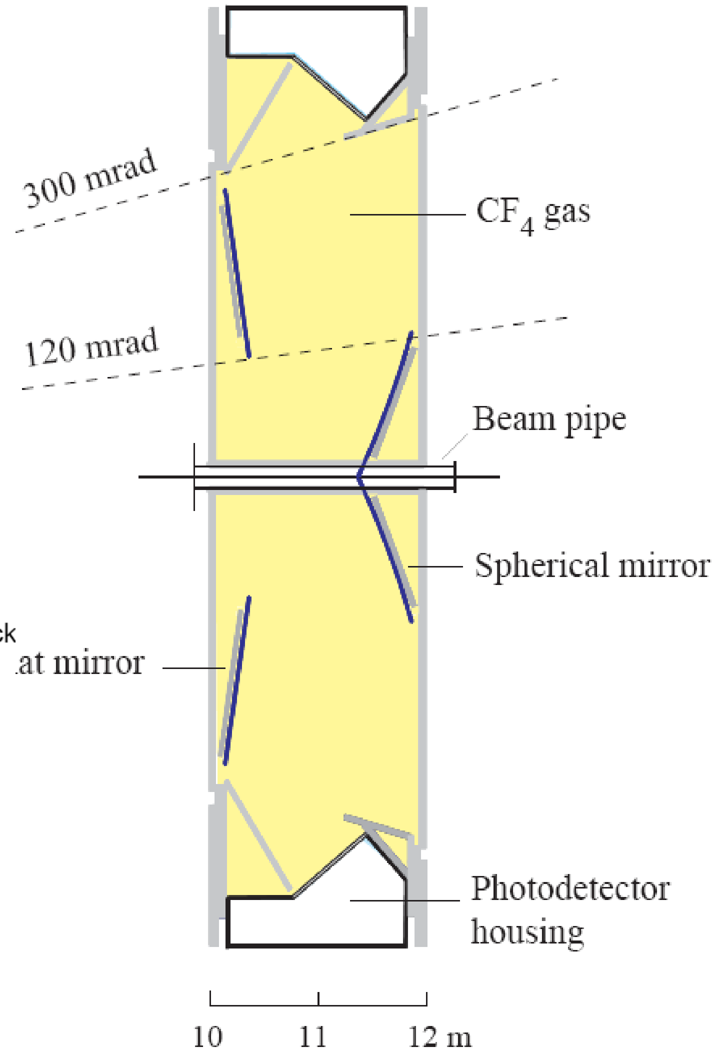


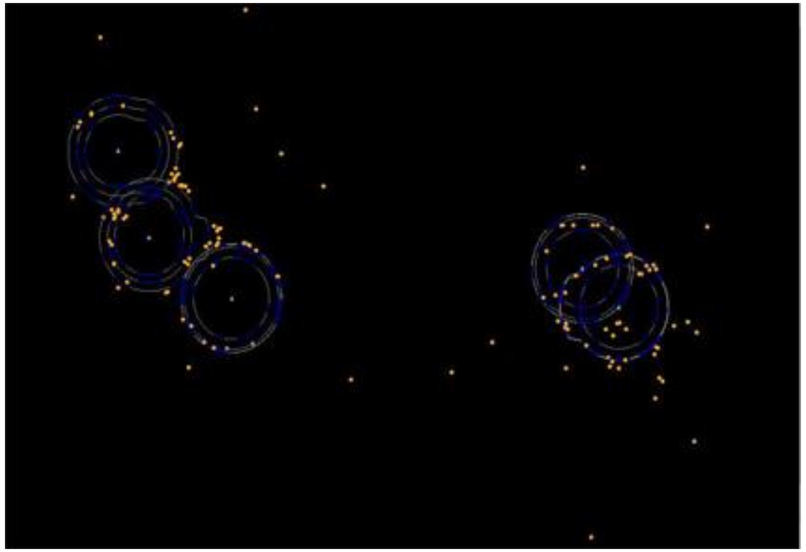
medium	n	θ_{max} (deg.)	N_{ph} (eV^{-1} cm^{-1})
air*	1.000283	1.36	0.208
isobutane*	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

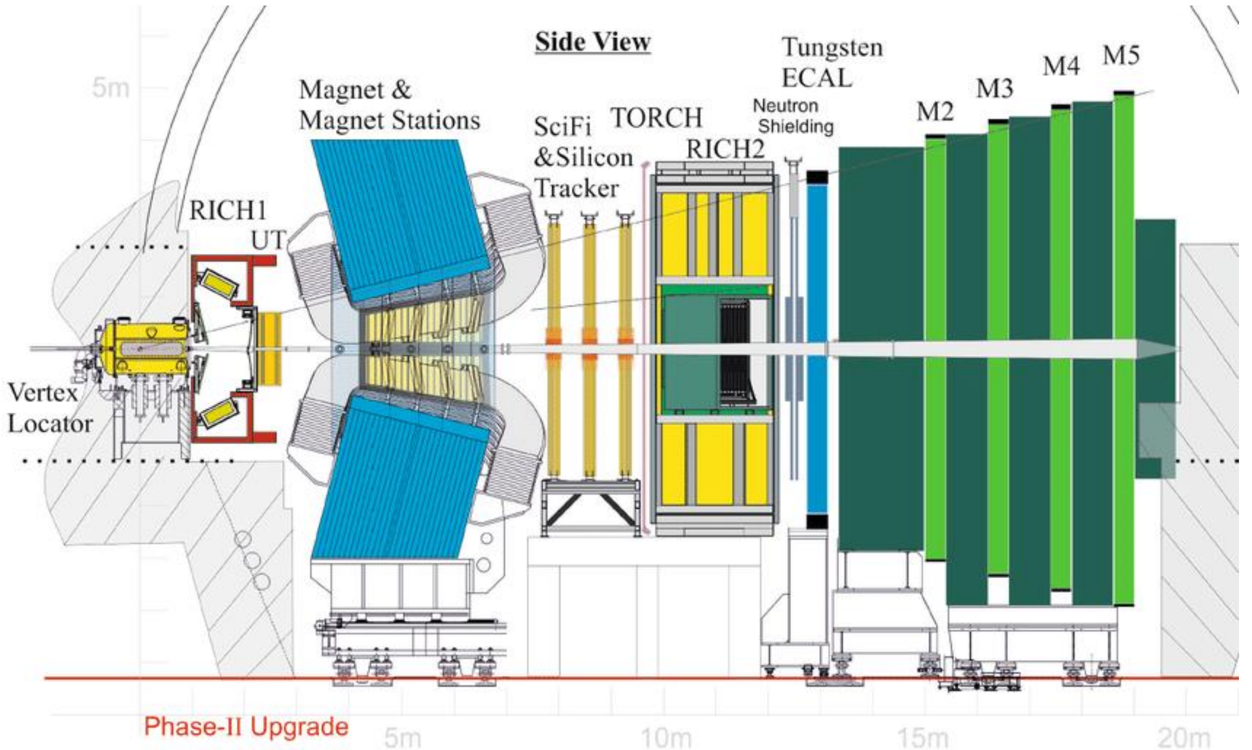
There are only 'a few' photons per event →one needs highly sensitive photon detectors to measure the rings !

LHCb RICH

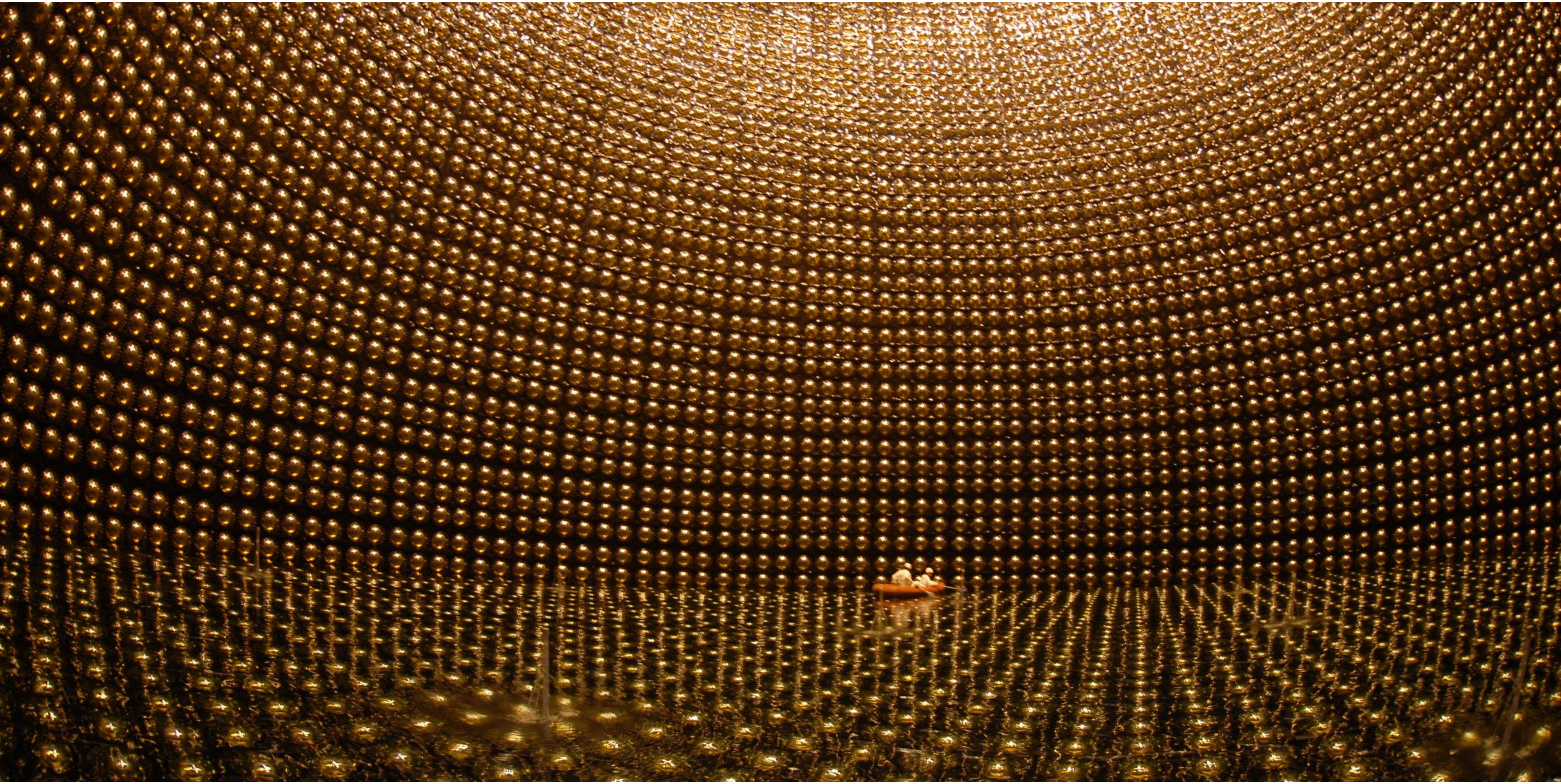




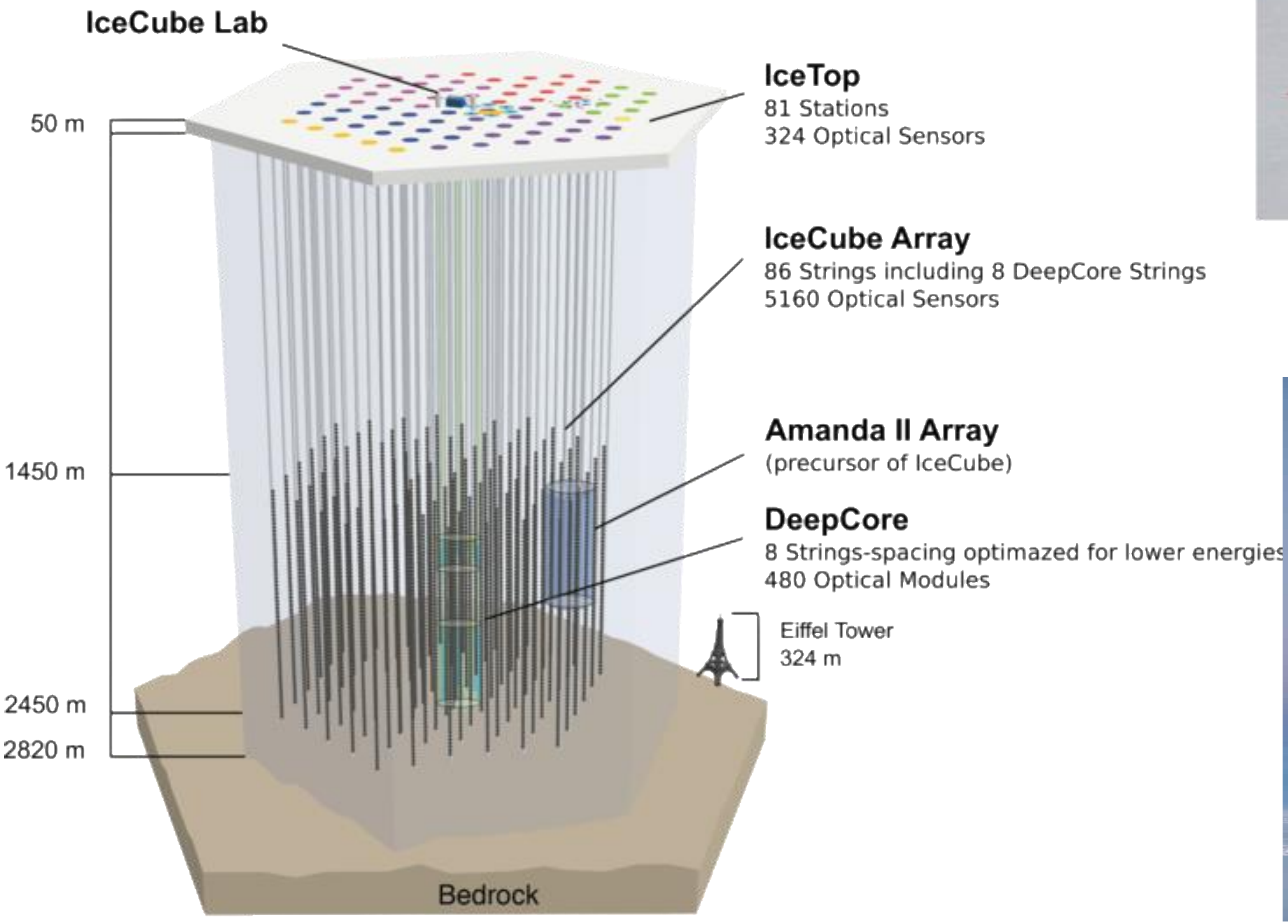




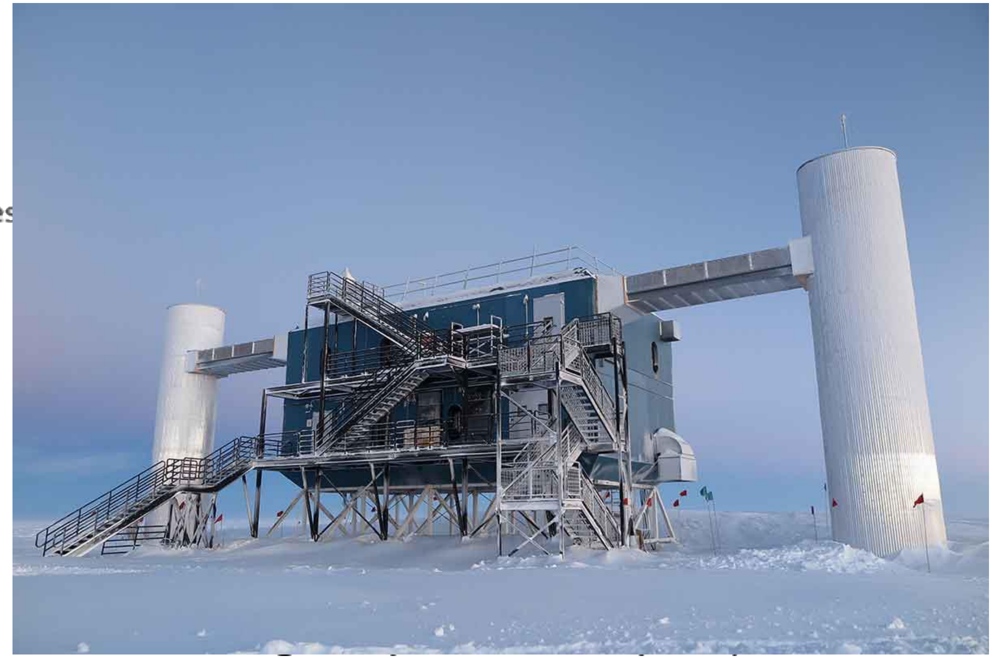
W. Riegler/CERN



Superkamiokande Cherekov Detector

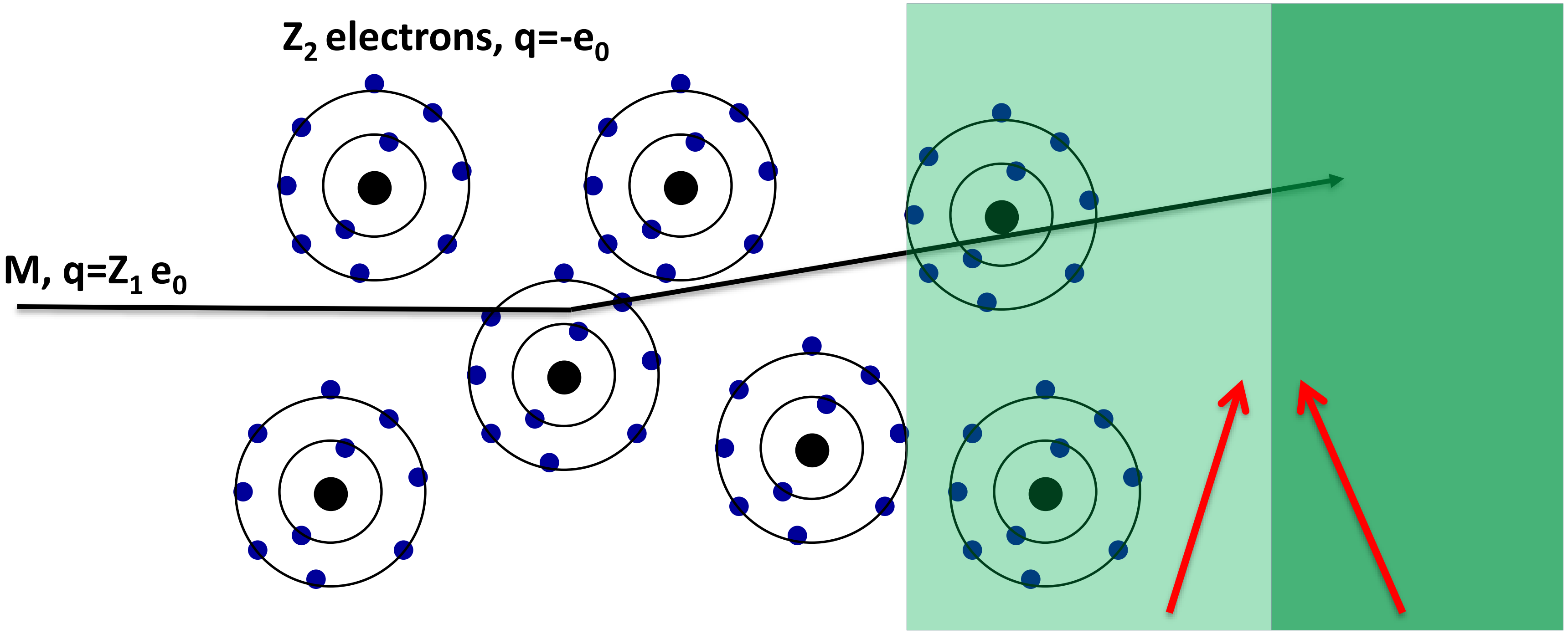


IceCube Aerial View



IceCube Cherekov Detector

Transition Radiation



When a highly relativistic particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called <u>Transition radiation</u>.

Transition Radiation

Radiation (~keV) emitted by ultra-relativistic Particles when they traverse the boarder of 2 Materials of different Dielectric Permittivity ($\varepsilon_1, \varepsilon_2$)

Classical Picture

$q = Z_1 e$

$$I = \frac{1}{3} \alpha Z_1^2 (\hbar\omega_p) \gamma$$ ... Radiated Energy per Transition

$\hbar\omega_p$ .... plasma Frequency of the Medium
.... ~20 eV for Styrene

About half the Energy is radiated between

$$0.1\,\hbar\omega_p\gamma < \hbar\omega < \hbar\omega_p\gamma$$

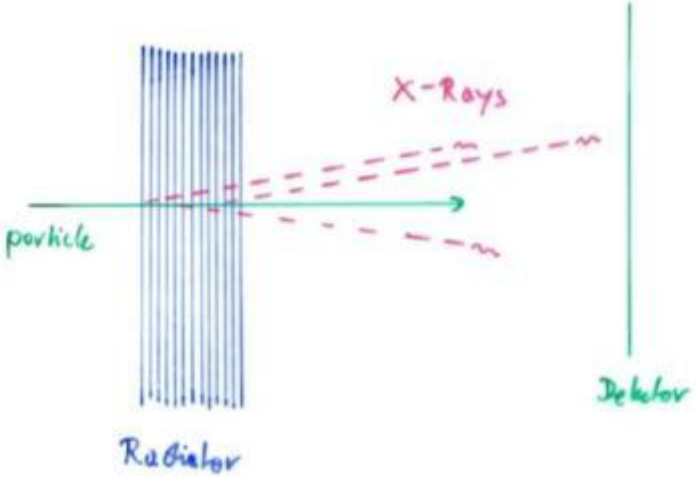
E.g. $\gamma = 1000$ 2-20 keV X-Rays

$$N_\gamma \sim \frac{2}{3}\alpha Z_1^2 \sim 5 \cdot 10^{-3} \cdot Z_1^2$$

γ - Dependence from hardening rather than N_γ

Emission Angle $\sim \frac{1}{\gamma}$

The Number of Photons can be increased by placing many foils of Material.



Electromagnetic Interaction of Particles with Matter

Ionization and Excitation:

Charged particles traversing material are exciting and ionizing the atoms.

The average energy loss of the incoming particle by this process is to a good approximation described by the Bethe Bloch formula.

The energy loss fluctuation is well approximated by the Landau distribution.

Multiple Scattering and Bremsstrahlung:

The incoming particles are scattering off the atomic nuclei.

Measuring the particle momentum by deflection of the particle trajectory in the magnetic field, this scattering imposes a lower limit on the momentum resolution of the spectrometer.

The deflection of the particle on the nucleus results in an acceleration that causes emission of Bremsstrahlungs-Photons. These photons in turn produced e+e- pairs in the vicinity of the nucleus, which causes an EM cascade. This effect depends on the 2nd power of the particle mass, so it is only relevant for electrons.

Electromagnetic Interaction of Particles with Matter

Cherenkov Radiation:

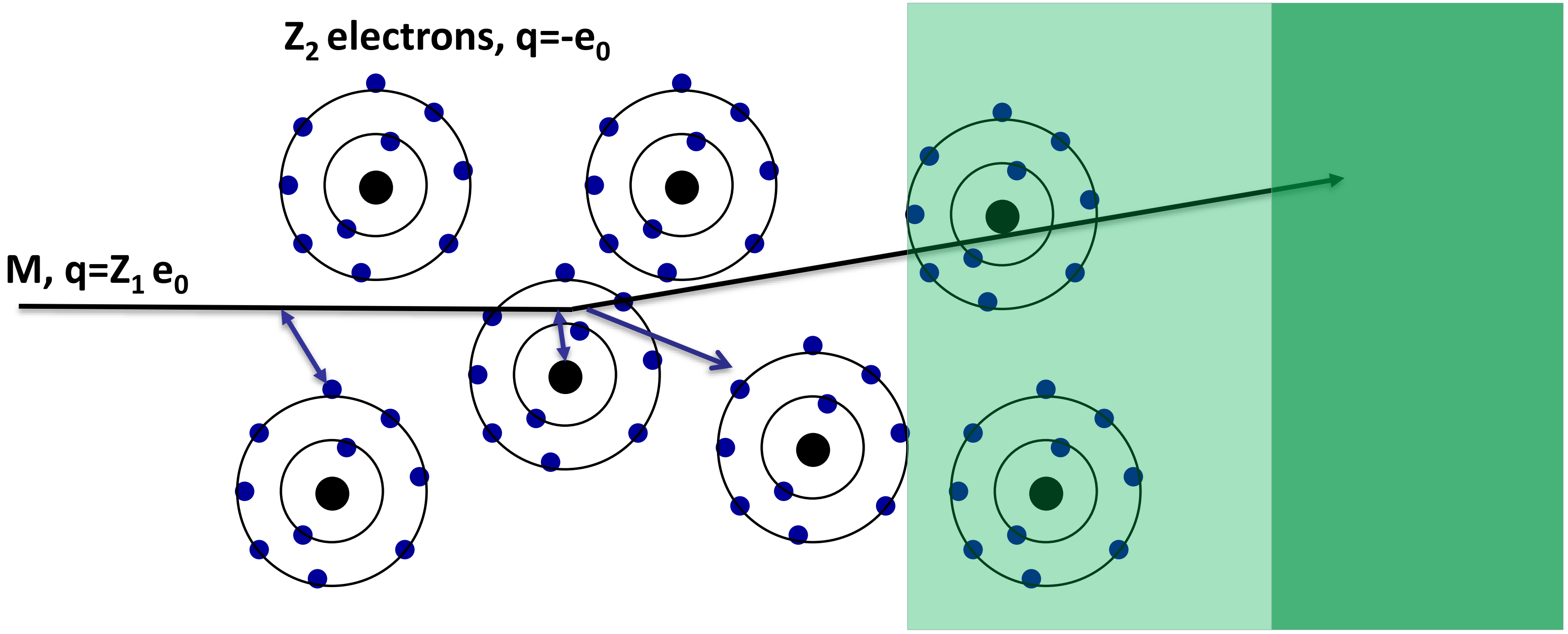
If a particle propagates in a material with a velocity larger than the speed of light in this material, Cherenkov radiation is emitted at a characteristic angle that depends on the particle velocity and the refractive index of the material.

Transition Radiation:

If a charged particle is crossing the boundary between two materials of different dielectric permittivity, there is a certain probability for emission of an X-ray photon.

→ The strong interaction of an incoming particle with matter is a process which is important for Hadron calorimetry and will be discussed later.

Electromagnetic Interaction of Particles with Matter



Now that we know all the Interactions we can talk about Detectors !

Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called Transition radiation.

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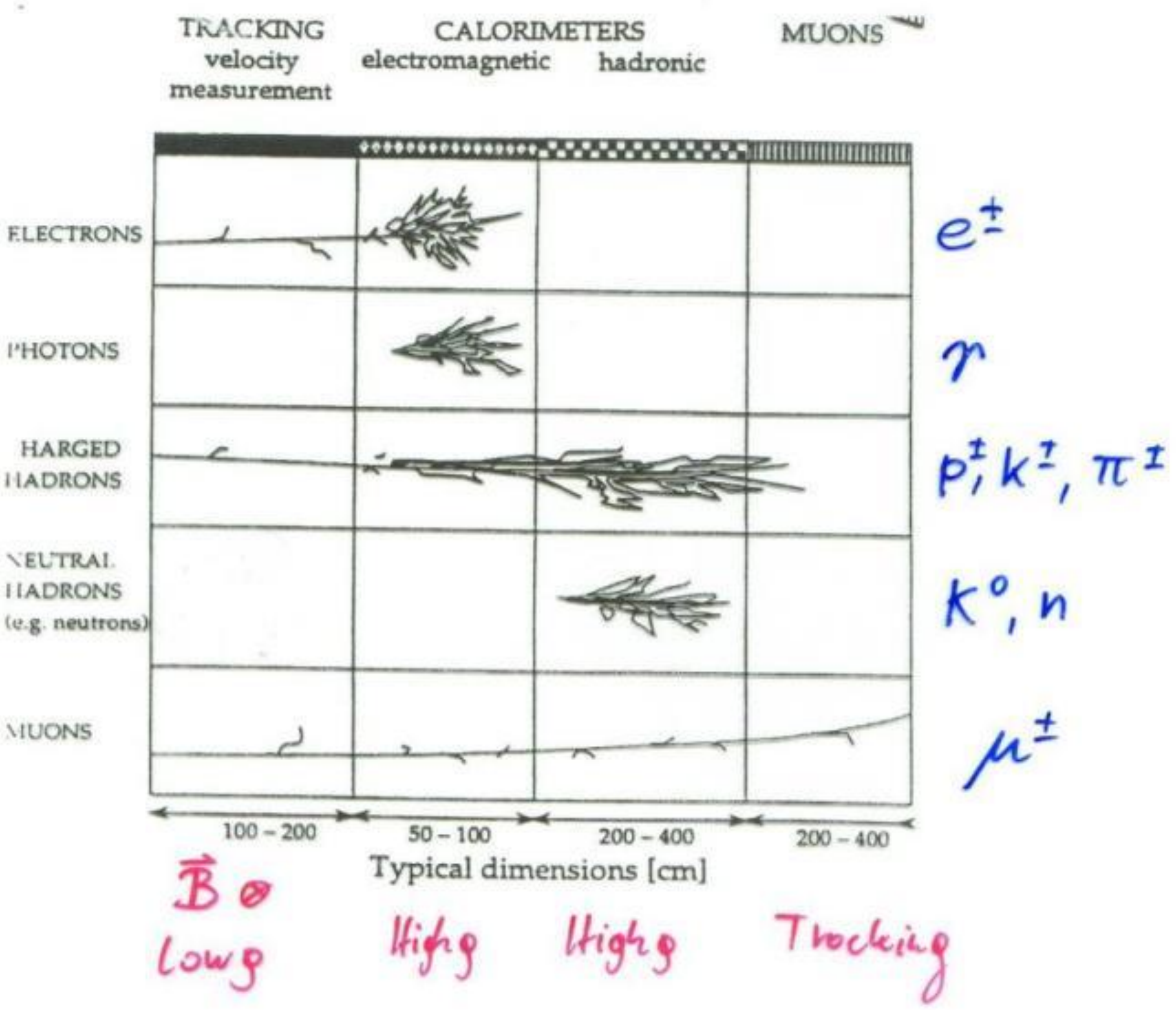
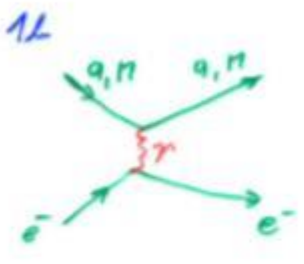


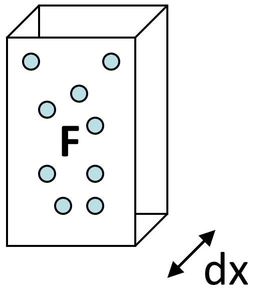
Intermezzo: Crossection



Crossection σ: Material with Atomic Mass A and density ρ contains n Atoms/cm^3

$$n[\mathrm{cm}^{-3}] = \frac{N_A[\mathrm{mol}^{-1}]\,\rho[\mathrm{g/cm}^3]}{A[\mathrm{g/mol}]} \qquad\qquad N_A = 6.022\times 10^{23}\,\mathrm{mol}^{-1}$$

E.g. Atom (Sphere) with Radius R: Atomic Crossection $\sigma = R^2\pi$



A volume with surface F and thickness dx contains N=nFdx Atoms.
The total 'surface' of atoms in this volume is N σ.
The relative area is $p = N\sigma/F = N_A\rho\,\sigma/A\,dx$ =
Probability that an incoming particle hits an atom in dx.

What is the probability P that a particle hits an atom between distance x and x+dx ?
P = probability that the particle does NOT hit an atom in the m=x/dx material layers and that the particle DOES hit an atom in the mth layer

$$P(x)dx = (1-p)^m\,p \approx e^{-m}\,p = \exp\left(-\frac{N_A\rho\sigma}{A}x\right)\frac{N_A\rho\sigma}{A}dx = \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right)dx \qquad \lambda = \frac{A}{N_A\rho\sigma}$$

Mean free path $\displaystyle = \int_0^\infty xP(x)dx = \int_0^\infty \frac{x}{\lambda}e^{-\frac{x}{\lambda}}dx = \lambda$

Average number of collisions/cm $\displaystyle = \frac{1}{\lambda} = \frac{N_A\rho\sigma}{A}$

Intermezzo: Differential Crossection

Differential Crossection: $\frac{d\sigma(E,E')}{dE'}$

→ Crossection for an incoming particle of energy E to lose an energy between E' and E'+dE'

Total Crossection: $\sigma(E) = \int \frac{d\sigma(E,E')}{dE'} dE'$

Probability P(E) that an incoming particle of Energy E loses an energy between E' and E'+dE' in a collision:

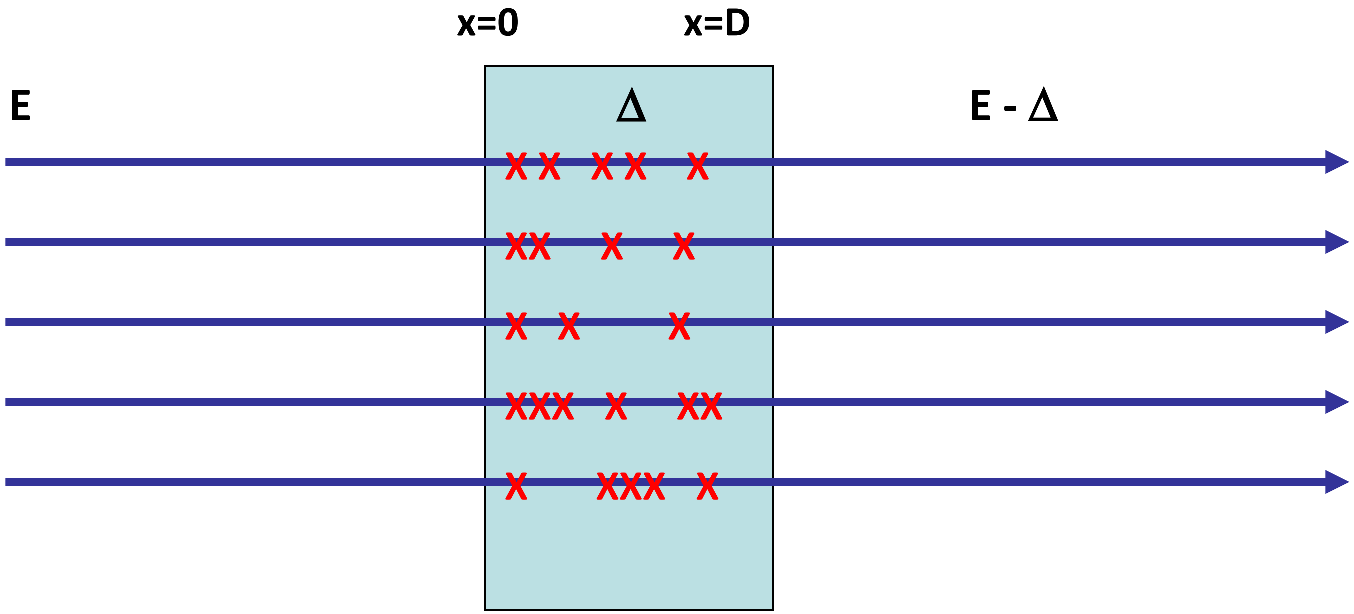
$$P(E,E')dE' = \frac{1}{\sigma(E)} \frac{d\sigma(E,E')}{dE'} dE'$$

Average number of collisions/cm causing an energy loss between E' and E'+dE' $= \frac{N_A \rho}{A} \frac{d\sigma(E,E')}{dE'}$

Average energy loss/cm: $\frac{dE}{dx} = -\frac{N_A \rho}{A} \int E' \frac{d\sigma(E,E')}{dE'} dE'$

Fluctuation of Energy Loss

Up to now we have calculated the average energy loss. The energy loss is however a statistical process and will therefore fluctuate from event to event.



P(Δ) = ? Probability that a particle loses an energy Δ when traversing a material of thickness D

We have see earlier that the probability of an interaction occuring between distance x and x+dx is exponentially distributed

$$P(x)dx = \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right)dx \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

Probability for n Interactions in D

We first calculate the probability to find n interactions in D, knowing that the probability to find a distance x between two interactions is P(x)dx = 1/ λ exp(-x/λ) dx with $\lambda = A/ N_A \rho\, \sigma$

Probability to have no interaction between 0 und D:

$$P(x > D) = \int_D^\infty P(x_1) dx_1 = e^{-\frac{D}{\lambda}}$$

Probability to have one interaction at x_1 and no other interaction:

$$P(x_1, x_2 > D) = \int_D^\infty P(x_1) P(x_2 - x_1) dx_2 = \frac{1}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have one interaction independently of x_1:

$$\int_0^D P(x_1, x_2 > D) = \frac{D}{\lambda} e^{-\frac{D}{\lambda}}$$

Probability to have the first interaction at x_1, the second at x_2 .... the n^{th}
x_n and no other interaction:

$$P(x_1, x_2 ... x_n > D) = \int_D^\infty P(x_1) P(x_2 - x_1) ... P(x_n - x_{n-1}) dx_n = \frac{1}{\lambda^n} e^{-\frac{D}{\lambda}}$$

Probability for n interactions independently of $x_1, x_2 ... x_n$

$$\int_0^D \int_0^{x_{n-1}} \int_0^{x_{n-1}} ... \int_0^{x_1} P(x_1, x_2 ..., x_n > D) dx_1 ... dx_{n-1} = \frac{1}{n!} \left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}}$$

Probability for n Interactions in D

For an interaction with a mean free path of λ , the probability for n interactions on a distance D is given by

$$P(n) = \frac{1}{n!}\left(\frac{D}{\lambda}\right)^n e^{-\frac{D}{\lambda}} = \frac{\overline{n}^n}{n!}e^{-\overline{n}} \qquad \overline{n} = \frac{D}{\lambda} \qquad \lambda = \frac{A}{N_A \rho \sigma}$$

→Poisson Distribution !

How do we find the energy loss distribution ?

If f(E) is the probability to lose the energy E' in an interaction, the probability p(E) to lose an energy E over the distance D ?

$$f(E) = \frac{1}{\sigma}\frac{d\sigma}{dE}$$

$$p(E) = P(1)f(E) + P(2)\int_0^E f(E-E')f(E')dE' + P(3)\int_0^E\int_0^{E'} f(E-E'-E'')f(E'')f(E')dE''dE' + ...$$

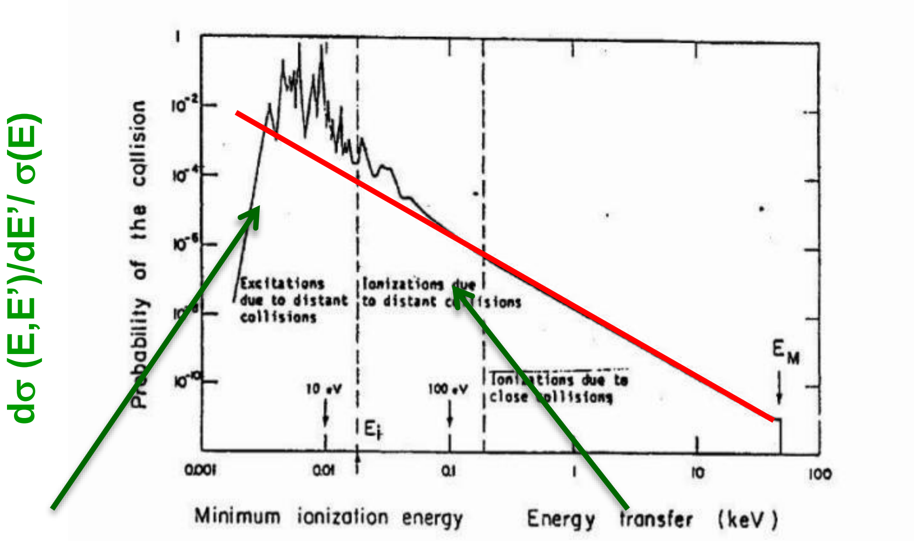
$$F(s) = \mathcal{L}\left[f(E)\right] = \int_0^\infty f(E)e^{-sE}dE$$

$$\mathcal{L}\left[p(E)\right] = P(1)F(s) + P(2)F(s)^2 + P(3)F(s)^3 + ... = \sum_{n=1}^{\infty} P(n)F(s)^n = \sum_{n=1}^{\infty}\frac{\overline{n}^n F^n}{n!}e^{-\overline{n}} = e^{\overline{n}(F(s)-1)} - 1 \approx e^{\overline{n}(F(s)-1)}$$

$$p(E) = \mathcal{L}^{-1}\left[e^{\overline{n}(F(s)-1)}\right] = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{\overline{n}(F(s)-1)+sE}ds$$

Fluctuations of the Energy Loss

Probability f(E) for loosing energy between E' and E'+dE' in a single interaction is given by the differential crossection dσ (E,E')/dE'/ σ(E) which is given by the Rutherford crossection at large energy transfers



$$\frac{d\sigma}{dE'} \approx \frac{2\pi Z_1^2 e^4}{m_e c^2 \beta^2 E'^2}$$

Excitation and ionization

Scattering on free electrons

$$p(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \exp(s\log s + xs)\, ds.$$

$$= \frac{1}{\pi}\int_0^{\infty} \exp(-t\log t - xt)\sin(\pi t)\, dt.$$

$$x = \frac{E}{\overline{n}\epsilon} + C_\gamma - 1 - \ln\overline{n} \qquad \overline{n} = \frac{N_A \rho Z_2 k D}{A\epsilon}$$

$$\ln\epsilon = \ln\frac{I^2}{E_{max}} + 2\beta^2$$